Cusp spectroscopy:

Experimental studies of threshold cusps & hadron-hadron scattering length

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@BARYONS2025

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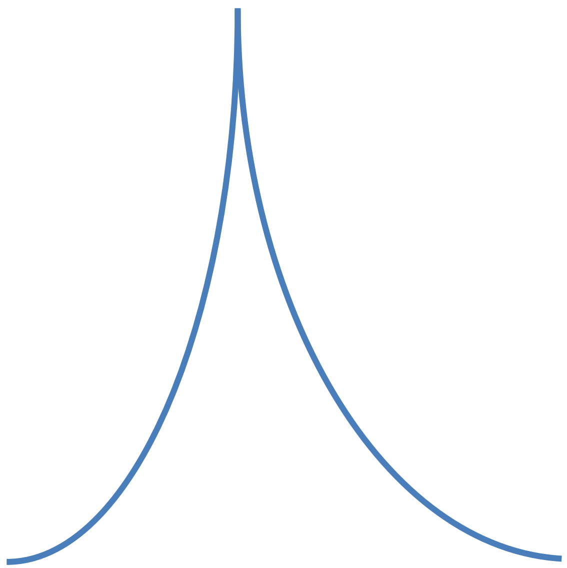
Threshold cusp

- Jump in strength ($|amp|^2$) in the (L+1)th derivative
  - In the widest sense, cusp ALWAYS appears at thresholds.
  - Practically, cusp appears only in S-wave
- Interesting case is the 1st derivative changes sign, especially from positive to negative
  - Cusp in the narrow sense.
  - In principle, can be distinguished from usual peak by the derivative at the peak, but practically there is experimental resolution.
  - Very few identified cases



Cusp & scattering length

- Cusp occurs at a threshold
  - The statistics is highest at the threshold
  - Low energy seen from the threshold
    → Behavior is (roughly) determined by the complex scattering length of the threshold channel
- A simple calculation:

$f_0 \sim \frac{1}{\frac{1}{kA} - i}$, A→complex (a+ib)

$\sigma_0 = \frac{4\pi}{k^2} |f_0|^2$

- Above the threshold:

$$\sigma_0 \propto \frac{1}{(1+kb)^2+(ka)^2} \sim 1-2kb$$

- Below the threshold:

$$\sigma_0 \propto \frac{1}{(1+|k|a)^2+(|k|b)^2} \sim 1-2|k|a$$

with $k=i\sqrt{2\mu|E|}$ is pure imaginary.

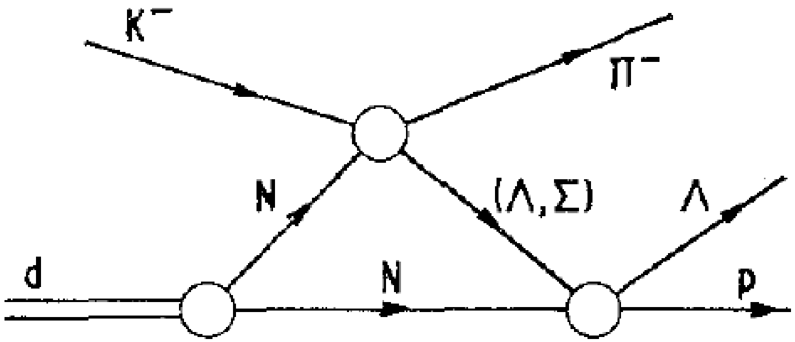
- a > 0 (attractive interaction) → cusp
  - Pole: $k \sim i/A$ is virtual
  - E < 0, but in different Riemann sheet
  - Interaction is attractive, but not strong enough to make a bound state

Other factors

- Two more issues to consider
  - Production (decay) mechanism
  - Simple ERE with complex scattering length is actually NOT sufficient.
- Production mechanism
  - An example: Dalitz theory for $K^{-}d \rightarrow \Lambda p \pi^{-}$
    [Czech. J. Phys. B 32 (1982)]

Dalitz model

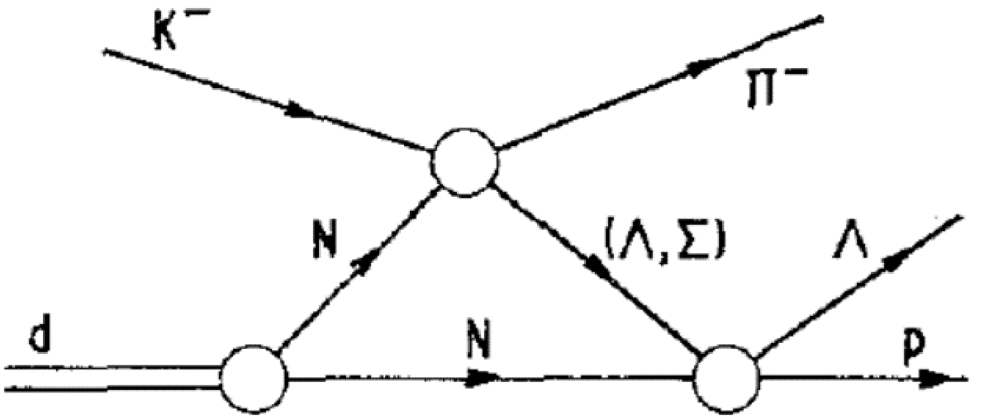
Fig. 3. The triangle graphs for the process $K^- d \to \Lambda p \pi^-$.



- 1st step: Σ production by N(K^-,π^-)Σ reaction
- 2nd step: rescattering with remaining N to form ΛN final state
- The simple ERE model treats only the 2nd step, while the 1st step can also produce cusp
  - Cusp from form factor

Coupled-channels EFT

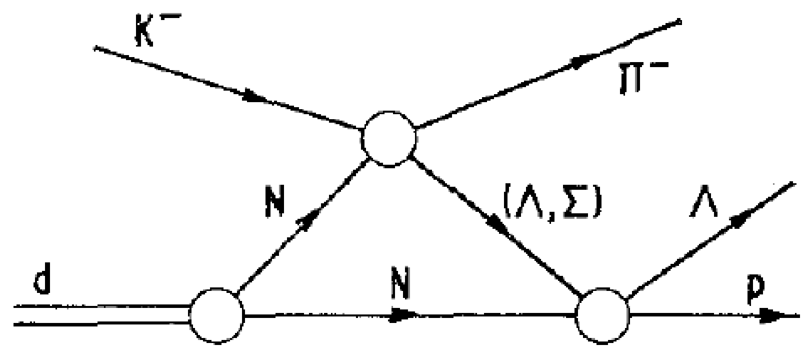
- Treating multi-channels explicitly in an effective field theory approach by Dong-Guo-Zou [PRL**126.**152001(2021)] and Sone-Hyodo
- Spectrum shape is different in the same final state (channel 1, Λp), depending on the intermediate state (ΛN or ΣN)
  - Dip structure may appear due to interference



Then, what shall we do?

- Use reactions with known mechanism! → J-PARC
- Best: elastic scattering
  → J-PARC E72 for Λη threshold
  & E45 for Nη, ΛK, etc.
- 2-step reactions may be also good.

