Recent progress on Bosonic HEFT:

Renormalization, Matching and Colliders

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Content of this talk based on results in:

LO

2312.03877, EPJC 84 (2024)5, 503, Davila, Domenech, Herrero, Morales
(a,b) = (κ_V, κ_{2V}) correlations

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez

NLO

2208.05452, Phys. Rev. D 106 (2022)11, 115027, Domenech, Herrero, Morales, Ramos
(a_i NLO coeffs, (η, δ) the most relevant)

NLO+loops

2405.05385 Anisha, Domenech, Englert, Herrero, Morales (gg to HH and HHH)

→ **HH (HHH) within HEFT**

Renorm. in R_ξ

2208.05900, Phy.Rev.D 106 (2022)7, 073008, Herrero, Morales (WW to HH)
(Series of works in R_ξ : 2005.03537, 2107.07890, 2208.05900, 2405.05385)

→ **Introducing loops via 1PIs**

Matching Amplitudes

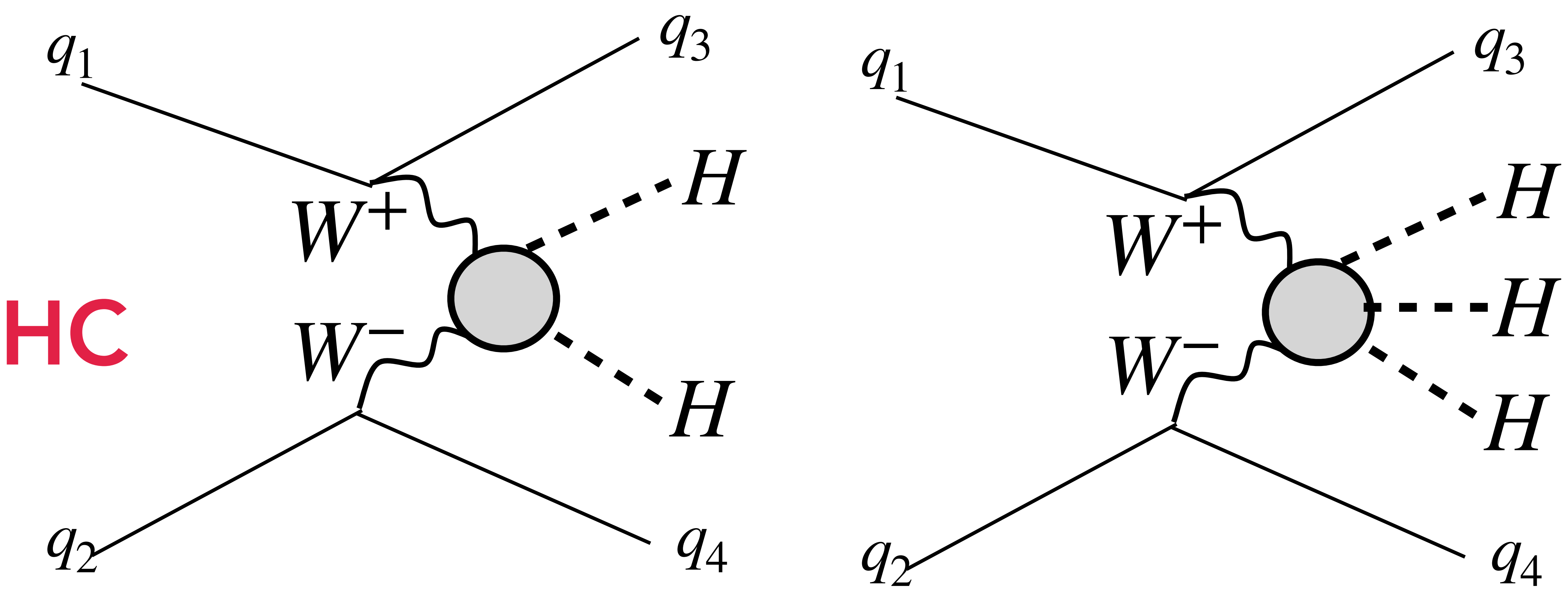
2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales → **Matching HEFT-2HDM**

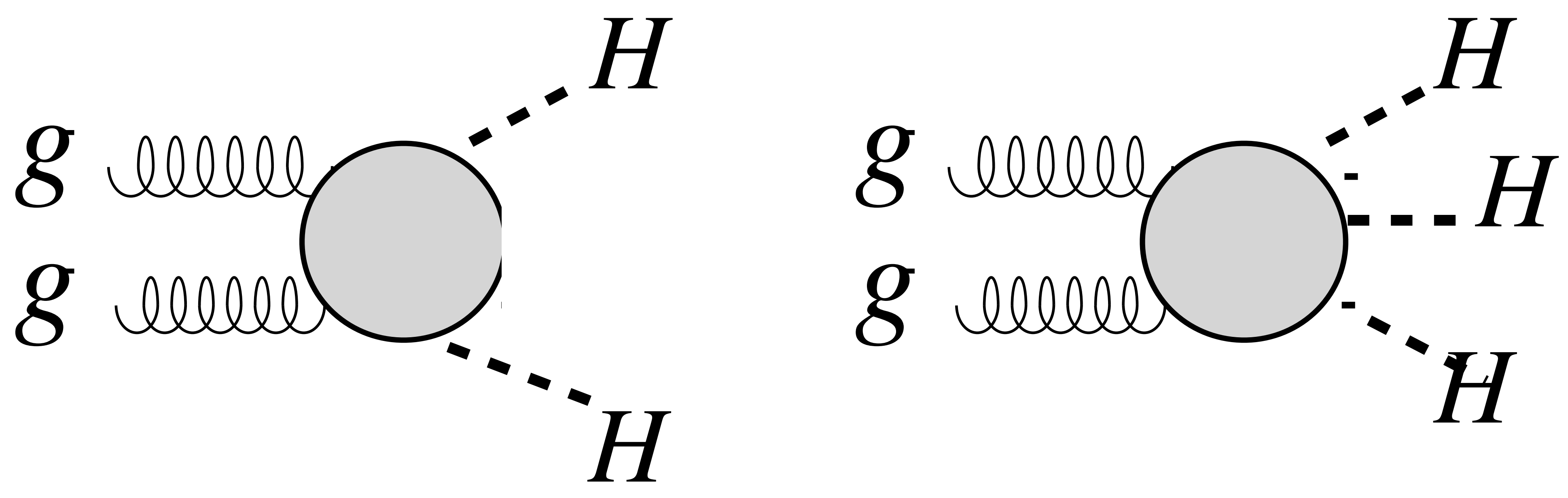
2208.05452, Phys. Rev. D 106 (2022)11, 115027, Domenech, Herrero, Morales, Ramos → **Matching HEFT-SMEFT**

Tools used: FeynArts, FeynRules, FormCalc, Looptools, MG5, VBFNLO, HEFT model file included

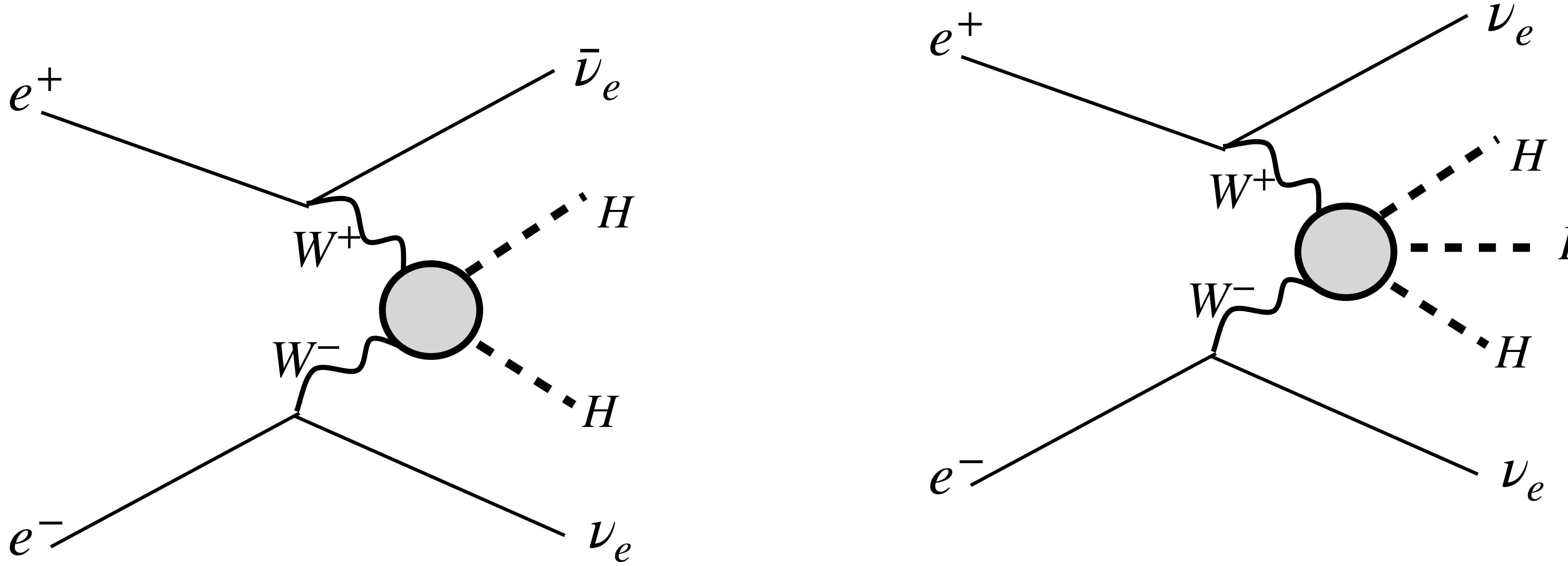
Focus here: relevant HEFT issues for HH and HHH

LHC





e^+e^-



Correlations within SM :

$V_{HWW} = vV_{HHWW}$

$V_{HHH} = vV_{HHHH}$

What about BSM?

Do we learn anything comparing HH and HHH production?

Bosonic HEFT (=EChL): proper tool for BSM MultiHiggs at pp and ee. The issue of H being a singlet has relevant consequences. The issue of non-linearity has relevant consequences.
These (V_{HWW}, V_{HHWW}) and (V_{HHH}, V_{HHHH}) are uncorrelated in the HEFT (in contrast to SMEFT, 2HDM, ..)
Easy connection of HEFT with kappa formalism. Fermionic sector assumed here to be as in the SM.

HH and HHH (EW) production with LO-HEFT: a, b, κ_3 c, κ_4 ...

SM: $\mathbf{a = b = \kappa_3 = \kappa_4 = 1, c = 0}$

$$\mathscr{L}^{LO}_{HEFT} = \frac{v^2}{4}\left[1 + 2a\left(\frac{H}{v}\right) + b\left(\frac{H}{v}\right)^2 + c\left(\frac{H}{v}\right)^3\right] \mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] - \kappa_3 \lambda v H^3 - \frac{1}{4}\kappa_4 \lambda H^4 - \frac{1}{2} m_H^2 H^2$$

$$+ \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2g^2}\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] - \frac{1}{2g'^2}\mathrm{Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

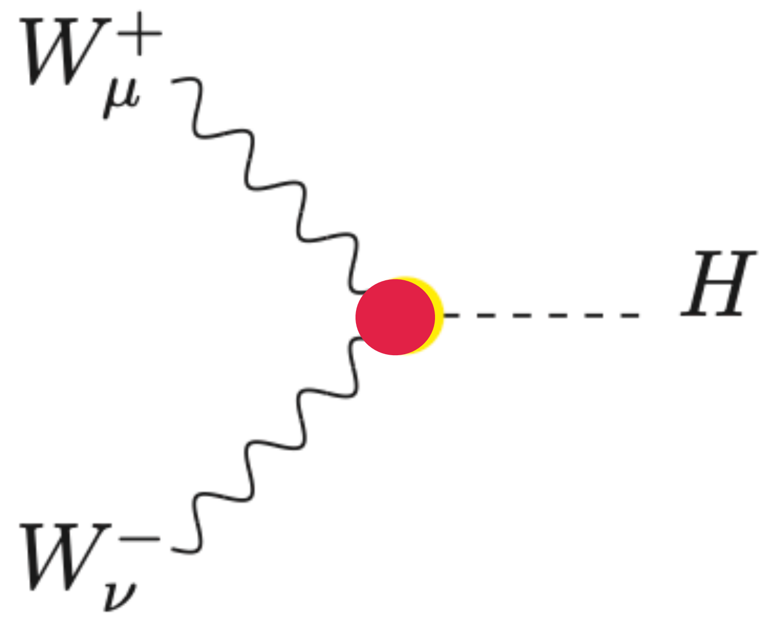
$$D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu$$

$$U(\omega^a) = e^{\omega^a \tau^a / v}$$

$\omega^a = \pi^a$ — Non-Linear GBs

H — Higgs in Polynomials — Higgs is singlet

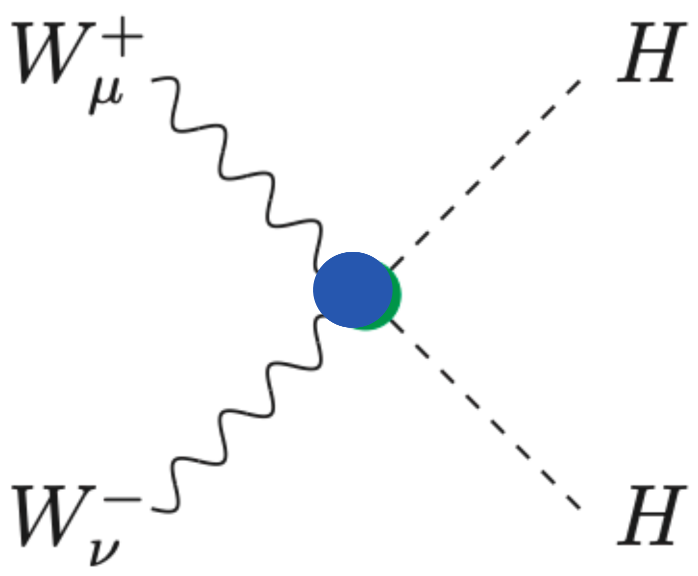
$m_H^2 = 2\lambda v^2$; $m_W = gv/2$; $m_Z = m_W/c_W$



$a = \kappa_V$

$$i\Gamma_{WWH} = iagm_W g_{\mu\nu}$$

$a^{exp} \in [0.97, 1.13]$[1]



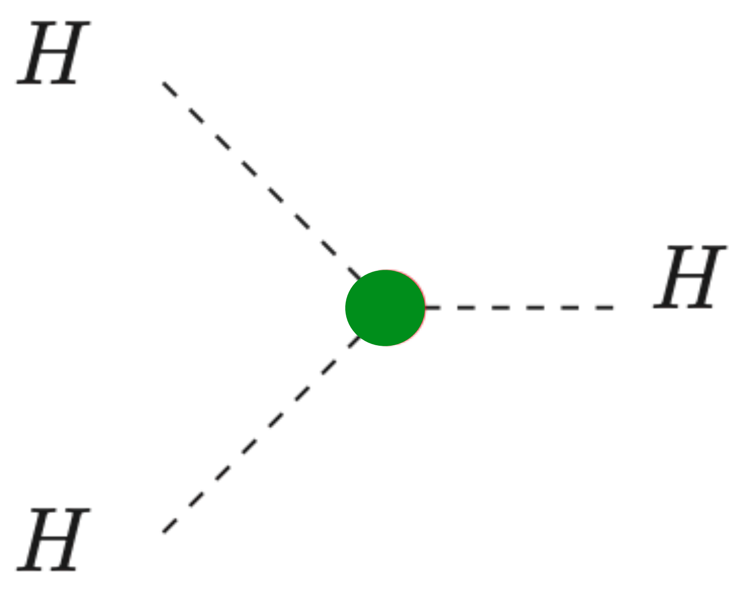
$b = \kappa_{2V}$

$$i\Gamma_{WWHH} = \frac{ibg^2}{2} g_{\mu\nu}$$

$b^{exp} \in [0.0, 2.1]$[2a]

$b^{exp} \in [-0.1, 2.2]$(2b)

$b^{exp} \in [-0.03, 2.11]$(2c)

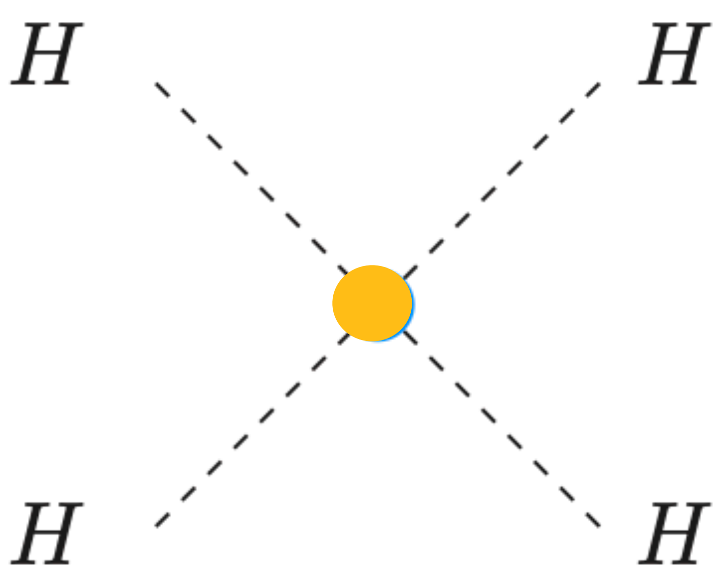


$\kappa_3 = \kappa_\lambda$

$$i\Gamma_{HHH} = -i\kappa_3 6\lambda v$$

$\kappa_3^{exp} \in [-0.4, 6.3]$[3a]

$\kappa_3^{exp} \in [-1.24, 6.49]$[3b]



κ_4

$$i\Gamma_{HHHH} = -i\kappa_4 6\lambda$$

κ_4^{exp} unconstrained

LO uncorrelated coeffs.

a versus b

κ_3 versus κ_4

In contrast to
SM, SMEFT, 2HDM,..
(where H is in a doublet)

[1]ATLAS, Phys. Rev. D **101** (2020) [1909.02845]

[2a]CMS, PLB 842,137531 (2023) [2206.09401]

(2b) CMS, PRL129, 081802(2022) [2202.09617]

(2c) ATLAS, PRD108, 052003(2023) [2301.03212]

[3a]ATLAS (PLB 843 (2023)137745

[3b]CMS (Nature 607, 7917, 2022)

HH (EW) with NLO-HEFT (15 $a_i's$ coeffs. + LO a, b, κ_3)

2208.05900, Phy.Rev.D 106(2022)7, 073008, Herrero, Morales (WW to HH)

$$\mathscr{L}_{\rm HEFT}^{\rm NLO} = \,..\, -a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}{\rm Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v} + a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right){\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]$$
$$-\left(a_{HWW}\frac{H}{v} + a_{HHWW}\frac{H^2}{v^2}\right){\rm Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big] + i\left(a_{d2} + a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}{\rm Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$
$$+\left(a_{\Box\mathcal{V}\mathcal{V}} + a_{H\Box\mathcal{V}\mathcal{V}}\frac{H}{v}\right)\frac{\Box H}{v}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}^\mu\Big] + a_{d3}\frac{\partial^\nu H}{v}{\rm Tr}\Big[\mathcal{V}_\nu\mathcal{D}_\mu\mathcal{V}^\mu\Big]$$
$$+\left(a_{\Box\Box} + a_{H\Box\Box}\frac{H}{v}\right)\frac{\Box H\,\Box H}{v^2} + a_{dd\Box}\frac{\partial^\mu H\,\partial_\mu H\,\Box H}{v^3} + a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$

$$\mathcal{V}_\mu = (D_\mu U)U^\dagger$$

e.o.m

$$\Box H = -m_h^2 H - \tfrac{3}{2}\kappa_3 m_h^2\tfrac{H^2}{v} - \tfrac{a}{2}v{\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big] - \tfrac{b}{2}H{\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]$$
$${\rm Tr}\Big[\tau^j\mathcal{D}_\mu\mathcal{V}^\mu\Big] = -{\rm Tr}\Big[\tau^j\mathcal{V}^\mu\Big]\tfrac{2a}{v}\partial_\mu H$$

Full operators list given in the literature (see, for instance, Brivio et al 1311.1823) →

summarized by: $a_{dd\mathcal{V}\mathcal{V}1} \leftrightarrow c_8$, $a_{dd\mathcal{V}\mathcal{V}2} \leftrightarrow c_{20}$, $a_{11} \leftrightarrow c_9$, $a_{HWW} \leftrightarrow a_W$, $a_{HHWW} \leftrightarrow b_W$, $a_{d2} \leftrightarrow c_5$, $a_{Hd2} \leftrightarrow a_5$, $a_{\Box\mathcal{V}\mathcal{V}} \leftrightarrow c_7$, $a_{H\Box\mathcal{V}\mathcal{V}} \leftrightarrow a_7$, $a_{d3} \leftrightarrow c_{10}$, $a_{Hd3} \leftrightarrow a_{10}$, $a_{\Box\Box} \leftrightarrow c_{\Box H}$, $a_{H\Box\Box} \leftrightarrow a_{\Box H}$, $a_{dd\Box} \leftrightarrow c_{\Delta H}$, $a_{H\mathcal{V}\mathcal{V}} \leftrightarrow a_C$ and $a_{HH\mathcal{V}\mathcal{V}} \leftrightarrow b_C$.

$$\mathscr{L}_{\rm HEFT}^{\rm NLO+e.o.m} = \,..\, -a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}{\rm Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v} + a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right){\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big] + a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$
$$-\left(a_{HWW}\frac{H}{v} + a_{HHWW}\frac{H^2}{v^2}\right){\rm Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big] + i\left(a_{d2} + a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}{\rm Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$

The most relevant are $a_{ddVV1} \equiv \eta$, $a_{ddVV2} \equiv \delta$

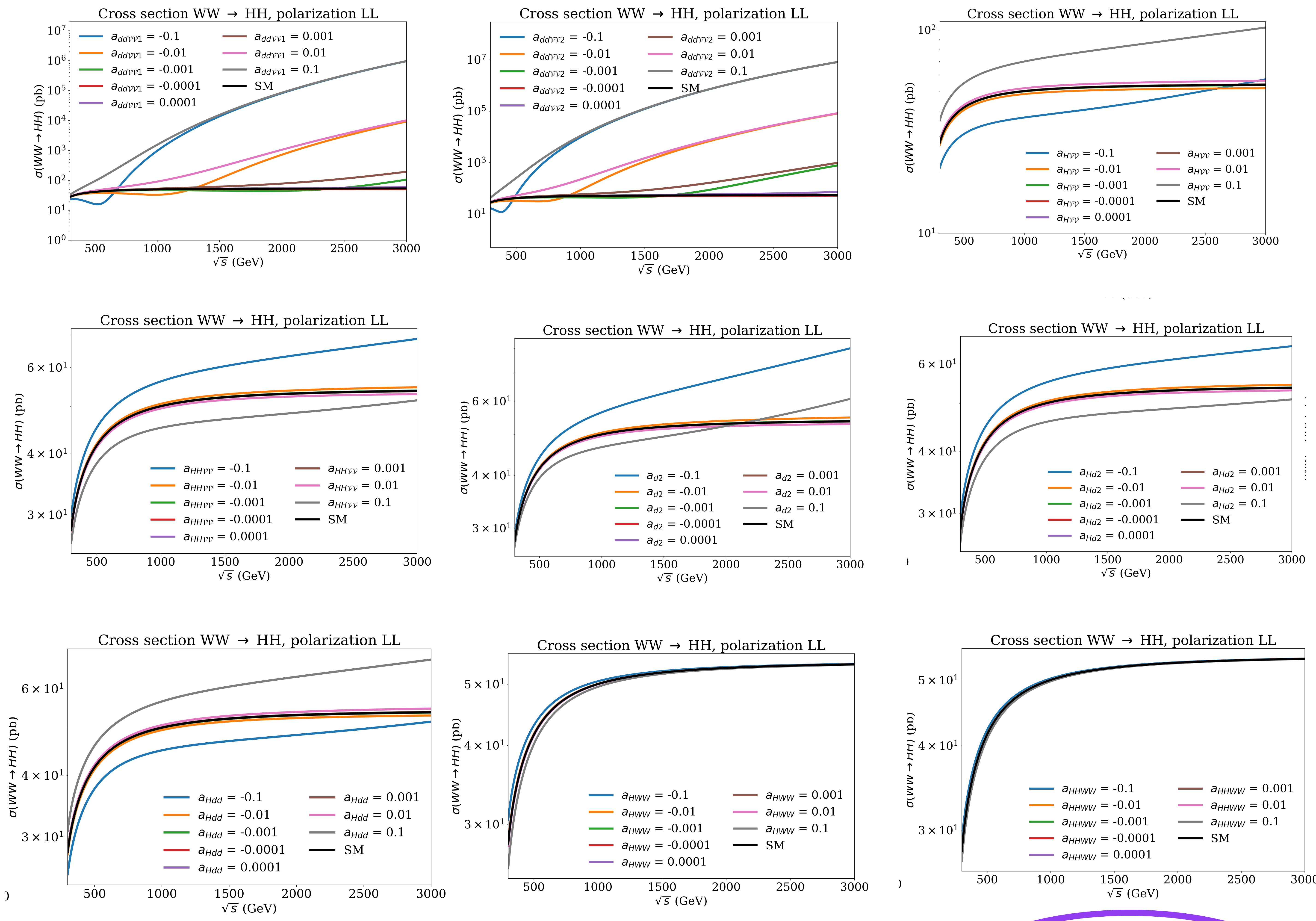
These operators contain 4 derivatives !!

Reduction to 9 $a_i's$ NLO coefficients entering into $WW \to HH$

Comparing the relevance of the various NLO $a_i's$ at $WW \to HH$

$a_{ddVV1} \equiv \eta$ $\quad$ $a_{ddVV2} \equiv \delta$

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos



For similar size of the $a_i's$ we find the largest xsections for

$a_{ddVV1} \equiv \eta$, $a_{ddVV2} \equiv \delta$

by several orders of magnitud !!

$$a_{ddVV1}(1/v^2)\,\partial^\mu H \partial^\nu H \,\mathrm{Tr}\,[(D_\mu U^+)(D_\nu U)] + a_{ddVV2}(1/v^2)\,\partial^\mu H \partial_\mu H \,\mathrm{Tr}\,[(D^\nu U^+)(D_\nu U)]$$

NLO: (η, δ) faster growth $A \sim \mathcal{O}(s^2)$

than **LO:** (a, b) $\quad A \sim \mathcal{O}(s)$

Compare sensitivities to LO/NLO coefficients

LO $\Delta a, \Delta b, \Delta k_3 \sim \mathcal{O}(10^{-1})$ 2312.03877 CLIC

$\Delta\kappa_4 \sim \mathcal{O}(10)$ $WW \to HHH$ 2011.13195 CLIC

NLO $\eta, \delta \sim \mathcal{O}(10^{-3})$ 2208.05452 CLIC

These results can be translated from HEFT to SMEFT using our results from matching

In particular using the relations of chdim 4 in HEFT with dim 8 in SMEFT (see later)

$$\eta \quad a_{ddVV1} = \frac{v^4}{4\Lambda^4}[a^{(1)}_{\phi^4} + a^{(2)}_{\phi^4}]$$

$$\delta \quad a_{ddVV2} = \frac{v^4}{4\Lambda^4} a^{(3)}_{\phi^4}$$

Relevance of (η, δ) easily understood in ET

These operators contain 4 derivatives !!

Including loop corrections within bosonic-HEFT

(Herrero and Morales Series of works in R_ξ : 2005.03537, 2107.07890, 2208.05900)

★ Developed a practical program to include one-loop HEFT corrections by means of Green functions 1PIs

★ Easy to implement in physical scattering procceses

★ Based on computation of one-loop FDs (graphical/intuitive) easy to implement with usual tools FeynRules, FormCalc, Looptools etc..

★ Renormalization of the involved 1PI Green functions in generic R_ξ gauges, with generic off-shell legs (Running Wilson coeffs. is not enough. Complete Loop computation needed)

★ Master equation to compute renormalized 1PI function within NLO HEFT

$$\hat{\Gamma}^{\rm NLO} = \Gamma^{\rm LO} + \Gamma^{a_i} + \Gamma^{\rm Loop} + \Gamma^{\rm CT}$$

$\hat{\Gamma}^{\rm NLO}$: Finite for all external (off-shell) momenta

Better not to use e.o.m, all operators needed

We use renorm. conditions: OS for W, Z,H…, MSbar for HEFT coefficients

$\Gamma^{\rm LO}$: From $\mathscr{L}^{\rm LO}$ FRs
$a, b, \kappa_3, \kappa_4, \ldots$

Γ^{a_i}: From $\mathscr{L}^{\rm NLO}$ FRs
$a_i \to a_i + \delta a_i$ → needed as new CTs to cancel new divergences from loops

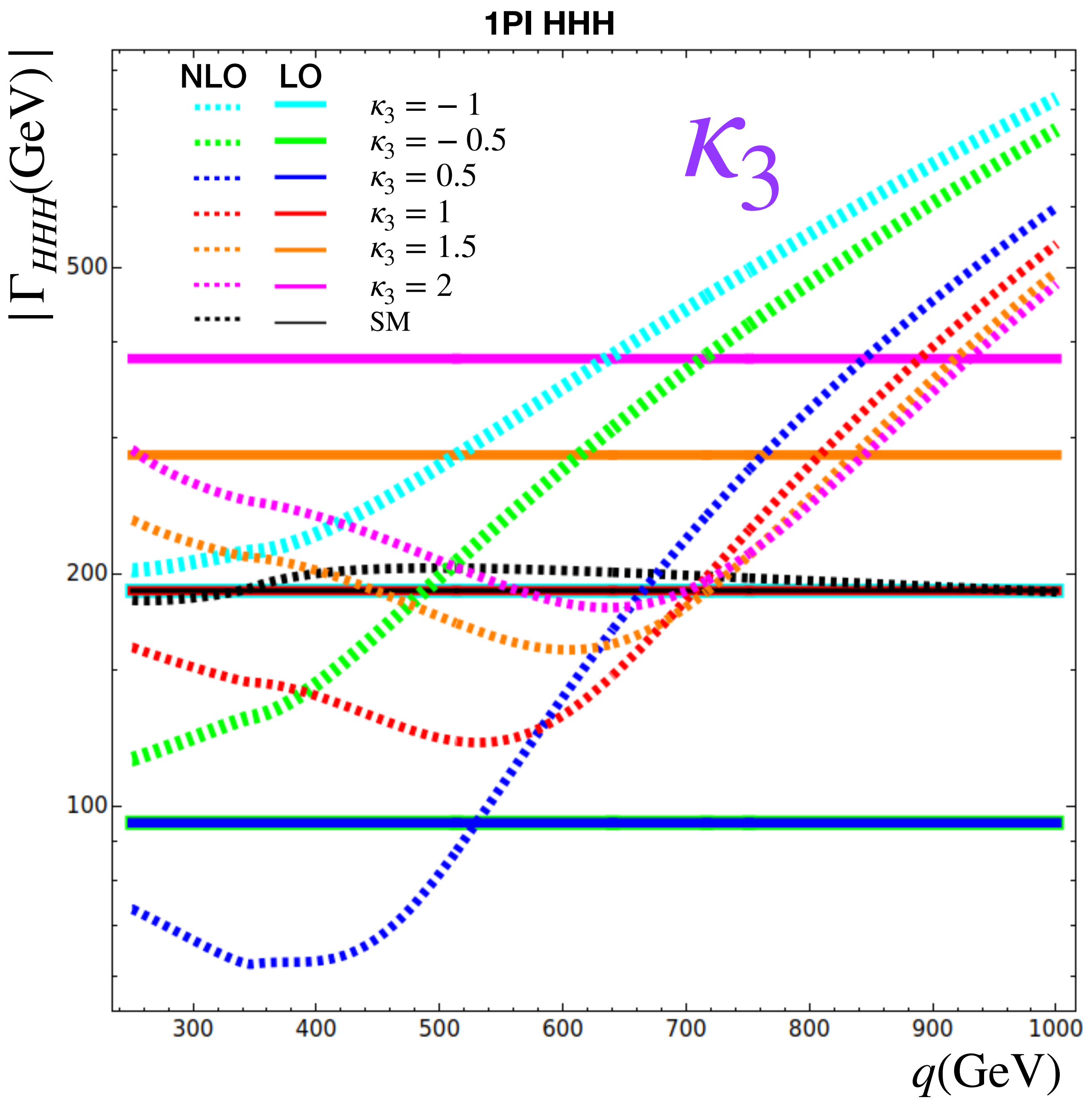
$\Gamma^{\rm Loop}$: From loop diagrams computed with $\mathscr{L}^{\rm LO}$ FRs

