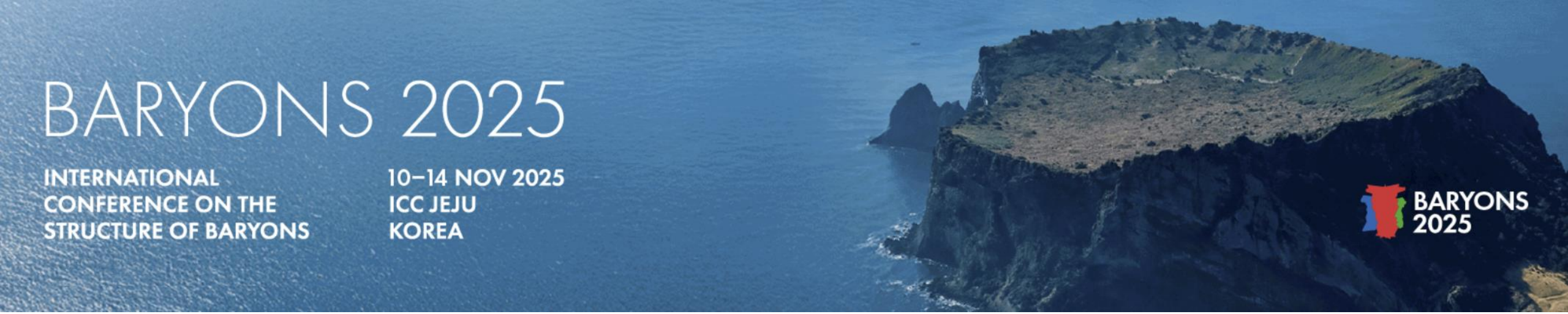


Uncovering the Source of Resonant States

Jia-Jun Wu (University of Chinese Academy of Sciences)

Collaborator: W. Hao(UCAS), J.-L. Fu(UCAS), G.-J. Wang(KEK), Z. Yang(UESTC), M. Oka(RIKEN), S.-L. Zhu(PKU)

PRD 112 056021, Scib.2024.07.012, PRL 128 112001

Baryon2025 **South Korea** **2025.11.13**

中国科学院大学
University of Chinese Academy of Sciences

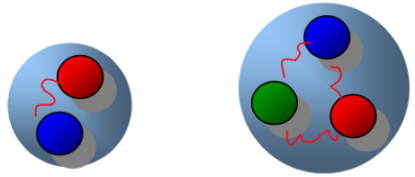
Outline

- Background
- Toy model with One bare state & One channel
- Examples: $X(3872)$ & $D_{s0}(2317)$
- Summary and Outlook

Background

Traditional Quark model

convential hadron

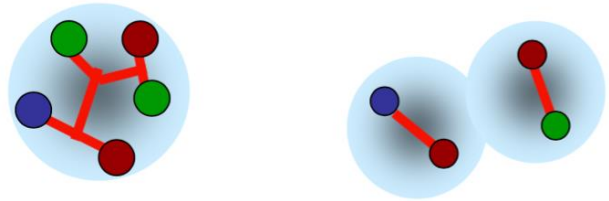


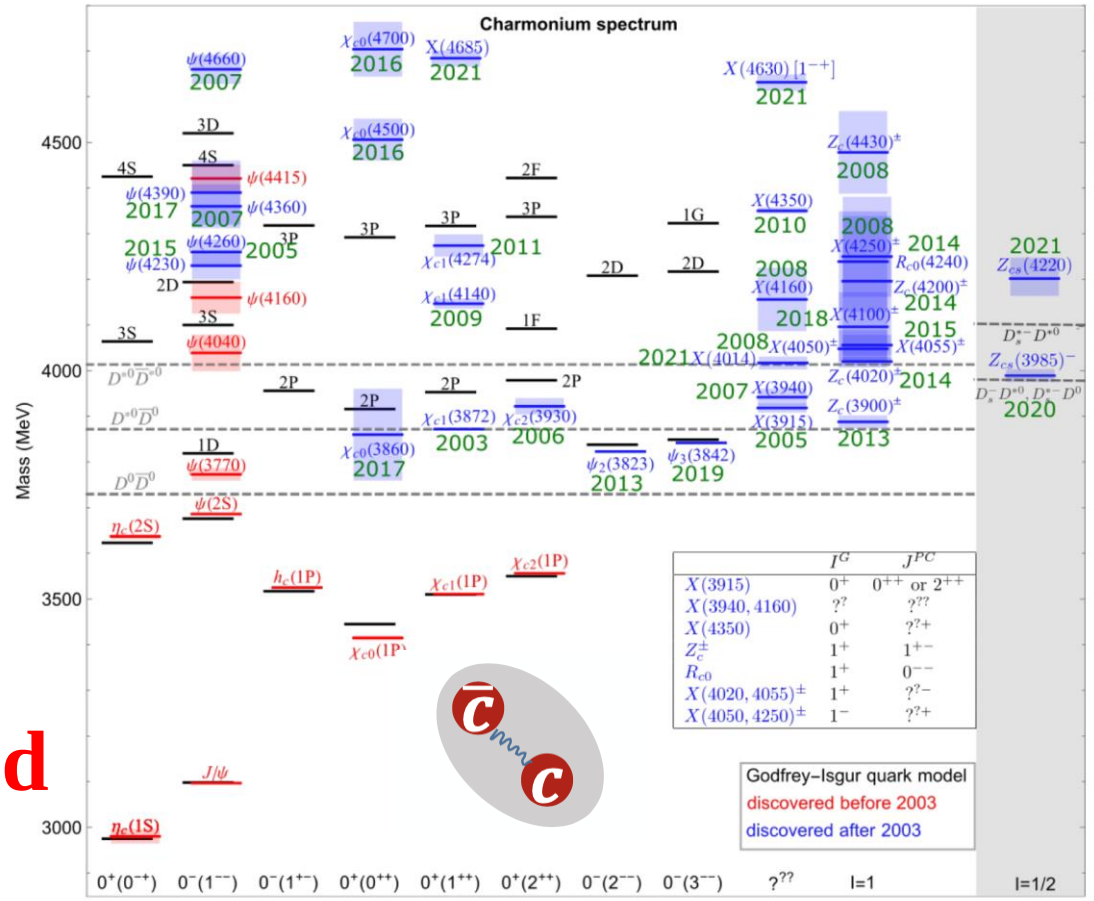
(q $\bar{q}$)　　(qqq)

Exotic

Hybrid　　Glueball　　Tetraquark

Pentaquark　　Hadronic molecule





Question： How is a hadron composed of these possible components?

Background

Question：How is a hadron composed of these possible components?

strong interaction

Colorful → quark level

Colorless → hadron level

Background

Question：How is a hadron composed of these possible components?

strong interaction

Colorful → quark level

Colorless → hadron level

A more comprehensive framework for systematically describing hadrons

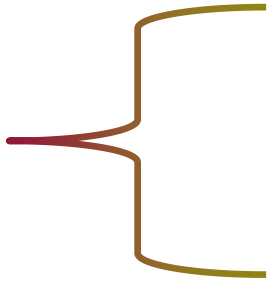
Include quark level and hadron level.

Not only one, at least a set of hadron

Background

Question： How is a hadron composed of these possible components?

strong interaction



Colorful → quark level

Colorless → hadron level

A more comprehensive framework for systematically describing hadrons
Resonance can decay, especially for the strong interaction decay.

Non-zero width shows the interaction with hadron channels

quark level
hadron level

A coupled channel model is necessary !

Strong interaction

{ Colorful → quark level
  Colorless → hadron level

Introduction of HEFT

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

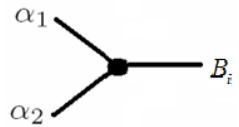
$|B_i\rangle$ bare state， bare mass m_i ⟷ Quark level

$|\alpha(k_\alpha)\rangle$ non-interaction channels ⟷ Hadron level

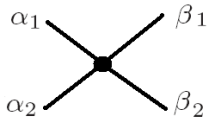
$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



3P0 model, ……



One boson exchange, ……

Resonance
(Mass, width, pole position, coupling)

HEFT

SEE Nov. 14, Friday, Zhan-wei Liu's talk

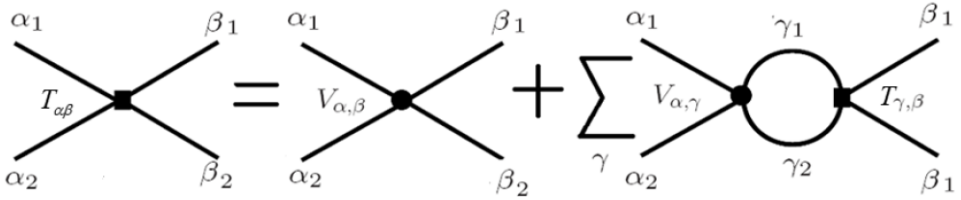
T matrix
(phase shift, Inelascity)