Fig. 3. The triangle graphs for the process
$K^{-}d \rightarrow \Lambda p \pi^{-}$.



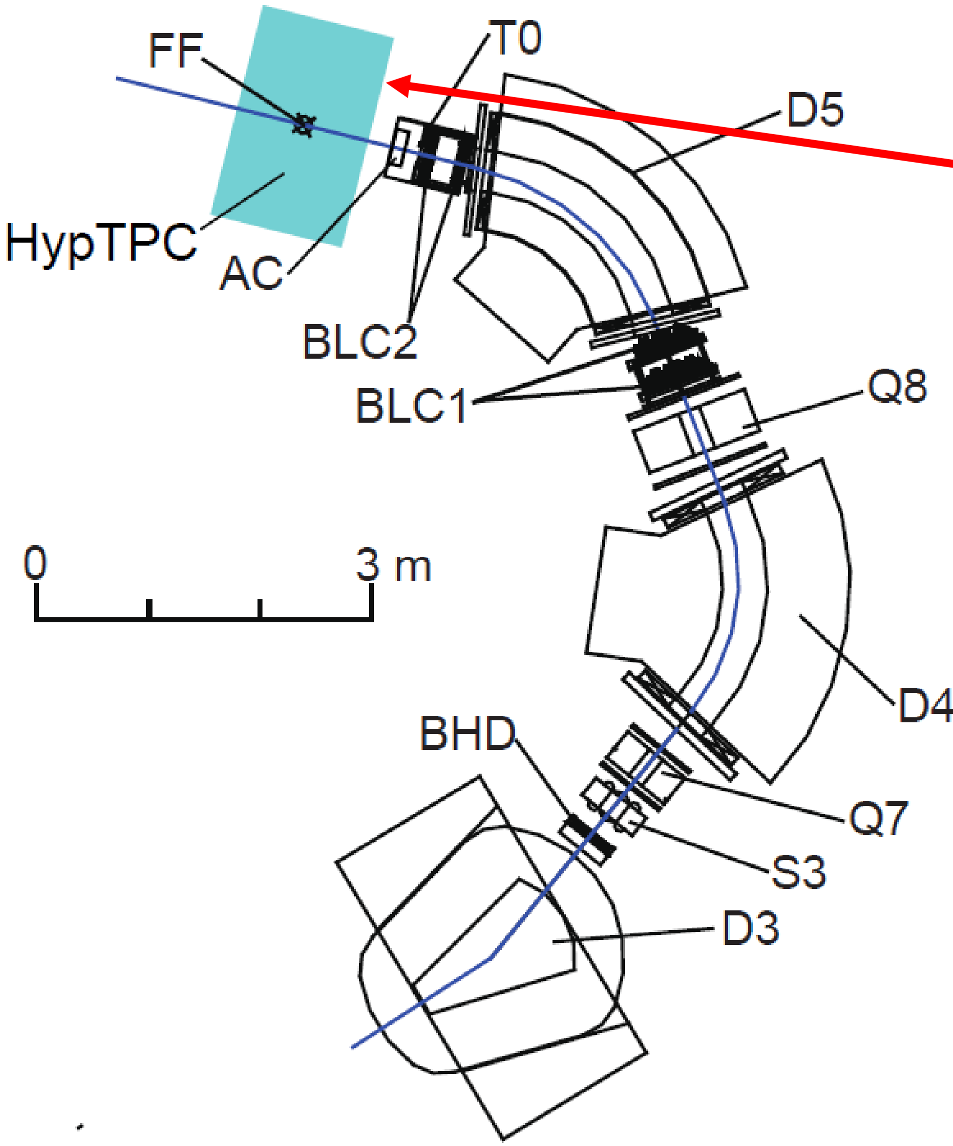
→ J-PARC E90 for ΣN threshold

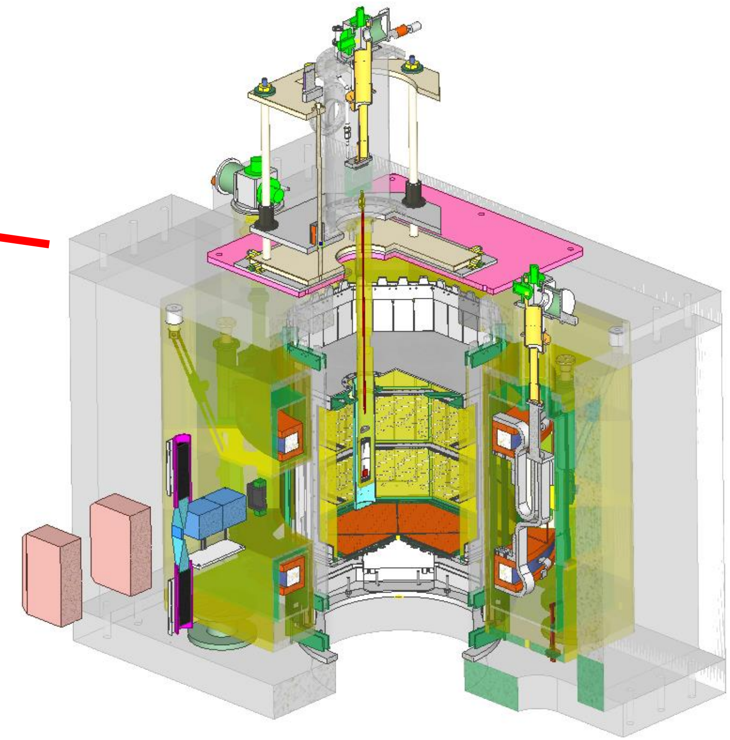
1. J-PARC E72 & E45

J-PARC E72 overview

- Kp → $\Lambda\eta$ scattering with a large acceptance detector, i.e., TPC (HypTPC) near the $\Lambda\eta$ threshold
  - – Search for a new resonance, $\Lambda(1665)$
  - – Determine parity by Λ polarization measurement
- Principle
  - – K beam momentum: 720-770 MeV/c
  - – Momentum resolution: 1 MeV/c or better
    → Can identify narrow resonance of Γ=1.5 MeV
  - – Detect Λ → $p\pi^-$, identify η by missing mass
- Physics run is starting NOW!

