Heavy Dark Matter EFT for any Spin

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ongoing work

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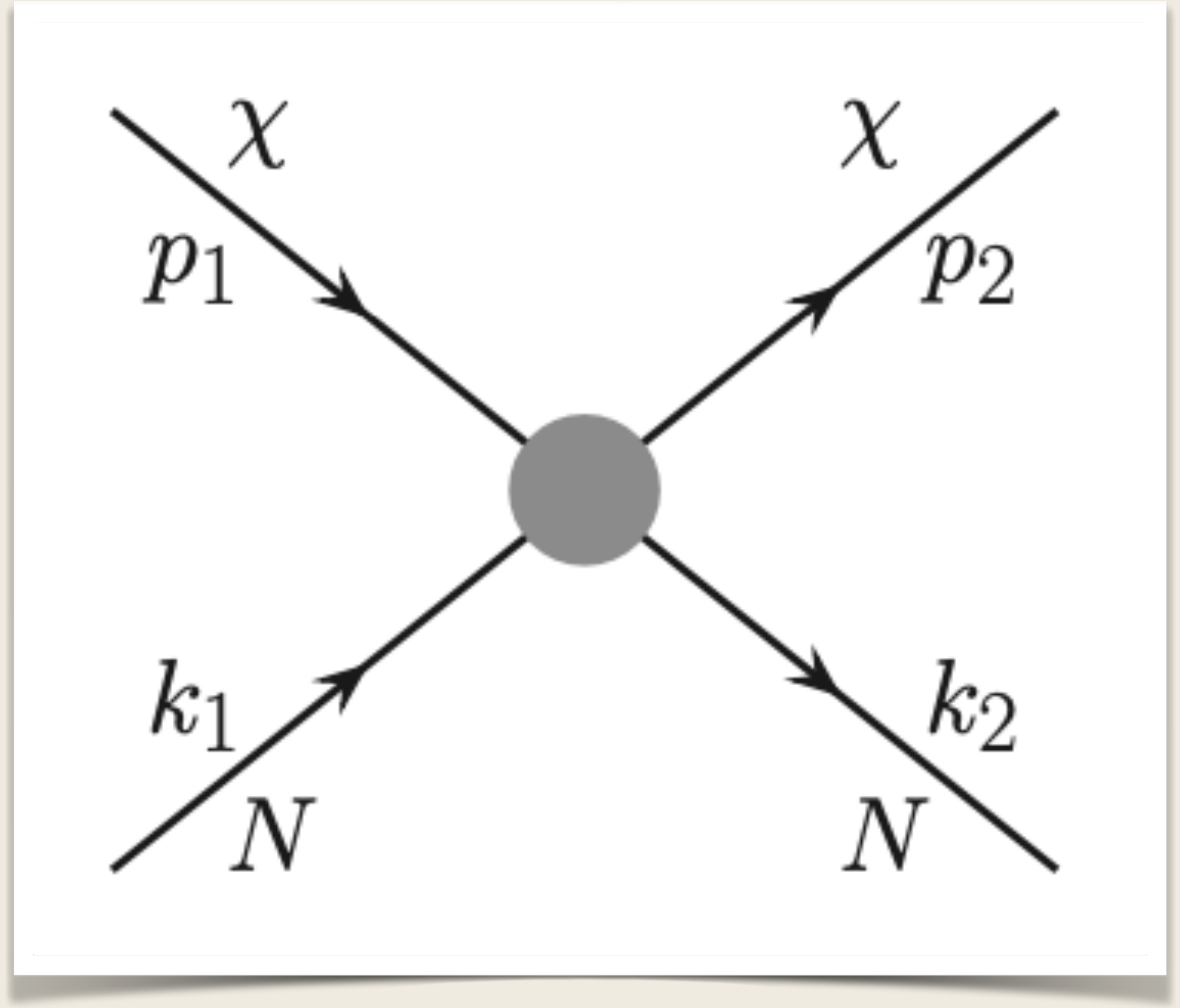
Introduction

