Which Standard Model?

— the SM gauge group, SMEFT, and generalized symmetries

Ling-Xiao Xu

The Abdus Salam International Centre for Theoretical Physics

Based on hep-ph/2404.04229 in collaboration with **Hao-Lin Li**

Renaissance of global symmetries

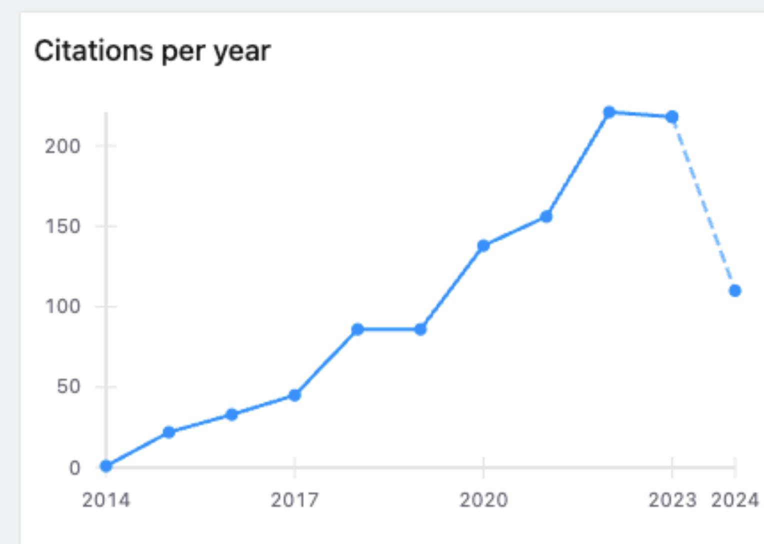
- Generalized symmetries: a mini revolution happened in the last 10 years in hep-th and condensed matter community.
- Global symmetries in QFT are defined as topological operators/defects. In this view, people found many generalizations.

Generalized Global Symmetries

Davide Gaiotto (Perimeter Inst. Theor. Phys.), Anton Kapustin (Stony Brook U.), Nathan Seiberg (Princeton, Inst. Advanced Study), Brian Willett (Princeton, Inst. Advanced Study)
Dec 16, 2014

77 pages
Published in: *JHEP* 02 (2015) 172
Published: Feb 26, 2015
e-Print: 1412.5148 [hep-th]
DOI: 10.1007/JHEP02(2015)172
View in: ADS Abstract Service, AMS MathSciNet

pdf cite claim reference search 1,116 citations

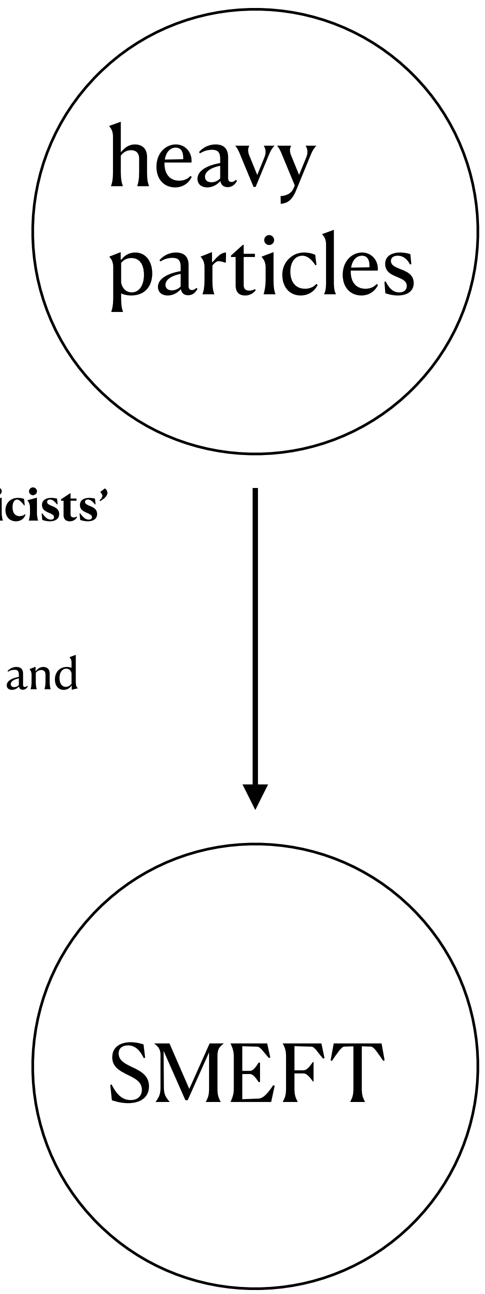


Renaissance of global symmetries

— a particle physicist's view

- Can generalized symmetries be used to solve open problems in particle physics?
- Are there implications of generalized symmetries in particle physics?