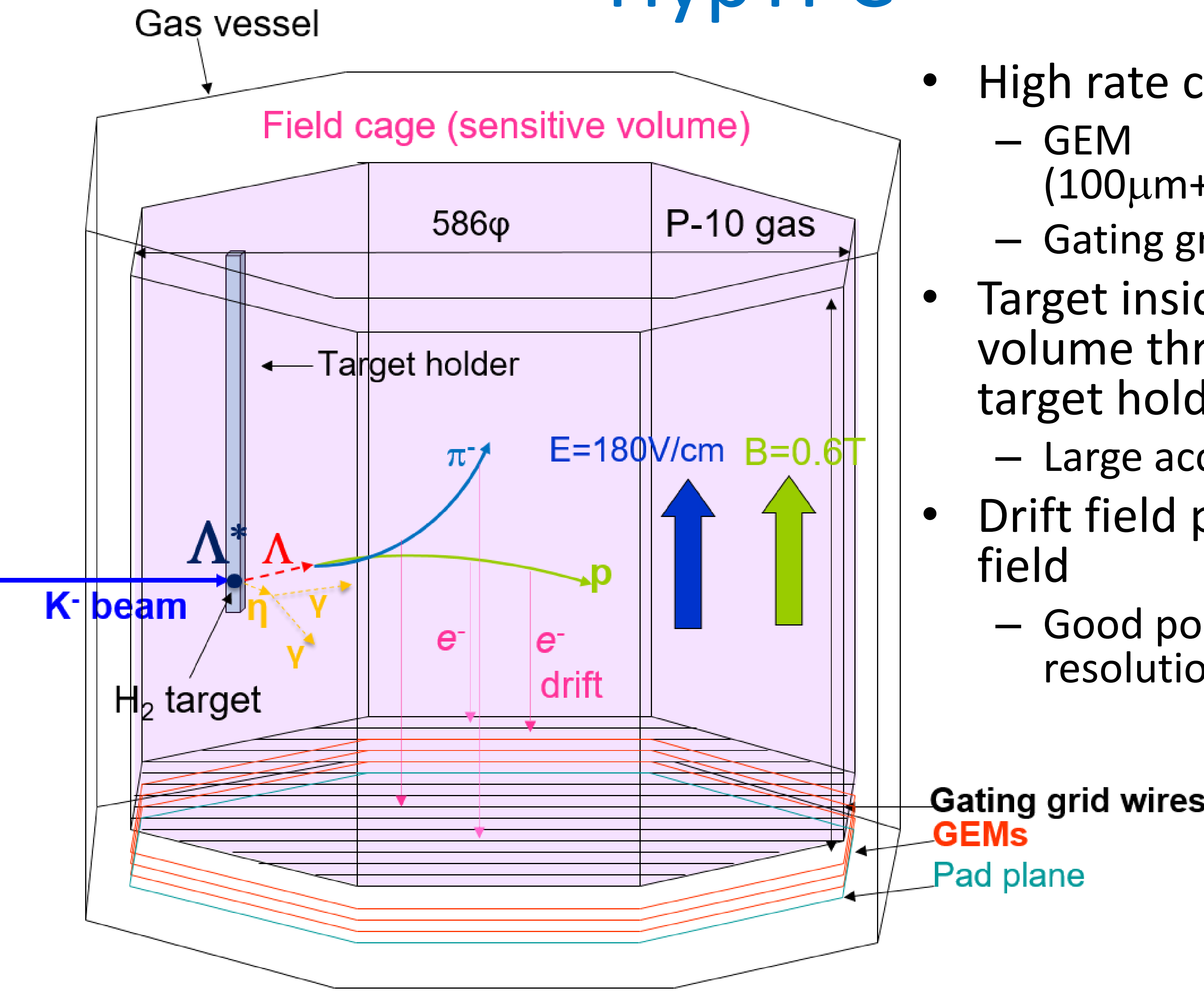
Setup Overview





- Kaon beam provided by K1.8BR
- Hyperon spectrometer to detect Λ

HypTPC



- High rate capability
  - GEM (100μm+50μm+50μm)
  - Gating grid
- Target inside the drift volume through the target holder
  - Large acceptance
- Drift field parallel to B-field
  - Good position resolution

Some pictures

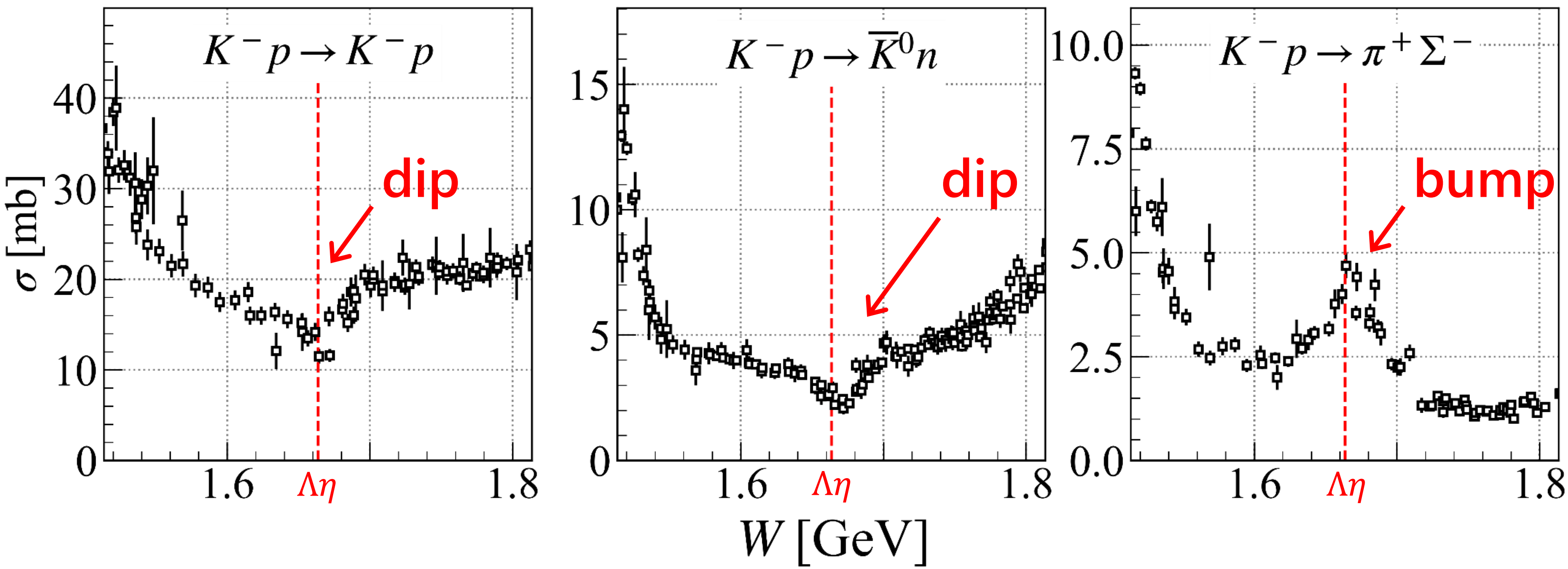
- Physics run is starting now.

Byproduct – threshold behavior

- We can take data not only on $K^-p \rightarrow \Lambda\eta$, but most other reaction channels such as $K^-p \rightarrow K^-p$ (elastic), K^0n, $\Sigma\pi$, $\Lambda\pi^0$, etc.
  - Study threshold behavior
  - The same excellent mass resolution (0.5 MeV)
- Determine pole position for $\Lambda(1670)$ and $\Lambda\eta$ scattering length
- Dip may appear in $K^-p \rightarrow K^-p$ and other channels.

