Avoiding gauge-variant operators

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supported by SNF

EOM and gauge-variant operators

Annoying Operators
and annoying divergences

subdivergences & CT diagrams

The Low-Energy EFT

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QED+QCD}} + \sum_{d\geq 5}\sum_i L_i^{(d)} \mathcal{O}_i^{(d)} \qquad U(1) \times SU(3) \text{ invariant}$$

$\longrightarrow$ no top, Higgs and EW gauge bosons

- Operators up to dimension six
  [Jenkins, Manohar, Stoffer, 2018]
- One-loop RGE
  [Jenkins, Manohar, Stoffer, 2018]
- Matching at one loop to SMEFT
  [Dekens, Stoffer, 2019]

Renormalization at two loops

At two loops, there are

~ 500 2-point

~ 3k 3-point

~ 15k 4-point

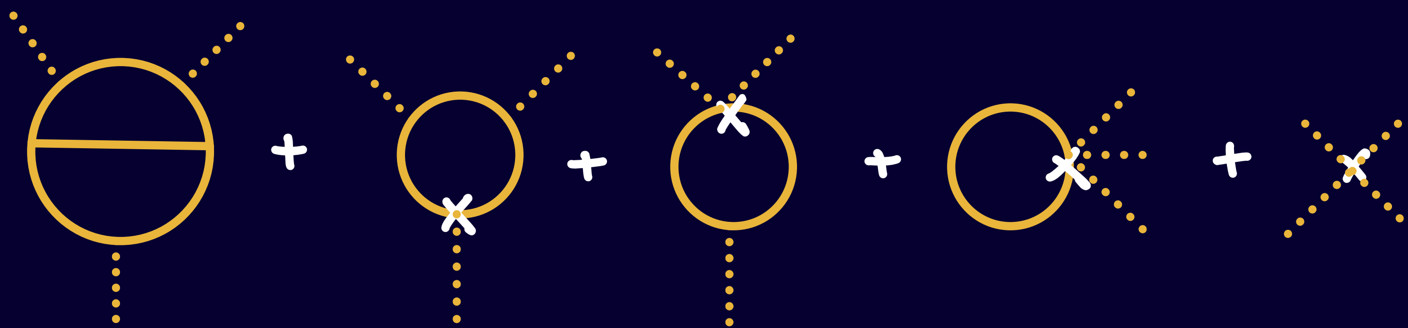
So ~ 20k diagrams with physical insertions

~ 500 terms per diagram

→ ~ 10 million terms

Renormalization at two loops

R-operation:



⇒ ~ 60k 1-loop diagrams with

- GI EOM operators (3x)
- Evanescent operators (16x)
- GV operators (?)

[Naterop, Stoffer, 2023]

Renormalization at two loops

R-operation:

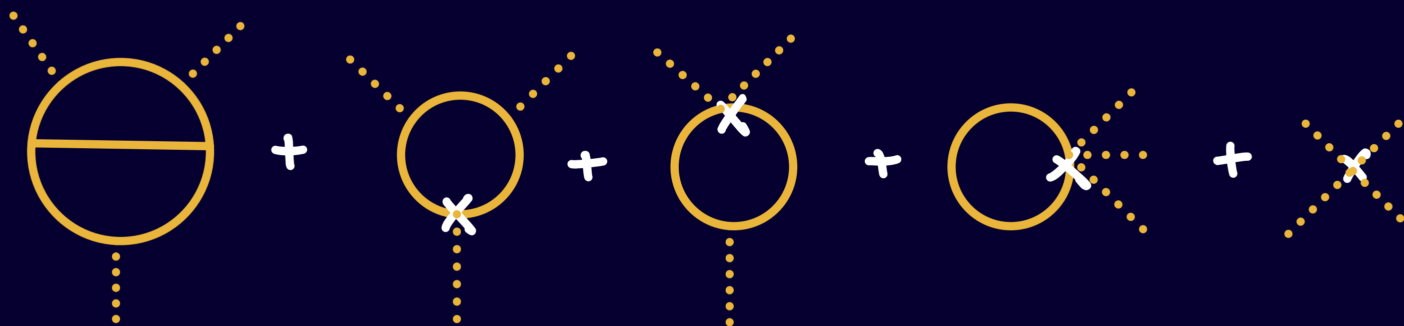
⇒ ~ 60k 1-loop diagrams with

- GI EOM operators (3x)
- ~~Evanescent operators (10x)~~
- GV operators (?)

[Naterop, Stoffer, 2023]

Renormalization at two loops

R-operation:



$\Rightarrow$ ~ 60k 1-loop diagrams with

- ~~GI EOM operators (3x)~~
- ~~Evanescent operators (10x)~~
- ~~GV operators (?)~~

[Naterop, Stoffer, 2023]

GV operators

QCD BF gauge [Abbott, 1982]

→ only invariant w.r.t. BG-field GT

So in subgraphs (= quantum fields) can generate

$$\bar{\Psi}\,\hat{\not{G}}\,\Psi,\ \bar{\Psi}\,\hat{\not{G}}^2\,\Psi,\ \bar{\Psi}\,\hat{\not{G}}\,i\not{D}\,\Psi,\ \bar{\Psi}\,i\not{D}\,\hat{\not{G}}\,\Psi,\ \ldots$$

$$\hat{\not{G}} = \hat{G}^a_\mu T^a \gamma^\mu$$

↑ quantum gluon

[Misiak, Muenz, 1994]

