Construction and Conversion of Operator Bases in EFTs

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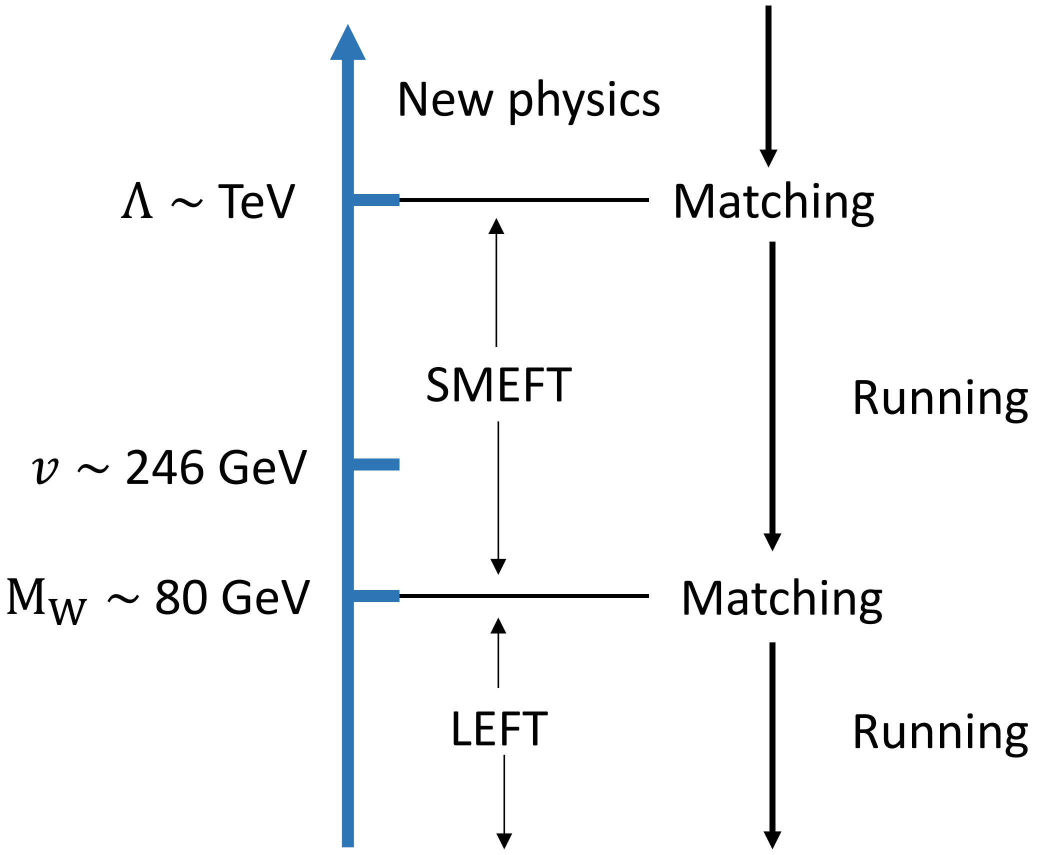
Outline

Why EFT?

01 Energy gap between Λ and v



02 Systematic method

physical effect of new physics

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \sum_{d>4} \left(\frac{1}{\Lambda}\right)^{d-4} \sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$$

physical effect of new physics and the heavy particles in the SM

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QCD+QED}}^{(4)} + \sum_{d>4} \left(\frac{1}{M_W}\right)^{d-4} \sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$$

01

On-shell Operator Basis

SMEFT Operator Basis

Dim 5: [S. Weinberg, 1979]

Dim 6: [W. Buchmuller and D. Wyler, 1986]
[B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, 2010]

Dim 7: [L. Lehman, 2014]
[B. Henning, X. Lu, T. Melia, H. Murayama, 2015]
[Y. Liao, X.-D. Ma, 2016]

Dim 8: [H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu, Y.-H. Zheng, 2020]
[C. W. Murphy, 2020]

Systematic method

Dim 9: [H.-L. Li, Z. Ren, M.-L. Xiao, J.-H. Yu, Y.-H. Zheng, 2020]
[Y. Liao, X.-D. Ma, 2020]

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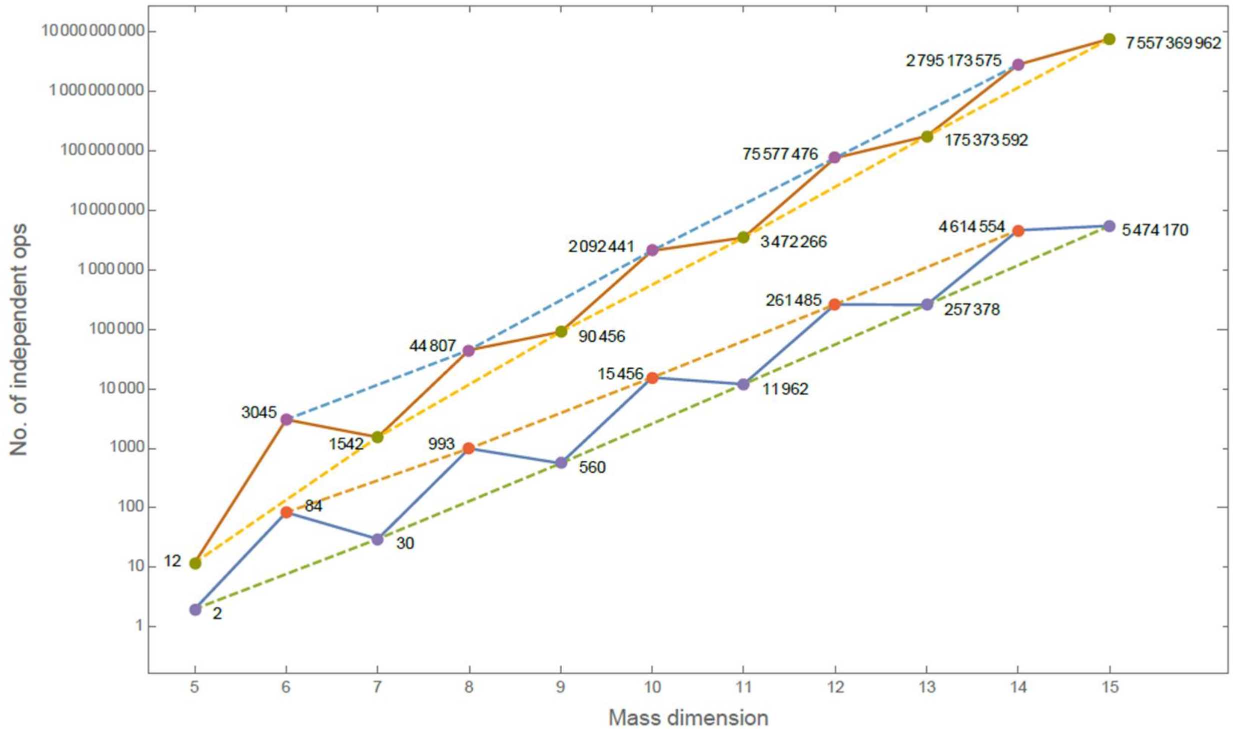


Algebraic Identities

Fierz identity (Schouten identity),

Bianchi identity,

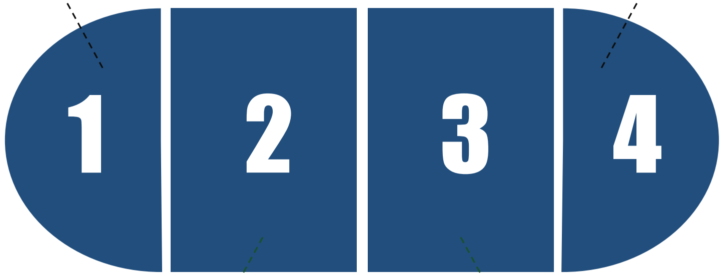
Jacobi identity,

...

Equation of Motion (EOM)

A special case of field redefinitions that leave the S-matrix invariant.

$$D^2\phi,\ \not{D}\psi,\ D_\mu F^{\mu\nu} \to 0$$



Integration by Part (IBP)

$$X\,DY = D(XY) - DX\,Y$$

Covariant Derivative Commutator (CDC)

$$[D_\mu, D_\nu] = igF_{\mu\nu}$$

Amplitude-Operator Correspondence

$$\begin{array}{ccc} F_{\mathrm{L/R}\,i} & \Leftrightarrow & \lambda_i^2/\tilde{\lambda}_i^2 \\ \psi_i/\psi_i^\dagger & \Leftrightarrow & \lambda_i/\tilde{\lambda}_i \\ \phi_i & \Leftrightarrow & 1 \\ D_i & \Leftrightarrow & -i\lambda_i\tilde{\lambda}_i \end{array}$$

$$p_{i\alpha\dot{\alpha}} = \lambda_{i\alpha}\tilde{\lambda}_{i\dot{\alpha}}$$

Amplitude-Operator Correspondence

$$F_{\mathrm{L/R}\,i} \Leftrightarrow \lambda_i^2/\tilde{\lambda}_i^2$$
$$\psi_i/\psi_i^\dagger \Leftrightarrow \lambda_i/\tilde{\lambda}_i$$
$$\phi_i \Leftrightarrow 1$$
$$D_i \Leftrightarrow -i\lambda_i\tilde{\lambda}_i$$

$$p_{i\alpha\dot{\alpha}} = \lambda_{i\alpha}\tilde{\lambda}_{i\dot{\alpha}}$$

linear isomorphism?

