RTG 2575:
Rethinking
Quantum Field Theory

Field redefinitions and infinite field anomalous dimensions

Jasper Roosmale Nepveu

with Aneesh V. Manohar and Julie Pagès

Based on JHEP 05 (2024) 018 [2402.08715]

Higgs and Effective Field Theory – HEFT 2024

University of Bologna, 12 June 2024

From UV to IR

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_4 + \sum_{d,i} \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Energy

$\mu = \Lambda$

UV theory

Matching

Effective Field Theory

$C_i(\Lambda)$

"anomalous dimensions"

$$\mu \frac{d\,C_i(\mu)}{d\mu} = \frac{1}{16\pi^2} \sum_j \gamma_{ij} C_j(\mu)$$

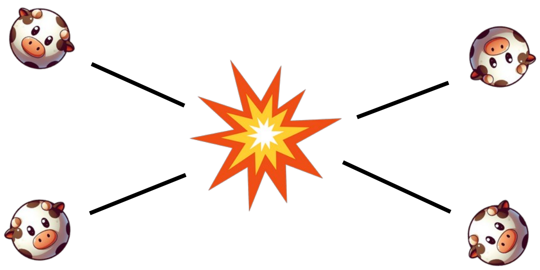
+ higher loops
+ non-linear in the EFT parameters

Running

$\mu = m_W$

$C_i(m_W)$

Mapping to observables



Talks at HEFT2024 about RG running

Guedes
Naterop
Fuentes-Martin
C. Bresciani
Ventura
Sutherland
Herrero

See also Henning, Lu, Murayama (2014)

Renormalization of EFTs

Recent renormalization calculations In e.g. the Standard Model EFT:

Jenkins, Manohar, Trott (2013) (2013) ; Alonso, Jenkins, Manohar, Trott (2013); Alonso, Chang, Jenkins, Manohar, Shotwell (2014); Davidson, Gorbahn, Leak (2018); Liao, Ma (2016) (2019); Chala, Guedes, Ramos, Santiago (2021); Chala, Titov (2021); Accettulli Huber, De Angelis (2021); Das Bakshi, Chala, Díaz-Carmona, Guedes (2022); Helset, Jenkins, Manohar (2022); Zhang (2023); Wang, Zhang, Zhou (2023); Das Bakshi, Díaz-Carmona (2023); Assi, Helset, Manohar, Pagès, Shen (2023).

Structure and zeros in the anomalous dimensions:

Alonso, Jenkins, Manohar (2014); Elias-Miro, Espinosa, Pomarol (2014); Cheung, Shen (2015); Bern, Parra-Martinez, Sawyer (2019); (2020); Jiang, Shu, Xiao, Zheng (2020); Baratella, Haslehner, Ruhdorfer, Serra, Weiler (2021); Machado, Renner, Sutherland (2022); Cao, Herzog, Melia, JRN (2021) (2023); Chala (2023); Chala, Li (2023).

Remarkably, the anomalous dimension of the field in the $\mathrm{O(n)}$ scalar model was found to be **infinite** [Jenkins, Manohar, Naterop, Pagès (2023) & (2023)] :

$$\gamma_\phi\, \phi(\mu) \overset{\text{def}}{=} \mu \frac{d\phi(\mu)}{d\mu} = \frac{\text{cnst}_1}{16\pi^2} + \frac{\text{cnst}_2}{(16\pi^2)^2} + \frac{1}{\epsilon}\frac{m^2}{\Lambda^2}\frac{\text{cnst}_3}{(16\pi^2)^2}$$

$$(\text{in } 4-2\epsilon \text{ spacetime dimensions})$$

Renormalization of EFTs

Recent renormalization calculations In e.g. the Standard Model EFT:

Jenkins, Manohar, Trott (2013) (2013) ; Alonso, Jenkins, Manohar, Trott (2013); Alonso, Chang, Jenkins, Manohar, Shotwell (2014); Davidson, Gorbahn, Leak (2018); Liao, Ma (2016) (2019); Chala, Guedes, Ramos, Santiago (2021); Chala, Titov (2021); Accettulli Huber, De Angelis (2021); Das Bakshi, Chala, Díaz-Carmona, Guedes (2022); Helset, Jenkins, Manohar (2022); Zhang (2023); Wang, Zhang, Zhou (2023); Das Bakshi, Díaz-Carmona (2023); Assi, Helset, Manohar, Pagès, Shen (2023).

