



中国科学技术大学

交叉学科理论研究中心
Interdisciplinary Center for Theoretical Study

Nucleon weak elastic and transition form factors at small and large virtualities

Chen Chen

Interdisciplinary Center for Theoretical Study
&
Peng Huanwu Center for Fundamental Theory

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BARYONS 2025

INTERNATIONAL
CONFERENCE ON THE
STRUCTURE OF BARYONS

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Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics – Two emergent phenomena**
 - **Confinement:** Colored particles have never been seen isolated
 - ✓ Explain how quarks and gluons bind together
 - **Dynamical Chiral Symmetry Breaking (DCSB):** Hadrons do not follow the chiral symmetry pattern
 - ✓ Explain the most important mass generating mechanism for visible matter in the Universe
- Neither of these phenomena is apparent in QCD's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

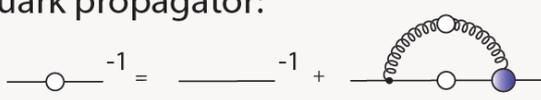
Non-Perturbative QCD:

- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).

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- Dressed-quark propagator:

Quark propagator:

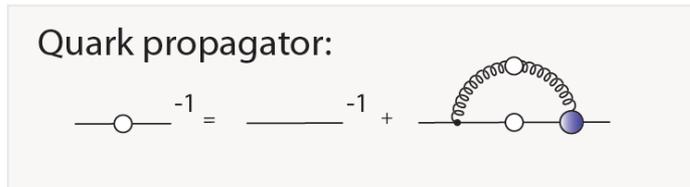

$$\text{Quark propagator}^{-1} = \text{Bare quark propagator}^{-1} + \text{Dressed quark propagator}^{-1}$$

- Mass generated from the interaction of quarks with the gluon.
- Light quarks acquire a **HUGE** constituent mass.
- Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

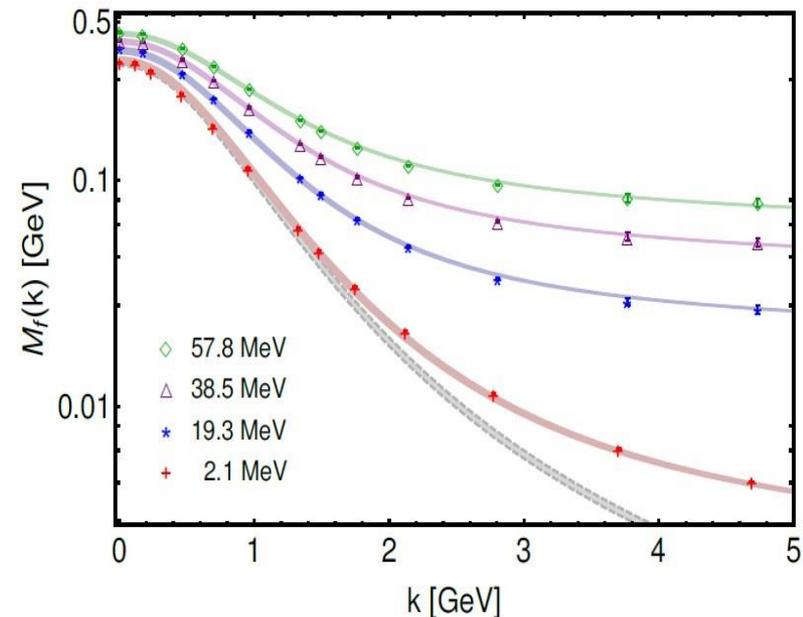
Non-Perturbative QCD:

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➤ Dressed-quark propagator:



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- Lei Chang, Yu-Bin Liu, Khépani Raya, J. Rodríguez-Quintero, and Yi-Bo Yang, Phys. Rev. D 104, no.9, 094509 (2021)

Dyson-Schwinger equations (DSEs)

Quark propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost-gluon vertex:

$$\text{---}\bigcirc\text{---} = \text{---}\text{---} + \text{---}\bigcirc\text{---}$$

Gluon propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

$$+ \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$+ \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$+ \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Quark-gluon vertex:

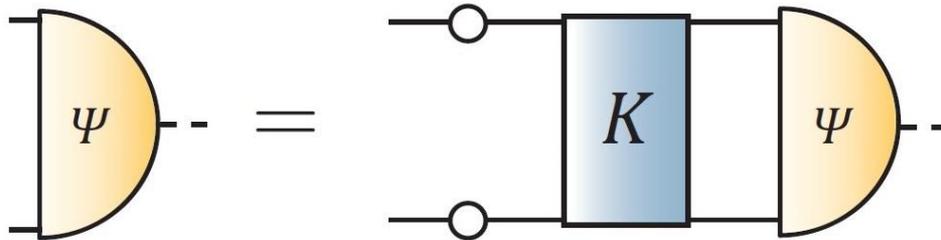
$$\text{---}\bigcirc\text{---} = \text{---}\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Hadrons: Bound-states in QFT

➤ **Mesons:** a 2-body bound state problem in QFT

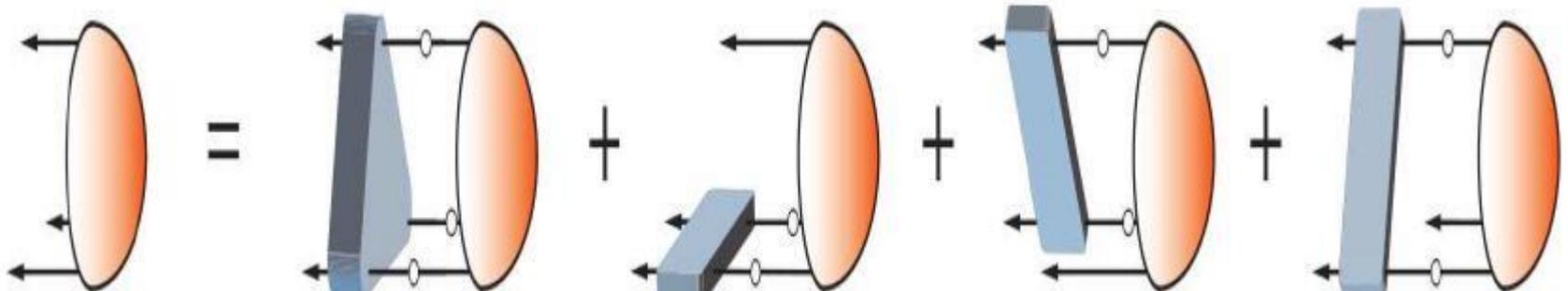
➤ Bethe-Salpeter Equation

➤ **K** - fully amputated, two-particle irreducible, quark-antiquark scattering kernel



➤ **Baryons:** a 3-body bound state problem in QFT

➤ Faddeev equation: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.



2-body correlations

- Mesons: quark-antiquark correlations -- color-singlet
- Diquarks: quark-quark correlations within a color-singlet baryon.
- Diquark correlations:
 - In our approach: non-pointlike color-antitriplet and fully interacting.
 - Diquark correlations are soft, they possess an electromagnetic size.
 - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous $J^{\{-P\}}$ meson.

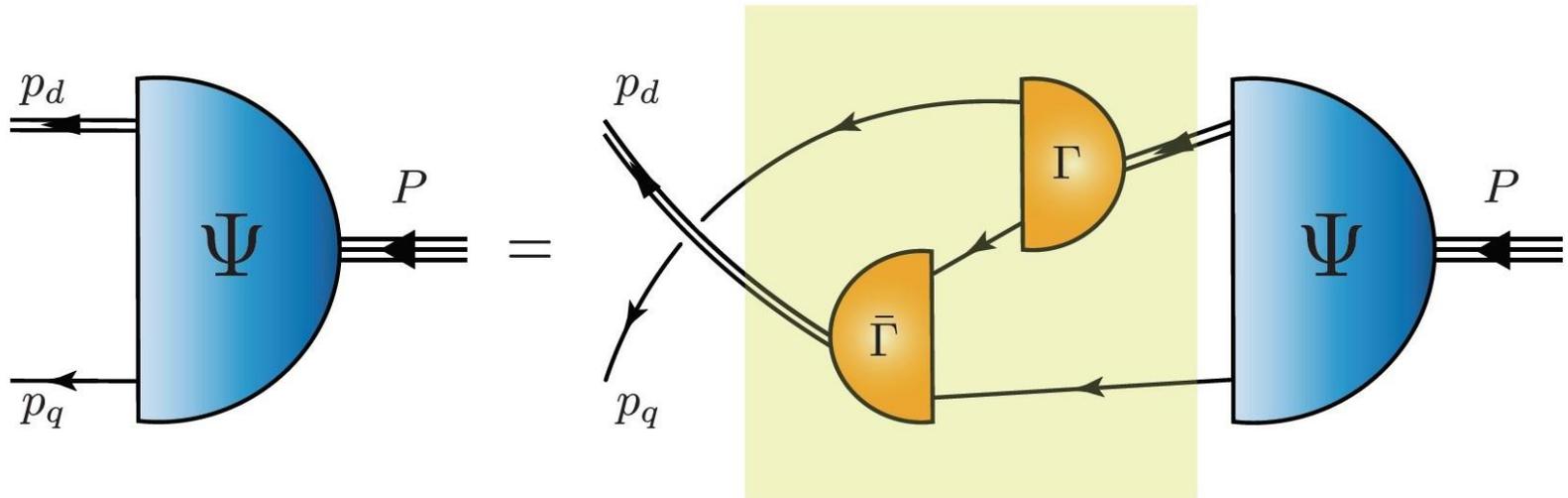
$$\Gamma_{q\bar{q}}(p; P) = - \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{q\bar{q}}(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu$$
$$\Gamma_{qq}(p; P) C^\dagger = -\frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{qq}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Quark-diquark Faddeev equation

- Quantum numbers:
 - $(I=0, J^P=0^{++})$: isoscalar-scalar diquark
 - $(I=1, J^P=1^{++})$: isovector-pseudovector diquark
 - $(I=0, J^P=0^{-})$: isoscalar-pseudoscalar diquark
 - $(I=0, J^P=1^{-})$: isoscalar-vector diquark
 - $(I=1, J^P=1^{-})$: isovector-vector diquark

- Three-body bound states  Quark-diquark two-body bound states

- ✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Phys. Rev. D 36 (1987) 2804
- ✓ R.T. Cahill, Craig D. Roberts, J. Praschifka, Austral.J.Phys. 42 (1989) 129-145





Progress in Particle and Nuclear Physics

Volume 116, January 2021, 103835

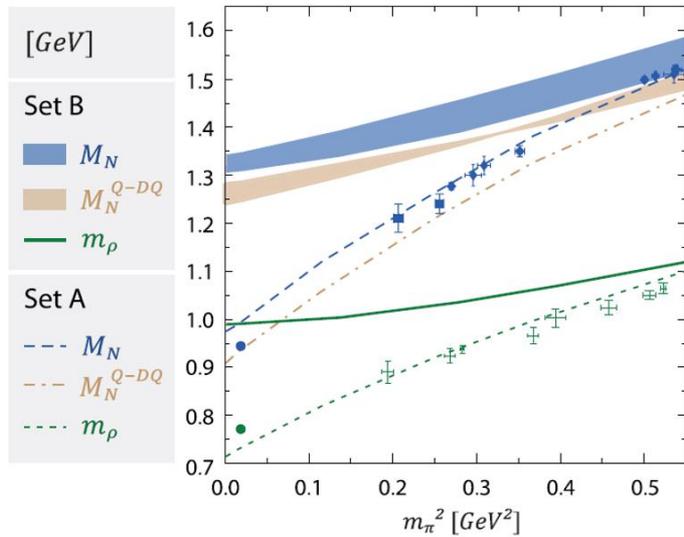


Review

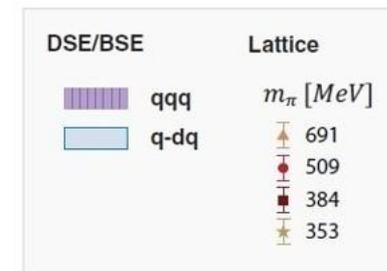
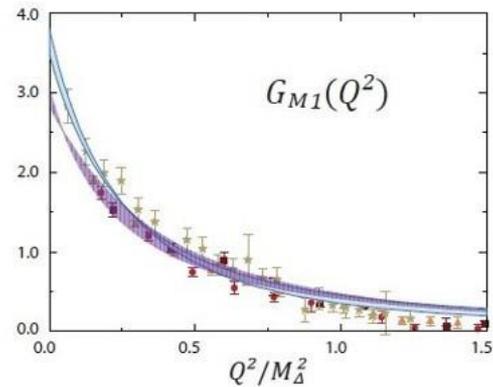
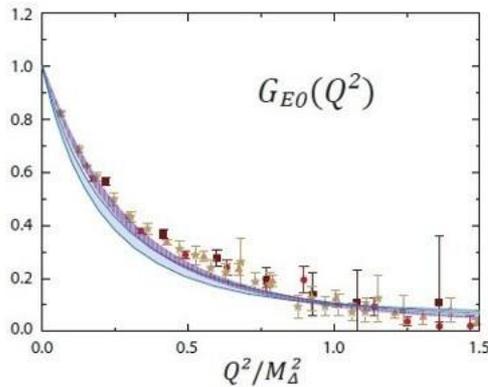
Diquark correlations in hadron physics: Origin, impact and evidence

M.Yu. Barabanov¹, M.A. Bedolla², W.K. Brooks³, G.D. Cates⁴, C. Chen⁵, Y.
Chen^{6,7}, E. Cisbani⁸, M. Ding⁹, G. Eichmann^{10,11}, R. Ent¹², J. Ferretti¹³
✉, R.W. Gothe¹⁴, T. Horn^{15,12}, S. Liuti⁴, C. Mezrag¹⁶, A. Pilloni⁹, A.J.R.
Puckett¹⁷, C.D. Roberts^{18,19} ✉ ... B.B. Wojtsekhowski¹² ✉

Quark-diquark Faddeev equation



- ✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, and C. S. Fischer, Prog. Part. Nucl. Phys. 91, 1 (2016)



Quark-diquark Faddeev equation

- Solution to the **60** year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is **50%** greater and it is unstable...

