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Calculation of $\overline{B}_s$ and Λ_b semileptonic decays in heavy quark effective theory to probe symmetry between anti-s quark and ud diquark

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Symmetry between $\bar{s}$ quark and ud diquark

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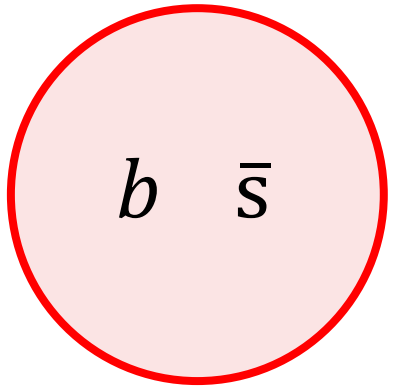
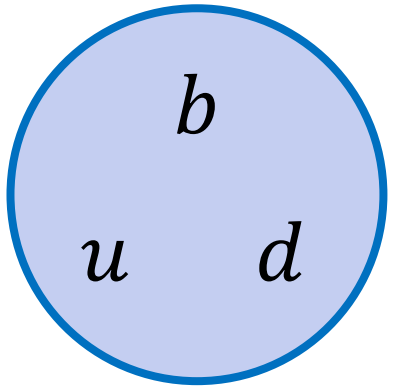
$\bar{B}_s^{(*)}$



- 5366 MeV, 5415 MeV
- ± 0 charge

Λ_b



- 5620 MeV
- ± 0 charge

- $\bar{s}$ quark and ud diquark have **same** electric charge and color charge.
- **Similar** constituent masses.
- A dynamical "**supersymmetry**" between $\bar{s}$ and ud has been proposed[1].
- This can be investigated by the **decays** of $\bar{B}_s^{(*)}$ and Λ_b.

[1] T. Amano and D. Jido, Prog. Theor. Exp. Phys. 2019, 093D02 (2019).

Decay rates of semileptonic decay(1/2)

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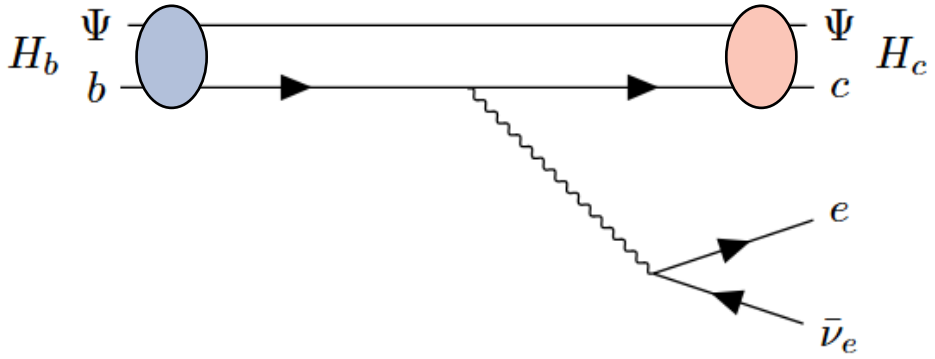
- Considering the **decay process** : H_b decays into H_c and two leptons.



* H_b = b-hadron

H_c = c-hadron

- Here, $\Psi = (\bar{s}, ud)$. Under the assumption of $\bar{s}, ud$ symmetry, **their wave functions are symmetric** about Ψ.

- Previous research [2] separates the decay amplitude into a hadronic part and a leptonic part.

$$\mathcal{M} = \mathcal{M}^{\mu}_{\text{hadron}} \mathcal{M}_{\text{lepton},\mu}$$

[2] T. Amano, D. Jido, S. Leupold, Phys. Rev. D105, L051504 (2022)

Decay rates of semileptonic decay(2/2)

- $\mathcal{M}^{\mu}_{\text{hadron}}$ can be described by $b \to c$ current $\mathcal{M}^{\mu}_{\alpha\beta}$, because Ψ is a spectator.

$$\mathcal{M}^{\mu}_{\alpha\beta} = \langle c^{(\beta)} | J^{\mu}_{h} | b^{(\alpha)} \rangle = \bar{u}^{(\beta)}_{c} \gamma^{\mu} (A + B\gamma_5) u^{(\alpha)}_{b}$$