operator space ⟺ amplitude space

$$D^2\phi_i \Leftrightarrow p_i^2 = 0$$

$$[D_\mu, D_\nu] \Leftrightarrow [p_\mu, p_\nu] = 0$$

Amplitude-Operator Correspondence

$$
\begin{array}{ccc}
F_{\mathrm{L/R}\,i} & \Leftrightarrow & \lambda_i^2/\tilde{\lambda}_i^2 \\
\psi_i/\psi_i^\dagger & \Leftrightarrow & \lambda_i/\tilde{\lambda}_i \\
\phi_i & \Leftrightarrow & 1 \\
D_i & \Leftrightarrow & -i\lambda_i\tilde{\lambda}_i
\end{array}
$$

$$p_{i\alpha\dot{\alpha}} = \lambda_{i\alpha}\tilde{\lambda}_{i\dot{\alpha}}$$

linear isomorphism?

operator space ⟺ amplitude space

$$D^2\phi_i \Leftrightarrow p_i^2 = 0$$

$$[D_\mu, D_\nu] \Leftrightarrow [p_\mu, p_\nu] = 0$$

linear isomorphism

the subspace where EOM and CDC are removed ⟺ amplitude space

$$D_{[\alpha\dot{\alpha}}D_{\beta]\dot{\beta}} = D_\mu D_\nu \sigma^\mu_{[\alpha\dot{\alpha}}\sigma^\nu_{\beta]\dot{\beta}} = -D^2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} + \frac{i}{2}[D_\mu, D_\nu]\epsilon_{\alpha\beta}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}},$$

$$D_{[\alpha\dot{\alpha}}\psi_{\beta]} = D_\mu \sigma^\mu_{[\alpha\dot{\alpha}}\psi_{\beta]} = -\epsilon_{\alpha\beta}(D_\mu\sigma^\mu\psi)_{\dot{\alpha}},$$

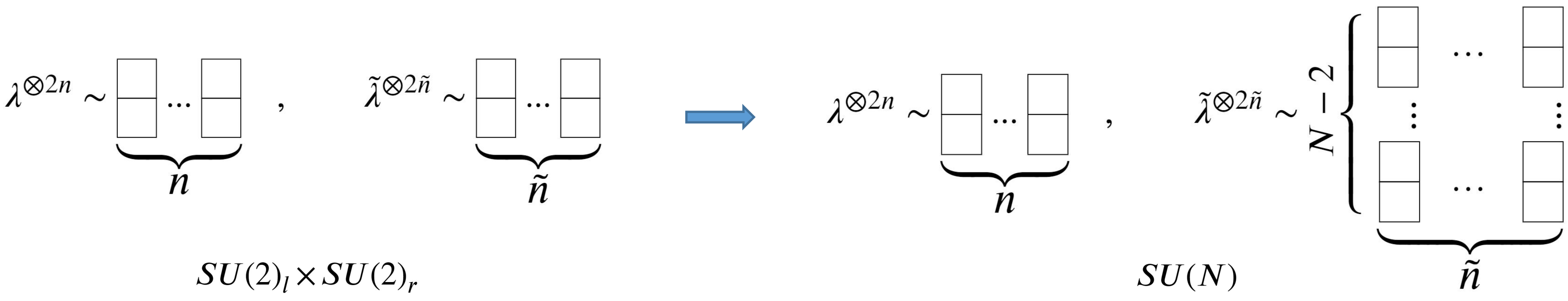
$$D_{[\alpha\dot{\alpha}}F_{\mathrm{L}\beta]\gamma} = \frac{i}{2}D_\mu F_{\mathrm{L}\nu\rho}\sigma^\mu_{[\alpha\dot{\alpha}}\sigma^{\nu\rho}_{\beta]\gamma} = iD^\mu F_{\mathrm{L}\mu\nu}\epsilon_{\alpha\beta}\sigma^\nu_{\gamma\dot{\alpha}},$$

SU(N) Group

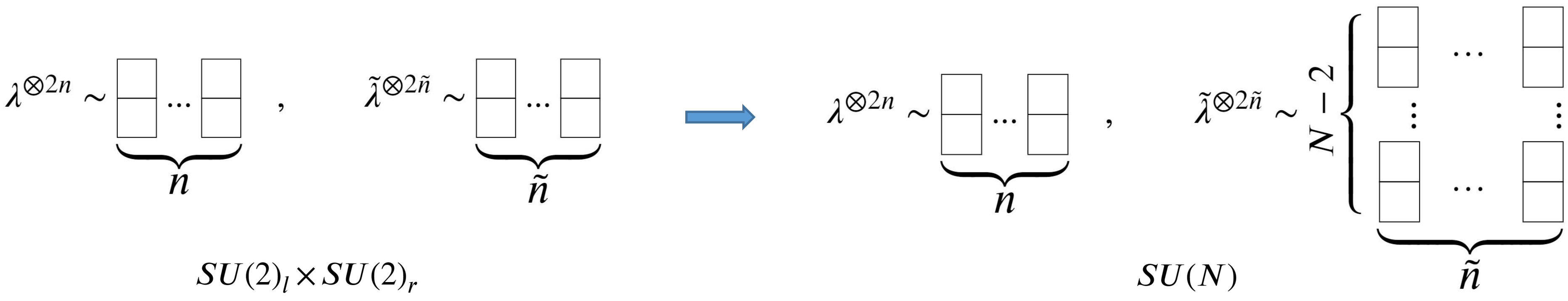
$$\lambda_i \to \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k \tilde{\lambda}^k, \qquad \sum_i \lambda_i \tilde{\lambda}^i \to \sum_i \sum_j \sum_k U_i^j U^{\dagger i}{}_k \lambda_j \tilde{\lambda}^k = \sum_j \lambda_j \tilde{\lambda}^j$$

SU(N) Group

$$\lambda_i \to \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k \tilde{\lambda}^k, \qquad \sum_i \lambda_i \tilde{\lambda}^i \to \sum_i \sum_j \sum_k U_i^j U^{\dagger i}{}_k \lambda_j \tilde{\lambda}^k = \sum_j \lambda_j \tilde{\lambda}^j$$



$$\lambda_i \to \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k \tilde{\lambda}^k, \qquad \sum_i \lambda_i \tilde{\lambda}^i \to \sum_i \sum_j \sum_k U_i^j U^{\dagger i}{}_k \lambda_j \tilde{\lambda}^k = \sum_j \lambda_j \tilde{\lambda}^j$$



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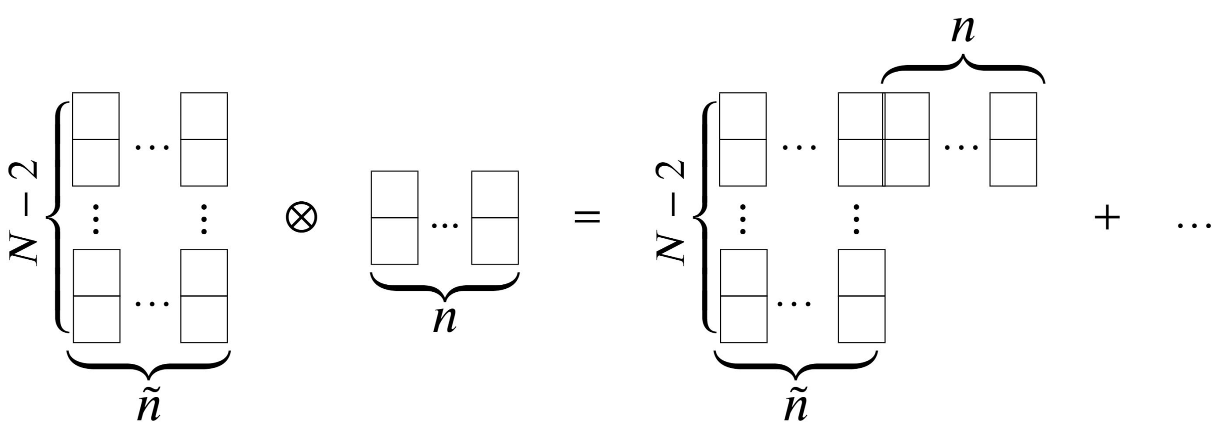
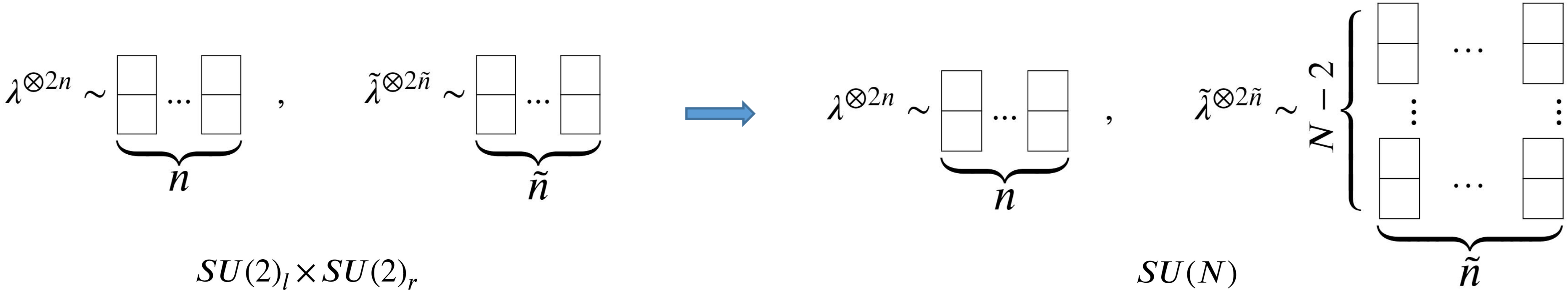


