Precision KKMC predictions for Z-boson SM and anomalous couplings.

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- **Motivation:** not only μ's also signatures of τ lepton offer precison window for:

  (i) measurement of Standard Model couplings parameters

  (ii) New Physics signatures: new particles, new interactions

  (iii) Future, man power, expertise preservation.

- **The τ pair production:** similar to μ-s. **But: (i)** negligible for μ- (e-) $\sim m_\tau/E_{beam}$ terms can not be neglected **(ii)** spin state $\leftrightarrow$ designing obervables $\leftrightarrow$ event record formats $\leftrightarrow$ ME/factorization savvy reference frames.

- **New Physics interactions** τ is heavy $\rightarrow$ Yukawa couplings large

- **τ decays and spin response** Modelling of τ decays rely on data fits.

- **Decay products** of non-observable τs are measured, except neutrinos which may be (partly) reconstructed from event kinematic and decay vertex position.

General conditions

1. KKMC **Monte Carlo :** is build around expansion with respect to **Eikonal QED**.
   Concern: perturbation calculation need to be re-ordered (the β_i's of YFS).
2. **Advantage**: Full, exact phase space coverage for all observable photons accompanying dedicated process, such as $e^+e^- \to l^+l^-$.
3. **QED matrix elements** can be then installed order by order, thanks to fudamental work of Yennie Frautchi Suura. It took us several challenging years to have it in Monte Carlo. See e.g.: Phys.Rev.D 63 (2001) 113009.
4. **Precision for QED**: $wt = 1$ events, auxiliary wt-s for physics ambiguity studies.
5. **Non QED effects:** separating out amplitudes into QED and other components: *(i)* hadronic (strong interaction) contribution to photon and Z boson propagators, *(ii)* genuine weak corrections.
6. **Definition of genuine weak corrections,** was not easy. Complex mass schemes have to be supported with careful proof that anti-analytic (optical theorem like) field theory constraints are not broken. Already massive effort for one loop level: Comput.Phys.Commun. 72 (1992) 175, Comput.Phys.Commun. 59 (1990) 303

1. **Anomalous couplings,** can be treated with approximations, if SM predictions are not compromised. Usually, New Interactions are of distinct energy scale.
2. **Measurable SM coupling?**
   (a) What does it mean?
   (b) Process dependent quantities?
   (c) How to define,
   (d) what are the limitations.
3. Reference for numerical results: Eur.Phys.J.Plus 137 (2022)
4. References, why Born like factor can be identified within multi-photon, multi-jest differential distributions: (i) E. Mirkes and J. Ohnemus,Phys. Rev.D51(1995) 4891, (ii) R. Kleiss,Nucl. Phys. B347(1990) 67 (iii) Precision electroweak measurements on the Z resonance, ALEPH, DELPHI, L3, OPAL and LEP Electroweak Collaborations, Phys.Rept. 427 (2006) 257
5. Reference for numerical results: Eur.Phys.J.Plus 137 (2022)

Improved Born[a]

$$
\begin{aligned}
ME_{Born+EW} \qquad &= \mathcal{N}\frac{\alpha}{s}\{[\bar{u}\gamma^{\mu}vg_{\mu\nu}\bar{v}\gamma^{\nu}u]\cdot(q_e\cdot q_f)\cdot\Gamma_{V_{\Pi}}\cdot\chi_{\gamma}(s)\\
&+[\bar{u}\gamma^{\mu}vg_{\mu\nu}\bar{\nu}\gamma^{\nu}u\cdot(v_e\cdot v_f\cdot vv_{ef})\\
&+\bar{u}\gamma^{\mu}vg_{\mu\nu}\bar{\nu}\gamma^{\nu}\gamma^{5}u\cdot(v_e\cdot a_f)\\
&+\bar{u}\gamma^{\mu}\gamma^{5}vg_{\mu\nu}\bar{\nu}\gamma^{\nu}u\cdot(a_e\cdot v_f) \qquad (1)\\
&+\bar{u}\gamma^{\mu}\gamma^{5}vg_{\mu\nu}\bar{\nu}\gamma^{\nu}\gamma^{5}u\cdot(a_e\cdot a_f)]\cdot Z_{V_{\Pi}}\cdot\chi_Z(s)\ \},
\end{aligned}
$$

In the formula $u,\ v$ stand for spinors - fermions wave functions, and $\mathcal{N}$ is a normalization factor which is convention dependent (e.g. for wave functions normalization). The (G_μ, M_Z, $\alpha(0)$)input scheme is used.

$$
\begin{aligned}
v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2 \cdot K_e(s,t))/\Delta, \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s,t))/\Delta, \\
a_e &= (2 \cdot T_3^e)/\Delta, \qquad s_W^2 = (1 - c_W^2) = 1 - M_W^2/M_Z^2, \\
a_f &= (2 \cdot T_3^f)/\Delta, \qquad \Delta = 4 s_W c_W, \\
\chi_Z(s) &= \frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}, \\
\Gamma_{V_\Pi} &= \frac{1}{2 - (1 + \Pi_{\gamma\gamma}(s))}, \qquad Z_{V_\Pi} = \rho_{\ell f}(s,t), \qquad \chi_\gamma(s) = 1,
\end{aligned}
$$

$$
\begin{aligned}
vv_{ef} = &\quad \frac{1}{v_e \cdot v_f}[(2 \cdot T_3^e)(2 \cdot T_3^f) - 4 \cdot q_e \cdot s_W^2 \cdot K_f(s,t) \\
-4 \cdot q_f \cdot s_W^2 &\quad \cdot K_e(s,t) + (4 \cdot q_e \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{ef}(s,t)] \frac{1}{\Delta^2}.
\end{aligned}
$$

Input includes also $\Pi_{\gamma\gamma}(s)$, t_t, m_h and that M_W is calculated iteratively. With $\Pi_{\gamma\gamma}(s)$ (in particular) comes issue of parametric ambiguity. Measurement of $\alpha_{QED}(M_Z^2)$ may help?