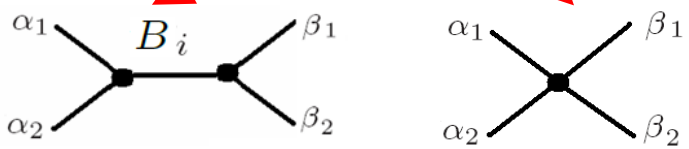
Lattice spectrum

J. M. M. Hall et al., PRD 87(2013), 094510
J.-j. Wu et al., PRC90 (2014), 055206
C. D. Abell et al., PRD 106 (2022) 3, 034506

Introduction of HEFT

$$T_{\alpha\beta}(k_\alpha,k_\beta,E)=V_{\alpha\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma^{\,2}dk_\gamma\,\frac{V_{\alpha\gamma}(k_\alpha,k_\gamma)T_{\gamma\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$





$G_\alpha^{B*}\dfrac{1}{E-m_B}G_\beta^B$

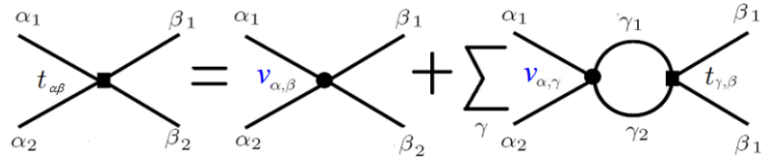
$v_{\alpha,\beta}$

$$\det\left[A^{-1}\right]=0$$

$$T_{\alpha\beta}(k,k';E)=t_{\alpha\beta}(k,k';E)+T_{\alpha\beta}^{bare}(k,k';E)$$

A. Matsuyama,T. Sato, T.-S.H. Lee,
Phys.Rept. 439 (2007) 193 Argonne-Osaka Analysis

$$t_{\alpha\beta}(k,k',E)=v_{\alpha\beta}(k,k')+\sum_\gamma\int q^2dq\,\frac{v_{\alpha\gamma}(k,q)t_{\gamma\beta}(q,k',E)}{E-\varpi_\gamma(q)+i\varepsilon}$$



$$T_{\alpha\beta}^{\ \text{bare}}(k,k',E)=\sum_{B,B'}G_{\alpha B}(k,E)A_{BB'}(E)\bar{G}_{B'\beta}(k',E)$$

$$G_{\alpha B}(k,E)=G_\alpha^{B*}(k)+\sum_\beta\int q^2dq\,\frac{t_{\alpha\beta}(k,q,E)G_\beta^{B*}(q)}{E-\varpi_\beta(q)+i\varepsilon}$$ +

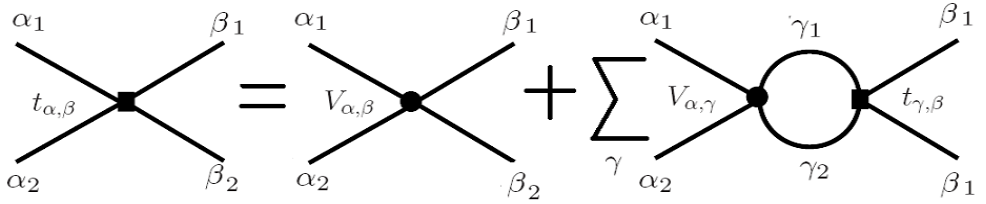
$$\bar{G}_{B\alpha}(k,E)=G_\alpha^B(k)+\sum_\beta\int q^2dq\,\frac{G_\beta^B(q)t_{\beta\alpha}(q,k,E)}{E-\varpi_\beta(q)+i\varepsilon}$$ +

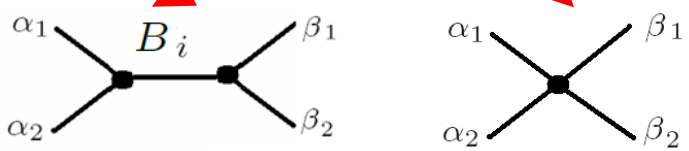
$$\left(A^{-1}\right)_{BB'}(E)=\delta_{BB'}\left(E-m_B\right)-\Sigma_{BB'}(E)$$

$$\Sigma_{BB'}(E)=\sum_\beta\int q^2dq\,\frac{G_\beta^B(q)G_{\beta B'}(k,E)}{E-\varpi_\beta(q)+i\varepsilon}$$ +

Toy model : ~~One bare state~~ & One channel

$$T_{\alpha\beta}(k_\alpha,k_\beta,E)=V_{\alpha\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma^{\,2}dk_\gamma\,\frac{V_{\alpha\gamma}(k_\alpha,k_\gamma)T_{\gamma\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$





$$\frac{g^2u(k)u(k')}{E-m_B}$$

$$vf(k)f(k')$$

$$\begin{bmatrix}M_{ff}(E) & M_{fu}(E)\\ M_{uf}(E) & M_{uu}(E)\end{bmatrix}=\int\frac{q^2dq}{E-\varpi(q)}\begin{bmatrix}f^2(q) & f(q)u(q)\\ u(q)f(q) & u^2(q)\end{bmatrix}$$

$$t(k,k',E)=vf(k)f(k')+\int q^2dq\;\frac{vf(k)f(q)}{E-\varpi(q)}t(q,k',E)$$

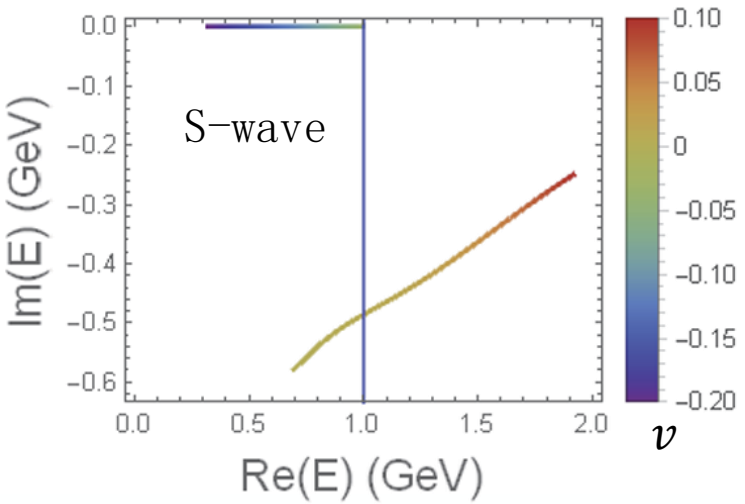
$$t(k,k',E)=f(k)\tilde{t}(E)f(k'),\quad \tilde{t}(E)=(1-vM_{ff}(E))^{-1}v$$

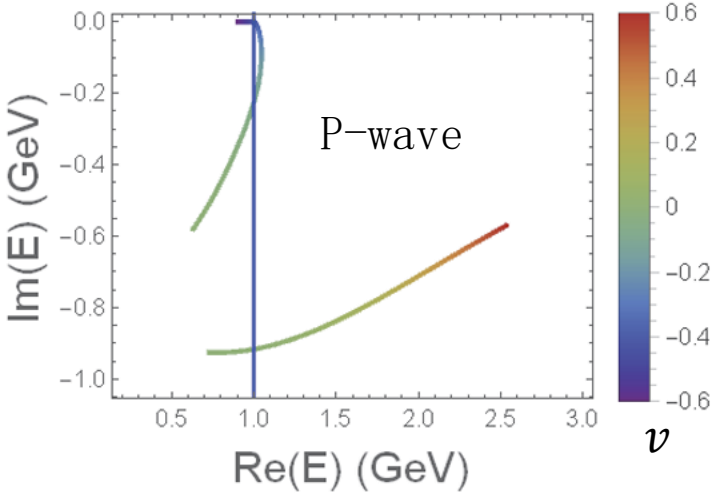
$$0=1-vM_{ff}$$

S-wave: $f(q)=5u(q)$

P-wave: $f(q)=qu(q)/m_M$

$$u(q)=\frac{1}{\varpi_M(q)}\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2$$

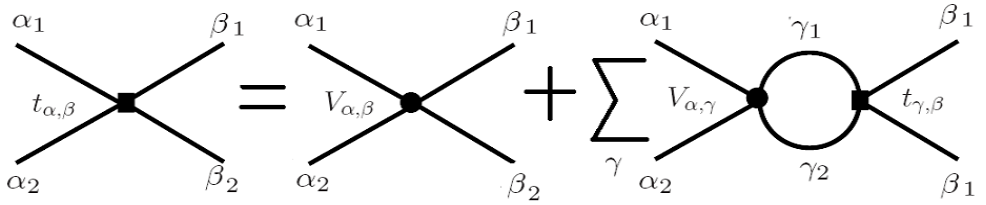


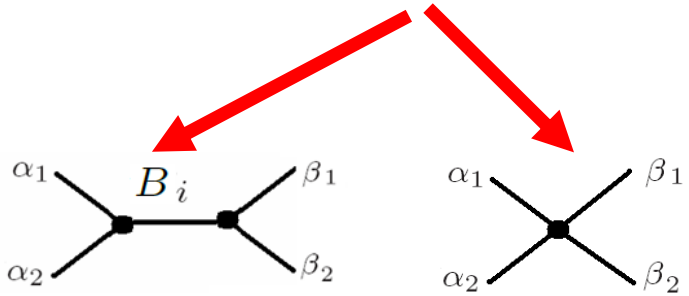


W. Hao, J.-l. Fu, J.-j. Wu, PRD 112 (2025) 056021

Toy model : One bare state & One channel

$$T_{\alpha\beta}(k_\alpha,k_\beta,E)=V_{\alpha\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma^{\,2}dk_\gamma\frac{V_{\alpha\gamma}(k_\alpha,k_\gamma)T_{\gamma\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$





