Jet Bundle Geometry of Scalar EFTs

Mohammad Alminawi (Speaker) - University of Zurich
Ilaria Brivio – University of Bologna
Joe Davighi – CERN

Higgs and Effective Field Theory (HEFT) 2024

Scalar Effective Field Theories

$$L = V + \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$$

Outline

- Motivation for geometric formalism
- Motivation for bundle formalism
- Introduction to bundles and jets
- Non-derivative field redefinitions as diffeomorphisms
- Amplitude calculations on 0-Jet bundle

Motivation for Geometric Formalism

- SMEFT and HEFT are the main way to extend the standard model

$$SM \subset SMEFT \subset HEFT$$

- Map from SMEFT to HEFT is well defined. Inverse is tricky.

- Exploit geometric techniques to identify when HEFT is needed.

[T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2008.08597]
[R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1602.00706]
[R. Gomez-Ambrosio et al., arXiv:2204.01763]

Motivation for Bundle Geometry

- Previous geometric formulations

[R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1605.03602]

$$L = \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V + O(\partial^4)$$

[A. Helset, A. Martin and M. Trott, arXiv:2001.01453]

- Using jet bundles

$$L = \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V + O(\partial^4)$$

[M. Alminawi, I. Brivio and J. Davighi , arXiv:2308.00017]

Motivation for Bundle Geometry

- Full Lagrangian obtained from geometry

$$L = \frac{1}{2}\langle \eta^{-1} , \; (j^n\phi)^* \, g \,\rangle$$

- Transformation rules of physical amplitudes indicate that they are combinations of momenta and tensors

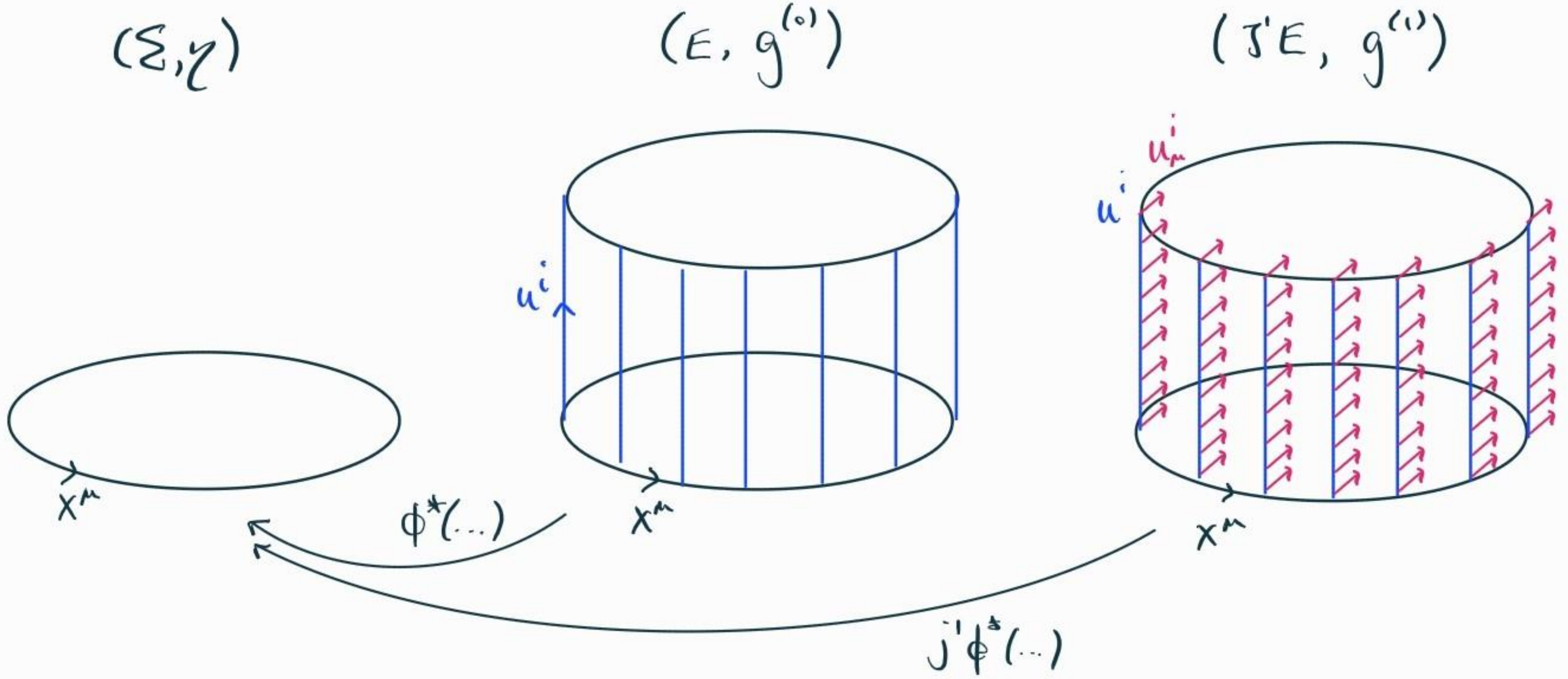
$$\overline{V}_{;(\alpha_1\alpha_2\alpha_3\alpha_4)} + \frac{2}{3}\left(s_{12}\overline{R}_{\alpha_1(\alpha_3\alpha_4)\alpha_2} + s_{13}\overline{R}_{\alpha_1(\alpha_2\alpha_4)\alpha_3} + s_{14}\overline{R}_{\alpha_1(\alpha_2\alpha_3)\alpha_4}\right)$$