- Goal: model independent description of direct detection experimental results
- Consider:
    - non-relativistic DM
    - scattering processes
    - small momentum exchanges, i.e. $p_1 - p_2 = q \ll m_\chi$
    - heavy mediators
    - arbitrary spin



- E.g. DM bound states, composite DM

Existing approaches

EFT for relativistic DM: $\mathscr{L}_\chi = \sum_{a,d} \hat{\mathscr{C}}_a^d \mathscr{O}_a^d\,, \qquad \hat{\mathscr{C}}_a^d = \frac{\mathscr{C}_a^d}{\Lambda^{d-4}}$

- $\mathscr{O}_a^d$: $(\bar{\chi}\chi)(\bar{q}q)\,, (\bar{\chi}\gamma^\mu\chi)(\bar{q}\gamma_\mu q)\,, \ldots$ up to dim 7 for spin 0, 1/2, 1
- EFT above EW scale - relate to indirect exp., LHC
- No higher spins

[Bishara *et al*, *JCAP* 02 (2017) 009]
[Goodman *et al*, *Phys. Rev. D* 82 (2010) 116010]

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EFT for non-relativistic DM interacting with non-relativistic nucleons: $$\mathscr{L}_{NR} = \sum_{i,N} c_i^N(q^2)\mathscr{O}_i^N$$

- $\mathscr{O}_i^N$: $\mathbf{1}_\chi \mathbf{1}_N\,, \vec{S}_\chi \cdot \vec{S}_N\,, \dots,$ for any spin
- Requires $\vec{q} \ll m_\pi$

[Fitzpatrick *et al*, *JCAP* 02 (2013) 004]
[Jenkins, Manohar, *Phys. Lett. B* 255 (1991) 558-562]

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=> Combine the best of both: HDMEFT - NR DM for any spin with Lorentz symmetry and couplings to SM fields

Heavy DM EFT - Little Group

- Little group is a subgroup of Lorentz transformations that is isomorphic to $SO(3)$ group of rotations and leaves time-like vector v invariant [Heinonen *et al*, *Phys. Rev. D* 86 (2012) 094020]
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- Build Lorentz inv. HDMEFT by
    - embedding DM fields into little group rep. and defining trans. under Lorentz group
    - requiring rotational invariance: $R^{\mu}_{\nu}v^{\nu} = v^{\mu}$ (sufficient for LO in $1/m_\chi$)
    - requiring inv. under small boosts: $\chi(x) \to \exp(i\vec{\eta}\cdot\vec{K})\chi(x') = e^{-iq\cdot x}(1+\mathcal{O}(1/m_\chi^2))\chi_v(x')$, where $\vec{\eta} = -\vec{q}/M$ => RPI relates operators of different dimensions

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- Choose v such that $v^2 = 1,$ e.g. $v^\mu = (1,\vec{0})$
- Define spin operator: $S_\mu = -\frac{1}{2}\epsilon_{\mu\alpha\beta\gamma} J^{\alpha\beta} v^\gamma,$ e.g. spin - 1/2: $S_\mu = \gamma_\mu \gamma_5 / 2$

Constructing the HDMEFT basis

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

- Operators: SM fields, DM fields, S^μ, v^μ and derivatives
- $(v \cdot \ldots)$ longitudinal and $\ldots_\perp$ perpendicular components
- EOM and Bianchi identities remove higher order terms

- Write down the possible operators

…

$$\mathcal{P}_{3f}^{(6)} = (\chi_v^\dagger S_\perp{}^\mu \chi_v)(\bar{f}\gamma_{\perp\mu}\gamma_5 f)$$

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$$\mathcal{P}_{23f}^{(7)} = (\chi_v^\dagger i(S_\perp \cdot \overleftrightarrow{\partial}_\perp)\chi_v)(\bar{f}\not{v}\gamma_5 f)$$