$\frac{g^2u(k)u(k')}{E-m_B}$ $vf(k)f(k')$

$$\begin{bmatrix}M_{ff}(E) & M_{fu}(E)\\ M_{uf}(E) & M_{uu}(E)\end{bmatrix}=\int\frac{q^2dq}{E-\varpi(q)}\begin{bmatrix}f^2(q) & f(q)u(q)\\ u(q)f(q) & u^2(q)\end{bmatrix}$$

$$T(k,k',E)=t(k,k',E)+T^{\text{bare}}(k,k',E)$$

$$t(k,k',E)=f(k)\tilde{t}(E)f(k'),\quad \tilde{t}(E)=(1-vM_{ff}(E))^{-1}v$$

$$T^{\text{bare}}(k,k',E)=G(k,E)A(E)\bar{G}(k',E)$$

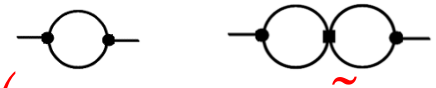
$$G(k,E)=\bar{G}(k,E)=g\left(u(k)+f(k)\tilde{t}(E)M_{fu}(E)\right)$$

$$A^{-1}(E)=E-m_B-g^2\left(M_{uu}(E)+M_{uf}(E)\tilde{t}(E)M_{fu}(E)\right)$$

When $\tilde{t}\to\infty$, $T^{bare}\to -t, T\to 0$,so the pole is only from

$$0=E-m_B-g^2\left(M_{uu}+M_{uf}\tilde{t}M_{fu}\right)$$

Toy model : One bare state & One channel



$$0 = E - m_B - g^2\left(M_{uu} + M_{uf}\,\tilde{t}M_{fu}\right)$$

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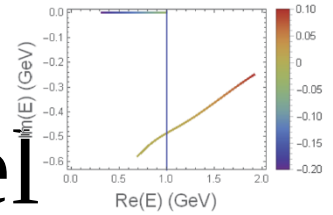
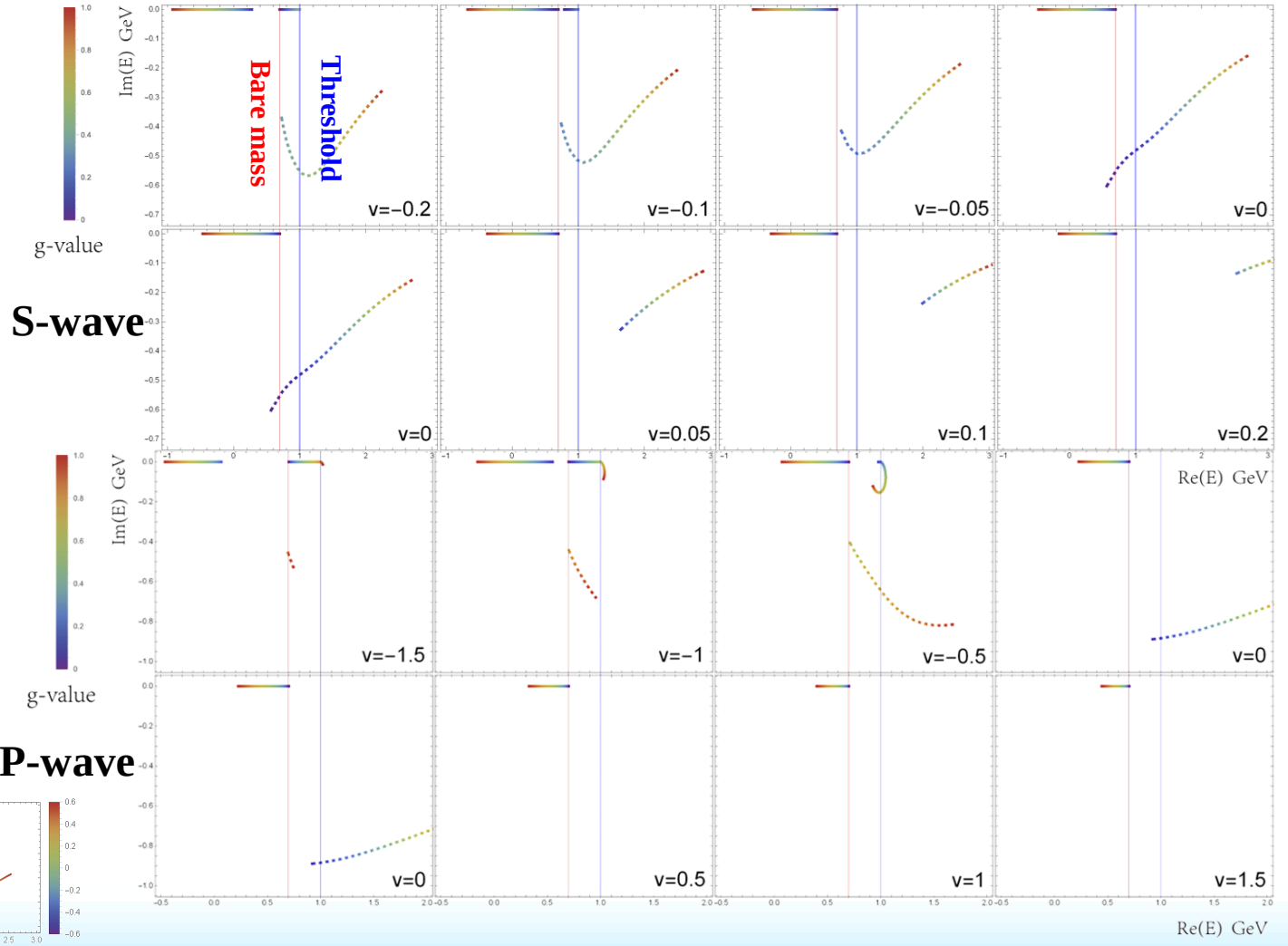
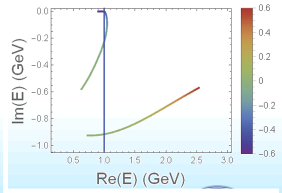
$$u(q) = \frac{1}{\varpi_M(q)}\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2$$