SU(N) Group

$$\lambda_i \to \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k \tilde{\lambda}^k, \qquad \sum_i \lambda_i \tilde{\lambda}^i \to \sum_i \sum_j \sum_k U_i^j U^{\dagger i}{}_k \lambda_j \tilde{\lambda}^k = \sum_j \lambda_j \tilde{\lambda}^j$$



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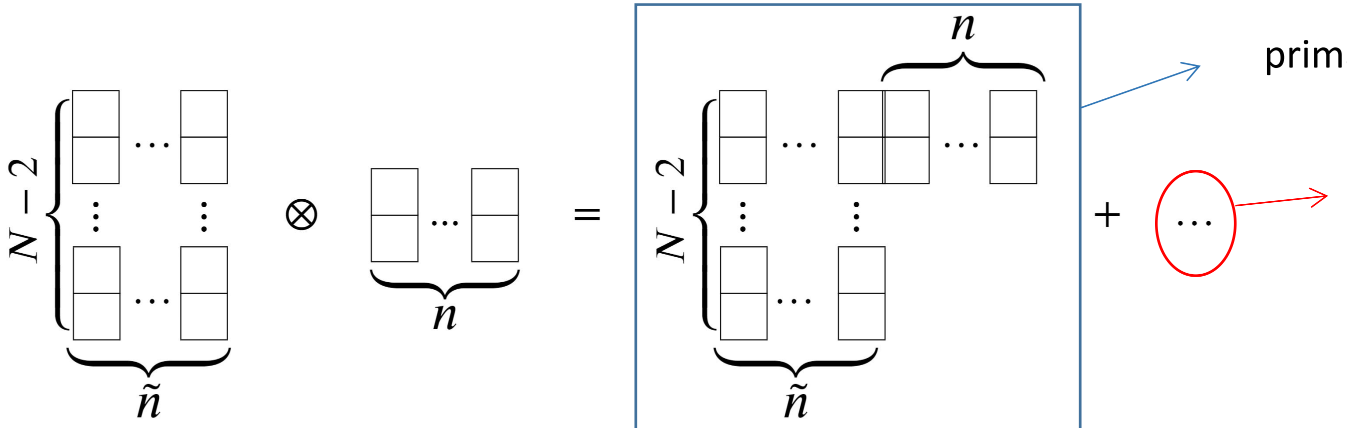


primary Young diagram

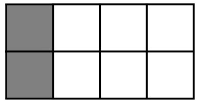
proportional to total momentum $P = \sum_i p_i = 0$

Y-basis ("Y" for Young Tableau)

$W_L^2 H H^\dagger D^2$

$N = 4,\ \tilde{n} = 1,\ n = 3$

$\#i = \tilde{n} - 2h_i$



SSYT

1	1	1	3
2	2	2	4

$\mathcal{M}_1^{(y)} = \langle 12\rangle^2\langle 34\rangle[34]$

$\mathcal{M}_1^{(y)} = F_{L1}{}^{\alpha\beta} F_{L2\alpha\beta} (D\phi_3)^{\gamma}{}_{\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$

1	1	1	2
2	2	3	4

$\mathcal{M}_2^{(y)} = \langle 12\rangle\langle 13\rangle\langle 24\rangle[34]$

$\mathcal{M}_2^{(y)} = F_{L1}{}^{\alpha\beta} F_{L2\alpha}{}^{\gamma} (D\phi_3)_{\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$

Fock's condition 1

1	1	1	3
2	2	2	4

+

1	1	3	2
2	2	1	4

+

1	1	2	1
2	2	3	4

$= 0$

momentum conservation (IBP)

$$\langle 12\rangle^2\langle 34\rangle[34] + \langle 12\rangle\langle 31\rangle\langle 24\rangle[34] + \langle 12\rangle\langle 23\rangle\langle 14\rangle[34] = 0$$

Fock's condition 2

1	1	1	3
2	2	2	4

+

3	1	1	2
1	2	2	4

+

2	1	1	1
3	2	2	4

$= 0$

Schouten identity

$$\langle 12\rangle^2\langle 34\rangle[34] + \langle 12\rangle^2\langle 24\rangle[24] + \langle 12\rangle^2\langle 14\rangle[14] = 0$$

Y-basis ("Y" for Young Tableau)

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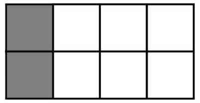
$$N = 4,\ \tilde{n} = 1,\ n = 3$$

$$\# i = \tilde{n} - 2h_i$$



SSYT

1	1	1	3
2	2	2	4

$$\mathcal{M}_1^{(y)} = \langle 12\rangle^2\langle 34\rangle[34]$$

$$\mathcal{M}_1^{(y)} = F_{\mathrm{L}1}{}^{\alpha\beta} F_{\mathrm{L}2\alpha\beta} (D\phi_3)^{\gamma}{}_{\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

1	1	1	2
2	2	3	4

$$\mathcal{M}_2^{(y)} = \langle 12\rangle\langle 13\rangle\langle 24\rangle[34]$$

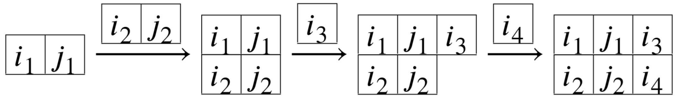
$$\mathcal{M}_2^{(y)} = F_{\mathrm{L}1}{}^{\alpha\beta} F_{\mathrm{L}2\alpha}{}^{\gamma} (D\phi_3)_{\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

$$(\tau^{I_1})_{i_1 j_1} W_1^{I_1} \equiv \epsilon_{j_1 m_1} (\tau^{I_1})_{i_1}^{m_1} W_1^{I_1} \sim \boxed{i_1 \,|\, j_1}\,,$$

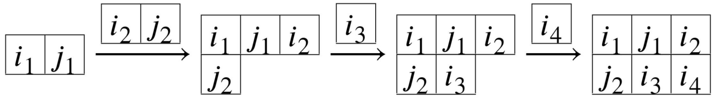
$$(\tau^{I_2})_{i_2 j_2} W_2^{I_2} \equiv \epsilon_{j_2 m_2} (\tau^{I_2})_{i_2}^{m_2} W_2^{I_2} \sim \boxed{i_2 \,|\, j_2}\,,$$

$$H_{3 i_3} \sim \boxed{i_3}\,, \quad \epsilon_{i_4 j_4} H_4^{\dagger j_4} \sim \boxed{i_4}\,.$$

L-R rule



$$\sim \epsilon^{i_1 i_2}\epsilon^{j_1 j_2}\epsilon^{i_3 i_4},$$



$$\sim \epsilon^{i_1 j_2}\epsilon^{j_1 i_3}\epsilon^{i_2 i_4}.$$

Y-basis ("Y" for Young Tableau)

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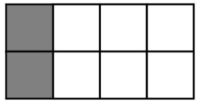


SSYT

$$\begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 3 \\ \hline 2 & 2 & 2 & 4 \\ \hline \end{array} \quad \Longleftrightarrow \quad \mathcal{M}_1^{(y)} = \langle 12\rangle^2\langle 34\rangle[34]$$

$$\mathcal{M}_1^{(y)} = F_{L1}{}^{\alpha\beta} F_{L2\alpha\beta} (D\phi_3)^{\gamma}{}_{\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

$$\begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 3 & 4 \\ \hline \end{array} \quad \Longleftrightarrow \quad \mathcal{M}_2^{(y)} = \langle 12\rangle\langle 13\rangle\langle 24\rangle[34]$$

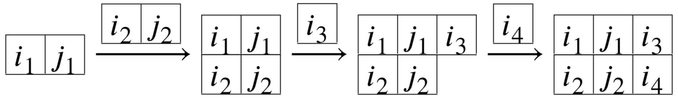
$$\mathcal{M}_2^{(y)} = F_{L1}{}^{\alpha\beta} F_{L2\alpha}{}^{\gamma} (D\phi_3)_{\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

$$(\tau^{I_1})_{i_1 j_1} W_1^{I_1} \equiv \epsilon_{j_1 m_1} (\tau^{I_1})_{i_1}^{m_1} W_1^{I_1} \sim \boxed{i_1}\boxed{j_1},$$

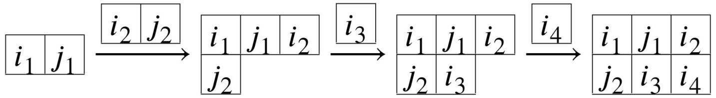
$$(\tau^{I_2})_{i_2 j_2} W_2^{I_2} \equiv \epsilon_{j_2 m_2} (\tau^{I_2})_{i_2}^{m_2} W_2^{I_2} \sim \boxed{i_2}\boxed{j_2},$$

$$H_{3 i_3} \sim \boxed{i_3}, \quad \epsilon_{i_4 j_4} H_4^{\dagger j_4} \sim \boxed{i_4}.$$

