EFT Matching in Spontaneously Broken Gauge Theories

Anders Eller Thomsen

Based on [2404.11640]

Higgs and Effective Field Theory
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On-shell EFT matching

Given $$\mathcal{L}_{\text{UV}}[\Phi, \phi] = \mathcal{L}_{\text{kin}}[\Phi, \phi] + \sum_a g_a\, Q_a[\Phi, \phi] \qquad M_\Phi \sim \Lambda \gg m_\phi$$

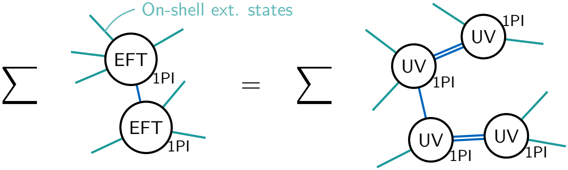
Heavy fields

determine $$\mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_{\text{kin}}[\phi] + \sum_k C_k(g)\, \mathcal{O}_k[\phi]$$

On-shell EFT matching

Weak (physical) matching condition

$$\langle f | S^{\text{EFT}}_{\text{mat.}} | i \rangle = \langle f | S^{\text{UV}}_{\text{mat.}} | i \rangle, \qquad \forall i, f \in \{\text{low energy}\}$$



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determine $\quad \mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_{\text{kin}}[\phi] + \sum_k C_k(g)\, \mathcal{O}_k[\phi]$

- Physical condition (works whenever decoupling is possible)
- Multiple solutions for $C_k(g)$: it is surprisingly difficult to determine an EFT basis
- **Challenging to compute on-shell matrix element!**

Off-shell matching

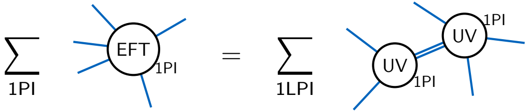
Strong matching condition

$$\mathcal{W}_{\text{EFT}}[J_\phi] = \mathcal{W}_{\text{UV}}[J_\Phi = 0,\ J_\phi] \Leftrightarrow$$

$$\Gamma_{\text{EFT}}[\hat{\phi}] = \Gamma_{\text{UV}}\big[\widehat{\Phi}[\hat{\phi}],\ \hat{\phi}\big], \qquad 0 = \frac{\delta \Gamma_{\text{UV}}}{\delta \Phi}\big[\widehat{\Phi}[\hat{\phi}],\ \hat{\phi}\big]$$

*The vacuum functional $\mathcal{W}$ generates all connected Green's functions

*The quantum effective action Γ generates all 1PI Green's functions



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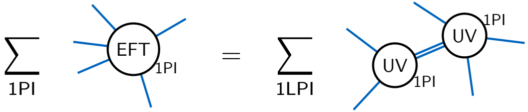

- Reduced number of diagrams
- Strong matching condition $\implies$ weak condition
- Non-trivial that a solution exists: **Green's functions depend on gauge choice**

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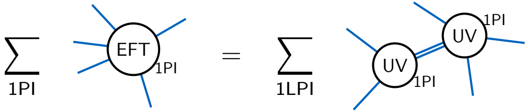

At tree-level we may **integrate out the heavy fields** with their EOM solution

$$S^{(0)}_{\text{EFT}}[\phi] = S_{\text{UV}}\big[\widehat{\Phi}[\phi],\, \phi\big], \qquad \frac{\delta S_{\text{UV}}}{\delta \Phi}\big[\widehat{\Phi}[\phi],\, \phi\big] = 0$$

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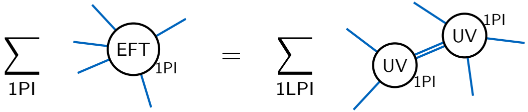

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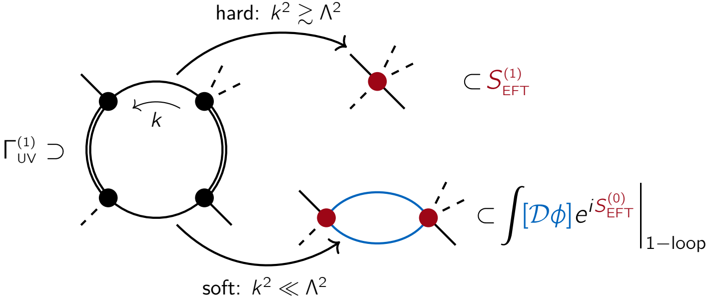
What about loop-level matching?