Peak vs Dip

[A. Baldini *et al.*, Landolt-Börnstein, Vol. 12, Springer-Verlag]



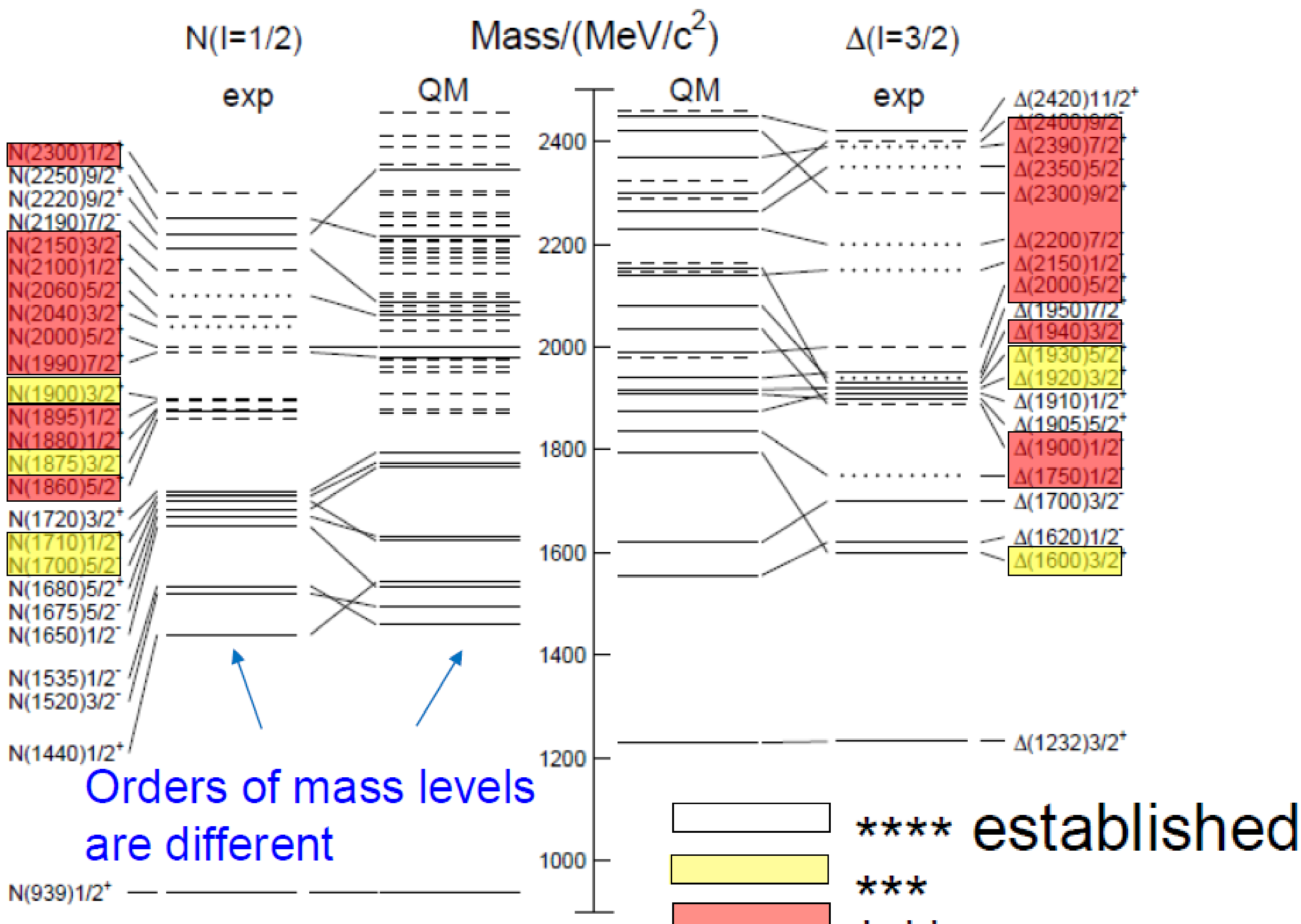
- E72 will give near-threshold data with higher statistics and resolution
  - Dip/peak position may be different from threshold
  - Partial wave analysis to separate S-wave contribution

→ Extraction of scattering length

2. J-PARC E45

Missing resonances

- A lot of states are predicted by QM, but not observed
- Measured by using mainly $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$ reactions



E45 setup

Measure (π,2π) in large acceptance TPC in dipole magnetic field

$\pi^- p \rightarrow \pi^+ \pi^- n,\ \pi^0 \pi^- p$ *2 charged particles* + *1 neutral particle*

$\pi^+ p \rightarrow \pi^0 \pi^+ p,\ \pi^+ \pi^+ n$ →*missing mass technique*

$\pi N \rightarrow KY$ (2-body reaction)