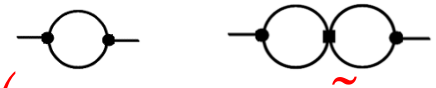
$$m_B < m_{\alpha 1} + m_{\alpha 2}$$







One bare state & One channel



$$0 = E - m_B - g^2\left(M_{uu} + M_{uf}\,\tilde{t}M_{fu}\right)$$

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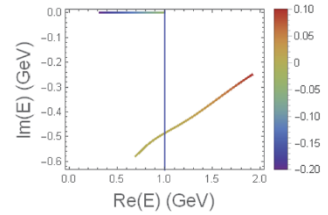
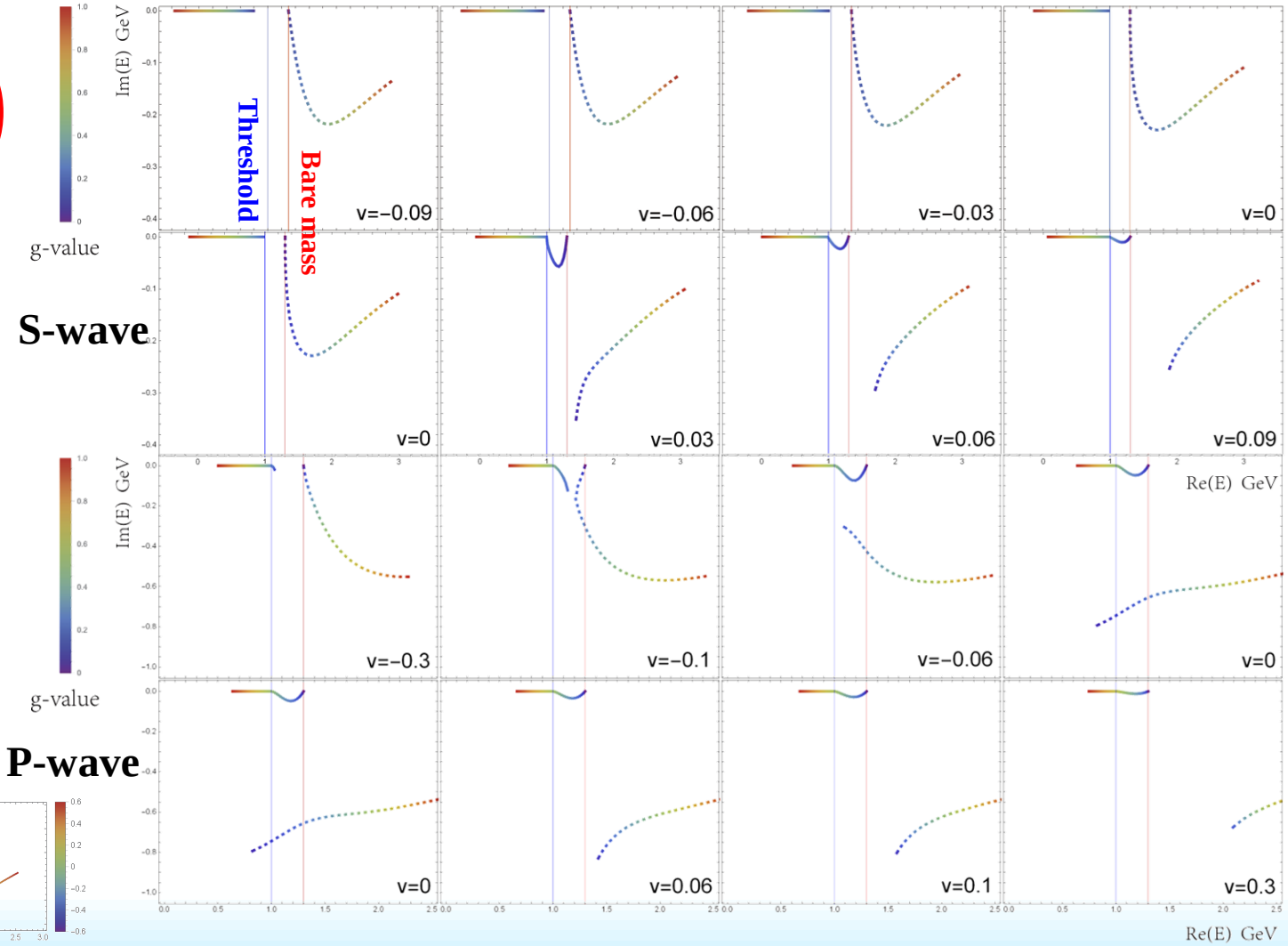
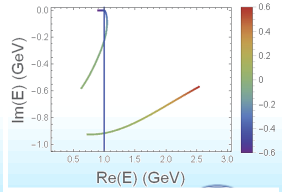
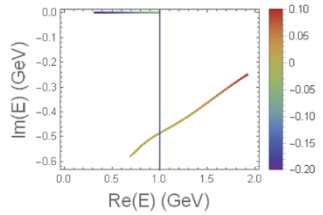
$$m_B > m_{\alpha 1} + m_{\alpha 2}$$







One bare state & One channel



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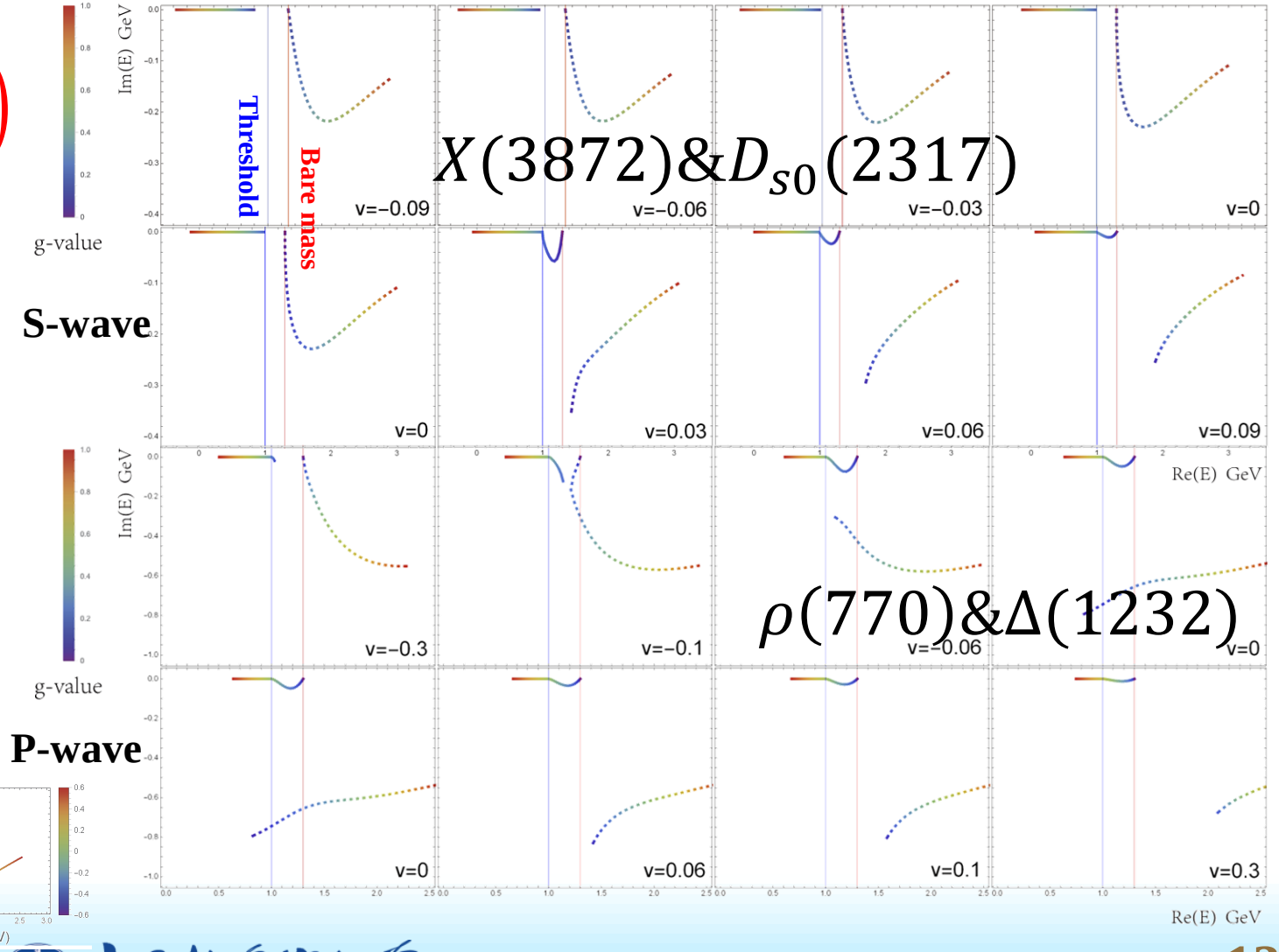
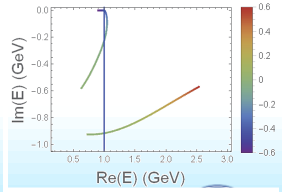
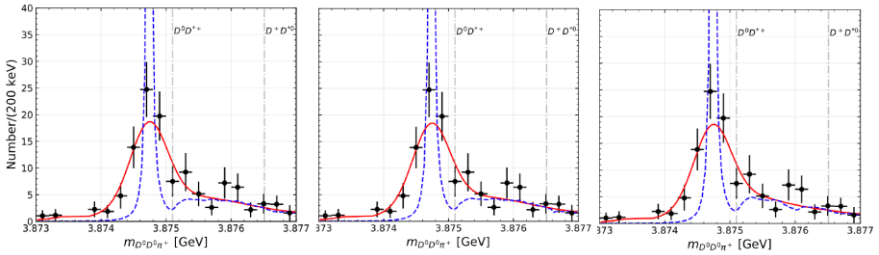




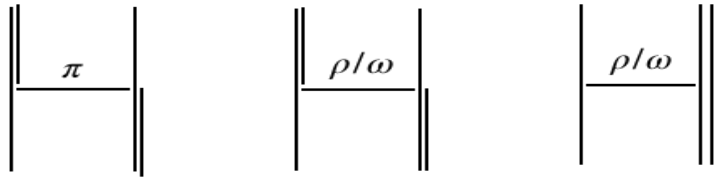
Λ (fixed)	λ（/GeV）	β
0.8 GeV	0.890 ± 0.20	0.810 ± 0.11
1 GeV	0.683 ± 0.025	0.687 ± 0.017
1.2 GeV	0.587±0.21	0.550±0.12
1.17 GeV	0. 56	0. 9

Example: $X(3872)$



T_{cc} → $DD^*, D\bar{D}^*$ interaction

The interaction between D and D^* OBE



Heavy Quark Symmetry

$c\bar{c}$ state ↓

X(3872)