L-R rule



$$\sim \epsilon^{i_1 i_2} \epsilon^{j_1 j_2} \epsilon^{i_3 i_4},$$



$$\sim \epsilon^{i_1 j_2} \epsilon^{j_1 i_3} \epsilon^{i_2 i_4}.$$

$$T^{(y)}_{SU3,1} = 1$$

$$\otimes$$

$$T^{(y)}_{SU2,1} = \epsilon^{i_1 i_2} \epsilon^{j_1 j_2} \epsilon^{i_3 i_4} \epsilon_{i_4 j_4} (\tau^{I_1})_{i_1 j_1} (\tau^{I_2})_{i_2 j_2} = 2\delta^{I_1 I_2} \delta^{i_3}_{j_4},$$

$$T^{(y)}_{SU2,2} = \epsilon^{i_1 j_2} \epsilon^{j_1 i_3} \epsilon^{i_2 i_4} \epsilon_{i_4 j_4} (\tau^{I_1})_{i_1 j_1} (\tau^{I_2})_{i_2 j_2} = \delta^{I_1 I_2} \delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J} (\tau^J)^{i_3}_{j_4}.$$

$$\otimes$$

$$\mathcal{M}_1^{(y)} = F_{L1}{}^{\alpha\beta} F_{L2\alpha\beta} (D\phi_3)^{\gamma}{}_{\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{M}_2^{(y)} = F_{L1}{}^{\alpha\beta} F_{L2\alpha}{}^{\gamma} (D\phi_3)_{\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}.$$

$$\mathcal{O}^{(y)}_{W_L^2 H H^\dagger D^2,1} = 2\delta^{I_1 I_2} \delta^{i_3}_{j_4} W_{L1}{}^{I_1\alpha\beta} W_{L2}{}^{I_2}{}_{\alpha\beta} (DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{O}^{(y)}_{W_L^2 H H^\dagger D^2,2} = 2\delta^{I_1 I_2} \delta^{i_3}_{j_4} W_{L1}{}^{I_1\alpha\beta} W_{L2}{}^{I_2}{}_{\alpha}{}^{\gamma} (DH_3)_{i_3\beta\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{O}^{(y)}_{W_L^2 H H^\dagger D^2,3} = \left(\delta^{I_1 I_2} \delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J} (\tau^J)^{i_3}_{j_4}\right) W_{L1}{}^{I_1\alpha\beta} W_{L2}{}^{I_2}{}_{\alpha\beta} (DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{O}^{(y)}_{W_L^2 H H^\dagger D^2,4} = \left(\delta^{I_1 I_2} \delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J} (\tau^J)^{i_3}_{j_4}\right) W_{L1}{}^{I_1\alpha\beta} W_{L2}{}^{I_2}{}_{\alpha}{}^{\gamma} (DH_3)_{i_3\beta\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha}.$$

Workflow

Y-basis ("Y" for Young tableau)

$$
\begin{aligned}
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,1} &= 2\delta^{I_1 I_2}\delta^{i_3}_{j_4} {W_{\rm L1}}^{I_1\alpha\beta} {W_{\rm L2}}^{I_2}{}_{\alpha\beta} (DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,2} &= 2\delta^{I_1 I_2}\delta^{i_3}_{j_4} {W_{\rm L1}}^{I_1\alpha\beta} {W_{\rm L2}}^{I_2}{}_{\alpha}{}^{\gamma} (DH_3)_{i_3\beta\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,3} &= \left(\delta^{I_1 I_2}\delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J}(\tau^J)^{i_3}_{j_4}\right) {W_{\rm L1}}^{I_1\alpha\beta} {W_{\rm L2}}^{I_2}{}_{\alpha\beta} (DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,4} &= \left(\delta^{I_1 I_2}\delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J}(\tau^J)^{i_3}_{j_4}\right) {W_{\rm L1}}^{I_1\alpha\beta} {W_{\rm L2}}^{I_2}{}_{\alpha}{}^{\gamma} (DH_3)_{i_3\beta\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha}.
\end{aligned}
$$

Workflow

Y-basis ("Y" for Young tableau)

$$
\begin{aligned}
\mathcal{O}^{(y)}_{W_{\rm L}^2HH^\dagger D^2,1} &= 2\delta^{I_1I_2}\delta^{i_3}_{j_4}W_{{\rm L}1}{}^{I_1\alpha\beta}W_{{\rm L}2}{}^{I_2}{}_{\alpha\beta}(DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2HH^\dagger D^2,2} &= 2\delta^{I_1I_2}\delta^{i_3}_{j_4}W_{{\rm L}1}{}^{I_1\alpha\beta}W_{{\rm L}2}{}^{I_2}{}_{\alpha}{}^{\gamma}(DH_3)_{i_3\beta\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2HH^\dagger D^2,3} &= \left(\delta^{I_1I_2}\delta^{i_3}_{j_4}-i\epsilon^{I_1I_2J}(\tau^J)^{i_3}_{j_4}\right)W_{{\rm L}1}{}^{I_1\alpha\beta}W_{{\rm L}2}{}^{I_2}{}_{\alpha\beta}(DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2HH^\dagger D^2,4} &= \left(\delta^{I_1I_2}\delta^{i_3}_{j_4}-i\epsilon^{I_1I_2J}(\tau^J)^{i_3}_{j_4}\right)W_{{\rm L}1}{}^{I_1\alpha\beta}W_{{\rm L}2}{}^{I_2}{}_{\alpha}{}^{\gamma}(DH_3)_{i_3\beta\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha}.
\end{aligned}
$$

$$
\mathcal{K}^{(my)} = \begin{pmatrix} -\frac{1}{8} & 0 & 0 & 0 \\ \frac{1}{16} & -\frac{1}{16} & 0 & 0 \\ \frac{i}{8} & 0 & -\frac{i}{4} & 0 \\ -\frac{i}{16} & \frac{i}{16} & \frac{i}{8} & -\frac{i}{8} \end{pmatrix}
$$

M-basis ("M" for monomial)

$$
\begin{aligned}
\mathcal{O}^{(m)}_{W_{\rm L}^2HH^\dagger D^2,1} &= W_{{\rm L}1}{}^{I}{}_{\nu\mu}W_{{\rm L}2}{}^{I\mu\nu}(D_\lambda H_{3i})(D^\lambda H_4^{\dagger i}),\\
\mathcal{O}^{(m)}_{W_{\rm L}^2HH^\dagger D^2,2} &= W_{{\rm L}1}{}^{I}{}_{\mu}{}^{\nu}W_{{\rm L}2}{}^{I\mu\lambda}(D_\lambda H_{3i})(D_\nu H_4^{\dagger i}),\\
\mathcal{O}^{(m)}_{W_{\rm L}^2HH^\dagger D^2,3} &= \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}1}{}^{I}{}_{\nu\mu}W_{{\rm L}2}{}^{J\mu\nu}(D_\lambda H_{3i})(D^\lambda H_4^{\dagger j}),\\
\mathcal{O}^{(m)}_{W_{\rm L}^2HH^\dagger D^2,4} &= \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}1}{}^{I}{}_{\mu}{}^{\nu}W_{{\rm L}2}{}^{J\mu\lambda}(D_\lambda H_{3i})(D_\nu H_4^{\dagger j}),
\end{aligned}
$$

Workflow

Y-basis ("Y" for Young tableau)

$$\begin{aligned}
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,1} &= 2\delta^{I_1 I_2}\delta^{i_3}_{j_4} W_{{\rm L}1}{}^{I_1\alpha\beta} W_{{\rm L}2}{}^{I_2}{}_{\alpha\beta} (DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,2} &= 2\delta^{I_1 I_2}\delta^{i_3}_{j_4} W_{{\rm L}1}{}^{I_1\alpha\beta} W_{{\rm L}2}{}^{I_2}{}_{\alpha}{}^{\gamma} (DH_3)_{i_3\beta\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,3} &= \left(\delta^{I_1 I_2}\delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J}(\tau^J)^{i_3}_{j_4}\right) W_{{\rm L}1}{}^{I_1\alpha\beta} W_{{\rm L}2}{}^{I_2}{}_{\alpha\beta} (DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},\\
\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,4} &= \left(\delta^{I_1 I_2}\delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J}(\tau^J)^{i_3}_{j_4}\right) W_{{\rm L}1}{}^{I_1\alpha\beta} W_{{\rm L}2}{}^{I_2}{}_{\alpha}{}^{\gamma} (DH_3)_{i_3\beta\dot\alpha} (DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha}.
\end{aligned}$$

$$\mathcal{K}^{(my)} = \begin{pmatrix} -\frac{1}{8} & 0 & 0 & 0 \\ \frac{1}{16} & -\frac{1}{16} & 0 & 0 \\ \frac{i}{8} & 0 & -\frac{i}{4} & 0 \\ -\frac{i}{16} & \frac{i}{16} & \frac{i}{8} & -\frac{i}{8} \end{pmatrix}$$

M-basis ("M" for monomial)

$$\begin{aligned}
\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,1} &= W_{{\rm L}1}{}^{I}_{\nu\mu} W_{{\rm L}2}{}^{I\mu\nu} (D_\lambda H_{3i})(D^\lambda H_4^{\dagger i}),\\
\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,2} &= W_{{\rm L}1}{}^{I}_{\mu}{}^{\nu} W_{{\rm L}2}{}^{I\mu\lambda} (D_\lambda H_{3i})(D_\nu H_4^{\dagger i}),\\
\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,3} &= \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}1}{}^{I}_{\nu\mu} W_{{\rm L}2}{}^{J\mu\nu} (D_\lambda H_{3i})(D^\lambda H_4^{\dagger j}),\\
\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,4} &= \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}1}{}^{I}_{\mu}{}^{\nu} W_{{\rm L}2}{}^{J\mu\lambda} (D_\lambda H_{3i})(D_\nu H_4^{\dagger j}),
\end{aligned}$$

$$\mathcal{K}^{(pm)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 1 \\ \frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