Two perspectives



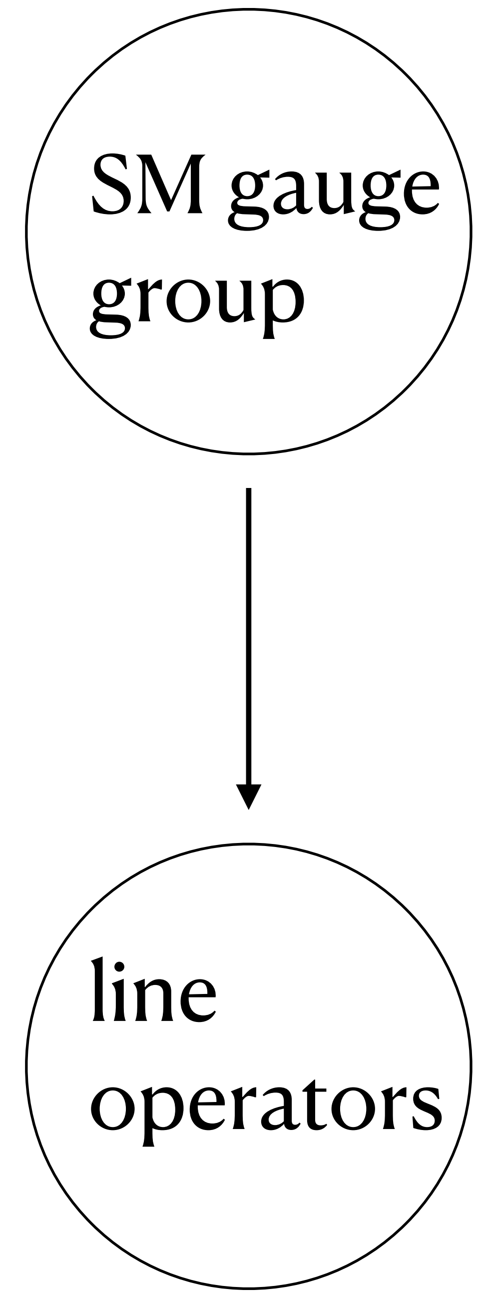
Particle physicists' viewpoint:

SM as an EFT and reductionism

Viewpoint of generalized symmetry:

Given the Lie algebra, there can be different Lie groups.

Gauge theories based on those Lie groups are considered as different theories, since they are distinguished by one-form symmetries acting on line operators.



heavy particles (with infinite mass) = line operators

Toy Model

Example: SU(2) versus SO(3) groups

- They are sometimes use interchangeably

Example: SU(2) versus SO(3) groups

- They are sometimes use interchangeably
- But we have to keep in mind they are not exactly the same, namely $SO(3) \sim \dfrac{SU(2)}{\mathbb{Z}_2}$, where $\mathbb{Z}_2 = (e^{i\pi}, e^{2\pi i} = 1)$ is the center

Example: SU(2) versus SO(3) groups

- They are sometimes use interchangeably

- But we have to keep in mind they are not exactly the same, namely $SO(3) \sim \dfrac{SU(2)}{\mathbb{Z}_2}$, where $\mathbb{Z}_2 = (e^{i\pi}, e^{2\pi i} = 1)$ is the center

- The consequence of the $\mathbb{Z}_2$ quotient:

  $SO(3)$ only has integer spin representations,

  $SU(2)$ can have both half-integer and integer spin representations

Example: SU(2) versus SO(3) groups

- They are sometimes use interchangeably
- But we have to keep in mind they are not exactly the same, namely $SO(3) \sim \frac{SU(2)}{\mathbb{Z}_2}$, where $\mathbb{Z}_2 = (e^{i\pi}, e^{2\pi i} = 1)$ is the center
- The consequence of the $\mathbb{Z}_2$ quotient:

  $SO(3)$ only has integer spin representations,

  $SU(2)$ can have both half-integer and integer spin representations
- In general, one can define $G \sim \frac{\tilde{G}}{H}$, where H is a subgroup of the center and all the allowed reps. are invariant under the H group

[Aharony, Seiberg, Tachikawa, 13]

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

- The gauge group appears to be $SU(2)$. But this is not quite true.

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

- The gauge group appears to be $SU(2)$. But this is not quite true.

- Instead, the gauge group can be either $SU(2)$ or $SO(3)$

- In fancier language, the gauge group $G = \dfrac{SU(2)}{\Gamma}$, where $\Gamma = 1,\ \mathbb{Z}_2$
  (The difference of the two theories can be rephrased in one-form symmetry.)

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

- The gauge group appears to be $SU(2)$. But this is not quite true.