[T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2108.03240]

- Only tensors that can be constructed from a metric with a torsion free connection are of the form $\nabla^n R^m$ where n, m are integers

[M. Alminawi, I. Brivio and J. Davighi , *in progress*]

What is a Bundle?



What is a Bundle?

- Consider two manifolds Σ and E with coordinate charts $\{x^\mu\}$ and $\{x^\mu, u^i\}$ and a map $\pi: \Sigma \to E$ then the triple (Σ, E, π) forms a bundle

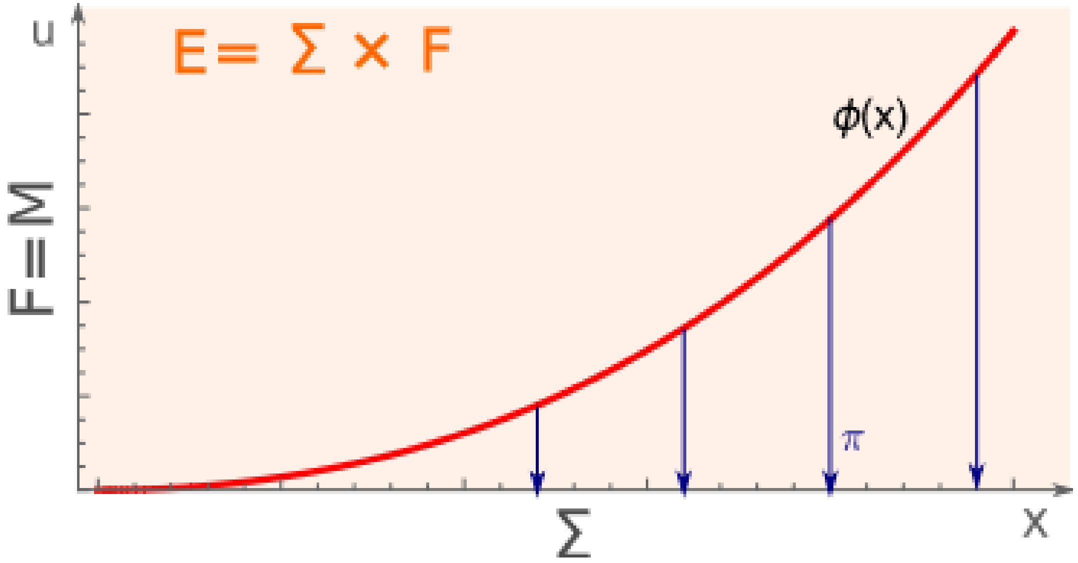
- Local inverses to the map π are called sections ϕ and they are defined by