Avoiding EOM operators

Ingredients:

(A) Green's functions are unchanged under field redefinitions

(B) GV operators are EOM operators

(C) EOM operators can be removed order by order

(D) In the end EOM operators do not mix into physical sector

Avoiding EOM operators

Example: Massless QED + dipole

$$\mathcal{L} = \bar\psi i\not{D}\psi + L\bar\psi\sigma_{\mu\nu}F^{\mu\nu}\psi + R\bar\psi(i\not{D})^2\psi + \bar{J}\psi + \bar\psi J$$

kill EOM operator: $\psi \to \psi - \frac{R}{2} i\not{D}\psi$

$$\mathcal{L}' = \bar\psi i\not{D}\psi + L\bar\psi\sigma_{\mu\nu}F^{\mu\nu}\psi + \bar{J}\left(\psi - \frac{R}{2} i\not{D}\psi\right) + \left(\bar\psi + \frac{R}{2}\bar\psi i\overleftarrow{\not{D}}\right)J$$

remove derivative sources: $\psi \to \psi + \frac{R}{2} J$

$$\mathcal{L}'' = \bar\psi i\not{D}\psi + L\bar\psi\sigma_{\mu\nu}F^{\mu\nu}\psi + \bar{J}\psi + \bar\psi J + R\bar{J}J$$

Avoiding EOM operators

$$Z[J] = \int \mathcal{D}\psi \, e^{iS[\psi,J]} = \int \overbrace{\det\left(\frac{\delta\psi}{\delta\psi'}\right)}^{1 \text{ for local FR}} \mathcal{D}\psi' \, e^{iS[\psi',J]}$$

$$\langle \psi(x_1)\bar{\psi}(x_2)\ldots\rangle = \frac{\delta}{i\delta J(x_1)}\frac{\delta}{i\delta J(x_2)}\cdots Z[J]\Big|_{J=0}$$

are left unchanged!

$R\bar{J}J$ term

$\mathcal{L} \supset R\bar{J}J$ term

- has no ψ, contributes only to $\langle\psi\bar{\psi}\rangle_{tree}$
- all other contributions are disconnected
- amputation: mult. with $(\not{p}-m)$

↳ goes into EOM operators in the end

$\langle\psi\bar{\psi}\rangle^{\mathcal{L}''}_{conn.} =$

from $R\bar{J}J$

Summary

- $\mathcal{L}$ and $\mathcal{L}''$ give same Green's functions
- $\mathcal{L}''$ has no EoM operators
- $R\bar{J}J$ term affects only tree-level diagrams
- General argument in upcoming paper
- Tested in QED + Dipole and QCD + dipole

$$\boxed{\text{Construction and loop-insertion of nuisance operators not needed}}$$

?

Backup: 2-loop RGEs

$\frac{1}{\epsilon^n}$ pole @ ℓ loops

$$\mathcal{L} = \sum_i C_i^{\text{bare}} O_i\,, \quad C_i^{\text{bare}} = \mu^{n_i \epsilon}(C_i + \delta_i)\,, \quad \delta_i = \sum_{\ell=1}^{\infty} \sum_{n=0}^{\ell} \frac{a^{(\ell,n)}}{\epsilon^n}$$

Using $\mu \frac{d}{d\mu} C_i^{\text{bare}} = 0$ and topological identities

$$\dot{C}_i^{(1)} = 2 a_i^{(1,1)}$$

$$\dot{C}_i^{(2)} = 4 a_i^{(2,1)} - 2 a_j^{(1,0)} \frac{\partial a_i^{(1,1)}}{\partial C_j} - 2 a_j^{(1,1)} \frac{\partial a_i^{(1,0)}}{\partial C_j}$$

where $\dot{C} = \mu \frac{d}{d\mu} C$.

→ depends on finite renormalizations

→ scheme dependence in RGEs @ two loops

Backup: Chiral symmetry

But D dimensions in HV

$$\bar{\psi} i\not{\partial} \psi = \bar{\psi}_L i\bar{\not{\partial}} \psi_L + \bar{\psi}_R i\bar{\not{\partial}} \psi_R + \underline{\bar{\psi}_L i\hat{\not{\partial}} \psi_R} + \underline{\bar{\psi}_R i\hat{\not{\partial}} \psi_L}$$

→ spurious symmetry-violating terms in intermediate steps

→ Need to cancel in relations between observables

finite effects on O_i are local

→ Can compensate in δL_i^{finite}

Backup: The looped LEFT

$$\mathcal{L}_{LEFT} = \mathcal{L}_{QED+QCD} + \sum_i L_i O_i + \sum_i L_i^{red} O_i^{red} + \sum_i K_i E_i$$

d-dim, 4-dim, redundant, evanescent

+ renormalization scheme:

$$L^{bare} = \mu^{n\epsilon}\Big(L^{ren} + \underbrace{\delta L^{div}}_{\text{mandatory}} + \underbrace{\delta L^{finite}_{ev} + \delta L^{finite}_{CSB}}_{\text{choice}}\Big)$$

→ deviation from $\overline{MS}$