[a] At the LO EW $K_e(s,t) = K_f(s,t) = \rho_{\ell f}(s,t) = 1,\ \Pi_{\gamma\gamma}(s) = 0$ and $\frac{G_\mu M_Z^2 \Delta^2}{\sqrt{2}\cdot 8\pi\alpha} = 1$. We use $s_W^2 = 1 - M_W^2/M_Z^2$ for the on-mass-shell definition. The $vv_{ef} = 1$ at LO too. The $vv_{ef} - 1$ bring correction which can not be attributed to coupling or to propagators; depends on whole process.

Effective Born

$$
\begin{aligned}
ME_{Born-eff} = \quad & \mathcal{N}\frac{\alpha}{s}\{[\bar{u}\gamma^\mu v g_{\mu\nu}\bar{v}\gamma^\nu u]\cdot(q_e\cdot q_f)\cdot\Gamma_{V_\Pi}\cdot\chi_\gamma(s) \\
+ \quad & [\bar{u}\gamma^\mu v g_{\mu\nu}\bar{\nu}\gamma^\nu u\cdot(v_e\cdot v_f\cdot vv_{ef}) \\
+ \quad & \bar{u}\gamma^\mu v g_{\mu\nu}\bar{\nu}\gamma^\nu\gamma^5 u\cdot(v_e\cdot a_f) \\
+ \quad & \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu}\bar{\nu}\gamma^\nu u\cdot(a_e\cdot v_f) \\
+ \quad & \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu}\bar{\nu}\gamma^\nu\gamma^5 u\cdot(a_e\cdot a_f)]\cdot Z_{V_\Pi}\cdot\chi_Z(s)\ \}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
v_e &= (2\cdot T_3^e - 4\cdot q_e\cdot s_W^2)/\Delta \\
v_f &= (2\cdot T_3^f - 4\cdot q_f\cdot s_W^2)/\Delta \\
a_e &= (2\cdot T_3^e)/\Delta, \quad \Delta = 4 s_w c_w, \quad a_f = (2\cdot T_3^f)/\Delta, \\
\chi_Z(s) &= \frac{G_\mu\cdot M_z^2\cdot\Delta^2}{\sqrt{2}\cdot 8\pi\cdot\alpha}\cdot\frac{s}{s-M_Z^2+i\cdot\Gamma_Z\cdot s/M_Z}, \\
\Gamma_{V_\Pi} &= 1, \qquad Z_{V_\Pi} = \mathrm{Re}\,\rho_{\ell f}(M_Z^2), \qquad \chi_\gamma(s)=1, \\
vv_{ef} &= 1, \qquad s_W^2 = (1-c_W^2) = \sin^2\theta_W^{eff}(M_Z^2), \\
\alpha &= \alpha(M_Z^2) = \frac{\alpha(0)}{2-(1+\mathrm{Re}\,\Pi_{\gamma\gamma}(M_Z^2))}.
\end{aligned}
$$

Note that $\sin^2\theta_W^{eff}(M_Z^2)$, $\rho_{\ell f}(M_Z^2)$ and $\alpha(M_Z^2)$ should now be understood as independent constants, even if in practice calculated in *on-mass-shell* OMS scheme. They absorb dominant parts of EW corrections; EW form-factors and vacuum polarization corrections. This useful approximation may take into account bulk of the EW effects, and couplings of fixed values are used. There is some level of uncertainty in the numerical values. The best match to Improved Born should correspond to the values predicted by these calculations. In particular, $\sin^2\theta_W^{eff}(M_Z^2) =$ Re$K(M_Z^2, -M_Z^2/2)s_W^2$, $s_W^2 = 1 - M_W^2/M_Z^2$, where M_W is a calculated quantity including EW corrections. The $s = M_Z^2$, $t = -M_Z^2/2$, corresponds to the Born-level with scattering angle $\theta = 0$. Alternatively, one can use best measured values, and Born expression, ignoring SM constraints.

Table 1: The EW parameters used for: the EW LO Born in $\alpha(0)$ scheme, and for variants of effective Born. The G_μ = $1.1663887 \cdot 10^{-5}$ GeV^{-2}, M_Z = 91.1876 GeV (M_Z = 91.1887 GeV Γ_Z=2.4952 for `Tauola`/LEP) and $K_f, K_e, K_{\ell f}$ = 1.

.