- Instead, the gauge group can be either $SU(2)$ or $SO(3)$

- In fancier language, the gauge group $G = \dfrac{SU(2)}{\Gamma}$, where $\Gamma = 1,\ \mathbb{Z}_2$
(The difference of the two theories can be rephrased in one-form symmetry.)

- When it's $SO(3)$, since $\Gamma = \mathbb{Z}_2$ acts trivially in the full theory, this implies all the heavy particles have to be in the integer spin representations.

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.
- The gauge group appears to be $SU(2)$. But this is not quite true.
- Instead, the gauge group can be either $SU(2)$ or $SO(3)$
- In fancier language, the gauge group $G = \dfrac{SU(2)}{\Gamma}$, where $\Gamma = 1,\ \mathbb{Z}_2$
  (The difference of the two theories can be rephrased in one-form symmetry.)
- When it's $SO(3)$, since $\Gamma = \mathbb{Z}_2$ acts trivially in the full theory, this implies all the heavy particles have to be in the integer spin representations.
- Distinguishing $SU(2)$ vs. $SO(3)$ requires to discover at least one heavy particle in the half-integer spin representation.

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

- The gauge group appears to be $SU(2)$. But this is not quite true.

- Instead, the gauge group can be either $SU(2)$ or $SO(3)$

- In fancier language, the gauge group $G = \dfrac{SU(2)}{\Gamma}$, where $\Gamma = 1,\ \mathbb{Z}_2$
  (The difference of the two theories can be rephrased in one-form symmetry.)

- When it's $SO(3)$, since $\Gamma = \mathbb{Z}_2$ acts trivially in the full theory, this implies all the heavy particles have to be in the integer spin representations.

- Distinguishing $SU(2)$ vs. $SO(3)$ requires to discover at least one heavy particle in the half-integer spin representation.

- Coming back to low-energy, heavy particle can be described by high dim. operators in EFT.

The Standard Model

The Standard Model

- The matter content (+ gauge fields in the adjoints)

Table 29.1 Charges of Standard Model fields. □ indicates that the field transforms in the fundamental representation, and – indicates that a field is uncharged.

Field	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	ν_R	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	H
SU(3)	–	–	–	□	□	□	–
SU(2)	□	–	–	□	–	–	□
$U(1)_Y$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

[M. Schwartz QFT & SM textbook]

- The $\tilde{G} = SU(3)_c \times SU(2)_L \times U(1)_Y$ appears to be the gauge group, naively

- Nonetheless, much like the $SU(2)$ in the toy model, we are not sure this is the genuine gauge group. To find the genuine gauge group, we need to take a quotient to remove the trivial group elements.

Which Standard Model?

- The ambiguity comes from the following $\mathbb{Z}_6$ group acting trivially on all SM fields. (This is analogous to the $\mathbb{Z}_2$ center in the toy model.)

[… O'Raifeartaigh, 86; … Tong, 17; …]

$$\mathbb{Z}_6 = \{\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 = 1\}$$

$$\alpha = \left(e^{\frac{2\pi i}{3}}\mathbb{1}_{3\times 3},\ e^{\pi i}\mathbb{1}_{2\times 2},\ e^{\frac{2\pi i}{6}}\right)$$

Which Standard Model?

- The ambiguity comes from the following $\mathbb{Z}_6$ group acting trivially on all SM fields. (This is analogous to the $\mathbb{Z}_2$ center in the toy model.)

[… O'Raifeartaigh, 86; … Tong, 17; …]

$$\mathbb{Z}_6 = \{\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 = 1\}$$

$$\alpha = \left(e^{\frac{2\pi i}{3}}\mathbb{1}_{3\times 3},\ e^{\pi i}\mathbb{1}_{2\times 2},\ e^{\frac{2\pi i}{6}}\right)$$

- The generator α act on a rep. (R_3, R_2, Q_Y) as

$$U_\alpha(R_3, R_2, Q_Y) = e^{\frac{2\pi i}{3}\mathcal{N}(R_3)+i\pi\mathcal{N}(R_2)+\frac{2\pi i}{6}(6Q_Y)} = e^{2\pi i\left(\frac{\mathcal{N}(R_3)}{3}+\frac{\mathcal{N}(R_2)}{2}+Q_Y\right)}$$

Which Standard Model?

- The ambiguity comes from the following $\mathbb{Z}_6$ group acting trivially on all SM fields. (This is analogous to the $\mathbb{Z}_2$ center in the toy model.)

