Universidad de Granada

High Energy Theory

On-Shell matching in effective field theories

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HEFT|2024

Junta de Andalucía

Green's basis vs physical basis

EFT Lagrangian : $\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$

Valid operators

- Local operators
- Preserve the symmetries of the Lagrangian

Finite number of operators → Integration by parts, Fierz identities (4D), etc. → **Green's basis**

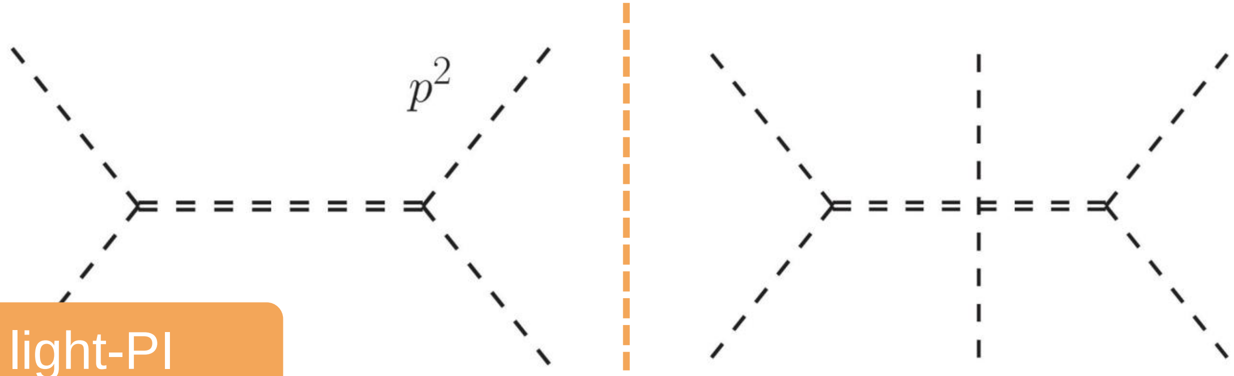
Some operators are still **redundant** on-shell → **Physical basis**

