Precision Scale Setting in Lattice QCD Using Baryon Spectra and Decay Constants

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Collaboration

- NME Collaboration: 2+1-flavor clover ensembles = clover-on-clover formulation

Members of NME

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Jun-Sik Yoo (Korea University)
Boram Yoon (NVIDIA)

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LANL NME 2+1 flavor clover ensembles

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CHROMA software package

Computing Resources:
INCITE award HEP133 on Frontier @ OLCF
ALCC award HEP145 on Frontier @ OLCF
ERCAP Awards on Perlmutter @ NERSC
LANL Institutional Computing
USQCD Allocations

Outline

- Motivation
- Methodology
- Result
- Summary

Motivation

Lattice scale propagates to every dimensionful observables as

$$\frac{\delta\mathcal{O}}{\mathcal{O}} = d\frac{\delta a}{a}$$

At sub-percent precision, scale error dominates systematics

Example)
dipole estimate of nucleon charge radius r_E^2

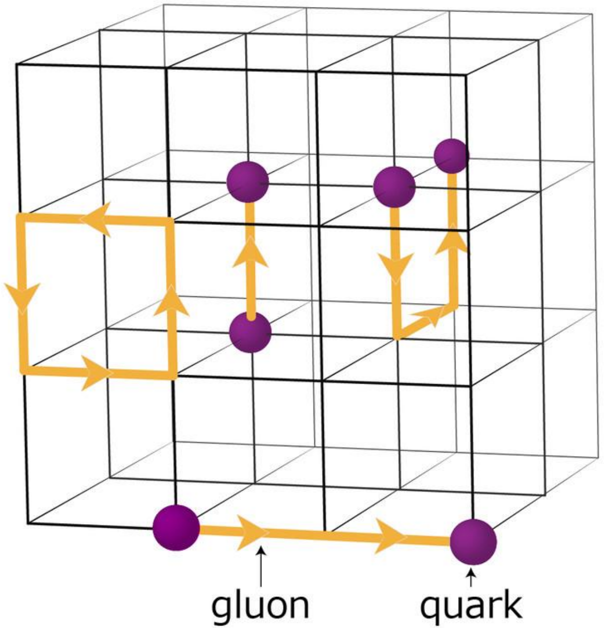
$0.586(17)(13)\ \mathrm{fm}^2$

PNDME, 2020, PRD 101, 014507

- Nucleon charge radius r_E^2 has units of fm² → 2× sensitivity to a.
- Due to precision determination, error from scale setting is small
- Precision tests demand sub-percent control.

For Clover-on-Clover, such determination is called for!

Lattice QCD (I)



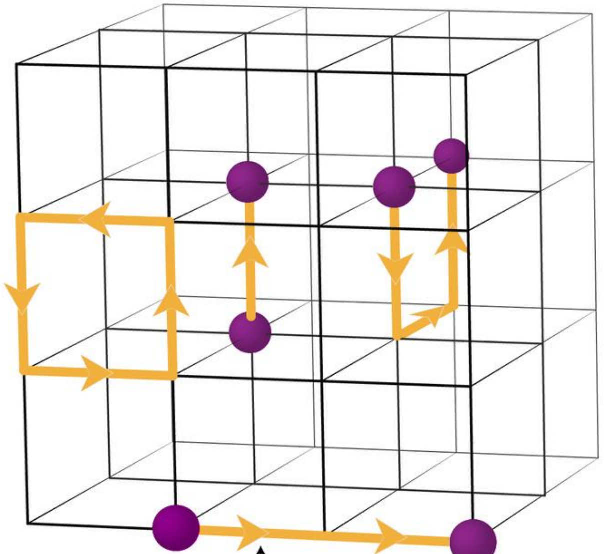
- quark fields ψ(x), $\bar{\psi}$(x) on the lattice sites
- gauge link U_μ(x) acts as parallel transporter connecting x $\rightarrow$x+aμ
- Lattice derivative

$$D_\mu\psi(x) = U_\mu(x)\psi(x+a\mu) - \psi(x),$$

- Path integral discretized, Wick rotated

$$\langle \mathcal{O}[U_\mu,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int\prod_{n,\mu} dU_\mu d\psi d\bar{\psi}\,\mathcal{O}[U_\mu,\psi,\bar{\psi}]\exp\{-\sum_n \mathcal{L}_{\mathrm{latt}}[U_\mu,\psi,\bar{\psi}]\}$$