[… O'Raifeartaigh, 86; … Tong, 17; …]

$$\mathbb{Z}_6 = \{\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 = 1\}$$

$$\alpha = \left(e^{\frac{2\pi i}{3}}\mathbb{1}_{3\times 3},\ e^{\pi i}\mathbb{1}_{2\times 2},\ e^{\frac{2\pi i}{6}}\right)$$

- The generator α act on a rep. (R_3, R_2, Q_Y) as

$$U_\alpha(R_3, R_2, Q_Y) = e^{\frac{2\pi i}{3}\mathcal{N}(R_3) + i\pi\mathcal{N}(R_2) + \frac{2\pi i}{6}(6Q_Y)} = e^{2\pi i\left(\frac{\mathcal{N}(R_3)}{3} + \frac{\mathcal{N}(R_2)}{2} + Q_Y\right)}$$

- Hence the condition for the $\mathbb{Z}_6$ group acting trivially, i.e. $U_\alpha = 1$, is

$$\mathcal{N}(R_3) = 6Q_Y \bmod 3 \qquad \text{and} \qquad \mathcal{N}(R_2) = 6Q_Y \bmod 2$$

- All SM fields are invariant under the $\mathbb{Z}_6$ group (check it!)

Which Standard Model?

- There are ***<u>four</u>*** SM models, they differ by the global form of the gauge group (or one-form sym):

$$G = \frac{\tilde{G}}{\Gamma} = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{\Gamma} \qquad \Gamma = \mathbb{Z}_6,\ \mathbb{Z}_3,\ \mathbb{Z}_2,\ 1$$

Which Standard Model?

- There are ***four*** SM models, they differ by the global form of the gauge group (or one-form sym):

$$G = \frac{\tilde{G}}{\Gamma} = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{\Gamma} \qquad \Gamma = \mathbb{Z}_6,\ \mathbb{Z}_3,\ \mathbb{Z}_2,\ 1$$

- Here $\mathbb{Z}_2$ and $\mathbb{Z}_3$ are the two nontrivial subgroup of $\mathbb{Z}_6$, which are generated by α^3 and α^2, respectively. They acts trivially when

$\mathbb{Z}_2:$ $\mathcal{N}(R_2) = 6Q_Y \text{ mod } 2$ and R_3 unconstrained

$\mathbb{Z}_3:$ $\mathcal{N}(R_3) = 6Q_Y \text{ mod } 3$ and R_2 unconstrained

Which Standard Model?

- There are ***four*** SM models, they differ by the global form of the gauge group (or one-form sym):

$$G = \frac{\tilde{G}}{\Gamma} = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{\Gamma} \qquad \Gamma = \mathbb{Z}_6,\ \mathbb{Z}_3,\ \mathbb{Z}_2,\ 1$$

- Here $\mathbb{Z}_2$ and $\mathbb{Z}_3$ are the two nontrivial subgroup of $\mathbb{Z}_6$, which are generated by α^3 and α^2, respectively. They acts trivially when

$$\mathbb{Z}_2: \quad \mathcal{N}(R_2) = 6Q_Y \bmod 2 \quad \text{and} \quad R_3 \text{ unconstrained}$$

$$\mathbb{Z}_3: \quad \mathcal{N}(R_3) = 6Q_Y \bmod 3 \quad \text{and} \quad R_2 \text{ unconstrained}$$

- Different realizations of Γ have different constraints on the rep. of heavy particles! (This is in analogous to integer spin rep. vs. half-interger spin rep. in the toy model.)

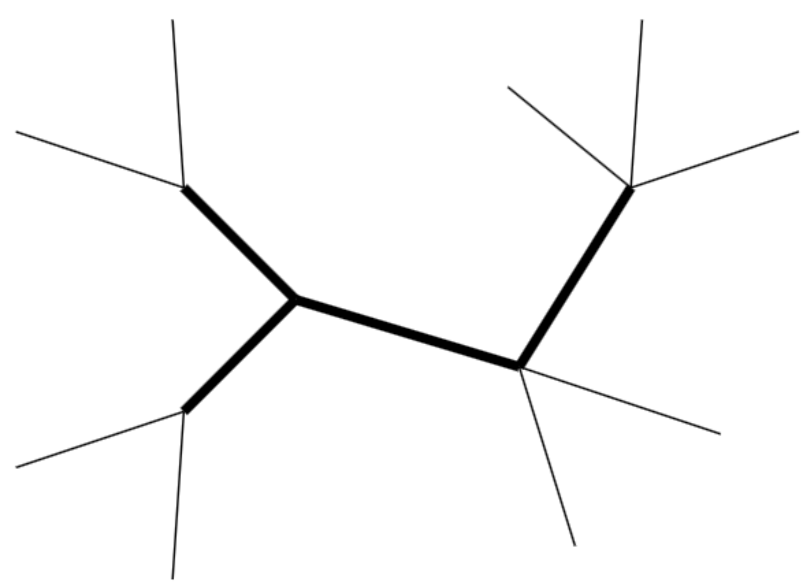
Heavy Particles & SMEFT

- Distinguishing them requires to discover new particles not invariant under $\mathbb{Z}_6$. (In the paper we call them "$\mathbb{Z}_6$ ***exotics***".) One can use SMEFT if they are heavy and have decoupling limit.