Matching theories: Off-Shell vs On-shell

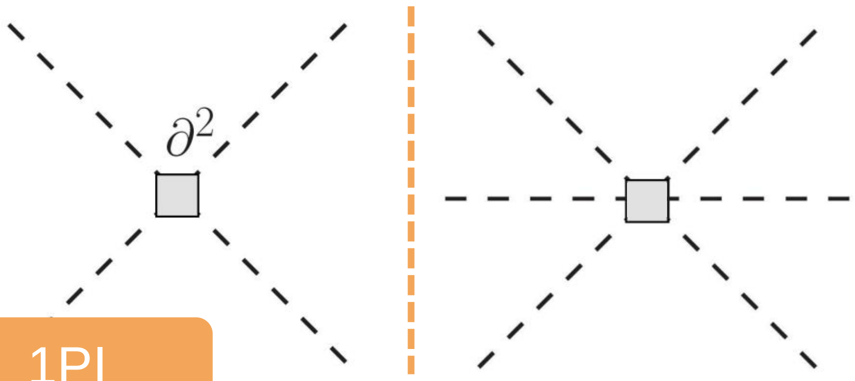
UV

IR

Off-shell

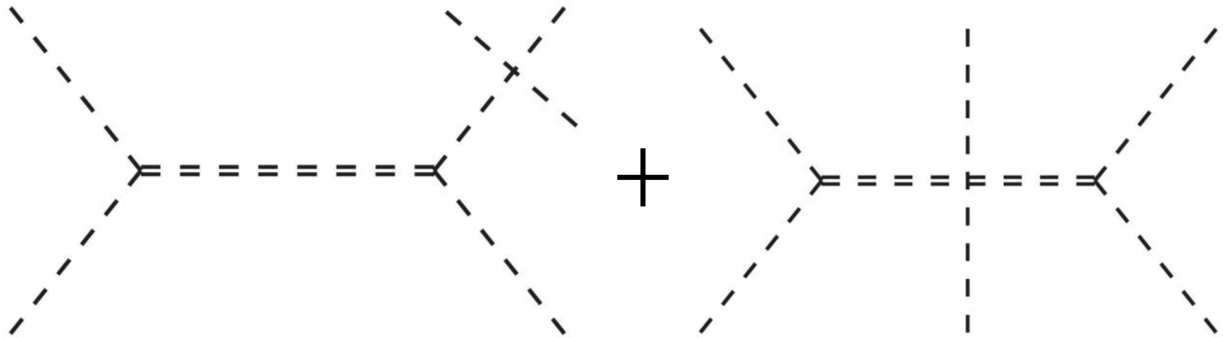


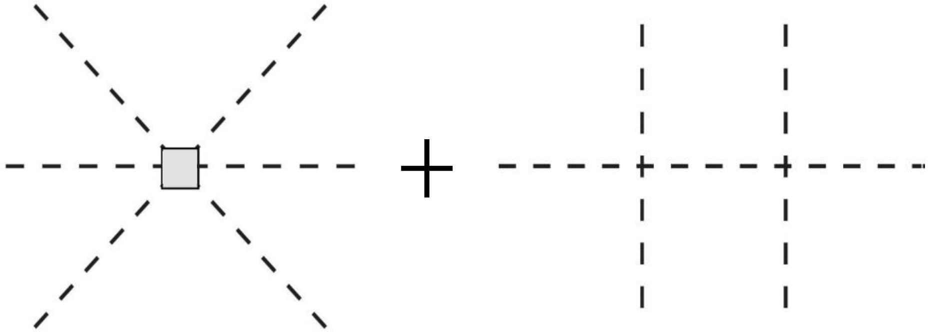
1 light-PI



1PI

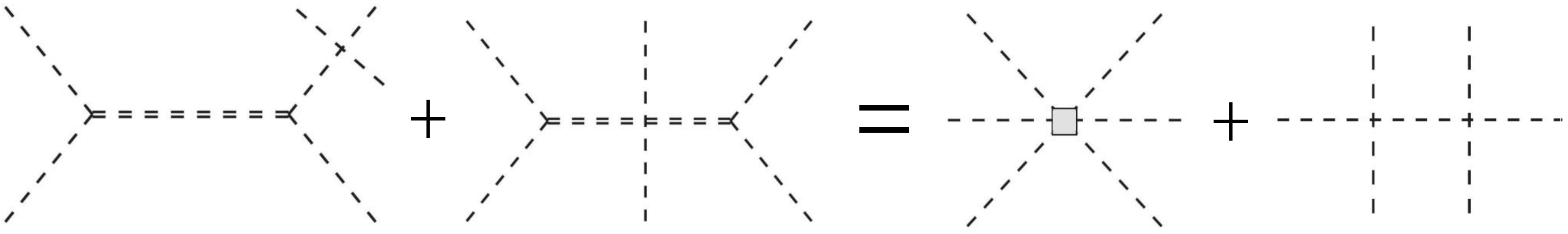
On-shell





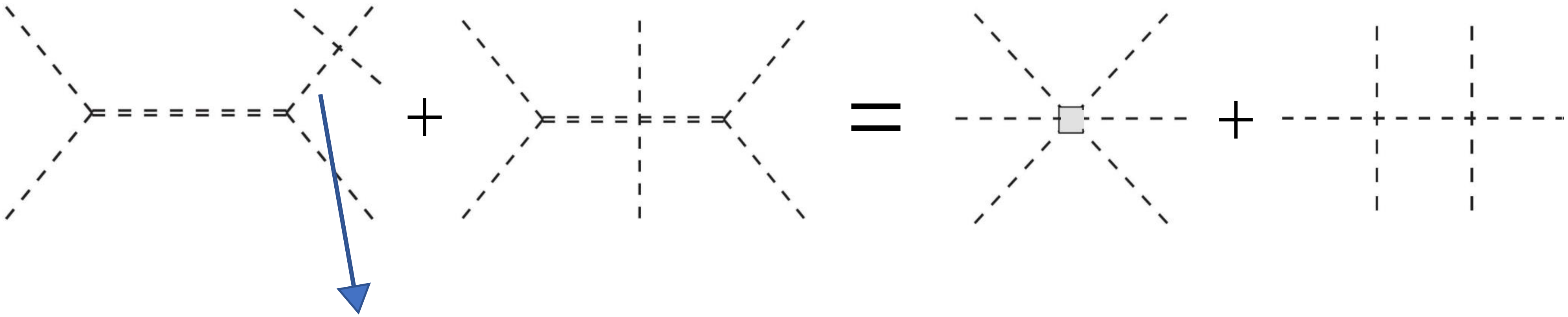
Matching theories: Off-Shell vs On-shell