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- Find RPI relations by performing a Lorentz boost $\mathscr{B}(q) = \Lambda(v - q/M, v)$ that shifts v^μ

$$\chi_v \xrightarrow{\mathcal{B}(q)} e^{iq\cdot x}\left[1 + \frac{iq\cdot\partial_\perp}{2m_\chi^2} - \frac{\epsilon_{\mu\nu\rho\sigma}v^\mu S_\perp{}^\nu q^\rho \partial_\perp^\sigma}{2m_\chi^2}\right]\chi_v + \mathcal{O}(1/m_\chi^4)$$

$$D_\perp^\mu \longrightarrow D_\perp^\mu - \frac{1}{m_\chi} q^\mu (v\cdot D) + \mathcal{O}(q^2)$$

Reparametrization invariance

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

- All RPI's up to $\mathcal{O}(q)$
- Dim 5 up to $\mathcal{O}(1/m_\chi^2)$
- Dim 6 up to $\mathcal{O}(1/m_\chi)$
- Dim 7 up to $\mathcal{O}(1/m_\chi^0)$

Operators	Lorentz boost
$\mathcal{P}_{3f}^{(6)} = (\chi_v^\dagger S_\perp^\mu \chi_v)(\bar{f}\gamma_{\perp\mu}\gamma_5 f)$	$\to \mathcal{P}_{3f}^{(6)} - \frac{q_\mu}{m_\chi}(\chi_v^\dagger S_\perp^\mu \chi_v)(\bar{f}\not{v}\gamma_5 f)$
$\mathcal{P}_{23f}^{(7)} = (\chi_v^\dagger i(S_\perp \cdot \overleftrightarrow{\partial}_\perp)\chi_v)(\bar{f}\not{v}\gamma_5 f)$	$\to \mathcal{P}_{23f}^{(7)} - 2q_\mu(\chi_v^\dagger S_\perp^\mu \chi_v)(\bar{f}\not{v}\gamma_5 f)$

- Each arrow corresponds to Lorentz boost, i.e.

$$\chi_v \longrightarrow e^{iq\cdot x}\chi_v + \mathcal{O}(1/m_\chi^2)$$

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RPI relation => $w_{23f}^{(7)} = -\frac{1}{2} w_{3f}^{(6)}$

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HDMEFT basis

The basis is organised as follows

- Minimal basis up to dim 7 consisting of operators with linearly independent Wilson coefficients
- Operators related via RPI
- Operators removed by the RPI

Note

- The final presentation will depend on the matching onto the nucleon theory

HDMEFT basis

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

Minimal basis with linearly independent WC's

- Dimension 5

$$\mathcal{P}_{1\gamma}^{(5)} = \frac{e}{16\pi^2}(\chi_v^\dagger S_\perp^\mu \chi_v)(v\cdot\widetilde{F})_\mu\,, \qquad \mathcal{P}_{2\gamma}^{(5)} = \frac{e}{16\pi^2}(\chi_v^\dagger S_\perp^\mu \chi_v)(v\cdot F)_\mu$$

$j_\chi \geq 1/2$

- Dimension 6

$$\mathcal{P}_{1f}^{(6)} = (\chi_v^\dagger \chi_v)(\bar{f}\not{v}f), \qquad \mathcal{P}_{2f}^{(6)} = (\chi_v^\dagger \chi_v)(\bar{f}\not{v}\gamma_5 f),$$

$$\mathcal{P}_{3f}^{(6)} = (\chi_v^\dagger S_\perp^\mu \chi_v)(\bar{f}\gamma_{\perp\mu}\gamma_5 f), \qquad \mathcal{P}_{4f}^{(6)} = (\chi_v^\dagger S_\perp^\mu \chi_v)(\bar{f}\gamma_{\perp\mu} f),$$

$$\mathcal{P}_{1\gamma}^{(6)} = \frac{e}{16\pi^2}\partial_{\perp\mu}(\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v)(v\cdot F)_\nu, \qquad \mathcal{P}_{2\gamma}^{(6)} = \frac{e}{16\pi^2}\partial_{\perp\mu}(\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v)(v\cdot \widetilde{F})_\nu$$