Heavy Particles & SMEFT

- Distinguishing them requires to discover new particles not invariant under $\mathbb{Z}_6$. (In the paper we call them "$\mathbb{Z}_6$ ***exotics***".) One can use SMEFT if they are heavy and have decoupling limit.

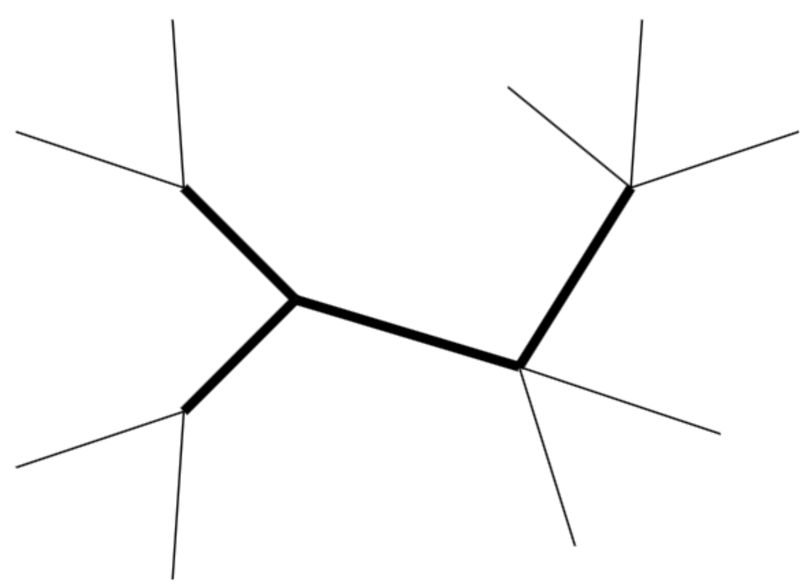
- No "$\mathbb{Z}_6$ ***exotics***" in tree-level UV completions. (The result is valid for operators of all mass dimensions.)



Heavy Particles & SMEFT

- Distinguishing them requires to discover new particles not invariant under $\mathbb{Z}_6$. (In the paper we call them "$\mathbb{Z}_6$ ***exotics***".) One can use SMEFT if they are heavy and have decoupling limit.

- No "$\mathbb{Z}_6$ ***exotics***" in tree-level UV completions. (The result is valid for operators of all mass dimensions.)



- Considering loop-level UV completion becomes **mandatory**!

Heavy Particles & SMEFT

- Example: adding one heavy complex scalar

$$\mathcal{L}_\phi \supset \ (D_\mu \phi^\dagger)(D^\mu \phi) - M^2 \phi^\dagger \phi - \lambda_{\mathbf{3}} (H^\dagger \sigma^I H)(\phi^\dagger T^I \phi) - \lambda_{\mathbf{1}} (H^\dagger H)(\phi^\dagger \phi)$$

One-loop matching (covariant derivative expansion) results:

$$c_{3G} = \frac{g_3^3}{(4\pi)^2 180 M^2}\ \mu(R_3)\ d(R_2)\ , \quad c_{3W} = \frac{g_2^3}{(4\pi)^2 180 M^2}\ \mu(R_2)\ d(R_3)\ ,$$

$$c_{HG} = \frac{g_3^2 \lambda_{\mathbf{1}}}{(4\pi)^2 12 M^2}\ \mu(R_3)\ d(R_2)\ , \quad c_{HW} = \frac{g_2^2 \lambda_{\mathbf{1}}}{(4\pi)^2 12 M^2}\ \mu(R_2)\ d(R_3)\ ,$$

$$c_{HB} = \frac{g_1^2 Y_\phi^2 \lambda_{\mathbf{1}}}{(4\pi)^2 12 M^2}\ d(R_2)\ d(R_3)\ , \quad c_{HWB} = \frac{g_1 g_2 Y_\phi \lambda_{\mathbf{3}}}{(4\pi)^2 6 M^2}\ \mu(R_2)\ d(R_3)\ ,$$

$$c_{H\Box} = -\frac{1}{(4\pi)^2 12 M^2}\left[d(R_2) d(R_3) \left(\lambda_{\mathbf{1}}^2 + \frac{g_1^4 Y_\phi^2}{20} \right) + \mu(R_2) d(R_3) \left(\frac{3 g_2^2}{80} - \lambda_{\mathbf{3}}^2 \right) \right],$$

$$c_{HD} = -\frac{1}{(4\pi)^2 3 M^2} \lambda_{\mathbf{3}}^2 \mu(R_2) d(R_3) - \frac{g_1^4}{(4\pi)^2 60 M^2} Y_\phi^2 d(R_3) d(R_2),$$

$$c_{ee} \equiv c_{ee}^{pppp} = -\frac{g_1^4 Y_\phi^2}{(4\pi)^2 60 M^2} d(R_2) d(R_3),$$

$$c_{ll} \equiv c_{ll}^{pppp} = -\frac{1}{(4\pi)^2 240 M^2}\left[g_2^4 \mu(R_2) d(R_3) + g_1^4 Y_\phi^2 d(R_3) d(R_2) \right],$$