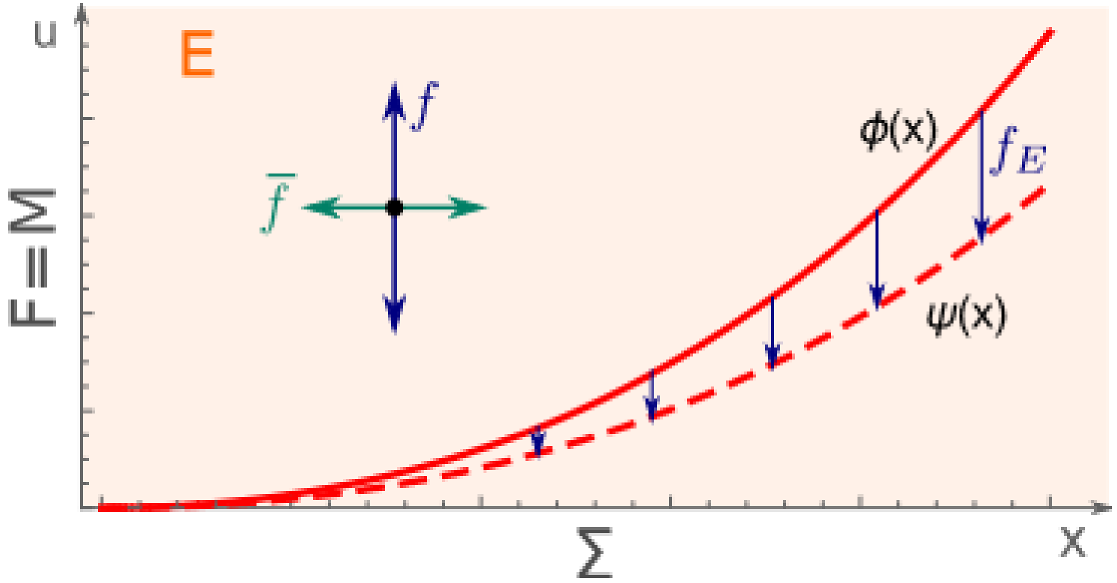
$$\phi \circ x^\mu = x^\mu$$
$$\phi \circ u^i = \phi^i$$

- Sections give us the tools to obtain fields and their derivatives from coordinates on bundles

D. J. Saunders, The Geometry of Jet Bundles, doi:10.1017/CBO9780511526411

What is a Bundle?





What is a Jet?

- Two sections ϕ, ψ are called 1-equivalent at some point $p \in E$ if we have

$$\phi(p) = \psi(p) \qquad \left.\frac{\partial(\phi \circ u^i)}{\partial x^\mu}\right|_p = \left.\frac{\partial(\psi \circ u^i)}{\partial x^\mu}\right|_p$$

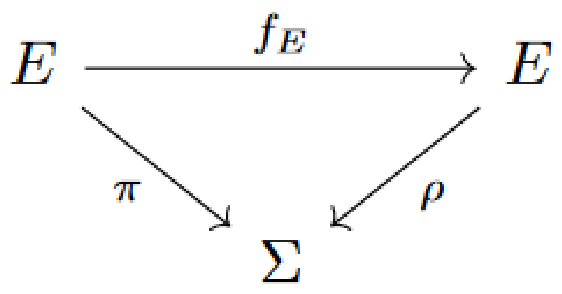
- The equivalence class containing ϕ at p is called the 1-jet and is denoted $j_p^1\phi$

- The set of all 1-jets is referred to as the 1-jet bundle and it naturally has the structure of a smooth manifold

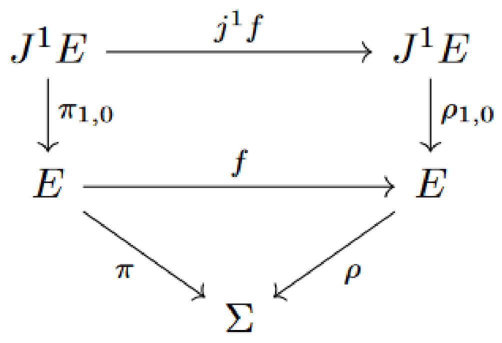
Field Redefinitions on Bundles

- A non-derivative field redefinition in the Lagrangian is equivalent to a diffeomorphism on the bundle