$j_\chi \geq 0$

$j_\chi \geq 1/2$

$j_\chi \geq 1$

Notation: $\gamma_{\perp\mu} = \gamma_\mu - v_\mu \not{v}$ $\quad \overline{S_\perp^{\mu\nu}} = S_\perp^{\{\mu} S_\perp^{\nu\}} - \frac{j_\chi(j_\chi+1)}{3}\hat{g}^{\mu\nu}$ $\quad \hat{g}_{\mu\nu} = g_{\mu\nu} - v_\mu v_\nu$

HDMEFT basis - dim 7

Fermions, photons and similar for gluons

$$\overline{S_\perp^{\mu\nu\rho}} = S_\perp^{\{\mu}S_\perp^{\nu}S_\perp^{\rho\}} - \frac{3j_\chi(j_\chi+1)-1}{5}\hat{g}^{\{\mu\nu}S_\perp^{\rho\}}$$

Operators that do not vanish for $j_\chi \geq 0$.

$$\mathcal{P}_{1f}^{(7)} = m_f(\chi_v^\dagger\chi_v)(\bar{f}f), \qquad \mathcal{P}_{2f}^{(7)} = m_f(\chi_v^\dagger\chi_v)(\bar{f}i\gamma_5 f),$$
$$\mathcal{P}_{3f}^{(7)} = (\chi_v^\dagger\chi_v)(\bar{f}\not{v}i(v\cdot\overleftrightarrow{D})f), \qquad \mathcal{P}_{4f}^{(7)} = (\chi_v^\dagger\chi_v)(\bar{f}\not{v}i(v\cdot\overleftrightarrow{D})\gamma_5 f),$$

Operators that do not vanish for $j_\chi \geq 1/2$.

$$\mathcal{P}_{5f}^{(7)} = m_f(\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}(v\cdot\sigma)_\mu i\gamma_5 f), \qquad \mathcal{P}_{6f}^{(7)} = m_f(\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}(v\cdot\sigma)_\mu f),$$
$$\mathcal{P}_{7f}^{(7)} = \partial_\perp^\mu(\chi_v^\dagger S_\perp^\nu\chi_v)\epsilon_{\mu\nu\rho\sigma}v^\rho(\bar{f}\gamma_\perp^\sigma f), \qquad \mathcal{P}_{8f}^{(7)} = \partial_\perp^\mu(\chi_v^\dagger S_\perp^\nu\chi_v)\epsilon_{\mu\nu\rho\sigma}v^\rho(\bar{f}\gamma_\perp^\sigma\gamma_5 f),$$
$$\mathcal{P}_{9f}^{(7)} = \partial_{\perp\mu}(\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}\not{v}\gamma_5 f), \qquad \mathcal{P}_{10f}^{(7)} = \partial_{\perp\mu}(\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}\not{v}f),$$
$$\mathcal{P}_{11f}^{(7)} = (\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}\not{v}\gamma_5 i\overleftrightarrow{D}_{\perp\mu}f), \qquad \mathcal{P}_{12f}^{(7)} = (\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}\not{v}i\overleftrightarrow{D}_{\perp\mu}f),$$
$$\mathcal{P}_{13f}^{(7)} = (\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}\gamma_{\perp\mu}i(v\cdot\overleftrightarrow{D})\gamma_5 f), \qquad \mathcal{P}_{14f}^{(7)} = (\chi_v^\dagger S_\perp^\mu\chi_v)(\bar{f}\gamma_{\perp\mu}i(v\cdot\overleftrightarrow{D})f),$$
$$\mathcal{P}_{15f}^{(7)} = (\chi_v^\dagger S_\perp^\mu\chi_v)\epsilon_{\mu\nu\rho\sigma}v^\nu(\bar{f}\gamma^\rho i\overleftrightarrow{D}_\perp^\sigma f), \qquad \mathcal{P}_{16f}^{(7)} = (\chi_v^\dagger S_\perp^\mu\chi_v)\epsilon_{\mu\nu\rho\sigma}v^\nu(\bar{f}\gamma^\rho\gamma_5 i\overleftrightarrow{D}_\perp^\sigma f),$$