Structure and zeros in the anomalous dimensions:

Alonso, Jenkins, Manohar (2014); Elias-Miro, Espinosa, Pomarol (2014); Cheung, Shen (2015); Bern, Parra-Martinez, Sawyer (2019); (2020); Jiang, Shu, Xiao, Zheng (2020); Baratella, Haslehner, Ruhdorfer, Serra, Weiler (2021); Machado, Renner, Sutherland (2022); Cao, Herzog, Melia, JRN (2021) (2023); Chala (2023); Chala, Li (2023).

Remarkably, the anomalous dimension of the field in the $\mathrm{O(n)}$ scalar model was found to be **infinite** [Jenkins, Manohar, Naterop, Pagès (2023) & (2023)] :

$$\gamma_\phi\, \phi(\mu) \overset{\text{def}}{=} \mu \frac{d\phi(\mu)}{d\mu} = \frac{\text{cnst}_1}{16\pi^2} + \frac{\text{cnst}_2}{(16\pi^2)^2} + \frac{1}{\epsilon}\frac{m^2}{\Lambda^2}\frac{\text{cnst}_3}{(16\pi^2)^2}$$

$$(\text{in } 4-2\epsilon \text{ spacetime dimensions})$$

In this talk, we will trace **infinite field anomalous dimensions** back to the use of field redefinitions and the removal of redundant parameters

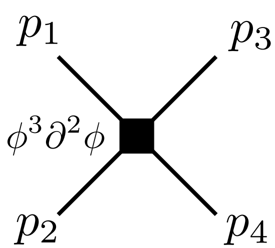
Field redefinitions

To exemplify redundancies in EFT, consider a single scalar theory defined by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4!}\lambda\phi^4 + \frac{D}{\Lambda^2}\frac{1}{3!}\phi^3\partial^2\phi + \frac{C}{\Lambda^2}\frac{1}{6!}\phi^6$$

Compute tree-level amplitudes at dimension six:

$$\mathcal{A}(p_1, p_2, p_3, p_4)\Big|_{1/\Lambda^2} =$$



$$= \frac{D}{\Lambda^2}\left(p_1^2 + p_2^2 + p_3^2 + p_4^2\right)$$

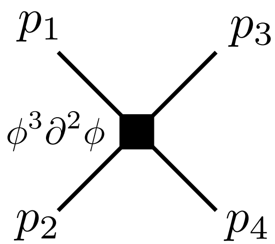
Field redefinitions

To exemplify redundancies in EFT, consider a single scalar theory defined by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4!}\lambda\phi^4 + \frac{D}{\Lambda^2}\frac{1}{3!}\phi^3\partial^2\phi + \frac{C}{\Lambda^2}\frac{1}{6!}\phi^6$$

Compute tree-level amplitudes at dimension six:

$$\mathcal{A}(p_1, p_2, p_3, p_4)\Big|_{1/\Lambda^2} =$$



$$= \frac{D}{\Lambda^2}\left(\cancelto{0}{p_1^2} + \cancelto{0}{p_2^2} + \cancelto{0}{p_3^2} + \cancelto{0}{p_4^2}\right) = 0$$

Field redefinitions

To exemplify redundancies in EFT, consider a single scalar theory defined by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4!}\lambda\phi^4 + \frac{D}{\Lambda^2}\frac{1}{3!}\phi^3\partial^2\phi + \frac{C}{\Lambda^2}\frac{1}{6!}\phi^6$$