On-Shell:



Off-Shell	On-Shell
Large number of operators (Green's basis)	Smaller set of operators (physical basis)
Small number of diagrams (1lPI in UV, 1PI in IR)	All diagrams (light bridges too)
Contribution directly local in momenta	Delicate cancelation of non-local contributions
Need of Background Field Method	
Need of evanescent operators	

Numerical on-shell matching



Cancel non-localities from light bridges by substitution of **numerical randomly-generated kinematics**

- The procedure is to be **numerical** but **exact** → Rational kinematics

Spinor Helicity Formalism [*arXiv:2304.01589, arXiv:2202.02681*]

$$p^\mu = \frac{1}{2}\langle p\, \sigma^\mu\, p]$$

$$\varepsilon^\mu = \frac{1}{\sqrt{2}} \frac{\langle r\, \sigma^\mu\, p]}{\langle r\, p\rangle}$$

$$|p\rangle = P_L u(p)$$
$$|p] = P_R u(p)$$

Applications: reduction of the Green's basis

Reduction of redundant operators in Green's basis via

- Field redefinitions
- EOMs (only valid up to linear order)

Non-trivial process

Hard to program it in a systematic way

- $\mathcal{L}_{Green}$ has to be **equivalent on-shell** to $\mathcal{L}_{phys}$.
- We can perform a **matching on-shell** between these theories and compute $c_i^{phys} = c_i^{phys}(c_j, r_k)$.

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

R. Fonseca [1907.12584]
J.C. Criado [1901.03501]

Some results in the SMEFT

BOSONIC SECTOR

Cross-check with [2003.12525v5]

$\boldsymbol{X^3}$		$\boldsymbol{X^2H^2}$		$\boldsymbol{H^2D^4}$	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G_{\mu\nu}^A G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{\widetilde{3G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}(H^\dagger H)$	$\boldsymbol{H^4D^2}$	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\square}$	$(H^\dagger H)\square(H^\dagger H)$
$\mathcal{O}_{\widetilde{3W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
$\boldsymbol{X^2D^2}$		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G_{\rho\nu}^A)$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W_{\rho\nu}^I)$	$\mathcal{O}_{HWB}$	$W_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$	$\boldsymbol{H^6}$	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		$\boldsymbol{H^2XD^2}$			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu^I H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

X^2H^2

$$c_{HB} \to c_{HB} - 2m_0^2 c_{HB} r_{DH}$$

$$c_{H\tilde{B}} \to c_{H\tilde{B}} - 2m_0^2 c_{H\tilde{B}} r_{DH}$$

H^4D^2

$$c_{H\square} \to c_{H\square} - \frac{1}{8}g'^2 r_{2B} + \frac{1}{2}g' r_{BDH} - m_0^2(4c_{H\square}r_{DH} + g' r_{BDH} r_{DH} + 2r_{DH} r'_{HD})$$

$$c_{HD} \to c_{HD} - \frac{1}{2}g'^2 r_{2B} + 2g' r_{BDH} - m_0^2(4c_{HD} r_{DH} + 4g' r_{BDH} r_{DH})$$

Reduction of dim. 6 operators up to dim. 8!!

