Froggatt-Nielsen models meet the SMEFT

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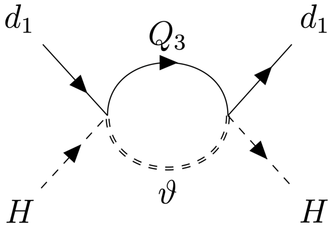
13 June 2024



Based on 2402.16940 in collaboration with Jim Talbert

Motivation

The flavour puzzle: What explains the dramatic hierarchies in fermion masses and mixings?

Patterns especially clear in the quark sector.

Quark masses:

$$\frac{m_u}{m_t} \sim 10^{-5}$$

CKM elements:

$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.009 & 0.04 & 1 \end{pmatrix}$$

$$\Rightarrow V_{11} \gg V_{21} \gg V_{31}$$

Yukawa sector of the SM

$$\mathcal{L} \supset y_{ij}\,\overline{\psi}_i\,H\,\psi_j \longrightarrow \frac{y_{ij}\,v_H}{\sqrt{2}}\,\overline{\psi}_i\,\psi_j$$

Two ingredients:

1. The Higgs vev v_H
2. Dimensionless Yukawa couplings y_{ij}

The mass hierarchies arise from the Yukawa couplings

Hierarchies in Yukawas could be generated anywhere between $\mathcal{O}(\text{TeV})$ and M_{Planck}

Potential solutions: introduce new symmetries, fields, extra dimensions, string theory etc.

No clear winner has emerged after decades of work.

Problems

Too many models available

Many models predict fermion masses by design.
How to falsify or distinguish between them?

Too much work to put bounds on all the different models.

$\longrightarrow$ Ideal situation to use the SMEFT.

Our goals

1. Take a simple model of fermion masses and mixings $\rightarrow$ Froggatt-Nielsen models
2. Match to the SMEFT
3. Study resulting operator and flavour structure

Froggatt-Nielsen Models[1]

One of the oldest and simplest models of flavour.

Setup:

SM fields & $\mathcal{G}_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$+$ new $U(1)$ symmetry (global or gauged)
$+$ heavy flavon field θ to break the symmetry

$+$ unknown UV dynamics: vector-like fermions? We remain agnostic about the details.

[1] Froggatt and Nielsen, 1979

Toy model charge assignments

An example model producing down-quark masses:

Field	$\overline{Q}_1$	$\overline{Q}_2$	$\overline{Q}_3$	d_1	d_2	d_3	H	θ
$U(1)_{\mathrm{FN}}$ charge	6	4	0	5	3	3	-3	-2

Which Yukawa-like terms are allowed?

dim-4: $y^d_{33}\overline{Q}_3 H d_3 + y^d_{32}\overline{Q}_3 H d_2$

dim-5: $c^d_{31}\,\overline{Q}_3 H d_1 \left(\frac{\theta}{\Lambda_{\mathrm{UV}}}\right)$

dim-6: $c^d_{23}\,\overline{Q}_2 H d_3 \left(\frac{\theta}{\Lambda_{\mathrm{UV}}}\right)^2 + c^d_{22}\,\overline{Q}_2 H d_2 \left(\frac{\theta}{\Lambda_{\mathrm{UV}}}\right)^2$

Yukawa sector

$$\mathcal{L} \supset y^d_{ij}\,\overline{Q}_i\,H\,d_j \longrightarrow \mathcal{L} \supset c^d_{ij}\,\overline{Q}_i\,H\,d_j\left(\frac{\theta}{\Lambda_{\mathrm{UV}}}\right)^{x_{ij}}$$

Lower generations come with more powers of $\theta/\Lambda_{\mathrm{UV}}$

Flavon takes a vev:

$$\theta = \frac{v_\theta + \vartheta}{\sqrt{2}}$$

Define $\lambda \equiv \dfrac{v_\theta}{\sqrt{2}\Lambda_{\mathrm{UV}}} \sim 0.1$

$\longrightarrow$ Yukawa matrices populated hierarchically.