Compute tree-level amplitudes at dimension six:

$$\mathcal{A}(p_1, p_2, p_3, p_4, p_5, p_6)\Big|_{1/\Lambda^2} =$$

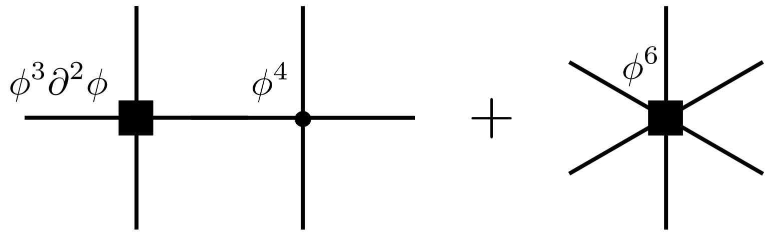
Field redefinitions

To exemplify redundancies in EFT, consider a single scalar theory defined by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \, - \, \frac{1}{4!}\lambda\phi^4 \, + \, \frac{D}{\Lambda^2}\frac{1}{3!}\phi^3\partial^2\phi \, + \, \frac{C}{\Lambda^2}\frac{1}{6!}\phi^6$$

Compute tree-level amplitudes at dimension six:

$$\mathcal{A}(p_1,p_2,p_3,p_4,p_5,p_6)\Big|_{1/\Lambda^2} =$$



$(+19 \text{ permutations})$

$$= \underbrace{C - 20\lambda D}_{\overline{C}}$$

The S-matrix depends only on the linear combination $\overline{C} = C - 20\lambda D$. In on-shell amplitudes, the $\phi^3\partial^2\phi$ operator “behaves like” ϕ^6.

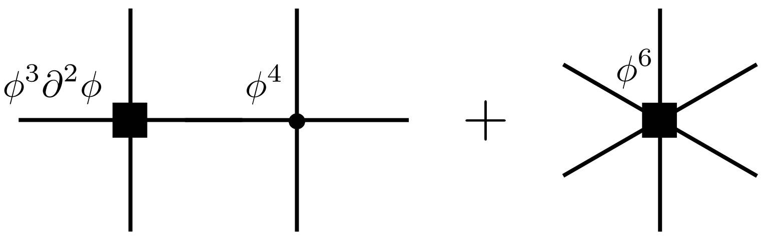
Field redefinitions

To exemplify redundancies in EFT, consider a single scalar theory defined by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4!}\lambda\phi^4 + \frac{D}{\Lambda^2}\frac{1}{3!}\phi^3\partial^2\phi + \frac{C}{\Lambda^2}\frac{1}{6!}\phi^6$$

Compute tree-level amplitudes at dimension six:

$$\mathcal{A}(p_1,p_2,p_3,p_4,p_5,p_6)\Big|_{1/\Lambda^2} =$$



(+19 permutations)

$$= \underbrace{C - 20\lambda D}_{\overline{C}}$$

The S-matrix depends only on the linear combination $\overline{C} = C - 20\lambda D$. In on-shell amplitudes, the $\phi^3\partial^2\phi$ operator "behaves like" ϕ^6. This follows (at any loop) from field redefinitions:

$$\mathcal{L}(\phi) \xrightarrow{\;\phi = \bar\phi + \frac{D}{\Lambda^2}\frac{1}{3!}\bar\phi^3\;} \mathcal{L}(\bar\phi) = \frac{1}{2}\partial_\mu\bar\phi\partial^\mu\bar\phi - \frac{1}{4!}\lambda\bar\phi^4 + \frac{\overline{C}}{\Lambda^2}\frac{1}{6!}\bar\phi^6 + O(1/\Lambda^4)$$

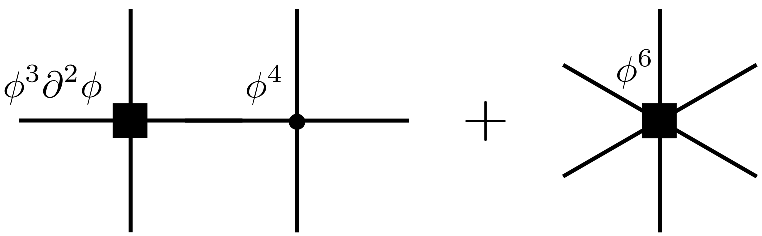
Field redefinitions

To exemplify redundancies in EFT, consider a single scalar theory defined by

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4!}\lambda\phi^4 + \frac{D}{\Lambda^2}\frac{1}{3!}\phi^3\partial^2\phi + \frac{C}{\Lambda^2}\frac{1}{6!}\phi^6$$