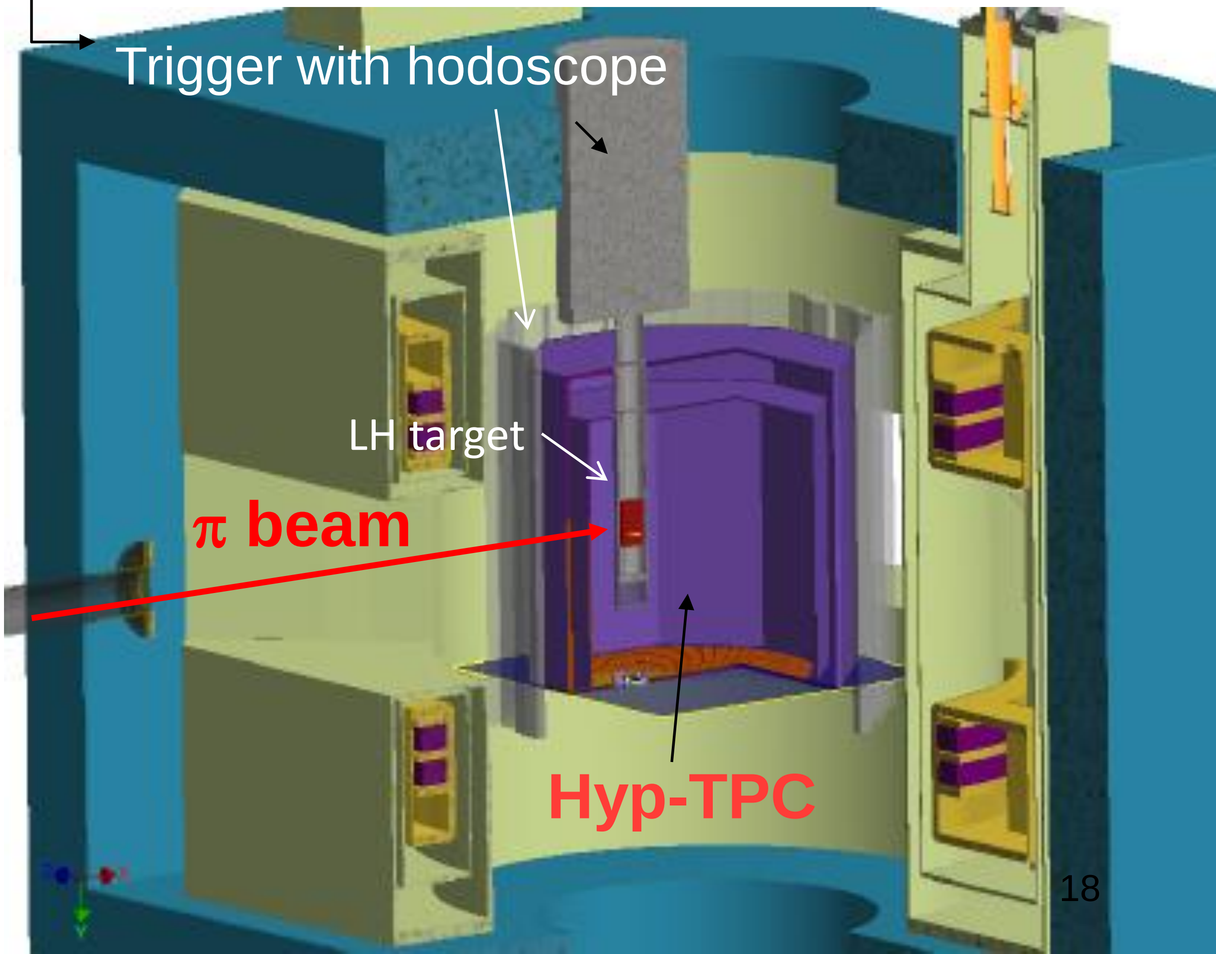
$\pi^- p \rightarrow K^0 \Lambda,$

$\pi^+ p \rightarrow K^+ \Sigma^+$ *(I=3/2, Δ*)*

π^{+-} beam on liquid-H target (p= 0.73 – 2.0 GeV/c W=1.5-2.15 GeV)

x100 more statistics than ever

Run expected in 2025-2027

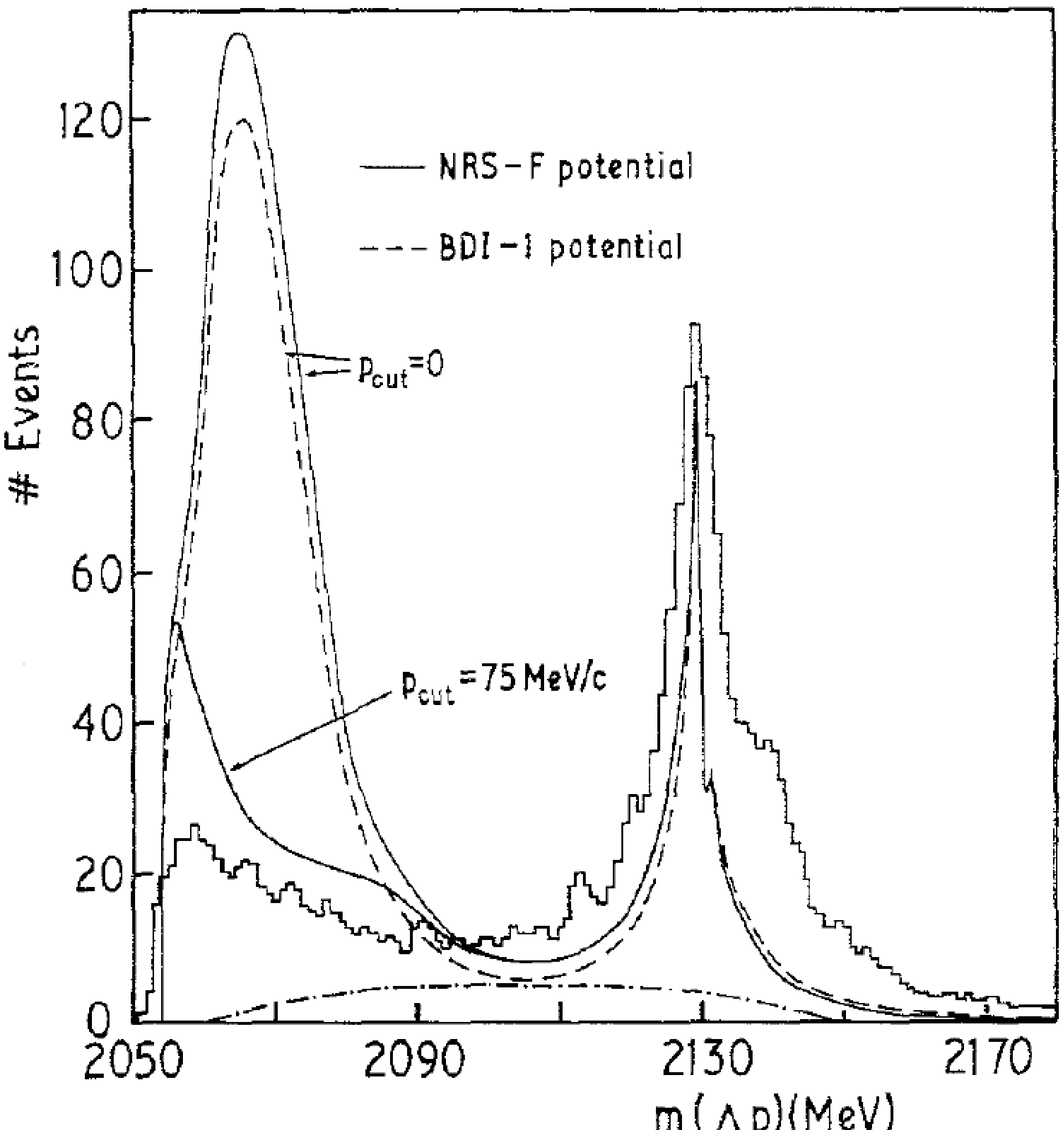


Possibilities in J-PARC E45

- There are several thresholds in πN scatterings:
  - ηN, ΛK, ΣK, η'N, ϕN
- Cusp could be seen in (π, π), (π, 2π), or others if the interaction is attractive
  - Bound state searches are often discussed, but there should be signal even for unbound cases.
- Cusps may be small
  → Need high statistics & excellent resolution
  - Possible at E45 – resolution better than 1 MeV
- Initial low mass (up to W～1.76 GeV) run
  - After E72: Nov. 2025 and Apr.-May 2026

3. J-PARC E90

ΣN cusp



- Seen in K^-(stopped)+d → $\Lambda p \pi^-$ and many others
- Maybe the cleanest cusp ever seen, but not confirmed.
  - Because the resolution is not enough

What should we do?