* α : b quark spin
β : c quark spin

- Specifying the final states, the spin combinations are as follows:

$$\sum_{\text{spin}} (\mathcal{M}^{\mu}_{\Lambda_c})^{*} \mathcal{M}^{\nu}_{\Lambda_c} = \frac{1}{2} (\mathcal{M}^{\mu *}_{11} \mathcal{M}^{\nu}_{11} + \mathcal{M}^{\mu *}_{22} \mathcal{M}^{\nu}_{22} + \mathcal{M}^{\mu *}_{12} \mathcal{M}^{\nu}_{12} + \mathcal{M}^{\mu *}_{21} \mathcal{M}^{\nu}_{21})$$

$$(\mathcal{M}^{\mu}_{D_s})^{*} \mathcal{M}^{\nu}_{D_s} = \frac{1}{4} (\mathcal{M}^{\mu *}_{11} \mathcal{M}^{\nu}_{11} + \mathcal{M}^{\mu *}_{22} \mathcal{M}^{\nu}_{22} + \mathcal{M}^{\mu *}_{11} \mathcal{M}^{\nu}_{22} + \mathcal{M}^{\mu *}_{22} \mathcal{M}^{\nu}_{11}) \quad \text{etc.}$$

- We can treat $\mathcal{M}^{\mu}_{\alpha\beta}$ as the same element because of $\bar{s}, ud$ symmetry.
(The wave functions are same due to $\bar{s}, ud$ symmetry.)

- By this property, the sum rule of decay rates is derived.

$$\Gamma(\Lambda_b \to \Lambda_c e \bar{\nu}_e) = \Gamma(\bar{B}_s \to D_s e \bar{\nu}_e) + \Gamma(\bar{B}_s \to D^{*}_{s} e \bar{\nu}_e)$$

What is Heavy Quark Effective Theory? (HQET)

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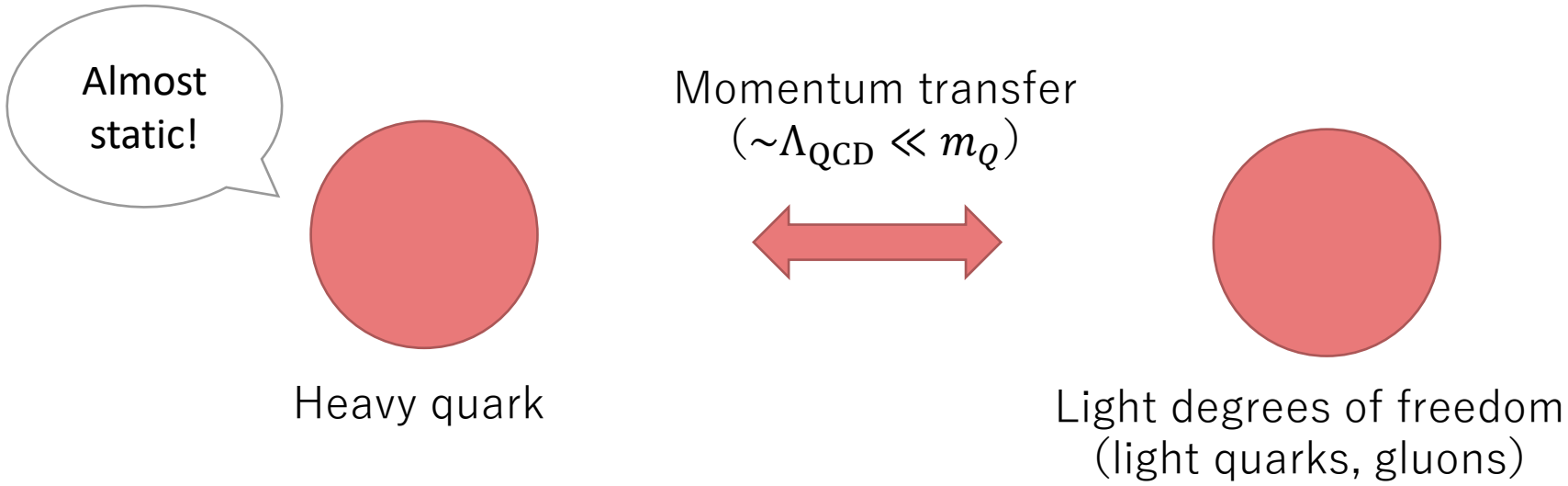
- HQET : Heavy Quark Effective Theory
- **Effective theory for heavy quarks** such as b and c.
- Since b and c are heavy, **its velocity is almost constant** even when interacting with light degrees of freedom (other light quarks and gluons)inside a hadron.
- It simplifies QCD by treating a heavy quark as a "**static color source**".