Operators that do not vanish for $j_\chi \geq 1$.

$$\mathcal{P}_{17f}^{(7)} = (\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(\bar{f}\gamma_{\perp\mu}i\overleftrightarrow{D}_{\perp\nu}f), \qquad \mathcal{P}_{18f}^{(7)} = (\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(\bar{f}\gamma_{\perp\mu}i\overleftrightarrow{D}_{\perp\nu}\gamma_5 f),$$
$$\mathcal{P}_{19f}^{(7)} = \partial_{\perp\mu}(\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(\bar{f}\gamma_{\perp\nu}f), \qquad \mathcal{P}_{20f}^{(7)} = \partial_{\perp\mu}(\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(\bar{f}\gamma_{\perp\nu}\gamma_5 f).$$

Operators that do not vanish for $j_\chi \geq 0$.

$$\mathcal{P}_{1\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger\chi_v)F_{\perp\mu\nu}F_\perp^{\mu\nu}, \qquad \mathcal{P}_{2\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger\chi_v)F_{\perp\mu\nu}\widetilde{F}_\perp^{\mu\nu},$$
$$\mathcal{P}_{3\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger\chi_v)(v\cdot F)^\mu(v\cdot F)_\mu,$$

Operators that do not vanish for $j_\chi \geq 1/2$.

$$\mathcal{P}_{4\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger S_\perp^\mu\chi_v)(v\cdot F)^\nu F_{\perp\mu\nu},$$
$$\mathcal{P}_{5\gamma}^{(7)} = \frac{e}{16\pi^2}\partial_\perp^2(\chi_v^\dagger S_\perp^\mu\chi_v)(v\cdot\widetilde{F})_\mu, \qquad \mathcal{P}_{6\gamma}^{(7)} = \frac{e}{16\pi^2}\partial_\perp^2(\chi_v^\dagger S_\perp^\mu\chi_v)(v\cdot F)_\mu,$$

Operators that do not vanish for $j_\chi \geq 1$.

$$\mathcal{P}_{7\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(v\cdot F)_\mu(v\cdot F)_\nu, \qquad \mathcal{P}_{8\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(v\cdot F)_\mu(v\cdot\widetilde{F})_\nu,$$
$$\mathcal{P}_{9\gamma}^{(7)} = \frac{\alpha_{\rm em}}{4\pi}(\chi_v^\dagger\overline{S_\perp^{\mu\nu}}\chi_v)(v\cdot\widetilde{F})_\mu(v\cdot\widetilde{F})_\nu,$$

Operators that do not vanish for $j_\chi \geq 3/2$.

$$\mathcal{P}_{10\gamma}^{(7)} = \frac{e}{16\pi^2}\partial_{\perp\nu}\partial_{\perp\rho}(\chi_v^\dagger\overline{S_\perp^{\mu\nu\rho}}\chi_v)(v\cdot\widetilde{F})_\mu, \qquad \mathcal{P}_{11\gamma}^{(7)} = \frac{e}{16\pi^2}\partial_{\perp\nu}\partial_{\perp\rho}(\chi_v^\dagger\overline{S_\perp^{\mu\nu\rho}}\chi_v)(v\cdot F)_\mu.$$

HDMEFT basis - RPI

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

For the Lorentz invariant basis one also needs to include operators related via reparametrization invariance

Dimension-six

$$\mathcal{P}_{3\gamma}^{(6)} = \frac{e}{16\pi^2}(\chi_v^\dagger S_\perp^\mu i\overleftrightarrow{\partial}_\perp^\nu \chi_v)\widetilde{F}_{\mu\nu}, \qquad \mathcal{P}_{4\gamma}^{(6)} = \frac{e}{16\pi^2}(\chi_v^\dagger S_\perp^\mu i\overleftrightarrow{\partial}_\perp^\nu \chi_v)F_{\mu\nu}.$$