H^6

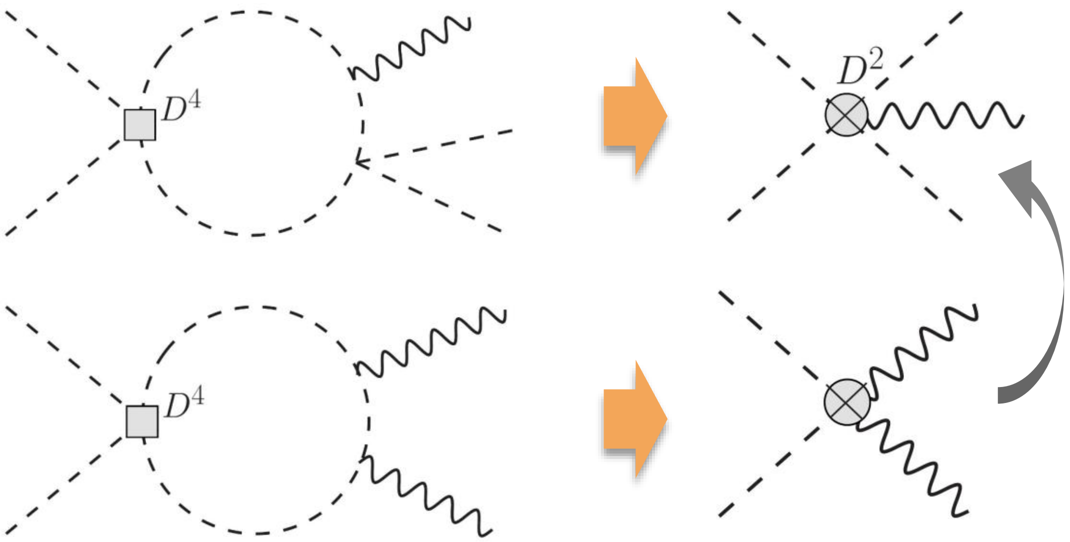
$$c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2\left(\frac{1}{4}g'^2 c_{HD} r_{2B} - \frac{1}{16}g'^4 r_{2B}^2 - \frac{1}{2}g' c_{HD} r_{BDH} + \frac{1}{2}g'^3 r_{2B} r_{BDH}\right.$$
$$- \frac{3}{4}g'^2 r_{BDH}^2 - 6c_H r_{DH} - \lambda c_{HD} r_{DH} + 8\lambda c_{H\square} r_{DH} + g'\lambda r_{BDH} r_{DH} - 11\lambda^2 r_{DH}^2$$
$$\left. -\frac{1}{2}c_{HD} r'_{HD} + 4c_{H\square} r'_{HD} + \frac{1}{2}g' r_{BDH} r'_{HD} - 9\lambda r_{DH} r'_{HD} - \frac{1}{4}r'^2_{HD} - r''^2_{HD}\right)$$

A theory expressed in terms of the physical basis can still generate redundant operators at the loop level through RGEs.

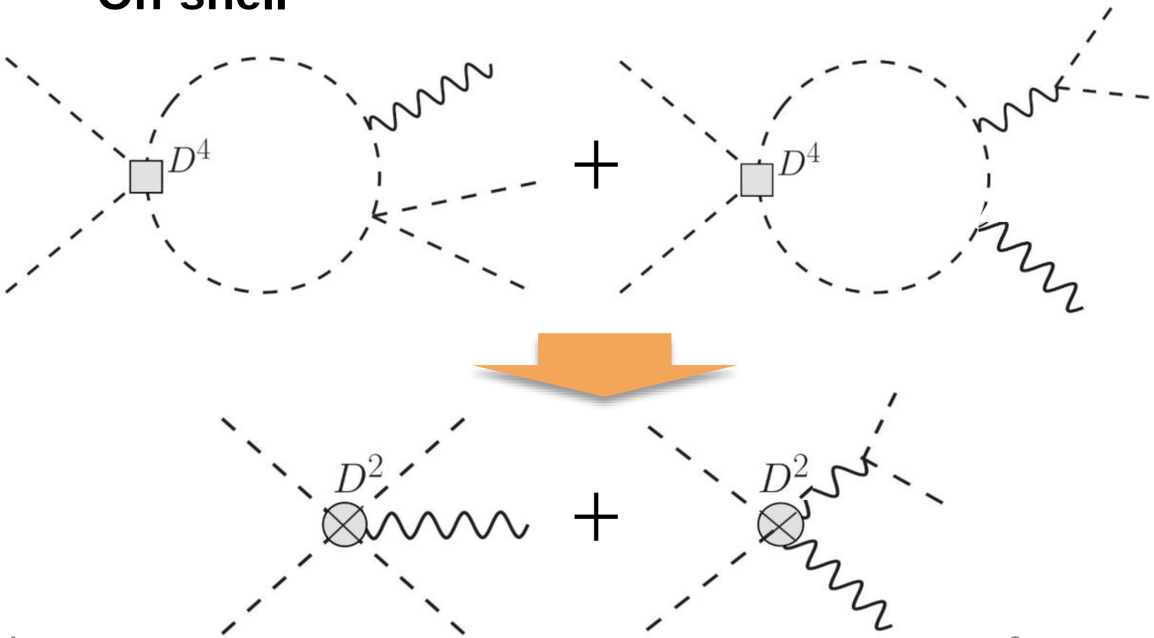
$$\mathcal{L}^{\text{div}} = \sum_j \frac{1}{16\pi^2\epsilon} c'_j(c)\mathcal{O}_j \quad \Rightarrow \quad 16\pi^2\mu\frac{\mathrm{d}c_i}{\mathrm{d}\mu} = -2c'_i \qquad \textit{(RGEs)}$$