Lattice QCD (II)



- Fermion integrated out

$$\langle \bar{\mathcal{O}}[U_\mu] \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_\mu \, \bar{\mathcal{O}}[U_\mu] \, \exp\left\{-S_{\text{latt}}^{\text{eff}}[U_\mu]\right\}$$

- Monte Carlo Simulation

$$\langle \bar{\mathcal{O}}[U_\mu] \rangle = \frac{1}{N} \sum_{i=1}^{N} \bar{\mathcal{O}}[U_\mu^{(i)}] + O\left(\frac{1}{\sqrt{N}}\right)$$

- Typical degree of freedom

$O(10^7 \sim 10^{10})$

Lattice Details

- 2+1 flavor Wilson clover ensemble
- tree-level tadpole improved symanzik gauge action
- Sheikholeslami-Wohlert coefficient tuned to $1/u_0^3$
- Quark mass aimed to have $(2M_{K^+}^2 - M_{\pi^+}^2)/M_{\Omega^-} = 0.1678$ but it varies from 0.17 to 0.20
- Lattice ensembles by many collaboration well summarized in Aoki, arXiv:2502.08303
- See also JHEP 05 (2023) 035 by G. Bali et al.

Lattice Details

ID	β	a (fm)	M_π (MeV)	M_K (MeV)	N_{KG}	σ	R_{RMS}/a	Lattices Analyzed	N_{HP}	N_{LP}	τ
$a127m285$	6.1	0.127	287(3)	476(5)	50	5	4.10(1)	3004	12,056	385,792	{8, 10, 12, 14}
$a094m270$	6.3	0.094			91	7	5.46(2)	2469	7,407	237,024	{10, 12, 14, 16}
$a094m270L$	6.3	0.094	269(3)	512(5)	91	7	5.49(3)	5453	18,040	577,280	{10, 12, 14, 16, 18}
$a093m220$	6.3	0.093	218(2)	543(6)	98	7	5.54(1)	2668	8,000	256,000	{10, 12, 14, 16, 18}
$a093m220X$	6.3	0.093	213(2)	500(5)	98	7	5.56(1)	2003	8,024	256,768	{10, 12, 14, 16, 18}
$a091m170$	6.3	0.091			91	7	5.41(2)	4012	16,048	513,536	{8, 10, 12, 14, 16}
$a091m170L$	6.3	0.091	167(2)	491(5)	91	7	5.49(3)	4014	20,000	640,000	{8, 10, 12, 14, 16}
$a073m270$	6.5	0.073	272(3)	575(6)	150	9	6.96(1)	5295	18,880	604,160	{11, 13, 15, 17, 19}
$a072m220$	6.5	0.072	221(2)	539(6)	200	10	7.53(1)	2350	14,100	225,600	{13, 15, 17, 19, 21}
$a071m170$	6.5	0.071	166(2)	538(6)	185	10	7.57(2)	3771	22,632	362,112	{13, 15, 17, 19, 21}
$a070m130$	6.5	0.070	128(1)	522(5)	200	10	7.57(1)	2500	15,000	240,000	{13, 15, 17, 19, 21}
$a056m280$	6.7	0.056	281(3)	526(6)	365	14	10.09(1)	3250	16,200	259,200	{15, 18, 21, 24, 27}
$a056m220$	6.7	0.056	217(2)	526(5)	392	14	10.18(1)	2485	15,300	244,800	{18, 21, 24, 27, 30}

Gradient flow observables

- Gradient flow smoothes field toward static point in Lattice action

$$\frac{dV(t)_{i,\mu}}{dt} = -g_0^2 \frac{\partial S(V)}{\partial V_{i,\mu}} V_{i,\mu}, \quad V_{i,\mu}(0) = U_{i,\mu}$$