Heavy Particles & SMEFT

Representation	Solution
$\phi\ (\cdot,\cdot,Y_\phi)$	$Y_\phi^2 = \frac{4c_{HB}^2}{5(4c_{H\Box}-c_{HD})c_{HD}}$
$\phi(R_3,\cdot,0)$	$\frac{\mu(R_3)}{d(R_3)} = -\frac{c_{HG}^2}{15g_3c_{H\Box}c_{3G}}$
$\phi(\cdot,R_2,0)$	$\frac{\mu(R_2)}{d(R_2)} = \frac{80c_{HW}^2c_{ll}}{225c_{3W}^2\left[g_2^2(4c_{H\Box}+c_{HD})-3c_{ll}\right]}$
$\phi\ (R_3,\cdot,Y_\phi)$	$Y_\phi^2 = \frac{4g_3^2c_{HG}^2c_{HD}}{45g_1^4c_{3G}^2(4c_{H\Box}-c_{HD})}$ $\quad \frac{\mu(R_3)}{d(R_3)} = \frac{4c_{HG}^3c_{HD}}{45g_1^2c_{3G}^2c_{HB}(4c_{H\Box}-c_{HD})}$
$\phi\ (\cdot,R_2,Y_\phi)$	$Y_\phi^2 = \frac{g_2c_{HWB}^2}{15g_1^2c_{3W}(c_{ee}-c_{HD})}$ $\quad \frac{\mu(R_2)}{d(R_2)} = -\frac{g_1^2c_{HWB}^2}{5g_2^2c_{ee}(c_{ee}-c_{HD})}$
$\phi\ (R_3,R_2,0)$	$\frac{\mu(R_3)}{d(R_3)} = -\frac{c_{HG}^2}{15g_3c_{3G}(c_{H\Box}-3c_{ll}/(4g_2^2)+c_{HD}/4)}$ $\frac{\mu(R_2)}{d(R_2)} = -\frac{c_{HW}^2}{15g_2c_{3W}(c_{H\Box}-3c_{ll}/(4g_2^2)+c_{HD}/4)}$

group theoretical data

measurable Wilson coeff.

- Step 1: find nonzero Wilson coefficients.
- Step 2: one needs loop-level dictionary to identify the rep. of heavy particles from Wilson coefficients.

- Q: What is the SM gauge group?
- A: We need to discover new heavy particles. There are four scenarios as follows:

  - All particles are invariant under $\mathbb{Z}_6$, Γ remains undetermined as in the SM. However, if this is the case it might be better to write $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y/\mathbb{Z}_6$.
  - At least one heavy particle is not invariant under $\mathbb{Z}_3$ but invariant under $\mathbb{Z}_2$ (hence not invariant under $\mathbb{Z}_6$), Γ can be either $\mathbb{Z}_2$ or 1.
  - At least one heavy particle is not invariant under $\mathbb{Z}_2$ but invariant under $\mathbb{Z}_3$ (hence not invariant under $\mathbb{Z}_6$), Γ can be either $\mathbb{Z}_3$ or 1.
  - At least one heavy particle is invariant under neither $\mathbb{Z}_2$ nor $\mathbb{Z}_3$ (hence not invariant under $\mathbb{Z}_6$), Γ is uniquely determined to be 1.

[See in the backup slides for examples]

Conclusion & outlook

- The global form of the SM gauge group is unknown, but we can potentially determine it by discovering heavy particles not invariant under $\mathbb{Z}_6$.
- If these particles are heavy enough, SMEFT is the ideal tool.
- In SMEFT, $\mathbb{Z}_6$ exotics cannot appear in tree-level UV completions, hence it becomes mandatory to study loop-level matching to identify these particles! Our result gives a strong motivation to carry on loop analysis in SMEFT.
- Scalars that can trigger EWSB cannot be $\mathbb{Z}_6$ exotics. Easy to prove in general, see in our paper or in the backup.
- Cosmological, astro-particle, and future collider studies are warranted. (We have a chance to bound the reheating temperature from above if any $\mathbb{Z}_6$ exotic particle is discovered.)