Effective Born TAUOLA/LEP	EW LO $\alpha(0)$ scheme	Effective Born v0	Effective Born v1	Effective Born v2
α = 1/128.6667471 s_W^2 = 0.23152 $\rho_{\ell f}$ = 1.0	α = 1/137.03599 s_W^2 = 0.21215 $\rho_{\ell f}$ = 1.0	α = 1/128.9503022 s_W^2 = 0.231499 $\rho_{\ell f}$ = 1.0	α = 1/128.9503022 s_W^2 = 0.231499 $\rho_{\ell f}$ = 1.005	α = 1/128.9503022 $s_W^{2\ \ell}$ = 0.231499 $s_W^{2\ up}$ = 0.231392 $s_W^{2\ down}$ = 0.231265 $\rho_{\ell up}$ = 1.005403 $\rho_{\ell down}$ = 1.005889

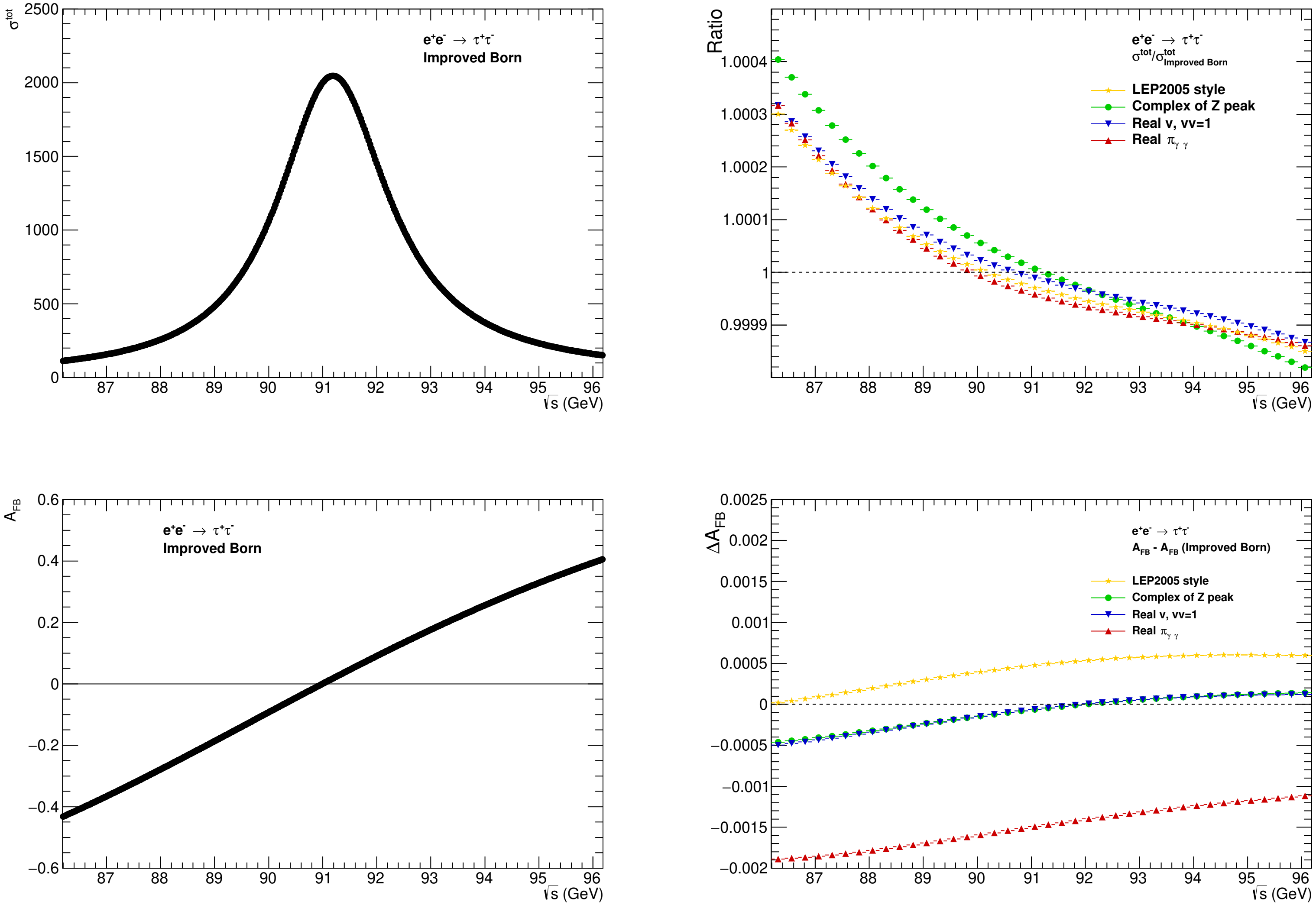


Figure 1: continuation on the next slide

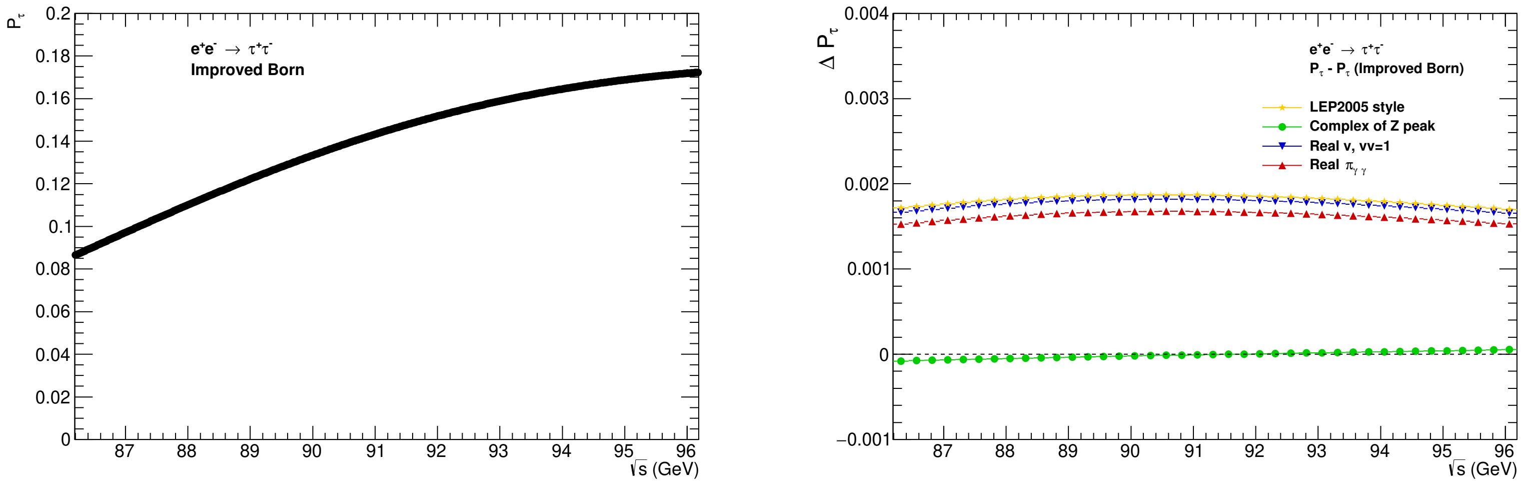


Figure 2: Left side plots, Improved Born-level and vicinity of the Z peak: σ^{tot} (top), A_{FB} (middle) and P_τ (bottom). Right side plots enumerate, with ratios or differences the effects of simplifications with respect to Improved Born results. Green points: instead of form-factors, their constant values calculated at s=M_Z^2, t=$-M_Z^2/2$ are used. Blue triangles: as for green ones, but in addition $vv_{\ell\ f} = 1$ and only real parts of v_e, v_f are used. Red triangles: as in blue triangles, but only real parts of $\Pi_{\gamma\gamma}$ and $\rho_{\ell f}$ are taken into account. Yellow stars: with respect to red triangles imaginary parts of $\Pi_{\gamma\gamma}$ are switched back on.

From these results, particularly for P_τ, we may conclude that approaches that rely on effective couplings may not work well for the $\sin^2\theta_W^{eff}$ precision tag better than about $20 \cdot 10^{-5}$. For further improvement revisiting EW effects in their complexity is required. Use of numerically adapted constants, which originally were multiplied by form-factors, does not suffice. For high precision, the picture of effective couplings is not universal: the choice appropriate for A_{FB} may not be optimal for P_τ. **Beware:** Effective/improved Born-s are important for fits and interpretation.