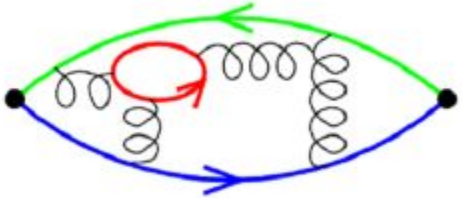
- Energy density at specific flow time, serves as a reference point

$$t_f^2 E(t_f)|_{t_f = t_0} = 0.3$$

- Lattice Measurement gives t_0/a^2
- Multiply by t_0/a^2 to certain power to keep observable dimensionless
- t_0 will be matched to t_0^{phys} at the physical point when extrapolating

Spectrum on the lattice

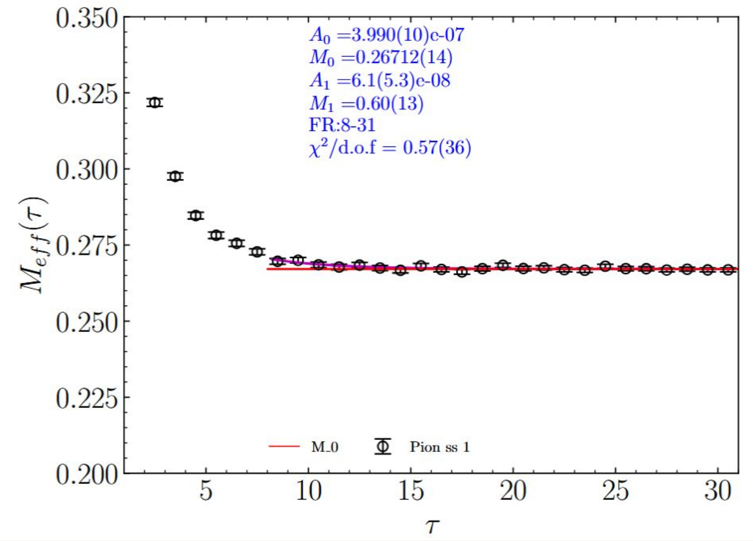
Euclidean two-point correlation function gives:



$$G(\vec{q},t) = \sum_{\vec{x}} e^{-i\vec{q}\vec{x}} \langle J(\vec{x},t) J^\dagger(\vec{0},0) \rangle$$

$$= \sum_{n=0,1\ldots} A_n e^{-E_n(\vec{q})t} \xrightarrow{t\to\infty} A_0 e^{-E_0(\vec{q})t}$$

where $J_{\pi^+}(x) = \bar{d}(x)\gamma_5 u(x)$



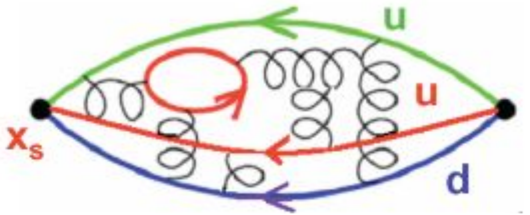
Spectrum on the lattice

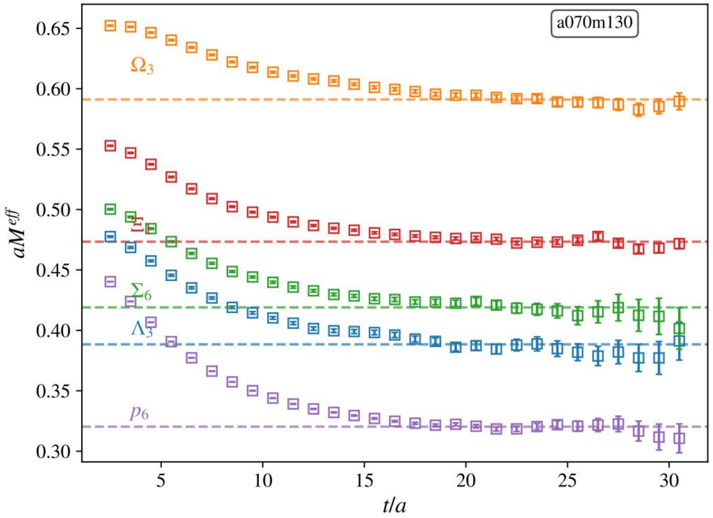
Euclidean two-point correlation function gives:

$$G(\vec{q},t) = \sum_{\vec{x}} e^{-i\vec{q}\vec{x}} \langle J(\vec{x},t) J^\dagger(\vec{0},0)\rangle$$

$$= \sum_{n=0,1\ldots} A_n e^{-E_n(\vec{q})t} \xrightarrow{t\to\infty} A_0 e^{-E_0(\vec{q})t}$$

where $J_p(x) = \epsilon_{abc}[\bar{u}^{aT}(x) C\gamma_5 d^b(x)] u^c(x)$





Analysis from Octet Baryon Masses

- Chiral model separated from lattice discretization effect

$$\sqrt{8t_0}M_O(M_\pi, M_K, a) = \sqrt{8t_0}M_O(M_\pi, M_K, a=0)\left\{1 + f(a)\left[c_o + \bar{c}_o(8t_0\overline{M}^2) + \delta c_O(8t_0\delta M^2)\right]\right\}$$

- lattice discretization modeled in three different shape

$$f(a) = \frac{a^2}{8t_0}, \quad \frac{\alpha_s a}{\sqrt{8t_0}}, \quad \frac{a}{\sqrt{8t_0}}$$

- LO, NLO, NNLO term is fit as LEC

$$\sqrt{8t_0}M_O(M_\pi, M_K) = \boldsymbol{M_0} + \bar{\boldsymbol{b}}\left(8t_0\overline{M}^2\right) + \boldsymbol{\delta b_O}\left(8t_0\delta M^2\right) + \left[\boldsymbol{g_{O,\pi}} f_O\left(\frac{\sqrt{8t_0}M_\pi}{\boldsymbol{M_0}}\right) + \boldsymbol{g_{O,K}} f_O\left(\frac{\sqrt{8t_0}M_K}{\boldsymbol{M_0}}\right) + \boldsymbol{g_{O,\eta_8}} f_O\left(\frac{\sqrt{8t_0}M_{\eta_8}}{\boldsymbol{M_0}}\right)\right]$$

- At physical point, we solve for t_0^{phys}

Analysis from Decay constants

- Chiral and discretization dependence modeled

$$\sqrt{8t_0} f_{\pi K}(M_\pi, M_K, a) =$$
$$\frac{A}{4\pi}\left[1 - \frac{7}{6}L\left(\frac{\phi_2}{A^2}\right) - \frac{4}{3}L\left(\frac{\phi_4 - \frac{1}{2}\phi_2}{A^2}\right) - \frac{1}{2}L\left(\frac{\frac{4}{3}\phi_4 - \phi_2}{A^2}\right) + B\phi_4\right]\left(1 + Cf\left(\frac{a}{\sqrt{8t_0}}\right)\right)$$

B. Strassberger, POS Lattice 2021

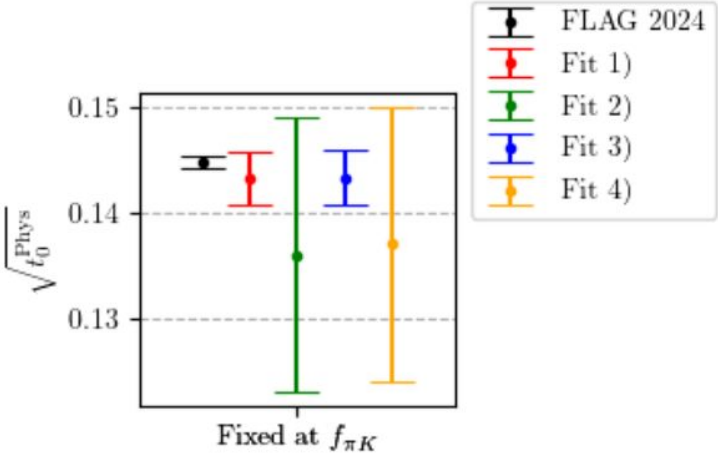
- Lattice discretization modeled in three different form

$$f(a) = \frac{a^2}{8t_0}, \quad \frac{\alpha_s a}{\sqrt{8t_0}}, \quad \frac{a}{\sqrt{8t_0}}$$

- Additional chiral dependence tested (Cont.)