PRL 115, 171801 (2015)

PHYSICAL REVIEW LETTERS

week ending
23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,¹ Bruno El-Bennich,^{2,3} Eduardo Rojas,^{2,4} Ian C. Cloët,⁵ Craig D. Roberts,⁵
Shu-Sheng Xu,⁶ and Hong-Shi Zong⁶

¹*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM),
Universidad de Salamanca, E-37008 Salamanca, Spain*

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⁴*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*

⁵*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

⁶*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801

PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

Quark-diquark Faddeev equation

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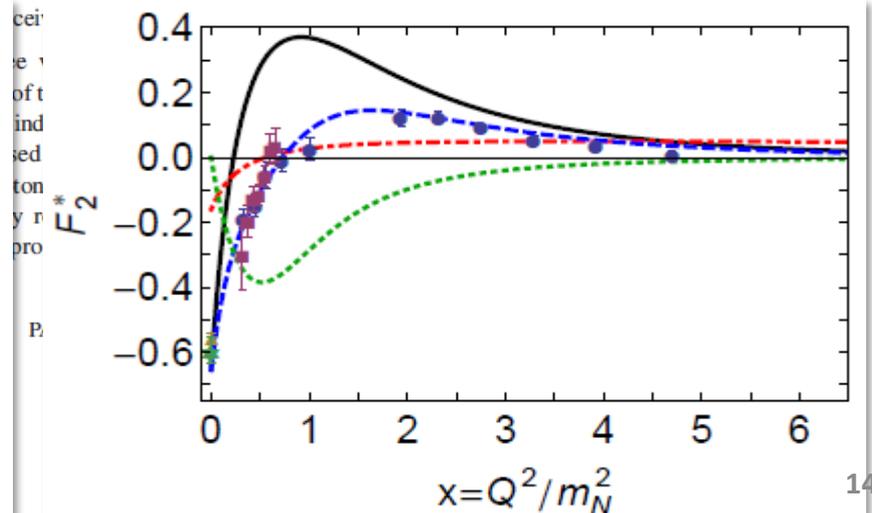
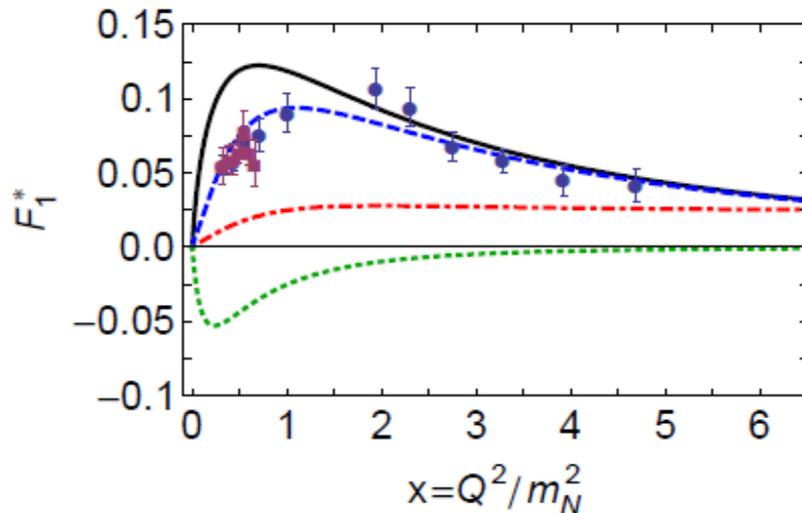
²Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil

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Quark-diquark Faddeev equation

- Roper resonance -- solution to the 60 year puzzle

REVIEWS OF MODERN PHYSICS

REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Colloquium: Roper resonance: Toward a solution to the fifty year puzzle

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Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA

Craig D. Roberts[†]

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

 (published 14 March 2019)

Quark-diquark Faddeev equation

- The electromagnetic nucleon-to- $\Delta(1600)$ transition form factors

PHYSICAL REVIEW D **100**, 034001 (2019)

Transition form factors: $\gamma^* + p \rightarrow \Delta(1232), \Delta(1600)$

Y. Lu,^{1,*} C. Chen,^{2,†} Z.-F. Cui,^{1,‡} C. D. Roberts,^{3,§} S. M. Schmidt,^{4,||} J. Segovia,^{5,¶} and H.-S. Zong^{1,6,**}

¹Department of Physics, Nanjing University, Nanjing, Jiangsu 210093, China

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⁴Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany

⁵Departamento de Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, E-41013 Sevilla, Spain

⁶Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing, Jiangsu 210093, China

- In 2023, those (parameter-free) predictions were confirmed in an analysis of data obtained using the CLAS detector at JLab

PHYSICAL REVIEW C **108**, 025204 (2023)

First results on nucleon resonance electroexcitation amplitudes from $ep \rightarrow e'\pi^+\pi^-p'$ cross sections at $W = 1.4\text{--}1.7$ GeV and $Q^2 = 2.0\text{--}5.0$ GeV²

V. I. Mokeev^{1,*}, P. Achenbach¹, V. D. Burkert¹, D. S. Carman¹, R. W. Gothe², A. N. Hiller Blin³,
E. L. Isupov⁴, K. Joo⁵, K. Neupane² and A. Trivedi²

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²University of South Carolina, Columbia, South Carolina 29208, USA

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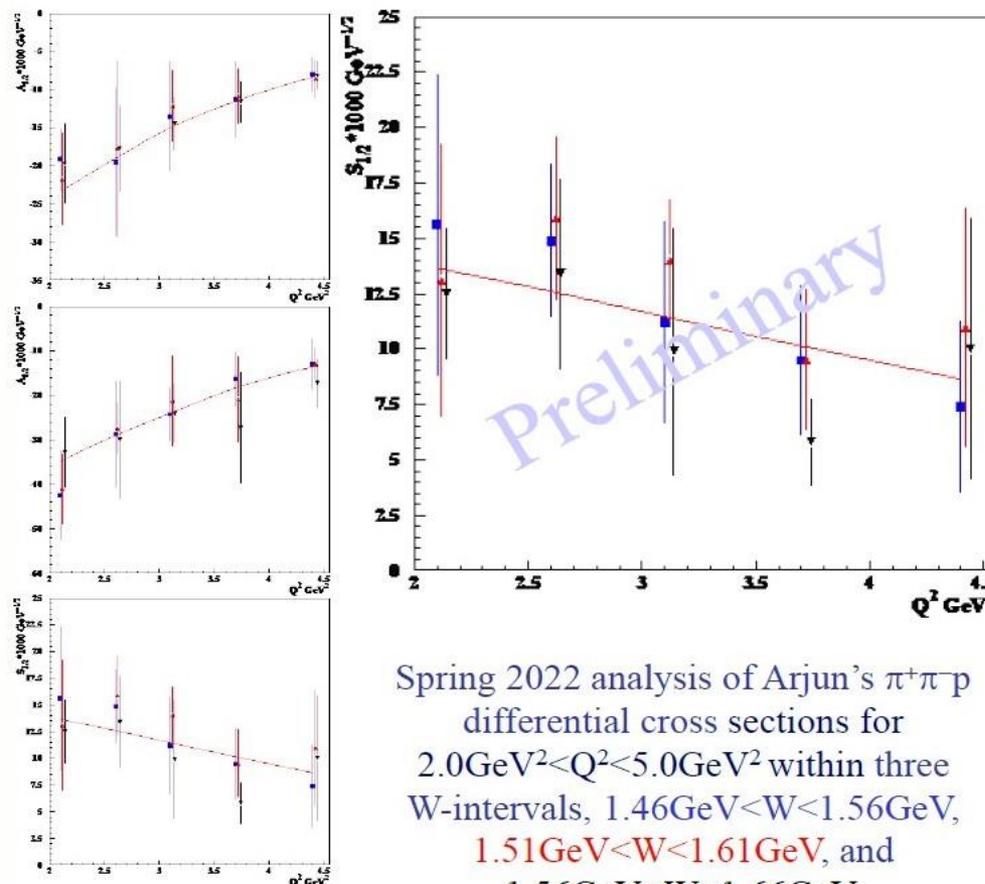
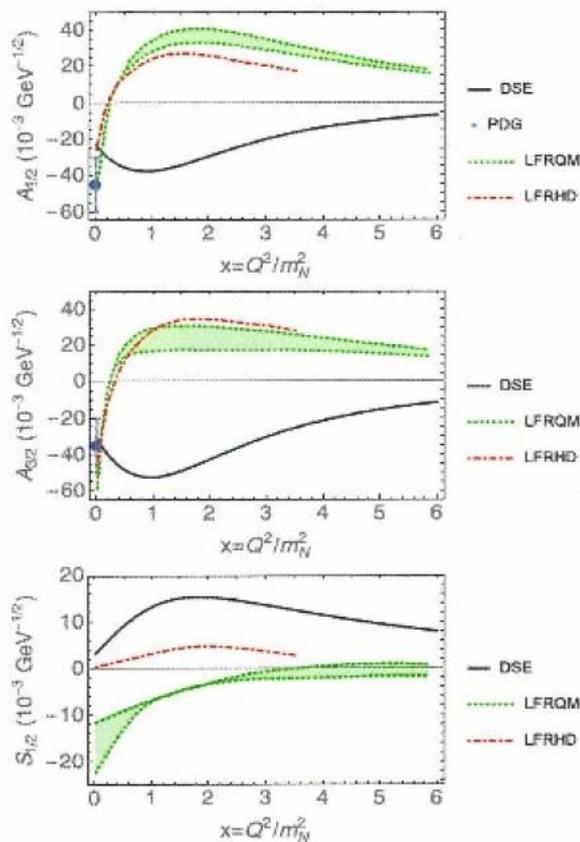
⁴Skobeltsyn Institute of Nuclear Physics and Physics Department, Lomonosov Moscow State University, 119234 Moscow, Russia

⁵University of Connecticut, Storrs, Connecticut 06269, USA

$\Delta(1600)3/2^+$ Form Factors in CSM Approach

Viktor Mokeev

CSM predictions of the $\Delta(1600)3/2^+$ electrocouplings



Spring 2022 analysis of Arjun's $\pi^+\pi^-\rho$ differential cross sections for $2.0\text{GeV}^2 < Q^2 < 5.0\text{GeV}^2$ within three W -intervals, $1.46\text{GeV} < W < 1.56\text{GeV}$, $1.51\text{GeV} < W < 1.61\text{GeV}$, and $1.56\text{GeV} < W < 1.66\text{GeV}$.

Ya Lu et al., PRD 100, 034001 (2019)

➤ arXiv:2412.15045 [hep-ph]

PHYSICAL REVIEW D **112**, 014022 (2025)

Screening masses of positive- and negative-parity hadron ground states, including those with strangeness

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²*Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*

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 (Received 27 December 2024; accepted 12 June 2025; published 9 July 2025)

Using a symmetry-preserving treatment of a vector \times vector contact interaction at nonzero temperature, we compute the screening masses of flavor-SU(3) ground-state $J^P = 0^\pm, 1^\pm$ mesons, and $J^P = 1/2^\pm, 3/2^\pm$ baryons. We find that all correlation channels allowed at $T = 0$ persist when the temperature increases, even above the QCD phase transition temperature. The results for mesons qualitatively agree with those obtained from the contemporary lattice-regularized quantum chromodynamics simulations. One of the most remarkable features is that each parity-partner-pair degenerates when $T > T_c$, with T_c being the critical temperature. For each pair, the screening mass of the negative parity meson increases monotonously with temperature. In contrast, the screening mass of the meson with positive parity is almost invariant on the domain $T \lesssim T_c/2$; when T gets close to T_c , it decreases but soon increases again and finally degenerates with its parity partner, which signals the restoration of chiral symmetry. We also find that the T -dependent behaviors of baryon screening masses are quite similar to those of the mesons. For baryons, the dynamical, nonpointlike diquark correlations play a crucial role in the screening mass evolution. We further calculate the evolution of the fraction of each kind of diquark within baryons respective to temperature. We observe that, at high temperatures, only $J = 0$ scalar and pseudoscalar diquark correlations can survive within $J^P = 1/2^\pm$ baryons.

Screening masses: mesons

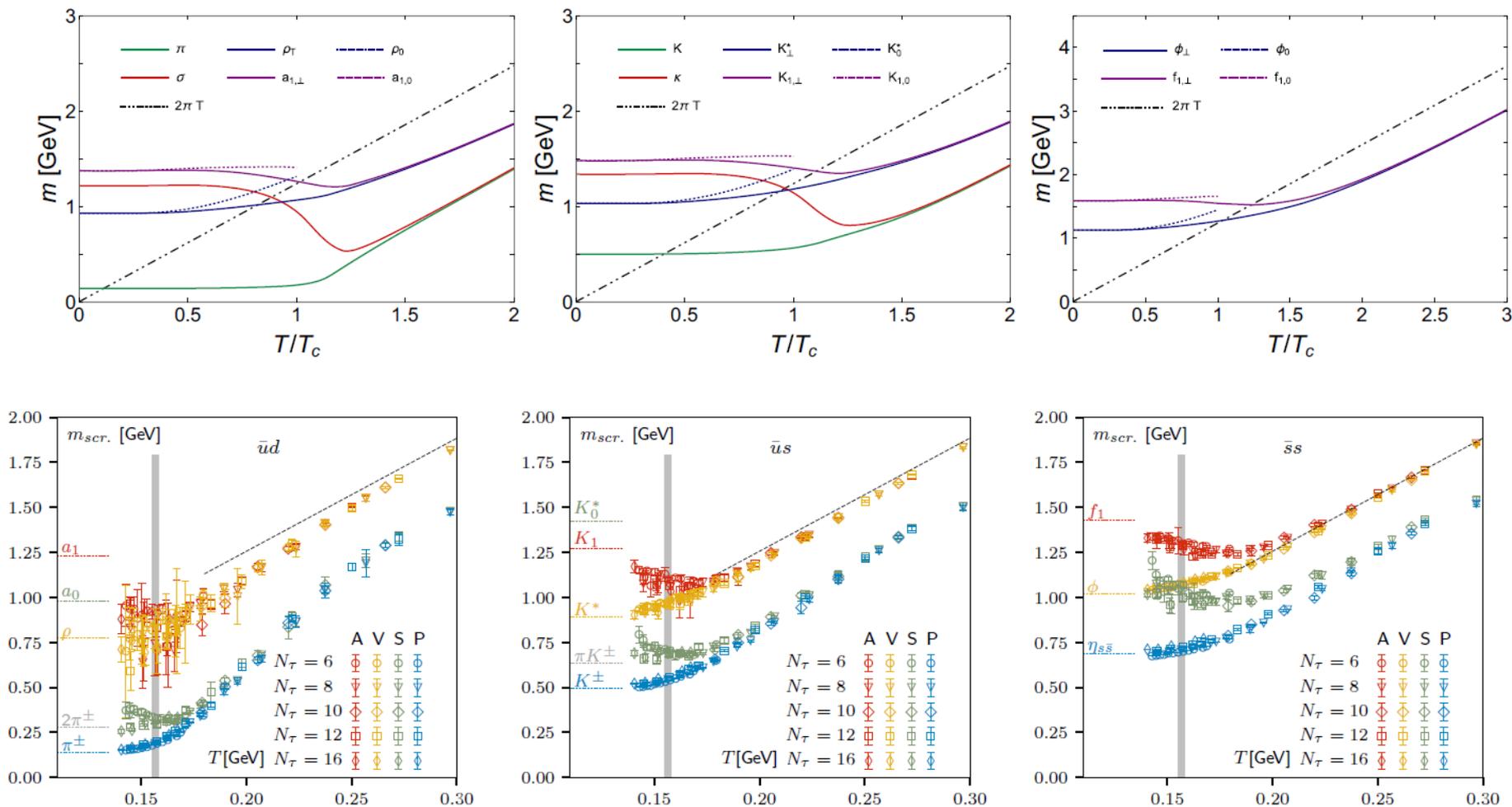


Figure 5. (Left to right) Results for all four screening masses for the $\bar{u}d$, $\bar{u}s$ and $\bar{s}s$ flavor combinations. The gray vertical band in all the figures represents the pseudo-critical temperature, $T_{pc} = 156.5(1.5)$ MeV [6]. The dashed lines corresponds to the free theory limit of $m = 2\pi T$.