Separation of scales

Decomposition of UV loops:



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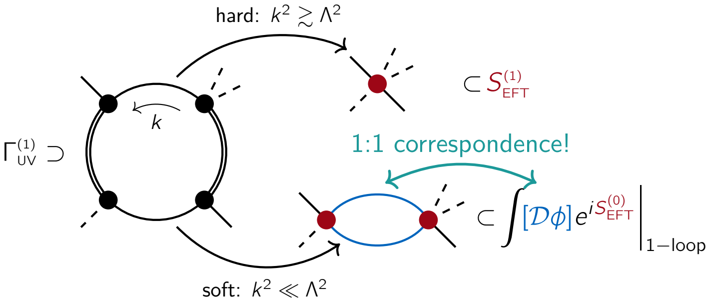

Hard-region matching formula

$$S_{\text{EFT}}[\phi] = \Gamma_{\text{UV}}[\widehat{\Phi}, \phi]\Big|_{\text{hard}}, \qquad \frac{\delta \Gamma_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\widehat{\Phi}, \phi] = 0$$

"hard" denotes the part without *any* soft loop momenta (it includes all tree-level contributions)

Fuentes-Martin, Palavrić, **AET** [2311.13630]

*Generalization of Fuentes-Martin *et al.* [1607.02142]; Zhang [1610.00710]

Matching gauge theories

Consider a gauge theory with **gauge group *G***

$$S_{\text{UV}}[\eta_g] = S_{\text{UV}}[\eta], \qquad \forall g \in G$$

With no Higgs-mechanism, we are looking for a low-energy EFT action with the same symmetry

$$S_{\text{EFT}}[\phi_g] = S_{\text{EFT}}[\phi], \qquad \forall g \in G$$

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What about the hard-region matching formula?

$$\underbrace{S_{\text{EFT}}[\phi]}_{G\text{-inv.}} \overset{?}{=} \overbrace{\Gamma_{\text{UV}}[\hat{\eta}]\Big|_{\text{hard}}}^{\text{BRST invariant?}}, \qquad \frac{\delta \Gamma_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\hat{\eta}] = 0$$

Γ_{UV} loses G invariance for the smaller BRST invariance with ordinary gauge-fixing

Gauge-invariant effective action

Gauge-invariant effective action of the **background field (BF) gauge**

(anti-)ghosts

Gauge-fixing η (quantum) using $\overline{\eta}$ (bkg.)

$$\overline{\Gamma}[\overline{\eta}] = -i \log \int \mathcal{D}\eta\, \mathcal{D}\boldsymbol{\omega}\, \exp\left[i \left(S[\eta + \overline{\eta}] + S^{G}_{\text{fix}}[\eta + \overline{\eta},\, \boldsymbol{\omega},\, \overline{\eta}] + \int_x J_I \eta^I \right) \right]$$

bkg. G invariance: $\quad \overline{\delta}_\alpha \overline{\eta}^I = D^I{}_A[\overline{\eta}]\alpha^A, \quad \overline{\delta}_\alpha \eta^I = D^I{}_{A,J}\alpha^A \eta^J$

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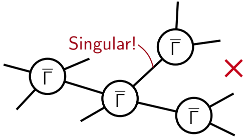
$\overline{\Gamma}[\overline{\eta}]$

Not the Legendre trans. of a vacuum functional!