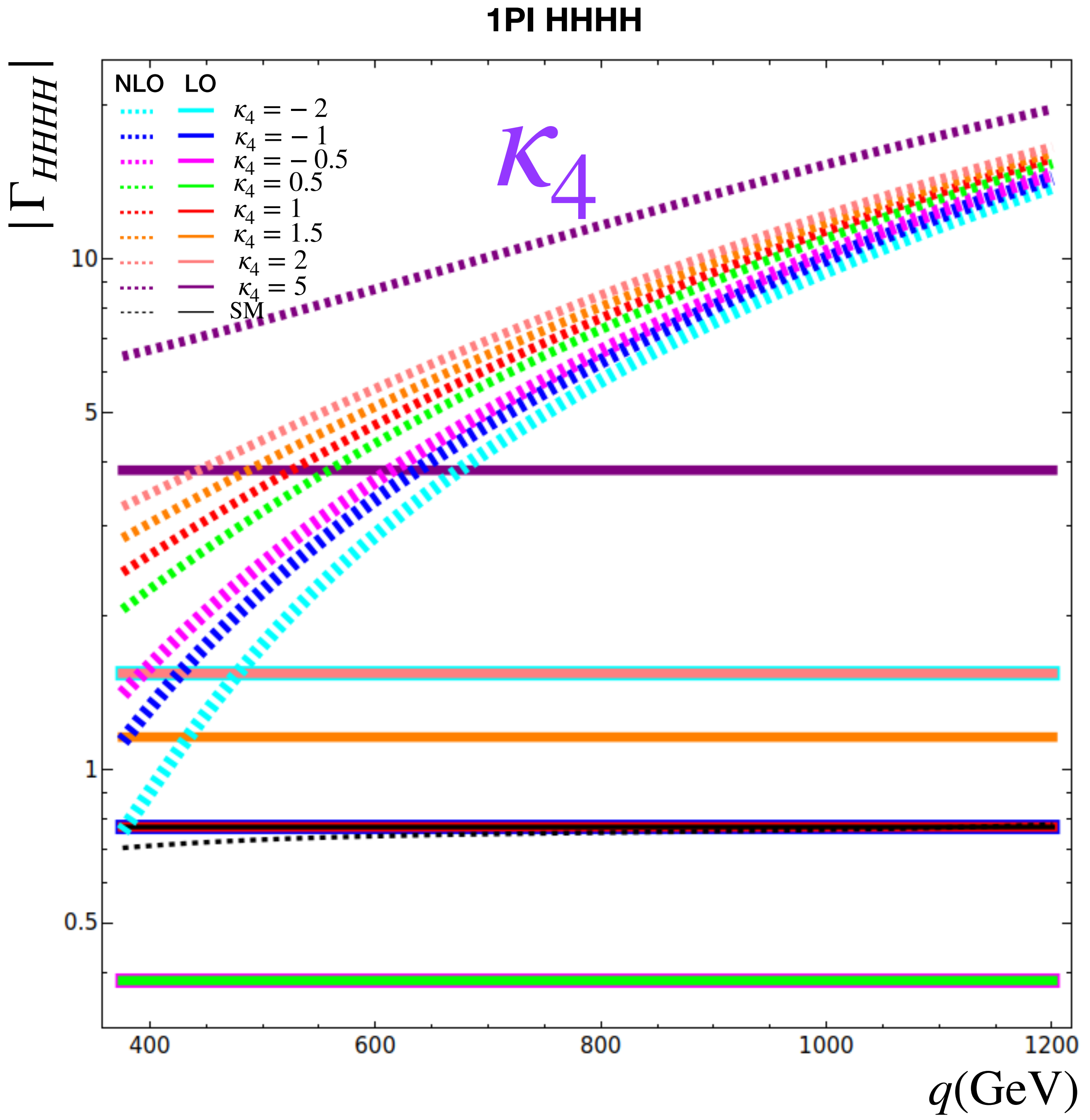
$\Gamma^{\rm CT}$: From $\mathscr{L}^{\rm LO}$ counterterms
$\delta Z_{W,Z,H..}\ \delta g, \delta g', \delta a, \delta b, \delta\kappa_3, \delta\kappa_4 \ldots$

Series of works in R_ξ: 2005.03537 (H decays to $\gamma\gamma$ and γZ), 2107.07890 (WZ to WZ), 2208.09334 (WW to HH) 2405.05385 (gg to HH, gg to HHH)

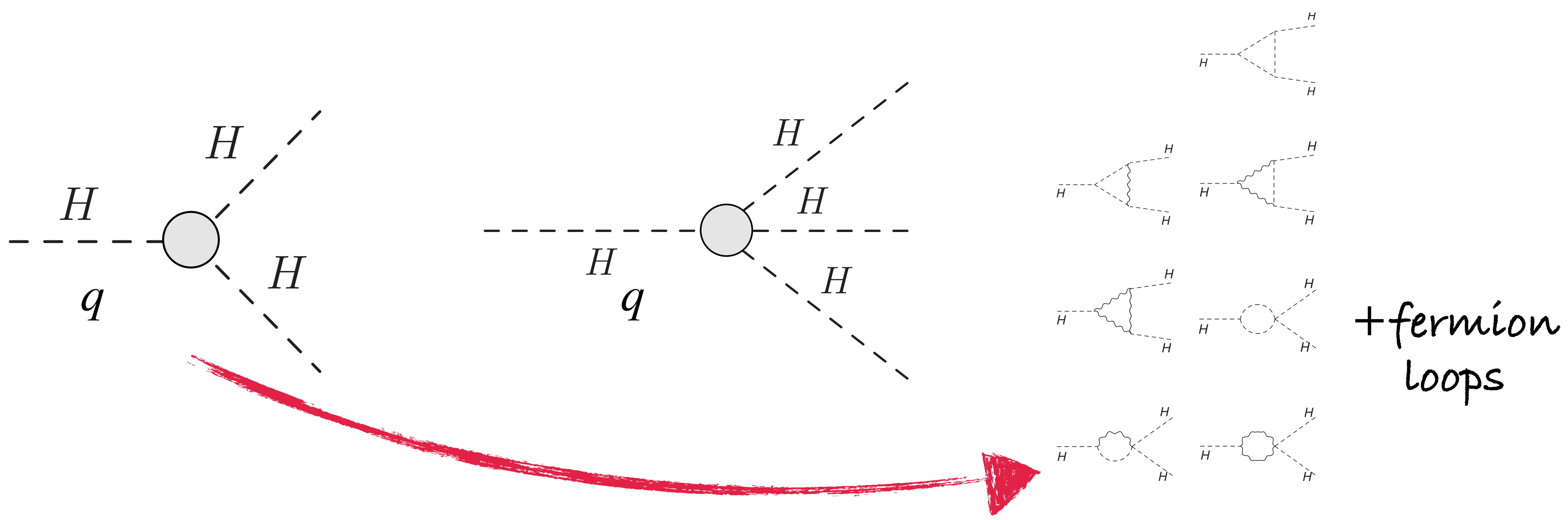
One-loop corrections in 1PIs: the case of Γ_{HHH} and Γ_{HHHH}

(details in 2208.05900 and 2405.05385)





In general, departures respect to the SM grow with offshellness q



Loops of bosons and tops included (Feynman gauge)

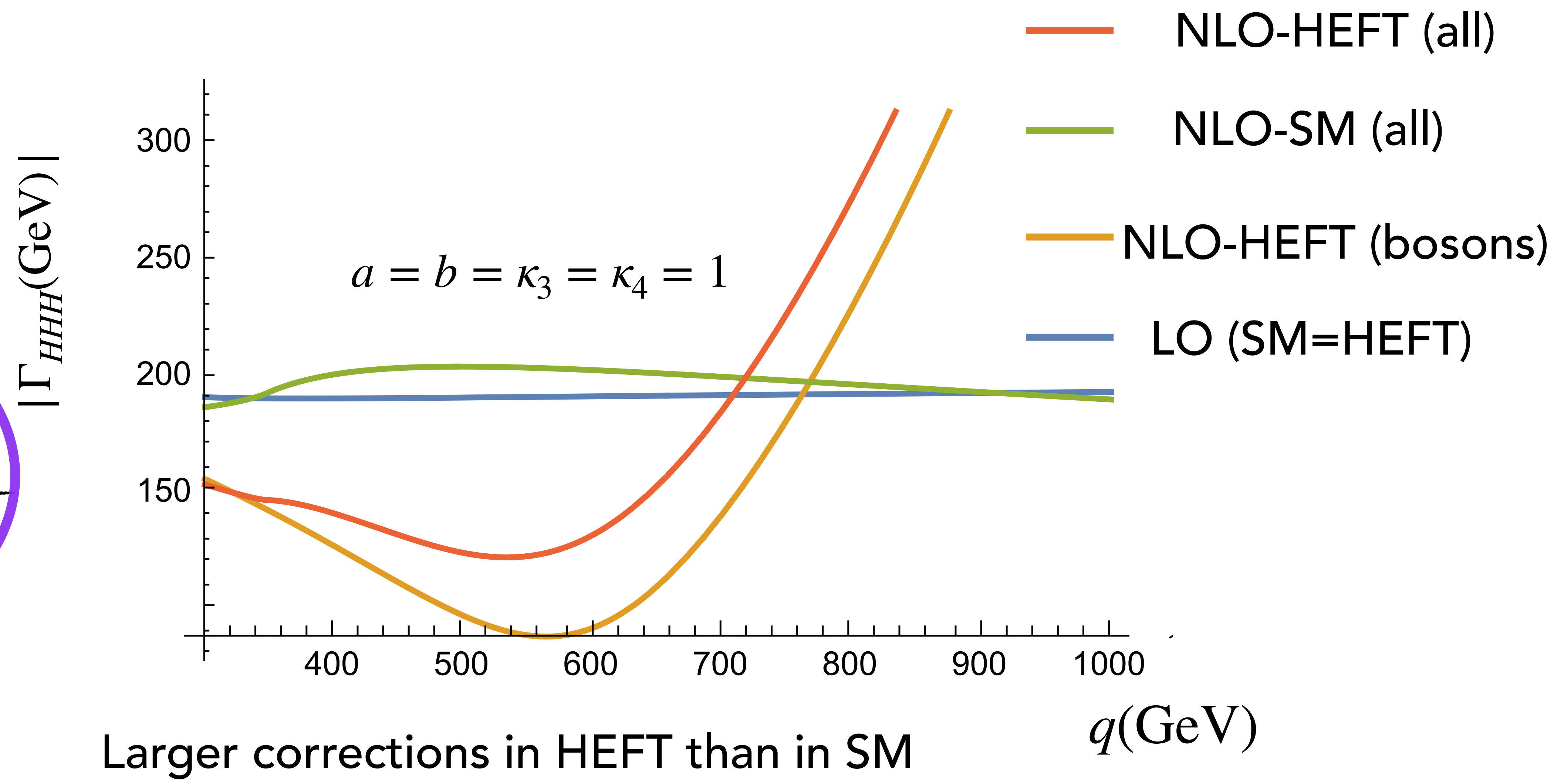
Loops of bosons clearly dominate if fermions assumed SM like

The size of the corrections can be large at large virtuality q

$\lvert\Gamma^{NLO}_{HHH}\rvert/\lvert\Gamma^{LO}_{HHH}\rvert$			$\lvert\Gamma^{NLO}_{HHHH}\rvert/\lvert\Gamma^{LO}_{HHHH}\rvert$		
κ_3	$q = 251$ GeV	$q = 1000$ GeV	κ_4	$q = 376$ GeV	$q = 1000$ GeV
-1	1.1	4.4	-2	0.49	6.2
-0.5	1.2	7.9	-1	1.5	13
0.5	0.77	6.3	-0.5	3.7	27
1	0.84	2.8	0.5	5.4	29
1.5	0.82	1.7	1	3.2	15
2	0.76	1.3	1.5	2.5	10
			2	2.1	7.9
			5	1.7	4.0
SM	0.97	1.0	SM	0.91	0.99

Non-linearity
H-singlet
growing with energy
of interactions within HEFT
are the resons for this

All CTs fixed
$\delta\kappa_3, \delta\kappa_4, \delta a_i's$ .. (see paper)



Larger corrections in HEFT than in SM

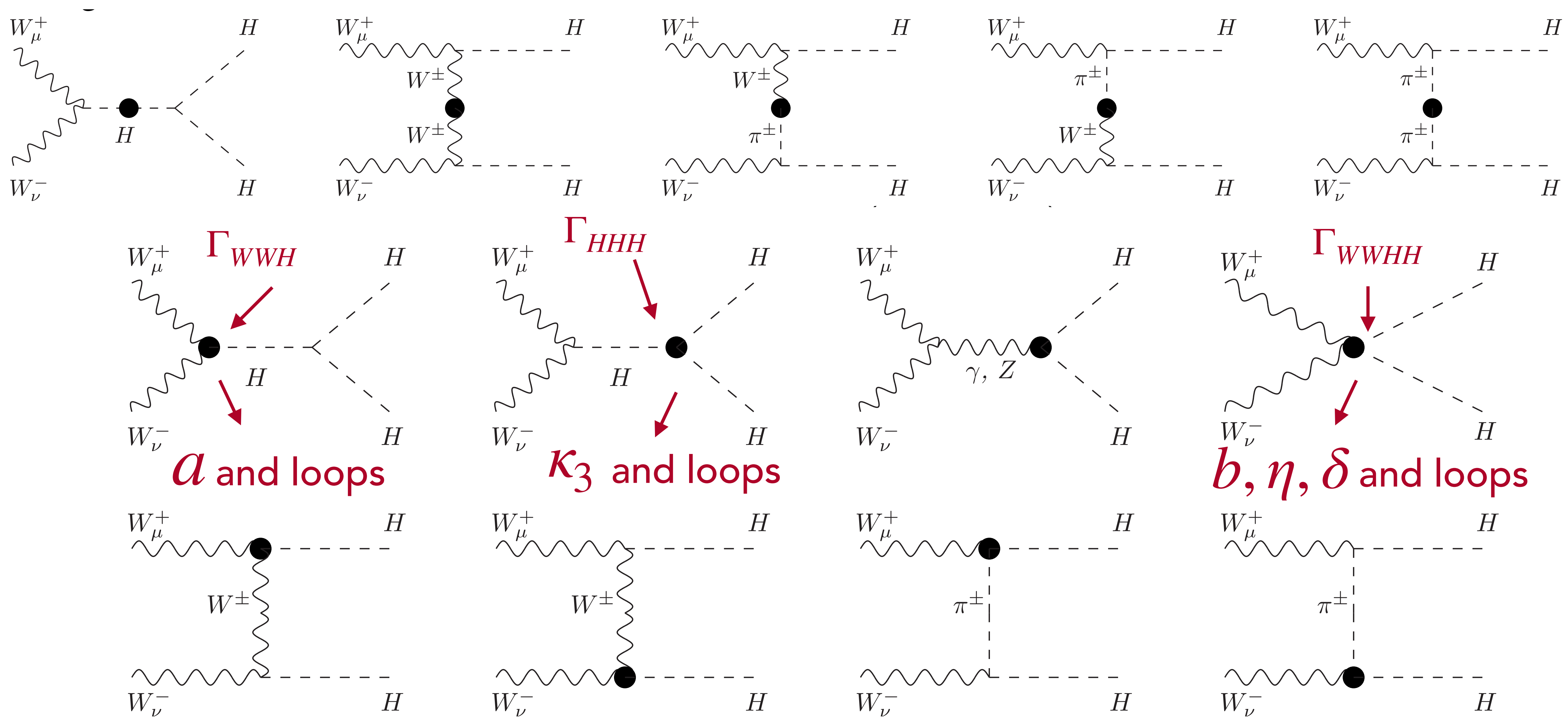
SM corrections almost flat with virtuality q

HEFT corrections highly sensitive to virtuality q

One-loop NLO-HEFT corrections in $WW \to HH$

M.J. Herrero and R.A Morales, PRD106,073008(2022) 2208.05900

Renormalized one-loop 1PIs $\hat{\Gamma}^{\rm NLO}_{\rm HEFT}$ computed in the R_ξ gauges = black balls contain all the loops

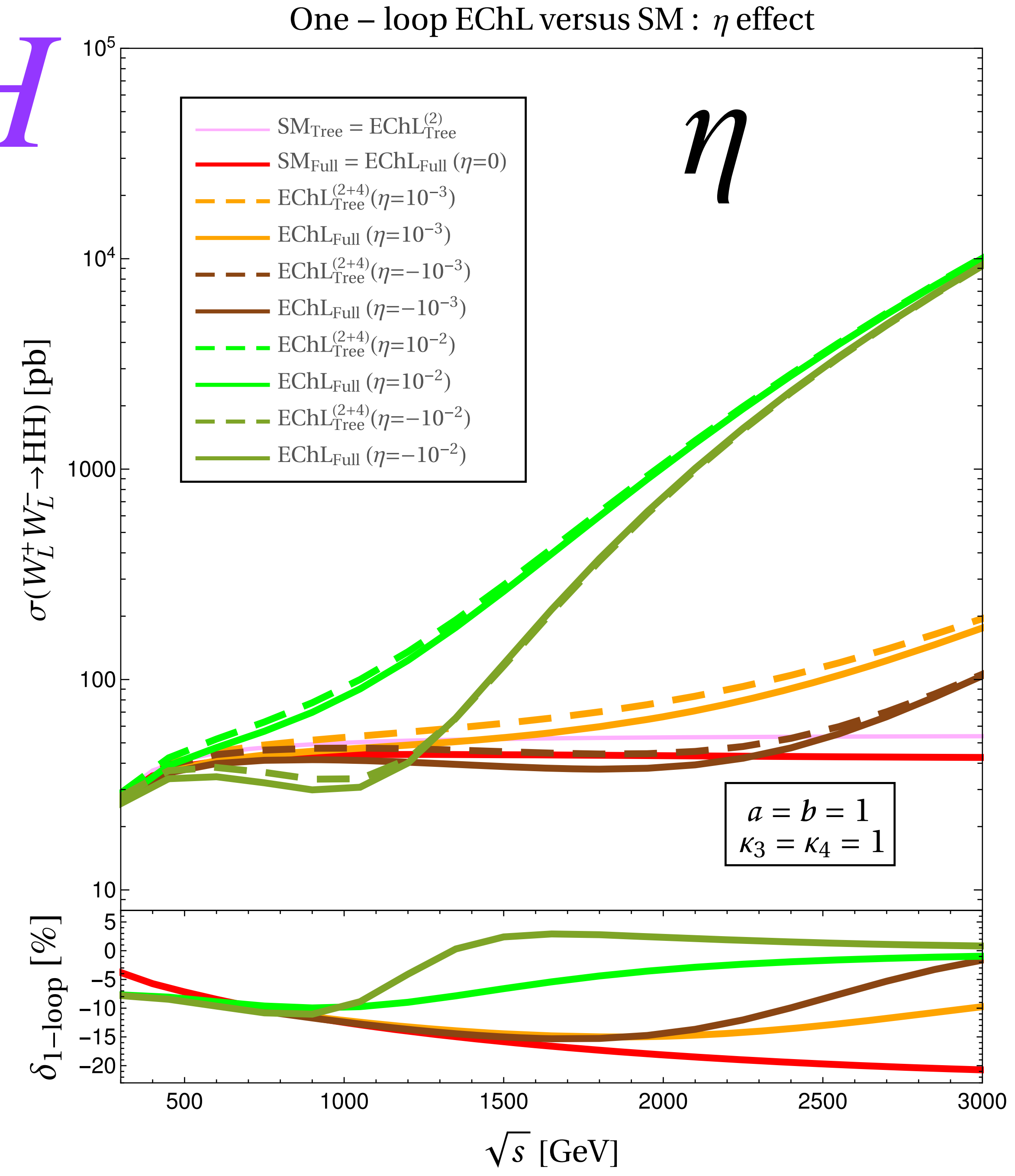


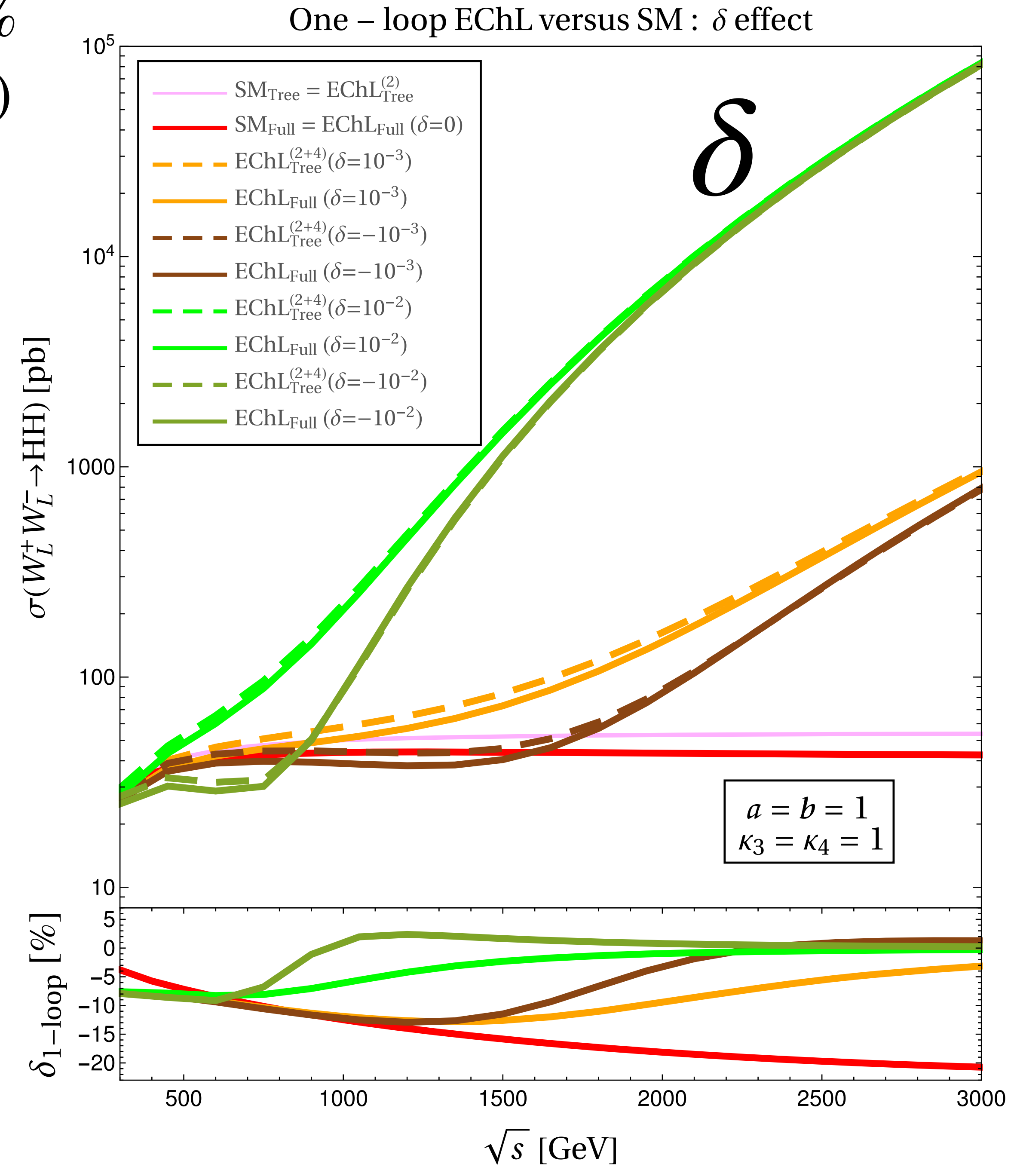
$a = b = \kappa_3 = 1$

in these plots

Size of $\delta_{1-{\rm loop}} \in (5, -15)\,\%$ comparable to SM $(-20\,\%)$

But different behaviour with energy





We have extracted all the needed CTs

In particular, all the involved $\delta a_i' s$

ξ independence checked

RGEs for all the involved HEFT coefficients derived

We checked some $\delta a_i' s$ with previous results in specific limits (pure scalar, isospin limit $m_W = m_Z$)

Others were unknown before our work (see paper)

Interesting RGE invariants for $(a^2 = b)$

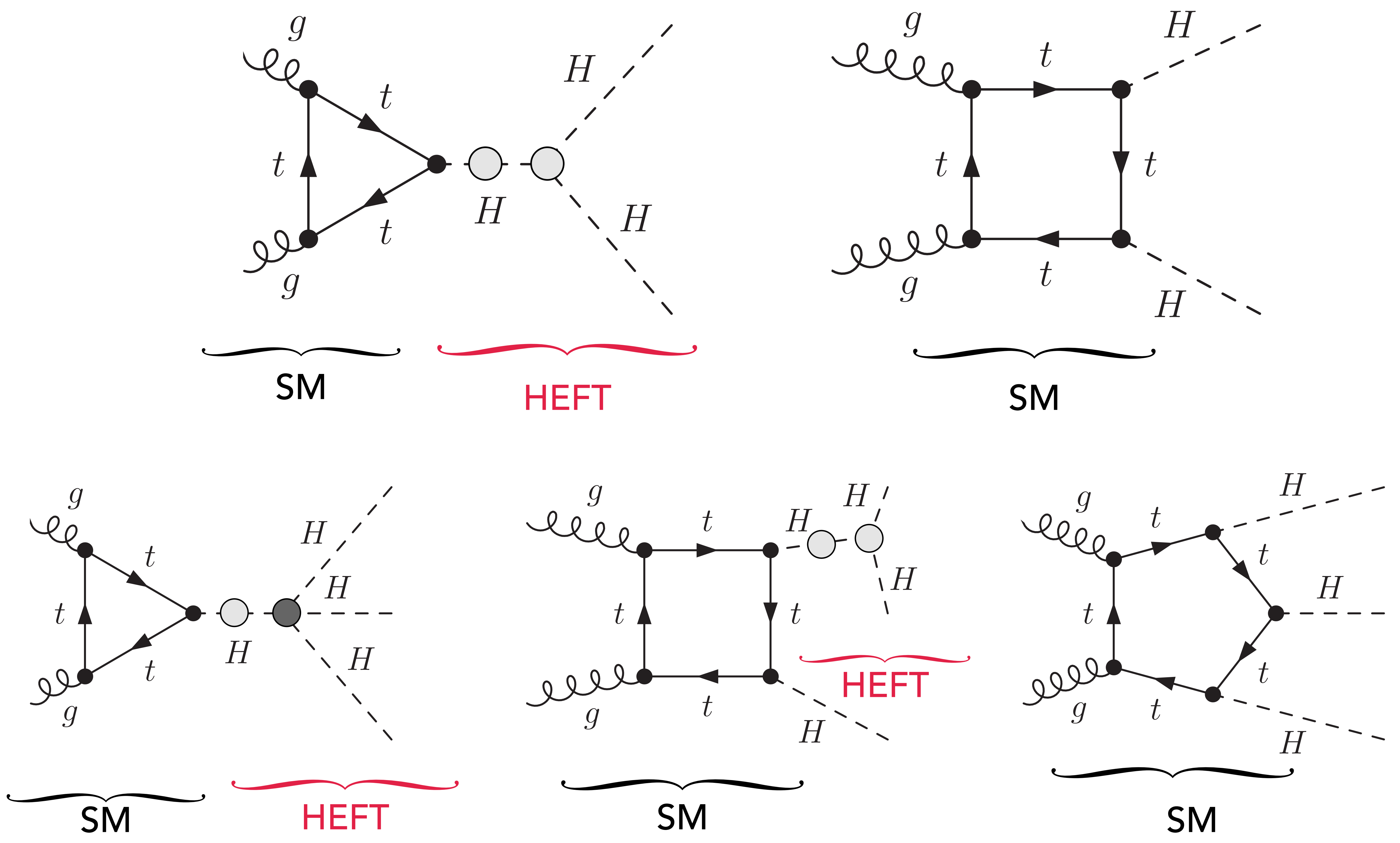
$$\eta(\mu) = \eta(\mu') - \frac{1}{16\pi^2}\frac{1}{3}(a^2-b)^2 \log\left(\frac{\mu^2}{\mu'^2}\right),$$

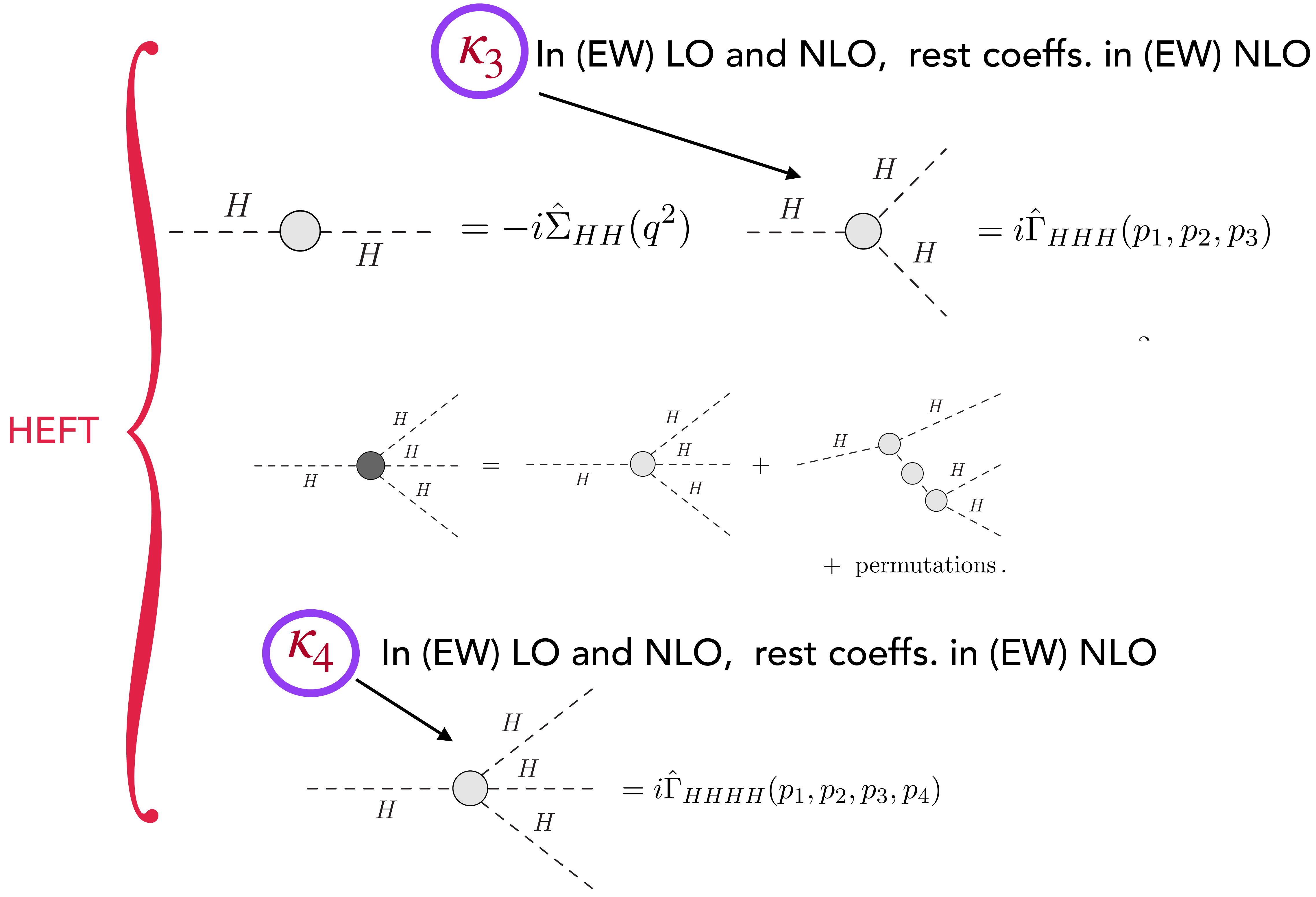
$$\delta(\mu) = \delta(\mu') + \frac{1}{16\pi^2}\frac{1}{12}(a^2-b)(7a^2-b-6)\log\left(\frac{\mu^2}{\mu'^2}\right)$$

EW-NLO loop corrections in $gg \to HH$ and in $gg \to HHH$

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385 (numerical estimates with VBFNLO)

Renormalized one-loop 1PIs $\hat{\Gamma}^{\rm NLO}_{\rm HEFT}$ computed in Feynman 'tHooft gauge = shaded balls contain the EW-NLO loops within HEFT





The loops in HHH and HHHH vertices and the non-trivial off-shell momenta dependencies produce relevant changes respect to LO

Renormalization of κ_3, κ_4 and of new $a_i's$ also set, RGEs etc (see paper)

OK with others in simplified limits

$$\delta_\epsilon \kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\rm H}^2 v^2}\Big(\kappa_3(a^2 - b + 9\kappa_3^2 - 6\kappa_4)m_{\rm H}^4 - 3(1-a^2)\kappa_3 m_{\rm H}^2(m_{\rm W}^2 + m_{\rm Z}^2)$$
$$+6(-2ab + 2a^2\kappa_3 + b\kappa_3)(2m_{\rm W}^4 + m_{\rm Z}^4)\Big) \,,$$
$$\delta_\epsilon \kappa_4 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\rm H}^2 v^2}\Big(\kappa_4(2a^2 - 2b + 9\kappa_3^2 - 6\kappa_4)m_{\rm H}^4 - 6(1-a^2)\kappa_4 m_{\rm H}^2(m_{\rm W}^2 + m_{\rm Z}^2)$$
$$+6(-2b^2 + 2a^2\kappa_4 + b\kappa_4)(2m_{\rm W}^4 + m_{\rm Z}^4)\Big) \,,$$

1311.5993,14091571 (pure scalar)
2109.02673 ($m_W = m_Z$)