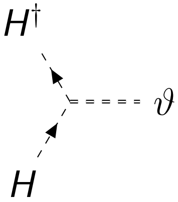
Scalar potential

$$V(H,\theta) = -\mu_H^2 H^\dagger H - \mu_\theta^2 \theta^* \theta + \lambda_{20}(H^\dagger H)^2 + \lambda_{02}(\theta^*\theta)^2 + \lambda_{11}\theta^*\theta H^\dagger H$$

After symmetry breaking:

$$\theta = \frac{v_\theta + \vartheta}{\sqrt{2}}$$

$$V(H,\theta) \supset -\lambda_{11} v_\theta \vartheta H^\dagger H \longrightarrow$$



Matching strategy

1) Write down a Froggatt-Nielsen EFT up to a given operator dimension. At dimension-4:

$$\mathcal{L}_{\mathrm{FN}} \supset y^d_{33}\overline{Q}_3 H d_3 + y^d_{32}\overline{Q}_3 H d_2 - \lambda_{11}(\theta^*\theta)\big(H^\dagger H\big).$$

At dimension-5:

$$\mathcal{L}_{\mathrm{FN}} \supset y^d_{33}\overline{Q}_3 H d_3 + y^d_{32}\overline{Q}_3 H d_2 - \lambda_{11}(\theta^*\theta)\big(H^\dagger H\big)$$

$$+ c^d_{31}\,\overline{Q}_3 H d_1 \left(\frac{\theta}{\Lambda_{\mathrm{UV}}}\right)$$

and so on.

2) Break the $U(1)_{\mathsf{FN}}$ symmetry:

$$\theta = \frac{v_\theta + \vartheta}{\sqrt{2}}$$

3) Integrate out ϑ and match to the SMEFT up to a given operator dimension.

Organisation

We need to approach the matching systematically.
We can:

1. Go to higher operator dimensions in $\mathcal{L}_{\mathsf{FN}}$

2. Go to higher operator dimensions in the SMEFT

3. Match at tree-level, one-loop, two-loop...?

Organisation

We need to approach the matching systematically.
We can:

1. Go to higher operator dimensions in $\mathcal{L}_{\text{FN}}$
   $d_{\text{FN}} = 4, 5$
2. Go to higher operator dimensions in the SMEFT
   $d_{\text{SMEFT}} = 6$
3. Match at tree-level, one-loop, two-loop...?
   Tree- and one-loop-level

Technical details

We have obtained our tree-level results manually and loop-level results using `Matchete`[2] which uses the functional method.

Have manually cross-checked loop-level results using diagrammatic matching.

[2] Fuentes-Martin et al., 2212.04510

$d_{\mathsf{FN}} = 4$; $d_{\mathsf{SMEFT}} = 6$; tree-level

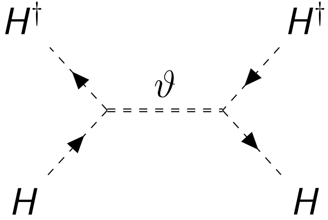
The only non-trivial Lagrangian term comes from the scalar potential:

$$\mathcal{L}_{\mathsf{FN}}^{d=4} \supset y_{33}^d \overline{Q}_3 H d_3 + y_{32}^d \overline{Q}_3 H d_2 - \lambda_{11} \theta^* \theta H^\dagger H$$

After SSB:

$$\mathcal{L}_{\mathsf{FN}}^{d=4} \supset y_{33}^d \overline{Q}_3 H d_3 + y_{32}^d \overline{Q}_3 H d_2 - \lambda_{11} v_\theta \vartheta \left(H^\dagger H\right) - \frac{\lambda_{11}}{2} \vartheta^2 \left(H^\dagger H\right)$$

Integrate out ϑ:

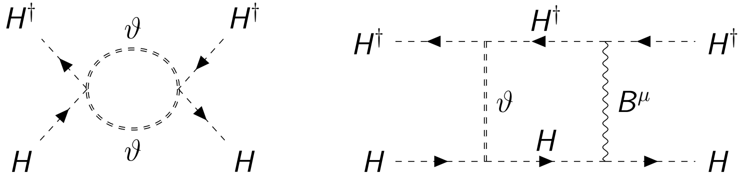


$$\longrightarrow \left(H^\dagger H\right)\Box\left(H^\dagger H\right)$$

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{\lambda_{11}^2 v_\theta^2}{2m_\theta^4}\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$$

$d_{\mathsf{FN}} = 4$; $d_{\mathsf{SMEFT}} = 6$; loop-level

Many more diagrams. E.g.