KKMC use form-factors, directly from within QED amplitudes. That arrangement was a challenge to achieve.

Now: anomalous couplings ....

- They are expected to be small or nonexistent. This precision is not fundamentapl, unless precison of SM part is compromised.

- Use of Monte Carlo internal variables is easier. Use of events stored in production files is more flexible, but format perils... I will go after first option now.

Formalism for $\tau^+\tau^-$: phase space $\times$ M.E. squared

- Because narrow τ width (τ propagator works as Dirac δ), cross-section for $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads (norm. const. dropped):

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Pauli matrices orthogonality $\delta^{\lambda'}_{\lambda}\delta^{\bar{\lambda}'}_{\bar{\lambda}} = \sum_\mu \sigma^\mu_{\lambda\bar{\lambda}}\sigma^{\lambda'\bar{\lambda}'}_\mu$ completes condition for production/decay separation with τ spin states.

- **core formula of spin algorithms, wt is product of density matrices of production and decays**, $0 < wt < 4$, $< wt >= 1$ useful properties.

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

To complete definitions

(**beware:** conventions for use of particle/antiparticle indices may be perilous):

$$R_{\mu\nu} = \sum_{\lambda_1\bar\lambda_1\lambda_2\bar\lambda_2=1}^{2} \sigma_\mu^{\lambda_1\bar\lambda_1}\sigma_\nu^{\lambda_2\bar\lambda_2}\mathcal{M}^{prod}_{\lambda_1\lambda_2}\,\bar{\mathcal{M}}^{prod}_{\bar\lambda_1\bar\lambda_2}$$

$$h^-_\mu = \sum_{\lambda'\bar\lambda'=1}^{2} \sigma_\mu^{\lambda'\bar\lambda'}\,\mathcal{M}^{\tau^-}_{\lambda'}\bar{\mathcal{M}}^{\tau^-}_{\bar\lambda'}$$