χ_{c1}(2P, 3940) + $D\bar{D}^*$

G.-J. Wang, Z. Yang, J.-J. Wu, M. Oka, S.-L. Zhu Scib.2024.07.012

Λ (fixed)	λ (/GeV)	β
0.8 GeV	0.890 ± 0.20	0.810 ± 0.11
1 GeV	0.683 ± 0.025	0.687 ± 0.017
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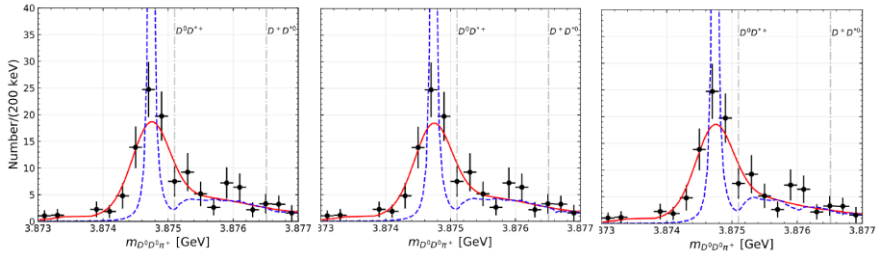
1.17 GeV	0. 56	0. 9

Example: $X(3872)$

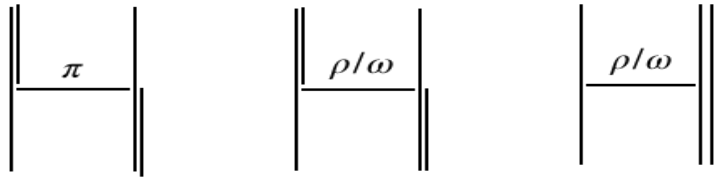


T_{cc} → $DD^*, D\bar{D}^*$ interaction → $c\bar{c}$ state → X(3872)

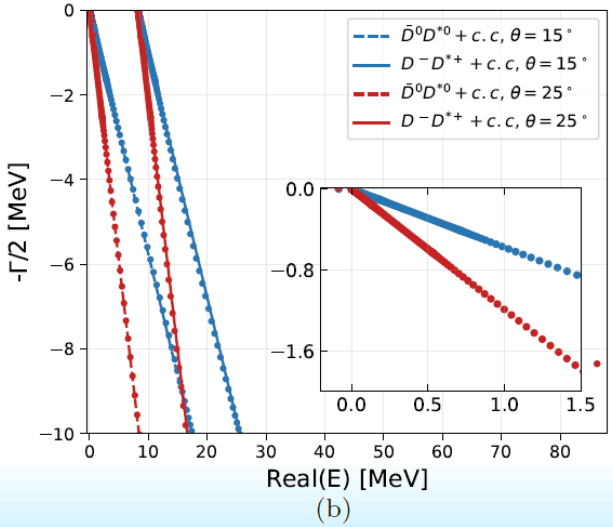
G.-J. Wang, Z. Yang, J.-J. Wu, M. Oka, S.-L. Zhu Scib.2024.07.012

χ_{c1}(2P, 3940) + $D\bar{D}^*$

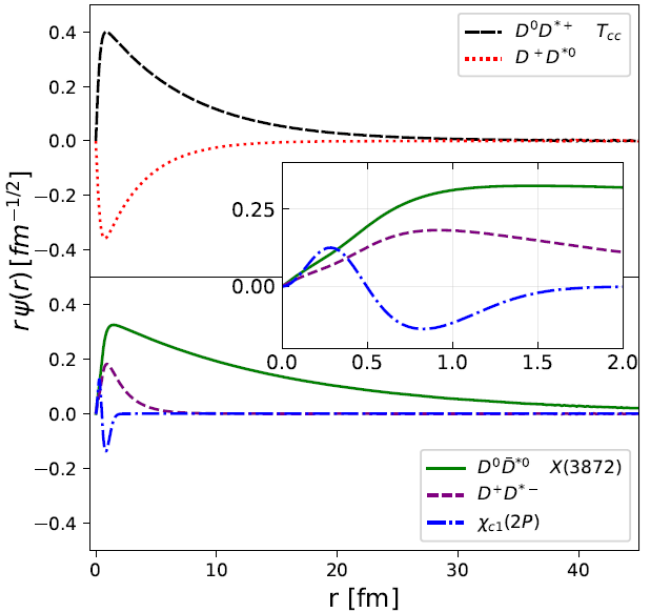
The interaction between D and D^* OBE



Heavy Quark Symmetry



(b)



- **T_{cc} bound state of $D^{*+}D^0$**

$\Delta E = -393$ keV $\Gamma_{T_{cc}} = 70.4$ keV

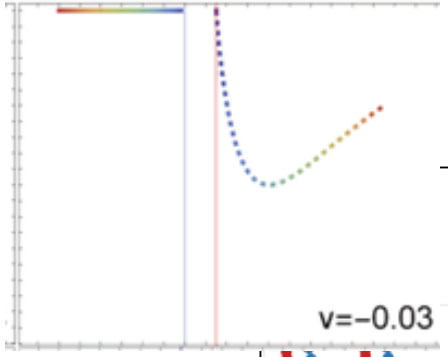
- $\sqrt{\langle r^2 \rangle} = 4.7\ fm$
- $70.0\% D^{*+}D^0$, $30.0\% D^+D^{*0}$

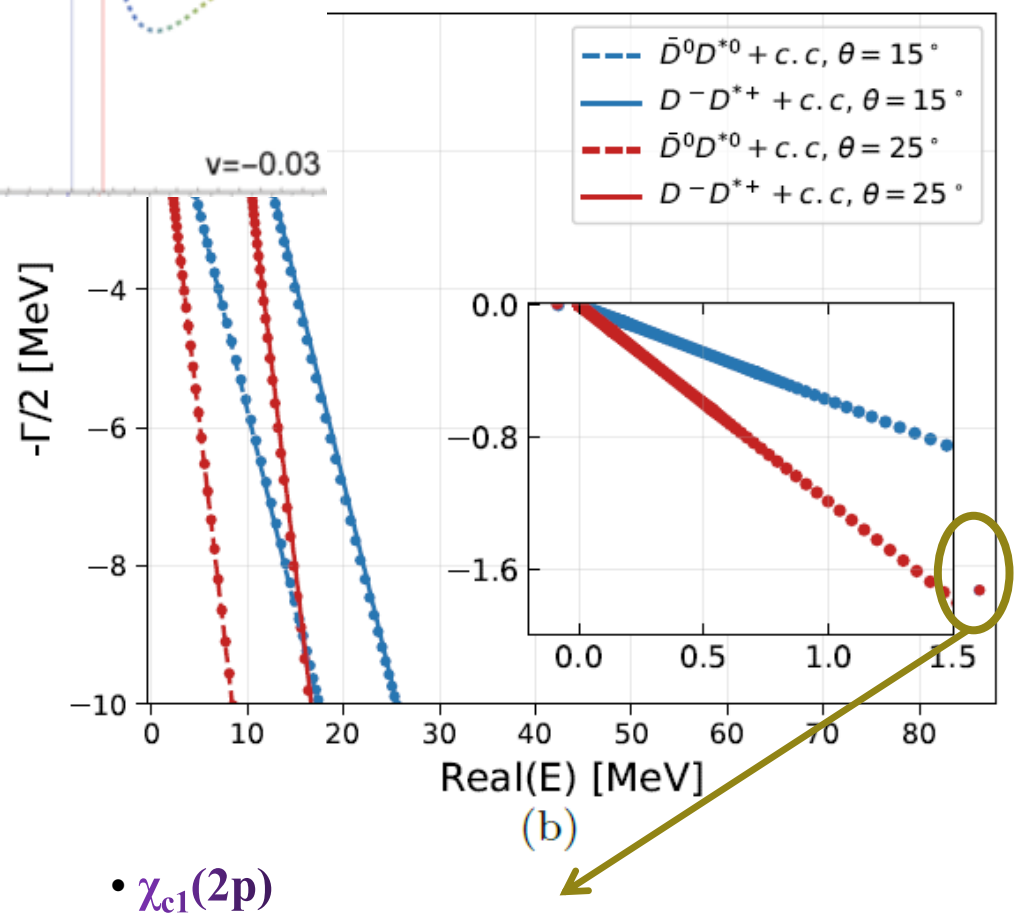
- **X(3872) bound state of $\bar{D}^{*0}D^0$**

$\Delta E = -80.4$ keV $\Gamma_{X(3872)} = 32.5$ keV

- $\sqrt{\langle r^2 \rangle} = 11.2\ fm$
- $94.0\% \bar{D}^{*0}D^0$, $4.8\% D^{*-}D^+$, $1.2\%\ c\bar{c}$

Prediction





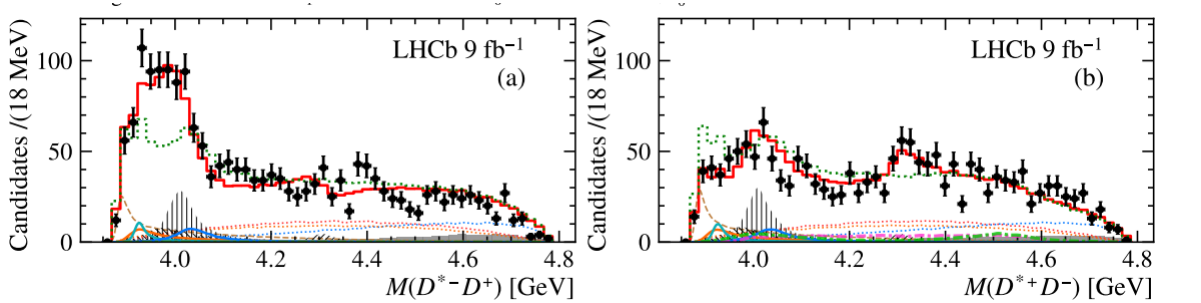
- $\chi_{c1}(2p)$
  M = 3957.9 MeV $\quad \Gamma_{\chi_{c1(2P)}} = 16.7$ MeV
- Main decay channel: $\bar{D}^*D$