- Consider transformations that leave spacetime unchanged



Morphism on 0-Jet Bundle
Equivalent to $\psi = f_E \circ \phi$



Morphism on 1-Jet Bundle
Equivalent to $j^1\psi = j^1f \circ j^1\phi$

Diffeomorphism vs. Coordinate Transformation

- Tensors are coordinate independent, thus a coordinate transformation $x \rightarrow y(x)$ leaves the metric unchanged

$$g = g_{ij}(x)dx^i dx^j \rightarrow g'_{ab}(y(x))dy^a dy^b = g_{ij}(x)\frac{\partial x^i}{\partial y^a}\frac{\partial x^j}{\partial y^b}dy^a dy^b = g$$

- In contrast a diffeomorphism of the form $x \rightarrow y(x)$ transforms the metric as follows

$$g = g_{ij}(x)dx^i dx^j \rightarrow g_{ab}\big(y(x)\big)\frac{\partial y^a}{\partial x^i}\frac{\partial y^b}{\partial x^j}dx^i dx^j$$

- Where now $g_{ab}\big(y(x)\big) \neq g_{ij}(x)\frac{\partial x^i}{\partial y^a}\frac{\partial x^j}{\partial y^b}$

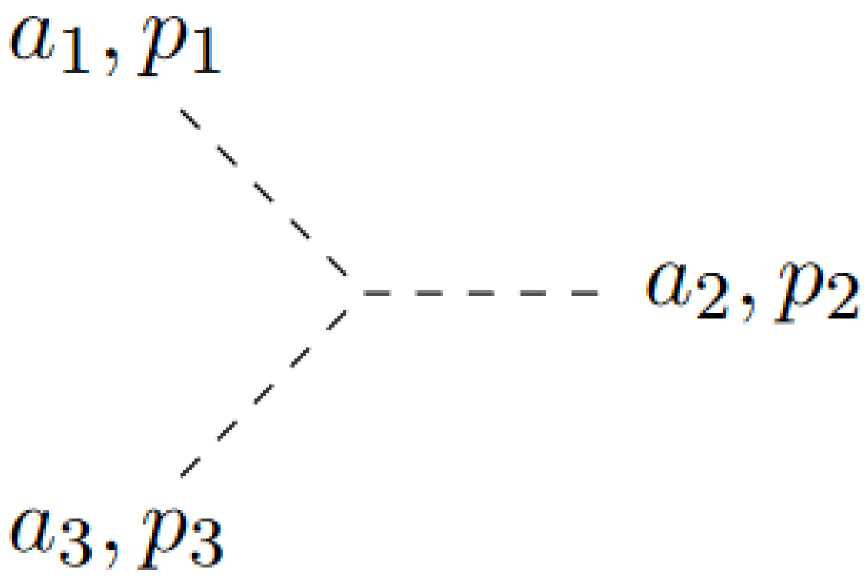
Amplitudes on 0-Jet

- Poincare invariance implies that our metric is block diagonal

$$\begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{ij} \end{pmatrix}$$

- Where $g_{\mu\nu} = -\frac{1}{2}\eta_{\mu\nu}V$ has dimensions determined by spacetime and g_{ij} has dimensions determined by the number of fields

Three Point Amplitude on 0-Jet Bundle



Three Point Amplitude on 0-Jet Bundle

- Label the particles 1,2,3 and their flavors by $a_1, a_2\, , a_3$

- Label quantities evaluated at the vacuum (typically $u^i = 0$) with a bar $\overline{g_{ij}} = g_{ij}(0)$. Derivatives denoted by a comma $\partial_k g_{ij} = g_{ij,k}$

- The Feynman rule for a 3-point interaction is given by

$$\frac{1}{12}\eta^{\mu\nu}\bar{g}_{\mu\nu,a_1a_2a_3} + \frac{1}{2}\bar{g}_{a_1a_2,a_3}p_1 \cdot p_2 + \frac{1}{2}\bar{g}_{a_1a_3,a_2}p_1 \cdot p_3 + \frac{1}{2}\bar{g}_{a_2a_3,a_1}p_2 \cdot p_3$$