Dimension-seven operators for fermions

$$\mathcal{P}_{21f}^{(7)} = (\chi_v^\dagger i\overleftrightarrow{\partial}_\perp^\mu \chi_v)(\bar{f}\gamma_{\perp\mu}f), \qquad \mathcal{P}_{22f}^{(7)} = (\chi_v^\dagger i\overleftrightarrow{\partial}_\perp^\mu \chi_v)(\bar{f}\gamma_{\perp\mu}\gamma_5 f),$$

$$\mathcal{P}_{23f}^{(7)} = (\chi_v^\dagger i(S_\perp \cdot \overleftrightarrow{\partial}_\perp)\chi_v)(\bar{f}\not{v}\gamma_5 f), \qquad \mathcal{P}_{24f}^{(7)} = (\chi_v^\dagger i(S_\perp \cdot \overleftrightarrow{\partial}_\perp)\chi_v)(\bar{f}\not{v}f),$$

$$\mathcal{P}_{25f}^{(7)} = (\chi_v^\dagger \overline{S_\perp^{\mu\nu}}\, i\overleftrightarrow{\partial}_{\perp\nu}\chi_v)(\bar{f}\gamma_{\perp\mu}f),$$

Dimension-seven operators for F

$$\mathcal{P}_{12\gamma}^{(7)} = \frac{e}{16\pi^2}(\chi_v^\dagger S_{\perp\nu} i\overleftrightarrow{\partial}_\perp^\nu i\overleftrightarrow{\partial}_\perp^\mu \chi_v)(v\cdot\widetilde{F})_\mu, \qquad \mathcal{P}_{13\gamma}^{(7)} = \frac{e}{16\pi^2}(\chi_v^\dagger S_{\perp\nu} i\overleftrightarrow{\partial}_\perp^\nu i\overleftrightarrow{\partial}_\perp^\mu \chi_v)(v\cdot F)_\mu,$$

$$\mathcal{P}_{14\gamma}^{(7)} = \frac{e}{16\pi^2}\partial_{\perp\mu}(\chi_v^\dagger \overline{S_\perp^{\mu\nu}}\, i\overleftrightarrow{\partial}_\perp^\rho \chi_v)F_{\perp\nu\rho}, \qquad \mathcal{P}_{15\gamma}^{(7)} = \frac{e}{16\pi^2}\partial_{\perp\mu}(\chi_v^\dagger \overline{S_\perp^{\mu\nu}}\, i\overleftrightarrow{\partial}_\perp^\rho \chi_v)\widetilde{F}_{\perp\nu\rho}.$$

RPI relations between Wilson Coefficients

$$w_{3\gamma}^{(6)} = -\frac{1}{2}w_{1\gamma}^{(5)}, \qquad w_{4\gamma}^{(6)} = -\frac{1}{2}w_{2\gamma}^{(5)},$$

$$w_{21f}^{(7)} = \frac{1}{2}w_{1f}^{(6)} - \frac{j(j+1)}{12}\frac{e}{16\pi^2}eQ_f w_{1\gamma}^{(5)}, \qquad w_{22f}^{(7)} = \frac{1}{2}w_{2f}^{(6)},$$

$$w_{23f}^{(7)} = -\frac{1}{2}w_{3f}^{(6)}, \qquad w_{24f}^{(7)} = -\frac{1}{2}w_{4f}^{(6)},$$

$$w_{25f}^{(7)} = -\frac{1}{4}\frac{e}{16\pi^2}eQ_f w_{1\gamma}^{(5)},$$

$$w_{12\gamma}^{(7)} = \frac{1}{8}w_{1\gamma}^{(5)}, \qquad w_{13\gamma}^{(7)} = \frac{1}{8}w_{2\gamma}^{(5)},$$

$$w_{14\gamma}^{(7)} = \frac{1}{4}(w_{1\gamma}^{(5)} - 2w_{1\gamma}^{(6)}), \qquad w_{15\gamma}^{(7)} = -\frac{1}{4}(w_{2\gamma}^{(5)} + 2w_{2\gamma}^{(6)}).$$