Observation of new charmonium(-like) states in $B^+ \to D^{*\pm}D^{\mp}K^+$ decays

LHCb Collaboration 2406.03156

$\chi_{c1}(4010) \quad J^{PC} = 1^{++}$

$m_0 = 4012.5\,^{+3.6\,+4.1}_{-3.9\,-3.7}\ \Gamma_0 = 62.7\,^{+7.0\,+6.4}_{-6.4\,-6.6}$



X(3872) Relevant $D\bar{D}^*$ Scattering in $N_f = 2$ Lattice QCD

H. Li, C. Shi, Y. Chen, M. Gong, J. Liang et al

CLQCD 2402.14541

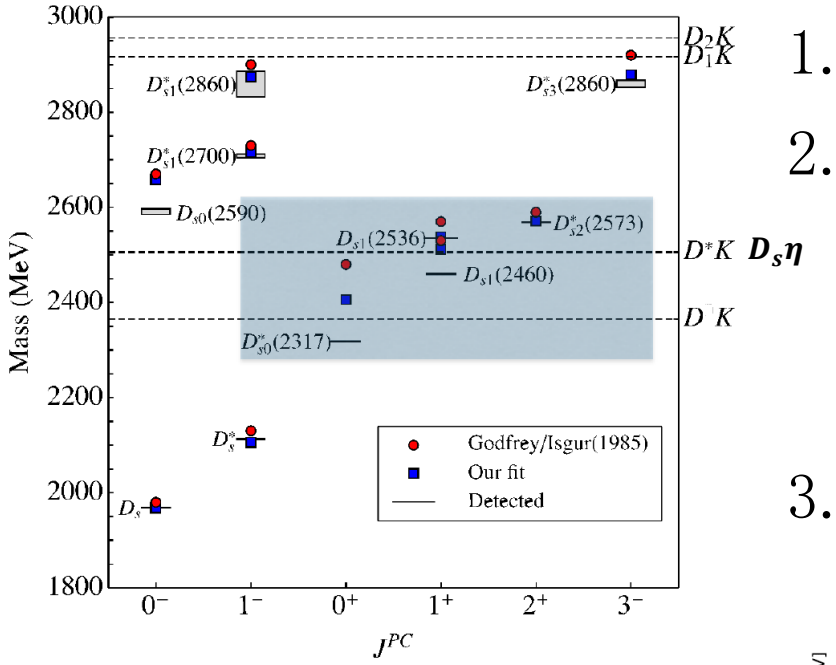
$$t(s) = \frac{K(s)}{1 - K(s)i\rho(s)}$$

$$K(s) = \frac{g}{M^2 - s} + \gamma$$

m_π(MeV)	250(3)	307(2)	362(1)	417(1)
Bound state from $E_{2,3}$				
E_B(MeV)	$-9.7^{+2.1}_{-2.2}$	$-9.7^{+1.9}_{-2.0}$	$-1.3^{+0.6}_{-0.8}$	$\mathbf{-1.3^{+0.8}_{-1.0}}$
BW fit from $E_{3,4}$				
m_R(MeV)	3924(5)	3926(6)	3969(4)	3995(4)
Γ_R(MeV)	63(23)	57(18)	37(13)	57(10)
Bound state pole and residual from $E_{2,3,4}$				
E_B(MeV)	−11(1)	−10(2)	−1.6(7)	−1.7(7)
Resonance pole and residue				
m_R(MeV)	4008(4)	4029(4)	4050(3)	4071(3)
Γ_R(MeV)	60(6)	38(9)	43(8)	50(7)
$\mathrm{Br}_{D\bar{D}^*}$(%)	∼ 100	∼ 100	∼ 100	∼ 100

$D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka PRL128(2022),112001



	$\bar{c}s$ cores		channel	
	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\|^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\|^1P_1\rangle - 0.74\|^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\|^1P_1\rangle - 0.68\|^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

1. Fix the bare mass and wave function from GI model;
2. The interaction of bare-channel and channel-channel;

$^3P_0$ model

at quark level

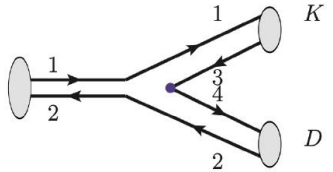
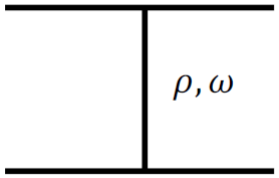
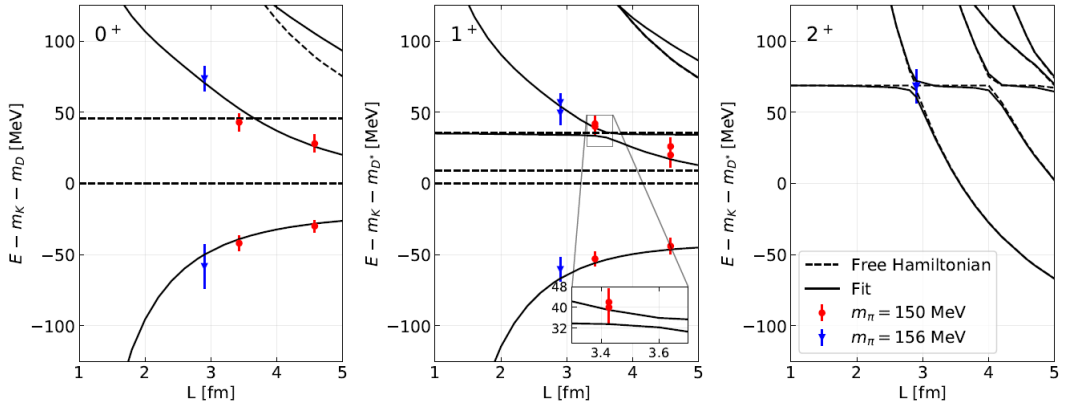


Figure 2: The diagram contribute to the process $D_s^*(2317) \to DK$.



	$P(c\bar{s})$
$D_{s0}^*(2317)$	$\approx 32.0\%$
$D_{s1}^*(2460)$	$\approx 52.4\%$
$D_{s1}^*(2536)$	$\approx 98.2\%$
$D_{s2}^*(2573)$	$\approx 95.9\%$

3. Fit Lattice data;



Conclusion:

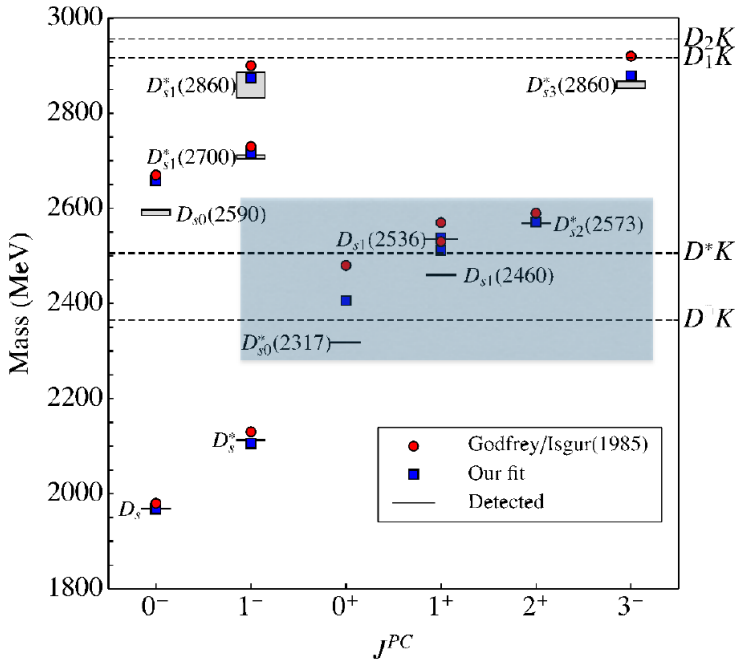
$D^*_{s0}(2317)$-DK
$D_{s1}(2460)$-D*K
S-wave