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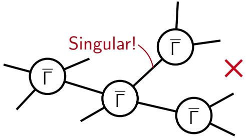
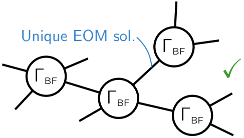


$$\Gamma_{\text{BF}}[\overline{\eta}] = \overline{\Gamma}[\overline{\eta}] + S^{G}_{\text{bg.}}[\overline{\eta}]$$



Vacuum functional is constructed with **gauge-fixed bkg. fields**:

$$\mathcal{W}_{\text{BF}}[\overline{J}] = \Gamma_{\text{BF}}[\overline{\eta}] + \int_x \overline{J}_I\overline{\eta}^I, \qquad \overline{J}_I = -\frac{\delta\Gamma_{\text{BF}}[\overline{\eta}]}{\delta\overline{\eta}^I},$$

Abbott *et al.* '83; Hart '83; Rebhan, Wirthumer '84

Matching in an ordinary BF gauge

$$\Gamma_{\mathrm{BF}} \xrightarrow{\text{Legendre trans.}} \mathcal{W}_{\mathrm{BF}} \xrightarrow{\text{on-shell}} S\text{-matrix}$$

A version of the strong matching condition is

$$\Gamma^{\mathrm{EFT}}_{\mathrm{BF}}[\overline{\phi}] \;=\; \Gamma^{\mathrm{UV}}_{\mathrm{BF}}[\overline{\eta}], \qquad\qquad \frac{\delta \Gamma^{\mathrm{UV}}_{\mathrm{BF}}}{\delta \Phi}[\overline{\eta}] = 0$$

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Choosing identical $S^{G}_{\text{bg.}}[\overline{\phi}]$ for UV and EFT yields

$S^{G}_{\text{bg.}}$ is Φ independent

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Quantum gauge-fixing can also be chosen identically for UV and EFT:
we can demonstrate a 1:1 correspondence of soft-region loops

Hard-region matching in unbroken gauge theories

$$S_{\text{EFT}}[\overline{\phi}] \;=\; \overline{\Gamma}_{\text{UV}}[\overline{\eta}]\Big|_{\text{hard}}, \qquad \frac{\delta \overline{\Gamma}_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\overline{\eta}] = 0$$

GIEA!

AET [2404.11640]

See also Henning, Lu, Murayama [1412.1837]; Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

SSB in BSM

- Many BSM scenarios involve **spontaneously broken gauge symmetries**
- Popular patterns include
  - $\mathrm{SU}(4) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SU}(4) \times \mathrm{SU}(3) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1) \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SU}(5) \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SO}(10) \longrightarrow G_\mathrm{SM}$
  - $\mathrm{SU}(2)_{12} \times \mathrm{SU}(2)_3 \longrightarrow \mathrm{SU}(2)_\mathrm{L}$
- Generically, the gauge group G is broken to a smaller group H in the IR
- Matching must accommodate the reduction in symmetry

Matching spontaneously broken gauge theories

Matching a spontaneously broken (Higgsed) gauge theory ($G \to H$)

$$
\begin{aligned}
S_{\text{UV}}[\eta_g] &= S_{\text{UV}}[\eta], && \forall g \in G \\
S_{\text{EFT}}[\phi_h] &= S_{\text{EFT}}[\phi], && \forall h \in H \subseteq G
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What happens to matching with the BF gauge?

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\underset{H\text{-inv.}}{S_{\text{EFT}}}[\phi] \overset{?}{=} \overset{G\text{-inv.}}{\overline{\Gamma}}_{\text{UV}}[\overline{\eta}]\Big|_{\text{hard}} \qquad\qquad \frac{\delta \overline{\Gamma}_{\text{UV}}\big|_{\text{hard}}}{\delta \Phi}[\overline{\eta}] = 0
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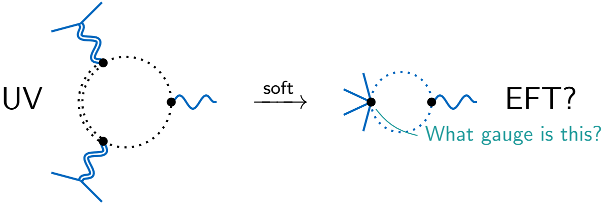
What about the 1:1 correspondence between soft UV and EFT loops?