P-basis ("P" for permutation)

$$\begin{aligned}
\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,1} &= \frac{1}{2}\mathcal{Y}\left[\begin{array}{|c|c|}\hline p & r \\ \hline\end{array}\right] W_{{\rm L}p}{}^{I}_{\nu\mu} W_{{\rm L}r}{}^{I\mu\nu} (D_\lambda H_{si})(D^\lambda H_t^{\dagger i}),\\
\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,2} &= \frac{1}{2}\mathcal{Y}\left[\begin{array}{|c|c|}\hline p & r \\ \hline\end{array}\right] \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}p}{}^{I}_{\mu}{}^{\nu} W_{{\rm L}r}{}^{J\mu\lambda} (D_\lambda H_{si})(D_\nu H_t^{\dagger j}),\\
\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,3} &= \frac{1}{2}\mathcal{Y}\left[\begin{array}{|c|}\hline p \\ \hline r \\ \hline\end{array}\right] W_{{\rm L}p}{}^{I}_{\mu}{}^{\nu} W_{{\rm L}r}{}^{I\mu\lambda} (D_\lambda H_{si})(D_\nu H_t^{\dagger i}),\\
\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,4} &= \frac{1}{2}\mathcal{Y}\left[\begin{array}{|c|}\hline p \\ \hline r \\ \hline\end{array}\right] \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}p}{}^{I}_{\nu\mu} W_{{\rm L}r}{}^{J\mu\nu} (D_\lambda H_{si})(D^\lambda H_t^{\dagger j}),
\end{aligned}$$

Workflow

Y-basis (“Y” for Young tableau)

$$\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,1} = 2\delta^{I_1 I_2}\delta^{i_3}_{j_4} W_{\rm L1}{}^{I_1\alpha\beta} W_{\rm L2}{}^{I_2}{}_{\alpha\beta}(DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,2} = 2\delta^{I_1 I_2}\delta^{i_3}_{j_4} W_{\rm L1}{}^{I_1\alpha\beta} W_{\rm L2}{}^{I_2}{}_{\alpha}{}^{\gamma}(DH_3)_{i_3\beta\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,3} = \left(\delta^{I_1 I_2}\delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J}(\tau^J)^{i_3}_{j_4}\right) W_{\rm L1}{}^{I_1\alpha\beta} W_{\rm L2}{}^{I_2}{}_{\alpha\beta}(DH_3)_{i_3}{}^{\gamma}{}_{\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha},$$

$$\mathcal{O}^{(y)}_{W_{\rm L}^2 H H^\dagger D^2,4} = \left(\delta^{I_1 I_2}\delta^{i_3}_{j_4} - i\epsilon^{I_1 I_2 J}(\tau^J)^{i_3}_{j_4}\right) W_{\rm L1}{}^{I_1\alpha\beta} W_{\rm L2}{}^{I_2}{}_{\alpha}{}^{\gamma}(DH_3)_{i_3\beta\dot\alpha}(DH_4^\dagger)^{j_4}{}_{\gamma}{}^{\dot\alpha}.$$

$$\mathcal{K}^{(my)} = \begin{pmatrix} -\frac{1}{8} & 0 & 0 & 0 \\ \frac{1}{16} & -\frac{1}{16} & 0 & 0 \\ \frac{i}{8} & 0 & -\frac{i}{4} & 0 \\ -\frac{i}{16} & \frac{i}{16} & \frac{i}{8} & -\frac{i}{8} \end{pmatrix}$$

M-basis (“M” for monomial)

$$\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,1} = W_{\rm L1}{}^{I}{}_{\nu\mu} W_{\rm L2}{}^{I\mu\nu}(D_\lambda H_{3i})(D^\lambda H_4^{\dagger i}),$$

$$\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,2} = W_{\rm L1}{}^{I}{}_{\mu}{}^{\nu} W_{\rm L2}{}^{I\mu\lambda}(D_\lambda H_{3i})(D_\nu H_4^{\dagger i}),$$

$$\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,3} = \epsilon^{IJK}(\tau^K)^i_j W_{\rm L1}{}^{I}{}_{\nu\mu} W_{\rm L2}{}^{J\mu\nu}(D_\lambda H_{3i})(D^\lambda H_4^{\dagger j}),$$

$$\mathcal{O}^{(m)}_{W_{\rm L}^2 H H^\dagger D^2,4} = \epsilon^{IJK}(\tau^K)^i_j W_{\rm L1}{}^{I}{}_{\mu}{}^{\nu} W_{\rm L2}{}^{J\mu\lambda}(D_\lambda H_{3i})(D_\nu H_4^{\dagger j}),$$

$$\mathcal{K}^{(pm)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 1 \\ \frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

P-basis (“P” for permutation)

F-basis (“F” for Flavor)

$$\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,1} = \frac{1}{2}\mathcal{Y}\left[\boxed{p\,|\,r}\right] W_{{\rm L}p}{}^{I}{}_{\nu\mu} W_{{\rm L}r}{}^{I\mu\nu}(D_\lambda H_{si})(D^\lambda H_t^{\dagger i}),$$

$$\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,2} = \frac{1}{2}\mathcal{Y}\left[\boxed{p\,|\,r}\right] \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}p}{}^{I}{}_{\mu}{}^{\nu} W_{{\rm L}r}{}^{J\mu\lambda}(D_\lambda H_{si})(D_\nu H_t^{\dagger j}),$$

$$\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,3} = \frac{1}{2}\mathcal{Y}\left[\boxed{\begin{smallmatrix}p\\r\end{smallmatrix}}\right] W_{{\rm L}p}{}^{I}{}_{\mu}{}^{\nu} W_{{\rm L}r}{}^{I\mu\lambda}(D_\lambda H_{si})(D_\nu H_t^{\dagger i}),$$

$$\mathcal{O}^{(p)}_{W_{\rm L}^2 H H^\dagger D^2,4} = \frac{1}{2}\mathcal{Y}\left[\boxed{\begin{smallmatrix}p\\r\end{smallmatrix}}\right] \epsilon^{IJK}(\tau^K)^i_j W_{{\rm L}p}{}^{I}{}_{\nu\mu} W_{{\rm L}r}{}^{J\mu\nu}(D_\lambda H_{si})(D^\lambda H_t^{\dagger j}),$$

02

Off-shell Green’s Basis

Motivation

(1) RGE calculation.

We usually consider 1PI diagrams and match off-shell to calculate RGE. Thus an off-shell operator basis is needed, known as the Green's basis.

(2) Operator reduction.

It is much easier to reduce operators to the Green's basis than to the physical basis once the conversion between the Green's basis and physical basis is known.

Background

Motivation

(1) RGE calculation.

We usually consider 1PI diagrams and match off-shell to calculate RGE. Thus an off-shell operator basis is needed, known as the Green's basis.

(2) Operator reduction.

It is much easier to reduce operators to the Green's basis than to the physical basis once the conversion between the Green's basis and physical basis is known.

SMEFT Green's basis

Dim 6 [V. Gherardi, D. Marzocca, E. Venturini, 2020]

Dim 7 [X.-X. Li, Z. Ren, J.-H. Yu, 2023], [D.-Zhang, 2023]

Dim 8 bosonic [M. Chala, Á. Díaz-Carmona, G. Guedes, 2021]

Dim 8 [Z. Ren, J.-H. Yu, 2022]

X^3	
$\mathcal{O}_{3G}$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{3W}$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
X^2D^2	
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$

X^2H^2	
$\mathcal{O}_{HG}$	$G^A_{\mu\nu} G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu} G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu} W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu} W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HB}$	$B_{\mu\nu} B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu} B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu} B^{\mu\nu}(H^\dagger \sigma^I H)$
$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu} B^{\mu\nu}(H^\dagger \sigma^I H)$
H^2XD^2	
$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i \overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i \overleftrightarrow{D}_\mu H)$

H^2D^4	
$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger (D_\nu D^\nu H)$
H^4D^2	
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger (H^\dagger D_\mu H)$
$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger (D^\mu H)$
$\mathcal{O}''_{HD}$	$(H^\dagger H) D_\mu (H^\dagger i \overleftrightarrow{D}^\mu H)$
H^6	
$\mathcal{O}_H$	$(H^\dagger H)^3$

off-shell ⟹ $D^2\phi,\ \not{D}\psi,\ D_\mu F^{\mu\nu}$ are kept.