Mixing of H^4D^4 into H^4BD^2

Off-shell



On-shell



Work in progress and future directions

- Finding **the reduction of any Green's basis** to any physical basis in a completely **automated way.**
  - ↳ Code in Mathematica to be released (based on FeynRules, FeynArts, FormCalc)

FIXED BUGS

- $(p_i \cdot p_j)^n$ structures
- $\epsilon^{\mu\nu\rho\sigma}\gamma_\mu$ structures
- Duplicate components in 'FFSSS'

- Finite matching and **evanescent operators**.
  - ↳ We do not need to add evanescent operators at 1 loop

- Operator-amplitude correspondence
  - ↳ Matching between amplitudes computed as in the amplitude formalism and computed from physical operators

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FTAE High Energy Theory

THANKS FOR YOUR ATTENTION !

Some results in the SMEFT

Dimension 4 up to dimension 8 (H and B)

$$m_0^2 \to m_0^2 - m_0^4 r_{DH} + 2m_0^6 r_{DH}^2$$

$$\lambda \to \lambda - m_0^2(4\lambda r_{DH} + 2r'_{HD}) + m_0^4(16\lambda r_{DH}^2 + 10 r_{DH} r'_{HD})$$

$$y_E \to y_E(1 - m_o^2 r_{DH} + \frac{5}{2} m_0^4 r_{DH}^2)$$

Some results in the SMEFT

Dimension 6 up to dimension 8 (H and B)

X^2H^2

$$c_{HB} \to c_{HB} - 2m_0^2 c_{HB} r_{DH}$$
$$c_{H\tilde{B}} \to c_{H\tilde{B}} - 2m_0^2 c_{H\tilde{B}} r_{DH}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8}g'^2 r_{2B} + \frac{1}{2}g' r_{BDH} - m_0^2(4c_{H\Box}r_{DH} + g' r_{BDH} r_{DH} + 2r_{DH}r'_{HD})$$
$$c_{HD} \to c_{HD} - \frac{1}{2}g'^2 r_{2B} + 2g' r_{BDH} - m_0^2(4c_{HD}r_{DH} + 4g' r_{BDH} r_{DH})$$

H^6

$$c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2 \left(\frac{1}{4}g'^2 c_{HD} r_{2B} - \frac{1}{16}g'^4 r_{2B}^2 - \frac{1}{2}g' c_{HD} r_{BDH} + \frac{1}{2}g'^3 r_{2B} r_{BDH} \right.$$
$$- \frac{3}{4}g'^2 r_{BDH}^2 - 6c_H r_{DH} - \lambda c_{HD} r_{DH} + 8\lambda c_{H\Box} r_{DH} + g'\lambda r_{BDH} r_{DH} - 11\lambda^2 r_{DH}^2$$
$$\left. - \frac{1}{2}c_{HD} r'_{HD} + 4c_{H\Box} r'_{HD} + \frac{1}{2}g' r_{BDH} r'_{HD} - 9\lambda r_{DH} r'_{HD} - \frac{1}{4}r'^2_{HD} - r''^2_{HD} \right)$$

Some results in the SMEFT

Dimension 8 (H and B)

XH^4D^2

$$c^{(1)}_{BH^4D^2} \to c^{(1)}_{BH^4D^2} - 4g' c_{HB} r_{2B} + \frac{1}{2} g'^3 r^2_{2B} + 8 c_{HB} r_{BDH} - 2 g'^2 r_{2B} r_{BDH} + 2 g' r^2_{BDH}$$