Matching done by Jiang et al.,1811.08878
and Haisch et al., 2003.05936

$d_{\mathsf{FN}} = 5$; $d_{\mathsf{SMEFT}} = 6$; tree-level

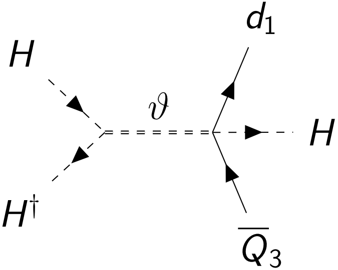
$$\mathcal{L}_{\mathsf{FN}}^{d=5} = \mathcal{L}_{\mathsf{FN}}^{d=4} + c_{31}^{d}\,\overline{Q}_3 H d_1 \left(\frac{\theta}{\Lambda_{\mathsf{UV}}}\right)$$

After SSB:

$$\mathcal{L}_{\mathsf{FN}}^{d=5} = \mathcal{L}_{\mathsf{FN}}^{d=4} + c_{31}^{d}\lambda\,\overline{Q}_3 H d_1 + c_{31}^{d}\,\overline{Q}_3 H d_1 \left(\frac{\vartheta}{\Lambda_{\mathsf{UV}}}\right)$$

(Recall $\lambda \sim v_\theta/\Lambda_{\mathsf{UV}} \sim 0.1$)

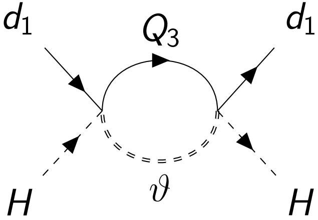
Matching



$$\longrightarrow \left(H^\dagger H\right)\left(\overline{Q}_3 H d_1\right)$$

$$\mathcal{L}_{\mathrm{SMEFT}} \supset \frac{\lambda\lambda_{11}c^d_{31}}{m_\theta^2}\left(H^\dagger H\right)\left(\overline{Q}_3 H d_1\right) + \mathrm{H.c.}$$

$d_{\mathsf{FN}} = 5$; $d_{\mathsf{SMEFT}} = 6$; loop-level



$$\longrightarrow C_{Hd}^{11}(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{d}_1\gamma^\mu d_1)$$

where

$$C_{Hd}^{11} = \frac{|c_{31}^d|^2\lambda^2}{64\pi^2 m_\theta^2}(1 + 2\mathbb{L}).$$

(Have defined $\mathbb{L} = \log \mu^2/m_\theta^2$)

Key findings at 1-loop

Main operator types:

Higgs-enhanced Yukawas: $(H^\dagger H)\,\overline{\psi}_i H \psi_j$

Higgs kinetic operators: $(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{\psi}_i \gamma^\mu \psi_j)$

4-fermion operators: $(\overline{\psi}_i \psi_j)(\overline{\psi}_k \psi_l)$

Flavour patterns controlled by powers of λ

Some operator classes appear at tree-level, others loop-suppressed

Conclusions

Goal: Understand the infrared imprint of Froggatt-Nielsen models.

Method: Systematically match a Froggatt-Nielsen EFT to the SMEFT.

Findings: Rich flavour structure especially in $(H^\dagger H)\overline{\psi}_i H\psi_j$, $(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{\psi}_i\gamma^\mu\psi_j)$ and $(\overline{\psi}_i\gamma^\mu\psi_j)(\overline{\psi}_k\gamma^\mu\psi_l)$ operators.

Wilson coefficients show hierarchies controlled by λ.

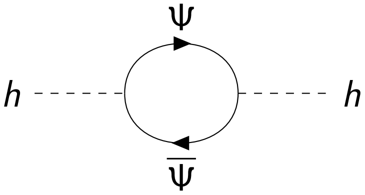
The End

Thank you for listening!

Back-up

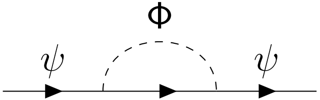
UV sensitivity of Yukawas

Note: The flavour puzzle is different from the *hierarchy problem* concerning the Higgs mass m_H^2.



$$\longrightarrow \quad \delta m_H^2 \sim M_\Psi^2$$

versus



$$\longrightarrow \delta m_\psi \sim m_\psi \log(M_\Phi / m_\psi)$$

Some model predictions

Nir and Seiberg noticed (hep-ph/9212278, hep-ph/9310320) that FN models necessarily predict e.g.

$$|V_{ub}| \sim |V_{us} V_{cb}|,$$

$$|V_{ij}| \gtrsim \frac{m_{u_i}}{m_{u_j}},$$

$$|V_{ij}| \gtrsim \frac{m_{d_i}}{m_{d_j}}.$$

These relations could fail in nature but they do not.
$\longrightarrow$ Promising for FN models.

Example of spurion analysis: Higgs kinetic operators

$$C_{Hd}^{ij}\left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\overline{d}_i\gamma^\mu d_j\right)$$

with

$$C_{Hd}^{ij} \sim \frac{1}{32\pi^2 m_\theta^2}\begin{pmatrix} 0.1\lambda_{11}^2 & \lambda\lambda_{11} & \lambda\lambda_{11} \\ \lambda\lambda_{11} & -\lambda_{11}^2 & -\lambda_{11}^2 \\ \lambda\lambda_{11} & -\lambda_{11}^2 & -\lambda_{11}^2 \end{pmatrix}$$

Recall:

Field	d_1	d_2	d_3	θ
$U(1)_{\mathrm{FN}}$ charge	5	3	3	-2

Goldstones: a threat or an opportunity?