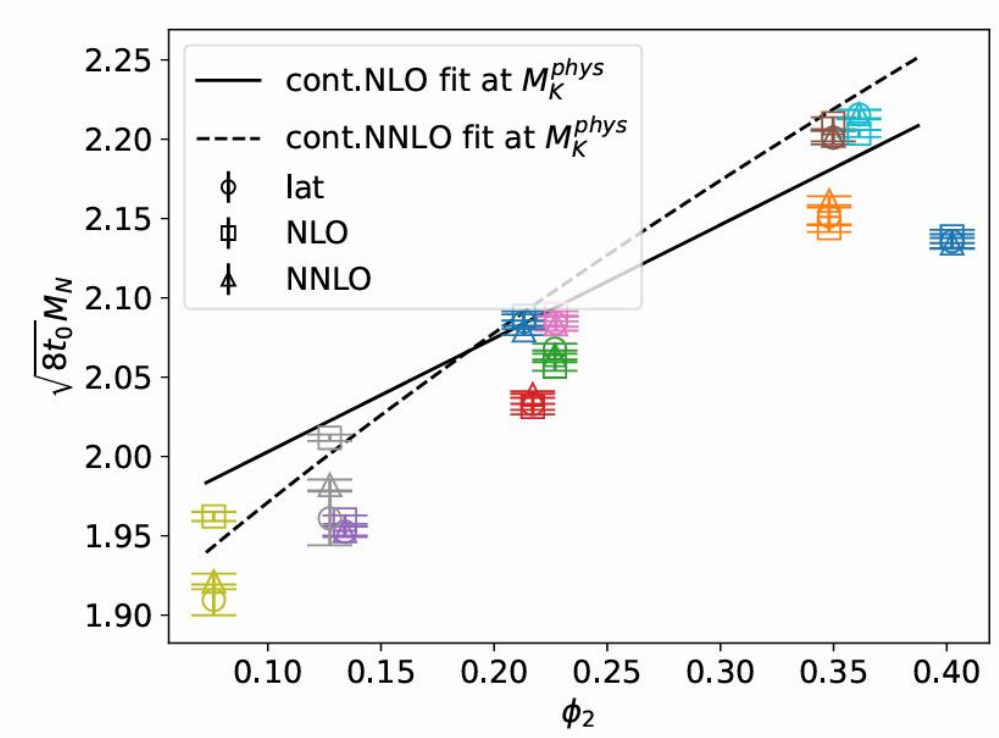
Analysis from Decay constants

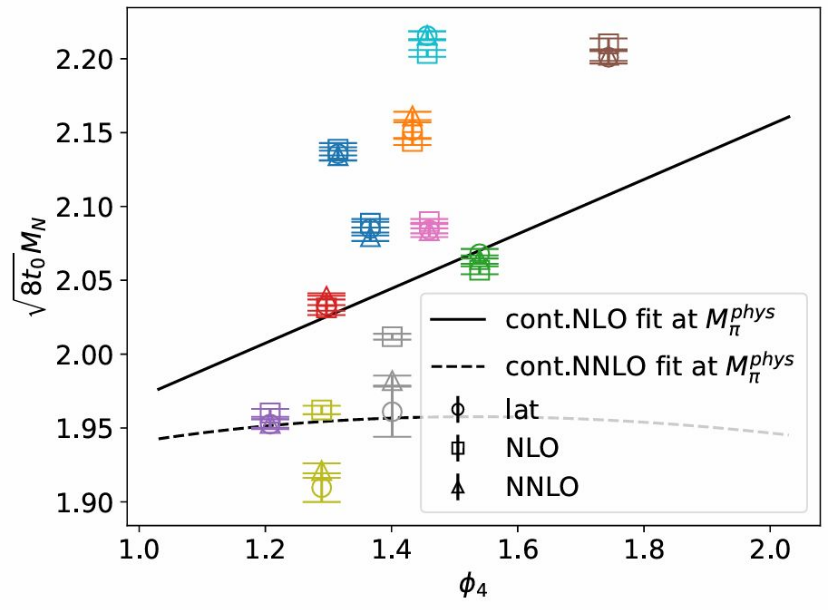
	ϕ_4 **dependence of** A **and** B
Fit 1) With A, B, C	$A(\phi_4) = A, \quad B(\phi_4) = B$
Fit 2) With A, B, C, D	$A(\phi_4) = A' + D\phi_4, \quad B(\phi_4) = B$
Fit 3) With A, B, C, E	$A(\phi_4) = A, \quad B(\phi_4) = B' + E\phi_4$
Fit 4) With A, B, C, D, E	$A(\phi_4) = A' + D\phi_4, \quad B(\phi_4) = B' + E\phi_4$