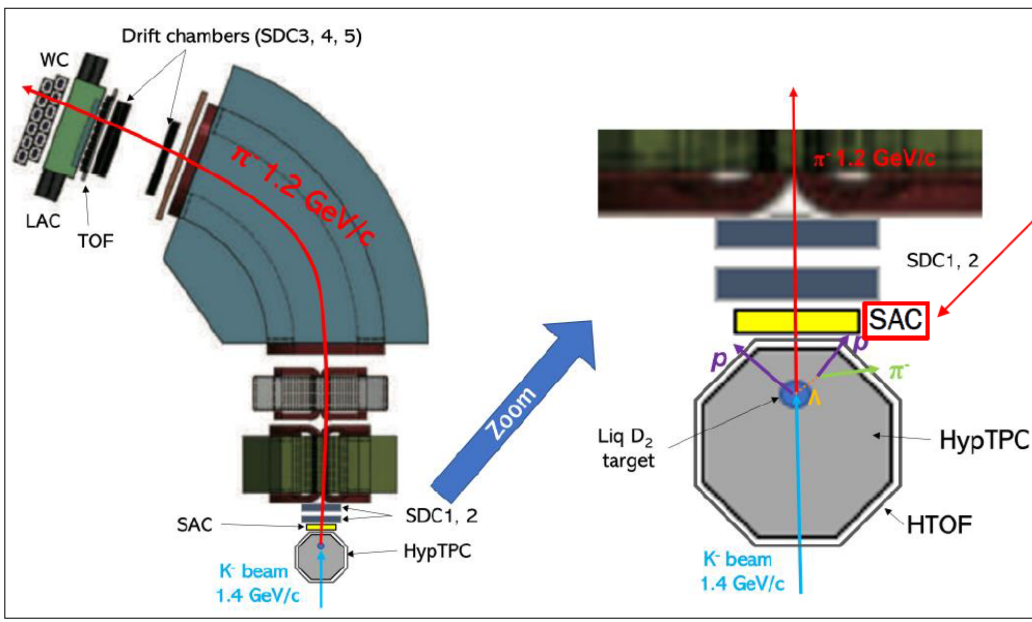
- Try even higher resolution
  - 0.4 MeV (σ) would be enough to see the cusp shape
- Tagging of the final state is necessary
  - Must be ΛN to derive ΣN (I=1/2) scattering length
  - ΣN (I=3/2) contaminate if not tagged
- J-PARC E90
  - 0.4 MeV resolution with d(K^-,p) reaction at $p_K \sim 1.4$ GeV/c thanks to the high resolution of S-2S spectrometer.
  - Tagging of decay particles by the Hyperon Spectrometer → 4π acceptance

E90 setup

SET UP

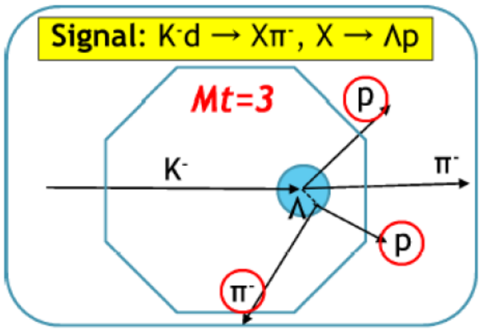
- **Reaction: $K^- d \to \Lambda p \pi^-$ at 1.4 GeV/c**
- **S-2S(developed for E70): π^- measurements → measurement of missing mass spectrum**
  - **Good mass resolution: ΔM ~ 0.4 MeV (σ), (Δp/p(K18)=3.3×10^{-4}(FWHM), Δp/p(S-2S)=6.0×10^{-4}(FWHM))**
- **HypTPC(developed for E42): Final state (Λp) restriction and background suppression**

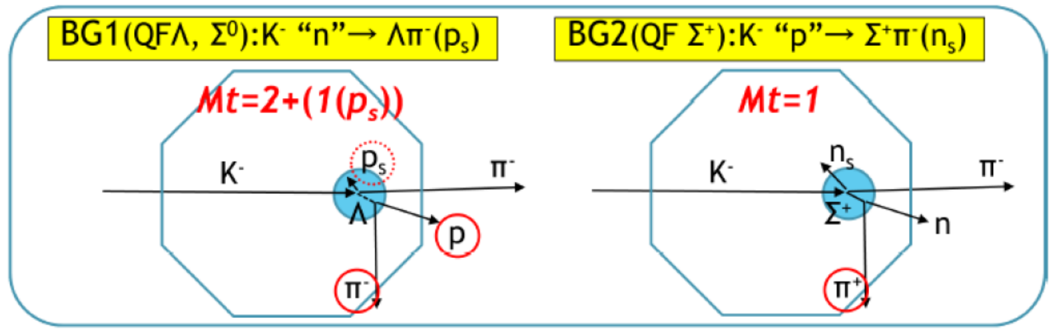
Momentum transfer
～200 MeV/c



New detector

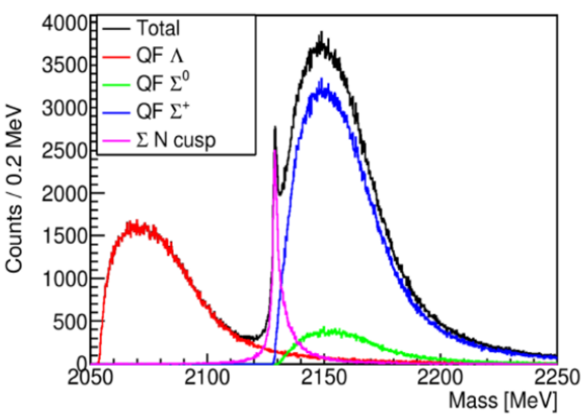
QF BACKGROUND SUPPRESSION BY HYPTPC



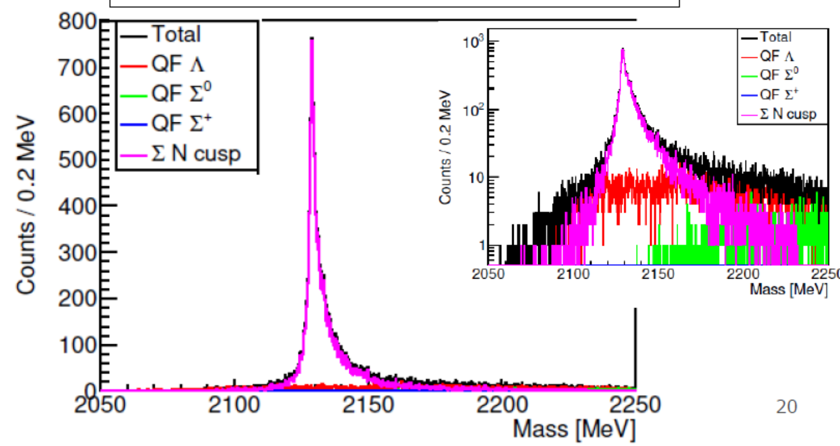


Simulated inclusive spectrum d(K^-, π^-)

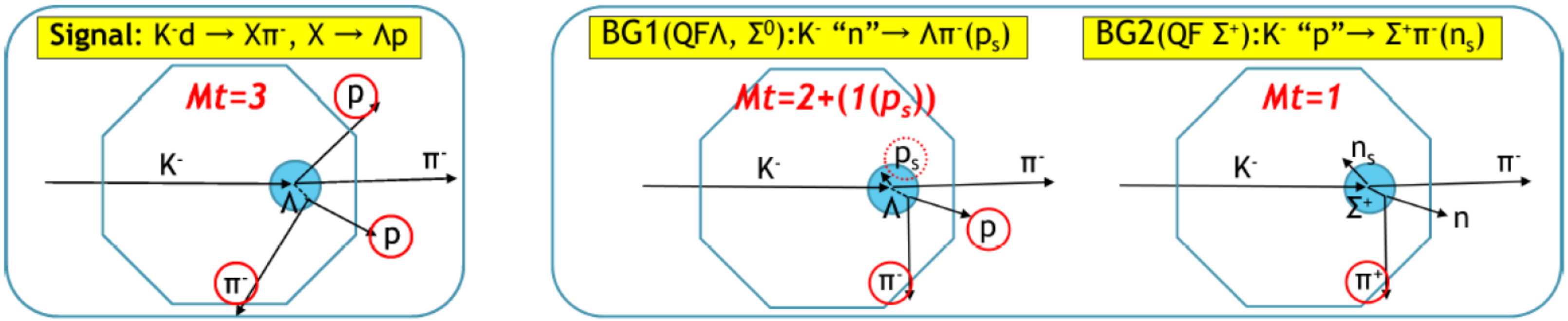
Expected spectrum for the 15 days beam time