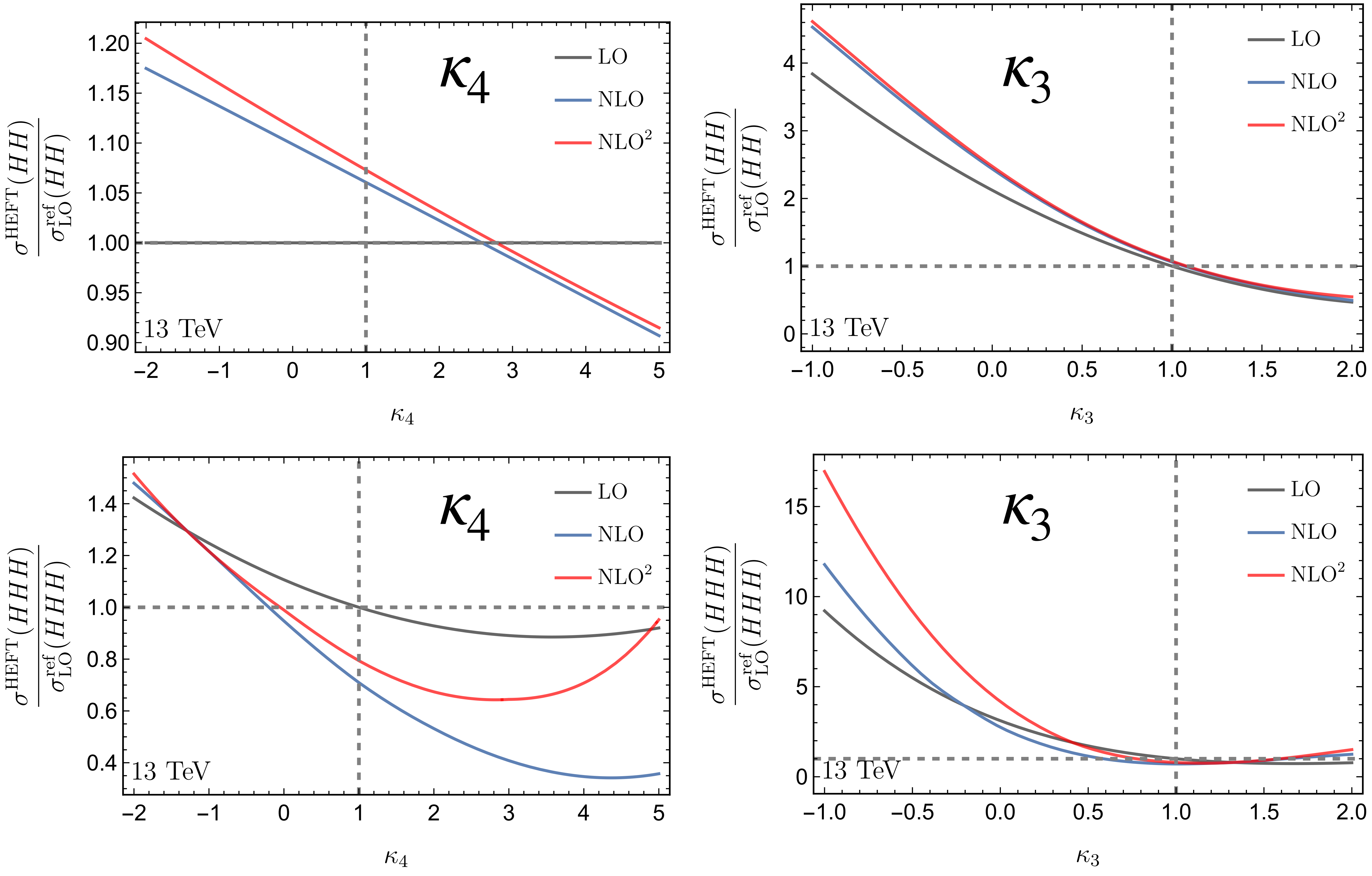
Size of the EW loops in $gg \to HH$ and in $gg \to HHH$

Corrections at LHC (13 TeV) cross sections

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385

$gg \to HH$

$gg \to HHH$



$$\sigma^{\rm SM}_{\rm LO}(HH) = \sigma^{\rm ref}_{\rm LO}(HH) = 17.40\,{\rm fb}\,;\, \sigma^{\rm SM}_{\rm LO}(HHH) = \sigma^{\rm ref}_{\rm LO}(HHH) = 0.041\,{\rm fb}$$

All simulations done with BVFNLO

Most important message:
(EW) loop corrections within NLO-HEFT change
the sensitivity to κ_3 and κ_4 in HH and HHH production at LHC

The most relevant change is in κ_3
For κ_3<0, we find relevant
enhancements in the NLO/LO prediction
$\sigma(HH)$ of $\sim 10\,\%$
and in
$\sigma(HHH)$ of $\sim 30\,\%$ ($\sim 80\,\%$ if NLO^2)

Also large changes in κ_4
For κ_4>0, we find relevant
reductions in the NLO/LO prediction
$\sigma(HHH)$ of $\sim 50\,\%$

Matching amplitudes

We do matching at amplitude level (more useful to compare with data).
In contrast to other approaches: matching Lagrangians, matching Effective Actions …etc

Matching amplitudes requires:

$$\mathscr{A}^{\mathrm{HEFT}} = \mathscr{A}^{\mathrm{UV}}(\mathrm{m_{heavy}} \gg \mathrm{m_{light}})$$

- Setting the HEFT order (LO, NLO,..)
- Setting the n-loop order $\mathscr{O}(\hbar^n)$, same in both sides
- Setting the input parameters, in both sides
- Setting the proper large mass expansion in the UV theory

2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales

Matching HEFT and 2HDM Amplitudes

Matching several amplitudes:

LO	$h \to WW^* \to Wf\bar{f}'$		tree
LO	$h \to ZZ^* \to Zf\bar{f}$		tree
LO	$W^+W^- \to hh$		tree
LO	$ZZ \to hh$		tree
LO	$hh \to hh$		tree
NLO	$h \to \gamma\gamma$	R_ξ	1-loop
NLO	$h \to \gamma Z$	R_ξ	1-loop

Choose input parameters:

(HEFT) $m_h, m_W, m_Z, c_i' s$

(2HDM)
- m_h, m_W, m_Z, m_{12} (light)
- $m_H, m_A, m_{H^\pm}$ (heavy)
- $\tan\beta,\ \cos(\beta - \alpha)$ (free)

Proper large mass expansion is in $\left(\dfrac{m_{\mathrm{light}}}{m_{\mathrm{heavy}}}\right)^n$. Other parameters are derived $(\lambda_{h_ih_jh_k}, \ldots.)$

Solution to the matching equations: HEFT versus 2HDM

2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales

Solving the matching equations implies identifying all momenta and Lorentz structures involved and extracting the corresponding HEFT $c_i's$ coeffs

$$(a = 1 - \Delta a\,, b = 1 - \Delta b\,, \kappa_3 = 1 - \Delta\kappa_3\,, \kappa_4 = 1 - \Delta\kappa_4)$$

$$
\begin{aligned}
\Delta a|_{2\mathrm{HDM}} &= 1 - s_{\beta-\alpha}\,,\\
\Delta b|_{2\mathrm{HDM}} &= -c^2_{\beta-\alpha}(1 - 2c^2_{\beta-\alpha} + 2c_{\beta-\alpha}s_{\beta-\alpha}\cot 2\beta)\,,\\
\Delta\kappa_3|_{2\mathrm{HDM}} &= 1 - s_{\beta-\alpha}(1 + 2c^2_{\beta-\alpha}) - c^2_{\beta-\alpha}\left(-2s_{\beta-\alpha}\frac{m^2_{12}}{m^2_h s_\beta c_\beta} + 2c_{\beta-\alpha}\cot 2\beta\left(1 - \frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\right),\\
\Delta\kappa_4|_{2\mathrm{HDM}} &= -\frac{c^2_{\beta-\alpha}}{3}\left(-7 + 64c^2_{\beta-\alpha} - 76c^4_{\beta-\alpha} + 12\left(1 - 6c^2_{\beta-\alpha} + 6c^4_{\beta-\alpha}\right)\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right.\\
&\quad +4c_{\beta-\alpha}s_{\beta-\alpha}\cot 2\beta\left(-13 + 38c^2_{\beta-\alpha} - 3(-5 + 12c_{\beta-\alpha})\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\\
&\quad \left. +4c^2_{\beta-\alpha}\cot^2 2\beta\left(3c^2_{\beta-\alpha} - 16s^2_{\beta-\alpha} + 3(-1 + 6s^2_{\beta-\alpha})\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\right),\\
a_{h\gamma\gamma}|_{2\mathrm{HDM}} &= -\frac{s_{\beta-\alpha}}{48\pi^2}\,,\\
a_{h\gamma Z}|_{2\mathrm{HDM}} &= -\frac{(2c^2_{\mathrm{w}} - 1)s_{\beta-\alpha}}{96c^2_{\mathrm{w}}\pi^2}\,.
\end{aligned}
$$

Posterior computations within HEFT are in agreement with ours: 2311.16897 (PC-3), 2312.13885

These contributions are $\mathcal{O}\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^0$

Leading terms in the large $m_{\rm heavy}$ expansion

Summarize the Non-Decoupling effects of the heavy Higgs bosons at low energies

They are valid for arbitrary $t_\beta\,, c_{\beta-\alpha}$

when $c_{\beta-\alpha} \ll 1$ is required (quasi-alignement)

These non-decoupling effects from the heavy bosons are not obtained in the SMEFT where all effects are decoupling

$$\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^n,\ n \geq 2$$

Interesting correlations found

$$
\begin{aligned}
\Delta a|^{\rm qal}_{2\mathrm{HDM}} &= -\frac{1}{2}\Delta b|^{\rm qal}_{2\mathrm{HDM}}\\
2\Delta\kappa_3|^{\rm qal}_{2\mathrm{HDM}} + \Delta\kappa_4|^{\rm qal}_{2\mathrm{HDM}} &= -\frac{2}{3}c^2_{\beta-\alpha}
\end{aligned}
$$

Matching amplitudes: HEFT versus SMEFT

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

Requiring matching of the amplitudes for WW—>HH (similar for ZZ —>HH) and identifying all momenta and Lorentz structures involved

$$\mathcal{A}(WW \to HH)|_{\rm HEFT} = \mathcal{A}^{(2)} + \mathcal{A}^{(4)} \quad \longleftrightarrow \quad \mathcal{A}(WW \to HH)_{\rm SMEFT} = \mathcal{A}_{\rm SM} + \mathcal{A}^{[6]} + \mathcal{A}^{[8]}$$

$$\mathcal{A}^{(2)}|_S = \frac{g^2}{2} 3a\kappa_3 \frac{m_{\rm H}^2}{S-m_{\rm H}^2}\epsilon_+\cdot\epsilon_-$$

$$\mathcal{A}^{(2)}|_T = g^2a^2\frac{m_{\rm W}^2\epsilon_+\cdot\epsilon_- + \epsilon_+\cdot k_1\epsilon_-\cdot k_2}{T-m_{\rm W}^2}$$

$$\mathcal{A}^{(2)}|_U = g^2a^2\frac{m_{\rm W}^2\epsilon_+\cdot\epsilon_- + \epsilon_+\cdot k_2\epsilon_-\cdot k_1}{U-m_{\rm W}^2}$$

$$\mathcal{A}^{(2)}|_C = \frac{g^2}{2} b\epsilon_+\cdot\epsilon_-$$

$$\begin{aligned}\mathcal{A}^{(4)}|_S = \frac{g^2}{2v^2}\frac{1}{S-m_{\rm H}^2}&(3\kappa_3 a_{d2}m_{\rm H}^2(S\epsilon_+\cdot\epsilon_- - 2\epsilon_+\cdot p_-\epsilon_-\cdot p_+)\\ &+6\kappa_3 a_{HWW}m_{\rm H}^2((S-2m_{\rm W}^2)\epsilon_+\cdot\epsilon_- - 2\epsilon_+\cdot p_-\epsilon_-\cdot p_+) - (3\kappa_3 a_{H\mathcal{V}\mathcal{V}}m_{\rm H}^4 + a a_{Hdd}m_{\rm H}^2(S+2m_{\rm H}^2))\epsilon_+\cdot\epsilon_-)\end{aligned}$$

$$\begin{aligned}\mathcal{A}^{(4)}|_T = \frac{g^2}{2v^2}\frac{a}{T-m_{\rm W}^2}&(a_{d2}(4m_{\rm W}^2m_{\rm H}^2\epsilon_+\cdot\epsilon_- + 2(T+3m_{\rm W}^2-m_{\rm H}^2)\epsilon_+\cdot k_1\epsilon_-\cdot k_2\\ &-4m_{\rm W}^2(\epsilon_+\cdot k_1\epsilon_-\cdot p_+ + \epsilon_+\cdot p_-\epsilon_-\cdot k_2))\\ &-8a_{HWW}m_{\rm W}^2((T+m_{\rm W}^2-m_{\rm H}^2)\epsilon_+\cdot\epsilon_- + \epsilon_+\cdot k_1\epsilon_-\cdot p_+ + \epsilon_+\cdot p_-\epsilon_-\cdot k_2)\\ &-4a_{H\mathcal{V}\mathcal{V}}m_{\rm H}^2(m_{\rm W}^2\epsilon_+\cdot\epsilon_- + \epsilon_+\cdot k_1\epsilon_-\cdot k_2))\end{aligned}$$

$$\mathcal{A}^{(4)}|_U = \mathcal{A}^{(4)}|_T \quad \text{with} \quad T\to U \quad \text{and} \quad k_1\leftrightarrow k_2$$

$$\begin{aligned}\mathcal{A}^{(4)}|_C = \frac{g^2}{2v^2}&(-2a_{dd\mathcal{V}\mathcal{V}1}(\epsilon_+\cdot k_2\epsilon_-\cdot k_1 + \epsilon_+\cdot k_1\epsilon_-\cdot k_2)\\ &+(-2a_{dd\mathcal{V}\mathcal{V}2}(S-2m_{\rm H}^2) + 4a_{HHWW}(S-2m_{\rm W}^2) + a_{Hd2}S - a_{HH\mathcal{V}\mathcal{V}}m_{\rm H}^2)\epsilon_+\cdot\epsilon_-\\ &-2(a_{Hd2}+4a_{HHWW})\epsilon_+\cdot p_-\epsilon_-\cdot p_+)\end{aligned}$$

$$\mathcal{L}_6 = \frac{a_{\phi\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \frac{a_{\phi D}}{\Lambda^2}(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi) + \frac{a_{\phi W}}{\Lambda^2}(\phi^\dagger\phi)W^a_{\mu\nu}W^{a\mu\nu} + \ldots$$

$$\mathcal{L}_8 = \frac{a^{(1)}_{\phi^6}}{\Lambda^4}(\phi^\dagger\phi)^2(D_\mu\phi^\dagger D^\mu\phi) + \frac{a^{(2)}_{\phi^6}}{\Lambda^4}(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)(D_\mu\phi^\dagger\sigma^I D^\mu\phi) + \frac{a^{(1)}_{\phi^4}}{\Lambda^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + \frac{a^{(2)}_{\phi^4}}{\Lambda^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + \frac{a^{(3)}_{\phi^4}}{\Lambda^4}(D_\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D^\nu\phi) + \ldots$$

$$\begin{aligned}\mathscr{A}_{\rm SM} &= \frac{g^2}{2}3\frac{m_{\rm H}^2}{S-m_{\rm H}^2}\epsilon_+\cdot\epsilon_- + g^2\frac{m_{\rm W}^2\epsilon_+\cdot\epsilon_- + \epsilon_+\cdot k_1\epsilon_-\cdot k_2}{T-m_{\rm W}^2}\\ &+g^2\frac{m_{\rm W}^2\epsilon_+\cdot\epsilon_- + \epsilon_+\cdot k_2\epsilon_-\cdot k_1}{U-m_{\rm W}^2} + \frac{g^2}{2}\epsilon_+\cdot\epsilon_- \qquad (2.17)\end{aligned}$$

$$\mathcal{A}^{[6]}|_S = \frac{g^2}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D}\frac{S+8m_{\rm H}^2}{S-m_{\rm H}^2}\epsilon_+\cdot\epsilon_- + 6\frac{v^2}{\Lambda^2}a_{\phi W}\frac{m_{\rm H}^2}{v^2}\frac{2\epsilon_-\cdot p_+\epsilon_+\cdot p_- - (S-2m_{\rm W}^2)\epsilon_+\cdot\epsilon_-}{S-m_{\rm H}^2}; \qquad \delta a_{\phi D} \equiv 4a_{\phi\Box} - a_{\phi D}$$

$$\begin{aligned}\mathcal{A}^{[6]}|_T &= \frac{g^2}{2}\frac{v^2}{\Lambda^2}\delta a_{\phi D}\frac{m_{\rm W}^2\epsilon_+\cdot\epsilon_- + (\epsilon_-\cdot p_+ - \epsilon_-\cdot k_1)\epsilon_+\cdot k_1}{T-m_{\rm W}^2}\\ &+2g^2\frac{v^2}{\Lambda^2}a_{\phi W}\frac{\epsilon_+\cdot\epsilon_-(-m_{\rm H}^2+m_{\rm W}^2+T) - \epsilon_-\cdot k_1\epsilon_+\cdot p_- + \epsilon_-\cdot p_+(\epsilon_+\cdot p_- + \epsilon_+\cdot k_1)}{T-m_{\rm W}^2};\end{aligned}$$

$$\mathcal{A}^{[6]}|_U = \mathcal{A}^{(6)}|_T \quad \text{with} \quad T\to U \quad \text{and} \quad k_1\leftrightarrow k_2$$

$$\mathcal{A}^{[6]}|_C = \frac{g^2}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D}\epsilon_+\cdot\epsilon_- + \frac{v^2}{\Lambda^2}a_{\phi W}\frac{1}{v^2}(-2(S-2m_W^2)\epsilon_+\cdot\epsilon_- + 4\epsilon_-\cdot p_+\epsilon_+\cdot p_-);$$

$$\mathcal{A}^{[8]}|_C = -\frac{g^2}{4}\frac{v^2}{\Lambda^4}((a^{(1)}_{\phi^4}+a^{(2)}_{\phi^4})(\epsilon_-\cdot p_+\epsilon_+\cdot k_1 + \epsilon_-\cdot k_1(\epsilon_+\cdot p_- - 2\epsilon_+\cdot k_1)) + a^{(3)}_{\phi^4}\epsilon_+\cdot\epsilon_-(S-2m_H^2))$$

Interesting correlations found

$$\Delta b|_{\rm SMEFT} = 4\Delta a|_{\rm SMEFT}$$

$$a_{HWW}|_{\rm SMEFT} = 2a_{HHWW}|_{\rm SMEFT}$$

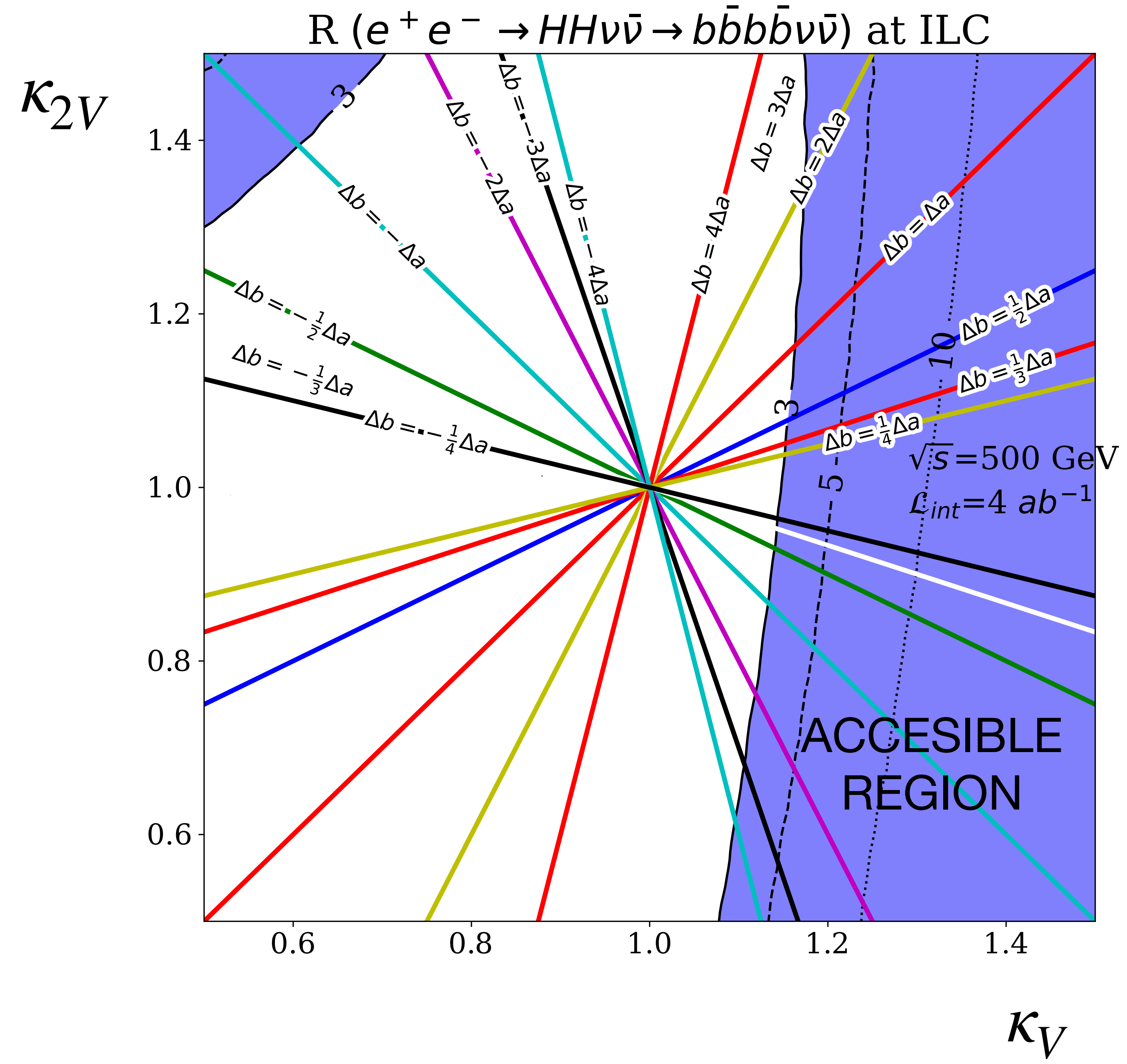
Solutions to the matching
Tree level

$$a-1 = \frac{1}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D} \qquad \kappa_3 - 1 = \frac{5}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D} \qquad a_{HWW} = -\frac{v^2}{2m_{\rm W}^2}\frac{v^2}{\Lambda^2}a_{\phi W} \qquad \eta \qquad a_{dd\mathcal{V}\mathcal{V}1} = \frac{v^4}{4\Lambda^4}[a^{(1)}_{\phi^4} + a^{(2)}_{\phi^4}]$$

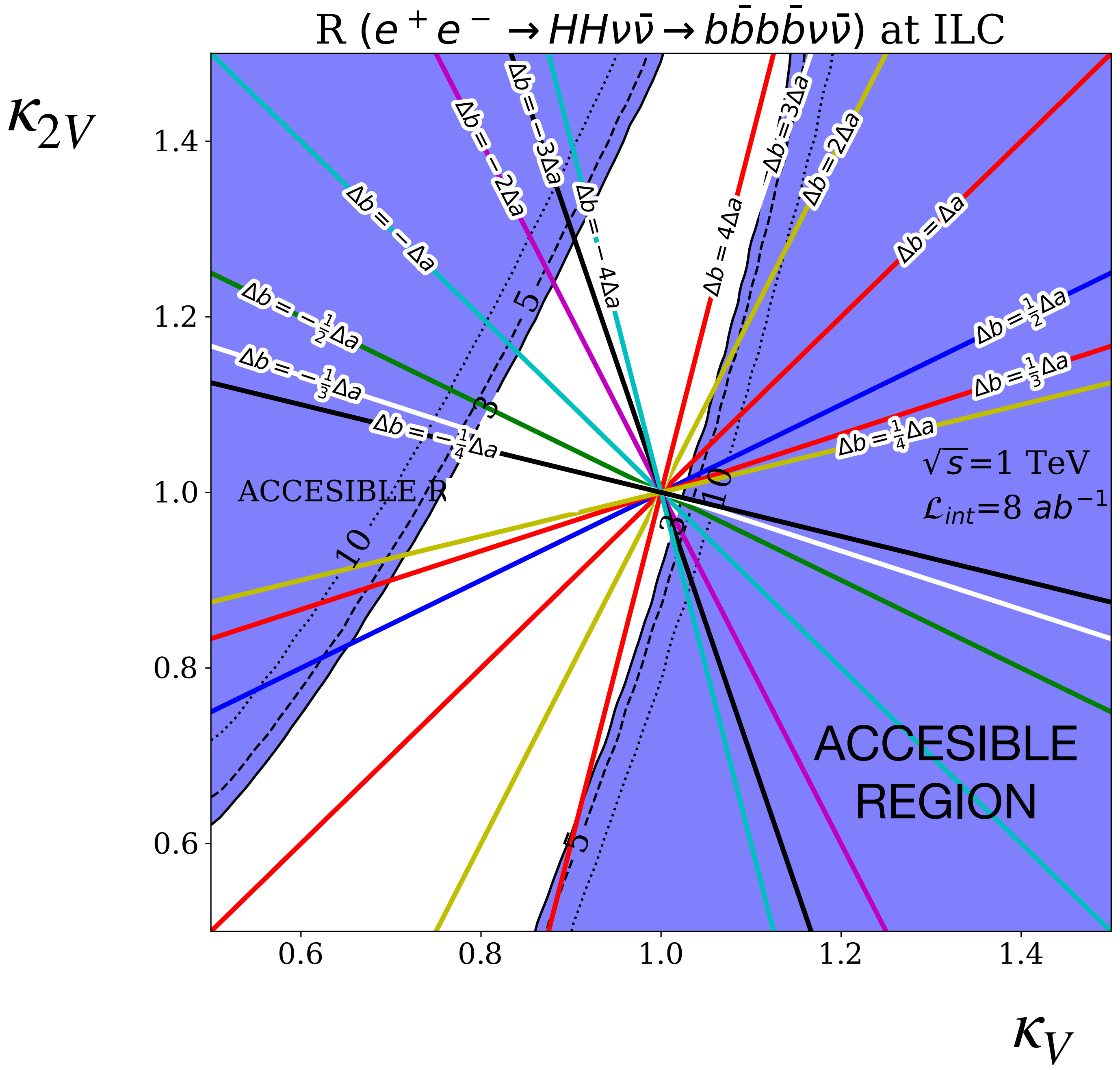
$$b-1 = \frac{v^2}{\Lambda^2}\delta a_{\phi D} \qquad a_{HHWW} = -\frac{v^2}{4m_{\rm W}^2}\frac{v^2}{\Lambda^2}a_{\phi W} \qquad \delta \qquad a_{dd\mathcal{V}\mathcal{V}2} = \frac{v^4}{4\Lambda^4}a^{(3)}_{\phi^4}$$

Access to (κ_V, κ_{2V}) and correlations in $e^+e^- \to HH\nu\bar{\nu} \to 4bjets + E_T^{miss}$

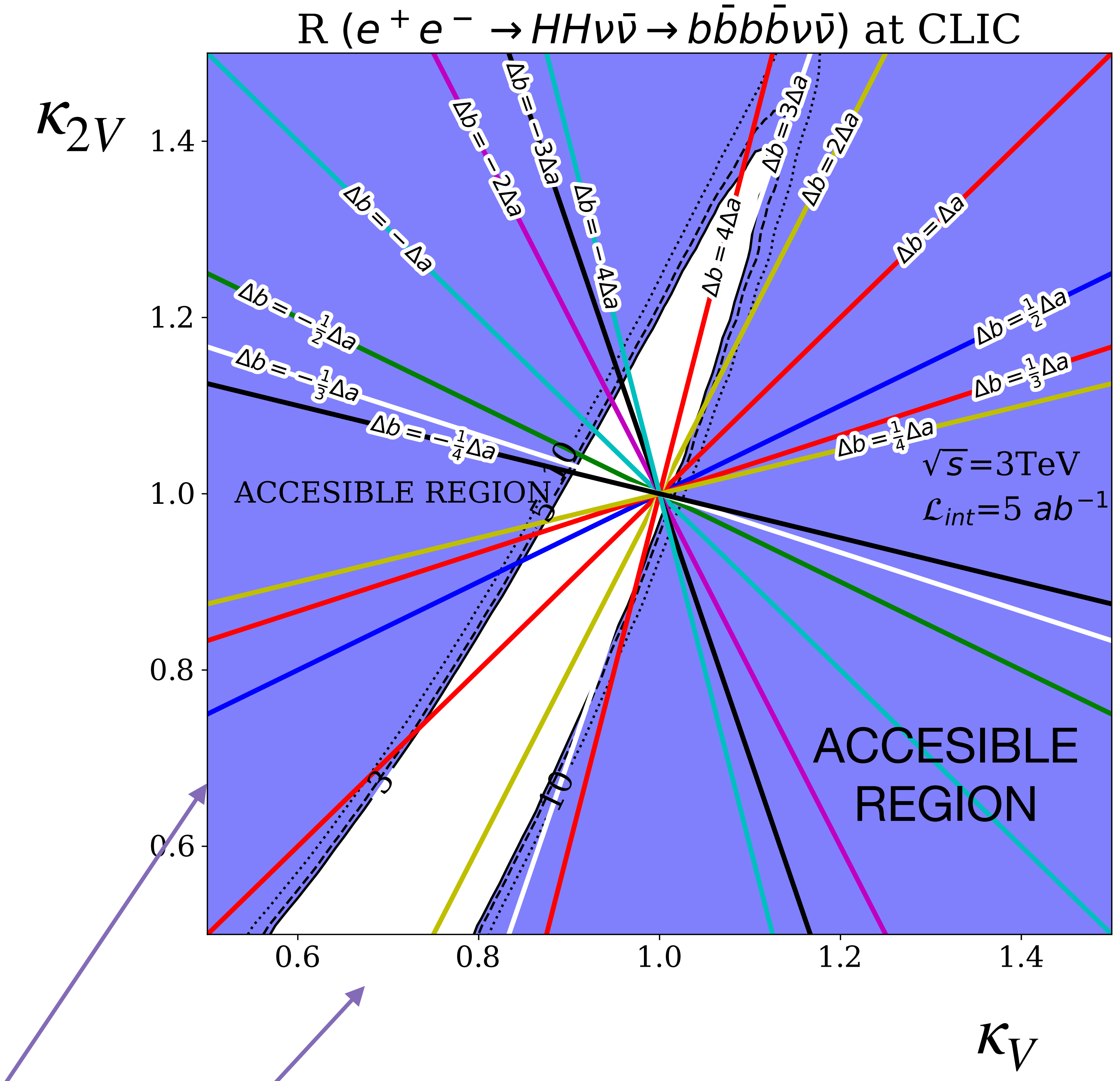
ILC 500 GeV



ILC 1 TeV



CLIC 3 TeV



Accessible Regions (in purple) defined as $R = \frac{N_{\rm BSM} - N_{\rm SM}}{\sqrt{N_{\rm SM}}} > 3$

Largest regions in $(a, b) = (\kappa_V, \kappa_{2V})$ in CLIC, up to $\Delta\kappa \sim \mathcal{O}(10^{-1})$

Correlations $b \neq a^2$ defined by lines $\Delta b = C\Delta a$; $b = 1 - \Delta b$; $a = 1 - \Delta a$

Some correlations better tested, for instance $C = 1/4, 1/3, 1/2, 1$ if $\kappa_{V,2V} < 1$

In contrast to moving in line $b = a^2$ (equiv to $\Delta b = 2\,\Delta a$, yellow lines)

N=EVENTS with 4b+ETmiss

4-btagged jets $\epsilon_b = 0.8$

✓ $p_T^j > 20$ GeV ✓ $\Delta R_{jj} > 0.4$

✓ $|\eta^j| < 2$ ✓ $E_T^{\rm mis} > 20$ GeV

Interesting cases: $\Delta b|_{\rm 2HDM} = -2\Delta a|_{\rm 2HDM}$ (magenta lines)

$\Delta b|_{\rm SMEFT} = 4\Delta a|_{\rm SMEFT}$ (red lines)

More difficult if $\kappa_{2V} > 1$

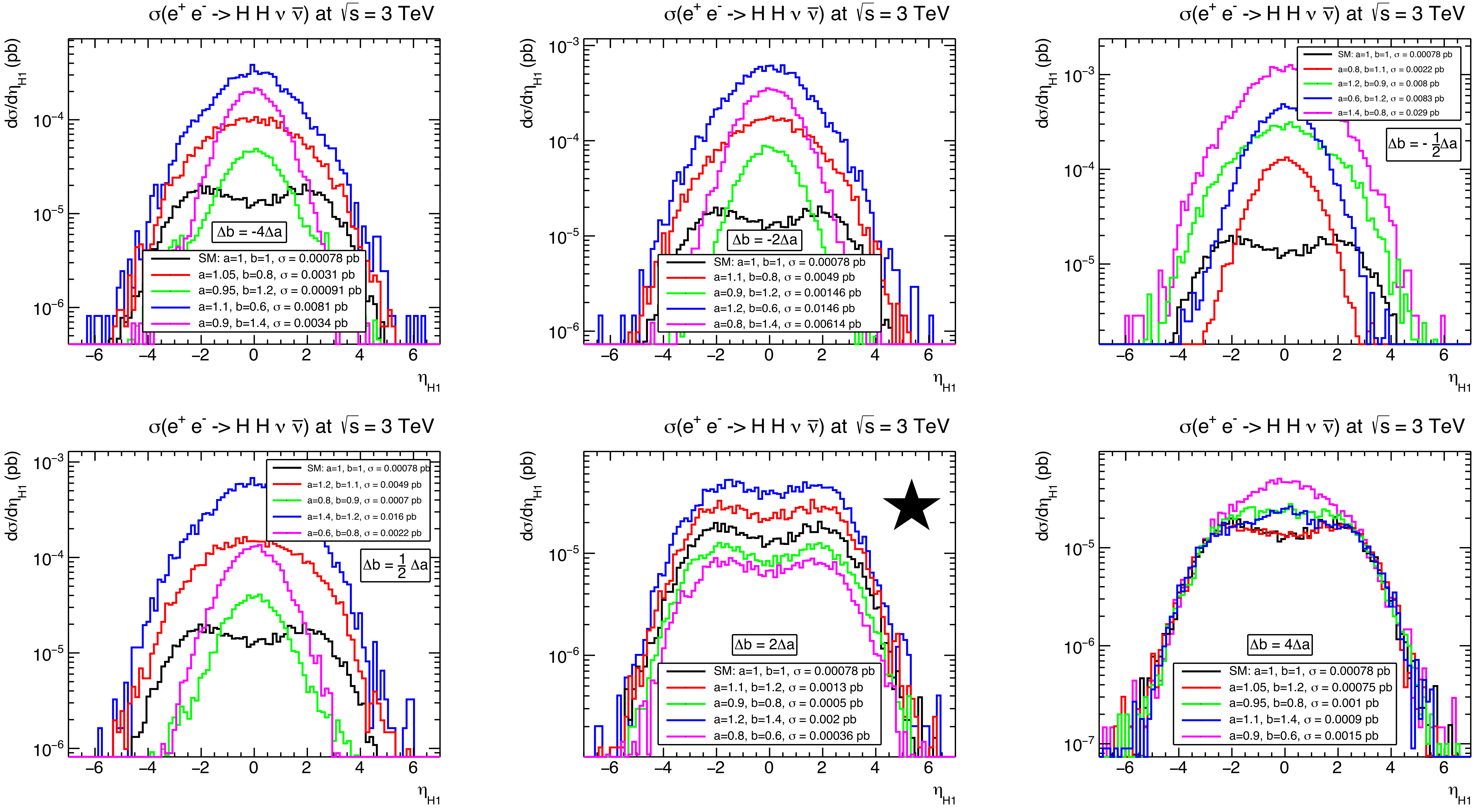
2312.03877 Davila, Domenech, Herrero, Morales, EPJC 84 (2024)5, 503 ; 2011.13195 2307.15693 2208.05452

Exploring correlations (κ_V, κ_{2V}) at $e^+e^- \to HH\nu\bar{\nu}$ in $d\sigma/d\eta_H$

In general going BSM with $\kappa_{2V} \neq 1$; $\kappa_V \neq 1$ distorts the dist. in η_H producing peaks at $\eta_H = 0$

Except close to $\kappa_{2V} = \kappa_V^2$ ★

$e^+e^-(3\,\mathrm{TeV})$



Very characteristic BSM events with $(\kappa_V^2 - \kappa_{2V}) \neq 0$

larger $(\kappa_V^2 - \kappa_{2V}) \to$ higher peaks $\to$ more transverse Higgs !!!