Compute tree-level amplitudes at dimension six:

$$\mathcal{A}(p_1,p_2,p_3,p_4,p_5,p_6)\Big|_{1/\Lambda^2} =$$



$$= \underbrace{C - 20\lambda D}_{\overline{C}}$$

(+19 permutations)

The S-matrix depends only on the linear combination $\overline{C} = C - 20\lambda D$. In on-shell amplitudes, the $\phi^3\partial^2\phi$ operator "behaves like" ϕ^6. This follows (at any loop) from field redefinitions:

$$\mathcal{L}(\phi) \xrightarrow{\;\phi = \bar\phi + \frac{D}{\Lambda^2}\frac{1}{3!}\bar\phi^3\;} \mathcal{L}(\bar\phi) = \frac{1}{2}\partial_\mu\bar\phi\partial^\mu\bar\phi - \frac{1}{4!}\lambda\bar\phi^4 + \frac{\overline{C}}{\Lambda^2}\frac{1}{6!}\bar\phi^6 + O(1/\Lambda^4)$$

It is well known that redundant parameters are generated by off-shell renormalization. We include them from the start and follow their imprint on quantities in a minimal basis.

Renormalization of the couplings

Consider the scalar $\mathrm{O(n)}$ model defined by

$$\mathcal{L}_{O(n)} = \frac{1}{2}(\partial_\mu \phi \cdot \partial^\mu \phi) - \frac{1}{2}m^2(\phi\cdot\phi) - \frac{1}{4}\lambda(\phi\cdot\phi)^2 + C_4(\phi\cdot\phi)(\partial_\mu\phi\cdot\partial^\mu\phi) + C_6(\phi\cdot\phi)^3$$
$$+D_2(\partial^2\phi\cdot\partial^2\phi) + D_4(\phi\cdot\partial_\mu\phi)^2$$

Two-loop renormalization of (for example) the $(\phi\cdot\phi)^3$ operator gives the counterterms

$$Z_{C_6}C_6 = C_6 + \begin{bmatrix} 1 \\ C_4 \\ C_6 \\ D_4 \\ D_2 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 & 0 & 0 \\ (n+8)\lambda^2 & -3(5n+58)\lambda^3 & \left(3n^2+47n+274\right)\lambda^3 \\ 3(n+14)\lambda & -\frac{3}{2}(53n+394)\lambda^2 & 3(n+14)(2n+25)\lambda^2 \\ 9\lambda^2 & -(23n+166)\lambda^3 & 4(8n+73)\lambda^3 \\ (n+26)\lambda^3 & -(61n+506)\lambda^4 & 3(n+11)(n+26)\lambda^4 \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon}\frac{1}{16\pi^2} \\ \\ \frac{1}{\epsilon}\frac{1}{(16\pi^2)^2} \\ \\ \frac{1}{\epsilon^2}\frac{1}{(16\pi^2)^2} \end{bmatrix}$$

Renormalization of the couplings

Consider the scalar $\mathrm{O(n)}$ model defined by

$$\mathcal{L}_{O(n)} = \frac{1}{2}(\partial_\mu\phi\cdot\partial^\mu\phi) - \frac{1}{2}m^2(\phi\cdot\phi) - \frac{1}{4}\lambda(\phi\cdot\phi)^2 + C_4(\phi\cdot\phi)(\partial_\mu\phi\cdot\partial^\mu\phi) + C_6(\phi\cdot\phi)^3 + D_2(\partial^2\phi\cdot\partial^2\phi) + D_4(\phi\cdot\partial_\mu\phi)^2$$