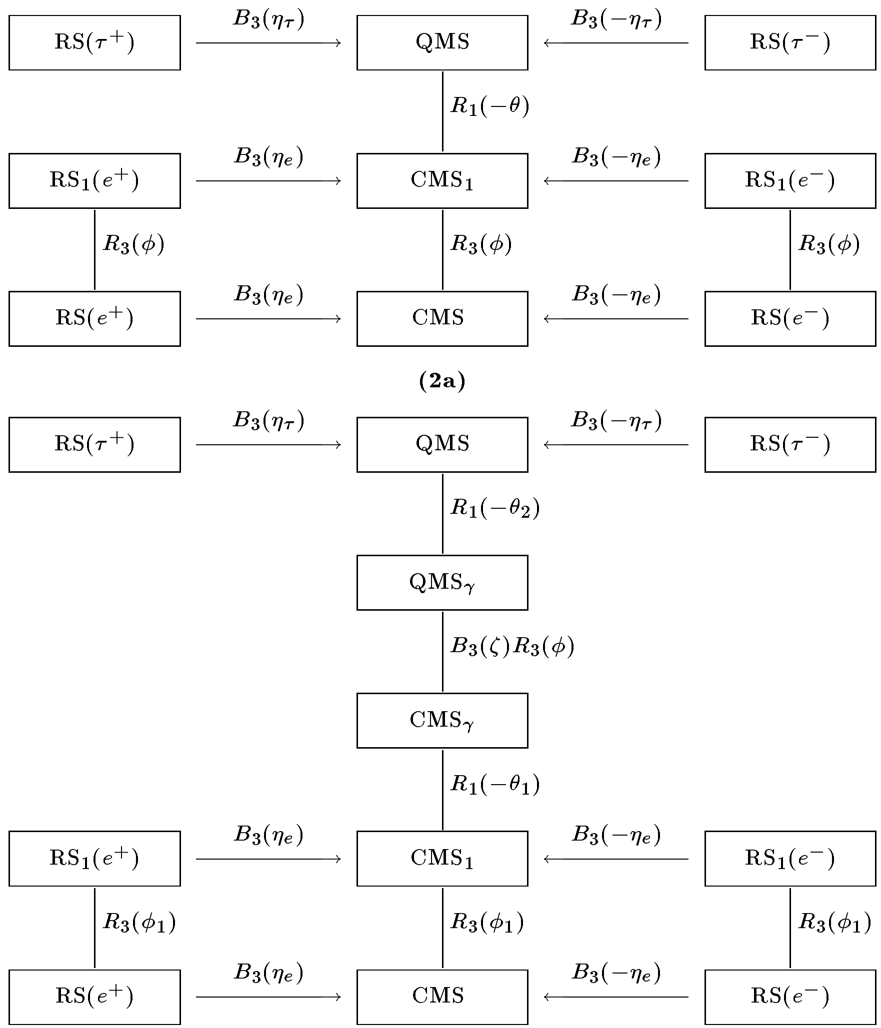
$$h^+_\mu = \sum_{\lambda'\bar\lambda'=1}^{2} \sigma_\mu^{\lambda'\bar\lambda'}\,\mathcal{M}^{\tau^+}_{\lambda'}\bar{\mathcal{M}}^{\tau^+}_{\bar\lambda'}$$

- The $R_{\mu\nu}$ depend on kinematic of τ-pair production, $h^\pm_\mu$ on $\tau^\pm$ decays.
- **Important:** reference frame orientation in which these objects are defined.
- In some of our programs, frames help exposing properties of matrix elements.
- Useful to visualize factorization properties (even if in principle, not needed).

Frames for spin: help expose properties of production and decay ME's.

Often ignored... but essential for event record standards interfaces and pheno intuition.

Figure 2



(2b)

Comments

1. **Exact universal formulae of previous slides:** are the templates only.
2. **Reference frames**: why so many? Useful for phenomenology. GPS in KKMC Eur.Phys.J.C 22(2001)423.
3. **Decay matrix element (hadronic currents)** of sufficient precision are needed.
4. **Production amplitudes:** The same amplitudes can be used for calculation of differential cross section of τ-pair production, and for calculation of spin effects.
5. **However,** for spin amplitudes complex phases are needed:
   - in many programs phases of Kleiss-Stirling amplitudes are not controlled,
   - for production cross section modules only, phases ignored $\leftrightarrow$ simplification.

Reweight with New Physics effects

Simplified kinematic for NP implementation is sufficient.

Cross section weight:

$$wt_{ME} = \Big(\sum_{spin} |\mathcal{M}^{prod\ SM+NP}|^2\Big) / \Big(\sum_{spin} |\mathcal{M}^{prod\ SM}|^2\Big)$$

Complicated spin correlation weight:

$$wt_{spin} = \Big(\sum_{ij} R_{ij}^{SM+NP} h_+^i h_-^j\Big) / \Big(\sum_{ij} R_{ij}^{SM} h_+^i h_-^j\Big)$$

Spin quantization frames orientation must be the same for production and decay.

Challenge for interfaces, frame useful for optimal variable investigation.

We use KKMC $h_\pm^i$ and its boosting from τ's rest- to lab- frame. Another routine is used to transfer $h_\pm^i$ back to $\tau^\pm$ frame but oriented as in New Physics calculation.

In this way reference frames are OK and impact of photons on phase space parametrisations is under control too.

Solution works for all τ decay modes!