Dávila, Domenech, Herrero, Morales [2312.03877] EPJC (2024)

Similar study ongoing at LHC for $pp \to HHjj$
with good prospects (Cepeda, Domenech, García-Mir, Herrero)

Summary /Conclusions

Including one-loop corrections within HEFT predictions is important

Sensitivity to the HEFT parameters may change in a relevant way

Particularly relevant the change in sensitivity to κ_3 and κ_4

a versus b, κ_3 versus κ_4 uncorrelated in HEFT because H is a singlet but correlated in other specific scenarios.
Ex.: 2HDM, SMEFT, …H is part of a doublet, they are correlated

Both HL-LHC (14 TeV) and CLIC (3TeV) will give the best access to HEFT coeffs. Studying specific difxsections clue in exploring HEFT/SMEFT diffs. Ex: In HH (EW) prod. $d\sigma/d\eta_H$ for $\kappa_V^2 \leftrightarrow \kappa_{2V}$ ($a^2 \leftrightarrow b$)

Back up slides

HH and HHH production from gluon-gluon with NLO-HEFT

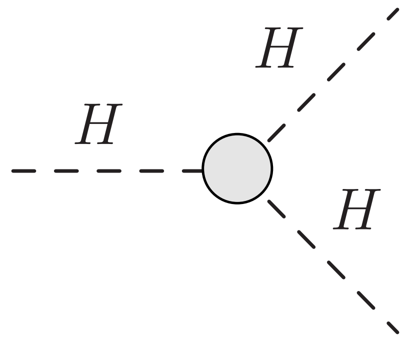
Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385

The relevant NLO operators are:

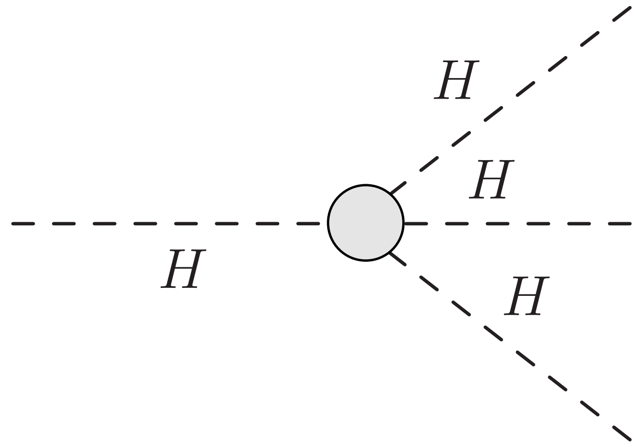
$$\mathscr{L}_{\mathrm{HEFT}}^{\mathrm{NLO}} = .. \left(a_{\Box\Box} + a_{H\Box\Box}\frac{H}{v} + a_{HH\Box\Box}\frac{H^2}{v^2}\right)\frac{\Box H\,\Box H}{v^2}$$
$$+a_{dd\Box}\frac{\partial^\mu H\,\partial_\mu H\,\Box H}{v^3} + \left(a_{Hdd}\frac{m_{\mathrm{H}}^2}{v^2} + a_{ddW}\frac{m_{\mathrm{W}}^2}{v^2} + a_{ddZ}\frac{m_{\mathrm{Z}}^2}{v^2}\right)\frac{H}{v}\partial^\mu H\,\partial_\mu H$$
$$+a_{Hdd\Box}\frac{H\partial^\mu H\,\partial_\mu H\,\Box H}{v^4} + \left(a_{HHdd}\frac{m_{\mathrm{H}}^2}{v^2} + a_{HddW}\frac{m_{\mathrm{W}}^2}{v^2} + a_{HddZ}\frac{m_{\mathrm{Z}}^2}{v^2}\right)\frac{H^2}{v^2}\partial^\mu H\,\partial_\mu H$$
$$+a_{dddd}\frac{\partial^\mu H\,\partial_\mu H\,\partial^\nu H\,\partial_\nu H}{v^4}$$

These modify the HHH and HHHH interactions (with non-trivial momenta dependencies) entering in $gg \to HH, HHH$ via the NLO 1PIs:

$$\Gamma_{HHH}^{\mathrm{NLO}}(p_1,p_2,p_3) = \Gamma_{HHH}^{\mathrm{LO}} + \Delta\Gamma_{HHH}^{a_i's}(p_1,p_2,p_3)\ ,\quad \Gamma_{HHH}^{\mathrm{LO}} = -\,\kappa_3\frac{m_H^2}{v}$$



$$\Gamma_{HHHH}^{\mathrm{NLO}}(p_1,p_2,p_3,p_4) = \Gamma_{HHHH}^{\mathrm{LO}} + \Delta\Gamma_{HHHH}^{a_i's}(p_1,p_2,p_3,p_4)\ ,\quad \Gamma_{HHHH}^{\mathrm{LO}} = -\,\kappa_4\frac{m_H^2}{v^2}$$



These NLO deviations $\Delta\Gamma$ are relevant for phenomenology (see next)

CTs in NLO $WW \to HH$ and derived RGEs

M.J. Herrero and R.A Morales , PRD106,073008(2022) 2208.05900

$$\delta_\epsilon a = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{2v^2}\left((a^2-b)(a-\kappa_3)m_{\rm H}^2 + a((1-3a^2+2b)m_{\rm W}^2 + (1-a^2)m_{\rm Z}^2)\right),$$

$$\boldsymbol{\delta_\epsilon b} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2v^2}\Big((a^2-b)(8a^2-2b-12a\kappa_3+3\kappa_4)m_{\rm H}^2 + 6a^2b(2m_{\rm W}^2+m_{\rm Z}^2) - 6b(m_{\rm W}^2+m_{\rm Z}^2) - 6b^2m_{\rm W}^2\Big),$$

$$\delta_\epsilon\kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\rm H}^2v^2}\Big(\kappa_3(a^2-b+9\kappa_3^2-6\kappa_4)m_{\rm H}^4 - 3(1-a^2)\kappa_3 m_{\rm H}^2(m_{\rm W}^2+m_{\rm Z}^2) + 6(-2ab+2a^2\kappa_3+b\kappa_3)(2m_{\rm W}^4+m_{\rm Z}^4)\Big)\,,$$

$$\boldsymbol{\delta_\epsilon a_{dd\mathcal{V}\mathcal{V}1}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a^4+a^2b+b^2}{3}, \qquad \boldsymbol{\delta_\epsilon a_{dd\mathcal{V}\mathcal{V}2}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)(2a^2+b+6)}{12},$$

$$\delta_\epsilon a_{11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a^2}{4}, \qquad \delta_\epsilon a_{H11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{2}, \qquad \boldsymbol{\delta_\epsilon a_{HH11}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{4},$$

$$\delta_\epsilon a_{HWW} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{12}, \qquad \boldsymbol{\delta_\epsilon a_{HHWW}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{24},$$

$$\delta_\epsilon a_{d2} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{6}, \qquad \boldsymbol{\delta_\epsilon a_{Hd2}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{6},$$

$$\delta_\epsilon a_{\Box\mathcal{V}\mathcal{V}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(2+a^2)}{4}, \qquad \boldsymbol{\delta_\epsilon a_{H\Box\mathcal{V}\mathcal{V}}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4+a^2(4-3b)-2b}{4},$$

$$\delta_\epsilon a_{d3} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2+b)}{2}, \qquad \boldsymbol{\delta_\epsilon a_{Hd3}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{-4a^4+a^2b+b^2}{2},$$

$$\delta_\epsilon a_{\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{4}, \qquad \boldsymbol{\delta_\epsilon a_{H\Box\Box}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a(2a^2-b)}{2},$$

$$\boldsymbol{\delta_\epsilon a_{dd\Box}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3(a^2-b)}{2}, \qquad \boldsymbol{\delta_\epsilon a_{Hdd}} = 0, \qquad \boldsymbol{\delta_\epsilon a_{ddW}}/2 = \boldsymbol{\delta_\epsilon a_{ddZ}} = -\frac{\Delta_\epsilon}{16\pi^2}3a(a^2-b),$$

$$\boldsymbol{\delta_\epsilon a_{H\mathcal{V}\mathcal{V}}} = \boldsymbol{\delta_\epsilon a_{HH\mathcal{V}\mathcal{V}}} = 0, \qquad \Delta_\epsilon = \frac{2}{\epsilon} - \gamma_E + \log(4\pi).$$

Comment: $\delta_\epsilon\kappa_4$ and others $\delta_\epsilon a_i's$ are fixed in NLO $gg \to HH$ and $gg \to HHH$ (see next)

Combinations appearing in scattering amplitude :
(=use of e.o.m)

$$\delta_\epsilon\eta = \delta_\epsilon\tilde{a}_{dd\mathcal{V}\mathcal{V}1} = \delta_\epsilon(a_{dd\mathcal{V}\mathcal{V}1} - 4a^2a_{11} + 2aa_{d3}) = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)^2}{3},$$

$$\delta_\epsilon\delta = \delta_\epsilon\tilde{a}_{dd\mathcal{V}\mathcal{V}2} = \delta_\epsilon\left(a_{dd\mathcal{V}\mathcal{V}2} + \frac{a}{2}a_{dd\Box}\right) = \frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)(7a^2-b-6)}{12},$$

$$\delta_\epsilon(a_{H\mathcal{V}\mathcal{V}} - 2a_{\Box\mathcal{V}\mathcal{V}} + 2aa_{\Box\Box}) = \frac{\Delta_\epsilon}{16\pi^2}a(1-a^2),$$

$$\delta_\epsilon(a_{HH\mathcal{V}\mathcal{V}} - 6\kappa_3 a_{\Box\mathcal{V}\mathcal{V}} - 4a_{H\Box\mathcal{V}\mathcal{V}} + 4ba_{\Box\Box} + 6\kappa_3 aa_{\Box\Box} + 4aa_{H\Box\Box}) = \frac{\Delta_\epsilon}{16\pi^2}(3\kappa_3 a(1-a^2) + 2b - 2a^2(2+3b) + 8a^4),$$

$$\delta_\epsilon(a_{Hdd} - a_{dd\Box}) = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a(a^2-b)}{2}.$$

RGE easily derived for all these $c_i's$ HEFT coefficients

$$c_i(\mu) = c_i(\mu') + \frac{1}{16\pi^2}\gamma_{c_i}\log\left(\frac{\mu^2}{\mu'^2}\right)\,, \qquad \delta_\epsilon c_i = \frac{\Delta_\epsilon}{16\pi^2}\gamma_{c_i}$$

We checked some $\delta c_i's$ with previous
results in specific limits :
pure scalar (1311.5993,14091571)
isospin limit $m_W = m_Z$ (2109.02673)
Others were unknown
before our work (see paper)

CTs in NLO $gg \to HH$, HHH and derived RGEs

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385

NLO Bosonic HEFT operators involved

$\mathcal{O}_{\Box\Box}$	$a_{\Box\Box}\,\frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{H\Box\Box}$	$a_{H\Box\Box}\left(\frac{H}{v}\right)\frac{\Box H \Box H}{v^2}$
$\mathcal{O}_{Hdd}$	$a_{Hdd}\,\frac{m_H^2}{v^2}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H$	$\mathcal{O}_{HHdd}$	$a_{HHdd}\,\frac{m_H^2}{v^2}\left(\frac{H^2}{v^2}\right)\partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddW}$	$a_{ddW}\,\frac{m_W^2}{v^2}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddW}$	$a_{HddW}\,\frac{m_W^2}{v^2}\left(\frac{H^2}{v^2}\right)\partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddZ}$	$a_{ddZ}\,\frac{m_Z^2}{v^2}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddZ}$	$a_{HddZ}\,\frac{m_Z^2}{v^2}\left(\frac{H^2}{v^2}\right)\partial^\mu H \partial_\mu H$
$\mathcal{O}_{dd\Box}$	$a_{dd\Box}\,\frac{1}{v^3}\,\partial^\mu H\,\partial_\mu H\,\Box H$	$\mathcal{O}_{Hdd\Box}$	$a_{Hdd\Box}\,\frac{1}{v^3}\left(\frac{H}{v}\right)\partial^\mu H\,\partial_\mu H\,\Box H$
$\mathcal{O}_{HH\Box\Box}$	$a_{HH\Box\Box}\left(\frac{H^2}{v^2}\right)\frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{dddd}$	$a_{dddd}\,\frac{1}{v^4}\partial^\mu H\,\partial_\mu H\,\partial^\nu H\,\partial_\nu H$

CTs

$$\delta_\epsilon \kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_H^2 v^2}\Big(\kappa_3(a^2 - b + 9\kappa_3^2 - 6\kappa_4)m_H^4 - 3(1-a^2)\kappa_3 m_H^2(m_W^2 + m_Z^2)$$
$$+6(-2ab + 2a^2\kappa_3 + b\kappa_3)(2m_W^4 + m_Z^4)\Big)\ ,$$

$$\delta_\epsilon \kappa_4 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_H^2 v^2}\Big(\kappa_4(2a^2 - 2b + 9\kappa_3^2 - 6\kappa_4)m_H^4 - 6(1-a^2)\kappa_4 m_H^2(m_W^2 + m_Z^2)$$
$$+6(-2b^2 + 2a^2\kappa_4 + b\kappa_4)(2m_W^4 + m_Z^4)\Big)\ ,$$

$$\delta_\epsilon a_{\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{4}\,, \qquad \delta_\epsilon a_{H\Box\Box} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a(2a^2-b)}{2}\,,$$

$$\delta_\epsilon a_{dd\Box} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a(a^2-b)}{2}\,, \quad \delta_\epsilon a_{Hdd} = 0\,, \quad \delta_\epsilon a_{ddW}/2 = \delta_\epsilon a_{ddZ} = -\frac{\Delta_\epsilon}{16\pi^2}3a(a^2-b)$$

$$\delta_\epsilon a_{dddd} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3(a^2-b)^2}{4}\,, \quad \delta_\epsilon a_{HH\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3(12a^4 - 10a^2 b + b^2)}{4}\,,$$

$$\delta_\epsilon a_{Hdd\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3(6a^4 - 7a^2 b + b^2)}{2}\,, \quad \delta_\epsilon a_{HHdd} = 0\,,$$

$$\delta_\epsilon a_{HddW} = \frac{\Delta_\epsilon}{16\pi^2}3(4a^4 - 5a^2 b + b^2)\,, \quad \delta_\epsilon a_{HddZ} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3(4a^4 - 5a^2 b + b^2)}{2}$$

$$c_i(\mu) = c_i(\mu') + \frac{1}{16\pi^2}\gamma_{c_i}\log\left(\frac{\mu^2}{\mu'^2}\right)\,, \qquad \delta_\epsilon c_i = \frac{\Delta_\epsilon}{16\pi^2}\gamma_{c_i}$$

Notice the non-trivial dependencies on the external momenta generically off-shell typical from non-linearity

This has relevant pheno implications at colliders, particularly at LHC

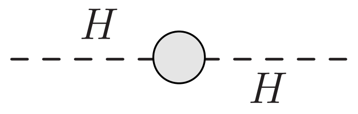
Interesting RGEs for κ_3 and κ_4

For instance, for $a = b = 1$

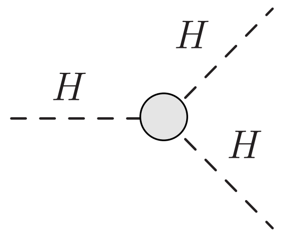
$$\gamma_{\kappa_3} = -\frac{1}{2m_H^2 v^2}\left(\kappa_3(9\kappa_3^2 - 6\kappa_4)m_H^4 + 6(3\kappa_3 - 2)(2m_W^4 + m_Z^4)\right)$$

$$\gamma_{\kappa_4} = -\frac{1}{2m_H^2 v^2}\left(\kappa_4(9\kappa_3^2 - 6\kappa_4)m_H^4 + 6(3\kappa_4 - 2)(2m_W^4 + m_Z^4)\right)$$

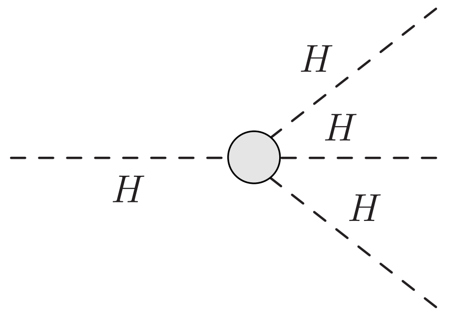
Renormalized 1PIs involved



$$= -i\hat{\Sigma}_{HH}(q^2)$$
$$= -i\Sigma_{HH}^{\rm Loop}(q^2) + i\left(\delta Z_H(q^2 - m_H^2) - \delta m_H^2\right) + i\frac{2a_{\Box\Box}}{v^2}q^4$$



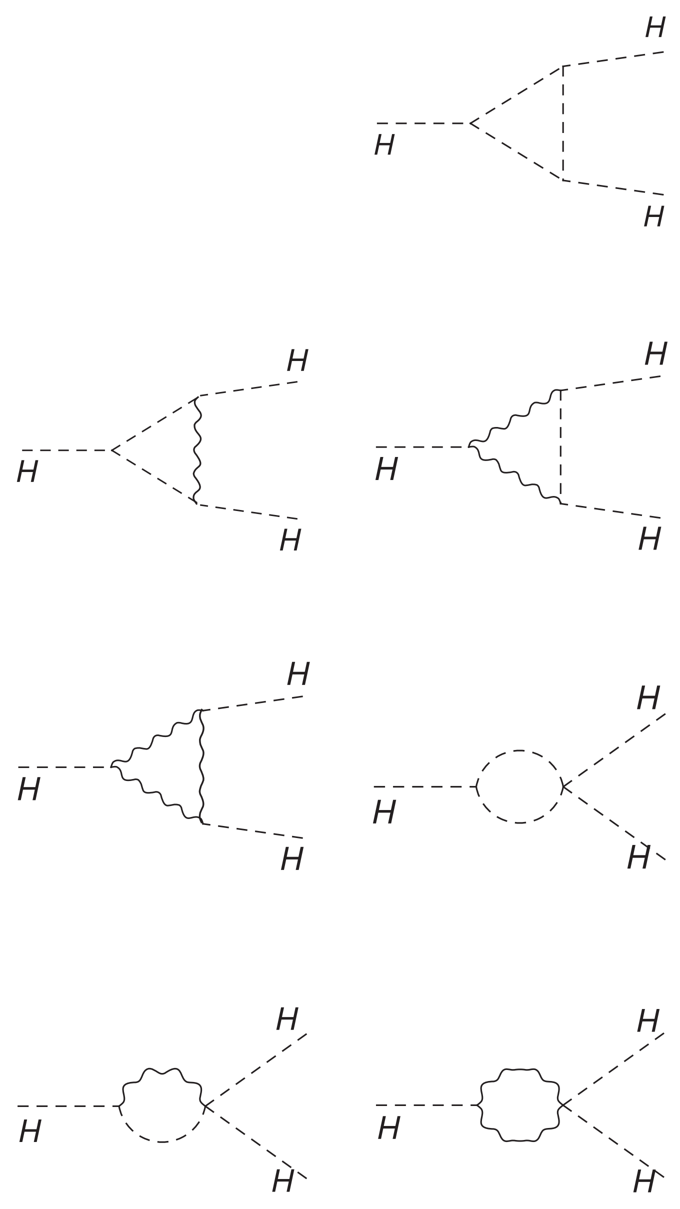
$$= i\hat{\Gamma}_{HHH}(p_1,p_2,p_3)$$
$$= -3i\kappa_3\frac{m_H^2}{v} + i\Gamma_{HHH}^{\rm Loop} - 3i\kappa_3\frac{m_H^2}{v}\left(\frac{\delta\kappa_3}{\kappa_3} + \frac{\delta m_H^2}{m_H^2} - \frac{\delta Z_\pi}{2} - \frac{\delta v}{v} + \frac{3\delta Z_H}{2}\right)$$
$$+\frac{i}{v^3}\big(a_{dd\Box}(p_1^4 + p_2^4 + p_3^4) + 2(a_{H\Box\Box} - a_{dd\Box})(p_1^2p_2^2 + p_2^2p_3^2 + p_3^2p_1^2)$$
$$+(a_{Hdd}\,m_H^2 + a_{ddW}\,m_W^2 + a_{ddZ}\,m_Z^2)(p_1^2 + p_2^2 + p_3^2)\big)\ ,$$



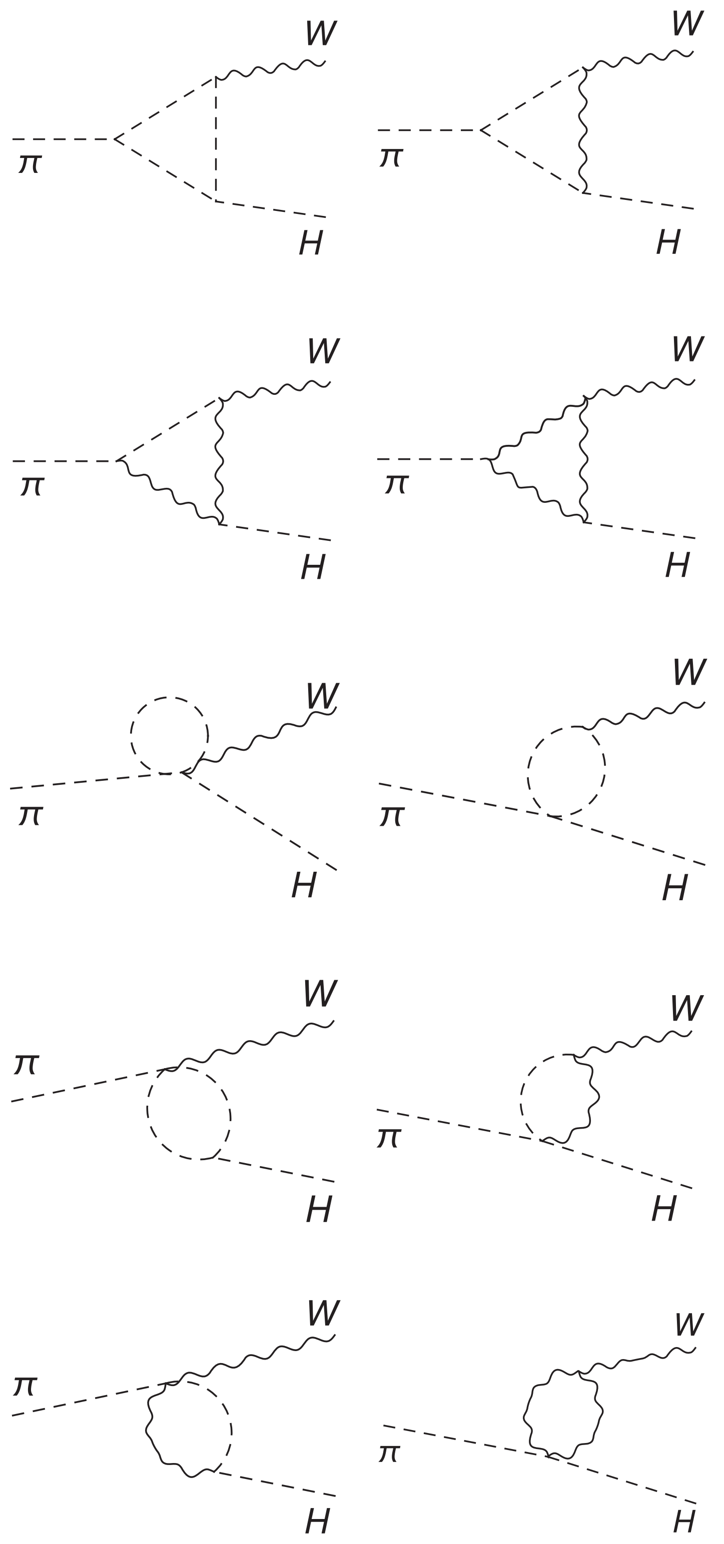
$$= i\hat{\Gamma}_{HHHH}(p_1,p_2,p_3,p_4)$$
$$= -3i\kappa_4\frac{m_H^2}{v^2} + i\Gamma_{HHHH}^{\rm Loop} - 3i\kappa_4\frac{m_H^2}{v^2}\left(\frac{\delta\kappa_4}{\kappa_4} + \frac{\delta m_H^2}{m_H^2} - 2\left(\frac{\delta Z_\pi}{2} + \frac{\delta v}{v}\right) + 2\delta Z_H\right)$$
$$+\frac{i}{v^4}\big(a_{Hdd\Box}(p_1^4 + p_2^4 + p_3^4 + p_4^4 - 2(p_1^2p_2^2 + p_1^2p_3^2 + p_1^2p_4^2 + p_2^2p_3^2 + p_2^2p_4^2 + p_3^2p_4^2))$$
$$+4a_{HH\Box\Box}(p_1^2p_2^2 + p_1^2p_3^2 + p_1^2p_4^2 + p_2^2p_3^2 + p_2^2p_4^2 + p_3^2p_4^2)$$
$$+2(a_{HHdd}\,m_H^2 + a_{HddW}\,m_W^2 + a_{HddZ}\,m_Z^2)(p_1^2 + p_2^2 + p_3^2 + p_4^2)$$
$$+4a_{dddd}\big((p_1+p_2)^2(p_1+p_3)^2 + (p_1+p_2)^2(p_2+p_3)^2 + (p_1+p_3)^2(p_2+p_3)^2$$
$$-(p_1^2p_2^2 + p_1^2p_3^2 + p_1^2p_4^2 + p_2^2p_3^2 + p_2^2p_4^2 + p_3^2p_4^2)\big)\big)\ .$$