Two-loop renormalization of (for example) the $(\phi\cdot\phi)^3$ operator gives the counterterms

$$Z_{C_6}C_6 = C_6 + \begin{bmatrix} 1 \\ C_4 \\ C_6 \\ D_4 \\ D_2 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 & 0 & 0 \\ (n+8)\lambda^2 & -3(5n+58)\lambda^3 & \left(3n^2+47n+274\right)\lambda^3 \\ 3(n+14)\lambda & -\frac{3}{2}(53n+394)\lambda^2 & 3(n+14)(2n+25)\lambda^2 \\ 9\lambda^2 & -(23n+166)\lambda^3 & 4(8n+73)\lambda^3 \\ (n+26)\lambda^3 & -(61n+506)\lambda^4 & 3(n+11)(n+26)\lambda^4 \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon}\frac{1}{16\pi^2} \\ \frac{1}{\epsilon}\frac{1}{(16\pi^2)^2} \\ \frac{1}{\epsilon^2}\frac{1}{(16\pi^2)^2} \end{bmatrix}$$

The S-matrix depends on the parameters $\overline{m}^2,\ \bar{\lambda},\ \overline{C}_4,\ \overline{C}_6$, e.g., $\overline{C}_6 = C_6 + \frac{1}{2}\lambda D_4 + \lambda^2 D_2$.
Field and parameter redefinitions result in

$$Z_{\overline{C}_6}\overline{C}_6 = \overline{C}_6 + \begin{bmatrix} 1 \\ \overline{C}_4 \\ \overline{C}_6 \\ D_4 \\ D_2 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 & 0 & 0 \\ 10\bar{\lambda}^2 & -\frac{2}{3}(23n+259)\bar{\lambda}^3 & 5(7n+62)\bar{\lambda}^3 \\ 3(n+14)\bar{\lambda} & -\frac{21}{2}(7n+54)\bar{\lambda}^2 & 3(n+14)(2n+25)\bar{\lambda}^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon}\frac{1}{16\pi^2} \\ \frac{1}{\epsilon}\frac{1}{(16\pi^2)^2} \\ \frac{1}{\epsilon^2}\frac{1}{(16\pi^2)^2} \end{bmatrix}$$

Redundant parameters do not renormalize the couplings in the minimal basis!

Renormalization of the field

In contrast to the couplings, the renormalization of the field depends on all parameters.

In an overcomplete basis (also called Green's basis):

$$Z_\phi = 1 + \begin{bmatrix} 1 \\ C_4 \\ C_6 \\ D_4 \\ D_2 \end{bmatrix}^\intercal \begin{bmatrix} 0 & -\frac{1}{2}(n+2)\lambda^2 & 0 \\ 2m^2 n & -(n+2)\lambda m^2 & 2(n+1)(n+2)\lambda m^2 \\ 0 & 0 & 0 \\ 2m^2 & -(n+2)\lambda m^2 & 4(n+2)\lambda m^2 \\ 0 & 6(n+2)\lambda^2 m^2 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon}\frac{1}{16\pi^2} \\ \frac{1}{\epsilon}\frac{1}{(16\pi^2)^2} \\ \frac{1}{\epsilon^2}\frac{1}{(16\pi^2)^2} \end{bmatrix}$$

and in the minimal basis:

$$Z_{\bar\phi} = 1 + \begin{bmatrix} 1 \\ \overline{C}_4 \\ \overline{C}_6 \\ D_4 \\ D_2 \end{bmatrix}^\intercal \begin{bmatrix} 0 & -\frac{1}{2}(n+2)\bar\lambda^2 & 0 \\ 2n\overline{m}^2 & -(n+2)\bar\lambda\overline{m}^2 & 2(n+1)(n+2)\bar\lambda\overline{m}^2 \\ 0 & 0 & 0 \\ (n+2)\overline{m}^2 & -\frac{7}{2}(n+2)\bar\lambda\overline{m}^2 & (n+2)(n+5)\bar\lambda\overline{m}^2 \\ 0 & -2(n+2)\bar\lambda^2\overline{m}^2 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon}\frac{1}{16\pi^2} \\ \frac{1}{\epsilon}\frac{1}{(16\pi^2)^2} \\ \frac{1}{\epsilon^2}\frac{1}{(16\pi^2)^2} \end{bmatrix}$$