EFT ghost interactions are determined by the gauge-fixing condition

(not a tree-level matching condition)

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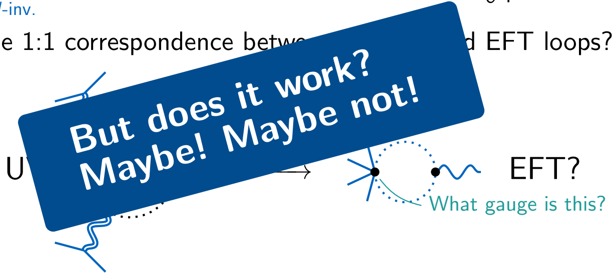

EFT ghost interactions are determined by the gauge-fixing condition

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Partial gauge fixing

Gauge-fixing a la Faddeev–Popov

$$Z = \int \mathcal{D}\eta\, e^{iS[\eta]} = \int \mathcal{D}\eta\, \delta\big(\mathcal{G}^A[\eta]\big)\, \mathrm{Det}\big(\mathcal{G}^A{}_{,I}[\eta] D^I{}_B[\eta]\big)\, e^{iS[\eta]}$$

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Factorization of the gauge-fixing condition:

Weinberg '80

$$G \to H, \qquad A^A_\mu = (\underbrace{B^\alpha_\mu}_{H\text{ gauge fields}}, V^i_\mu), \qquad \mathcal{G}^A[\eta] = \big(\mathcal{G}^\alpha[\eta], \underbrace{\mathcal{G}^i[\eta]}_{H\text{-covariant}}\big)$$

$$Z = \int \mathcal{D}\eta \, \delta\big(\mathcal{G}^\alpha[\eta]\big) \delta\big(\mathcal{G}^i[\eta]\big) \, \mathrm{Det}\begin{pmatrix} \mathcal{G}^\alpha{}_{,I}[\eta] D^I{}_\beta[\eta] & \mathcal{G}^\alpha{}_{,I}[\eta] D^I{}_j[\eta] \\ \cancel{-f^i{}_{\beta k} \mathcal{G}^k} \to \mathbf{0} & \mathcal{G}^i{}_{,I}[\eta] D^I{}_j[\eta] \end{pmatrix} e^{iS[\eta]}$$

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Clever manipulations and the introduction of an auxiliary field yields

$$Z = \int \mathcal{D}\eta \mathcal{D}\mathbf{u} \, \underbrace{\delta\big(\mathcal{G}^\alpha[\eta]\big) \, \mathrm{Det}\big(\mathcal{G}^\alpha{}_{,I}[\eta] D^I{}_\beta[\eta]\big)}_{H\text{ gauge-fixing}} \exp\Big[i \Big(\underbrace{S[\eta] + S^{G/H}_{\text{fix}}[\eta, \mathbf{u}]}_{H\text{-inv.}} \Big) \Big]$$

where Ferrari [1308.6802]

$$S^{G/H}_{\text{fix}}[\eta, \mathbf{u}] = -\int_x \Big(\frac{1}{2\zeta} \mathcal{G}_i[\eta] \mathcal{G}^i[\eta] + \overline{u}_i \big(\mathcal{G}^i{}_{,I}[\eta] D^I{}_j[\eta] + f^i{}_{jk} \mathcal{G}^k[\eta] \big) u^j - \frac{\zeta}{2} \hat{a}^{\alpha\beta} f^i{}_{j\alpha} f^k{}_{\ell\beta} \overline{u}_i u^j \overline{u}_k u^\ell \Big)$$

Partially fixed BF gauge

Proposal: Combine the partial fixing of G/H with a BF gauge for H

bkg. H inv.