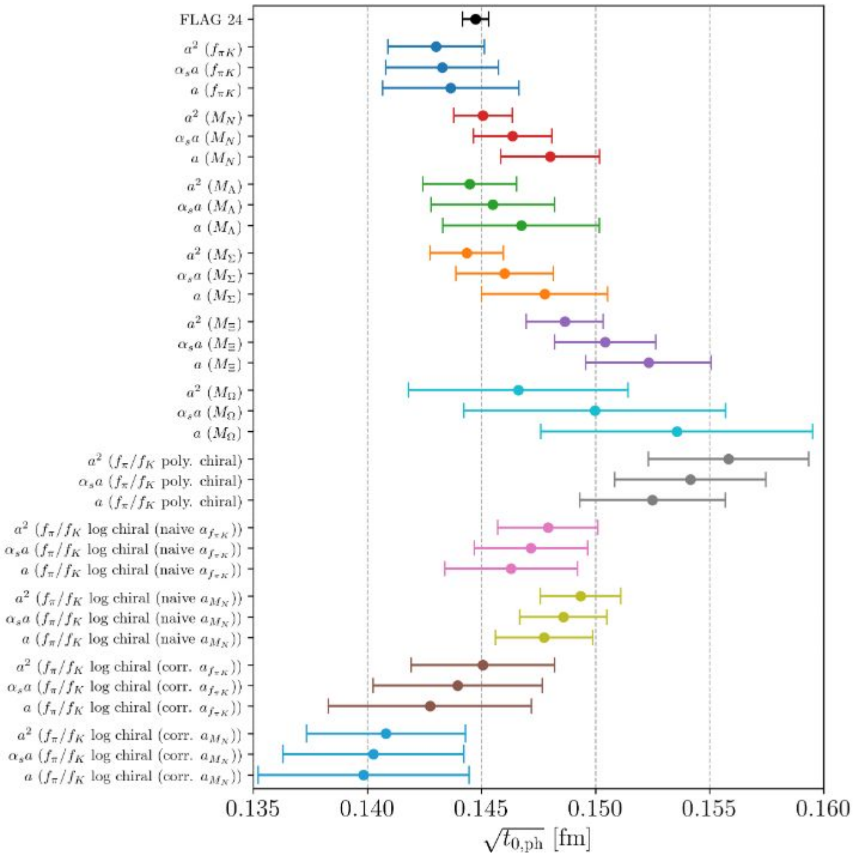
Analysis from Octet Baryon Masses

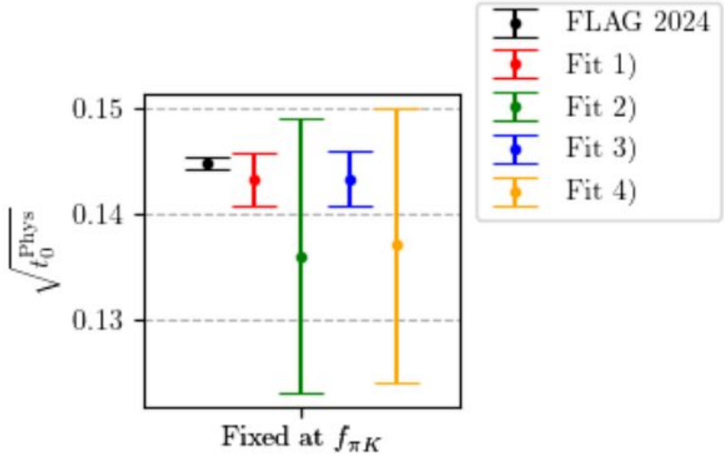
- Chiral and discretization dependence modeled



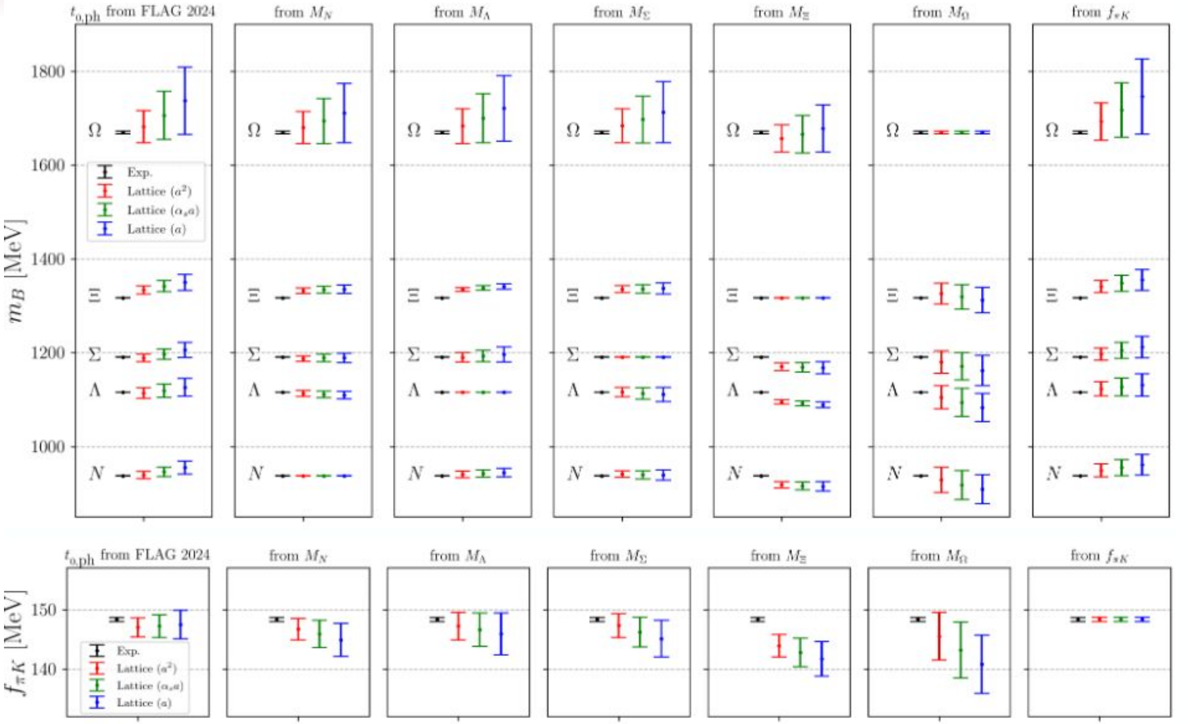


Analysis from spectra and decay constant





Final Result



Black: Experimental values

Red: Results with a^2

Green: Results with $\alpha_s a$

Blue: Results with a

Summary

- Flow time observable t0 and w0 is measured on Clover ensemble
- Meson and Baryon spectrum measured in precision
- Decay constar
- Small numbers
- Scale setting a
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Thank you for your attention