Three Point Amplitude on 0-Jet Bundle

- The momenta fulfill

$$p_3^2 = (p_1 + p_2)^2$$

- The Christoffel symbols are defined as

$$\Gamma_{IJK} = \frac{1}{2}(g_{IJ,K} + g_{IK,J} - g_{JK,I})$$

- For the momentum independent term

$$\eta^{\mu\nu}\bar{g}_{\rho\sigma,a_1\,a_2a_3} = \overline{\nabla_{a_3}R^{\mu}_{a_1\mu\,a_2}} - 2(m_1^2\,\overline{\Gamma_{a_1a_2a_3}} + m_2^2\,\overline{\Gamma_{a_2a_1a_3}} + m_3^2\,\overline{\Gamma_{a_3a_1a_2}})$$

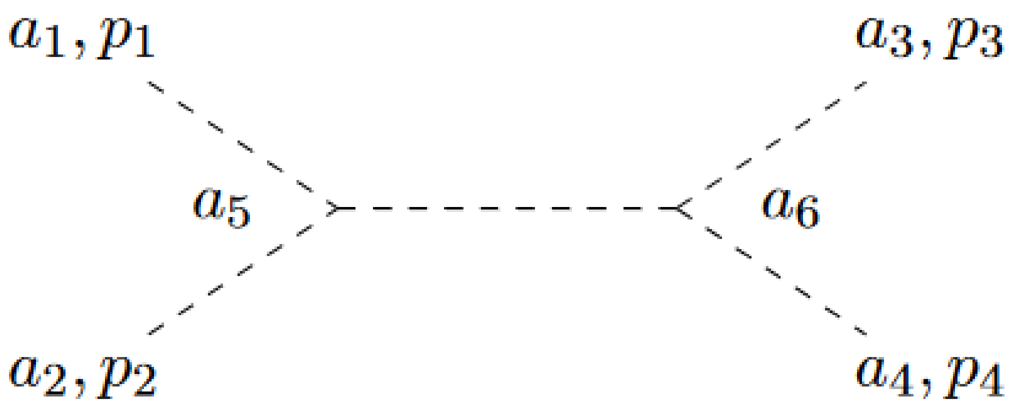
Three Point Amplitude on 0-Jet Bundle

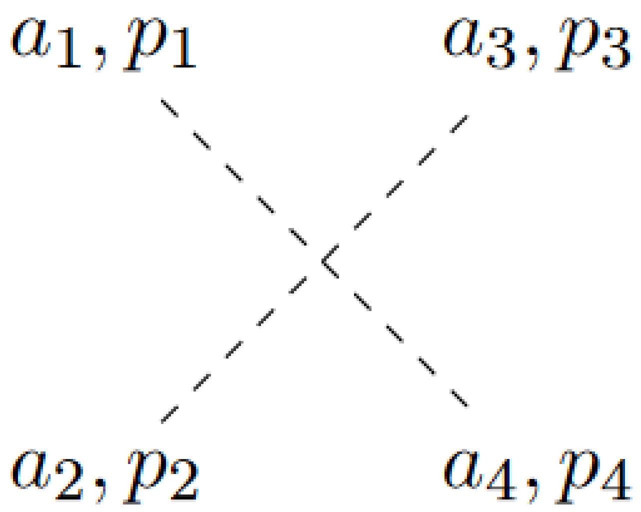
- Accounting for the symmetry factors the three-point amplitude is given by

$$i\left(\frac{1}{6}(\overline{\nabla_{a_3} R^{\mu}_{a_1 \mu a_2}} + \overline{\nabla_{a_2} R^{\mu}_{a_1 \mu a_3}} + \overline{\nabla_{a_1} R^{\mu}_{a_2 \mu a_3}}) \right.$$
$$\left. + (p_1^2 - m_1^2)\overline{\Gamma_{a_1 a_2 a_3}} + (p_2^2 - m_2^2)\overline{\Gamma_{a_2 a_1 a_3}} + (p_3^2 - m_3^2)\overline{\Gamma_{a_3 a_2 a_1}}\right)$$

- On-shell only the tensorial piece survives

Two to Two Scattering





Two to Two Scattering

- Contributions from gluing of three-point interactions and from contact terms

- Momenta degrees of freedom exist unlike the three-point amplitude

- On-shell the amplitude should be given by products of s_{12}, s_{13}, s_{14} and $\nabla^n R^m$ with $n, m \leq 2$