Loop diagramas involved in WW -> HH

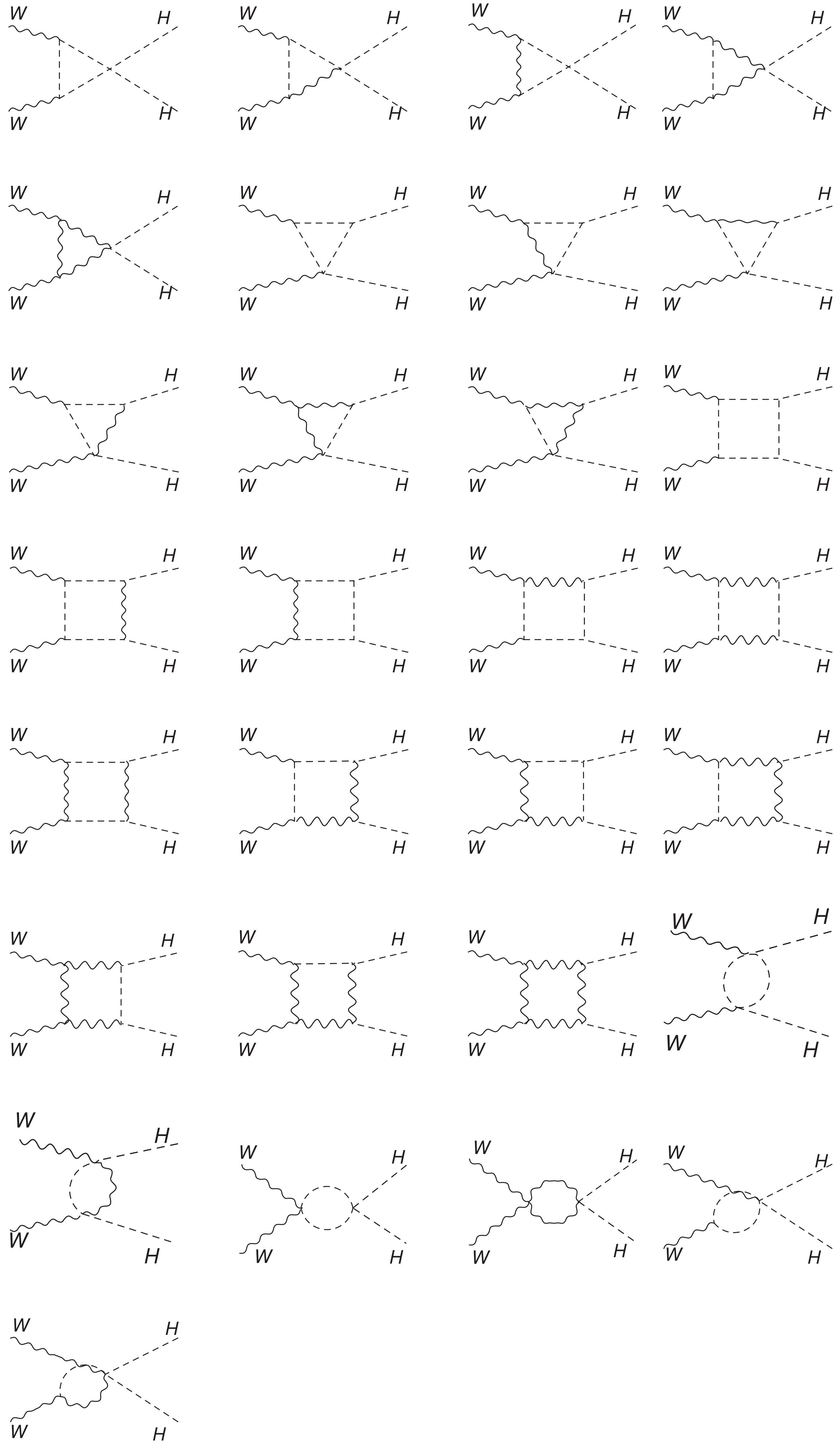
Γ_{HHH}



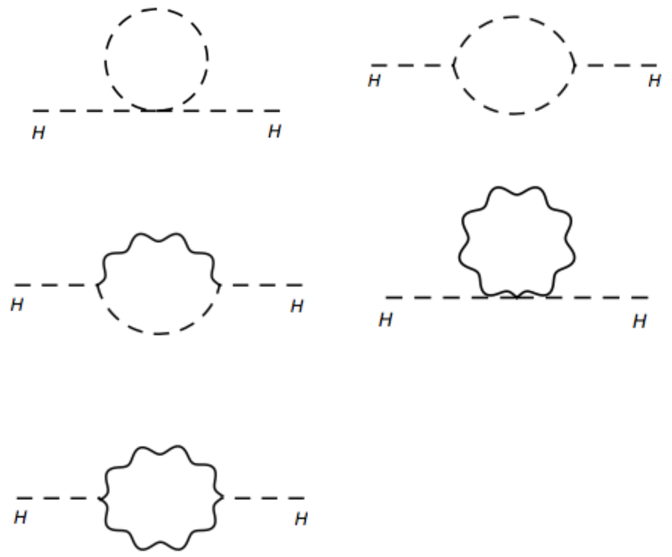
$\Gamma_{\pi WH}$



Γ_{WWHH}

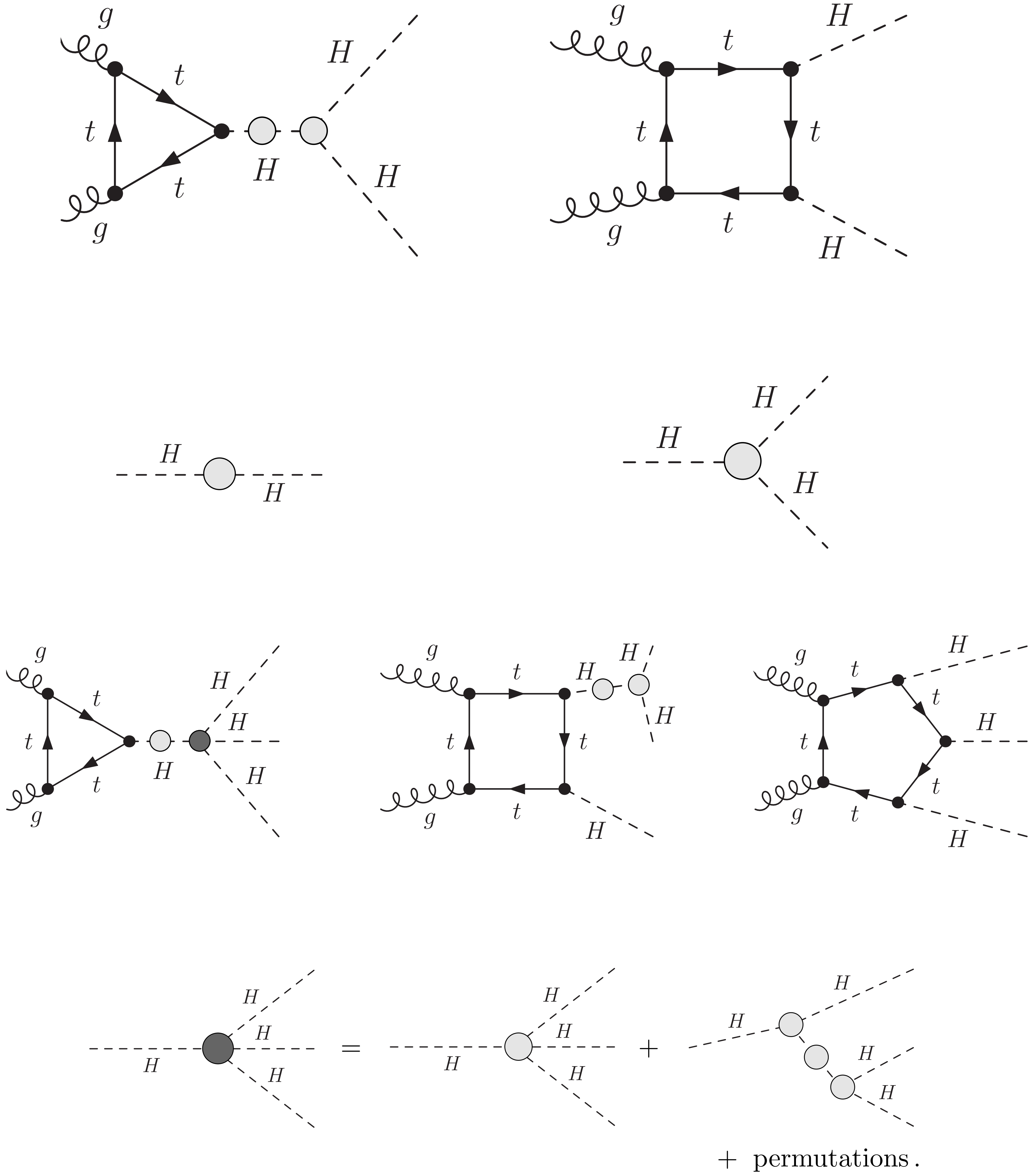


+ 2-point functions

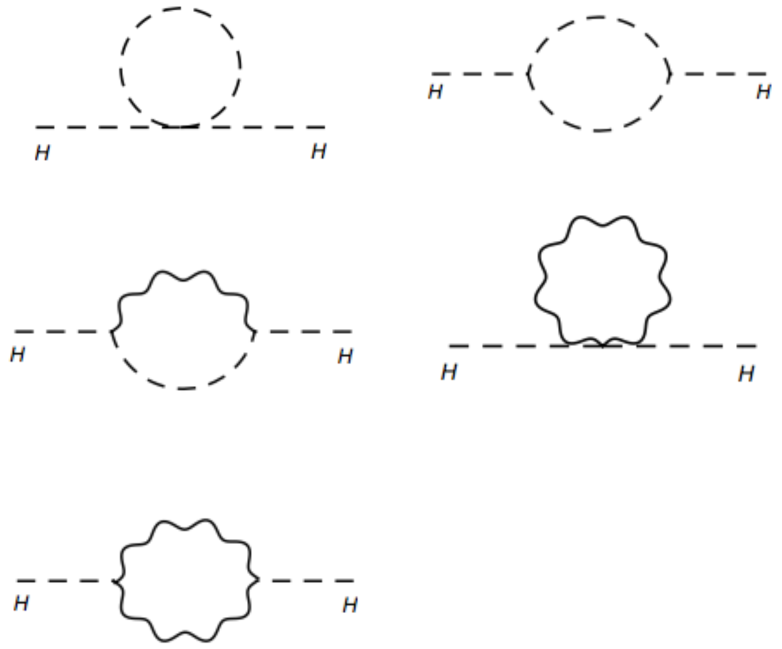


+ ....

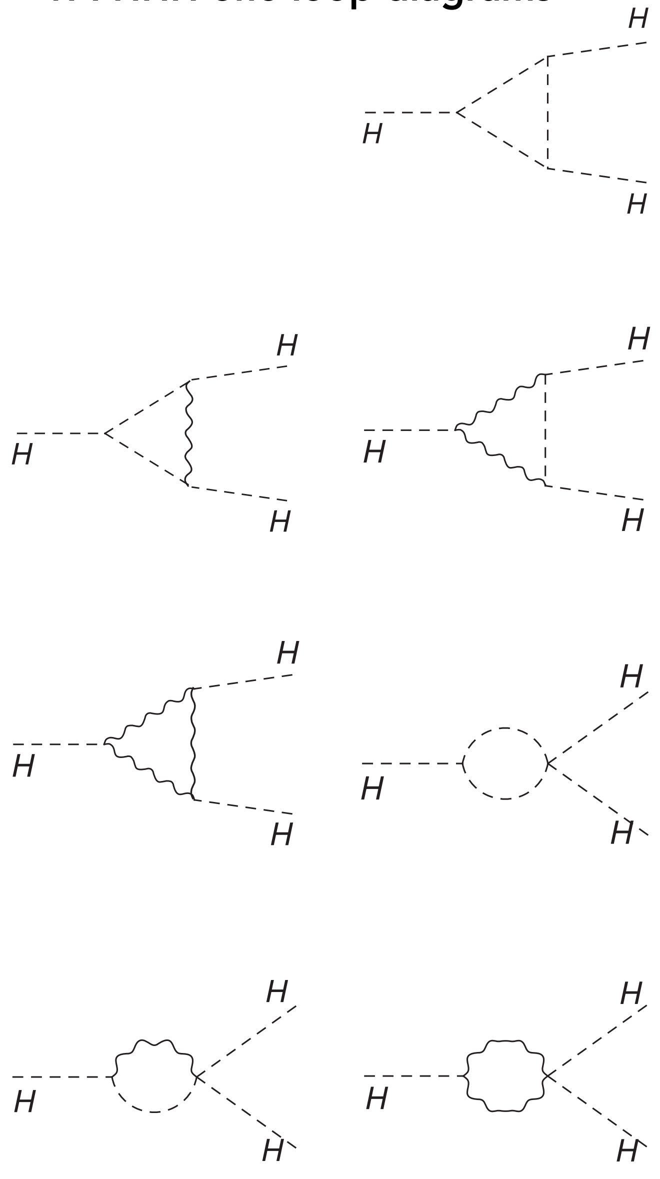
Loop diagramas involved in gg -> HH(HHH)



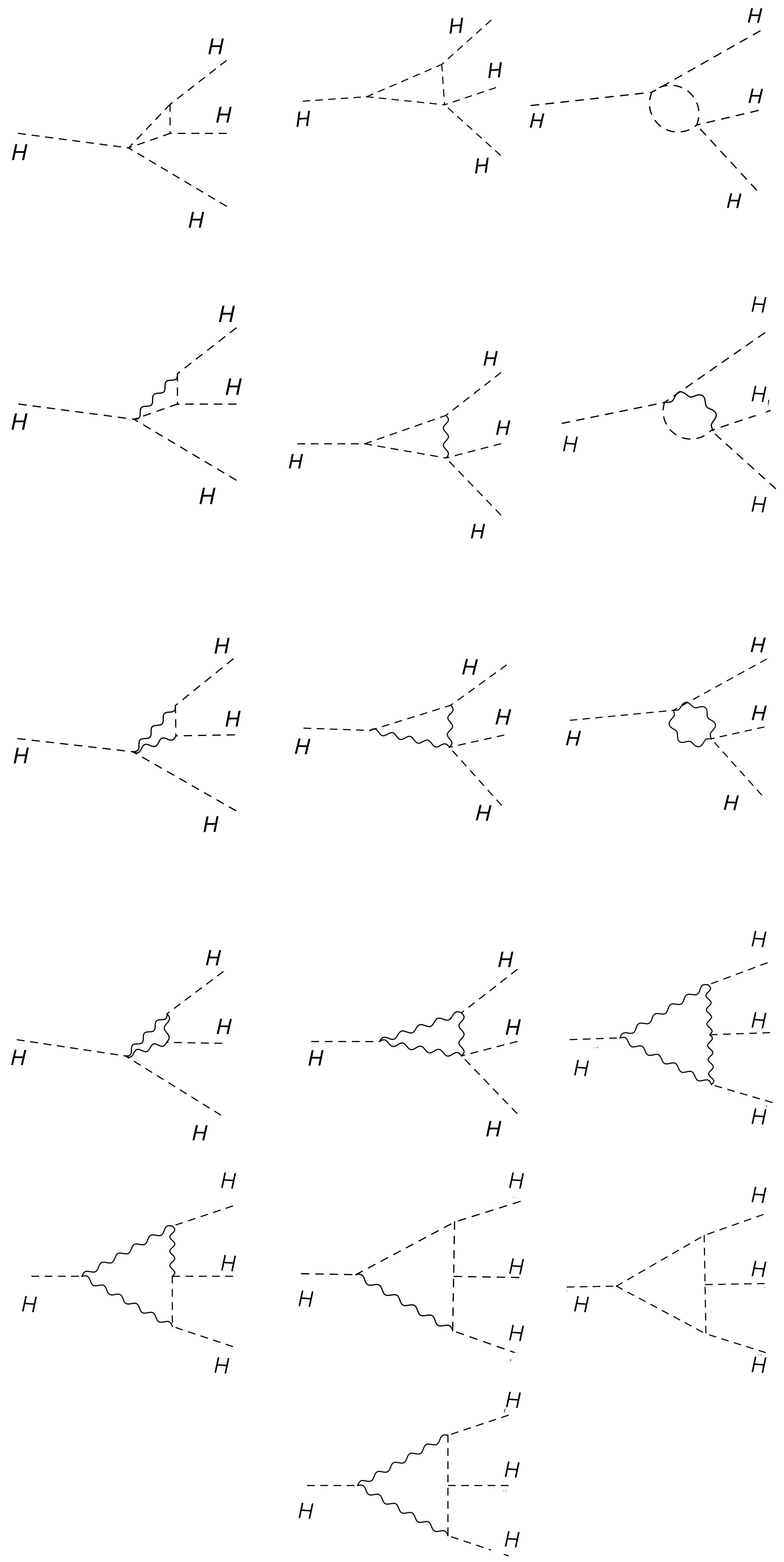
1PI HH one-loop diagrams



1PI HHH one-loop diagrams



1PI HHHH one-loop diagrams



Large effects from NLO coefficients

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385

$\mathcal{O}_{\Box\Box}$	$a_{\Box\Box}\frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{H\Box\Box}$	$a_{H\Box\Box}\left(\frac{H}{v}\right)\frac{\Box H \Box H}{v^2}$
$\mathcal{O}_{Hdd}$	$a_{Hdd}\frac{m_H^2}{v^2}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H$	$\mathcal{O}_{HHdd}$	$a_{HHdd}\frac{m_H^2}{v^2}\left(\frac{H^2}{v^2}\right)\partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddW}$	$a_{ddW}\frac{m_W^2}{v^2}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddW}$	$a_{HddW}\frac{m_W^2}{v^2}\left(\frac{H^2}{v^2}\right)\partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddZ}$	$a_{ddZ}\frac{m_Z^2}{v^2}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddZ}$	$a_{HddZ}\frac{m_Z^2}{v^2}\left(\frac{H^2}{v^2}\right)\partial^\mu H \partial_\mu H$
$\mathcal{O}_{dd\Box}$	$a_{dd\Box}\frac{1}{v^3}\partial^\mu H \partial_\mu H \Box H$	$\mathcal{O}_{Hdd\Box}$	$a_{Hdd\Box}\frac{1}{v^3}\left(\frac{H}{v}\right)\partial^\mu H \partial_\mu H \Box H$
$\mathcal{O}_{HH\Box\Box}$	$a_{HH\Box\Box}\left(\frac{H^2}{v^2}\right)\frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{dddd}$	$a_{dddd}\frac{1}{v^4}\partial^\mu H \partial_\mu H \partial^\nu H \partial_\nu H$

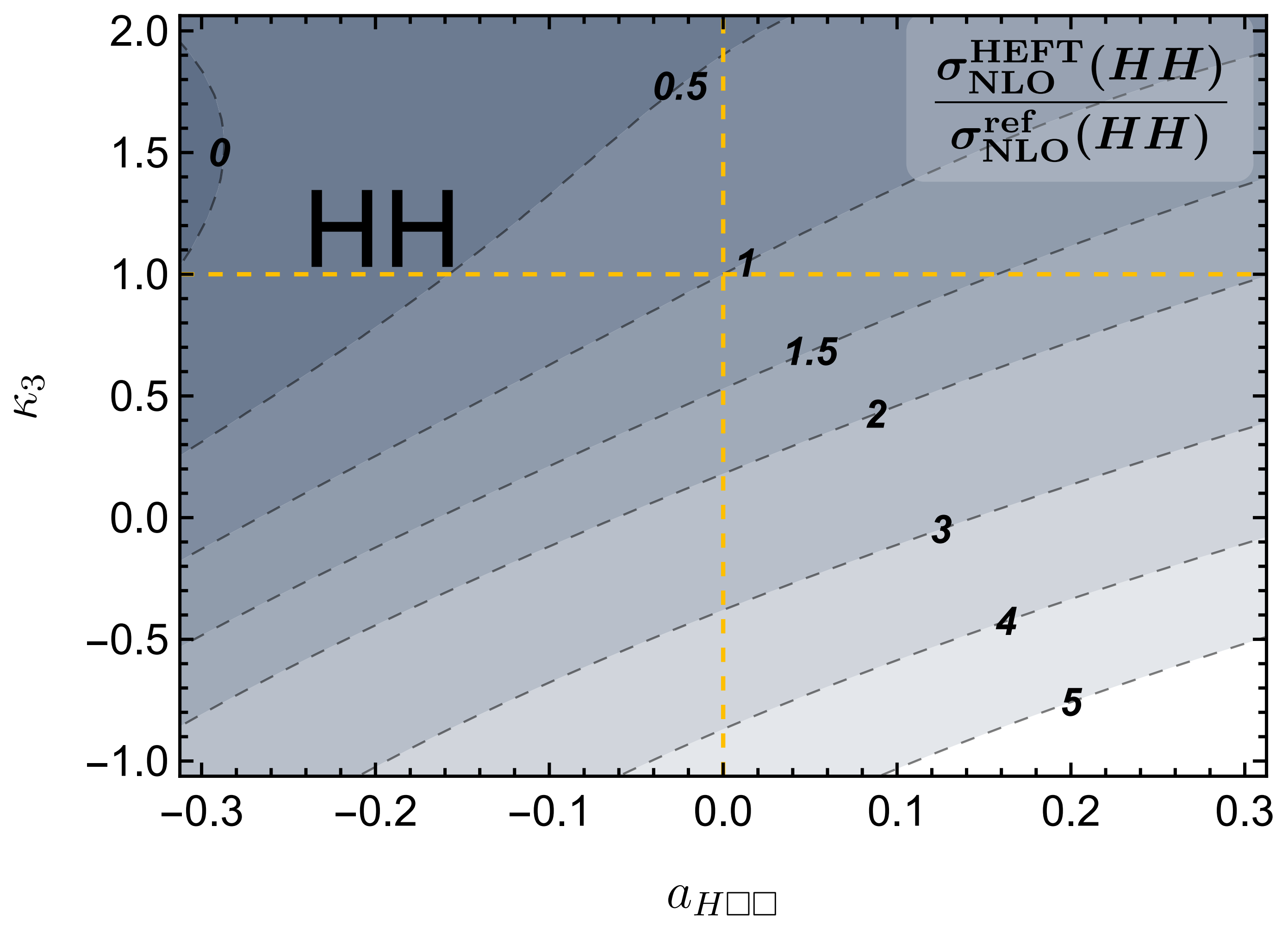
The largest effects are from operators with higher number of derivatives: $a_{dd\Box}, a_{H\Box\Box}, a_{dddd}\ldots$

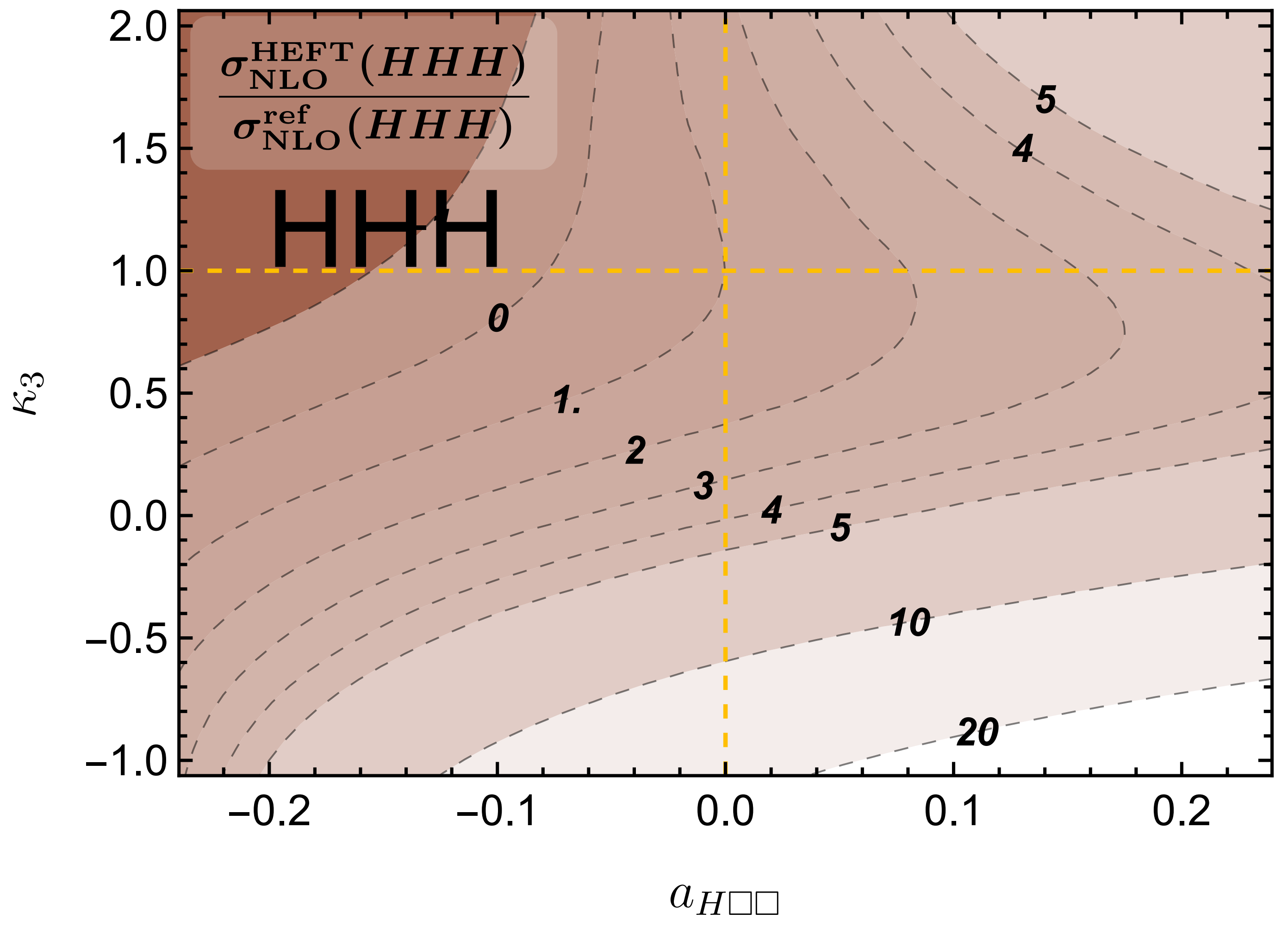
For instance, for $a_{H\Box\Box} = 0.1$ and $\kappa_3 = 1$

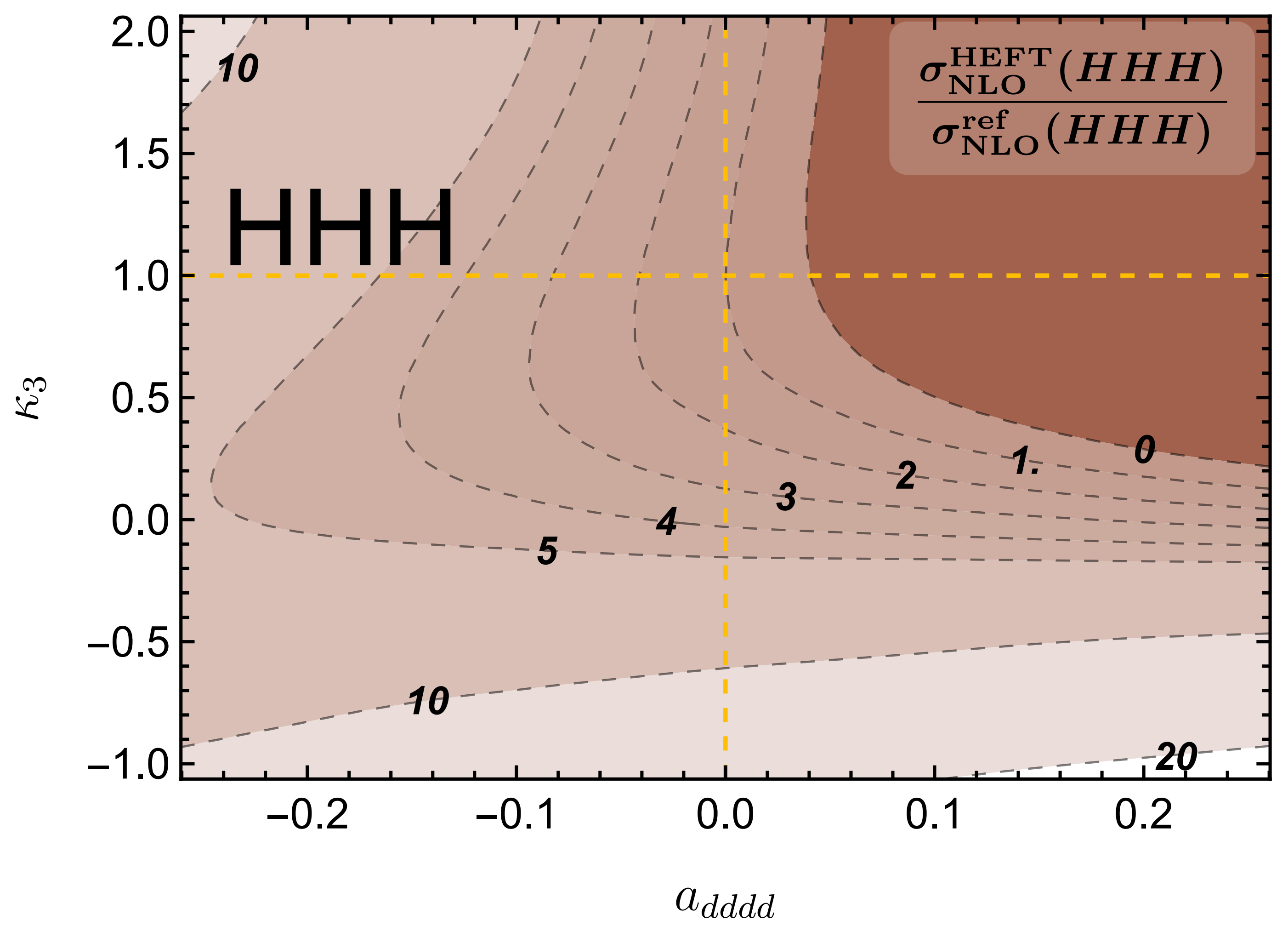
$$\sigma^{\rm HEFT}(HH) \sim 1.5\,\sigma^{\rm SM}(HH) \quad (50\%)$$

$$\sigma^{\rm HEFT}(HHH) \sim 1.8\,\sigma^{\rm SM}(HHH)\ (80\%)$$

Other 2D correlation plots in 2405.05385







Behavior with energy: subprocess (LO-HEFT)

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez

Subprocess level

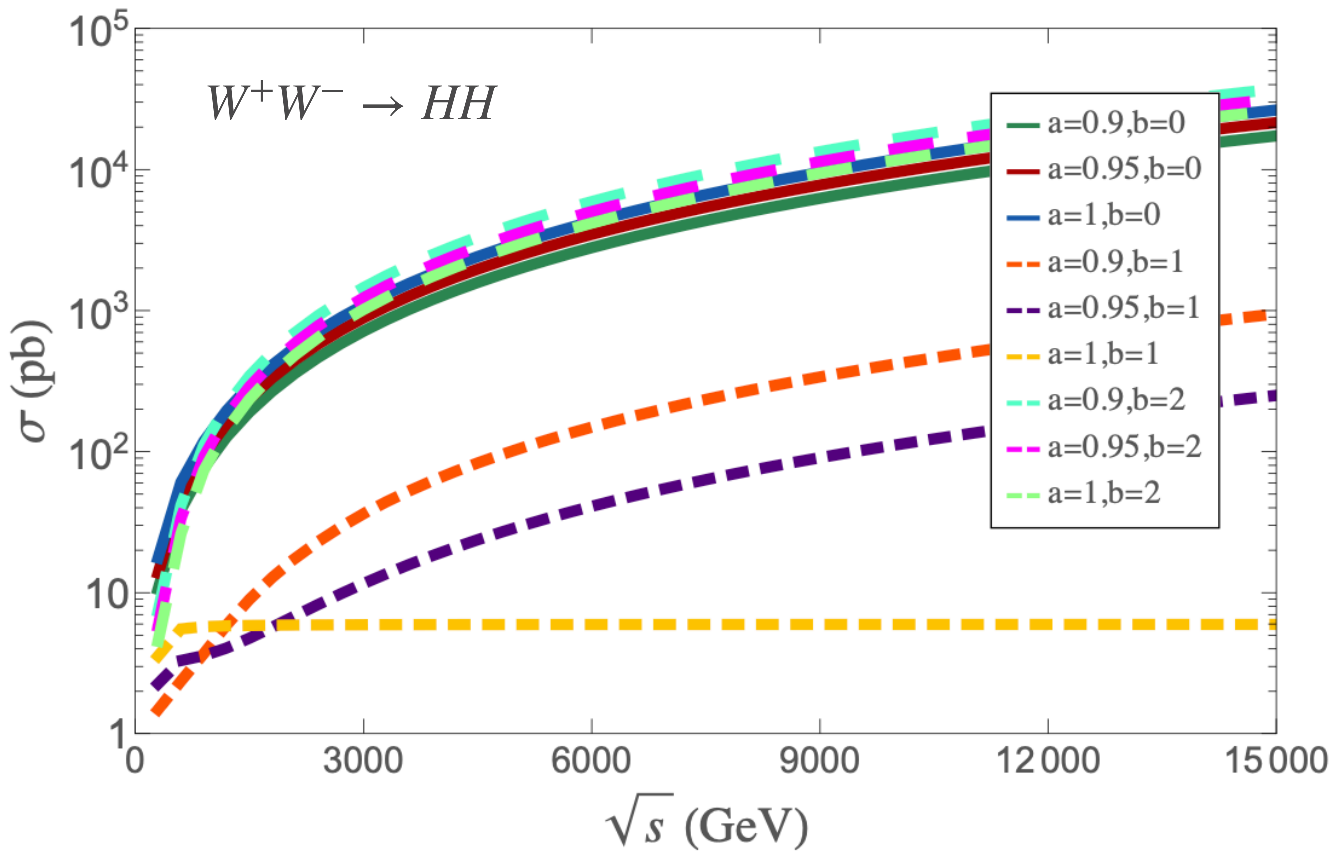
$WW \to HH$

Largest deviations respect to SM in LL modes

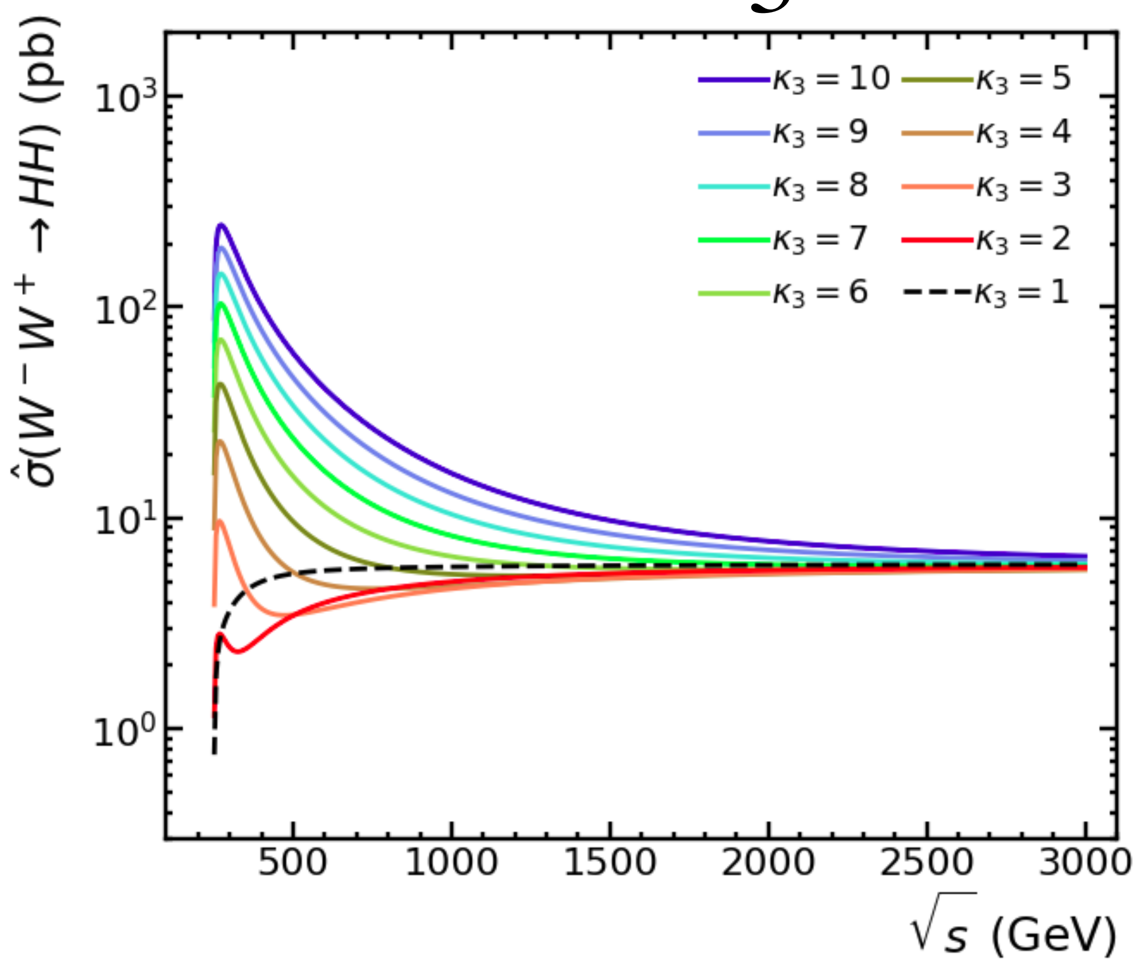
$WW \to HHH$

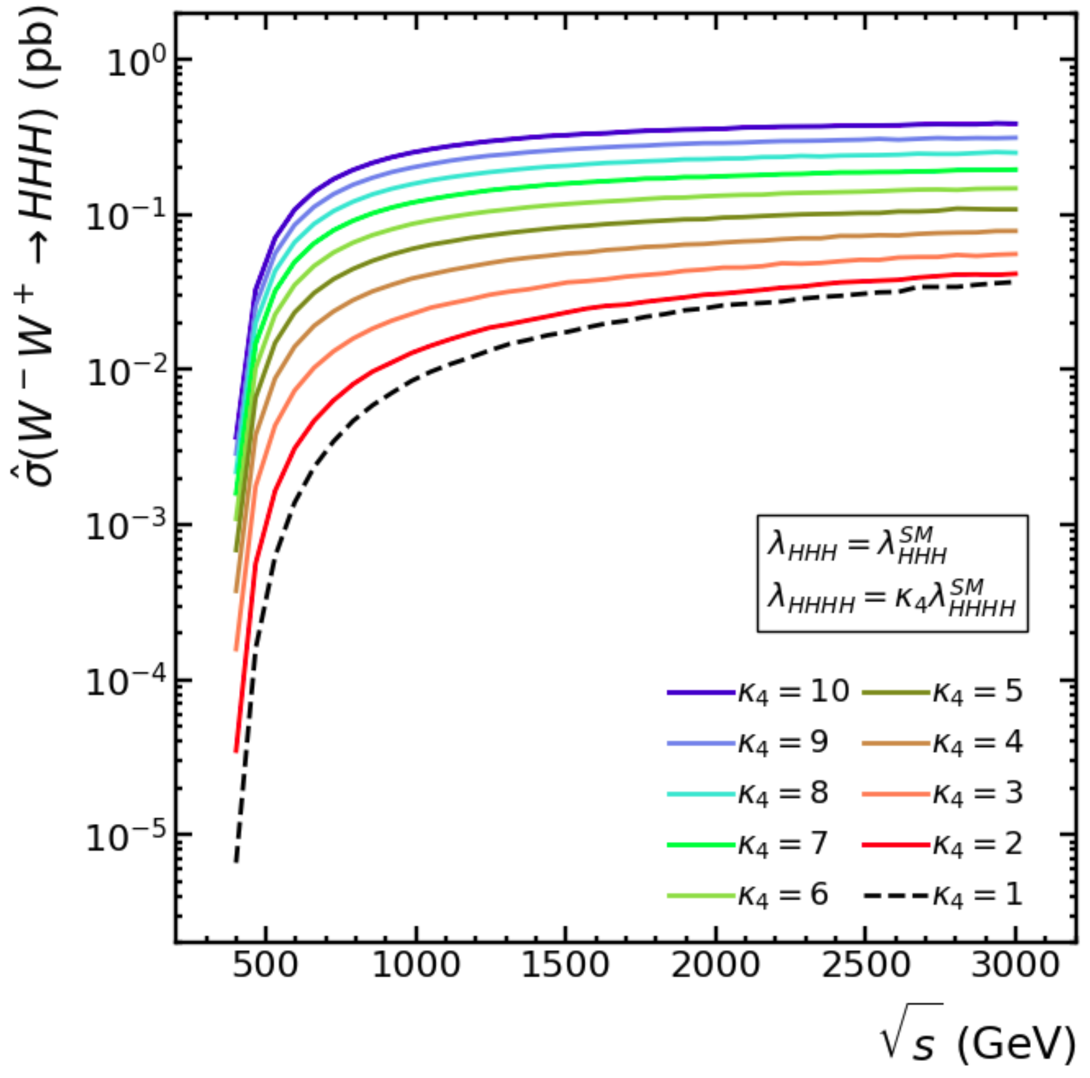
Idem ZZ—>HH(H)