Backup slides

Examples of heavy particles & SM gauge group

- (R_3, R_2, Q_Y) = (fundamental, fundamental, 0) is allowed when $\Gamma = 1$ but forbidden when $\Gamma = \mathbb{Z}_{2,3,6}$
- (R_3, R_2, Q_Y) = (fundamental, fundamental, 2/3) is allowed when $\Gamma = 1$ or $\mathbb{Z}_3$, but forbidden when $\Gamma = \mathbb{Z}_2$ or $\mathbb{Z}_6$
- (R_3, R_2, Q_Y) = (fundamental, fundamental, 1/2) is allowed when $\Gamma = 1$ or $\mathbb{Z}_2$, but forbidden when $\Gamma = \mathbb{Z}_3$ or $\mathbb{Z}_6$
- Some well-known realistic examples include the original KSVZ fermions in axions models, fractionally-charged and milli-charged particles.

Electroweak symmetry breaking

- Q: What about the scalars that can trigger EWSB?
- A: They don't decouple and they are not $\mathbb{Z}_6$ exotics.
- Proof: 1) Since color is unbroken, the scalars must be neutral under $SU(3)_c$ (i.e. singlet rep. has N-ality zero). 2) In the notation of (j, Q_Y) the quantum numbers are subject to the following constraints to accommodate a electric neutral component:

$$-j \leq Q_Y \leq j \qquad \text{and} \qquad j + Q_Y \in \mathbb{Z}$$

- Q_Y is either integer or half-integer since j is, hence

$$0 = 6Q_Y \mod 3 \longrightarrow \text{invariant under } \mathbb{Z}_3$$

- Furthermore, let's compute $2j - 6Q_Y$

$$2(j - Q_Y) - 4Q_Y = 0 \mod 2 \longrightarrow \text{invariant under } \mathbb{Z}_2$$

- Invariance under both $\mathbb{Z}_{2,3}$ implies invariance under $\mathbb{Z}_6$

Higher-form symmetries

- Free Maxwell theory with no matter:
  the Gauss law is understood as electric $U(1)$ 1-form symmetry

- Pure $SU(N)$ gauge theory with no matter:
  the center of the gauge group measures the N-ality of a Wilson line, which is understood as electric Z_N 1-form symmetry

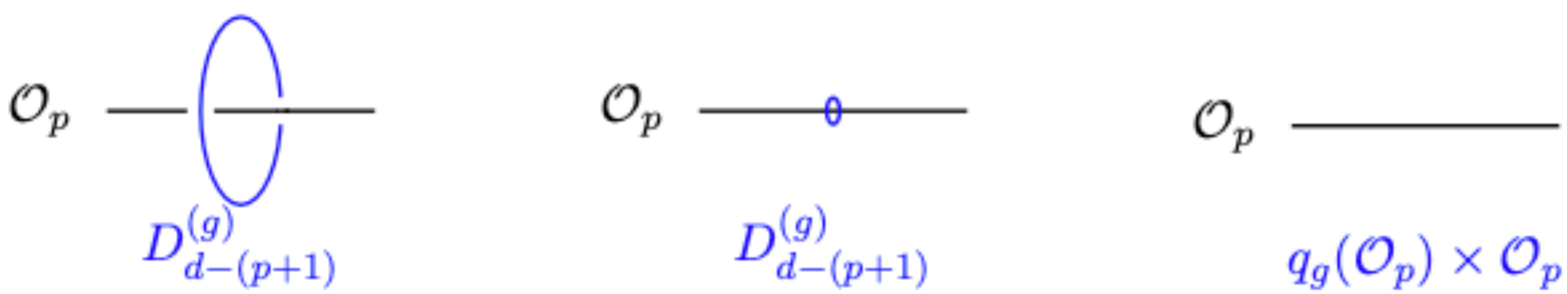
- Adding matter fields breaks the electric 1-form symmetry explicitly, i.e. Wilson lines can be screened/trivialized by particles.

- Nevertheless, the notions of electric 1-form symmetry and Wilson lines are still valid below the mass scale of the heavy particles that screen the Wilson lines. As such, the 1-form symmetry is viewed as accidental at low energy.