➡ **The renormalization of the field depends on redundant parameters, even in the minimal basis!**

The origin of infinite field anomalous dimensions

Recall: $\bar{\phi}_b = Z_{\bar{\phi}}\, \bar{\phi}(\mu)\,, \quad \gamma_{\bar{\phi}}\, \bar{\phi}(\mu) \overset{\text{def}}{=} \mu \frac{d\bar{\phi}(\mu)}{d\mu}$

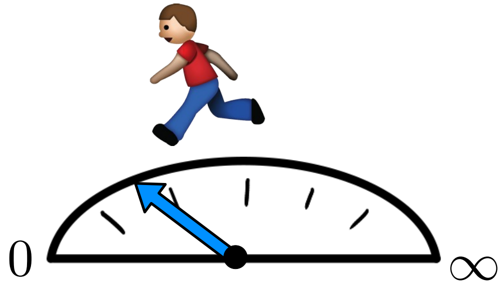
Since the counterterms of the field depend on all couplings (even in the minimal basis), a complete determination of $\gamma_{\bar{\phi}}$ needs to consider all of them:

$\neq 0$!!!

$$\gamma_{\bar{\phi}} = Z_{\bar{\phi}}^{-1}\, \dot{Z}_{\bar{\phi}} = Z_{\bar{\phi}}^{-1} \left(\frac{\partial Z_{\bar{\phi}}}{\partial \bar{\lambda}}\, \dot{\bar{\lambda}} + \frac{\partial Z_{\bar{\phi}}}{\partial \overline{m}^2}\, \dot{\overline{m}}^2 + \sum_i \frac{\partial Z_{\bar{\phi}}}{\partial \overline{C}_i}\, \dot{\overline{C}}_i + \sum_i \frac{\partial Z_{\bar{\phi}}}{\partial D_i}\, \dot{D}_i \right)$$

where $\dot{X} = \mu \frac{dX}{d\mu}$

resulting in a **finite** anomalous dimension of the field.



The origin of infinite field anomalous dimensions

Recall: $\bar{\phi}_b = Z_{\bar{\phi}}\,\bar{\phi}(\mu)\,,\quad \gamma_{\bar{\phi}}\,\bar{\phi}(\mu) \stackrel{\text{def}}{=} \mu\frac{d\bar{\phi}(\mu)}{d\mu}$

Since the counterterms of the field depend on all couplings (even in the minimal basis), a complete determination of $\gamma_{\bar{\phi}}$ needs to consider all of them:

$\neq 0$!!!

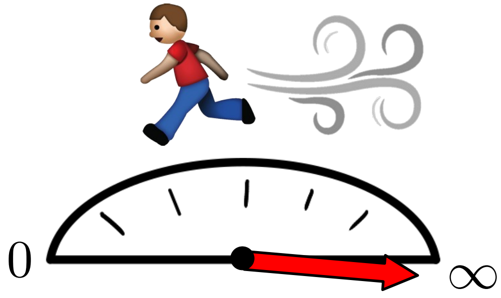
$$\gamma_{\bar{\phi}} = Z_{\bar{\phi}}^{-1}\,\dot{Z}_{\bar{\phi}} = Z_{\bar{\phi}}^{-1}\left(\frac{\partial Z_{\bar{\phi}}}{\partial\bar{\lambda}}\,\dot{\bar{\lambda}} + \frac{\partial Z_{\bar{\phi}}}{\partial\overline{m}^2}\,\dot{\overline{m}}^2 + \sum_i\frac{\partial Z_{\bar{\phi}}}{\partial\overline{C}_i}\,\dot{\overline{C}}_i + \sum_i\frac{\partial Z_{\bar{\phi}}}{\partial D_i}\,\dot{D}_i\right)$$

$$\text{where } \dot{X} = \mu\frac{dX}{d\mu}$$

resulting in a **finite** anomalous dimension of the field.