The flavon field θ contains two degrees of freedom

$$\theta = \frac{v_\theta + \vartheta + i\pi}{\sqrt{2}}$$

If $U(1)_{\mathrm{FN}}$ is a global symmetry, its breaking gives a light (pseudo-)Goldstone boson.

Calibbi et al., 1612.08040 and Ema et al., 1612.05492 identify π as the QCD axion.

Severe constraints on the axion mass and decay constant imply $v_\theta \gtrsim 10^{14}$ GeV

Gauged $U(1)_{\mathrm{FN}}$

Introduce a Z' boson which eats the π component of the scalar field.

The covariant derivatives

$$\mathcal{L} \supset \overline{\psi}_i i \not{D} \psi_i + (D_\mu H)^\dagger (D^\mu H)$$

yield interactions between the SM fields and the Z'.

Phenomenologically more viable than having a Goldstone.

All potential complications can be removed by assuming we have a *discrete* flavour symmetry instead of a $U(1)_{\mathrm{FN}}$.

For the toy model

$$\mathcal{L}_{\mathrm{FN}} \supset -g_{\mathrm{F}} Z'_\mu \big(\Omega_{ij} \overline{d}_i \gamma^\mu d_j + \xi_{ij} \overline{Q}_i \gamma^\mu Q_j\big) \,,$$

with two flavour matrices

$$\Omega = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \xi = -\begin{pmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Integrating out Z' gives

$$\mathcal{L}^{d=6}_{\mathrm{SMEFT}} \supset -\frac{g_{\mathrm{F}}^2\, \Omega_{ij}\Omega_{kl}}{2M_{Z'}^2} \big(\overline{d}_i \gamma^\mu d_j\big)\big(\overline{d}_k \gamma_\mu d_l\big)$$
$$-\frac{g_{\mathrm{F}}^2\, \xi_{ij}\xi_{kl}}{2M_{Z'}^2} \big(\overline{Q}_i \gamma^\mu Q_j\big)\big(\overline{Q}_k \gamma_\mu Q_l\big) - \frac{g_{\mathrm{F}}^2\, \Omega_{ij}\xi_{kl}}{M_{Z'}^2} \big(\overline{d}_i \gamma^\mu d_j\big)\big(\overline{Q}_k \gamma_\mu Q_l\big).$$

The Higgs covariant derivatives contribute to:

$$\mathcal{L}^{d=6}_{\text{SMEFT}} \supset -\frac{9g_{\text{F}}^2}{2M_{Z'}^2}\big(H^\dagger H\big)\Box\big(H^\dagger H\big) - \frac{18g_{\text{F}}^2}{M_{Z'}^2}\big(H^\dagger D^\mu H\big)^*\big(H^\dagger D_\mu H\big)$$

Finally, can use the Z' to "connect" the two covariant derivatives:

$$\mathcal{L}^{d=6}_{\text{SMEFT}} \supset -\frac{3g_{\text{F}}^2\Omega_{ij}}{M_{Z'}^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\overline{d}_i\gamma^\mu d_j) - \frac{3g_{\text{F}}^2\xi_{ij}}{M_{Z'}^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\overline{Q}_i\gamma^\mu Q_j).$$

Basics of EFT matching

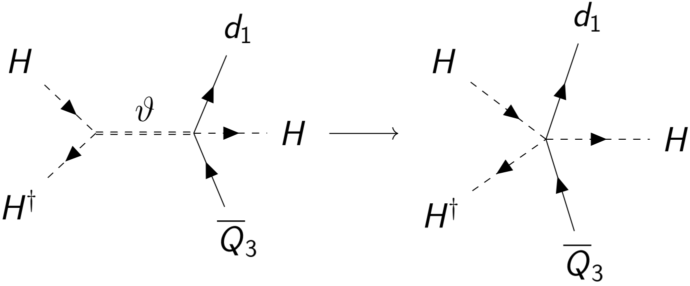
There are two main ways to match UV theories to EFTs:

1) Diagrammatic matching
Require that at low energies, UV theory and EFT give identical scattering amplitudes

$$i\mathcal{A}_{\text{EFT}}(p_i) = i\mathcal{A}_{\text{UV}}(p_i) \quad \text{for } p_i \ll \Lambda_{\text{UV}}$$