Screening masses: $J^P = 1/2^\pm$ baryons

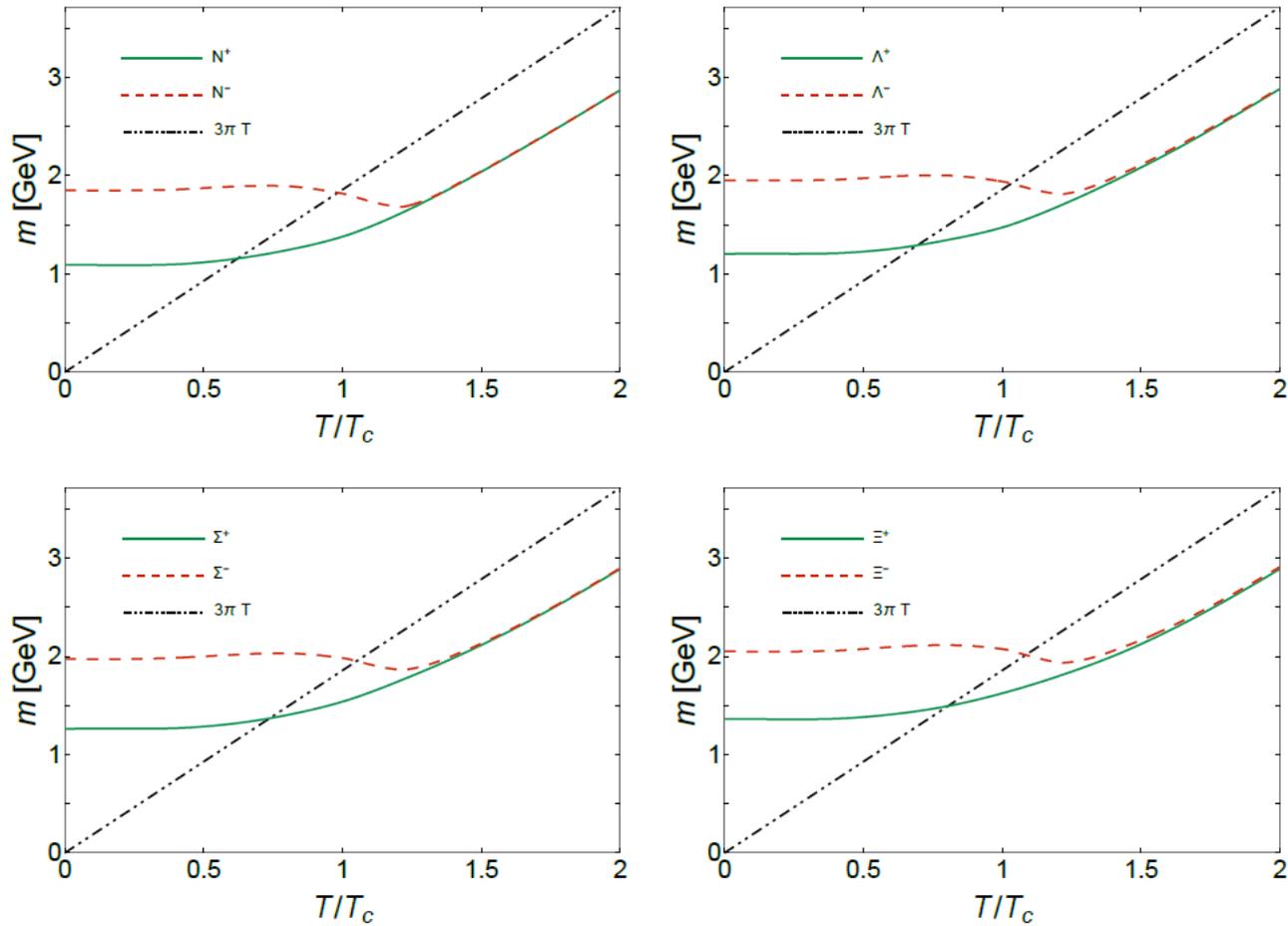


FIG. 6. Screening masses of $J^P = 1/2^\pm$ baryon ground states. In each figure: *solid green curve*: positive-parity baryon; *dashed red curve*: negative-parity baryon; and *black dot-dot-dashed curve*: free theory limit of $m = 3\pi T$.

Screening masses: $J^P = 3/2^\pm$ baryons

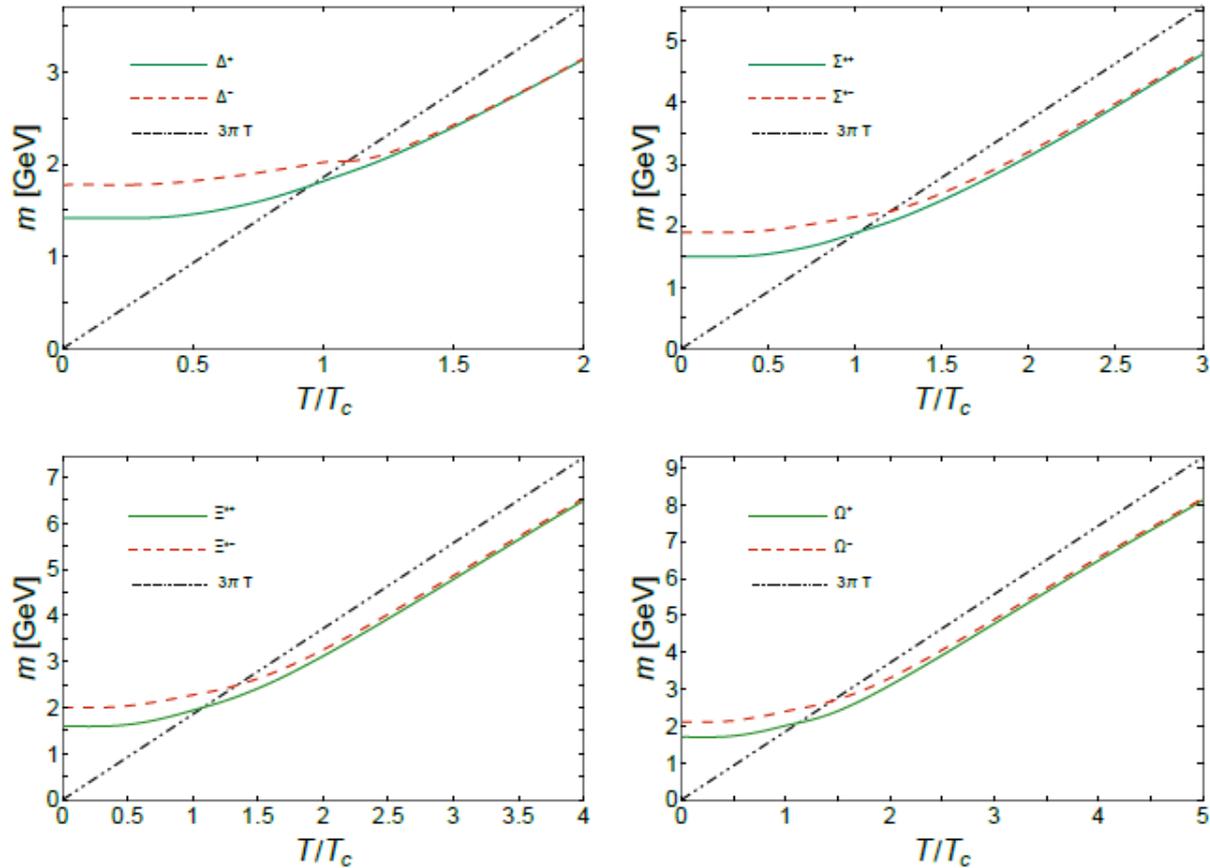
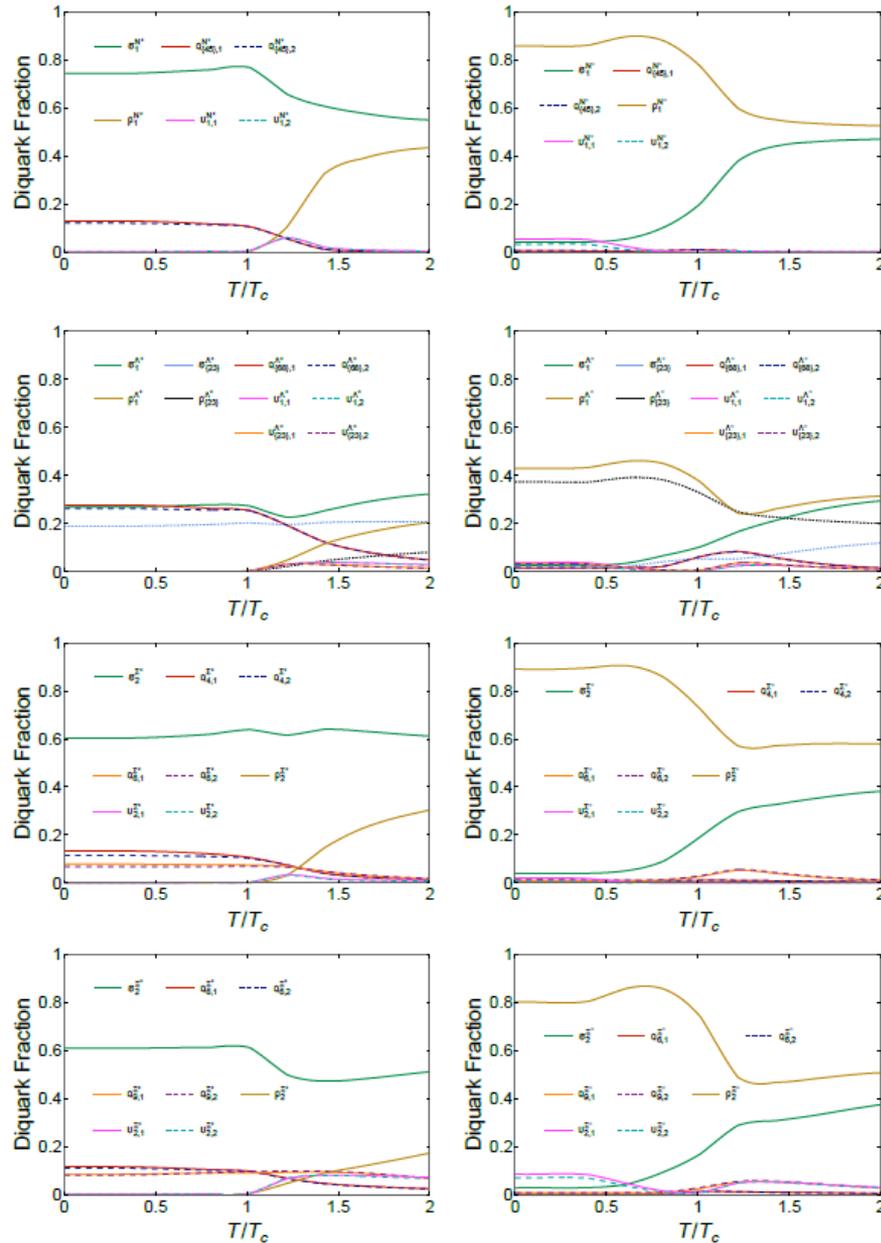


FIG. 8. Screening masses of $J^P = 3/2^\pm$ baryon ground states. In each figure: *solid green curve*: positive-parity baryon; *dashed red curve*: negative-parity baryon; and *black dot-dot-dashed curve*: free theory limit of $m = 3\pi T$.

Diquark fractions: $J^P = 1/2^\pm$ baryons



Axial Form Factors

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Axial Form Factors

PHYSICAL REVIEW D **105**, 094022 (2022)

Nucleon axial-vector and pseudoscalar form factors and PCAC relations

Chen Chen (陈晨)^{1,2,3,4,*} Christian S. Fischer^{3,4,†} Craig D. Roberts^{5,6,‡} and Jorge Segovia^{7,6,§}

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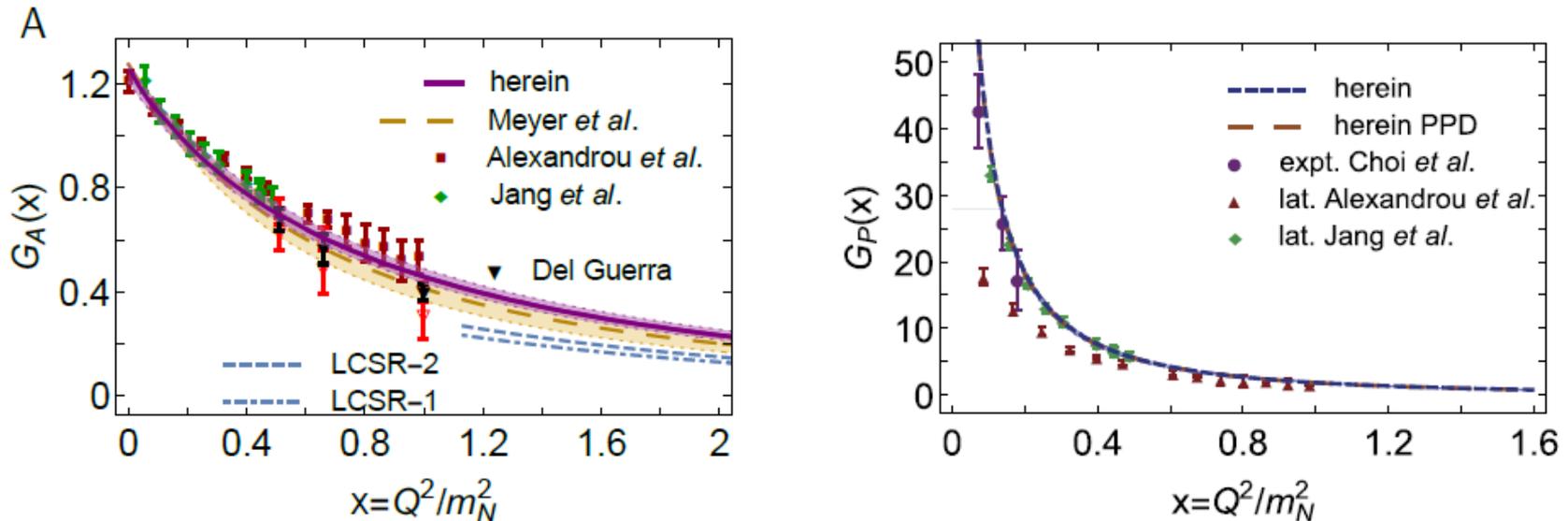
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Nucleon axial form factor at large momentum transfers