Removing the redundant parameters from the theory requires an additional **infinite** field redefinition (for some a and b)

$$\bar{\phi} = \left(1 + \frac{a}{\epsilon}\,D_2 + \frac{b}{\epsilon}\,D_4\right)\tilde{\phi}$$



resulting in an **infinite** field with **infinite** anomalous dimension.
This field redefinition does not affect the S-matrix, nor the counterterms of the couplings.
It is implicit when redundant parameters are ignored from the start.

Conclusion

- Field redefinitions leave the S-matrix invariant and can be used to transform an EFT Lagrangian to a minimal basis

$$m^2,\ \lambda,\ C_4,\ C_6,\ D_2,\ D_4 \quad \Longleftrightarrow \quad \overline{m}^2,\ \bar{\lambda},\ \overline{C}_4,\ \overline{C}_6,\ D_2,\ D_4$$

  with e.g. $\overline{C}_4 = C_4 - \frac{1}{2}D_4,\ \overline{C}_6 = C_6 + \frac{1}{2}\lambda D_4 + \lambda^2 D_2$.

  The S-matrix and the counterterms of the couplings in the minimal basis are independent of the redundant couplings D_2 and D_4 .

- The counterterms of the fields depend on the redundant couplings. Ignoring them corresponds to an (implicit) infinite field redefinition, leaving the S-matrix and the couplings invariant but generating an infinite anomalous dimension.

- We exemplified this in the scalar $\mathrm{O(n)}$ model, but the arguments extend more generally in the minimal subtraction scheme of dimensional regularization.

- After field redefinitions, off-shell Green's functions generally diverge. The S-matrix is insensitive to the choice of field, but its computation (through the LSZ reduction formula) involves the normalization of the two-point function,

  $\mathcal{R} \sim \langle\Omega|T\,\phi(x)\,\phi(y)|\Omega\rangle$, which becomes non-trivial.

Thank you!

Backup slide: explicit values

$O(n)$ model Lagrangian:

$$\mathcal{L}_{O(n)} = \frac{1}{2}(\partial_\mu\phi\cdot\partial^\mu\phi) - \frac{1}{2}m^2(\phi\cdot\phi) - \frac{1}{4}\lambda(\phi\cdot\phi)^2 + C_4(\phi\cdot\phi)(\partial_\mu\phi\cdot\partial^\mu\phi) + C_6(\phi\cdot\phi)^3$$

In Jenkins, Manohar, Naterop, Pagès (2023), the anomalous dimension of the field was found to be

$$\gamma_\phi = \frac{-4(n-1)m^2C_4}{16\pi^2} + \frac{(n+2)\lambda^2 - \frac{8}{3}(n+2)\lambda m^2 C_4}{(16\pi^2)^2} + \frac{1}{\epsilon}\,\frac{4(n-1)(n+2)\lambda m^2 C_4}{(16\pi^2)^2}\,. \qquad (1)$$

In Manohar, Pagès, JRN (2024), we found

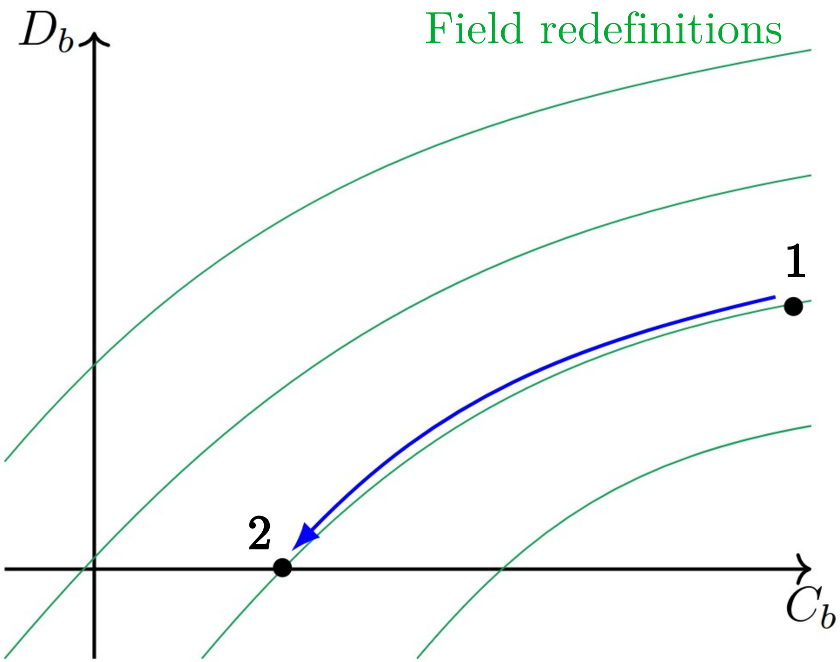
$$\gamma_\phi = \frac{-2nm^2C_4}{16\pi^2} + \frac{(n+2)\lambda^2 + 2(n+2)\lambda m^2 C_4}{(16\pi^2)^2}\,,$$