Multiplicity = 3 without (K^-, π^-)

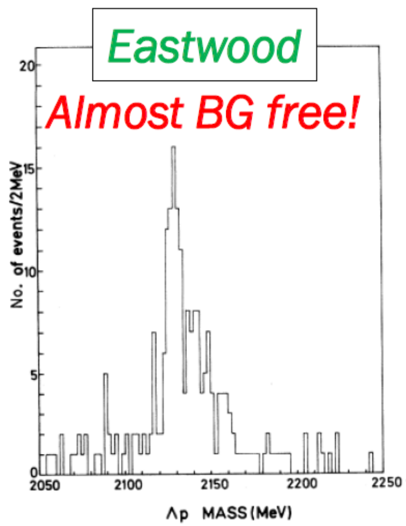


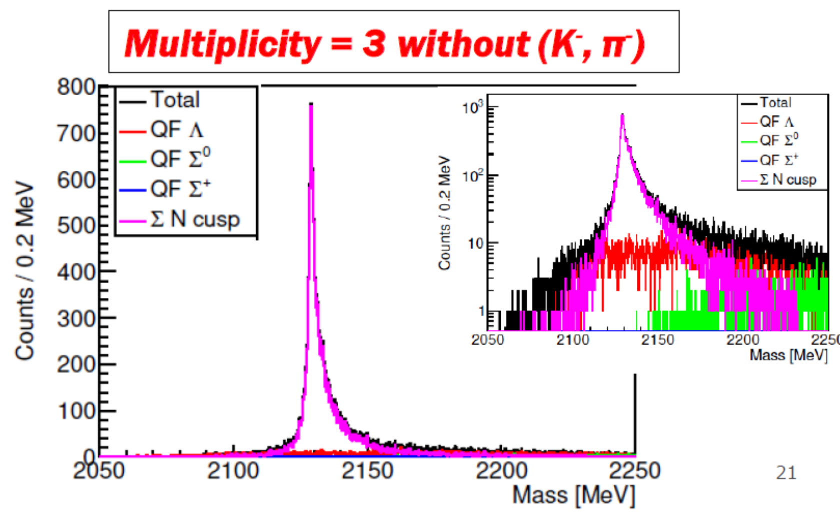
QF BACKGROUND SUPPRESSION BY HYPTPC



$K^- d \to \Lambda p \pi^-$
@1.45 and 1.65 GeV/c
(Bubble chamber)

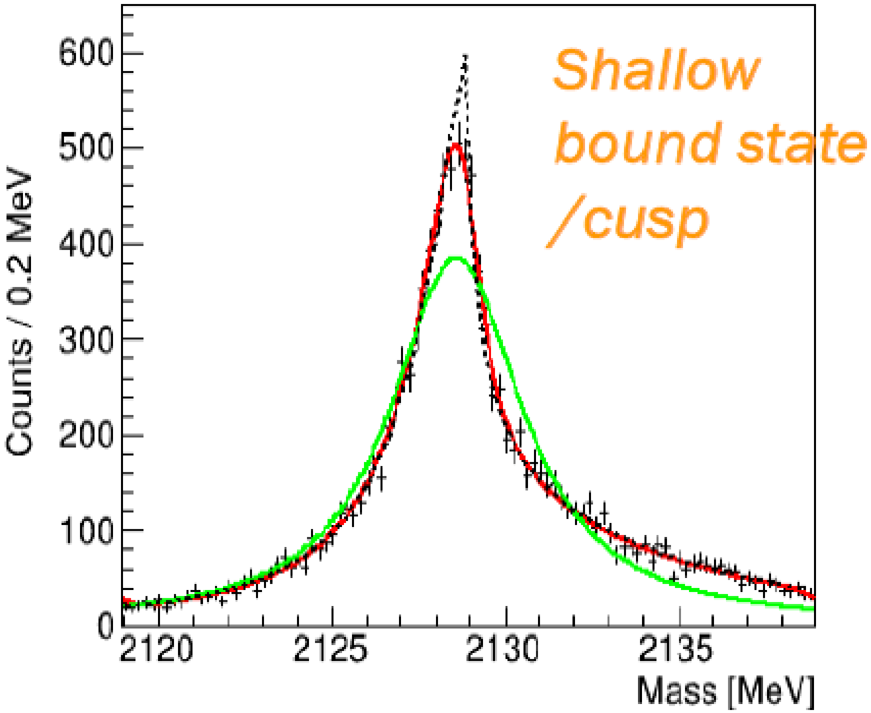
$\cos\theta_{CM} > 0.9$
$p_{proton} > 150$ MeV/c