What is Heavy Quark Symmetry?(HQS)

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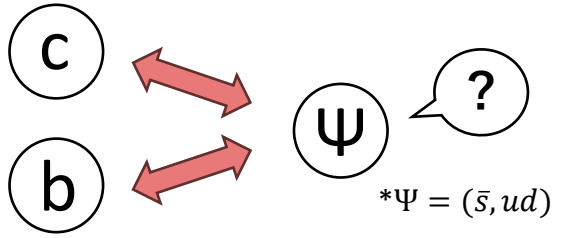
- HQS : Heavy Quark Symmetry
- In the limit that the heavy quark mass $m_Q \to \infty$, **two symmetries** emerge for heavy quarks :



1. Heavy Quark **Flavor** Symmetry
   - In the $m_Q \to \infty$, the specific masses of c and b do not affect the physics.
   - As a result, the light degrees of freedom **do not distinguish the heavy quark flavor**.
2. Heavy Quark **Spin** Symmetry
   - The spin-spin interaction between heavy quarks↔light degrees of freedom is suppressed by $1/m_Q$.
   - In the $m_Q \to \infty$, the light degrees of freedom **do not distinguish the heavy quark spin**.

Meson decay : $\bar{B}_s \to D_s^{(*)}$ form factors (1/2)

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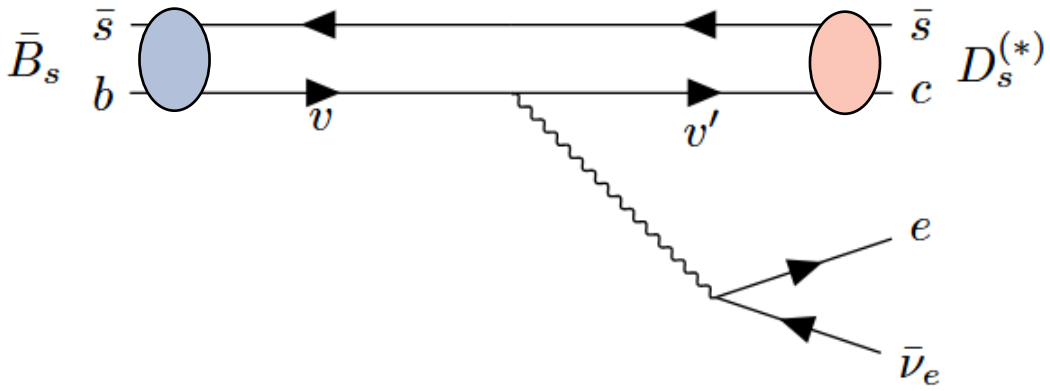
- Considering the decay : $\bar{B}_s \to D_s^{(*)} e \bar{\nu}_e$



- The currents for $\bar{B}_s \to D_s^{(*)}$ are expressed by w and **six** Lorentz-invariant form factors $h_{\pm}(w), h_V(w), h_{A_{1,2,3}}(w)$

$$\langle D_s(v')|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}_s(v)\rangle = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

$$\langle D_s^*(v',\epsilon)|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}_s(v)\rangle = h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\langle D_s^*(v',\epsilon)|\,\bar{c}_{v'}\gamma^\mu\gamma_5 b_v\,|\bar{B}_s(v)\rangle = -ih_{A_1}(w)(w+1)\epsilon^{*\mu} + ih_{A_2}(w)(\epsilon^*\cdot v)v^\mu + ih_{A_3}(w)(\epsilon^*\cdot v)v'^\mu$$

- Here, w is inner product of the four-velocities of heavy quarks as a HQET parameter.

$$w = v\cdot v' \qquad (v = p/m_H)$$

Meson decay : $\bar{B}_s \to D_s^{(*)}$ form factors(2/2)

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$$\langle D_s(v')|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}_s(v)\rangle = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

$$\langle D_s^*(v',\epsilon)|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}_s(v)\rangle = h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\langle D_s^*(v',\epsilon)|\,\bar{c}_{v'}\gamma^\mu\gamma_5 b_v\,|\bar{B}_s(v)\rangle = -ih_{A_1}(w)(w+1)\epsilon^{*\mu} + ih_{A_2}(w)(\epsilon^*\cdot v)v^\mu + ih_{A_3}(w)(\epsilon^*\cdot v)v'^\mu$$

- Using HQS and HQET, the six Lorentz-invariant form factors can be reduced to a single **Isgur-Wise function** $\xi(w)$ [3].