$$\overline{\Gamma}[\overline{\eta}] = -i \log \int \mathcal{D}\eta\, \mathcal{D}\mathbf{c}\, \mathcal{D}\mathbf{u} \exp\left[i \left(S[\eta + \overline{\eta}] + S^{G/H}_{\text{fix}}[\eta + \overline{\eta}, \mathbf{u}] + S^{H}_{\text{fix}}[\eta + \overline{\eta}, \mathbf{c}, \overline{\eta}] + \int_x J_I \eta^I \right) \right]$$

with the BF effective action

$$\Gamma_{\text{BF}}[\overline{\eta}] = \overline{\Gamma}[\overline{\eta}] + S^{H}_{\text{bg.}}[\overline{\eta}]$$

$\overline{\Gamma}_{\text{UV}}$ of the partially fixed BF gauge possesses the symmetries of S_{EFT}!

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The gauge-fixing terms of the PFBF gauge are "manageable:"

$$\mathcal{L}_{\text{vec.}}^{G/H} = -\frac{1}{2\zeta}(d_\mu V_i^\mu)(d^\nu V_\nu^i) + M_i \chi_i d^\mu V_\mu^i - \frac{\zeta}{2} M_i^2 \chi_i \chi^i ,$$

$$\mathcal{L}_{\text{gh.}}^{G/H} = -\overline{u}_i(d^2 + \zeta M_i^2) u^i + \overline{u}_i (f^i{}_{jk} V_\mu^k d^\mu + f^i{}_{k\alpha} f^\alpha{}_{\ell j} V^{k\mu} V_\mu^\ell) u^j + \zeta \overline{u}_i (i f^i{}_a x_{jb}^a \varphi^b + f^i{}_{jk} M_k \chi^k) u^j + \frac{\zeta}{2} \hat{a}^{\alpha\beta} f^i{}_{j\alpha} f^k{}_{\ell\beta} \overline{u}_i u^j \overline{u}_k u^\ell ,$$

$$\mathcal{L}_{\text{vec.}}^{H} = -\frac{1}{2\xi} \hat{a}_{\alpha\beta}^{-1} \overline{d}^\mu B_\mu^\alpha \overline{d}^\nu B_\nu^\beta , \qquad \mathcal{L}_{\text{gh.}}^{H} = -\overline{c}_\alpha \overline{d}^\mu (\overline{d}_\mu c^\alpha + f^\alpha{}_{\beta\gamma} B_\mu^\beta c^\gamma)$$