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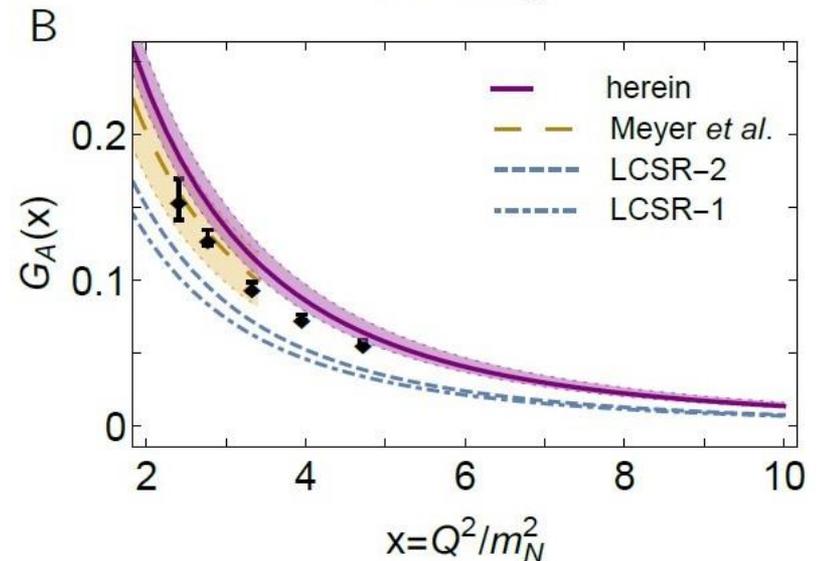
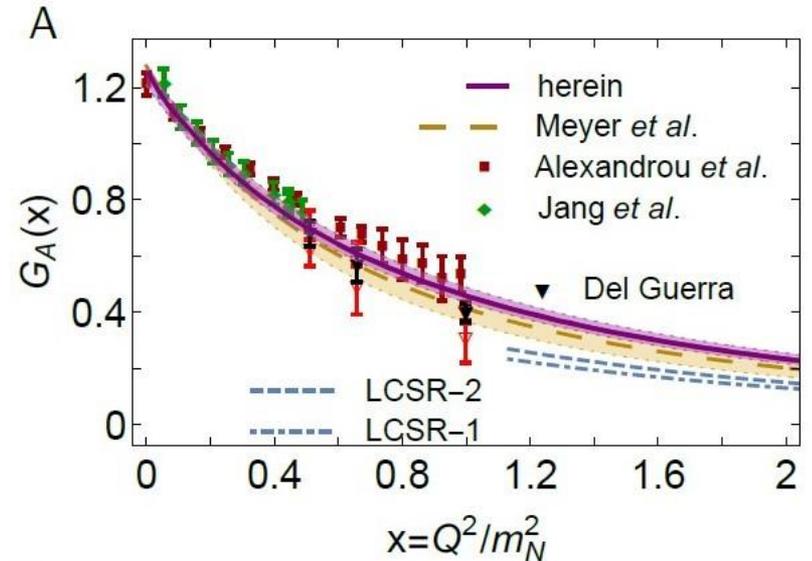
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Large Q^2 Nucleon Axial Form Factor

➤ Parameter-free DSEs predictions to

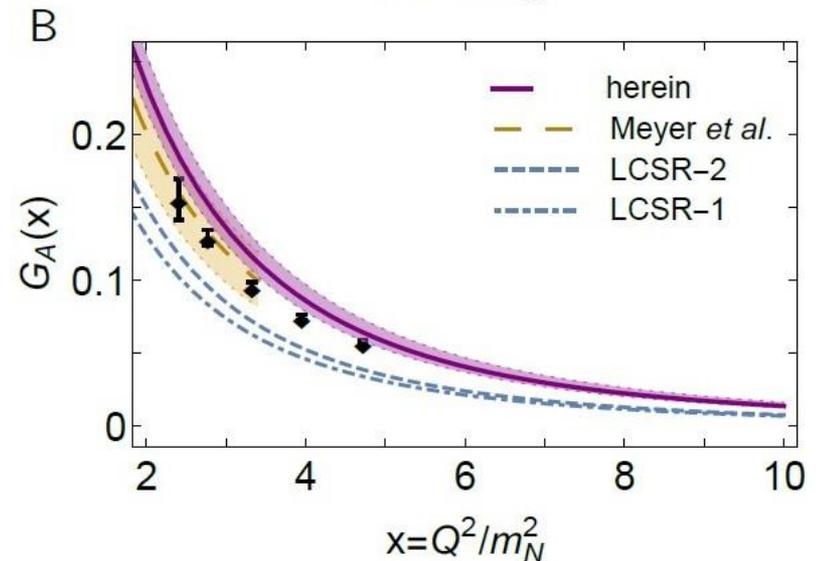
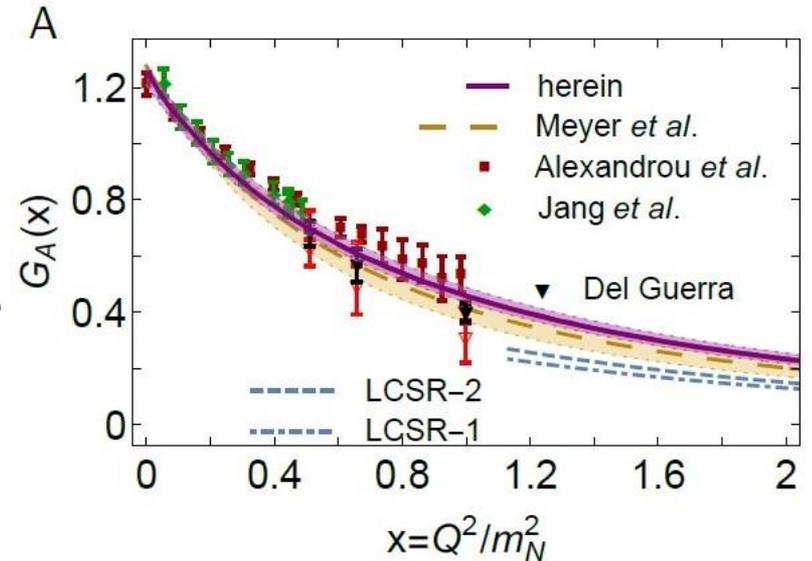
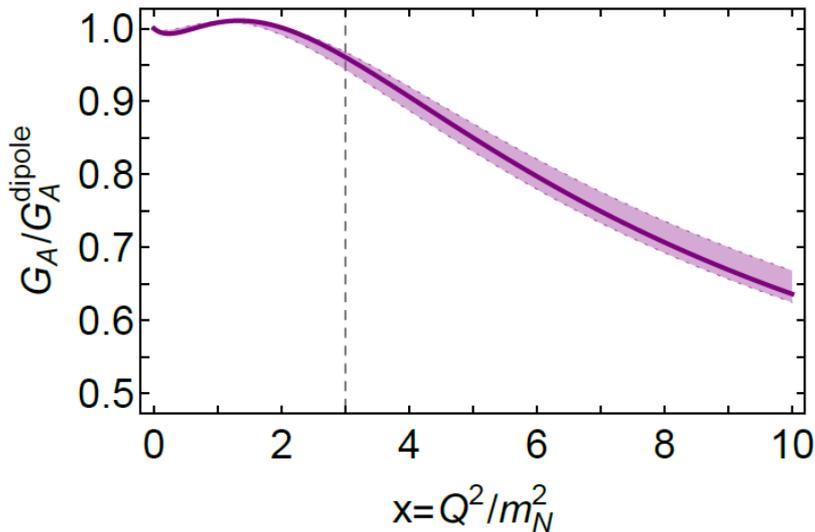
$$x = \frac{Q^2}{m_N^2} = 10$$

➤ DSEs prediction agrees with available data: small & large x



Large Q^2 Nucleon Axial Form Factor

- Parameter-free DSEs predictions to $x = \frac{Q^2}{m_N^2} = 10$
- DSEs prediction agrees with available data: small & large x
- The dipole Ansatz: it could be used to provide reasonable representation of $G_A(x)$ on $x \in [0, 3]$. Outside the fitted domain, however, the quality of approximation deteriorates quickly.





Δ -Baryon axialvector and pseudoscalar form factors, and associated PCAC relations

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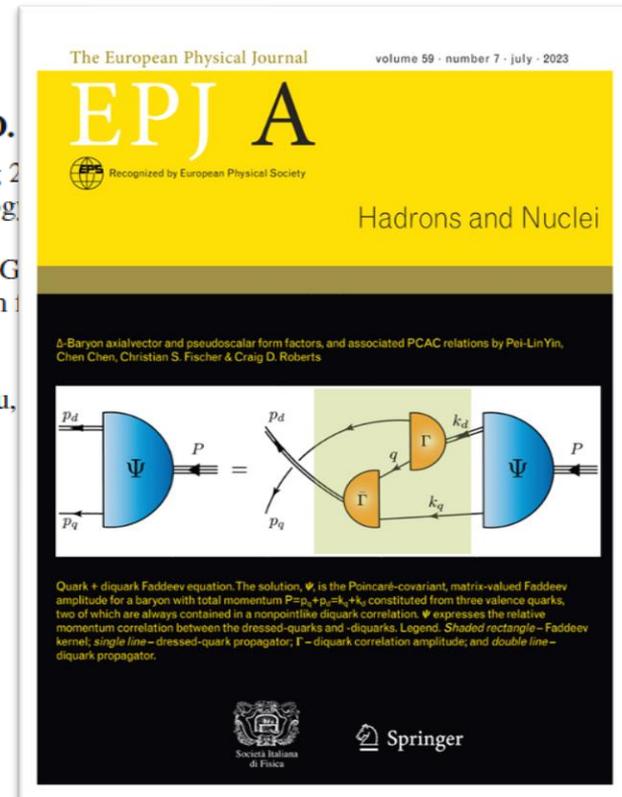
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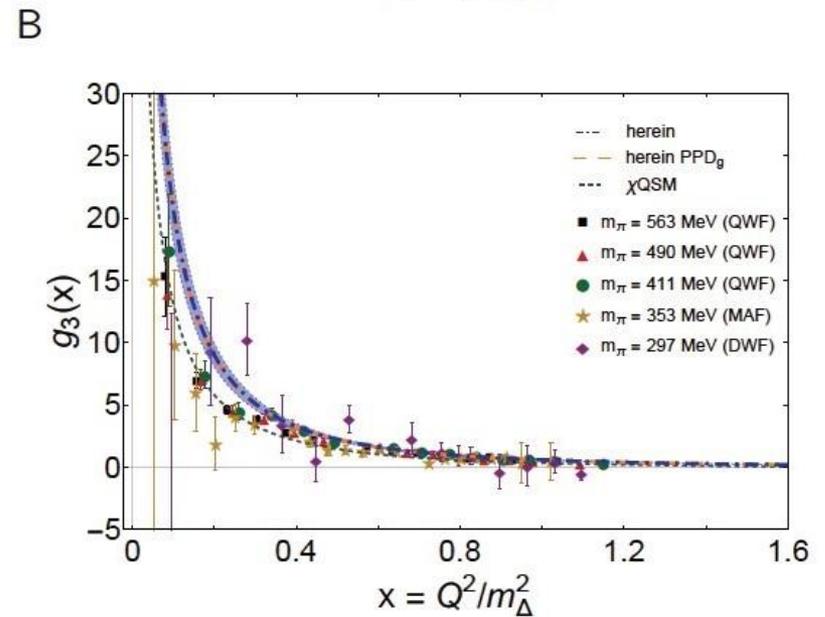
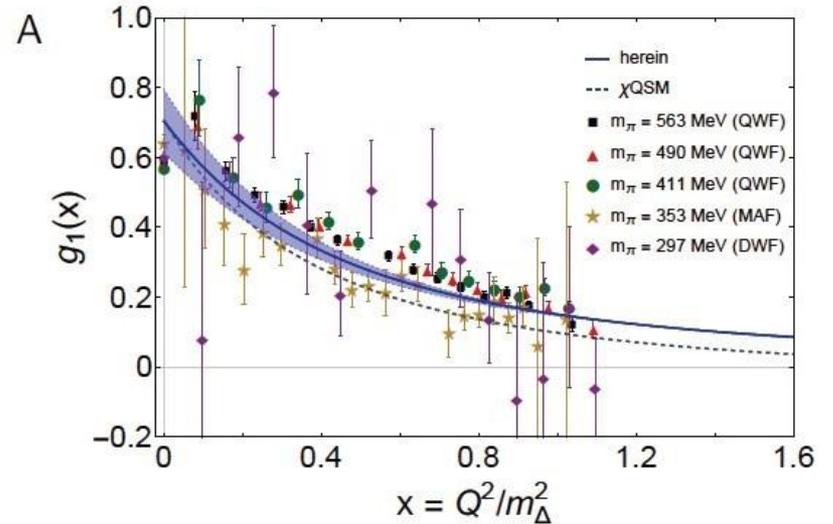
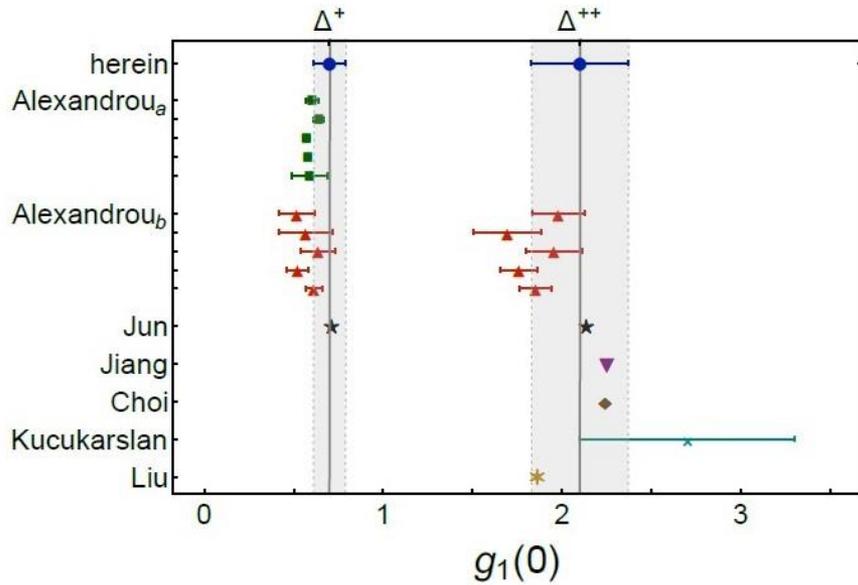
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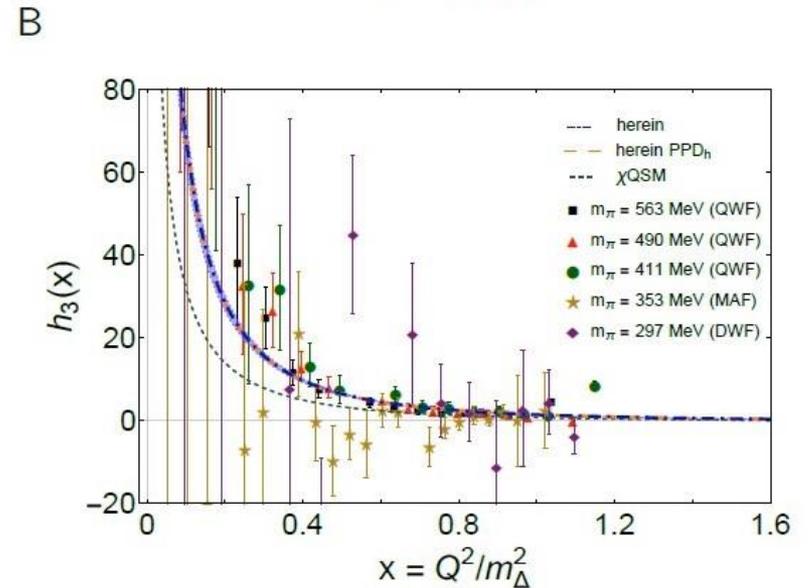
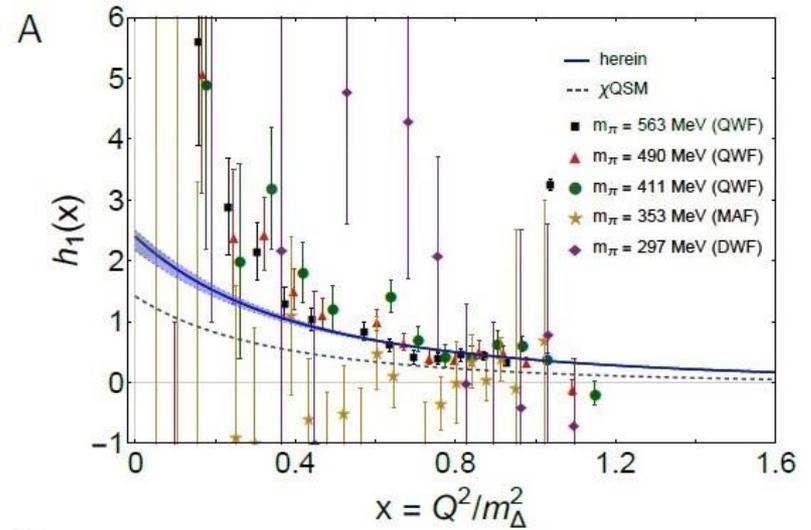
$\Delta(1232)$ Axial Form Factors