Two to Two Scattering

$$
\begin{aligned}
i\Bigg(\frac{1}{12}\Bigg(&\overline{\nabla_{a_1}\nabla_{a_2}R^{\mu}_{a_3\mu a_4}}+\overline{\nabla_{a_2}\nabla_{a_1}R^{\mu}_{a_3\mu a_4}}+\overline{\nabla_{a_1}\nabla_{a_3}R^{\mu}_{a_4\mu a_2}}+\overline{\nabla_{a_3}\nabla_{a_1}R^{\mu}_{a_4\mu a_2}}+\overline{\nabla_{a_1}\nabla_{a_4}R^{\mu}_{a_3\mu a_2}}\\
&+\overline{\nabla_{a_4}\nabla_{a_1}R^{\mu}_{a_3\mu a_2}}+\overline{\nabla_{a_2}\nabla_{a_3}R^{\mu}_{a_1\mu a_4}}+\overline{\nabla_{a_3}\nabla_{a_2}R^{\mu}_{a_1\mu a_4}}+\overline{\nabla_{a_2}\nabla_{a_4}R^{\mu}_{a_1\mu a_3}}+\overline{\nabla_{a_4}\nabla_{a_2}R^{\mu}_{a_1\mu a_3}}\\
&+\overline{\nabla_{a_3}\nabla_{a_4}R^{\mu}_{a_1\mu a_2}}+\overline{\nabla_{a_4}\nabla_{a_3}R^{\mu}_{a_1\mu a_2}}\Bigg)-\frac{1}{6}\Bigg(\overline{R^{\mu}_{a_1\nu a_2}R^{\nu}_{a_3\mu a_4}}+\overline{R^{\mu}_{a_1\nu a_3}R^{\nu}_{a_2\mu a_4}}+\overline{R^{\mu}_{a_1\nu a_4}R^{\nu}_{a_2\mu a_3}}\Bigg)\\
&-\frac{1}{3}\Bigg(s_{12}\Bigg(\overline{R_{a_1a_4a_3a_2}}+\overline{R_{a_2a_4a_3a_1}}\Bigg)+s_{13}\Bigg(\overline{R_{a_1a_4a_2a_3}}+\overline{R_{a_3a_4a_2a_1}}\Bigg)+s_{14}\Bigg(\overline{R_{a_1a_2a_3a_4}}+\overline{R_{a_4a_2a_3a_1}}\Bigg)\Bigg)\\
&-\frac{1}{36}\Bigg(\frac{\overline{g^{a_5a_6}}}{s_{12}-m_5^2}(\overline{\nabla_{a_5}R^{\mu}_{a_1\mu a_2}}+\overline{\nabla_{a_1}R^{\mu}_{a_5\mu a_2}}+\overline{\nabla_{a_2}R^{\mu}_{a_1\mu a_5}})(\overline{\nabla_{a_6}R^{\mu}_{a_3\mu a_4}}+\overline{\nabla_{a_3}R^{\mu}_{a_6\mu a_4}}+\overline{\nabla_{a_4}R^{\mu}_{a_3\mu a_6}})\\
&+(2\leftrightarrow 3)+(2\leftrightarrow 4)\Bigg)\Bigg)
\end{aligned}
$$

Diffeomorphisms and Tensors

- Under a general diffeomorphism f the Riemann tensor is not invariant

$$R_{IJKL}(x)dx^I dx^J dx^K dx^L \to R_{IJKL}(f(x))\frac{\partial(f\circ x^I)}{\partial x^A}\frac{\partial(f\circ x^J)}{\partial x^B}\frac{\partial(f\circ x^K)}{\partial x^C}\frac{\partial(f\circ x^L)}{\partial x^D}dx^A dx^B dx^C dx^D$$

- A diffeomorphism of the form $u \to f(u) = u + c_n u^n$ with $n \geq 2$ is special since at the point $u = 0$ we have

$$\lim_{u\to 0} f(u) = \lim_{u\to 0} u = 0 \qquad \lim_{u\to 0}\frac{\partial(f\circ u^i)}{\partial u^j} = \delta^i_j$$

- Tensors are invariant under such a transformation at the vacuum just like amplitudes

Scalar Curvature

- The Ricci Scalar R is also not invariant under a diffeomorphism f. It transforms according to

$$R(u) \to R(f(u))$$

- At the vacuum, a diffeomorphism of the form discussed earlier leaves the scalar invariant since

$$\lim_{u \to 0} R(f(u)) = \lim_{u \to 0} R(u)$$

- Disagreement of Ricci scalars at the vacuum indicates that the physical amplitudes are different.