Background field gauge

The gauge-fixing terms in ordinary BF gauge for $G \to H$ gauge theories

$$\begin{aligned}
\mathcal{L}^{G}_{\text{vec.}} = &-\frac{1}{2\xi}\hat{a}^{-1}_{\alpha\beta}\overline{\partial}^{\mu}B^{\alpha}_{\mu}\overline{\partial}^{\nu}B^{\beta}_{\nu} - \frac{1}{2\xi}\overline{\partial}^{\mu}V^{i}_{\mu}\overline{\partial}_{\nu}V^{\nu}_{i} + M_i\chi_i\overline{\partial}^{\mu}V^{i}_{\mu} - \frac{\xi}{2}M_i^2\chi_i\chi^i - \frac{1}{\xi}\overline{\partial}^{\mu}B^{\alpha}_{\mu}\hat{a}^{-1}_{\alpha\beta}f^{\beta}{}_{jk}\overline{V}^{j}_{\nu}V^{k\nu}\\
&-\frac{1}{2\xi}\hat{a}^{-1}_{\alpha\beta}f^{\alpha}{}_{ij}\overline{V}^{i}_{\mu}V^{j\mu}f^{\beta}{}_{k\ell}\overline{V}^{k}_{\nu}V^{\ell\nu} + \left(\overline{\partial}^{\mu}B^{\alpha}_{\mu} + f^{\alpha}{}_{ij}\overline{V}^{i}_{\mu}V^{j\mu}\right)\left(\chi_k f^{k}{}_{\alpha\ell}\overline{\chi}^{\ell} - i\varphi_{h\,a}t^{a}_{\alpha b}\overline{\varphi}^{b}_{h}\right)\\
&-\frac{\xi}{2}\hat{a}^{\alpha\beta}\left(\chi_i f^{i}{}_{\alpha j}\overline{\chi}^{j} - i\varphi_{h\,a}t^{a}_{\alpha b}\overline{\varphi}^{b}_{h}\right)\left(\chi_k f^{k}{}_{\beta\ell}\overline{\chi}^{\ell} - i\varphi_{h\,c}t^{a}_{\beta d}\overline{\varphi}^{d}_{h}\right) - \frac{1}{\xi}\overline{\partial}^{\mu}V^{i}_{\mu}\overline{V}^{j}_{\nu}\left(f_{ij\alpha}B^{\alpha\nu} + f_{ijk}V^{k\nu}\right)\\
&-\frac{1}{2\xi}\overline{V}^{j}_{\mu}\overline{V}^{\ell}_{\nu}\left(f_{ij\alpha}B^{\alpha\mu} + f_{ijk}V^{k\mu}\right)\left(f^{i}{}_{\ell\beta}B^{\beta\nu} + f^{i}{}_{\ell m}V^{m\nu}\right) + \frac{\xi}{2}\left(\varphi_a x^{a}_{ib}\overline{\varphi}^{b}\right)\kappa^{ij}\left(\varphi_c x^{c}_{jd}\overline{\varphi}^{d}\right)\\
&+\left(M_i\chi_i - i\varphi_a x^{a}_{ib}\overline{\varphi}^{b}\right)\overline{V}^{j}_{\mu}\left(f^{i}{}_{j\alpha}B^{\alpha\mu} + f^{i}{}_{jk}V^{k\mu}\right) - i\overline{\partial}^{\mu}V^{i}_{\mu}\varphi_a x^{a}_{ib}\overline{\varphi}^{b} + i\xi M_i\chi^i\varphi_a x^{a}_{ib}\overline{\varphi}^{b},
\end{aligned}$$