➤ Axial charge: $g_1(0) = 0.71(9)$



$\Delta(1232)$ Axial Form Factors

- $h_1(x)$: regular
- $h_1(0) = 2.25(17)$



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

PHYSICAL REVIEW LETTERS **133**, 131901 (2024)

Nucleon-to- Δ Axial and Pseudoscalar Transition Form Factors

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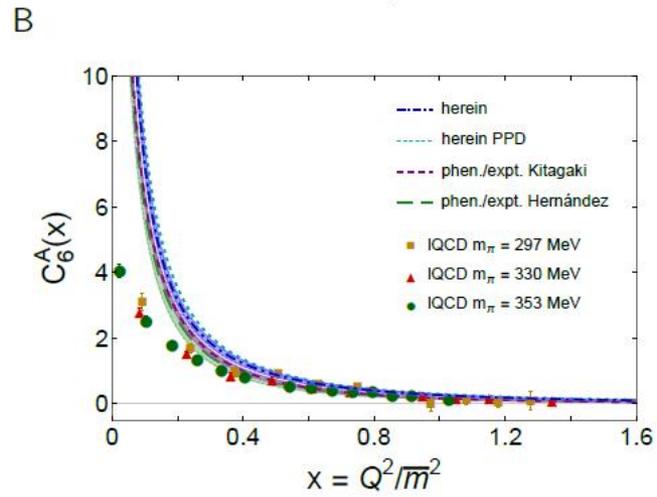
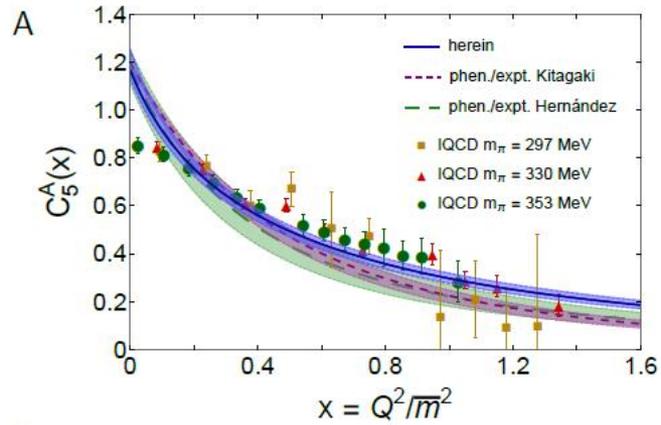
$$J_{\mu}^{\Delta N}(K, Q) = \bar{u}_{\alpha}^{\Delta}(P_f) \Gamma_{(5)(\mu), \alpha}(K, Q) u^N(P_i),$$

$$\Gamma_{\mu, \alpha}^{\text{EM}}(K, Q) = b \left[\frac{i\omega}{2\lambda_+} (G_M^* - G_E^*) \gamma_5 \varepsilon_{\alpha\mu\gamma\delta} K_{\gamma} \hat{Q}_{\delta} - G_E^* T_{\alpha\gamma}^Q T_{\gamma\mu}^K - \frac{i\tau}{\omega} G_C^* \hat{Q}_{\alpha} K_{\mu} \right],$$

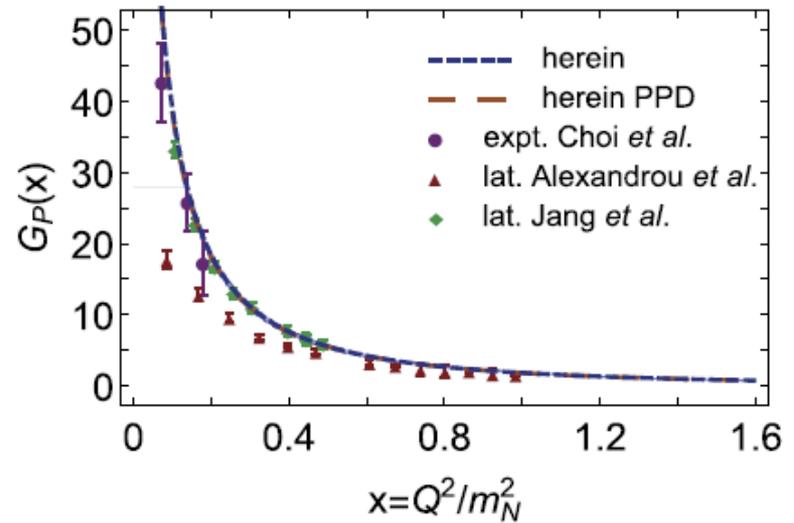
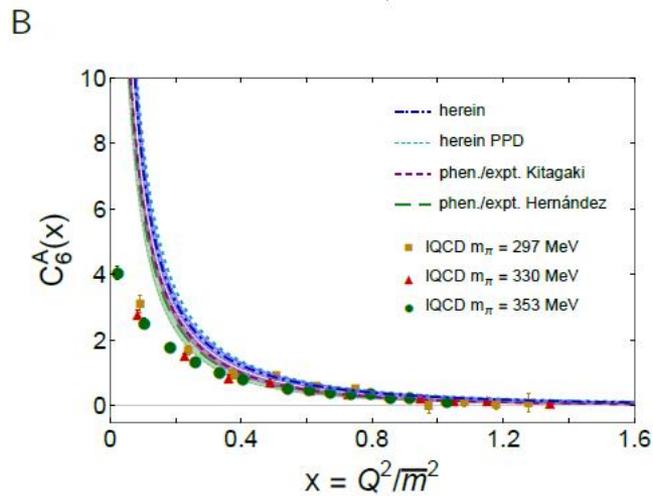
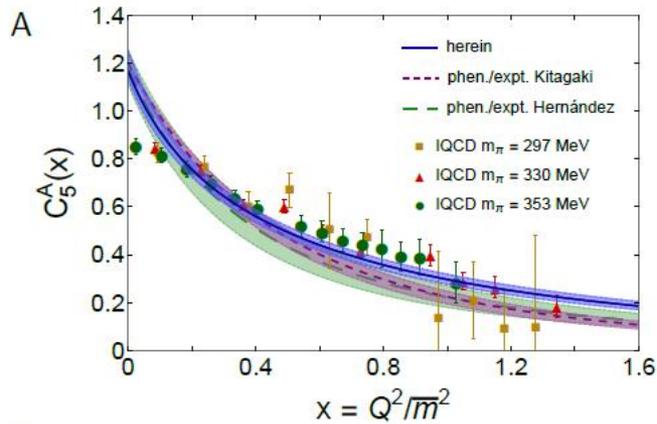
$$\Gamma_{5, \alpha}^{\text{AX}}(Q) = \sqrt{\frac{2}{3}} \left[i(\gamma_{\mu} Q_{\lambda} - \delta_{\mu\lambda} Q) \frac{C_3^A}{m_N} - (\delta_{\mu\lambda} (P_f \cdot Q) - P_{f\mu} Q_{\lambda}) \frac{C_4^A}{m_N^2} + \delta_{\mu\lambda} C_5^A - Q_{\mu} Q_{\lambda} \frac{C_6^A}{m_N^2} \right],$$

$$\Gamma_{5, \alpha}^{\text{PS}}(Q) = \sqrt{\frac{2}{3}} \left[i \frac{Q_{\lambda}}{4m_N} \frac{m_{\pi}^2}{Q^2 + m_{\pi}^2} \frac{f_{\pi}}{m_q} G_{\pi N \Delta}(Q^2) \right].$$

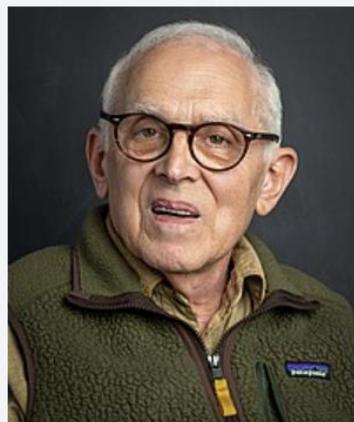
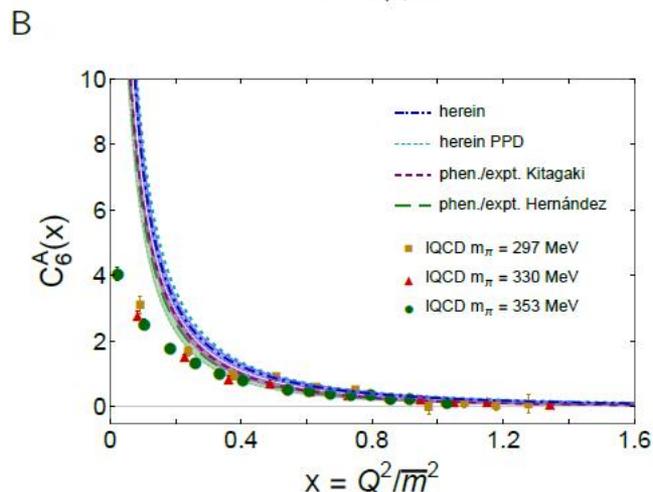
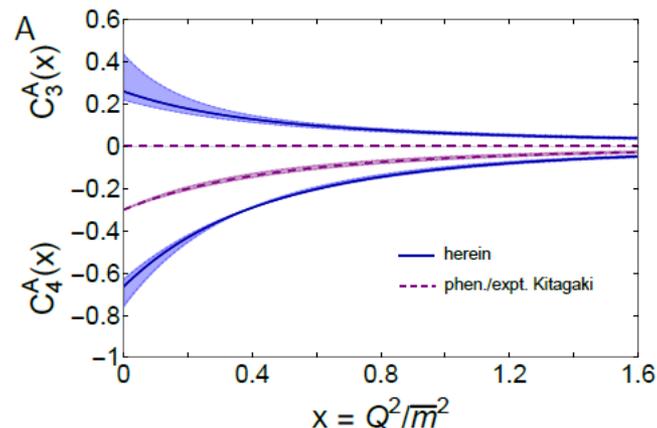
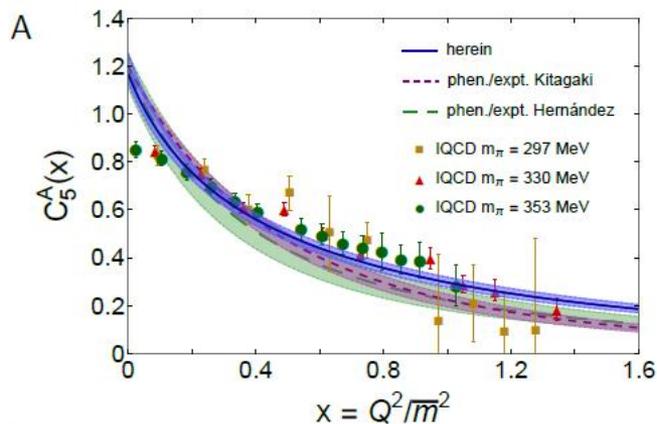
$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors



Stephen L. Adler

PROFESSOR EMERITUS

School of Natural Sciences
Particle Physics

Photoproduction, electroproduction and weak single pion production in the (3,3) resonance region

#1

Stephen L. Adler (Princeton, Inst. Advanced Study) (1968)

Published in: *Annals Phys.* 50 (1968) 189-311

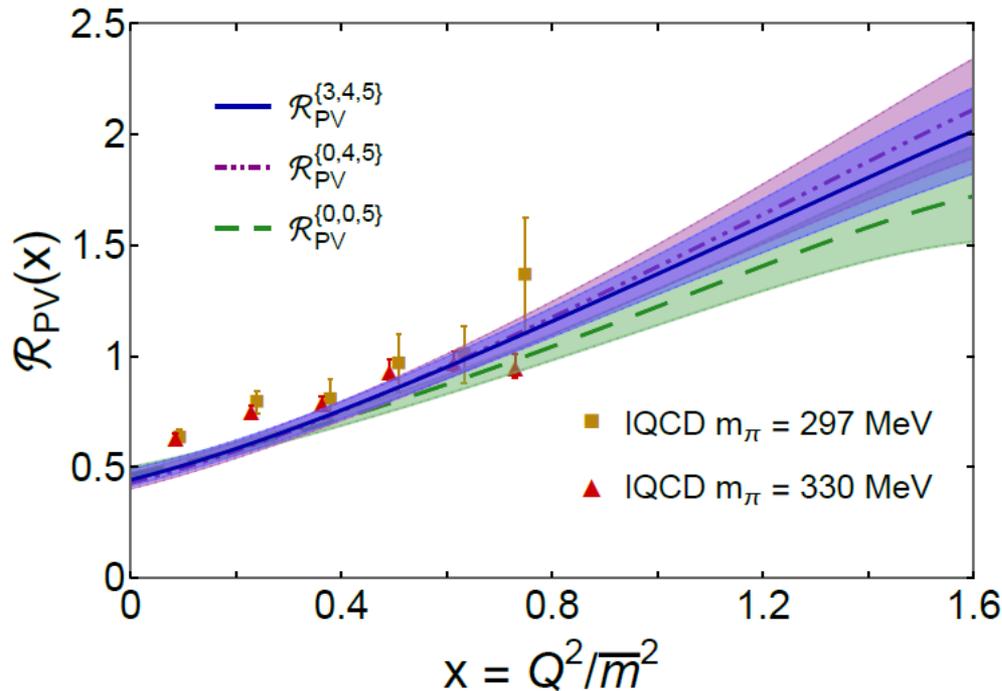
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[reference search](#) [493 citations](#)

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

➤ The parity violation asymmetry

$$\mathcal{R}_{PV}^{\{3,4,5\}}(Q^2) := \frac{C_5^A}{C_3^V} \left[1 + \frac{m_\Delta^2 - Q^2 - m_N^2}{2m_N^2} \frac{C_4^A}{C_5^A} - \frac{m_N^2 + Q^2 + 2m_N m_\Delta - 3m_\Delta^2}{4m_N m_\Delta} \frac{C_3^A}{C_5^A} \right]$$



Summary

- **Dyson-Schwinger equations, Bethe-Salpeter equations, Faddeev equation**
- **Diquark correlations in baryons**
- **Nucleon Axial Form Factor at large momentum transfers**
- **$\Delta(1232)$ Axial Form Factors**
- **$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Transition Form Factors**

Summary

- Dyson-Schwinger equations, Bethe-Salpeter equations, Faddeev equation
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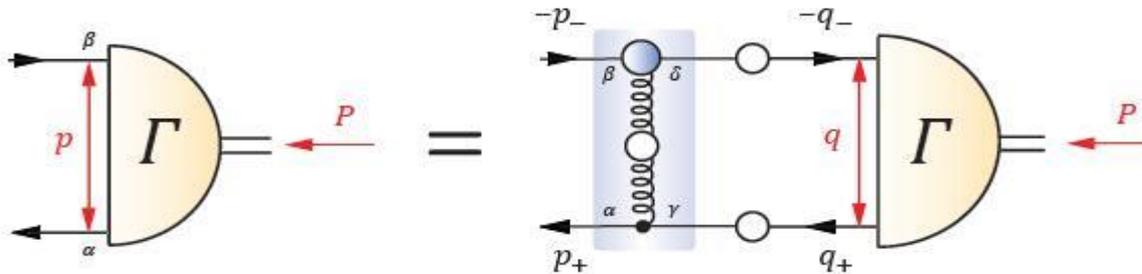
Thank you!