$b \neq a^2$

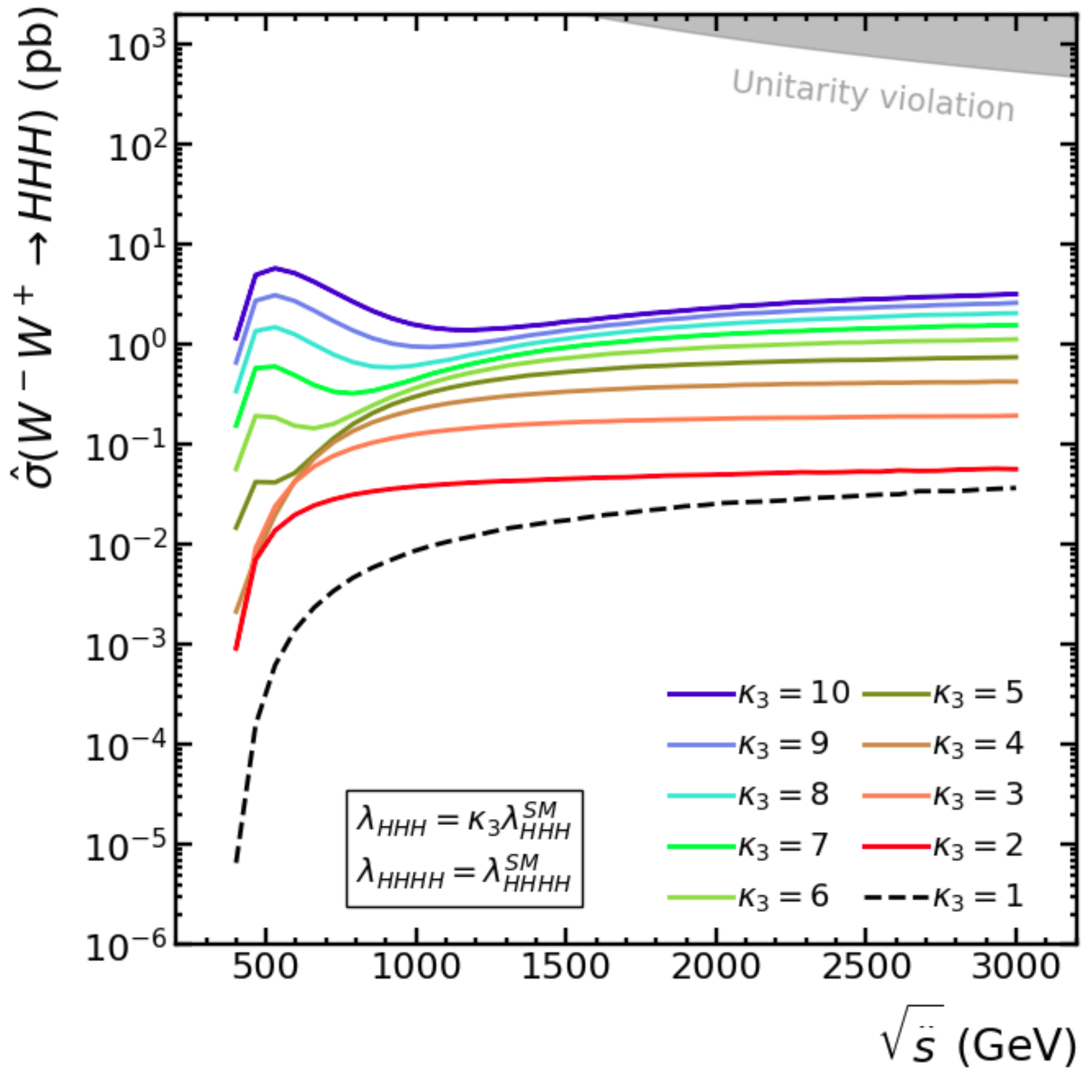


κ_3



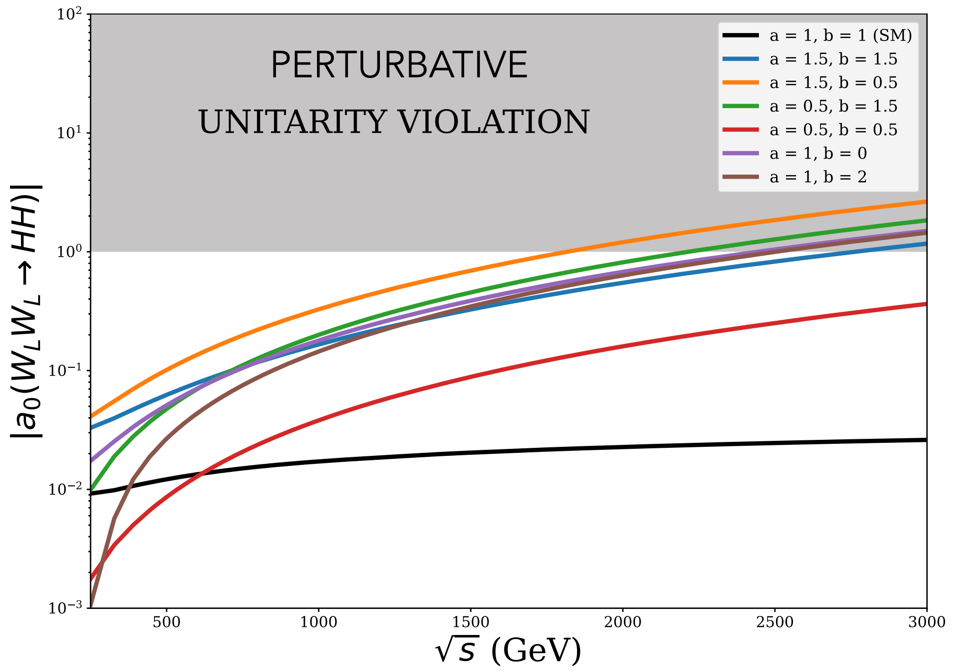


κ_4



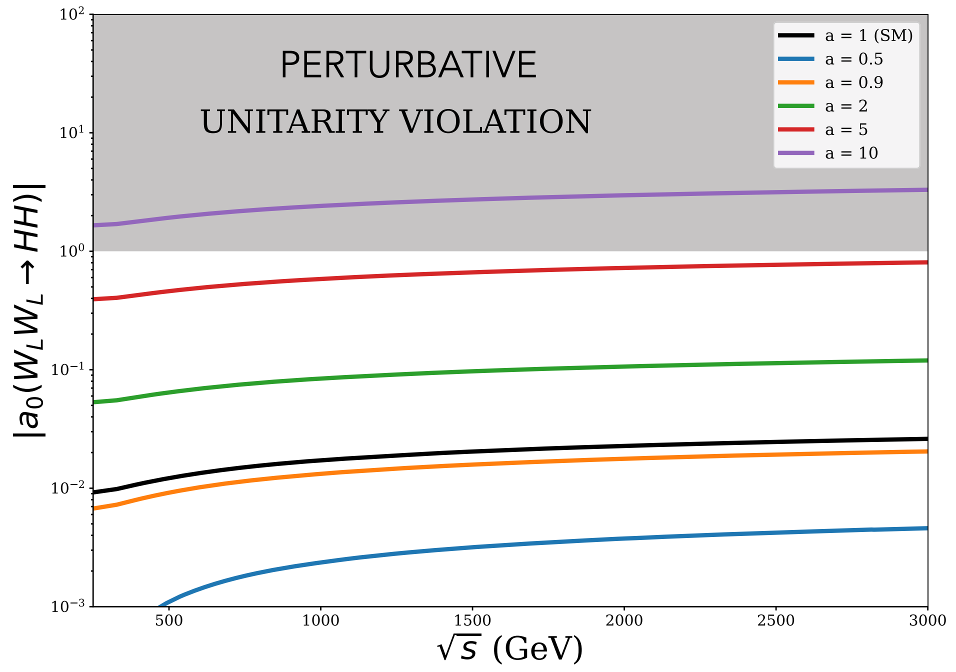
κ_3

$b \neq a^2$



HH

$b = a^2$



HH : Strong enhancement at large $\sqrt{s}$ for $b \neq a^2$
Pert. unitarity viol above few TeV
$\kappa_{2V} = 0$ viol unit. above 2.4 TeV !
Max sensitivity to κ_3 close to $2m_H$

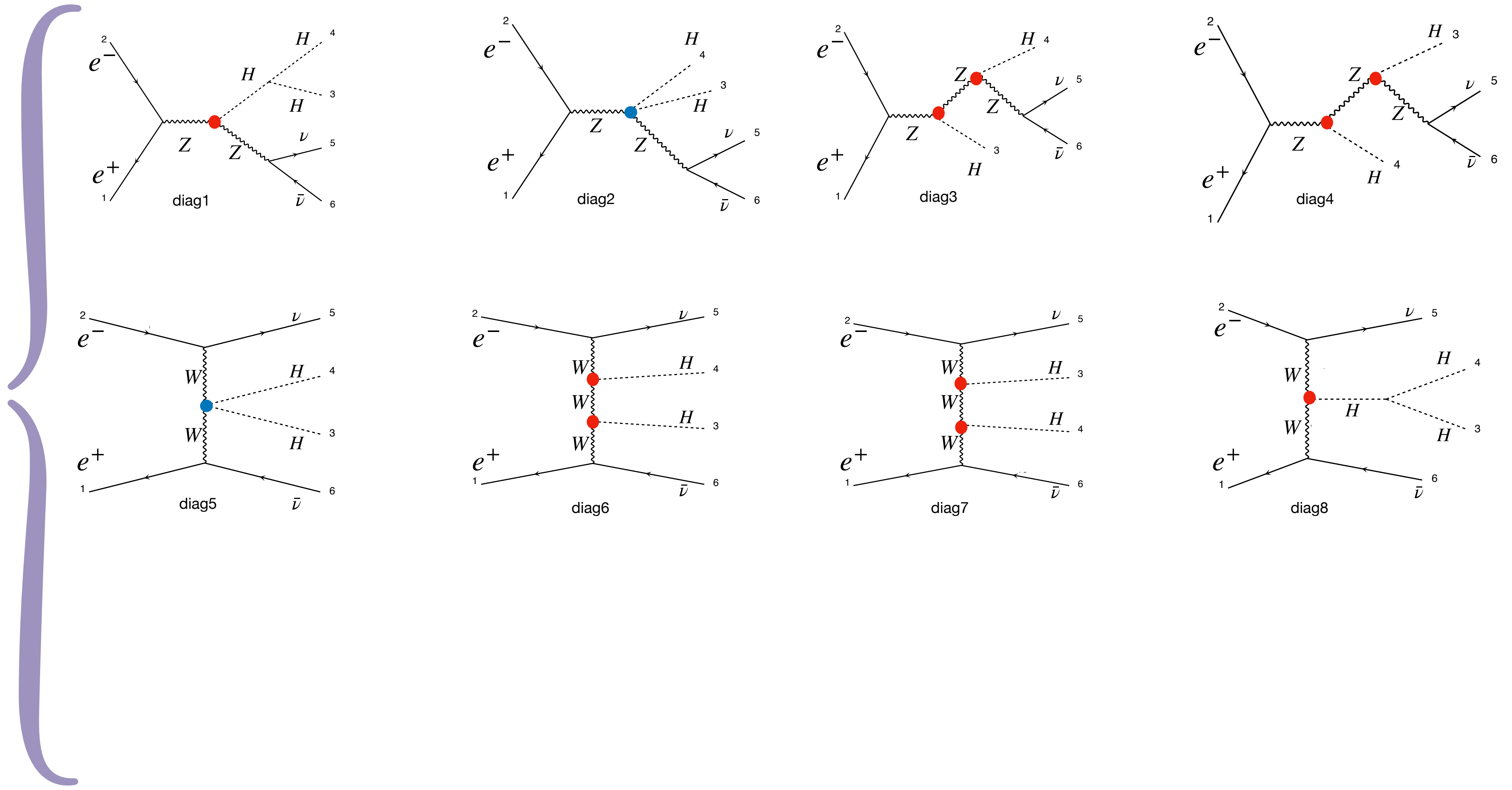
HHH : Similar behavior at large $\sqrt{s}$ as in the SM (shifted upwards)
No unitarity constraints on κ_3, κ_4
Max sensitivity to κ_3 close to $3m_H$

The largest sensitivity to k3 and k4 occurs in ggF, gg —>HH(HHH), see later

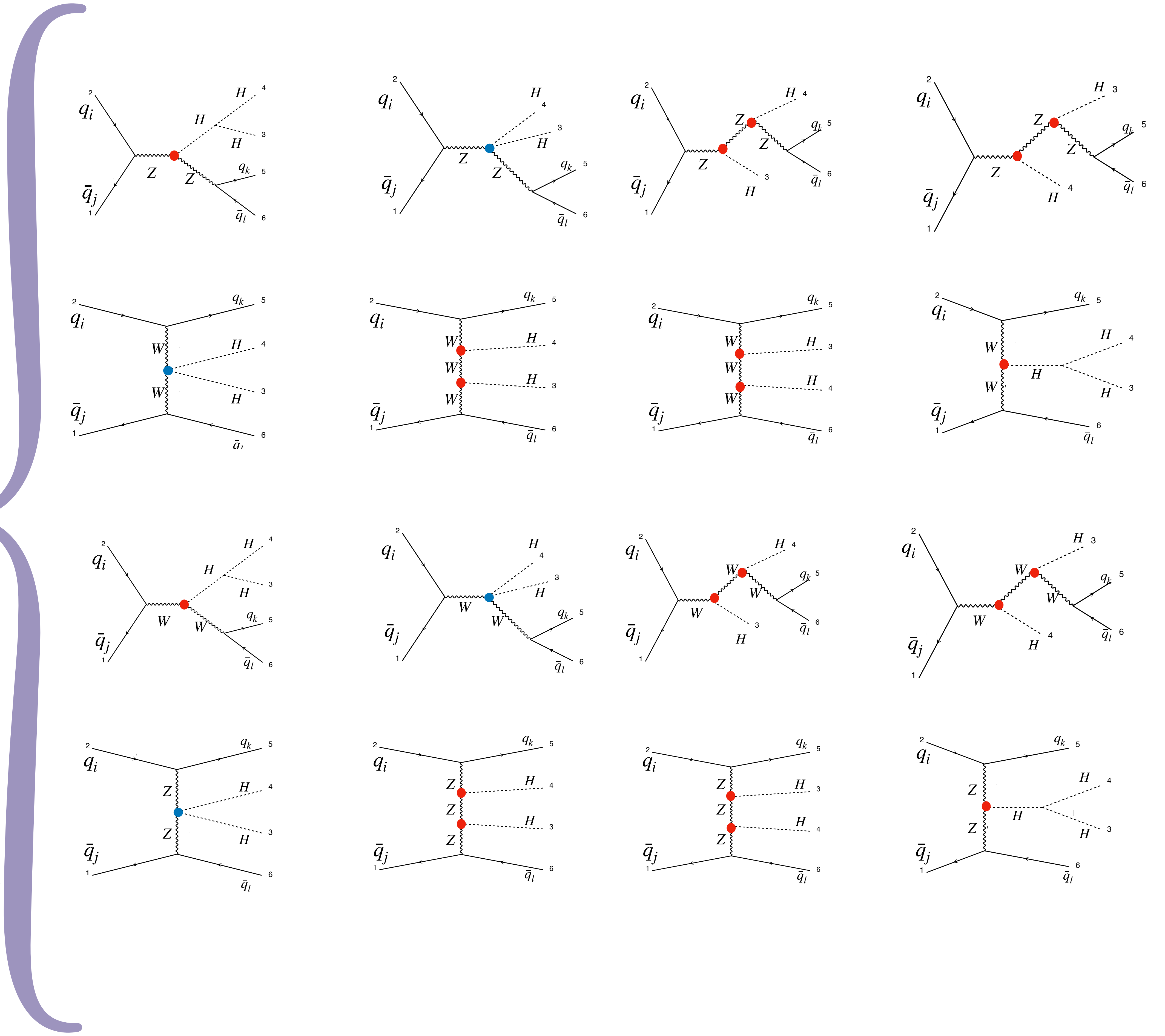
HH production: testing a=κ_V, b=κ_{2V} together at colliders (LO-HEFT)

Our Bosonic-HEFT model file is implemented in MG5

e^+e^- $\quad e^+e^- \rightarrow HH\nu\bar{\nu}$



LHC $\quad q_1\bar{q}_2 \rightarrow HHq_3\bar{q}_4$ (+ diags for $\bar{q}\bar{q}$ and for qq)



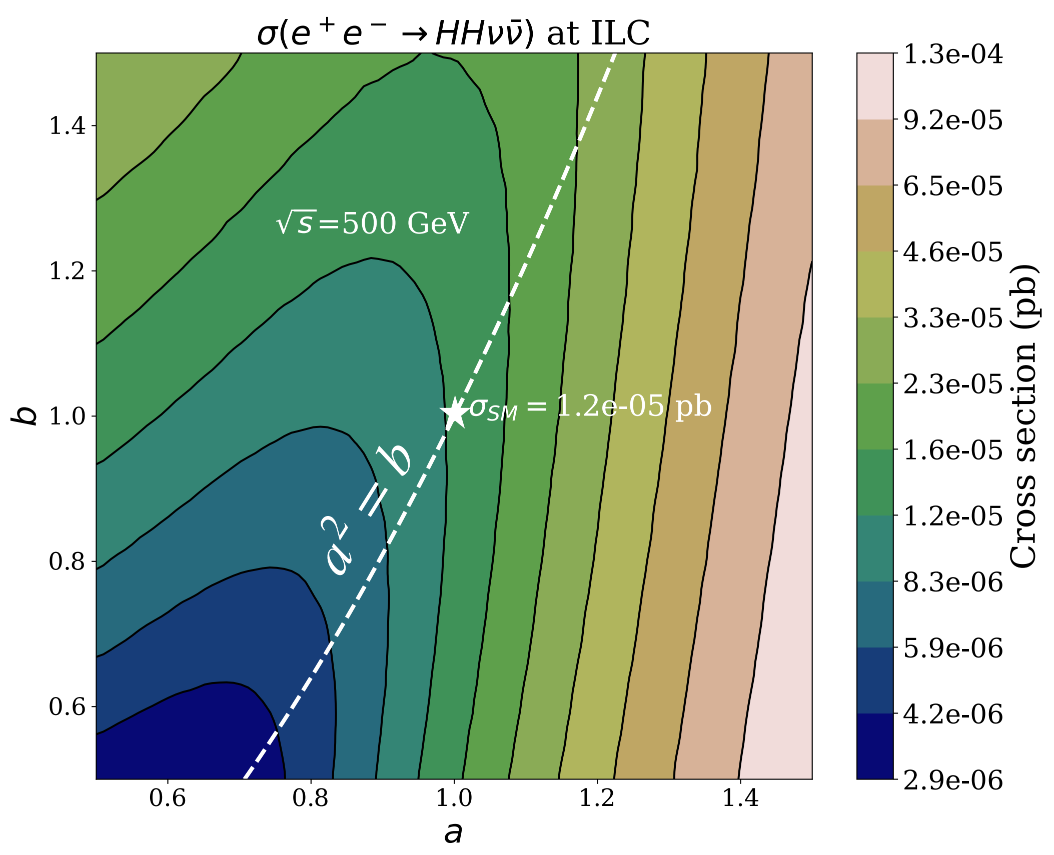
both $(a, b) = (\kappa_V, \kappa_{2V})$ involved

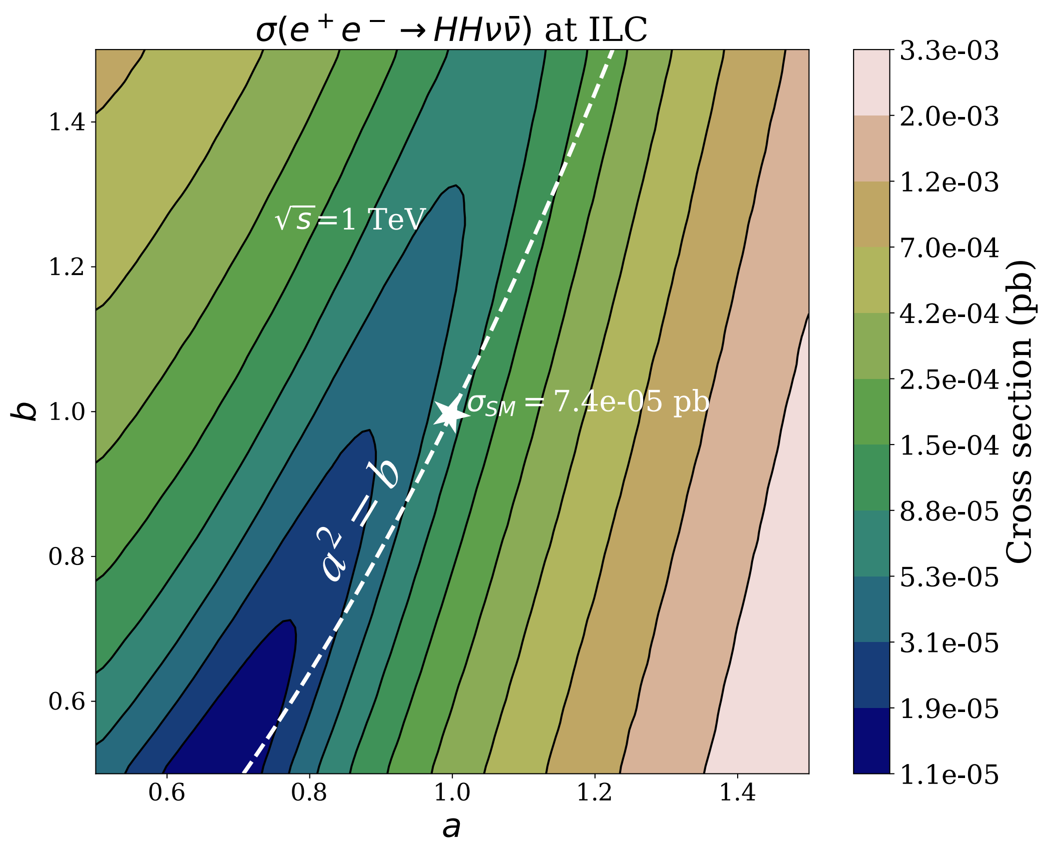
Explore in $HH\not{E}_T$ events. We require $\not{E}_T > 20\,\mathrm{GeV}$

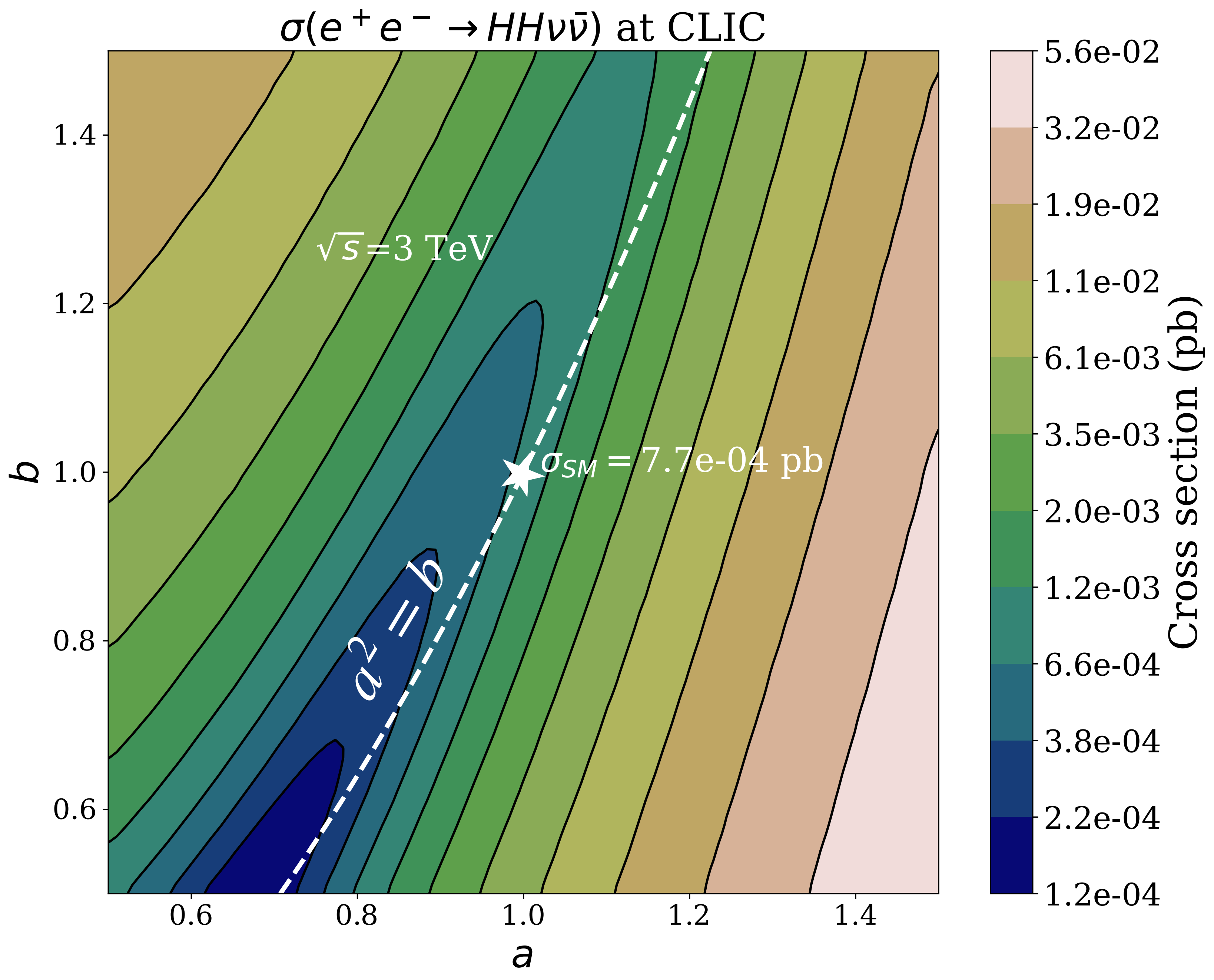
Explore in $HHjj$ events. We require WBF cuts, $\Delta\eta_{jj}$> 4, $M_{jj} > 500$ GeV

BSM signals means deviations in σ and in $d\sigma's$ respect the SM rates. We also explore correlations.