Mass moving vs GI Model

$D_{s1}(2536)$-D*K
$D_{s2}(2573)$-$D^{(*)}$K
D-wave

Mass stable vs GI model

Examples: $D_{s0}(2317)$



	$\bar{c}s$ cores		channel	
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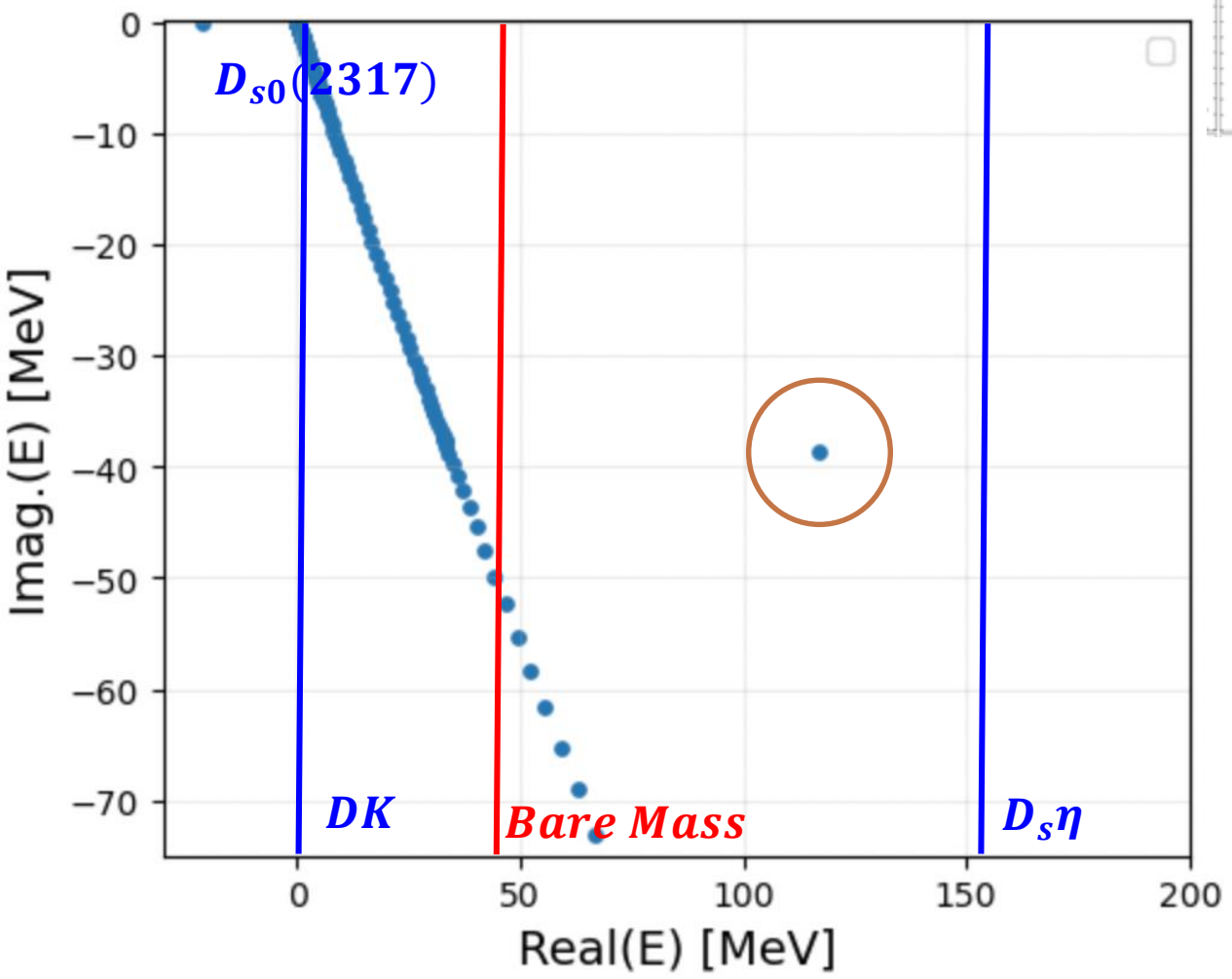
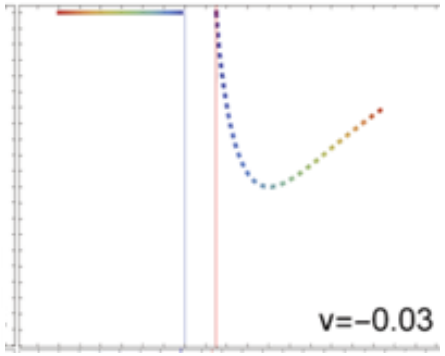
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

1

2

3





el;

–

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ion:

7)−DK

)−D*K S−wave

ving vs GI Model

)−D*K

)−D(*)K D−wave

able vs GI model

Summary and Outlook

- $m_B < m_{\alpha 1} + m_{\alpha 2}$, S-wave, strong attractive, two states
  S-wave, weak attractive&repulsive, one state
  P-wave, strong attractive, two states
  P-wave, weak attractive&repulsive, one state

  Below the threshold, the quark model prediction is quiet good.

- $m_B > m_{\alpha 1} + m_{\alpha 2}$, S-wave, strong&weak attractive, two states
  S-wave, repulsive, one state
  P-wave, strong attractive, two states
  P-wave, weak attractive&repulsive, one state

  Above the threshold, the quark model prediction is poor.

- Examples: $X(3872)$ & $D_{s0}(2317)$

Qualitative Analysis

Summary and Outlook

- $m_B < m_{\alpha 1} + m_{\alpha 2}$, S-wave, strong attractive, two states
  S-wave, weak attractive&repulsive, one state
  P-wave, strong attractive, two states
  P-wave, weak attractive&repulsive, one state

  Below the threshold, the quark model prediction is quiet good.

- $m_B > m_{\alpha 1} + m_{\alpha 2}$, S-wave, strong&weak attractive, two states
  S-wave, repulsive, one state
  P-wave, strong attractive, two states
  P-wave, weak attractive&repulsive, one state

  Above the threshold, the quark model prediction is poor.

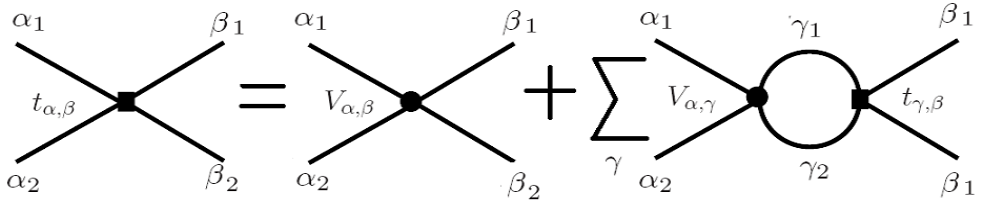
Qualitative Analysis

- Examples: $X(3872)$ & $D_{s0}(2317)$
- Outlook: How about two coupled-channels and two bare states?

Thanks for attention

~~One bare state~~ & One channel

$$T_{\alpha\beta}(k_\alpha,k_\beta,E)=V_{\alpha\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma^{\,2}dk_\gamma\,\frac{V_{\alpha\gamma}(k_\alpha,k_\gamma)T_{\gamma\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$



$$t(k,k',E)=vf(k)f(k')+\int q^2dq\,\frac{vf(k)f(q)}{E-\varpi(q)}t(q,k',E)$$

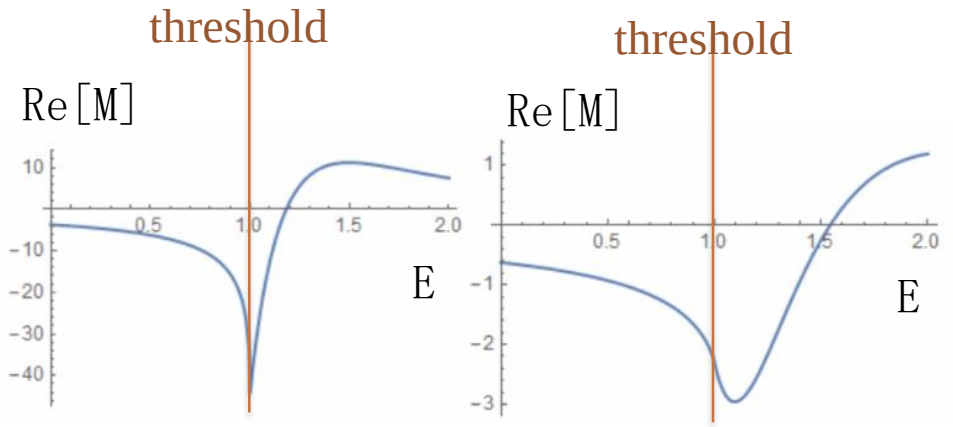
$$t(k,k',E)=f(k)\tilde{t}(E)f(k'),\qquad \tilde{t}(E)=(1-vM_{ff}(E))^{-1}v$$