Hadrons: Bound-states in QFT

➤ **Mesons:** a 2-body bound state problem in QFT

➤ Bethe-Salpeter Equation

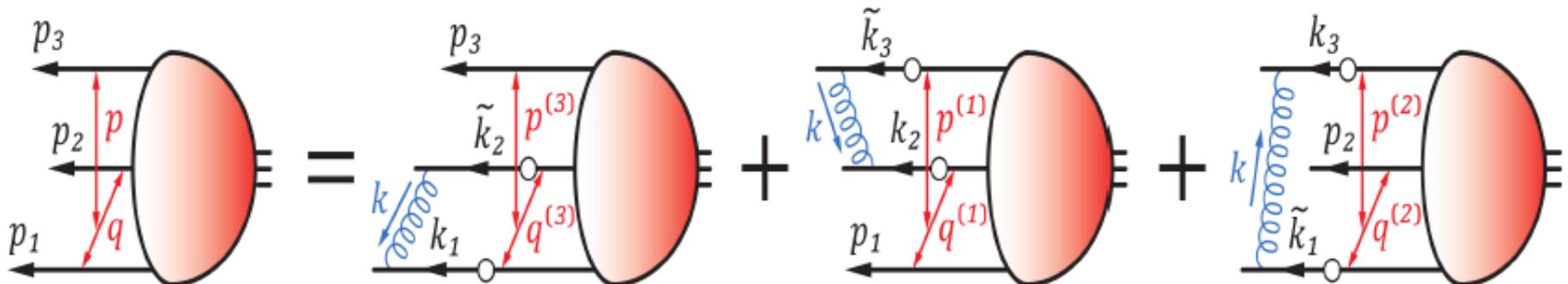
➤ **K** - fully amputated, two-particle irreducible, quark-antiquark scattering kernel



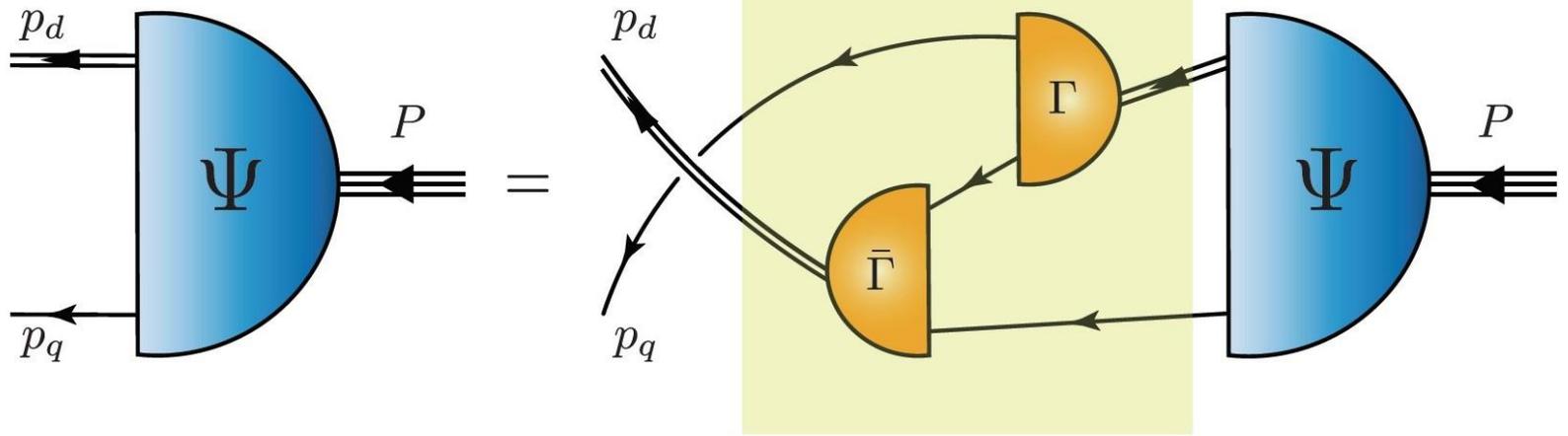
➤ **Baryons:** a 3-body bound state problem in QFT

➤ Faddeev equation: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.

Faddeev equation in rainbow-ladder truncation

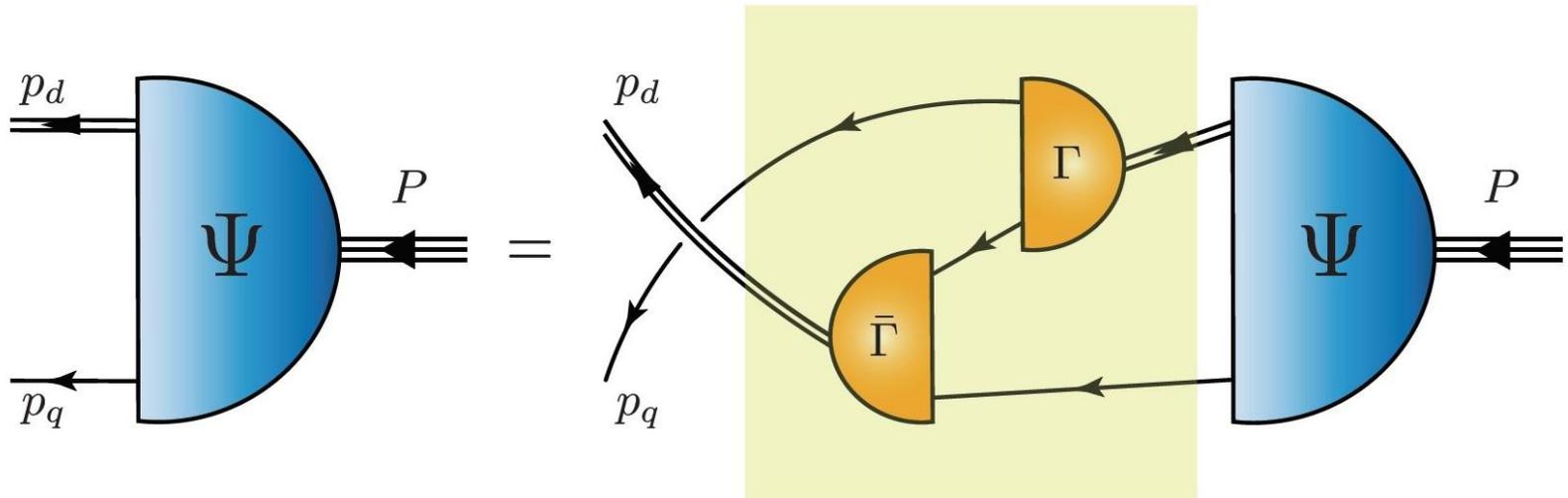


How to solve?



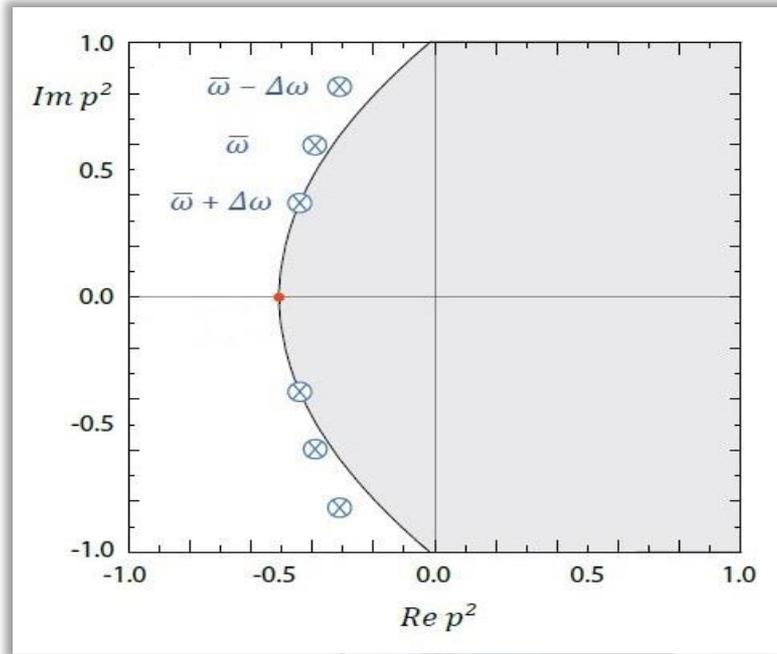
How to solve?

- ◆ The dressed-quark propagator
- ◆ Diquark amplitudes
- ◆ Diquark propagators
- ◆ **Faddeev amplitudes**

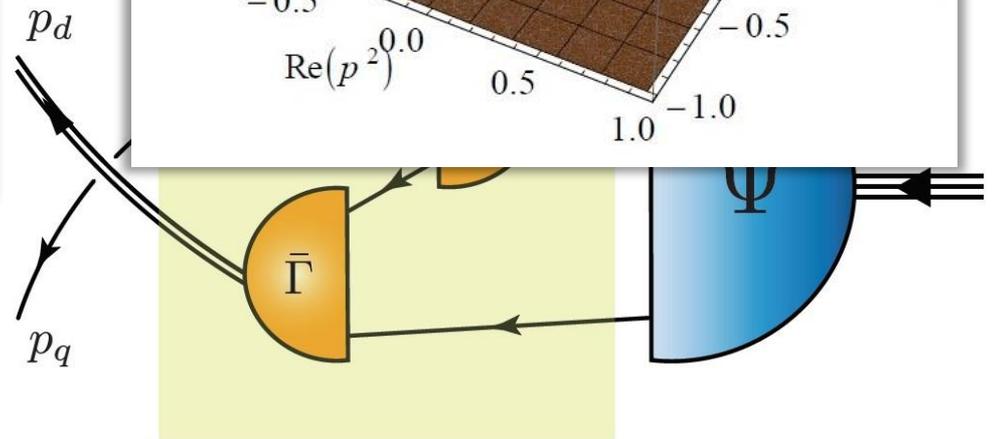
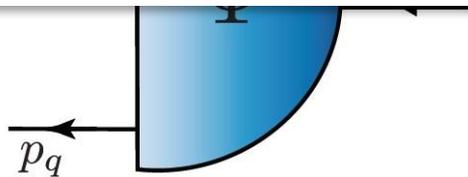
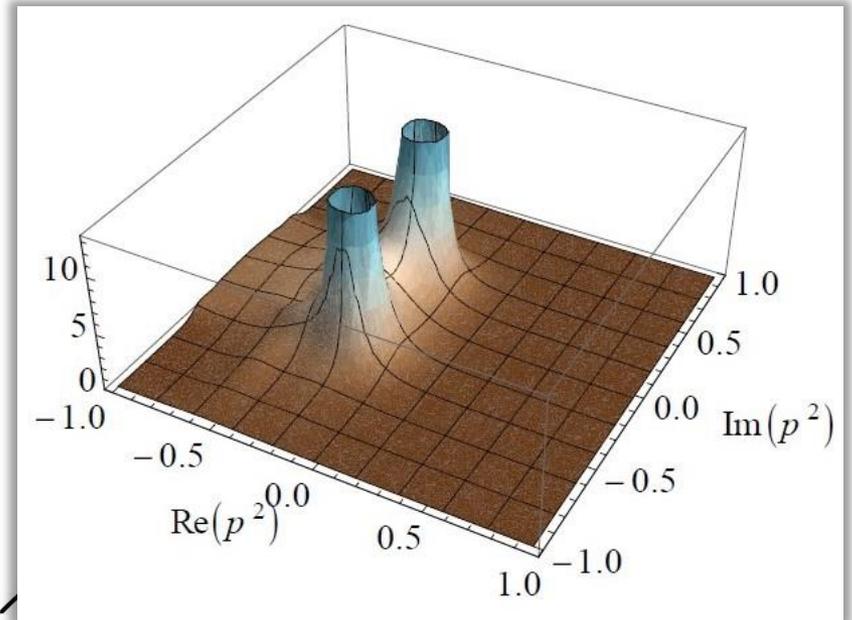


How to solve?

◆ The dressed-quark propagator

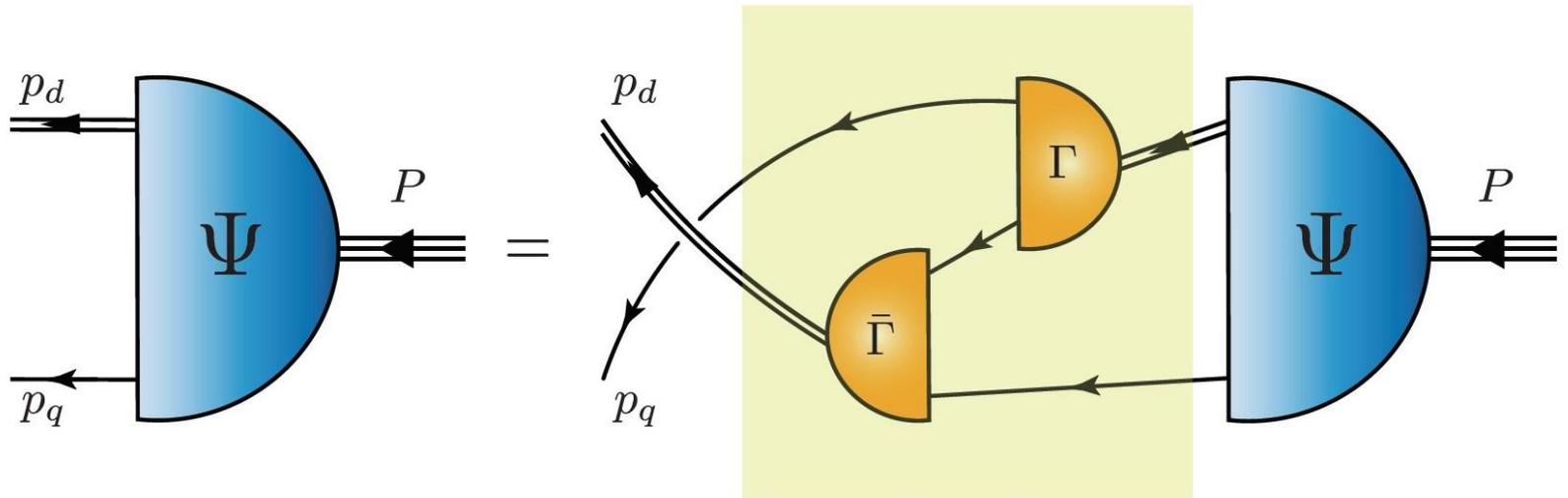


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QCD-kindred model

- ◆ The dressed-quark propagator
- ◆ Diquark amplitudes
- ◆ Diquark propagators
- ◆ **Faddeev amplitudes**