Sensitivity to a=κ_V, b=κ_{2V} in $e^+e^- \to HH\nu\bar{\nu}$







Largest sensitivity expected if

$$a^2 \neq b$$

producing the largest deviations compared to SM predictions

The best expectations are for CLIC (3 TeV) where

BSM/SM $\gtrsim \mathcal{O}(10)$

for yet allowed (a,b)

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez , 2312.03877 Davila, Domenech, Herrero, Morales, EPJC 84 (2024)5, 503

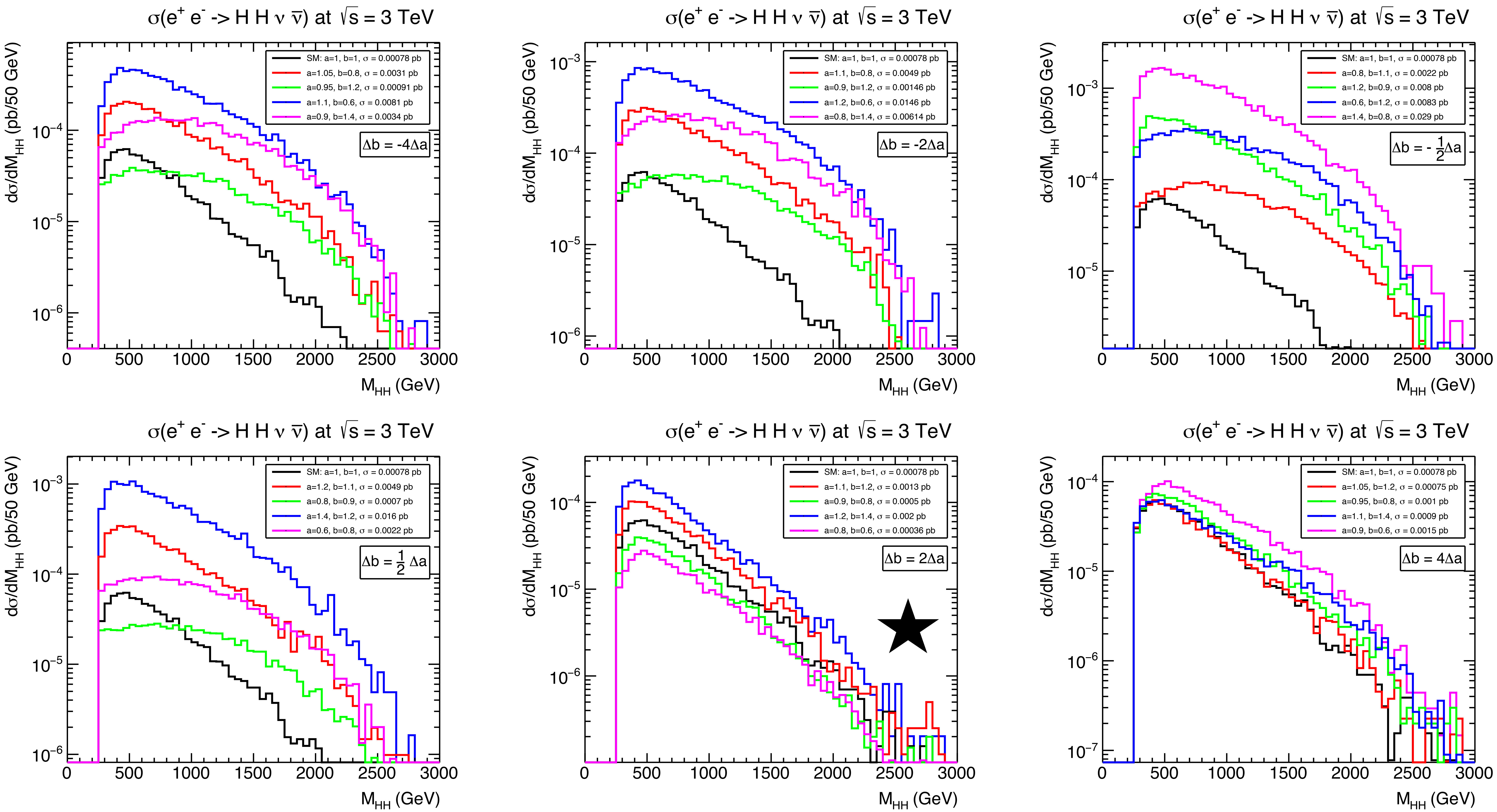
Exploring correlations (κ_V, κ_{2V}) at $e^+e^- \to HH\nu\bar{\nu}$ in $d\sigma/dM_{HH}$

$e^+e^-(3\,\mathrm{TeV})$

Dávila, Domenech, Herrero, Morales [2312.03877] EPJC 84 (2024)5, 503

In general going BSM with $\kappa_{2V} \neq 1$; $\kappa_V \neq 1$ distorts the dist. in M_{HH} producing bumps,

Except close to $\kappa_{2V} = \kappa_V^2$ ★



Close to

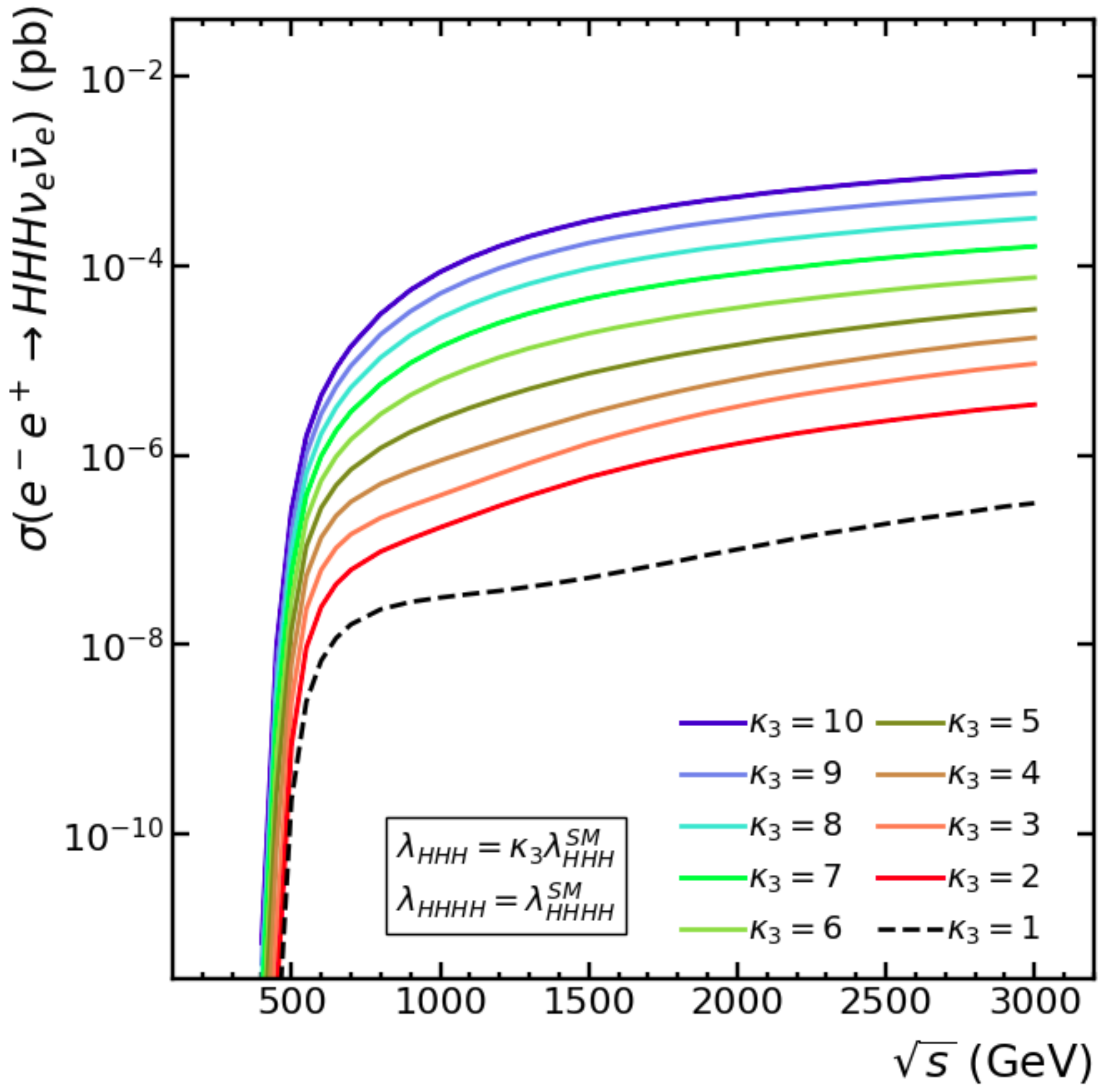
$$\kappa_{2V} = \kappa_V^2$$

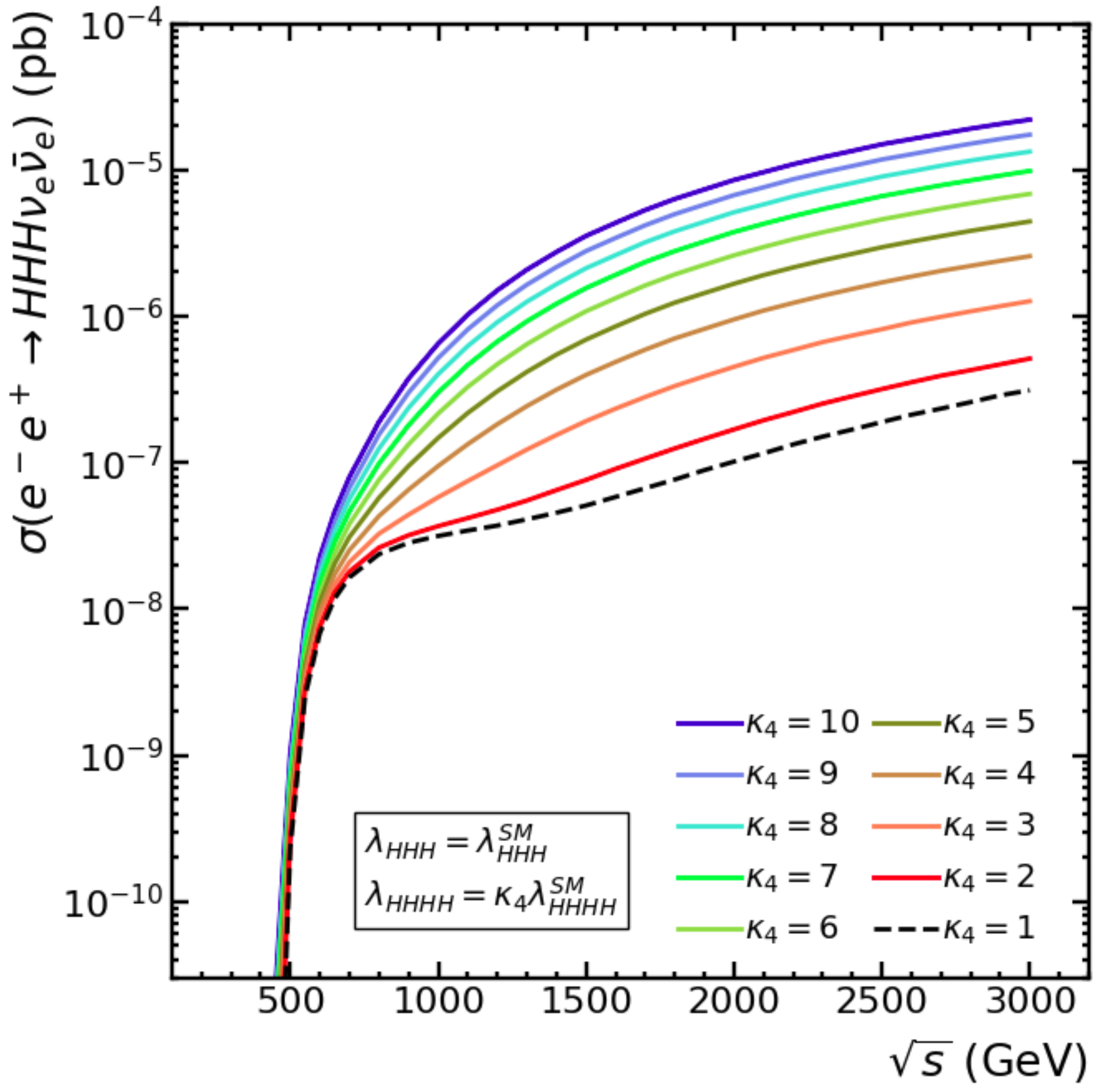
$$\Delta\kappa_{2V} = 2\Delta\kappa_V$$

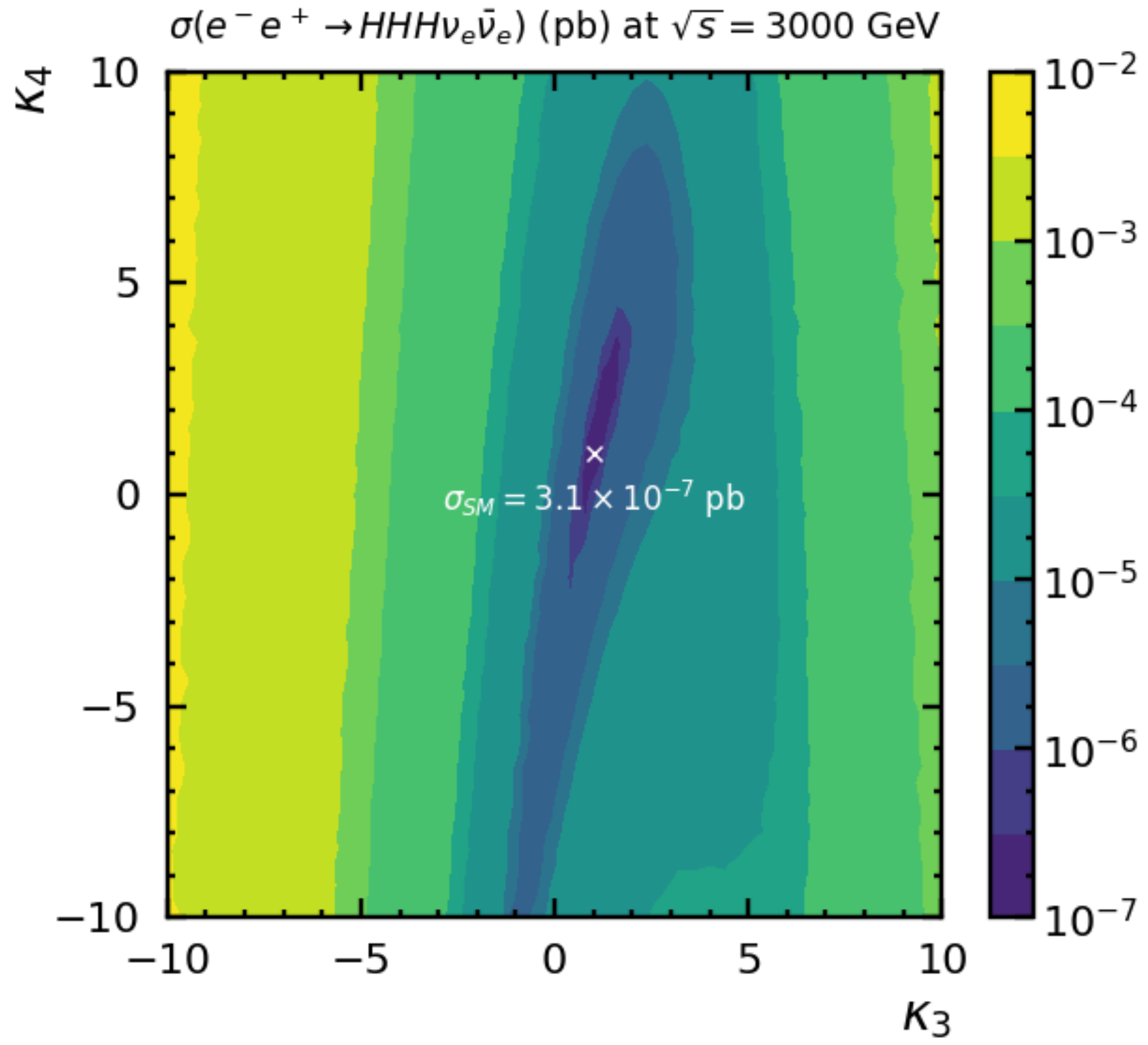
$$(\Delta b = 2\Delta a)$$

Sensitivity to κ_3 and κ_4 in $e^+e^- \to HHH\nu\bar{\nu}$

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez







The best expectations are for CLIC (3 TeV) where

BSM/SM $\gtrsim 10$ for $\kappa_3 \gtrsim 2$ $(\kappa_4 = 1)$

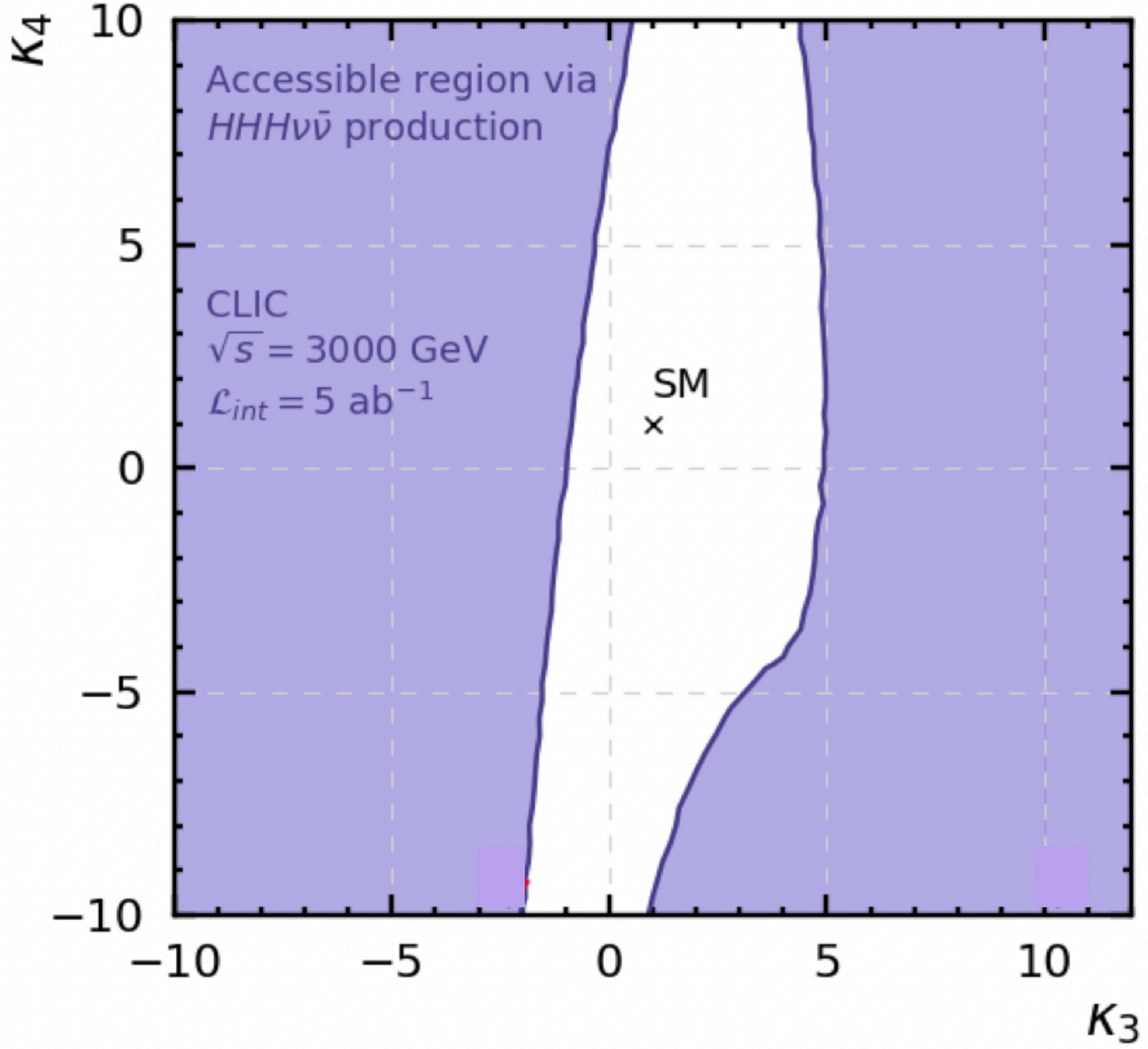
BSM/SM $\gtrsim 10$ for $\kappa_4 \gtrsim 4$ $(\kappa_3 = 1)$

Higher sensitivity to κ_3 than to κ_4 !!

Access to (κ_3, κ_4) in $e^+e^- \to HHH\nu\bar{\nu} \to 6b\text{jets} + E_T^{miss}$

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez

(cases with at least 10 events after applying all cuts)



$e^+e^- \to 6b + E_T^{mis}$

10 events required for accessibility

At least 5 btagged jets $\epsilon_b = 0.8$

✓ $p_T^j > 20$ GeV ✓ $N_j \geq 6$

✓ $|\eta^j| < 2.72$ ✓ $E_T^{mis} > 20$ GeV

The best expectations are for CLIC (3 TeV) where

BSM/SM $\gtrsim 10$ for $\kappa_3 \gtrsim 2$ $(\kappa_4 = 1)$

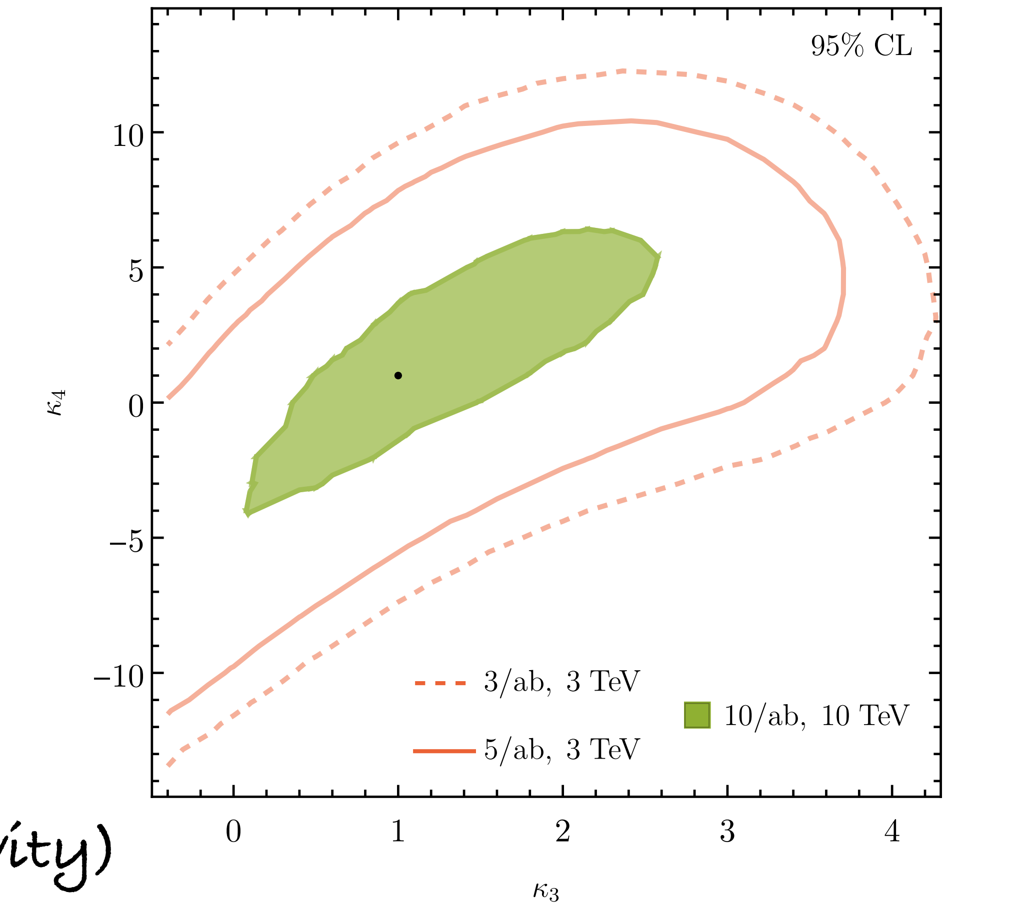
BSM/SM $\gtrsim 10$ for $\kappa_4 \gtrsim 4$ $(\kappa_3 = 1)$

$\sigma^{SM}(e^+e^- \to HHH\nu\bar{\nu})\,(3\,\text{TeV}) = 3 \times 10^{-7}\,\text{pb}$

Sensitivity at CLIC to both κ_3 and κ_4

Other studies of 5b's at CLIC

2312.04646 (Stylianou, Weiglein)



A recent study (more sophisticated and precise than ours) Is in agreement with our previous sensitivities found, solid red contours: reach at CLIC, $\kappa_3 \sim 3.5$, $\kappa_4 \sim 10$

Also compared with HL-LHC $3\,\text{ab}^{-1}$ (giving poorer sensitivity)

$\kappa_4 \sim 60$ already in the non-perturbative regime

Conclusions {
- Future expected sensitivity to κ_4 yet poor
- much higher sensitivity to κ_3 expected

Accessibility to NLO-HEFT (η, δ) at e^+e^-

Minimal detection cuts

$p_T^b > 20$ GeV $|\eta^b| < 2$

$\Delta R_{bb} > 0.4$ $\not{E}_T > 20$ GeV

b-tagging efficiency of 80%

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

Signal with greater statistics: e+ e- → HH$\nu\bar{\nu}$ → $b\bar{b}b\bar{b}\nu\bar{\nu}$

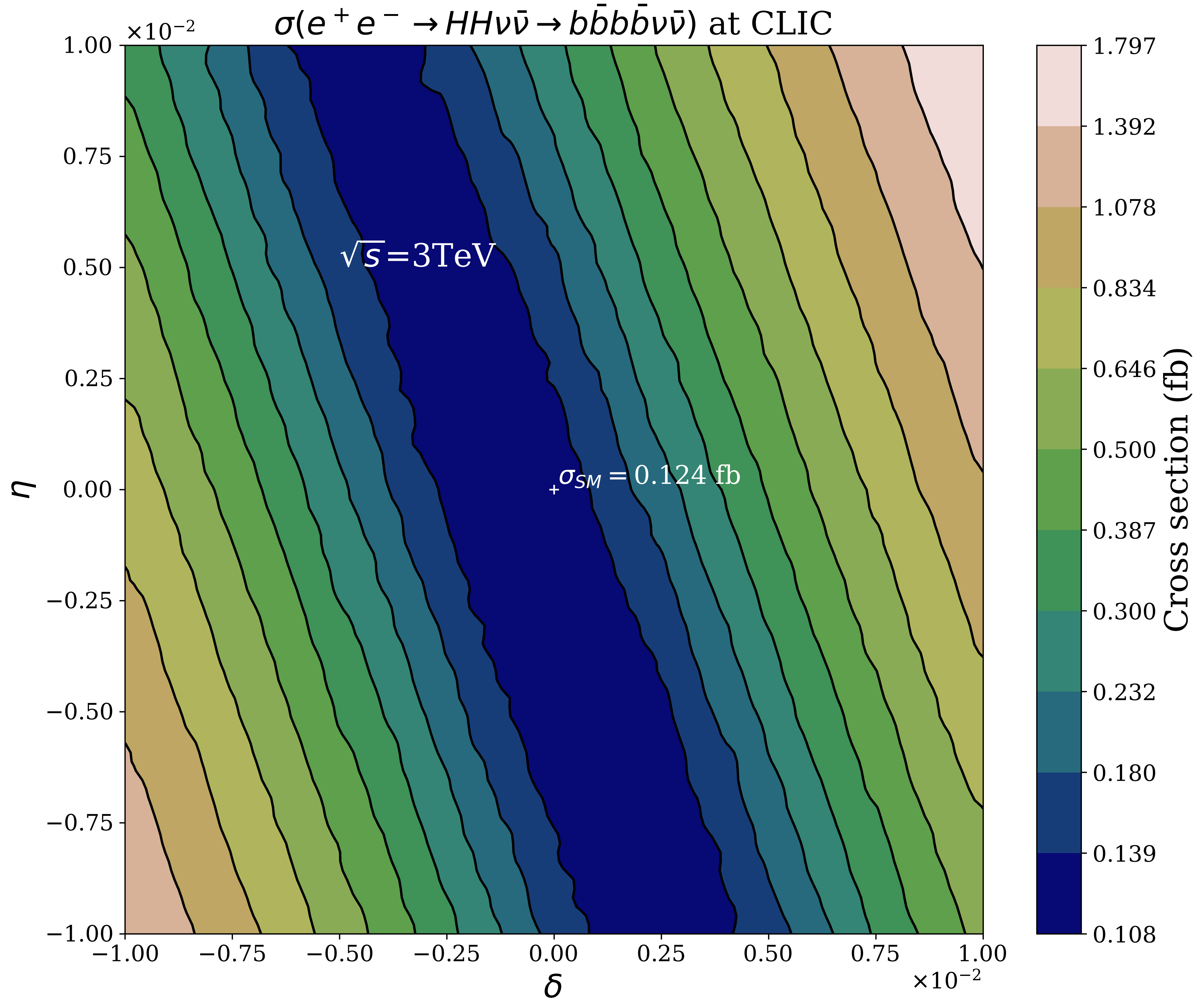
Greater accessibility in CLIC (3TeV)