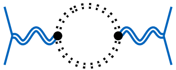
$$\begin{aligned}
\mathcal{L}^{G}_{\text{gh.}} = &-\overline{c}_{\alpha}\overline{\partial}^2 c^{\alpha} - \overline{u}_i\overline{\partial}^2 u^i + \overline{\partial}^{\mu}\overline{c}_{\alpha}f^{\alpha}{}_{\beta\gamma}B^{\beta}_{\mu}c^{\gamma} + \overline{\partial}^{\mu}\overline{u}_i f^{i}{}_{\alpha j}B^{\alpha}_{\mu}u^j + \overline{\partial}^{\mu}\overline{c}_{\alpha}f^{\alpha}{}_{ij}(\overline{V}^{i}_{\mu} + V^{i}_{\mu})u^j\\
&+\overline{\partial}^{\mu}\overline{u}_i(\overline{V}^{j}_{\mu} + V^{j}_{\mu})(f^{i}{}_{j\alpha}c^{\alpha} + f^{i}{}_{jk}u^k) - \overline{c}_{\alpha}f^{\alpha}{}_{ij}\overline{V}^{i}_{\mu}\overline{\partial}^{\mu}u^j - \overline{u}_i\overline{V}^{j}_{\mu}(f^{i}{}_{j\alpha}\overline{\partial}^{\mu}c^{\alpha} + f^{i}{}_{jk}\overline{\partial}^{\mu}u^k)\\
&-\overline{c}_{\alpha}f^{\alpha}{}_{ij}f^{j}{}_{\alpha k}\overline{V}^{i}_{\mu}B^{\alpha\mu}u^k - \overline{u}_i\overline{V}^{j}_{\mu}B^{\beta\mu}(f^{i}{}_{j\alpha}f^{\alpha}{}_{\beta\gamma}c^{\gamma} + f^{i}{}_{jk}f^{k}{}_{\beta\ell}u^{\ell})\\
&-\overline{V}^{k\mu}(\overline{V}^{\ell}_{\mu} + V^{\ell}_{\mu})[\overline{c}_{\alpha}f^{\alpha}{}_{km}f^{m}{}_{\ell\beta}c^{\beta} + \overline{c}_{\alpha}f^{\alpha}{}_{km}f^{m}{}_{\ell j}u^j + \overline{u}_i f^{i}{}_{km}f^{m}{}_{\ell\beta}c^{\beta} + \overline{u}_i(f^{i}{}_{k\alpha}f^{\alpha}{}_{\ell j} + f^{i}{}_{km}f^{m}{}_{\ell j})u^j]\\
&+\xi\overline{c}_{\alpha}\hat{a}^{\alpha\beta}[\overline{\chi}_i f^{i}{}_{\beta j}f^{j}{}_{\gamma k}(\overline{\chi} + \chi)^k - \overline{\varphi}_{h\,a}(t_{\alpha}t_{\beta})^{a}{}_{b}(\overline{\varphi}_h + \varphi_h)^b]c^{\gamma}\\
&+\xi\overline{c}_{\alpha}\hat{a}^{\alpha\beta}[M_j\chi_j f^{j}{}_{\alpha i} - \overline{\varphi}_a(t_{\alpha}x_i)^{a}{}_{b}(\overline{\varphi} + \varphi)^b]u^i\\
&-\xi\overline{u}_i[f^{i}{}_{\alpha j}M_j(\overline{\chi} + \chi)^j + \overline{\varphi}_a(x^i t_{\alpha})^{a}{}_{b}(\overline{\varphi} + \varphi)^b]c^{\alpha}\\
&-\xi\overline{u}_i[\delta^{i}{}_{j}M_j^2 - if^{i}{}_{a}x^{a}_{jb}(\overline{\varphi} + \varphi)^b - if_{ja}\kappa^{ik}x^{a}_{kb}\overline{\varphi}^b + \overline{\varphi}_a(x^i x_j)^{a}{}_{b}(\overline{\varphi} + \varphi)^b]u^j
\end{aligned}$$

Now *this* is an unmanageable mess!

Matching with the PFBF gauge

Massive ghosts of the PFBF gauge don't need to be mimicked by any EFT loops

$$\overline{\Gamma}_{\text{UV}} \supset \mathrm{Det}\big(\mathcal{G}^i_{,I} D^I{}_j\big) = \mathrm{Det}\big(\mathcal{G}^i_{,I} D^I{}_j\big)\Big|_{\text{hard}}$$

UV  $\xrightarrow{\text{soft}}$ 0 EFT

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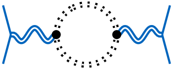
UV  $\xrightarrow{\text{soft}}$ 0 EFT

Integrating out heavy vectors at tree-level gives identical vertices to the soft-region of UV loops

Also the GF condition
for the quantum fields.

$$S^{(0)}_{\text{EFT}} = S_{\text{UV}} + S^{G/H}_{\text{fix}}, \qquad \frac{\delta\big(S_{\text{UV}} + S^{G/H}_{\text{fix}}\big)}{\delta\Phi} = 0$$

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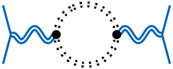
UV  $\xrightarrow{\text{soft}}$ 0 EFT

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Matching broken gauge theories with the PFBF gauge

$$S_{\text{EFT}}[\overline{\phi}] = \overline{\Gamma}_{\text{UV}}[\overline{\eta}]\Big|_{\text{hard}}, \qquad \frac{\delta\overline{\Gamma}_{\text{UV}}|_{\text{hard}}}{\delta\Phi}[\overline{\eta}] = 0$$

AET [2404.11640]

The soft-region cancellation of the one-loop functional traces can be explicitly demonstrated

Summary

- Practical matching relies on the hard-region matching formula
- The formula can be generalized to unbroken gauge theories with the BF gauge
- The partially fixed BF gauge allows for extension to broken gauge symmetries

Thank you!