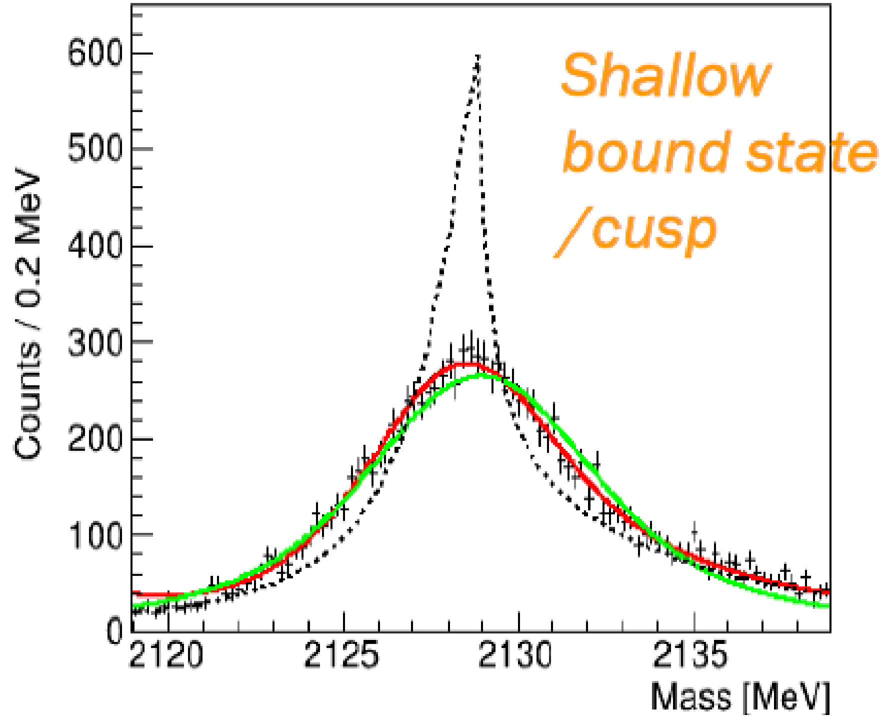


Importance of resolution

σ＝0.4 MeV



σ＝2 MeV



Identification possible with σ＝0.4 MeV, but not with 2 MeV

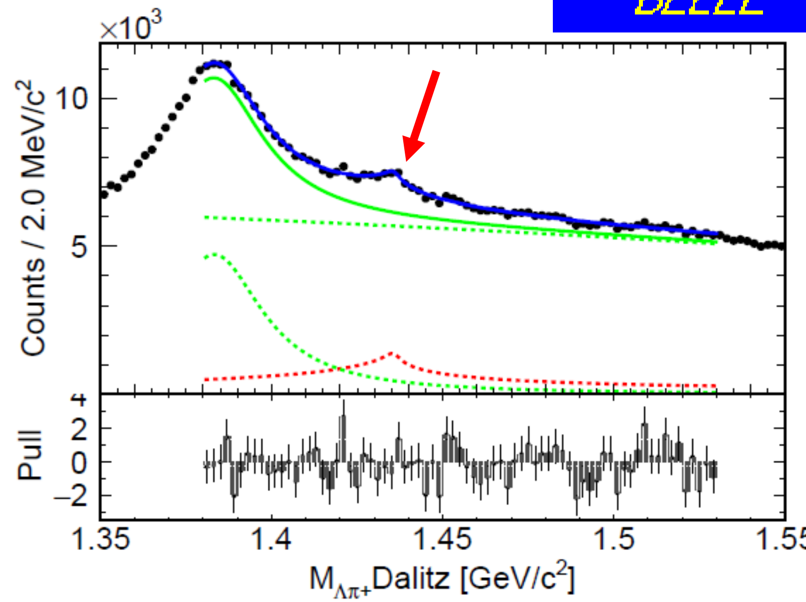
4. Possibility for $\bar{K}N(I=1)$ interaction via cusp spectroscopy

$\bar{K}N(I=1)$ scattering length

- Important, related to
  - Kaonic nuclei
  - Kaon condensation in neutron stars
- Dedicated experiments with a measurement of kaonic deuterium atom X rays.
  - J-PARC E57
  - Siddharta-2 at DAΦNE
- A new, independent measurement possible using threshold cusp.

$\Lambda\pi - \bar{K}N(I=1)$ cusp at J-PARC?

- We already saw a hint in Λ_c decay@Belle
  - very poor S/N
  - Also unknown production mechanism



[PRL130(2023)151903]

- Direct reaction is preferred
  - Two possibilities with $\Lambda\pi^\pm$ in the final state
    1. $p(K^-,\pi^\pm)\Lambda\pi^\mp$
    2. $d(K^-,p)\Lambda\pi^-$
  - reaction 2: small momentum transfer & controlled reaction mechanism – the same mechanism as J-PARC E31 [PLB837(2023)137637]

Summary

- Threshold cusp
  - A peak-like structure at a threshold
  - Shape is (mostly) determined by scattering length
- Plans at J-PARC
  - Cusp at $\Lambda\eta$ threshold with E72
  - E45 could observe cusps in $N\eta$ and other thresholds
  - E90: ΛN-ΣN cusp study for ΣN interaction. Bound state?
- **Many other possibilities!!**
  - Cusp should appear everywhere.
    Any near-threshold peaks could be threshold cusps

Backup

- Around the threshold of channel 2 (ΣN), $F_{ij}=1/a_{ij}$, $k_1=k_0$ (constants)

$$f = \frac{1}{\frac{1}{a_{12}^2} - \left(\frac{1}{a_{11}} - ik_0\right)\left(\frac{1}{a_{22}} - ik_2(E)\right)} \begin{pmatrix} \frac{-1}{a_{22}} + ik_2(E) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{-1}{a_{11}} + ik_0 \end{pmatrix}.$$

- f_{22} ($\Sigma N \rightarrow \Sigma N$) is consistent with simple ERE by taking

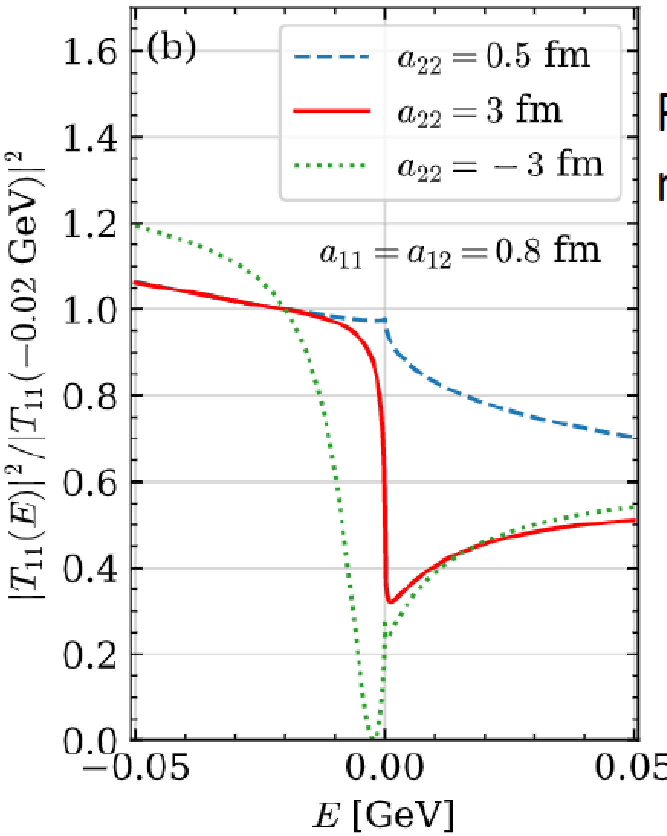
$$A = \frac{(1 - ia_{11}k_0)a_{12}^2 a_{22}}{(1 - ia_{11}k_0)a_{12}^2 - a_{11}a_{22}},$$

- $f_{12}=f_{21}$ ($\Sigma N \rightarrow \Lambda N$) is also (essentially) the same
  - With a difference by a constant factor

- The difference is in f_{11}

$$f_{11} = \frac{1}{\frac{1}{k_2 A} - i\,\frac{\frac{1}{a_{22}} - ik_2}{\frac{1}{a_{11}} - ik_0}}.$$

- Due to the 2nd term in numerator, dip may appear
  - However, the dip position is NOT exactly on the threshold.



Poles in complex momentum plane:

$(-0.08 - i0.37)$GeV

$(-0.08 - i0.04)$GeV

$(-0.08 \ - i0.09)$GeV

Interference?

- Higher partial waves (P,D,…) would not affect the cusp shape because
  - Discontinuity in the higher partial waves appear only in the second or higher derivatives
  - The interference with different L vanishes with an integral over the solid angle
  - S-wave interference is approximately considered with a constant.
- Confirmed by an amplitude analysis based on the LHCb result [PRD108(2023)012023]
  - Consistent results are obtained between the amplitude analysis & one-dimensional fit.