Off-shell Amplitude Formalism

$$
\begin{array}{ccc}
F_{\mathrm{L/R}\,i} & \Leftrightarrow & \lambda_i^2/\tilde{\lambda}_i^2 \\
\psi_i/\psi_i^\dagger & \Leftrightarrow & \lambda_i/\tilde{\lambda}_i \\
\phi_i & \Leftrightarrow & 1 \\
D_i & \Leftrightarrow & -i\lambda_i\tilde{\lambda}_i
\end{array}
$$

the subspace where EOM and CDC are removed ⟺ (linear isomorphism) on-shell amplitude space

Off-shell Amplitude Formalism

$$\begin{array}{ccc} F_{\mathrm{L/R}\,i} & \Leftrightarrow & \lambda_i^2/\tilde{\lambda}_i^2 \\ \psi_i/\psi_i^\dagger & \Leftrightarrow & \lambda_i/\tilde{\lambda}_i \\ \phi_i & \Leftrightarrow & 1 \\ D_i & \Leftrightarrow & -i\lambda_i\tilde{\lambda}_i \end{array}$$

$$\begin{array}{ccc} F_{\mathrm{L/R}\,i} & \Leftrightarrow & \lambda_{i,0}^2/\tilde{\lambda}_{i,0}^2 \\ \psi_i/\psi_i^\dagger & \Leftrightarrow & \lambda_{i,0}/\tilde{\lambda}_{i,0} \\ \phi_i & \Leftrightarrow & 1 \\ D_{i,d_i} & \Leftrightarrow & -i\lambda_{i,d_i}\tilde{\lambda}_{i,d_i} \end{array}$$

[Z. Ren, J.-H. Yu, 2022]

the subspace where EOM and CDC are removed ⟺ (linear isomorphism) on-shell amplitude space

operator space ⟺ (linear isomorphism) off-shell amplitude space

Off-shell Amplitude Formalism

$$\begin{array}{ccc} F_{\mathrm{L/R}\,i} & \Leftrightarrow & \lambda_i^2/\tilde{\lambda}_i^2 \\ \psi_i/\psi_i^\dagger & \Leftrightarrow & \lambda_i/\tilde{\lambda}_i \\ \phi_i & \Leftrightarrow & 1 \\ D_i & \Leftrightarrow & -i\lambda_i\tilde{\lambda}_i \end{array}$$

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[Z. Ren, J.-H. Yu, 2022]

the subspace where EOM and CDC are removed ⟺ (linear isomorphism) on-shell amplitude space

operator space ⟺ (linear isomorphism) off-shell amplitude space

$$\psi_1^\alpha(\sigma^\nu)_{\alpha\dot{\alpha}}D^\mu D_\mu D_\nu \psi_2^{\dagger\dot{\alpha}} = \frac{1}{2}\psi_1^\alpha D^{\beta\dot{\beta}}D_{\beta\dot{\beta}}D_{\alpha\dot{\alpha}}\psi_2^{\dagger\dot{\alpha}} \Leftrightarrow -\frac{i}{2}\langle 1_0 2_1\rangle\langle 2_3 2_2\rangle[2_1 2_0][2_3 2_2] \xrightarrow{\text{on-shell}} -\frac{i}{2}\langle 12\rangle\langle 22\rangle[22][22] = 0$$

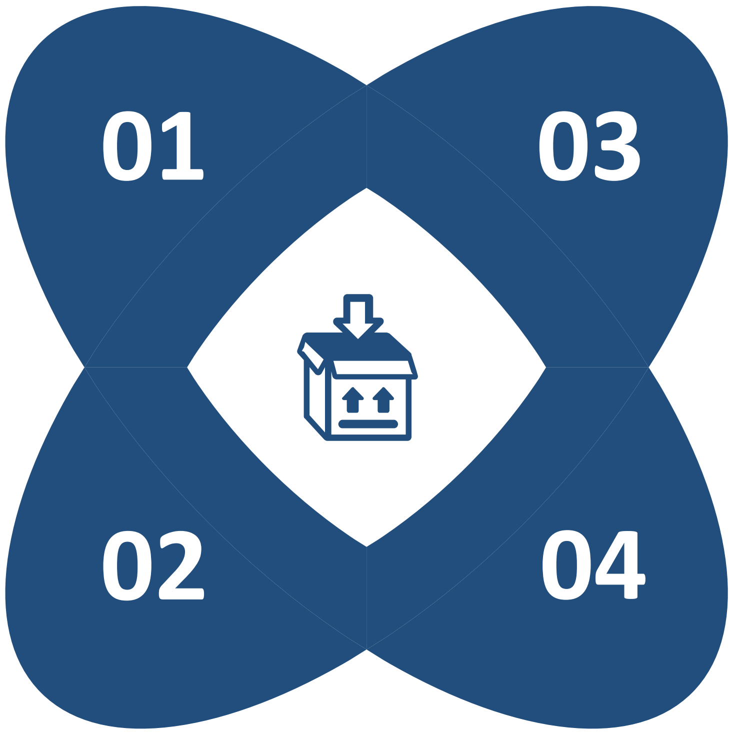
Redundancies

IBP

$$|i_{\hat{d}_i}\rangle[i_{\hat{d}_i}| = -\sum_{j=1,j\neq i}^{N} |j_{\hat{d}_j+1}\rangle[j_{\hat{d}_j+1}|$$

CDC

Use $\mathcal{Y}[\boxed{i_1}\boxed{i_2}\cdots\boxed{i_{\hat{d}_i}}]$ to symmetrize covariant derivatives



Schouten identity

$$\langle i_{x_i} l_{x_l}\rangle\langle j_{x_j} k_{x_k}\rangle \quad i_{x_i} < j_{x_j} < k_{x_k} < l_{x_l}$$

$$= -\langle i_{x_i} j_{x_j}\rangle\langle k_{x_k} l_{x_l}\rangle + \langle i_{x_i} k_{x_k}\rangle\langle j_{x_j} l_{x_l}\rangle$$

Bianchi identity

$$\epsilon^{\mu\nu\rho\sigma} D_\nu F_{\rho\sigma} = 0$$

$$\Downarrow$$

$$D_\mu F_{\mathrm{L}}^{\mu\nu} - D_\mu F_{\mathrm{R}}^{\mu\nu} = -iD_\mu \tilde{F}^{\mu\nu} = 0$$

Choose F_R to be on shell

Off-shell Construction

$$F_{L1}\psi_2\psi_3\phi_4 D^2$$

IBP

$$\lambda_{1,0}^2\lambda_{2,0}\lambda_{3,0} \times \begin{matrix} \lambda_{2,1}\lambda_{2,2}\tilde\lambda_{2,1}\tilde\lambda_{2,2}, & \lambda_{3,1}\lambda_{3,2}\tilde\lambda_{3,1}\tilde\lambda_{3,2}, & \lambda_{4,1}\lambda_{4,2}\tilde\lambda_{4,1}\tilde\lambda_{4,2}, \\ \lambda_{2,1}\lambda_{3,1}\tilde\lambda_{2,1}\tilde\lambda_{3,1}, & \lambda_{2,1}\lambda_{4,1}\tilde\lambda_{2,1}\tilde\lambda_{4,1}, & \lambda_{3,1}\lambda_{4,1}\tilde\lambda_{3,1}\tilde\lambda_{4,1}. \end{matrix}$$

$$\begin{matrix} \langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 2_2 3_0\rangle[2_1 2_2], & \langle 1_0 2_0\rangle\langle 1_0 2_2\rangle\langle 2_1 3_0\rangle[2_1 2_2], \\ \langle 1_0 2_0\rangle\langle 1_0 3_0\rangle\langle 2_1 2_2\rangle[2_1 2_2], & \langle 1_0 2_1\rangle\langle 1_0 2_2\rangle\langle 2_0 3_0\rangle[2_1 2_2], \\ \langle 1_0 2_1\rangle\langle 1_0 3_0\rangle\langle 2_0 2_2\rangle[2_1 2_2], & \langle 1_0 2_2\rangle\langle 1_0 3_0\rangle\langle 2_0 2_1\rangle[2_1 2_2]. \end{matrix}$$

Schouten identity

$$\langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 2_2 3_0\rangle[2_1 2_2], \quad \langle 1_0 2_0\rangle\langle 1_0 2_2\rangle\langle 2_1 3_0\rangle[2_1 2_2]$$

$$\langle 1_0 2_1\rangle\langle 1_0 2_2\rangle\langle 2_0 3_0\rangle[2_1 2_2].$$

$$\langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 2_2 3_0\rangle[2_1 2_2] \overset{\text{exchange } 2_1, 2_2}{\Longleftrightarrow} -\langle 1_0 2_0\rangle\langle 1_0 2_2\rangle\langle 2_1 3_0\rangle[2_1 2_2],$$

$$\langle 1_0 2_1\rangle\langle 1_0 2_2\rangle\langle 2_0 3_0\rangle[2_1 2_2] \overset{\text{exchange } 2_1, 2_2}{\Longleftrightarrow} -\langle 1_0 2_1\rangle\langle 1_0 2_2\rangle\langle 2_0 3_0\rangle[2_1 2_2].$$