Reweight with New Physics effects

From Phys.Rev.D 106 (2022) 11, 113010, a- magnetic dipole moment, b- electric dipole moment couplings.

$$
\begin{aligned}
R_{11} &= \frac{e^4}{4\gamma^2}\left(4\gamma^2\,\mathrm{Re}(a)+\gamma^2+1\right)\sin^2(\theta),\\
R_{12} &= -R_{21} = \frac{e^4}{2}\beta\sin^2(\theta)\,\mathrm{Re}(b),\\
R_{13} &= R_{31} = \frac{e^4}{4\gamma}\left[(\gamma^2+1)\mathrm{Re}(a)+1\right]\sin(2\theta),\\
R_{22} &= -\frac{e^4}{4}\beta^2\sin^2(\theta),\\
R_{23} &= -R_{32} = -\frac{e^4}{4}\beta\,\gamma\sin(2\theta)\,\mathrm{Re}(b),\\
R_{33} &= \frac{e^4}{4\gamma^2}\left[\left(4\gamma^2\,\mathrm{Re}(a)+\gamma^2+1\right)\cos^2(\theta)+\beta^2\gamma^2\right],\\
R_{14} &= -R_{41} = \frac{e^4}{4}\beta\,\gamma\sin(2\theta)\,\mathrm{Im}(b),\\
R_{24} &= R_{42} = \frac{e^4}{4}\beta^2\,\gamma\sin(2\theta)\,\mathrm{Im}(a),\\
R_{34} &= -R_{43} = -\frac{e^4}{2}\beta\sin^2(\theta)\,\mathrm{Im}(b),\\
R_{44} &= \frac{e^4}{4\gamma^2}\left[4\gamma^2\,\mathrm{Re}(a)+\beta^2\gamma^2\cos^2(\theta)+\gamma^2+1\right].
\end{aligned}
\tag{3}
$$

Reweight with New Physics effects

1) Anomalous magnetic and electric dipole moments spin correlations in τ-lepton pair production are taken from Sw. Banerjee, A.Yu. Korchin, Z. Was, Phys.Rev.D 106 (2022) 11, 113010

2) The observable exploits six-body final state : $\pi^-\pi^0 \;\; \pi^+\pi^0$ and two non-observable neutrinos.

- The CP parity properties may be useful to control background, even if ambiguity of SM simulation would be worse than required precision target.

3) Example decay channel: $\tau^\pm \to \pi^\pm\pi^0\nu$. Test distribution: acoplanarity of the visible decay products oriented half- planes. All in the rest frame of visible decay products system

$$y_1 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}, \qquad y_2 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}. \tag{4}$$

4) Observable does not rely on decay vertex position.

Reweight with New Physics effects

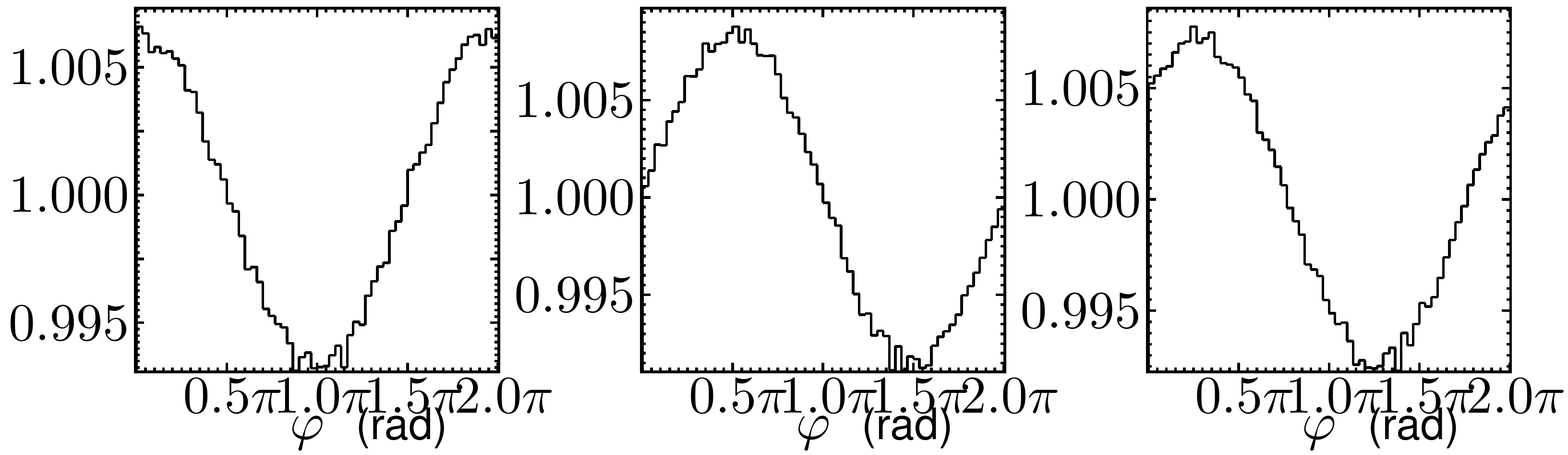


Figure 3: Distribution over acoplanarity angle φ of the ratio $wt_{spin}^{anomalous}$ for $\sqrt{s} = 10.5$ GeV. Constraint $y_1 y_2 > 0$ is imposed. Left: $\mathrm{Re}(a_{NP}) = 0.04$ and other couplings are zero, Center: $\mathrm{Re}(b_{NP}) = 0.04$ and other couplings are zero, Right: $\mathrm{Re}(a_{NP}) = 0.04\,\cos(\pi/4),\ \mathrm{Re}(b_{NP}) = 0.04\,\sin(\pi/4)$ and other couplings are zero. This is idealized (test of the principle) observable. In practice Machine Learning approach, helpful to combine impact from all τ decay channels will be more appropriate. Too many variables, too many cases for human eye. Also partial information on decay vertex position may be used.