QCD-kindred model

➤ **Diquark masses (in GeV):**

$$m_{[ud]_{0+}} = 0.80 \text{ GeV}$$

$$m_{\{uu\}_{1+}} = m_{\{ud\}_{1+}} = m_{\{dd\}_{1+}} = 0.89 \text{ GeV}$$

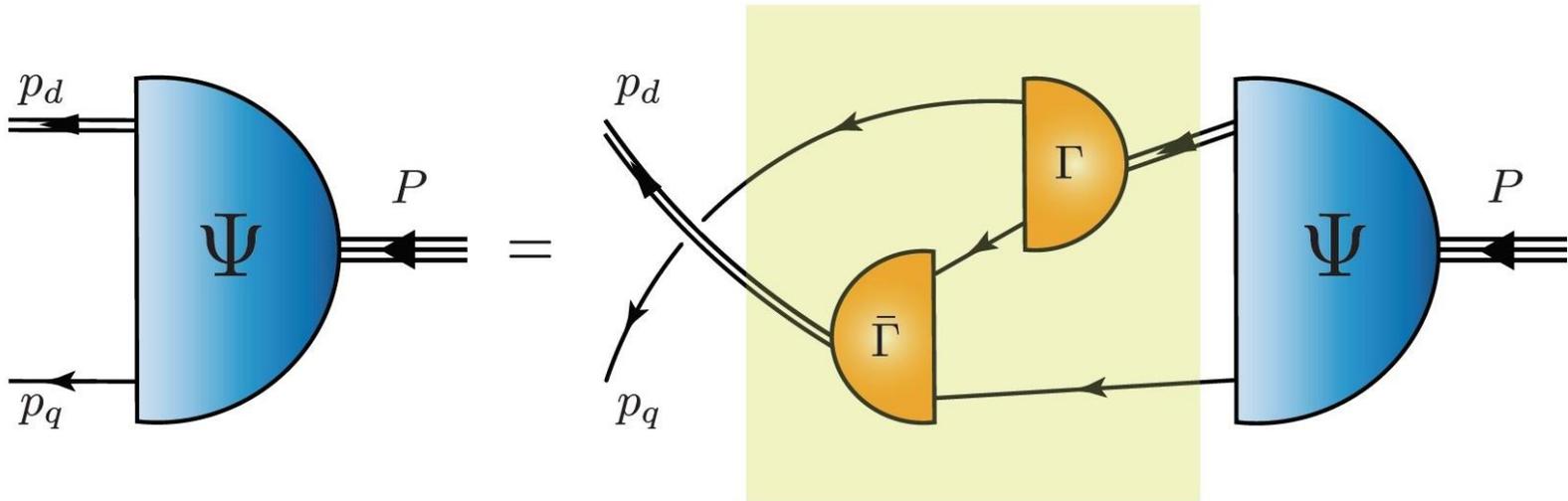
$$m_N = 1.18 \text{ GeV}$$

$$m_R = 1.72 \text{ GeV}$$

$$m_\Delta = 1.35 \text{ GeV}$$

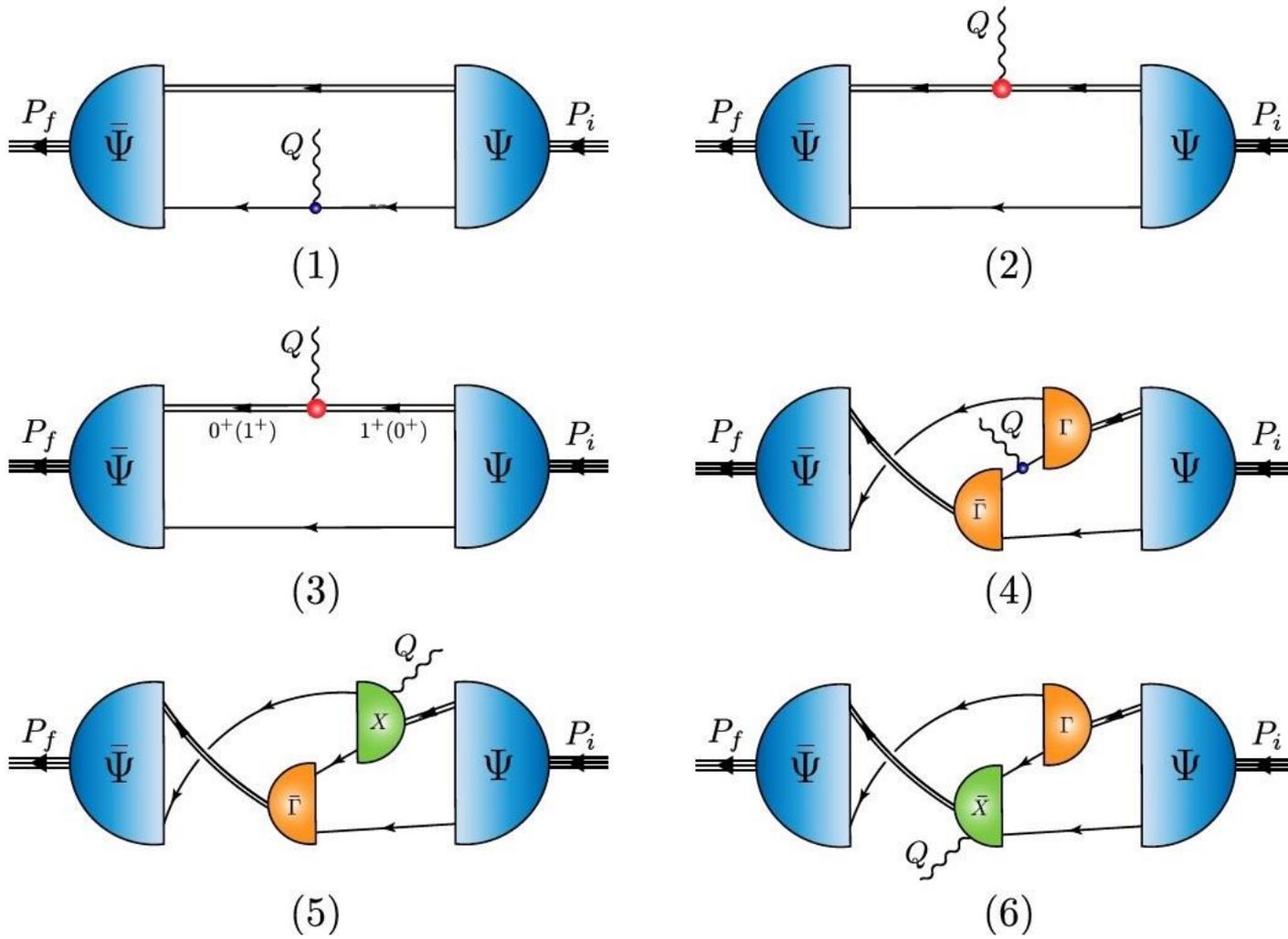


- **These two values provide for a good description of numerous dynamical properties of the nucleon, Δ -baryon and low-lying excitations, e.g., N(1440), N(1535), Δ (1600).**



How to compute Form Factors?

- In the quark-diquark framework, the associated symmetry-preserving current:



Proton Spin Structure

- Flavour separation of proton axial charge
- **d**-quark receives large contribution from probe+quark in presence of axialvector diquark

$$\frac{g_A^d}{g_A^u} = 0^+ \& 1^+ -0.32(2)$$

$$\frac{g_A^d}{g_A^u} = 0^+ \text{ only } -0.054(13)$$

- Experiment: **0.27(4)**
- Hadron scale: $g_A^u + g_A^d (+g_A^s = 0) = 0.65(2) \Rightarrow$ quarks carry **65%** of the proton spin
- Poincaré-covariant proton wave function: remaining **35%** lodged with quark+diquark orbital angular momentum



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Perspective on polarised parton distribution functions and proton spin

P. Cheng (程鹏)^{a,b}, Y. Yu (俞杨)^{a,b}, H.-Y. Xing (邢惠瑜)^{a,b}, C. Chen (陈晨)^{c,d,*},
Z.-F. Cui (崔著钊)^{a,b,*}, C.D. Roberts^{a,b,*}



Large Q^2 Nucleon Axial Form Factor

➤ Light-front transverse density profiles

➤ Scalar diquark only:

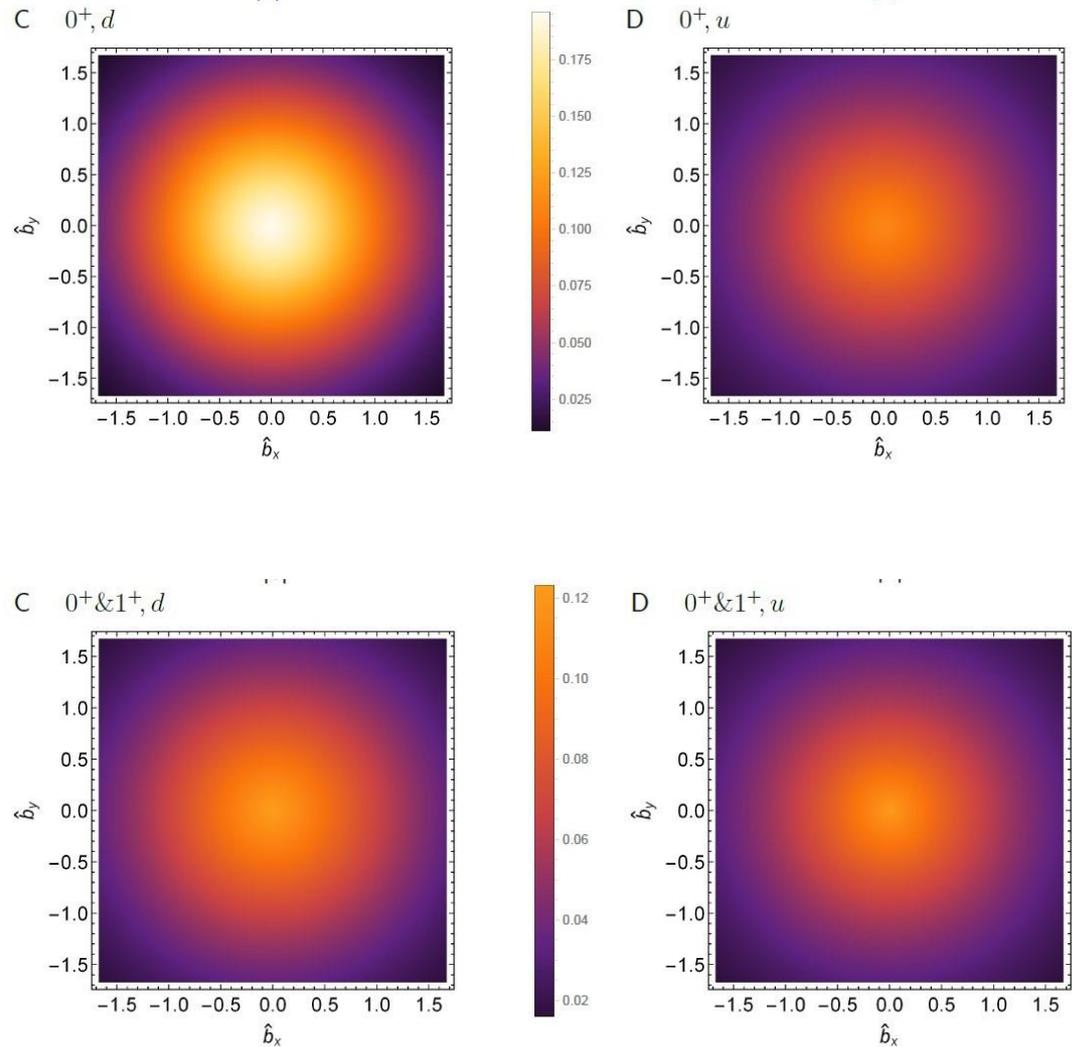
- magnitude of the d quark contribution to GA is just 10% of that from the u quark
- d quark is also much more localized

$$r_{A_d}^\perp \approx 0.5 r_{A_u}^\perp$$

➤ Scalar + axial-vector diquarks:

- d and u quark transverse profiles are quite similar

$$r_{A_d}^\perp \approx 0.9 r_{A_u}^\perp$$



The axial current – G_A & G_P

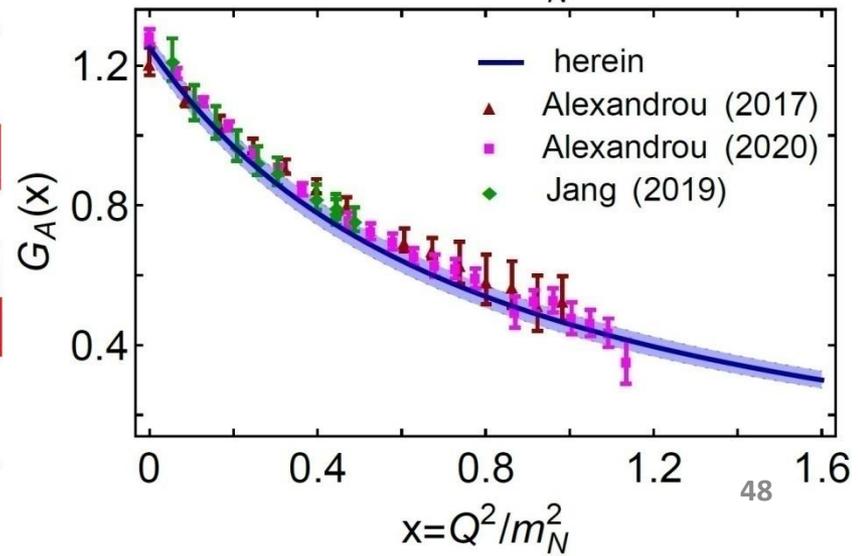
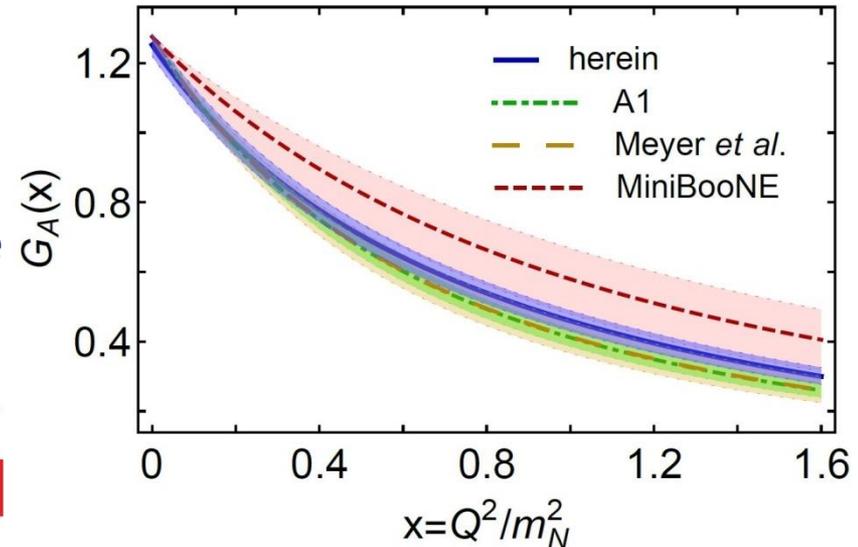
$$J_{5\mu}^j(K, Q) = \bar{u}(P_f) \frac{\tau^j}{2} \gamma_5 \left[\gamma_\mu G_A(Q^2) + i \frac{Q_\mu}{2m_N} G_P(Q^2) \right] u(P_i)$$