One-form symmetries and Line Operators

[ICTP lectures by Schafer-Nameki, 2023]

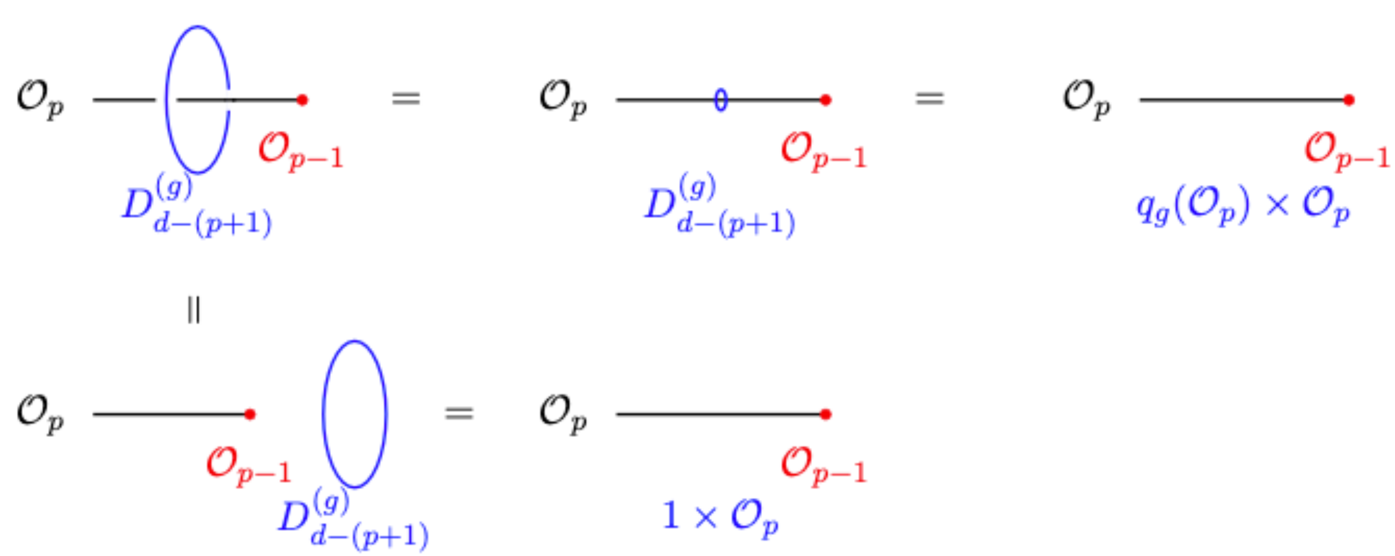
- A p-form global symmetry is generated by a dimension $(d-p-1)$ dimensional topological operator D_{d-p-1} acting on a p dimensional charged operator $\mathcal{O}_p$ as in the following:



(2.13)

- Higher form symmetries (i.e. $p > 1$) are abelian.

- Screening the charge: p-form symmetry can be screened (trivialized) by $p-1$ dimensional operators $\mathcal{O}_{p-1}$ which live at the end of $\mathcal{O}_p$



One-form symmetries and Line Operators

[ICTP lectures by Schafer-Nameki, 2023]

- One useful perspective is to think in terms of the equivalence relations between charged operators $\mathcal{O}_p$

$$\mathcal{O}_p^{(1)} \sim \mathcal{O}_p^{(2)} \quad \Leftrightarrow \quad \exists\ O_{p-1} \ \text{at the junction between } \mathcal{O}_p^{(1)} \text{ and } \mathcal{O}_p^{(2)} . \tag{2.28}$$

- Example: in a pure Yang-Mill theory with simply-connected gauge group G, Wilson lines of all possible charges under the center $\mathbb{Z}_G$ are allowed. Since the only local operators are in the adjoint which is not charged under center, all these $\mathbb{Z}_G$ charged Wilson lines are inequivalent and so the 1-form symmetry is the center. Also it's obvious that adding additional matter can trivialize some of the Wilson lines, hence breaking the 1-form symmetry to a subgroup.

- Taking the quotient Γ restricts the allowed Wilson lines, but it allows for more 't Hooft lines. There are different ways of adding the lines (called choices of "polarizations").

Centers for simply-connected groups

G	Z_G	$q(\boldsymbol{F})$
$SU(N)$	$\mathbb{Z}_N$	$1 \mod N$
$\mathrm{Spin}(4N)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1,1) \mod (2,2)$
$\mathrm{Spin}(4N+2)$	$\mathbb{Z}_4$	$2 \mod 4$
$\mathrm{Spin}(2N+1)$	$\mathbb{Z}_2$	$1 \mod 2$
E_6	$\mathbb{Z}_3$	$1 \mod 3$
E_7	$\mathbb{Z}_2$	$1 \mod 2$
E_8	$\mathbb{Z}_1$	$1 \mod 1$

Table 1: Simply-connected Lie groups G and their centers Z_G, as well as the charge of the fundamental representation $\boldsymbol{F}$ under the generator(s) of the center.

$$Z_G = \mathrm{Center}(G) = \{g \in G : \; gh = hg \text{ for all } h \in G\} \,. \qquad (2.18)$$

[ICTP lectures by Schafer-Nameki, 2023]