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$$

$$h_-(w) = h_{A_2}(w) = 0$$

- By applying this, each current can be described by only this single form factor ξ.

$$\langle D_s(v')|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}_s(v)\rangle = \xi(w)[v^\mu + v'^\mu]$$

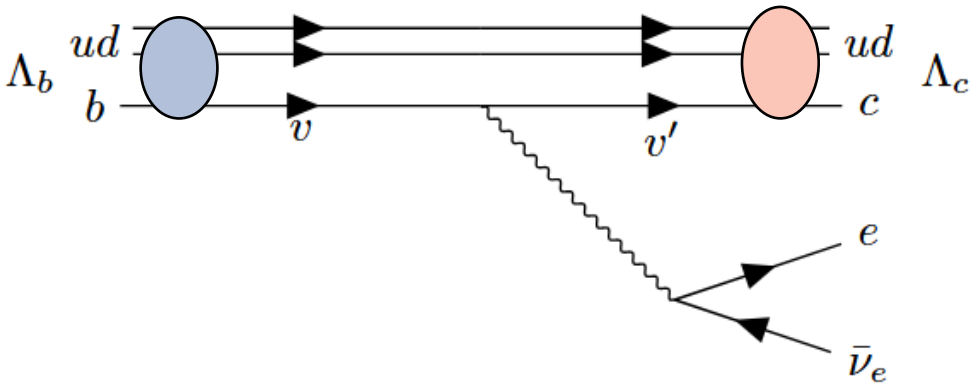
$$\langle D_s^*(v',\epsilon)|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}_s(v)\rangle = \xi(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\langle D_s^*(v',\epsilon)|\,\bar{c}_{v'}\gamma^\mu\gamma_5 b_v\,|\bar{B}_s(v)\rangle = -i\xi(w)[(1+w)\epsilon^{*\mu} - (\epsilon^*\cdot v)v'^\mu]$$

[3] A. V. Manohar and M. B. Wise, "Heavy Quark Physics", Cambridge University Press (2000).

Baryon decay : $\Lambda_b \to \Lambda_c$ form factors

- Similarly considering the decay : $\Lambda_b \to \Lambda_c e \bar{\nu}_e$



- Same as a meson decay, the **six** baryon decay form factors can be reduced to a single Isgur-Wise function $\xi'(w)$ [3].

$$\langle \Lambda_c(v',s')|\, \bar{c}_{v'} \gamma^\mu b_v \,|\Lambda_b(v,s)\rangle = \bar{u}(v',s')[F_1\gamma^\mu + F_2 v^\mu + F_3 v'^\mu]u(v,s)$$

$$\langle \Lambda_c(v',s')|\, \bar{c}_{v'} \gamma^\mu \gamma_5 b_v \,|\Lambda_b(v,s)\rangle = \bar{u}(v',s')[G_1\gamma^\mu + G_2 v^\mu + G_3 v'^\mu]\gamma_5 u(v,s)$$

$$\Downarrow$$

$$\langle \Lambda_c(v',s')|\, \bar{c}_{v'} \gamma^\mu b_v \,|\Lambda_b(v,s)\rangle = \xi'(w)\bar{u}(v',s')\gamma^\mu u(v,s)$$

$$\langle \Lambda_c(v',s')|\, \bar{c}_{v'} \gamma^\mu \gamma_5 b_v \,|\Lambda_b(v,s)\rangle = \xi'(w)\bar{u}(v',s')\gamma^\mu \gamma_5 u(v,s)$$

[3] A. V. Manohar and M. B. Wise, "Heavy Quark Physics", Cambridge University Press (2000).

In this study (1/2)

- Introducing **a relativistic (recoil) factor** for the relation between full QCD hadron state $|\mathrm{H}(p)\rangle$ and HQET hadron state $|\mathrm{H}(v)\rangle$.

$$|H(p)\rangle = \sqrt{m_H}[|H(v)\rangle + \mathcal{O}(1/m_Q)]$$

Original by A. V. Manohar and M. B. Wise[3]

$$|H(p)\rangle = \begin{cases} \sqrt{m_H}\dfrac{2m_H}{E_H + m_H}[|H(v)\rangle + \mathcal{O}(1/m_Q)] & \text{for meson} \\ \sqrt{m_H}\sqrt{\dfrac{2m_H}{E_H + m_H}}[|H(v)\rangle + \mathcal{O}(1/m_Q)] & \text{for hadron} \end{cases}$$

In this research

- Taking the spin average for initial state and the spin sum for final state.