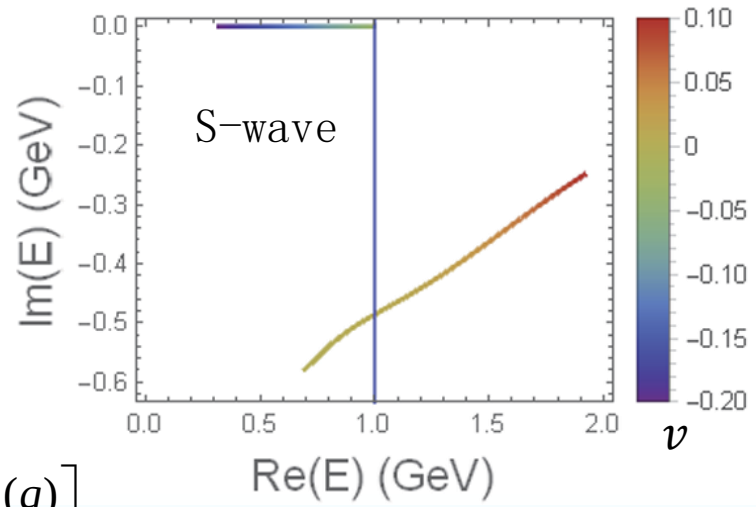
$$0=1-vM_{ff}$$

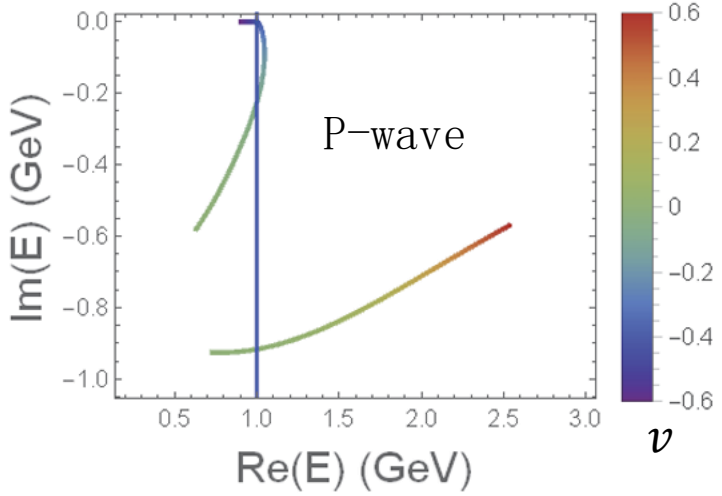
S-wave: $f(q)=5u(q)$

P-wave: $f(q)=qu(q)/m_M$

$$u(q)=\frac{1}{\varpi_M(q)}\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2$$



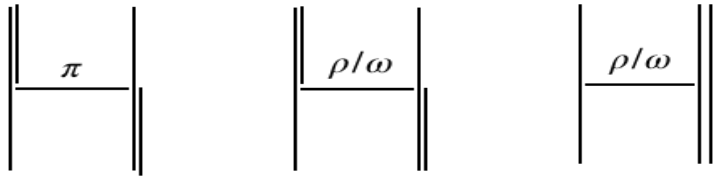




$$\begin{bmatrix}M_{ff}(E) & M_{fu}(E)\\ M_{uf}(E) & M_{uu}(E)\end{bmatrix}=\int\frac{q^2dq}{E-\varpi(q)}\begin{bmatrix}f^2(q) & f(q)u(q)\\ u(q)f(q) & u^2(q)\end{bmatrix}$$

Introduction of OBE and DD^* vs $D\bar{D}^*$

The interaction between D and D^* OBE



$$V_\pi = \frac{g^2}{f_\pi^2}\frac{(q\cdot\epsilon_\lambda)\left(q\cdot\epsilon_{\lambda'}^\dagger\right)}{q^2-m_\pi^2},$$

$$V^u_{\rho/\omega} = -2\lambda^2 g_V^2\frac{(\epsilon_{\lambda'}^\dagger\cdot q)(\epsilon_\lambda\cdot q)-q^2\left(\epsilon_\lambda\cdot\epsilon_{\lambda'}^\dagger\right)}{q^2-m^2_{\rho/\omega}},$$

$$V^t_{\rho/\omega} = \frac{\beta^2 g_V^2}{2}\frac{\left(\epsilon_\lambda\cdot\epsilon_{\lambda'}^\dagger\right)}{q^2-m^2_{\rho/\omega}}.$$

PWA Just S-wave

$$\mathcal{V}(l,l'S,j) = \frac{1}{(2\pi)^3}\sqrt{\frac{1}{2E_D^i 2E_D^f 2E_K^i 2E_K^f}}$$

$$2\pi\int dcos\theta V^v(\vec{p}_f,\vec{p}_i)\left(\frac{\Lambda^2}{\Lambda^2+p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2+p_i^2}\right)^2$$

- $g = 0.57$ from the decay width of $D^* \to D\pi$, while undetermined parameters $\lambda\&\beta$.

Heavy Quark Symmetry

$D^{(*)}D^{(*)}$

$$H_a^{(Q)} = \frac{1+\not{v}}{2}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right]$$

$$\bar{H}_a^{(Q)} \equiv \gamma_0 H^{(Q)\dagger}\gamma_0 = \left[P_a^{*\dagger\mu}\gamma_\mu + P_a^\dagger\gamma_5\right]\frac{1+\not{v}}{2}$$

$$P = (D^0, D^+, D_s^+)\ \&P^* = (D^{*0}, D^{*+}, D_s^{*+})$$

$$\mathcal{L}_{MH^{(Q)}H^{(Q)}} = ig\,\mathrm{Tr}\left[H_b^{(Q)}\gamma_\mu\gamma_5 A^\mu_{ba}\bar{H}_a^{(Q)}\right]$$

$$\mathcal{L}_{VH^{(Q)}H^{(Q)}} = i\beta\,\mathrm{Tr}\left[H_b^{(Q)}v_\mu\left(V^\mu_{ba}-\rho^\mu_{ba}\right)\bar{H}_a^{(Q)}\right]$$
$$+i\lambda\,\mathrm{Tr}\left[H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)_{ba}\bar{H}_a^{(Q)}\right]$$

$$H_a^{(\bar{Q})} \equiv C\left(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1}\right)^T C^{-1} = \left[P^{(\bar{Q})*}_{a\mu}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5\right]\frac{1-\not{v}}{2}$$

$$\bar{H}_a^{(\bar{Q})} \equiv \gamma_0 H_a^{(\bar{Q})\dagger}\gamma_0 = \frac{1-\not{v}}{2}\left[P^{(\bar{Q})*\dagger}_{a\mu}\gamma^\mu + P_a^{(\bar{Q})\dagger}\gamma_5\right]$$

$$\tilde{P} = (\bar{D}^0, D^-, D_s^-)\ \&\ \tilde{P}^* = (\bar{D}^{*0}, D^{*-}, D_s^{*-})$$

$$\mathcal{L}_{MH^{(\bar{Q})}H^{(\bar{Q})}} = ig\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\gamma_\mu\gamma_5 A^\mu_{ab}H_b^{(\bar{Q})}\right]$$

$$\mathcal{L}_{VH^{(\bar{Q})}H^{(\bar{Q})}} = -i\beta\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}v_\mu\left(V^\mu_{ab}-\rho^\mu_{ab}\right)H_b^{(\bar{Q})}\right]$$

$D^{(*)}\overline{D^{(*)}}$

$$+i\lambda\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\sigma_{\mu\nu}F'^{\mu\nu}_{ab}(\rho)H_b^{(\bar{Q})}\right]$$

	wave function	$I(J^{PC})$	$u-$channel : π	$u-$channel : ρ/ω	$t-$channel : ρ/ω
DD^*	$\frac{1}{\sqrt{2}}(D^+D^{*0}-D^0D^{*+})$	$0(1^+)$ $[T_{cc}^+]$	$\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u-\frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t+\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}(D^+D^{*0}+D^0D^{*+})$	$1(1^+)$	$\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u+\frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t+\frac{1}{2}V_\omega^t$
$D\bar{D}^*$	$\frac{1}{\sqrt{2}}\left([D^+D^{*-}]+[D^0\bar{D}^{*0}]\right)$	$0(1^{++})[X(3872)]$	$\frac{3}{2}V_\pi$	$-\frac{3}{2}V_\rho^u-\frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t-\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}\left([D^+D^{*-}]-[D^0\bar{D}^{*0}]\right)$	$1(1^{++})$	$-\frac{1}{2}V_\pi$	$\frac{1}{2}V_\rho^u-\frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t-\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}\left(\{D^+D^{*-}\}+\{D^0\bar{D}^{*0}\}\right)$	$0(1^{+-})[h_c]$	$-\frac{3}{2}V_\pi$	$\frac{3}{2}V_\rho^u+\frac{1}{2}V_\omega^u$	$-\frac{3}{2}V_\rho^t-\frac{1}{2}V_\omega^t$
	$\frac{1}{\sqrt{2}}\left(\{D^+D^{*-}\}-\{D^0\bar{D}^{*0}\}\right)$	$1(1^{+-})$ $[Z_c(3900)]$	$\frac{1}{2}V_\pi$	$-\frac{1}{2}V_\rho^u+\frac{1}{2}V_\omega^u$	$\frac{1}{2}V_\rho^t-\frac{1}{2}V_\omega^t$

The mass differences leads to isospin breaking, our calculations are based on the physical states of the particles.