Draw UV diagrams with heavy internal lines.
Expand heavy propagators as

$$\frac{i}{p_i^2 - m_\theta^2} \to \frac{-i}{m_\theta^2}\left(1 + \frac{p_i^2}{m_\theta^2} + \dots\right)$$



$\longrightarrow$ UV diagram captured by local EFT operator

$$C_{dH}^{31}\left(H^\dagger H\right)\left(\overline{Q}_3 H d_1\right)$$

2) Functional matching

Start with UV Lagrangian $\mathcal{L}_{\mathsf{UV}}$ and derive Euler-Lagrange equation for heavy field:

$$\partial_\mu \frac{\delta \mathcal{L}_{\mathsf{UV}}}{\delta(\partial_\mu \vartheta)} = \frac{\delta \mathcal{L}_{\mathsf{UV}}}{\delta \vartheta}$$

We get

$$\begin{aligned}(\Box + m_\theta^2)\vartheta = &- \lambda_{11} v_\theta \left(H^\dagger H\right) - \lambda_{11}\left(H^\dagger H\right)\vartheta \\ &+ \frac{c_{31}}{\Lambda_{\mathsf{UV}}} \overline{Q}_3 H d_1 + \dots\end{aligned}$$

Get a recursive equation for ϑ:

$$\vartheta = \frac{-\lambda_{11} v_\theta}{\Box + m_\theta^2}\left(H^\dagger H\right) - \frac{\lambda_{11}}{\Box + m_\theta^2}\left(H^\dagger H\right)\vartheta + \frac{c_{31}}{\Lambda_{\mathrm{UV}}(\Box + m_\theta^2)}\,\overline{Q}_3 H d_1 + \ldots$$

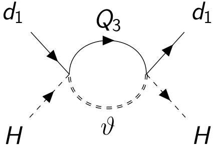
Plug back into $\mathcal{L}_{\mathrm{UV}}$ to eliminate ϑ from the theory.

The resulting Lagrangian is the EFT.

Diagrammatic and functional methods give identical results.

Both methods extend to loop-level.

Diagrammatically:



$\sim \int \frac{d^d k}{(2\pi)^d}(\ldots)$

The "hard region" $k^2 \sim m_\theta^2$ of the loop integral gives a contribution to a local EFT operator

$$\longrightarrow C_{Hd}^{11}(H^\dagger i \overleftrightarrow{D}_\mu H)\big(\overline{d}_1 \gamma^\mu d_1\big)$$

Functional method at 1-loop is more complicated.

Functional matching beyond tree-level

The following is copied from Cohen, Lu and Zhang, 2011.02484:

Write fields as $\varphi = (\Phi, \phi)$ where Φ stands for heavy fields.

Matching based on equating 1LPI effective actions of the EFT and UV theory:

$$\Gamma_{\mathrm{EFT}}[\phi] = \Gamma_{\mathrm{L,UV}}[\phi].$$

At tree-level,

$$\mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}[\phi] = \mathcal{L}_{\mathrm{UV}}[\Phi, \phi]\Big|_{\Phi=\Phi_c[\phi]}$$

At 1-loop:

$$\Gamma^{\text{1-loop}}_{\text{L,UV}}[\phi] = \Gamma^{\text{1-loop}}_{\text{L,UV}}[\phi]\Big|_{\text{hard}} + \Gamma^{\text{1-loop}}_{\text{L,UV}}[\phi]\Big|_{\text{soft}}$$

and

$$\Gamma^{\text{1-loop}}_{\text{EFT}}[\phi] = \mathcal{S}^{\text{1-loop}}_{\text{EFT}} + \Big(\text{1-loop contributions from } \mathcal{L}^{(\text{tree})}_{\text{EFT}}\Big)$$

Second terms in both expressions equal. Get

$$\mathcal{S}^{\text{1-loop}}_{\text{EFT}} = \Gamma^{\text{1-loop}}_{\text{L,UV}}[\phi]\Big|_{\text{hard}}$$

How does one evaluate the RHS?

$$\mathcal{S}_{\text{EFT}}^{\text{1-loop}} = \frac{i}{2} \log \text{Sdet}\left(-\frac{\delta^2 \mathcal{S}_{\text{UV}}}{\delta \phi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right)\bigg|_{\text{hard}}$$

$$= \frac{i}{2} \text{STr} \log \boldsymbol{K}\big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{Sdet}\Big[\big(\boldsymbol{K}^{-1} \boldsymbol{X} \big)^n \Big]\big|_{\text{hard}}$$

Truncate at desired order in EFT expansion.