$$\frac{d}{dw}\Gamma(\bar{B}_s \to D_s e\bar{\nu}_e) = \frac{G_F^2|V_{cb}|m_{B_s}^5}{48\pi^3}\left(\frac{2}{1+w}\right)^2\sqrt{w^2-1}r^3\xi^2(w)(-1-2r-r^2+w^2+2rw^2+r^2w^2)$$

$$\frac{d}{dw}\Gamma(\bar{B}_s \to D_s^* e\bar{\nu}_e) = \frac{G_F^2|V_{cb}|m_{B_s}^5}{48\pi^3}\left(\frac{2}{1+w}\right)^2\sqrt{w^2-1}r^{*3}\xi^2(w)(1-2r+r^2+6w-4rw$$
$$+6r^2w+5w^2-10rw^2+5r^2w^2-8rw^3)$$

$$\frac{d}{dw}\Gamma(\Lambda_b \to \Lambda_c e\bar{\nu}_e) = \frac{G_F^2|V_{cb}|m_{\Lambda_b}^5}{12\pi^3}\frac{2}{1+w}\sqrt{w^2-1}r_\Lambda^3\xi'^2(w)(-2r+3w+3r^2w-4rw^2)$$

In this study (2/2)

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- Assume that the Isgur-Wise functions $\xi(w), \xi'(w)$ are **identical**.

$$\xi(w) = \xi'(w)$$

- By also assuming **the mass ratio** of c-hadrons and b-hadrons $r = m_{H_c}/m_{H_b}$, and **the b-hadron masses** m_{H_b}, are identical, we obtain the following sum rule.

$$\Gamma(\Lambda_b \to \Lambda_c e\bar{\nu}_e) = \Gamma(\bar{B}_s \to D_s e\bar{\nu}_e) + \Gamma(\bar{B}_s \to D_s^* e\bar{\nu}_e)$$

- Previous research[2] : assumption of $\bar{s}, ud$ sym. → wave function → sum rule
  Our research : HQET,HQS → assumption of $\xi(w) = \xi'(w)$ → sum rule

[2] T. Amano, D. Jido, S. Leupold, Phys. Rev. D105, L051504 (2022)

Prospects

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- Calculate the Isgur-Wise functions ξ, ξ' using a specific model, and probe the mechanism of the dynamical symmetry between $\bar{s}$ quark and ud diquark.

- This discussion **has ignored $O(m^{-1})$ corrections**.
  We expect to find further properties of the sum rule by discussing these corrections.

- **Future experiments** can verify this sum rule.

Summary

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Purposes

- To verify the dynamical supersymmetry between $\bar{s}$ quark and ud diquark.
- To clarify the relationship between the decay rates of $\bar{B}_s$ and Λ_b.

Methods

- HQS and HQET

Results

- The sum rule derived from supersym. was also confirmed from the HQET under $\xi = \xi'$.

$$\Gamma(\Lambda_b \to \Lambda_c e\bar{\nu}_e) = \Gamma(\bar{B}_s \to D_s e\bar{\nu}_e) + \Gamma(\bar{B}_s \to D_s^* e\bar{\nu}_e)$$

Prospects

- Calculation of $\xi^{(\prime)}$ using a specific model
- Investigation of m^{-1} corrections
- Experimental verification

Back up slides 1

The original sum rule supposed by [2] is :

$$\Gamma\left(\Lambda_b \to \Lambda_c P^-\right) = \Gamma\left(\bar{B}_s^0 \to D_s^+ P^-\right) + \Gamma\left(\bar{B}_s^0 \to D_s^{*+} P^-\right).$$

P stands for the emitted particles such as π^- in pionic decay, K^- in kaonic decay.

Pionic decay and kaonic decay follow the sum rule well.

	Branching fraction	Γ_i [10^9/s]
Pionic decay		
$\bar{B}_s^0 \to \pi^- D_s^+$	$(3.00 \pm 0.23) \times 10^{-3}$	1.98 ± 0.15
$\bar{B}_s^0 \to \pi^- D_s^{*+}$	$(2.0 \pm 0.5) \times 10^{-3}$	1.3 ± 0.3
Sum		$[3.3 \pm 0.4]$
$\Lambda_b \to \pi^- \Lambda_c$	$(4.9 \pm 0.4) \times 10^{-3}$	3.3 ± 0.3
Kaonic decay		
$\bar{B}_s^0 \to K^- D_s^+$	$< (2.27 \pm 0.19) \times 10^{-4}$	$< (0.150 \pm 0.013)$
$\bar{B}_s^0 \to K^- D_s^{*+}$	$< (1.33 \pm 0.35) \times 10^{-4}$	$< (0.088 \pm 0.023)$
Sum		$[< (0.237 \pm 0.026)]$
$\Lambda_b \to K^- \Lambda_c$	$(3.59 \pm 0.30) \times 10^{-4}$	0.244 ± 0.020

Pionic and Kaonic decay rates[2]

[2] T. Amano, D. Jido, S. Leupold, Phys. Rev. D105, L051504 (2022)