$$c^{(2)}_{BH^4D^2} \to -4g' c_{H\tilde{B}} r_{2B} + 8 c_{H\tilde{B}} r_{BDH}$$

$X^2H^2D^2$

$$c^{(1)}_{B^2H^2D^2} \to 0$$

$$c^{(2)}_{B^2H^2D^2} \to 0$$

$$c^{(3)}_{B^2H^2D^2} \to 0$$

X^4

$$c^{(1)}_{B^4} \to 0$$

$$c^{(2)}_{B^4} \to 0$$

$$c^{(3)}_{B^4} \to 0$$

X^2H^4

$$c^{(1)}_{B^2H^4} \to -c_{HB} g'^2 r_{2B} + \frac{1}{16} g'^4 r^2_{2B} + 2 c_{HB} g' r_{BDH} - \frac{1}{4} g'^3 r_{2B} r_{BDH} + \frac{1}{4} g'^2 r^2_{BDH} - 2 c_{HB} \lambda r_{DH} - c_{HB} r'_{HD}$$

$$c^{(2)}_{B^2H^4} \to -g'^2 c_{H\tilde{B}} r_{2B} + 2 g' c_{H\tilde{B}} r_{BDH} - 2\lambda c_{H\tilde{B}} r_{BDH} - c_{H\tilde{B}} r'_{HD}$$

Some results in the SMEFT

Dimension 8 (H and B)

H^4D^4

$$c^{(1)}_{H^4} \to \frac{1}{2}g'^2 r^2_{2B} - 2g' r_{2B} r_{BDH} + 2r^2_{BDH}$$

$$c^{(2)}_{H^4} \to -\frac{1}{2}g'^2 r^2_{2B} + 2g' r_{2B} r_{BDH} - 2r^2_{BDH}$$

$$c^{(3)}_{H^4} \to 0$$

H^6D^2

$$c^{(1)}_{H^6} \to -\frac{3}{4}g'^2 c_{HD} r_{2B} + \frac{3}{16}g'^4 r^2_{2B} + \frac{3}{2}g' c_{HD} r_{BDH} - \frac{3}{2}g'^3 r_{2B} r_{BDH} + \frac{9}{4}g'^2 r^2_{BDH} - \lambda c_{HD} r_{DH} - 8\lambda c_{H\Box} r_{DH} - 3g'\lambda r_{BDH} r_{DH} + \lambda^2 r^2_{DH} - \frac{1}{2}c_{HD} r'_{HD} - 4c_{H\Box} r'_{HD} - \frac{3}{2}g' r_{BDH} r'_{HD} - 3\lambda r_{DH} r'_{HD} - \frac{7}{4}r'^2_{HD} + r''^2_{HD}$$

$$c^{(2)}_{H^6} \to -\frac{1}{2}g'^2 c_{HD} r_{2B} + \frac{1}{8}g'^4 r^2_{2B} + g' c_{HD} r_{BDH} - g'^3 r_{2B} r_{BDH} + \frac{3}{2}g'^2 r^2_{BDH} - 2\lambda c_{HD} r_{DH} - 2g'\lambda r_{BDH} r_{DH} - c_{HD} r'_{HD} - g' r_{BDH} r'_{HD}$$

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Massless momenta : $P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ ➡ $P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$

Massive momenta : $P^\mu := q^\mu + \frac{m^2}{2q \cdot k} k^\mu$

$$\begin{aligned} q^2, k^2 &= 0 \\ q_{\alpha\dot{\alpha}} &= \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} &= \mu_\alpha \tilde{\mu}_{\dot{\alpha}} \end{aligned}$$

Evanescent operators

$\mathcal{R} = \alpha\, \mathcal{O}$ → $d = 4 - 2\epsilon$ → $\mathcal{R} = \alpha\mathcal{O} + \mathcal{E}$

$\mathcal{O}(\epsilon)$

Additional finite local contributions in loop amplitudes

$$\boldsymbol{IR^{(0)} + IR^{(1)} = UV^{(0)} + UV^{(1)}}$$

$$\boldsymbol{IR^{(0)} + IR^{(1)}_{soft} = UV^{(0)} + UV^{(1)}_{hard} + UV^{(1)}_{soft}}$$

We take the hard region

$$\int \mathcal{R} - \mathcal{O} = \frac{1}{\epsilon}(b_{\mathcal{R}}\epsilon - b_{\mathcal{O}}\epsilon) = b$$

$$\int \mathcal{O} = \frac{1}{\epsilon}(a + b_{\mathcal{O}}\epsilon)$$

$$\int \mathcal{R} = \frac{1}{\epsilon}(a + b_{\mathcal{R}}\epsilon)$$