Reweight with New Physics effects

This was an example of how to precision simulation additional interaction can be added (without necessity to re-do work on SM interfering background.

Now re-weighting for FCC energies as well: e-Print 2307.03526

Another example (additional slides): of somewhat different type (imprinting extra particle into final state configuration) is covered in:

Symmetries of spin amplitudes: applications for factorization and Monte Carlo solutions, Zbigniew Was (Cracow, INP) DOI: 10.22323/1.406.0008, Published in: PoS CORFU2021 (2022), 008

and

Monte Carlo Event Generator updates, for τ pair events at Belle II energies, Sw. Banerjee, D. Biswas, T. Przedzinski, Z. Was 2111.05914 Contribution to: TAU2021 conference.

Summary

I have addressed some aspects of precision simulation for τ and μ and phenomenology of their couplings:

1) τ lepton production: not much more demanding in comparison to light lepton production: mass terms and better control of amplitudes phases. Issues of improved Born approximatuion and effective couplings was discussed.

2) Implementation of extra interactions: $\to$ large τ-mass may mean large Yukawa couplins to New Physic fields. Technicality: internal variables/methods for extra effects calculation. In fortran all is open. The C++ class private variables/methods usually not for external use.

3) Use of information stored in production files is interesting, offers flerxibility, but compatibility issues may be challenging.

4) KKMC Recent reference: *Multi-photon Monte Carlo event generator KKMCee for lepton and quark pair production in lepton colliders*, S. Jadach, B.F.L. Ward, Z. Was, S.A. Yost, A. Siodmok, (also Marcin Chrzaszcz and Jacek Holeczek) Comput.Phys.Commun. 283 (2023) 108556 BEWARE: expertise preservation and man power for future.

5) KKMC and future precision requirements: *Standard model theory for the FCC-ee Tera-Z stage* A. Blondel et al. Contribution to FCC-ee, 1809.01830.

• PHOTOS (by E.Barberio, B. van Eijk, Z. W., P. Golonka) is used since 1989 to simulate the effects of radiative corrections in decays.

Full events of complicated mother-daughter tree structure of consecutive decays are generated earlier. PHOTOS eventually modify decay (tree branching).

• Web pages of TAUOLA, PHOTOS and MC-TESTER projects:

• Phase-space is again exactÂ and parametrization under full control

• Matrix element: from factorization and with simplifications. Required lots of work.

• For lepton pair emission algorithm works similarly.

• It can be used not only for QED but for New Physics too. Dark photon, extra scalar/pseudo-scalar imprinting into final state. New Physics particles with consecutive decays to lepton pairs.

Phase Space Formula of Photos

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$
$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}\big). \quad (5)$$

1. One can verify that if $dLips_n(P)$ was exact, then this formula lead to exact parametrization of $dLips_{n+1}(P)$
2. Practical implementation: Take the configurations from n-body phase space.
3. Turn it back into some coordinate variables.
4. construct new kinematical configuration from all variables.
5. **Forget about temporary** $k_\gamma \theta \phi$. **From now on, only weight and four vectors count.**
6. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is possible and necessary/convenient if more than one charge is present in final state.

Phase Space: (main formula)

If we choose

$$G_n \;:\; M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \;\rightarrow\; \bar{k}_1 \dots \bar{k}_n \tag{6}$$

and

$$G_{n+1} \;:\; k_\gamma, \theta, \phi, M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \;\rightarrow\; k_1 \dots k_n, k_{n+1} \tag{7}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \dots, \bar{k}_n)). \tag{8}$$

The ratio of the Jacobians form the phase space weight W_n^{n+1} for the transformation. Such solution is universal and valid for any choice of G's. However, G_{n+1} and G_n has to match matrix element, otherwise algorithm will be inefficient (factor 10^{10} ...).

In case of PHOTOS G_n's

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\dots n}, M^2_{2\dots n}/M^2_{1\dots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\dots n}/M^2)}, \tag{9}$$

Add extra particles

once phase-space adjusted, again $M^{SM} \to M^{SM+NP}$ is enough.