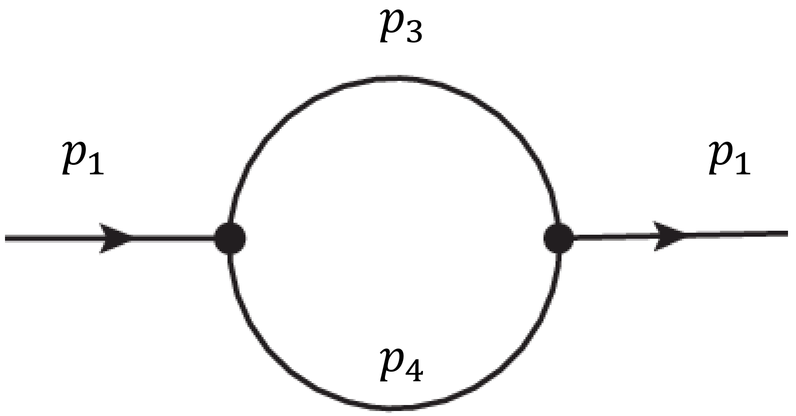
Conclusion and Outlook

- Jet bundles offer a path to write a Lagrangian of any derivative order in terms of geometry
- Amplitudes are combinations of geometric tensors
- Non-derivative field redefinitions are diffeomorphisms on bundle
- Derivative field redefinitions as maps between jet bundle orders (in progress)
- Incorporating gauge fields and fermions (future goal)

Thank you

Loop Diagrams

- Consider the 1-loop correction to the propagator



$$\int \frac{d^4p_3}{(2\pi)^4}\frac{\overline{g}^{a_3a_5}\overline{g}^{a_4a_6}}{(p_3^2-m_3^2)((p_1+p_3)^2-m_4^2)}$$

$$\left(\frac{1}{6}(\overline{\nabla_{a_5}R^{\mu}_{a_2\mu a_6}}+\overline{\nabla_{a_2}R^{\mu}_{a_5\mu a_6}}+\overline{\nabla_{a_6}R^{\mu}_{a_2\mu a_5}})+(p_3^2-m_3^2)\bar{\Gamma}_{a_5a_2a_6}+((p_1+p_3)^2-m_4^2)\bar{\Gamma}_{a_6a_2a_5}\right)$$

$$\left(\frac{1}{6}(\overline{\nabla_{a_3}R^{\mu}_{a_1\mu a_4}}+\overline{\nabla_{a_1}R^{\mu}_{a_3\mu a_4}}+\overline{\nabla_{a_4}R^{\mu}_{a_1\mu a_3}})+(p_3^2-m_3^2)\bar{\Gamma}_{a_3a_1a_4}+((p_1+p_3)^2-m_4^2)\bar{\Gamma}_{a_4a_1a_3}\right)$$

Riemannian Metric on Jet Bundle

$$L = \frac{1}{2}\langle \eta^{-1} , \ (j^n\phi)^* \, g \ \rangle$$

$$(j^1\phi)^* g = g_{\mu\nu} dx^\mu \otimes dx^\nu + g_{ij} d\phi^i \otimes d\phi^j + g_{ij}^{\mu\nu} d\phi^i_\mu \otimes d\phi^j_\nu$$
$$+ g_{i\mu} d\phi^i \otimes dx^\mu + g_{ij}^{\mu} d\phi^i_\mu \otimes d\phi^j + g_{i\nu}^{\mu} d\phi^i_\mu \otimes dx^\nu$$

$$g_{ij}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \subset L$$

$$g_{\mu\nu}\eta^{\mu\nu} = V(\phi) + \cdots \subset L$$

$$g_{ij}^{\mu\nu}\eta^{\rho\sigma}\partial_\rho\partial_\mu\phi^i\partial_\sigma\partial_\nu\phi^j \subset L$$