CDC

$$\mathcal{Y}[\boxed{2_1}\boxed{2_2}]\begin{pmatrix} \langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 2_2 3_0\rangle[2_1 2_2] \\ \langle 1_0 2_0\rangle\langle 1_0 2_2\rangle\langle 2_1 3_0\rangle[2_1 2_2] \\ \langle 1_0 2_1\rangle\langle 1_0 2_2\rangle\langle 2_0 3_0\rangle[2_1 2_2] \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 2_2 3_0\rangle[2_1 2_2] \\ \langle 1_0 2_0\rangle\langle 1_0 2_2\rangle\langle 2_1 3_0\rangle[2_1 2_2] \\ \langle 1_0 2_1\rangle\langle 1_0 2_2\rangle\langle 2_0 3_0\rangle[2_1 2_2] \end{pmatrix}.$$

$$\begin{aligned}
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,1} &= \mathcal{Y}[\boxed{2_1}\boxed{2_2}]\,\langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 2_2 3_0\rangle[2_1 2_2], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,2} &= \mathcal{Y}[\boxed{3_1}\boxed{3_2}]\,\langle 1_0 2_0\rangle\langle 1_0 3_0\rangle\langle 3_1 3_2\rangle[3_1 3_2], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,3} &= \mathcal{Y}[\boxed{4_1}\boxed{4_2}]\,\langle 1_0 2_0\rangle\langle 1_0 3_0\rangle\langle 4_1 4_2\rangle[4_1 4_2], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,4} &= \langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 3_0 3_1\rangle[2_1 3_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,5} &= \langle 1_0 2_0\rangle\langle 1_0 3_0\rangle\langle 2_1 3_1\rangle[2_1 3_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,6} &= \langle 1_0 2_1\rangle\langle 1_0 3_0\rangle\langle 2_0 3_1\rangle[2_1 3_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,7} &= \langle 1_0 2_0\rangle\langle 1_0 2_1\rangle\langle 3_0 4_1\rangle[2_1 4_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,8} &= \langle 1_0 2_0\rangle\langle 1_0 3_0\rangle\langle 2_1 4_1\rangle[2_1 4_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,9} &= \langle 1_0 2_1\rangle\langle 1_0 3_0\rangle\langle 2_0 4_1\rangle[2_1 4_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,10} &= \langle 1_0 2_0\rangle\langle 1_0 3_0\rangle\langle 3_1 4_1\rangle[3_1 4_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,11} &= \langle 1_0 2_0\rangle\langle 1_0 3_1\rangle\langle 3_0 4_1\rangle[3_1 4_1], \\
\mathcal{M}^{(\text{off-shell})\,(y)}_{F_L\psi^2\phi D^2,12} &= \langle 1_0 3_0\rangle\langle 1_0 3_1\rangle\langle 2_0 4_1\rangle[3_1 4_1].
\end{aligned}$$

03

Basis Conversion

Bases in the Literature

$$\mathcal{O}_{\partial H} = \tfrac{1}{2} D_\mu (H^\dagger H) D^\mu (H^\dagger H),$$
$$\mathcal{O}_H^{(1)} = (H^\dagger H)(D_\mu H^\dagger D^\mu H),$$
$$\mathcal{O}_H^{(2)} = (H^\dagger D^\mu H)(D_\mu H^\dagger H).$$

[W. Buchmuller and D. Wyler, 1986]

$$\mathcal{O}_{H\Box} = \left(H^\dagger H\right) \Box \left(H^\dagger H\right),$$
$$\mathcal{O}_{HD} = \left(H^\dagger D_\mu H\right)^\dagger \left(H^\dagger D^\mu H\right).$$

[B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, 2010]

$$\mathcal{O}_{SILH}^{(1)} = D^\mu (H^\dagger H) D_\mu (H^\dagger H),$$
$$\mathcal{O}_{SILH}^{(2)} = (H^\dagger \overleftrightarrow{D}^\mu H)(H^\dagger \overleftrightarrow{D}_\mu H),$$

[G. F. Giudice, C. Grojean, A. Pomarol, R. Rattazzi, 2007]

reduce →

$$\begin{pmatrix} \mathcal{O}_{\partial H} \\ \mathcal{O}_H^{(1)} \\ \mathcal{O}_H^{(2)} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{H\Box} \\ \mathcal{O}_{HD} \end{pmatrix}$$

← reduce

$$\begin{pmatrix} \mathcal{O}_{SILH}^{(1)} \\ \mathcal{O}_{SILH}^{(2)} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{H\Box} \\ \mathcal{O}_{HD} \end{pmatrix}$$

Systematic Reduction

operator → on-shell amplitude → Y-basis → M-basis → P-basis → F-basis

- operator → on-shell amplitude: amplitude-operator correspondence
- on-shell amplitude → Y-basis: amplitude reduction
- Y-basis → M-basis: $(K^{(my)})^{-1}$
- M-basis → P-basis: $(K^{(pm)})^{-1}$
- P-basis → F-basis: select column

$$\langle i1\rangle[1j] = -\sum_{k=2}^{N}\langle ik\rangle[kj],$$

$$[1|p_2|i\rangle = -\sum_{k=3}^{N}[1|p_k|i\rangle, \quad \langle 1|p_2|i] = -\sum_{k=3}^{N}\langle 1|p_k|i],$$

$$[1|p_3|2\rangle = -\sum_{k=4}^{N}[1|p_k|2\rangle, \quad \langle 1|p_3|2] = -\sum_{k=4}^{N}\langle 1|p_k|2],$$

$$p_2\cdot p_3 = \sum_{\substack{i,j\neq 1\\ \{i,j\}\neq\{2,3\}}} -p_i\cdot p_j,$$

momentum conservation and Schouten identity

$$\langle il\rangle\langle jk\rangle = \langle ik\rangle\langle jl\rangle - \langle ij\rangle\langle kl\rangle,$$

$$[il][jk] = [ik][jl] - [ij][kl].$$

Systematic Reduction

operator → on-shell amplitude → Y-basis → M-basis → P-basis → F-basis

- operator → on-shell amplitude: amplitude-operator correspondence
- on-shell amplitude → Y-basis: amplitude reduction
- Y-basis → M-basis: $(K^{(my)})^{-1}$
- M-basis → P-basis: $(K^{(pm)})^{-1}$
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$$[1|p_3|2\rangle = -\sum_{k=4}^{N}[1|p_k|2\rangle, \quad \langle 1|p_3|2] = -\sum_{k=4}^{N}\langle 1|p_k|2],$$

$$p_2 \cdot p_3 = \sum_{\substack{i,j\neq 1 \\ \{i,j\}\neq\{2,3\}}} -p_i \cdot p_j,$$

momentum conservation and Schouten identity

$$\langle il\rangle\langle jk\rangle = \langle ik\rangle\langle jl\rangle - \langle ij\rangle\langle kl\rangle,$$

$$[il][jk] = [ik][jl] - [ij][kl].$$

$\mathcal{O}^{(1)}_{H^4}$	$(D_\mu H^{\dagger i} D_\nu H_i)(D^\nu H^{\dagger j} D^\mu H_j)$
$\mathcal{O}^{(2)}_{H^4}$	$(D_\mu H^{\dagger i} D_\nu H_i)(D^\mu H^{\dagger j} D^\nu H_j)$
$\mathcal{O}^{(3)}_{H^4}$	$(D^\mu H^{\dagger i} D_\mu H_i)(D^\nu H^{\dagger j} D_\nu H_j)$

$\mathcal{O}^{(p)}_{H^2H^{\dagger 2}D^2,1}$	$\frac{1}{4}\mathcal{Y}\left[\begin{array}{\|c\|c\|}\hline p & r \\ \hline\end{array}, \begin{array}{\|c\|c\|}\hline s & t \\ \hline\end{array}\right] H_{pi}H_{rj}(D_\mu D_\nu H^{\dagger i}_s)(D^\mu D^\nu H^{\dagger j}_t)$
$\mathcal{O}^{(p)}_{H^2H^{\dagger 2}D^2,2}$	$\frac{1}{4}\mathcal{Y}\left[\begin{array}{\|c\|c\|}\hline p & r \\ \hline\end{array}, \begin{array}{\|c\|c\|}\hline s & t \\ \hline\end{array}\right] H_{pi}H^{\dagger i}_s(D_\mu D_\nu H_{rj})(D^\mu D^\nu H^{\dagger j}_t)$
$\mathcal{O}^{(p)}_{H^2H^{\dagger 2}D^2,3}$	$\frac{1}{4}\mathcal{Y}\left[\begin{array}{\|c\|c\|}\hline p & r \\ \hline\end{array}, \begin{array}{\|c\|c\|}\hline s & t \\ \hline\end{array}\right] H_{pi}(D_\mu H_{rj})(D_\nu H^{\dagger i}_s)(D^\mu D^\nu H^{\dagger j}_t)$
$\mathcal{O}^{(p)}_{H^2H^{\dagger 2}D^2,4}$	$\frac{1}{4}\mathcal{Y}\left[\begin{array}{\|c\|}\hline p \\ \hline r \\ \hline\end{array}, \begin{array}{\|c\|}\hline s \\ \hline t \\ \hline\end{array}\right] H_{pi}H_{rj}(D_\mu D_\nu H^{\dagger i}_s)(D^\mu D^\nu H^{\dagger j}_t)$
$\mathcal{O}^{(p)}_{H^2H^{\dagger 2}D^2,5}$	$\frac{1}{4}\mathcal{Y}\left[\begin{array}{\|c\|}\hline p \\ \hline r \\ \hline\end{array}, \begin{array}{\|c\|}\hline s \\ \hline t \\ \hline\end{array}\right] H_{pi}H^{\dagger i}_s(D_\mu D_\nu H_{rj})(D^\mu D^\nu H^{\dagger j}_t)$
$\mathcal{O}^{(p)}_{H^2H^{\dagger 2}D^2,6}$	$\frac{1}{4}\mathcal{Y}\left[\begin{array}{\|c\|}\hline p \\ \hline r \\ \hline\end{array}, \begin{array}{\|c\|}\hline s \\ \hline t \\ \hline\end{array}\right] H_{pi}(D_\mu H_{rj})(D_\nu H^{\dagger i}_s)(D^\mu D^\nu H^{\dagger j}_t)$

$$C^{(p)} = C^{(m)}\left(\mathcal{K}^{(pm)}\right)^{-1} = \begin{pmatrix} 1 & 1 & 2 & 1 & 1 & 2 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Off-shell Amplitude Reduction