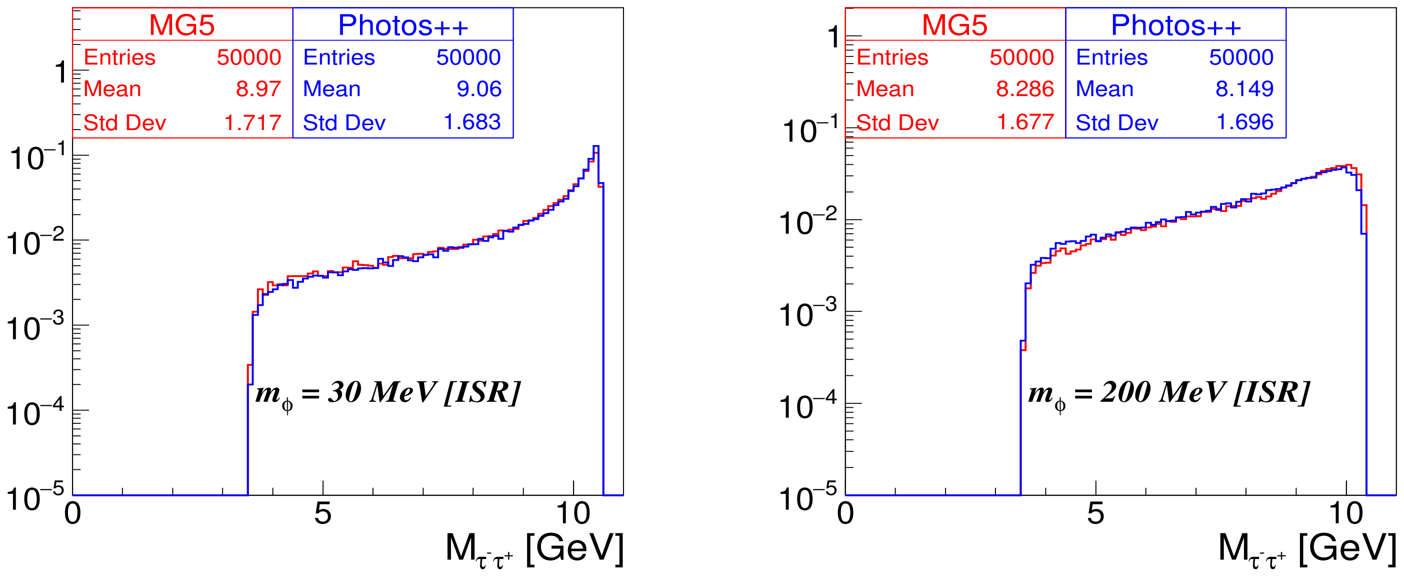


Figure 4: Belle 2 cms energy $e^-e^+ \to \tau^-\tau^+\phi_{\text{Dark Scalar}}(\to e^-e^+)$ Case of dark scalar of 30 and 200 MeV. Simulation of `KKMC+Photos` is compared with the one based on `MadGraph`. Q: Why not use `MadGraph` alone? A: Multiple photon emissions, τ decays with spin. Emission kernel was inspired from that comparison. At start, QED pair emission kernel was used. Spin correlations of τ-s modified by rotation of τ^- decay products.

General formalism for semi-leptonic decays

- Matrix element used in TAUOLA for semi-leptonic decay

$$\tau(P,s) \to \nu_\tau(N) X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N)\gamma^\mu (v + a\gamma_5) u(P) J_\mu$$

- J_μ the current depends on the momenta of all hadrons (o be taken from models and fits).

$$|\mathcal{M}|^2 = G^2 \frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma$$
$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$
$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2} m_\nu M (J^*\cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2} m_\nu\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma$$

Hadronic currents dominate uncertainty

- **For** $\tau \to \rho\nu \to \pi^{\pm}\pi^{0}\nu$ channel fits are straightforward: single 1-variable real function: $J^{\mu} = (p_{\pi^{\pm}} - p_{\pi^{0}})^{\mu} F_V(Q^2) + (p_{\pi^{\pm}} + p_{\pi^{0}})^{\mu} F_S(Q^2)$, ($F_S \simeq 0$).
- **For 3-scalar states:** 4 complex function of 3 variables each. Role of theoretical assumptions is larger. Fits of 1-dim distribution is a consistency check only.
- **No-go for model independent** approach? True, starting from **four scalars**? For three scalars, take all dimensions of data distributions. **(i)** Invariant masses Q^2, s_1, s_2 arguments of form-factors. **(ii)** Angular asymmetries help to separate currents: scalar $J_4^{\mu} \sim Q^{\mu} = (p_1 + p_2 + p_3)^{\mu}$, vector $J_1^{\mu} \sim (p_1 - p_3)^{\mu}|_{\perp Q}$ and $J_2^{\mu} \sim (p_2 - p_3)^{\mu}|_{\perp Q}$ and finally pseudo-vector $J_5^{\mu} \sim \epsilon(\mu, p_1, p_2, p_3)$.
- Model independent methods, template methods, neural networks, multidimensional signatures. **It was easier for Cleo.** There, τ's were produced nearly at rest, ν_τ four-momentum was easy to reconstruct. **But Belle data samples are to be huge.**
- Fitting in complex situation is ... **well complex !**
- **Input from Belle 2 data and collaboration with Belle 2 people indispensable:**

  S. Antropov, Sw. Banerjee (Belle 2), Z. Was, J. Zaremba Comput.Phys.Commun. 283 (2023), 108592

  Monte Carlo Event Generator updates, for τ pair events at Belle II energies Sw. Banerjee (Belle 2), D. Biswas (Belle 2), T. Przedzinski, Z. Was 2111.05914 [TAU2021 conference]

Why optimal variables remain important

Figure 5: Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don't.

From http://googleresearch.blogspot.com/2015/06/inceptionism-going-deeper-into-neural.html

Pattern recognition is an active field and deep concern and not only for us.

Thank you for listening