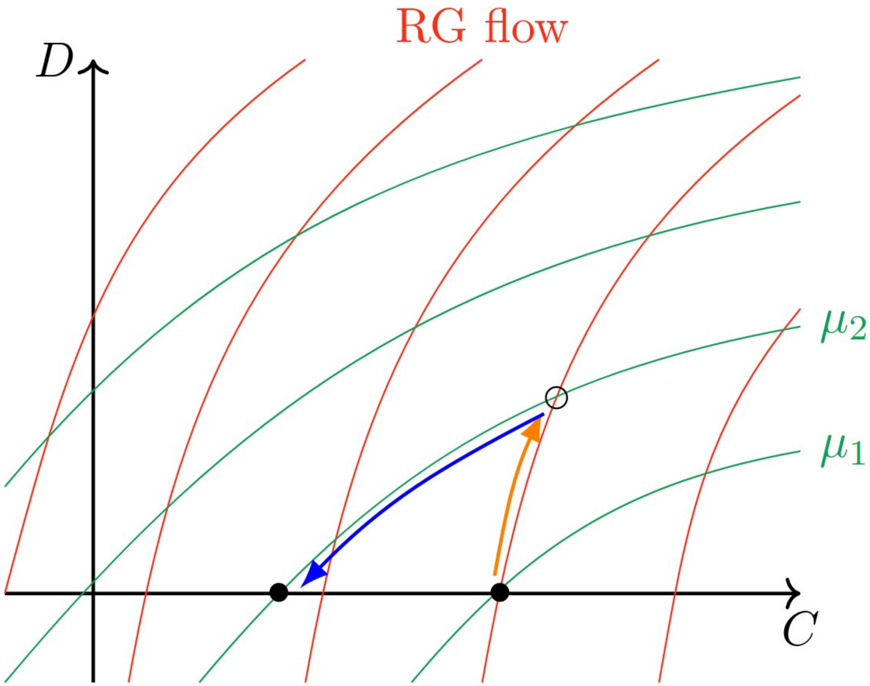
when including the anomalous dimension of the redundant operators (before setting them to zero), and

$$\gamma_\phi = \frac{-2nm^2C_4}{16\pi^2} + \frac{(n+2)\lambda^2 + 2(n+2)\lambda m^2 C_4}{(16\pi^2)^2} + \frac{1}{\epsilon}\,\frac{2(n^2-4)\lambda m^2 C_4}{(16\pi^2)^2}\,,$$

when the running of the redundant operators are ignored from the start. Note the difference with Eq.(1) in both the finite and infinite terms, already at one loop! This difference arises from the difference in the use of field redefinitions.

Backup slide: figures





Bare parameters of redundant operators D_b (w.r.t. the S-matrix) are generated by off-shell renormalization (point 1). Field redefinitions can be used to remove redundant parameters, i.e. absorb them into the parameters of a minimal basis C_b (point 2). The bare parameters do not depend on the renormalization scale

There are two complementary flows in the parameter space of renormalized couplings. The renormalized couplings are affected by field redefinitions, as well as the renormalization group equations.
Even starting with $D(\mu_1) = 0$, non-zero values for the redundant parameters are generated by the renormalization group (RG) flow (red arrow). This effect has to be countered by a field redefinition (blue arrow).