Further operators removed by the RPI

QM Basis and matching onto NR-nucleon EFT

1. Write the basis in the QM notation:

$$\chi_v^\dagger \chi_v \to 1_\chi, \qquad \chi_v^\dagger S_\perp^\mu \chi_v \to (0, \vec{S}_\chi),$$

$$\mathcal{P}_{1\gamma}^{(5)} \to \frac{e}{16\pi^2} \vec{S}_\chi \cdot \vec{B}, \qquad \mathcal{P}_{2\gamma}^{(5)} \to \frac{e}{16\pi^2} \vec{S}_\chi \cdot \vec{E}.$$

$$\begin{aligned}
\mathcal{P}_{1f}^{(6)} &\to 1_\chi(\bar{f}\gamma^0 f), & \mathcal{P}_{2f}^{(6)} &\to 1_\chi(\bar{f}\gamma^0\gamma_5 f),\\
\mathcal{P}_{3f}^{(6)} &\to -\vec{S}_\chi \cdot (\bar{f}\vec{\gamma}\gamma_5 f), & \mathcal{P}_{4f}^{(6)} &\to -\vec{S}_\chi \cdot (\bar{f}\vec{\gamma} f),\\
\mathcal{P}_{1\gamma}^{(6)} &\to \frac{e}{16\pi^2} iq^i \, \overline{S_\chi^{ij}} \, E^j, & \mathcal{P}_{2\gamma}^{(6)} &\to \frac{e}{16\pi^2} iq^i \, \overline{S_\chi^{ij}} \, B^j.
\end{aligned}$$

2. Hadronization and matching onto NR-nucleon EFT:

$$\mathcal{L}_{\rm NR} = \sum_{i,N} c_i^N(q^2) \mathcal{O}_i^N .$$

$$\begin{aligned}
\mathcal{O}_1^N &= \mathbb{1}_\chi \mathbb{1}_N, & \mathcal{O}_2^N &= (v_\perp)^2 \mathbb{1}_\chi \mathbb{1}_N,\\
\mathcal{O}_3^N &= \mathbb{1}_\chi \vec{S}_N \cdot \left(\vec{v}_\perp \times \frac{i\vec{q}}{m_N}\right), & \mathcal{O}_4^N &= \vec{S}_\chi \cdot \vec{S}_N,\\
\mathcal{O}_5^N &= \vec{S}_\chi \cdot \left(\vec{v}_\perp \times \frac{i\vec{q}}{m_N}\right) \mathbb{1}_N, & \mathcal{O}_6^N &= \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),\\
\mathcal{O}_7^N &= \mathbb{1}_\chi \left(\vec{S}_N \cdot \vec{v}_\perp\right), & \mathcal{O}_8^N &= \left(\vec{S}_\chi \cdot \vec{v}_\perp\right) \mathbb{1}_N,\\
\mathcal{O}_9^N &= \vec{S}_\chi \cdot \left(\frac{i\vec{q}}{m_N} \times \vec{S}_N\right), & \mathcal{O}_{10}^N &= -\mathbb{1}_\chi \left(\vec{S}_N \cdot \frac{i\vec{q}}{m_N}\right),\\
\mathcal{O}_{11}^N &= -\left(\vec{S}_\chi \cdot \frac{i\vec{q}}{m_N}\right) \mathbb{1}_N, & \mathcal{O}_{12}^N &= \vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{v}_\perp\right),\\
\mathcal{O}_{13}^N &= -\left(\vec{S}_\chi \cdot \vec{v}_\perp\right)\left(\vec{S}_N \cdot \frac{i\vec{q}}{m_N}\right), & \mathcal{O}_{14}^N &= -\left(\vec{S}_\chi \cdot \frac{i\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \vec{v}_\perp\right)
\end{aligned}$$

3. Obtain Wilson coefficients $c_i^N(q^2)$ and compute cross-sections - direct detection experiments.

Conclusions

- EFT for heavy Dark Matter of arbitrary spin at LO in $1/m_\chi$.
- Constraints on DM bound/composite states.
- DM fields embedded in little group rep. - rotational inv.
- Lorentz invariance is present through RPI relations.
- Matching onto NR-nucleon EFT.