➤ Two form factors:

- G_A – axial form factor
- G_P – induced pseudoscalar form factor

➤ G_A can reliably be represented by dipole characterised by mass-scale m_A

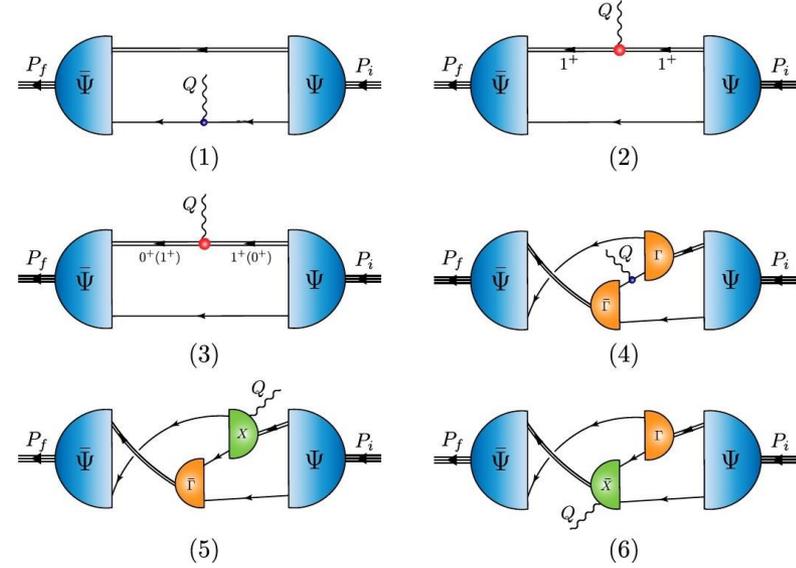
	g_A	$m_N \langle r_A^2 \rangle^{1/2}$	m_A/m_N
Herein	1.25(03)	3.25(04)	1.23(03)
Faddeev ₃ [31]	0.99(02)	2.63(06)	1.32(03)
Exp [4]	1.2756(13)	–	–
Exp [13]	–	3.02(11)	1.15(04)
Exp [14]	–	3.23(72)	1.15(08)
Exp [17]	–	2.41(31)	1.44(18)
IQCD [57]	1.21(3)(2)	2.45(08)(03)	1.41(04)(02)
IQCD [58]	1.30(6)	3.57(30)	0.97(16)
IQCD _d [59]	1.23(3)	2.48(15)	1.39(09)
IQCD _z [59]	1.30(9)	3.19(30)	1.09(11)



Fractions of $G_A(0)$, $G_P(0)$ and $G_5(0)$

TABLE I. Referring to Fig. 3, separation of $G_A(0)$, $G_P(0)$ and $G_5(0)$ into contributions from various diagrams, listed as a fraction of the total $Q^2 = 0$ value. Diagram (1): $\langle J \rangle_q^S$ – weak-boson strikes dressed-quark with scalar diquark spectator; and $\langle J \rangle_q^A$ – weak-boson strikes dressed-quark with axial-vector diquark spectator. Diagram (2): $\langle J \rangle_{qq}^{AA}$ – weak-boson interacts strikes axial-vector diquark with dressed-quark spectator. Diagram (3): $\langle J \rangle_{dq}^{SA+AS}$ – weak-boson mediates transition between scalar and axial-vector diquarks, with dressed-quark spectator. Diagram (4): $\langle J \rangle_{ex}$ – weak-boson strikes dressed-quark “in-flight” between one diquark correlation and another. Diagrams (5) and (6): $\langle J \rangle_{sg}$ – weak-boson couples inside the diquark correlation amplitude. The listed uncertainty in these results reflects the impact of $\pm 5\%$ variations in the diquark masses in Eq. (16), *e.g.* $0.71_{1\mp} \Rightarrow 0.71 \mp 0.01$.

	$\langle J \rangle_q^S$	$\langle J \rangle_q^A$	$\langle J \rangle_{qq}^{AA}$	$\langle J \rangle_{dq}^{SA+AS}$	$\langle J \rangle_{ex}$	$\langle J \rangle_{sg}$
$G_A(0)$	$0.71_{4\mp}$	$0.064_{2\pm}$	$0.025_{5\pm}$	0.130_{\mp}	$0.072_{32\pm}$	0
$G_P(0)$	$0.74_{4\mp}$	$0.070_{5\pm}$	$0.025_{5\pm}$	0.130_{\mp}	$0.22_{4\pm}$	$-0.19_{1\mp}$
$G_5(0)$	$0.74_{4\mp}$	$0.069_{5\pm}$	$0.025_{5\pm}$	0.130_{\mp}	$0.22_{4\pm}$	$-0.19_{1\mp}$



➤ Projections:

$$G_A = -\frac{1}{4(1+\tau)} \text{tr}_D [J_{5\mu} \gamma_5 \gamma_\mu^T],$$

$$G_P = \frac{1}{\tau} \left(G_A - \frac{Q_\mu}{4im_N \tau} \text{tr}_D [J_{5\mu} \gamma_5] \right),$$

$$G_5 = \frac{1}{2\tau} \text{tr}_D [J_5 \gamma_5],$$

➤ $G_P(0) \sim G_5(0)$

$$G_P \sim \frac{Q_\mu}{\tau^2} \text{tr}_D [J_{5\mu} \gamma_5] \sim \frac{1}{\tau} \text{tr}_D [J_5 \gamma_5] \sim G_5,$$

when $Q^2 \sim 0 \text{ GeV}^2$.

QCD-kindred model

➤ **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

➤ **algebraic form:**

$$\begin{aligned} \bar{\sigma}_S(x) = & 2\bar{m}\mathcal{F}(2(x + \bar{m}^2)) \\ & + \mathcal{F}(b_1x)\mathcal{F}(b_3x)[b_0 + b_2\mathcal{F}(\epsilon x)], \end{aligned} \quad (\text{A3a})$$

$$\bar{\sigma}_V(x) = \frac{1}{x + \bar{m}^2} [1 - \mathcal{F}(2(x + \bar{m}^2))], \quad (\text{A3b})$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1 - e^{-x}}{x}, \quad (\text{A4})$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$$\frac{\bar{m} \quad b_0 \quad b_1 \quad b_2 \quad b_3}{0.00897 \quad 0.131 \quad 2.90 \quad 0.603 \quad 0.185}, \quad (\text{A5})$$

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate- p^2 domains.]

QCD-kindred model

➤ The dressed-quark propagator

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

- Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
- Mass function has a real-world value at $p^2 = 0$, NOT the highly inflated value typical of **RL** truncation.
- Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from **RL** truncation.
- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
- Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introduced long beforehand.

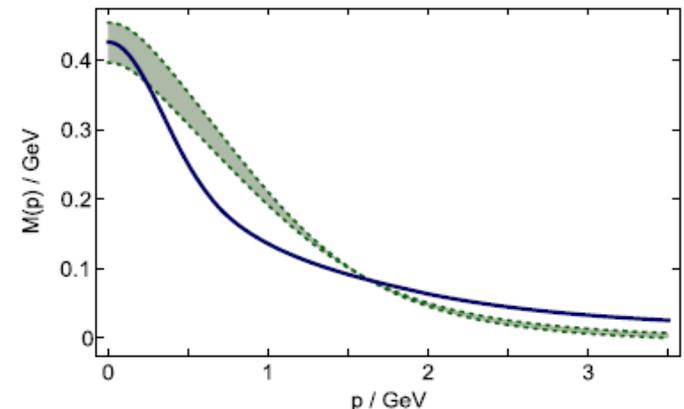


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and⁵¹used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes:** five types of correlation are possible in a $J=1/2$ bound state: isoscalar scalar ($I=0, J^P=0^+$), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.
- The **LEADING** structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),

$$\Gamma^{0+}(k; K) = g_{0+} \gamma_5 C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{0+}^2),$$

$$\vec{\Gamma}_{\mu}^{1+}(k; K) = i g_{1+} \gamma_{\mu} C \vec{\tau} \vec{H} \mathcal{F}(k^2 / \omega_{1+}^2),$$

$$\Gamma^{0-}(k; K) = i g_{0-} C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{0-}^2),$$

$$\Gamma_{\mu}^{1-}(k; K) = g_{1-} \gamma_{\mu} \gamma_5 C \tau^2 \vec{H} \mathcal{F}(k^2 / \omega_{1-}^2),$$

$$\vec{\Gamma}_{\mu}^{1-}(k; K) = i g_{1-} [\gamma_{\mu}, \gamma \cdot K] \gamma_5 C \vec{\tau} \vec{H} \mathcal{F}(k^2 / \omega_{1-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

➤ The diquark propagators

$$\Delta^{0\pm}(K) = \frac{1}{m_{0\pm}^2} \mathcal{F}(k^2/\omega_{0\pm}^2),$$

$$\Delta_{\mu\nu}^{1\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1\pm}^2} \right] \frac{1}{m_{1\pm}^2} \mathcal{F}(k^2/\omega_{1\pm}^2).$$

- The *\mathcal{F} -functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and $1/q^2$ evolution (UV) of meson propagators.
- Diquarks are **confined**.
- free-particle-like at spacelike momenta
 - pole-free on the timelike axis
 - This is **NOT** true of **RL** studies. It enables us to reach arbitrarily high values of momentum transfer.

QCD-kindred model

➤ The Faddeev amplitudes:

$$\begin{aligned}
 \psi^\pm(p_i, \alpha_i, \sigma_i) = & [\Gamma^{0+}(k; K)]_{\sigma_1 \alpha_1 \sigma_2 \alpha_2} \Delta^{0+}(K) [\varphi_{0^+}^\pm(\ell; P) u(P)]_{\sigma_3}^{\alpha_3} \\
 & + [\Gamma_\mu^{1+j}] \Delta_{\mu\nu}^{1+} [\varphi_{1^+}^{j\pm}(\ell; P) u(P)] \\
 & + [\Gamma^{0-}] \Delta^{0-} [\varphi_{0^-}^\pm(\ell; P) u(P)] \\
 & + [\Gamma_\mu^{1-}] \Delta_{\mu\nu}^{1-} [\varphi_{1^-}^\pm(\ell; P) u(P)], \quad (9)
 \end{aligned}$$

➤ Quark-diquark vertices:

$$\varphi_{0^+}^\pm(\ell; P) = \sum_{i=1}^2 s_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) \mathcal{G}^\pm,$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_D(\gamma_5)$ and

$$\varphi_{1^+}^{j\pm}(\ell; P) = \sum_{i=1}^6 a_i^{j\pm}(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_\nu^i(\ell; P) \mathcal{G}^\pm,$$

$$S^1 = \mathbf{I}_D, \quad S^2 = i\gamma \cdot \hat{\ell} - \hat{\ell} \cdot \hat{P} \mathbf{I}_D$$

$$\mathcal{A}_\nu^1 = \gamma \cdot \ell^\perp \hat{P}_\nu, \quad \mathcal{A}_\nu^2 = -i\hat{P}_\nu \mathbf{I}_D, \quad \mathcal{A}_\nu^3 = \gamma \cdot \hat{\ell}^\perp \hat{\ell}_\nu^\perp$$

$$\varphi_{0^-}^\pm(\ell; P) = \sum_{i=1}^2 r_i^\pm(\ell^2, \ell \cdot P) S^i(\ell; P) \mathcal{G}^\mp,$$

$$\mathcal{A}_\nu^4 = i\hat{\ell}_\nu^\perp \mathbf{I}_D, \quad \mathcal{A}_\nu^5 = \gamma_\nu^\perp - \mathcal{A}_\nu^3, \quad \mathcal{A}_\nu^6 = i\gamma_\nu^\perp \gamma \cdot \hat{\ell}^\perp - \mathcal{A}_\nu^4,$$

$$\varphi_{1^-}^\pm(\ell; P) = \sum_{i=1}^6 v_i^\pm(\ell^2, \ell \cdot P) \gamma_5 \mathcal{A}_\nu^i(\ell; P) \mathcal{G}^\mp,$$

QCD-kindred model

- Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.
- Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.
- In consequence, the manner by which the dressed quarks' spin, S , and orbital angular momentum, L , add to form the total momentum J , is **frame dependent**: L , S are not independently Poincare invariant.
- The set of baryon **rest-frame** quark-diquark angular momentum identifications:

$${}^2S: S^1, \mathcal{A}_v^2, (\mathcal{A}_v^3 + \mathcal{A}_v^5),$$

$${}^2P: S^2, \mathcal{A}_v^1, (\mathcal{A}_v^4 + \mathcal{A}_v^6),$$

$${}^4P: (2\mathcal{A}_v^4 - \mathcal{A}_v^6)/3,$$

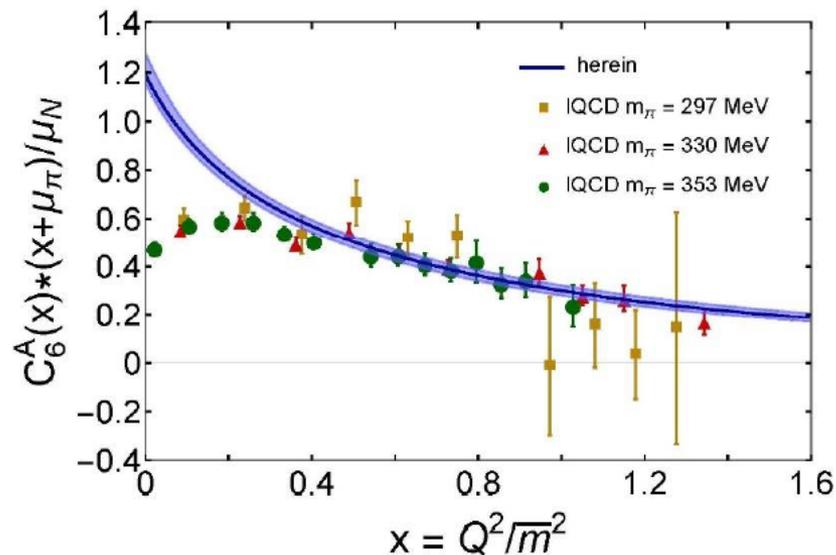
$${}^4D: (2\mathcal{A}_v^3 - \mathcal{A}_v^5)/3,$$

- The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

Thus, the discrepancy between our prediction and the IQCD results is not explained by the larger than physical pion masses used in their simulation. Instead, the existing simulations are affected by some as yet unknown and uncontrolled systematic lattice artifact.

The figure at right augments Figs. 6, 7 in the IQCD paper. It highlights the issue and shows that factorizing a pion pole component consistent with PPD accentuates the visual power of the discrepancy between our prediction and the IQCD results.



We elect to keep the original version of our figure because we are not in a position to explain systematic issues with lattice simulations, which have, furthermore, not been explained by the IQCD practitioners themselves. Notwithstanding that, we have highlighted the challenge in the revised manuscript, adding a closing sentence to the penultimate paragraph of Sec. 4:

It seems fair to judge that these $C_{5,6}^A$ discrepancies owe partly to the larger than physical pion masses which characterise the lattice configurations. However, there are other systematic issues with the IQCD results – see, *e.g.*, Ref. [37, Figs. 6, 7].

$N(940) \rightarrow \Delta(1232)$ Axial and Pseudoscalar Form Factors

➤ Pseudoscalar form factor