IBP

$$|1_{\hat{d}_1}\rangle[1_{\hat{d}_1}| \xRightarrow{\hat{d}_1\to\hat{d}_1-1} -\sum_{i=2}^{N}|i_{\hat{d}_i+1}\rangle[i_{\hat{d}_i+1}|$$

$$[1_{x_1}2_{\hat{d}_2}]\langle 2_{\hat{d}_2}i_{x_i}\rangle \xRightarrow{\hat{d}_2\to\hat{d}_2-1} \boxed{-[1_{x_1}1_{\hat{d}_1+1}]\langle 1_{\hat{d}_1+1}i_{x_i}\rangle} - \sum_{k=3}^{N}[1_{x_1}k_{\hat{d}_k+1}]\langle k_{\hat{d}_k+1}i_{x_i}\rangle,$$

$$\langle 1_{x_1}2_{\hat{d}_2}\rangle[2_{\hat{d}_2}i_{x_i}] \xRightarrow{\hat{d}_2\to\hat{d}_2-1} \boxed{-\langle 1_{x_1}1_{\hat{d}_1+1}\rangle[1_{\hat{d}_1+1}i_{x_i}]} - \sum_{k=3}^{N}\langle 1_{x_1}k_{\hat{d}_k+1}\rangle[k_{\hat{d}_k+1}i_{x_i}].$$

$$[1_{x_1}3_{\hat{d}_3}]\langle 3_{\hat{d}_3}2_{x_2}\rangle \xRightarrow{\hat{d}_3\to\hat{d}_3-1} \boxed{-[1_{x_1}1_{\hat{d}_1+1}]\langle 1_{\hat{d}_1+1}2_{x_2}\rangle}\boxed{-[1_{x_1}2_{\hat{d}_2+1}]\langle 2_{\hat{d}_2+1}2_{x_2}\rangle} - \sum_{k=4}^{N}[1_{x_1}k_{\hat{d}_k+1}]\langle k_{\hat{d}_k+1}2_{x_2}\rangle,$$

$$\langle 1_{x_1}3_{\hat{d}_3}\rangle[3_{\hat{d}_3}2_{x_2}] \xRightarrow{\hat{d}_3\to\hat{d}_3-1} \boxed{-\langle 1_{x_1}1_{\hat{d}_1+1}\rangle[1_{\hat{d}_1+1}2_{x_2}]}\boxed{-\langle 1_{x_1}2_{\hat{d}_2+1}\rangle[2_{\hat{d}_2+1}2_{x_2}]} - \sum_{k=4}^{N}\langle 1_{x_1}k_{\hat{d}_k+1}\rangle[k_{\hat{d}_k+1}2_{x_2}].$$

$$[3_{\hat{d}_3}2_{\hat{d}_2}]\langle 3_{\hat{d}_3}2_{\hat{d}_2}\rangle \xRightarrow[\hat{d}_3\to\hat{d}_3-1]{\hat{d}_2\to\hat{d}_2-1} \boxed{\frac{1}{2}[1_{\hat{d}_1+1}1_{\hat{d}_1+2}]\langle 1_{\hat{d}_1+1}1_{\hat{d}_1+2}\rangle}\boxed{-\frac{1}{2}\sum_{i=2}^{N}[i_{\hat{d}_i+1}i_{\hat{d}_i+2}]\langle i_{\hat{d}_i+1}i_{\hat{d}_i+2}\rangle} - \sum_{j=4,j>i}^{N}\sum_{i=2}^{N}[j_{\hat{d}_j+1}i_{\hat{d}_i+1}]\langle j_{\hat{d}_j+1}i_{\hat{d}_i+1}\rangle.$$

Schouten identity

$$\langle i_{x_i}l_{x_l}\rangle\langle j_{x_j}k_{x_k}\rangle \Rightarrow -\langle i_{x_i}j_{x_j}\rangle\langle k_{x_k}l_{x_l}\rangle + \langle i_{x_i}k_{x_k}\rangle\langle j_{x_j}l_{x_l}\rangle, \quad i<j<k<l.$$

$$\langle i_{x_i}k_{x_k}\rangle\langle j_{x_j}l_{x_l}\rangle \Rightarrow \langle i_{x_i}j_{x_j}\rangle\langle k_{x_k}l_{x_l}\rangle + \langle i_{x_i}l_{x_l}\rangle\langle j_{x_j}k_{x_k}\rangle,$$
$$\neg(i\neq j\neq k\neq l)\wedge(i_{x_i}<j_{x_j}<k_{x_k}<l_{x_l}).$$

Use IBP and Schouten identity to reduce off-shell amplitudes, collect EOM and CDC in the reduction.

[X.-X. Li, Z. Ren, J.-H. Yu, 2023]

Field Redefinition and EOM

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6$$

field redefinition

$$\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3$$

EOM

$$\partial^2\phi = -m^2\phi - \frac{1}{3!}\lambda\phi^3 + \mathcal{O}(\frac{1}{\Lambda^2})$$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6 \\ &\quad + \frac{c_1}{\Lambda^2}\phi^3\left(-\partial^2\phi - m^2\phi - \frac{1}{3!}\lambda\phi^3\right) + \mathcal{O}(\frac{1}{\Lambda^4}) \\ &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \left[\frac{1}{4!}\lambda + \frac{c_1}{\Lambda^2}m^2\right]\phi^4 + \left[\frac{c_6}{\Lambda^2} - \frac{c_1}{\Lambda^2}\frac{\lambda}{3!}\right]\phi^6 + \mathcal{O}(\frac{1}{\Lambda^4})\end{aligned}$$

Up to dim 6

Field Redefinition and EOM

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6$$

field redefinition

$$\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3$$

EOM

$$\begin{aligned}\partial^2\phi &= -m^2\phi - \frac{1}{3!}\lambda\phi^3 \\ &\quad + \frac{c_1}{\Lambda^2}\left(6\phi^2\partial^2\phi + 6\phi(\partial\phi)^2\right) + \frac{c_6}{\Lambda^2}6\phi^5\end{aligned}$$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6 \\ &\quad + \frac{c_1}{\Lambda^2}\phi^3\left(-\partial^2\phi - m^2\phi - \frac{\lambda}{3!}\phi^3\right) \\ &\quad + \frac{c_1^2}{\Lambda^4}\phi^3\left(6\phi^2\partial^2\phi + 6\phi(\partial\phi)^2\right) + \frac{c_1c_6}{\Lambda^4}6\phi^8 \\ &\quad + \frac{c_1^2}{\Lambda^4}\frac{1}{2}\phi^3\left(-3\phi^2\partial^2\phi - 6\phi(\partial\phi)^2 - m^2\phi^3 - \frac{\lambda}{2}\phi^5\right) + \mathcal{O}(\frac{1}{\Lambda^6})\end{aligned}$$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6 \\ &\quad + \frac{c_1}{\Lambda^2}\phi^3\left(-\partial^2\phi - m^2\phi - \frac{1}{3!}\lambda\phi^3\right) \\ &\quad + \frac{c_1^2}{\Lambda^4}\phi^3\left(6\phi^2\partial^2\phi + 6\phi(\partial\phi)^2\right) + \frac{c_1c_6}{\Lambda^4}6\phi^8\end{aligned}$$

Up to dim 8

Field Redefinition and EOM

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6$$

field redefinition

$$\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3$$

modified EOM

$$\frac{\partial \mathcal{L}^{(6)}}{\partial \phi}$$

$$\begin{aligned}\partial^2\phi &= -m^2\phi - \frac{1}{3!}\lambda\phi^3 \\ &+ \frac{c_1}{\Lambda^2}\left(6\phi^2\partial^2\phi + 6\phi(\partial\phi)^2\right) + \frac{c_6}{\Lambda^2}6\phi^5 \\ &+ \frac{c_1}{\Lambda^2}\frac{1}{2}\left(-3\phi^2\partial^2\phi - 6\phi(\partial\phi)^2 - m^2\phi^3 - \frac{\lambda}{2}\phi^5\right) + \mathcal{O}(\frac{1}{\Lambda^4})\end{aligned}$$

$$\frac{1}{2}\frac{\partial^2\mathcal{L}^{(4)}}{\partial\phi^2}\Delta\phi \qquad \Delta\phi = \frac{c_1}{\Lambda^2}\phi^3$$

Up to dim 8

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6 \\ &+ \frac{c_1}{\Lambda^2}\phi^3\left(-\partial^2\phi - m^2\phi - \frac{\lambda}{3!}\phi^3\right) \\ &+ \frac{c_1^2}{\Lambda^4}\phi^3\left(6\phi^2\partial^2\phi + 6\phi(\partial\phi)^2\right) + \frac{c_1 c_6}{\Lambda^4}6\phi^8 \\ &+ \frac{c_1^2}{\Lambda^4}\frac{1}{2}\phi^3\left(-3\phi^2\partial^2\phi - 6\phi(\partial\phi)^2 - m^2\phi^3 - \frac{\lambda}{2}\phi^5\right) + \mathcal{O}(\frac{1}{\Lambda^6})\end{aligned}$$

Conclusion

01. A systematic method to construct on-shell operator basis in general Lorentz-invariant EFTs at any mass dimension.

02. A systematic method to construct off-shell Green's basis in general Lorentz-invariant EFTs at any mass dimension.

03. A systematic method to reduce any operator to a certain on-shell operator basis.