Expected reach $\eta, \delta \sim \mathcal{O}(10^{-3})$

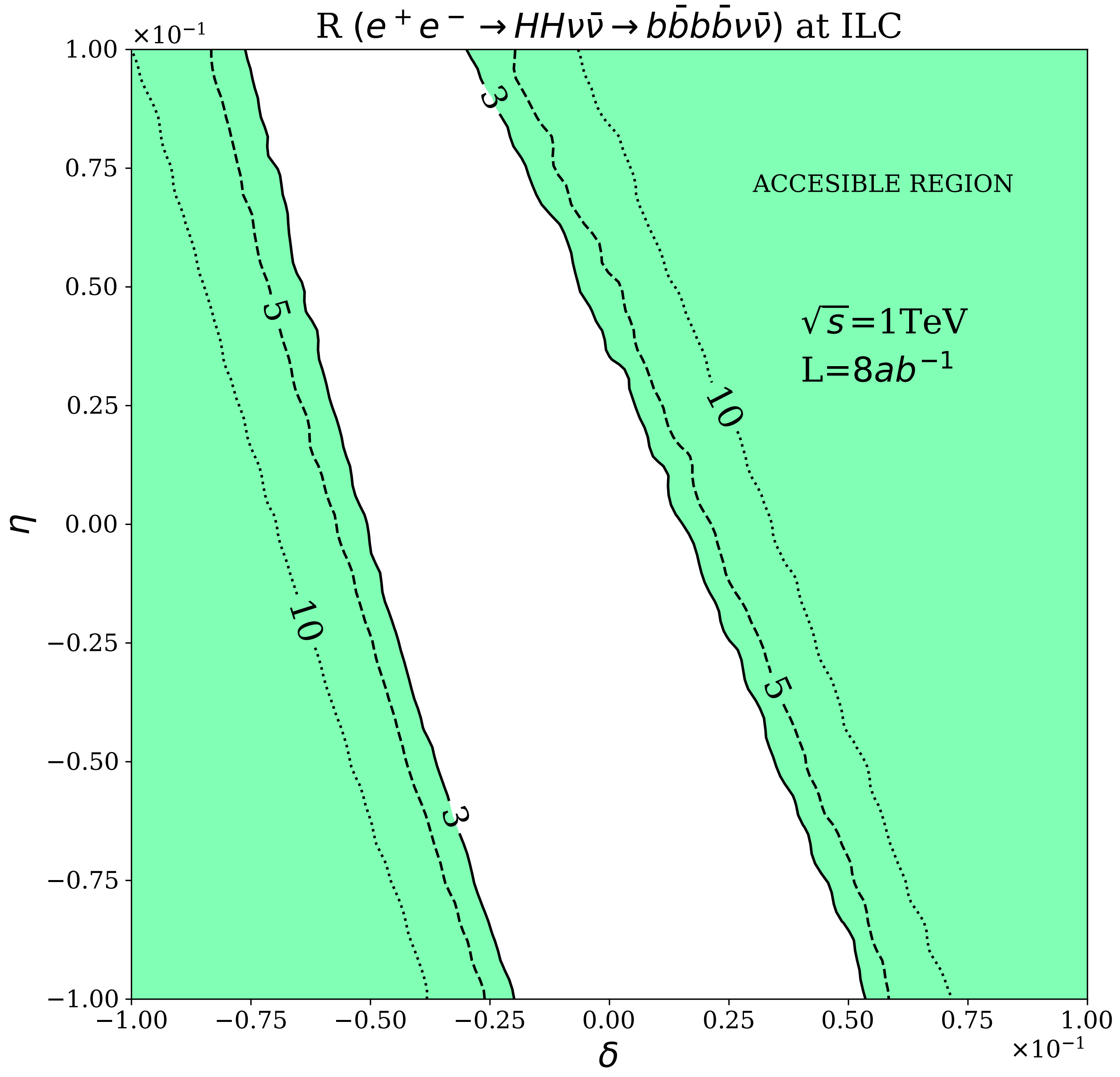
Accessibility parameter

$$R = \frac{N_{BSM} - N_{SM}}{\sqrt{N_{SM}}}$$

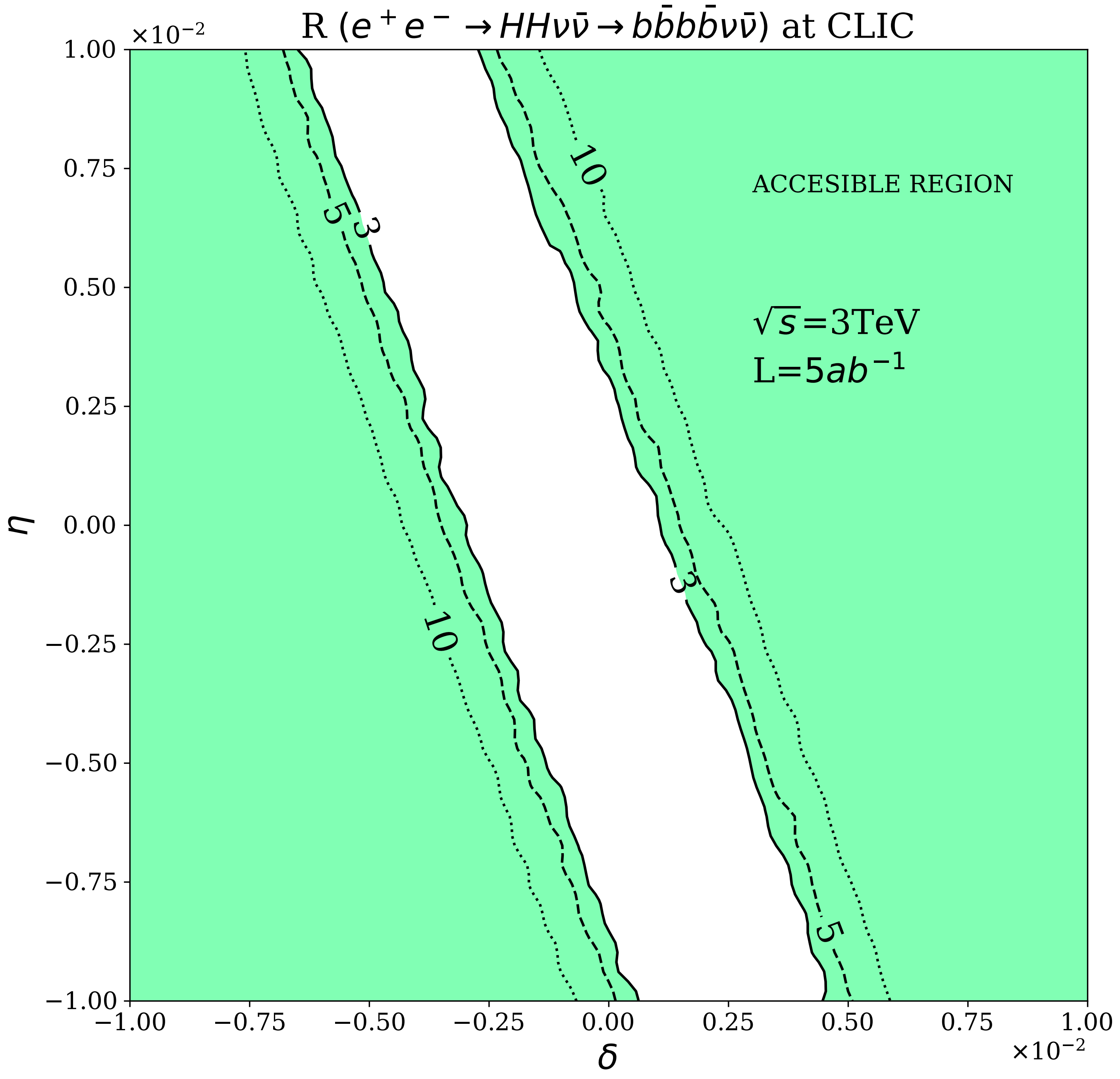
Accesible region: R > 3



ILC



CLIC

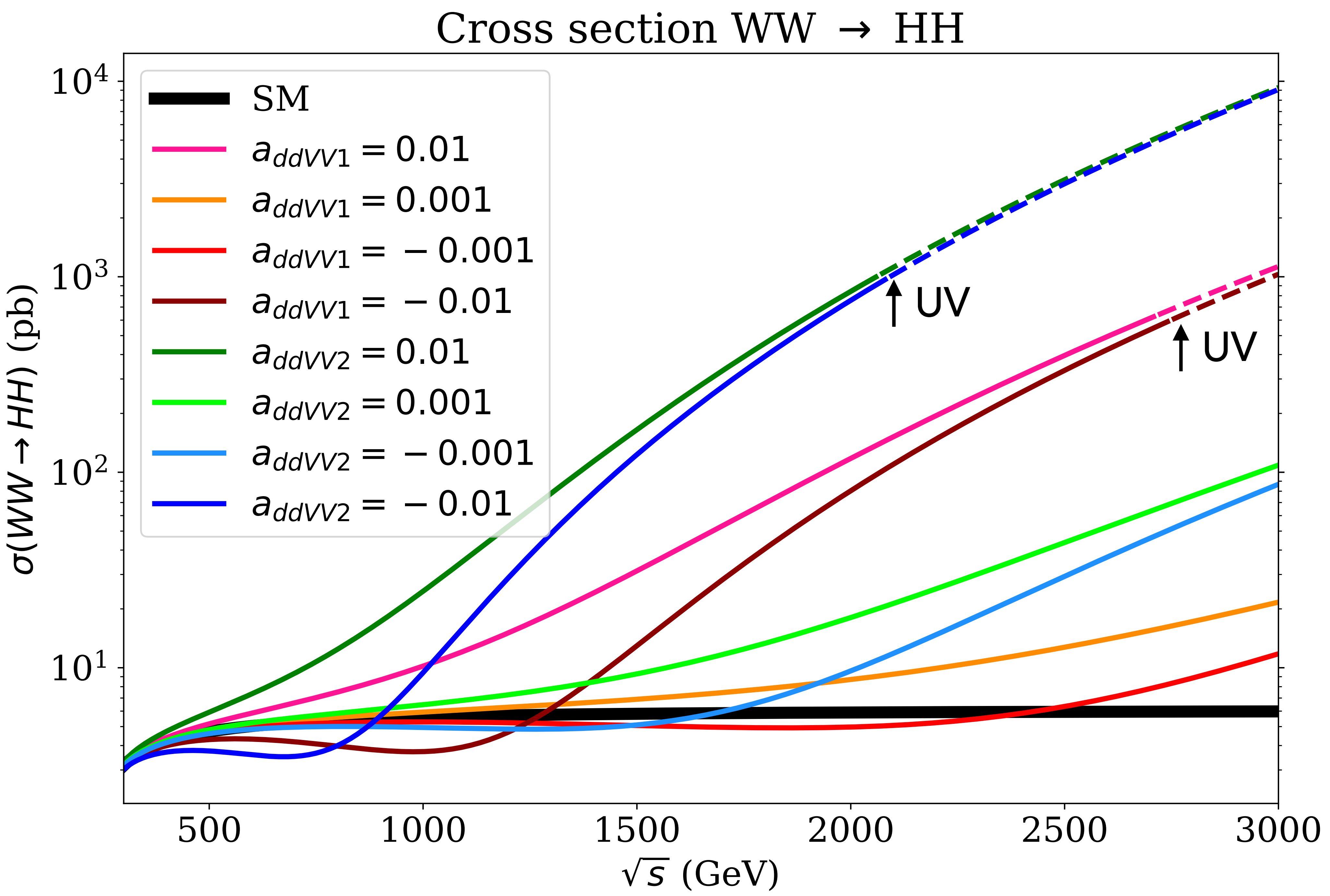


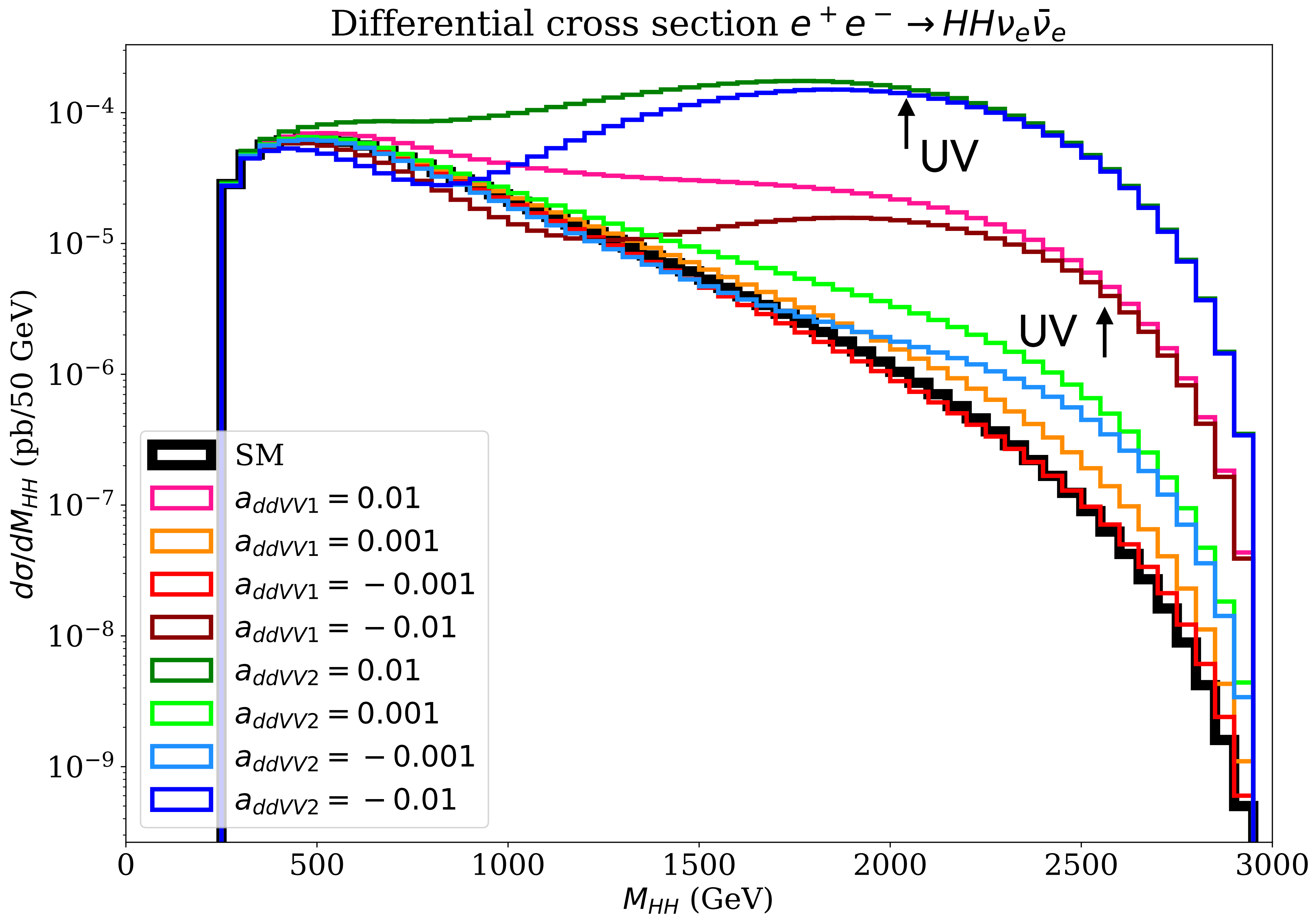
HH (WBF) with NLO-HEFT (focus on 2 most relevant coeff)

$$\mathscr{L}^{\text{NLO}}_{\text{HEFT}} = \ldots + \underbrace{a_{ddVV1}}_{=\eta=e} (1/v^2)\, \partial^\mu H \partial^\nu H \,\text{Tr}\left[(D_\mu U^+)(D_\nu U)\right] + \underbrace{a_{ddVV2}}_{=\delta=d} (1/v^2)\, \partial^\mu H \partial_\mu H \,\text{Tr}\left[(D^\nu U^+)(D_\nu U)\right] + \ldots$$

WW subprocess

example: e^+e^- process





enhancement in $WW \to HH$ at large $\sqrt{s}$ $\Rightarrow$ enhancement in $e^+e^- \to HH\bar{\nu}_e\nu_e$ at large invariant mass M_{HH}

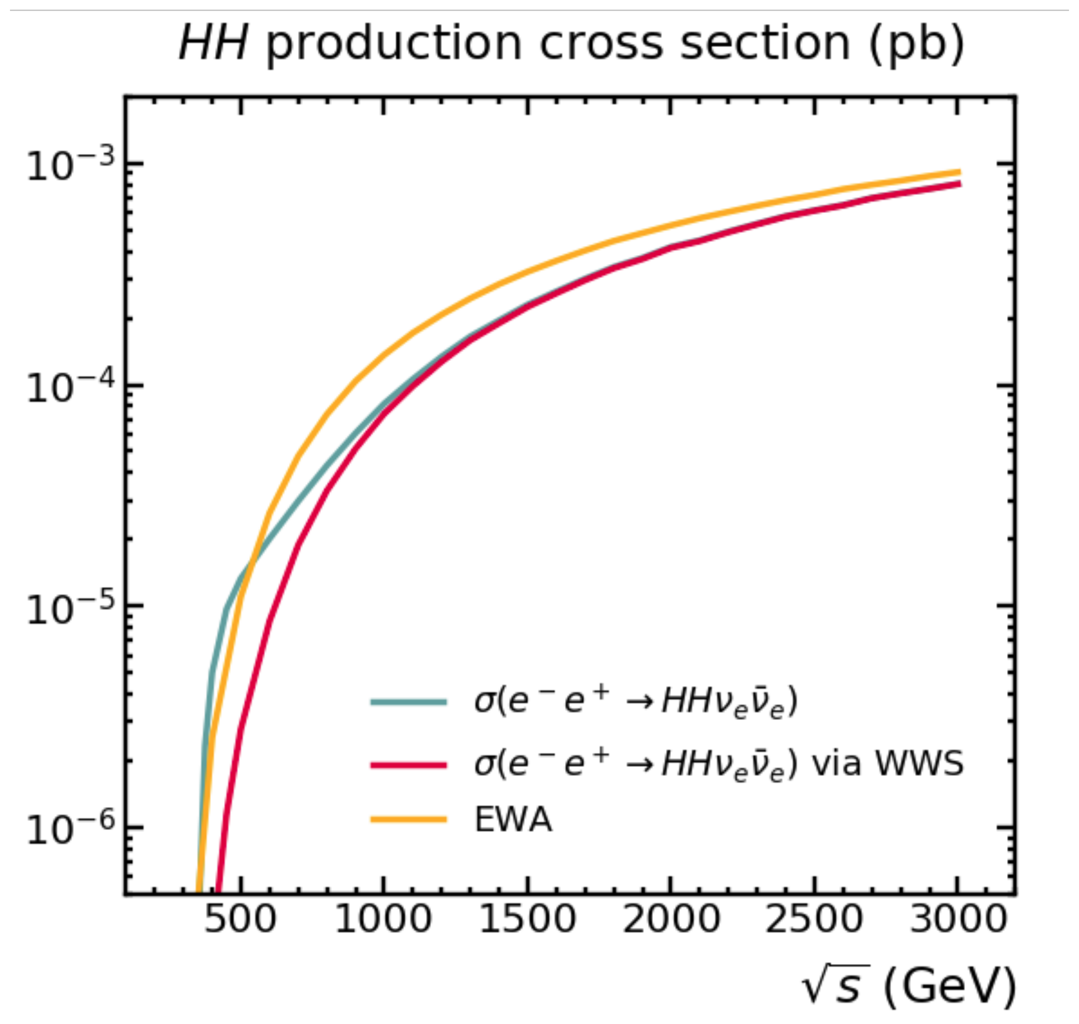
↑ UV= to the right of this point prediction enters in the Unitarity Violating region

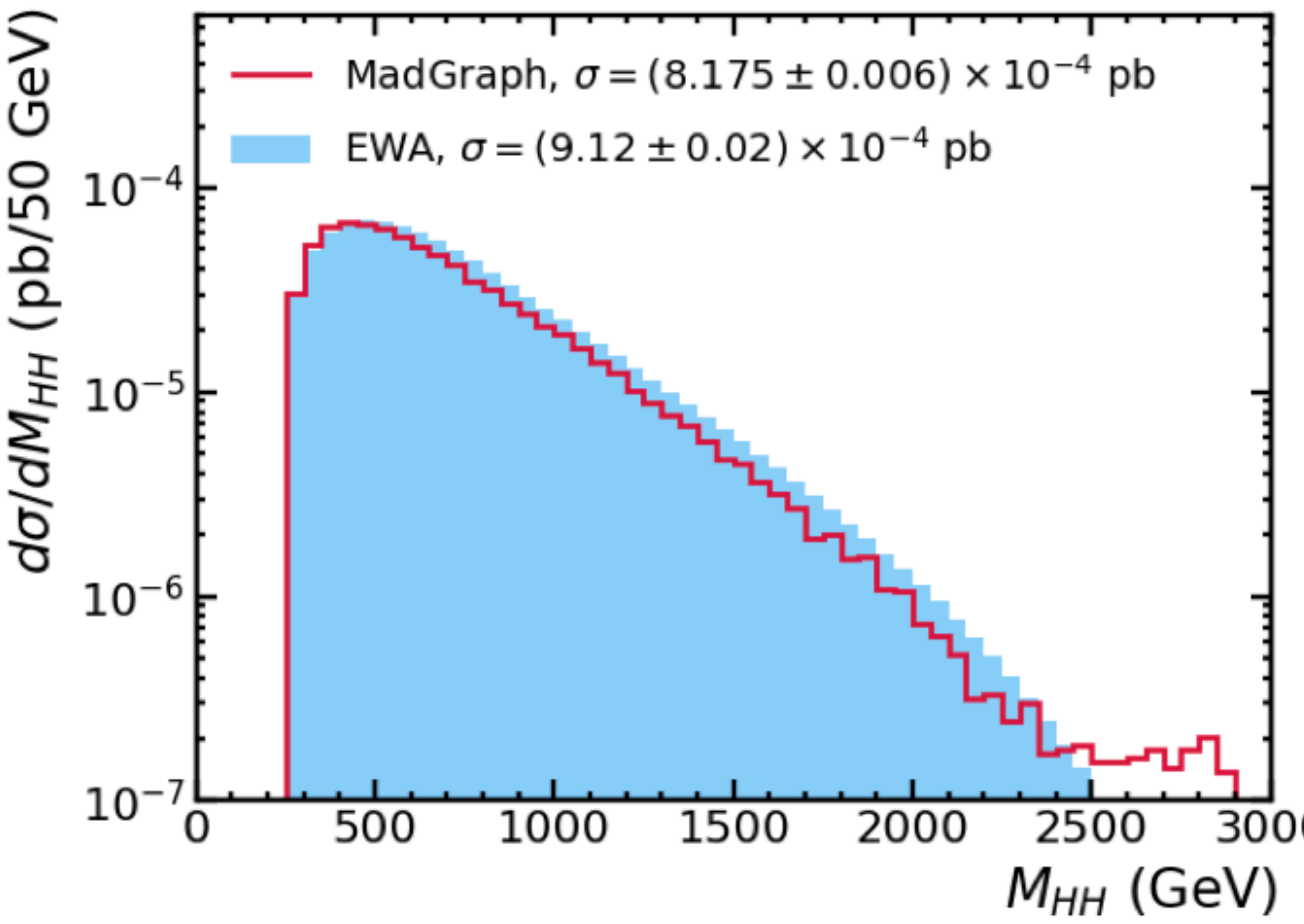
2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

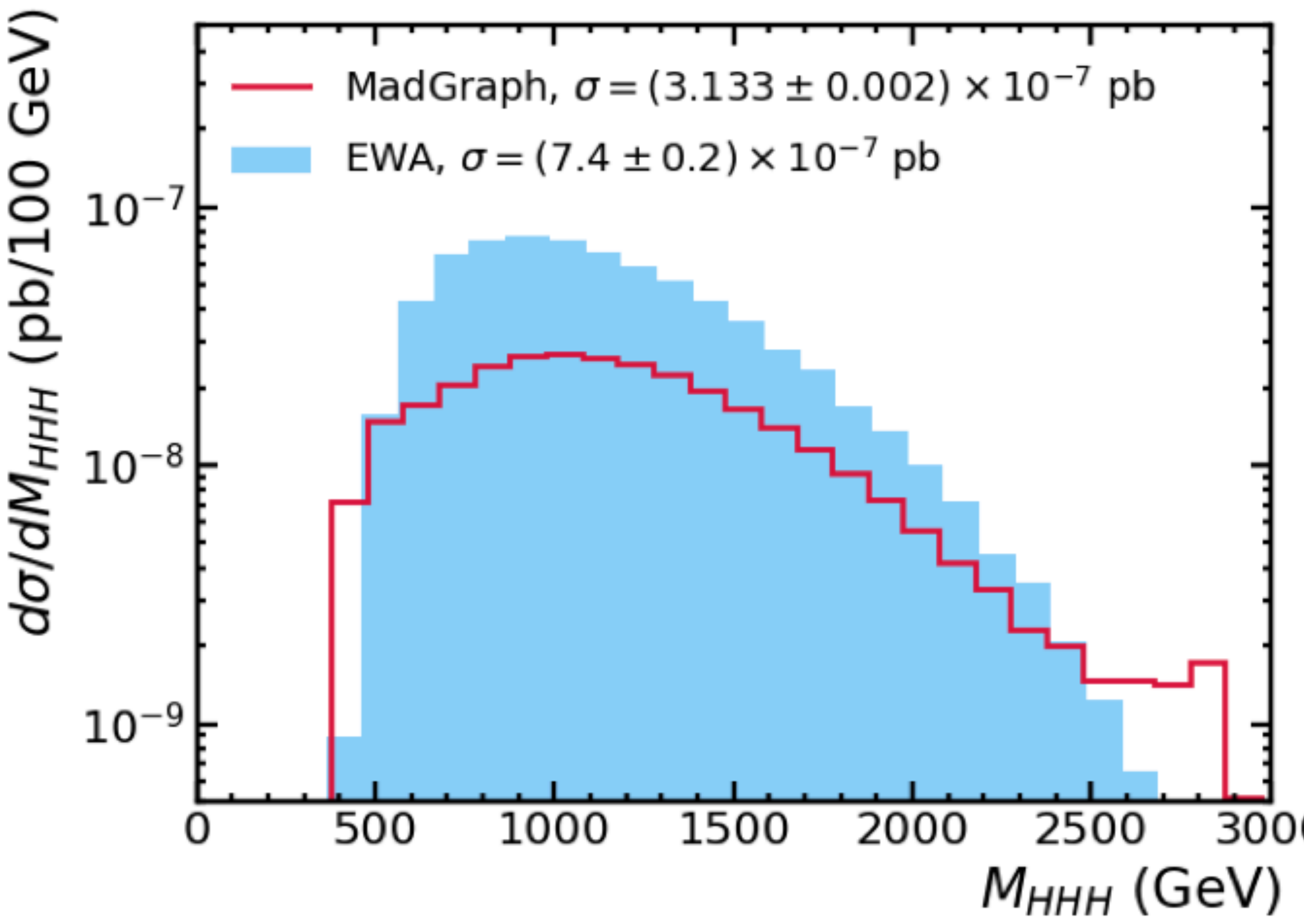
Comment:

EW production of HH and HHH at TeV colliders is dominated by VBS configurations but full computation of all diagrams and no use of EWA are required. Both for SM and BSM. We use MadGraph both for LHC and e^+e^-

Example in SM: $\sigma(e^+e^- \to HH\nu\bar{\nu})_{\rm EWA} = \int dx_1 \int dx_2 f_W(x_1) f_W(x_2) \hat{\sigma}(W^+W^- \to HH)$







EWA is a good approximation for HH, not so good for HHH 2011.13195

Determination of κ_λ at future e^+e^- colliders

- *Proposed* high-energy linear e^+e^- colliders: ILC and CLIC
- Projected sensitivity to κ_λ from hhZ and $hh\nu\bar{\nu}$ *(better than HL-LHC!)*:

(at 68% CL)

At ILC:

500 GeV (4 ab^{-1}):	±27%
500 GeV (4 ab^{-1}) + 1 TeV (5* ab^{-1}):	±10%

[Dürig, 16] [Fujii *et al*., 15]

At CLIC:

1.4 TeV (2.5 ab^{-1}):	-29%, +67%
1.4 TeV (2.5 ab^{-1}) + 3 TeV (5 ab^{-1}):	-8%, +11%

[CLICdp Collab., 15]

Cortesy by Francisco Arco

Comparing SMEFT and HEFT : LO and NLO

$WW \to HH$ subprocess

2208.05452, Phys. Rev. D 106 (2022)11, 115027, D. Domenech, M. Herrero, R. Morales, M. Ramos, 2022

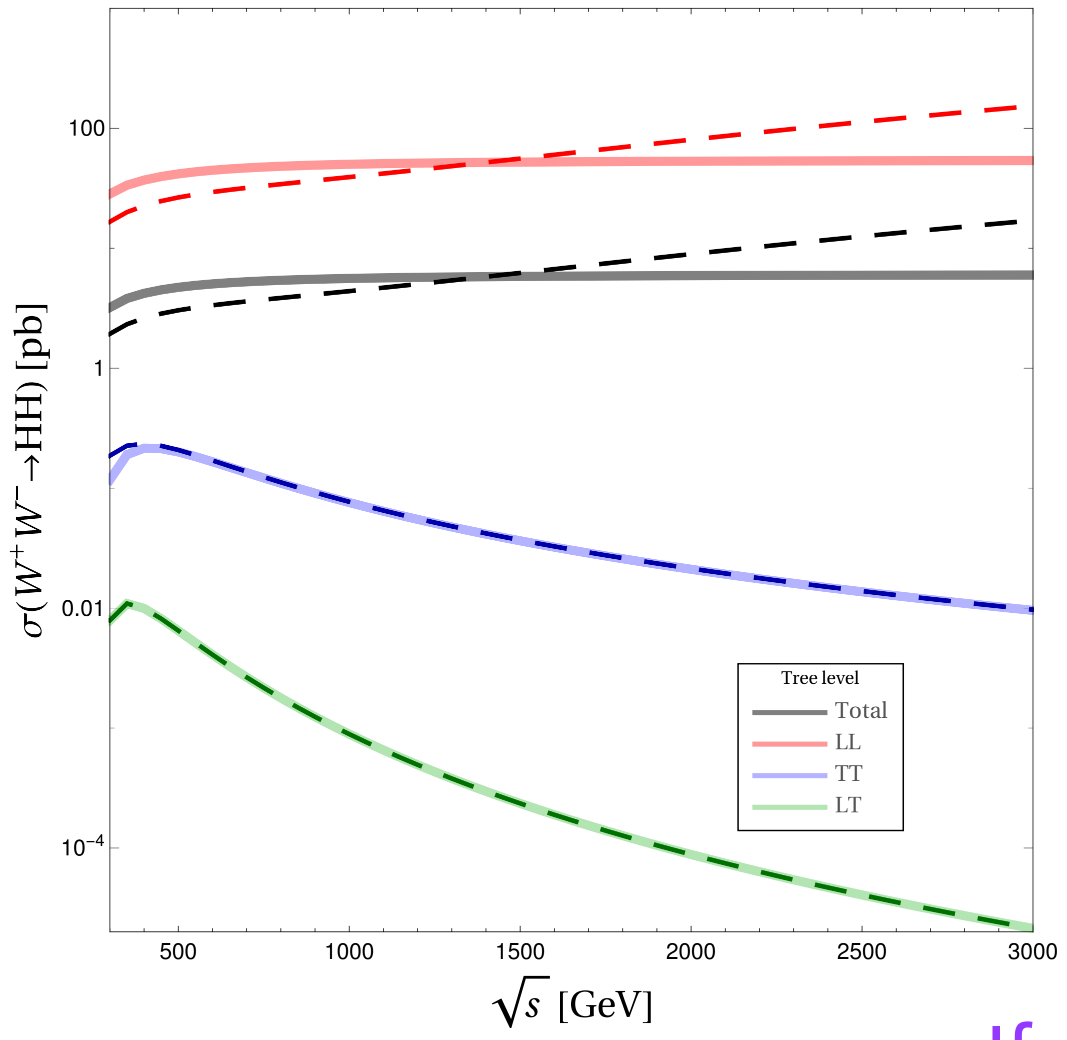
$$\mathcal{L}_6 \supset c_{\phi^6}(\phi^\dagger\phi)^3 + c_{\phi\Box}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + c_{\phi D}(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi) + c_{\phi W}(\phi^\dagger\phi)W^a_{\mu\nu}W^{a\mu\nu} \qquad c_i \equiv a_i/\Lambda^2$$

$$\mathcal{L}_8 \supset c^{(1)}_{\phi^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + c^{(2)}_{\phi^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + c^{(3)}_{\phi^4}(D_\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D^\nu\phi) + \ldots \qquad c_i \equiv a_i/\Lambda^4$$

Again: the largest BSM deviations in Longitudinal modes $W_L W_L \to HH$ Transverse modes are less affected. At TeV: dim8 compete with dim6 !!

$\Lambda = 1\,\mathrm{TeV}$ In these plots

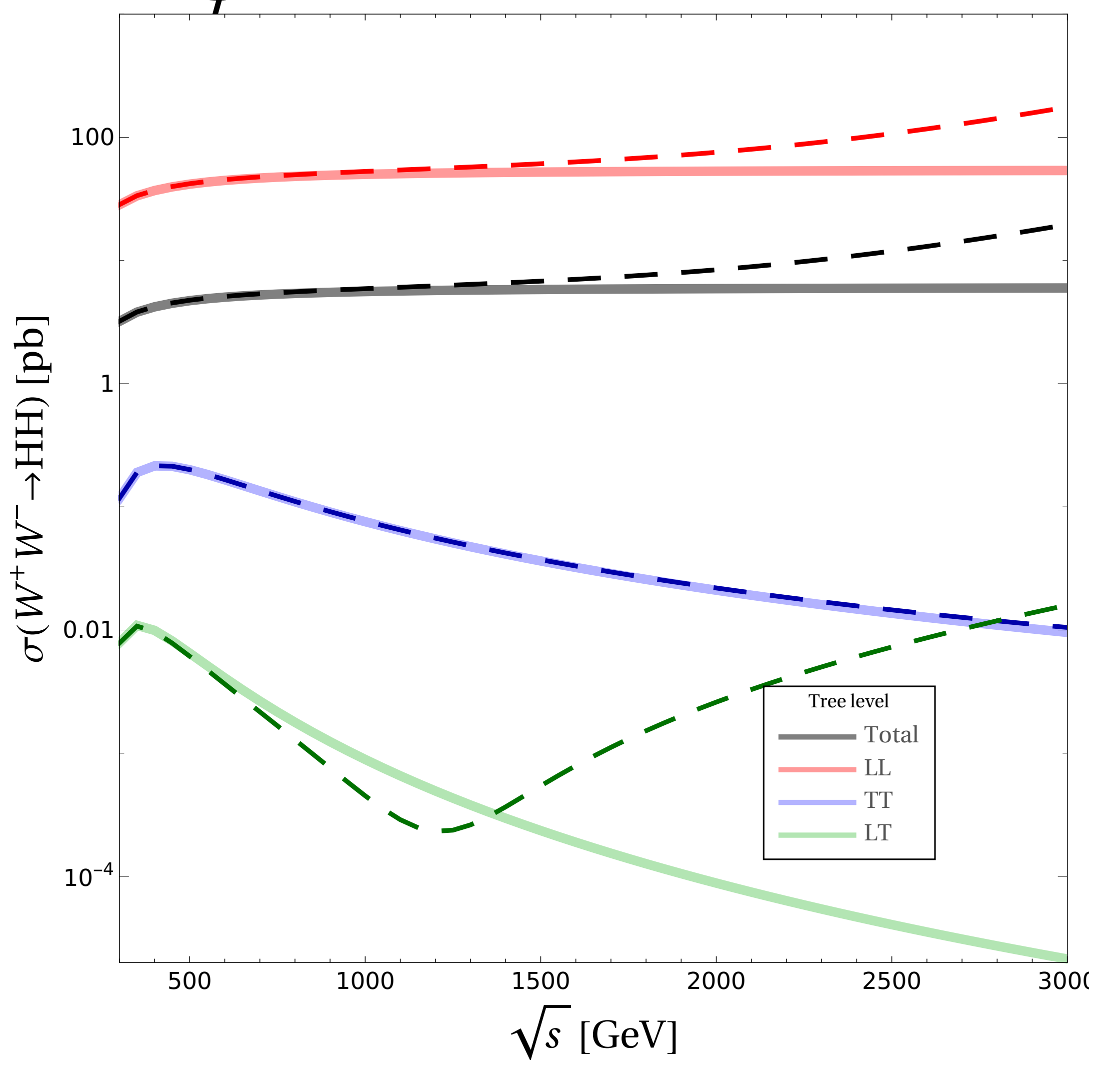
$a_{\phi\Box} = 1.0$



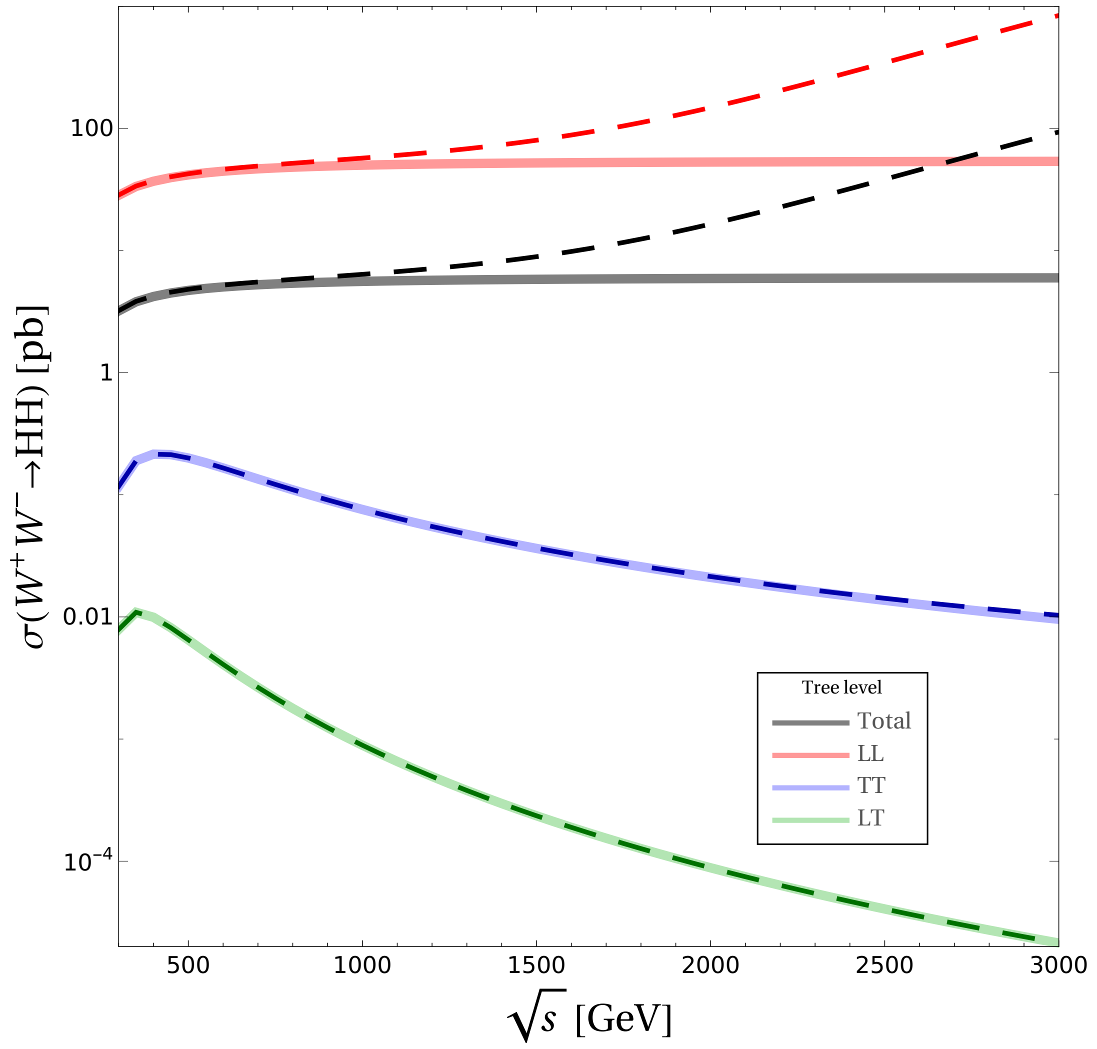
dim 6

$a_{\phi\Box}$ compares to $a\,(\kappa_V)$ and $b\,(\kappa_{2V})$

$a^{(1)}_{\phi^4} = 1.0$



$a^{(3)}_{\phi^4} = 1.0$



dim 8

$a^{(1)}_{\phi^4}, a^{(3)}_{\phi^4}$ compare to a_{ddVV1}, a_{ddVV2}

If matching in amplitudes according to behavior with energy: SMEFT dim 8 (6) <—> HEFT chi-dim 4 (2)