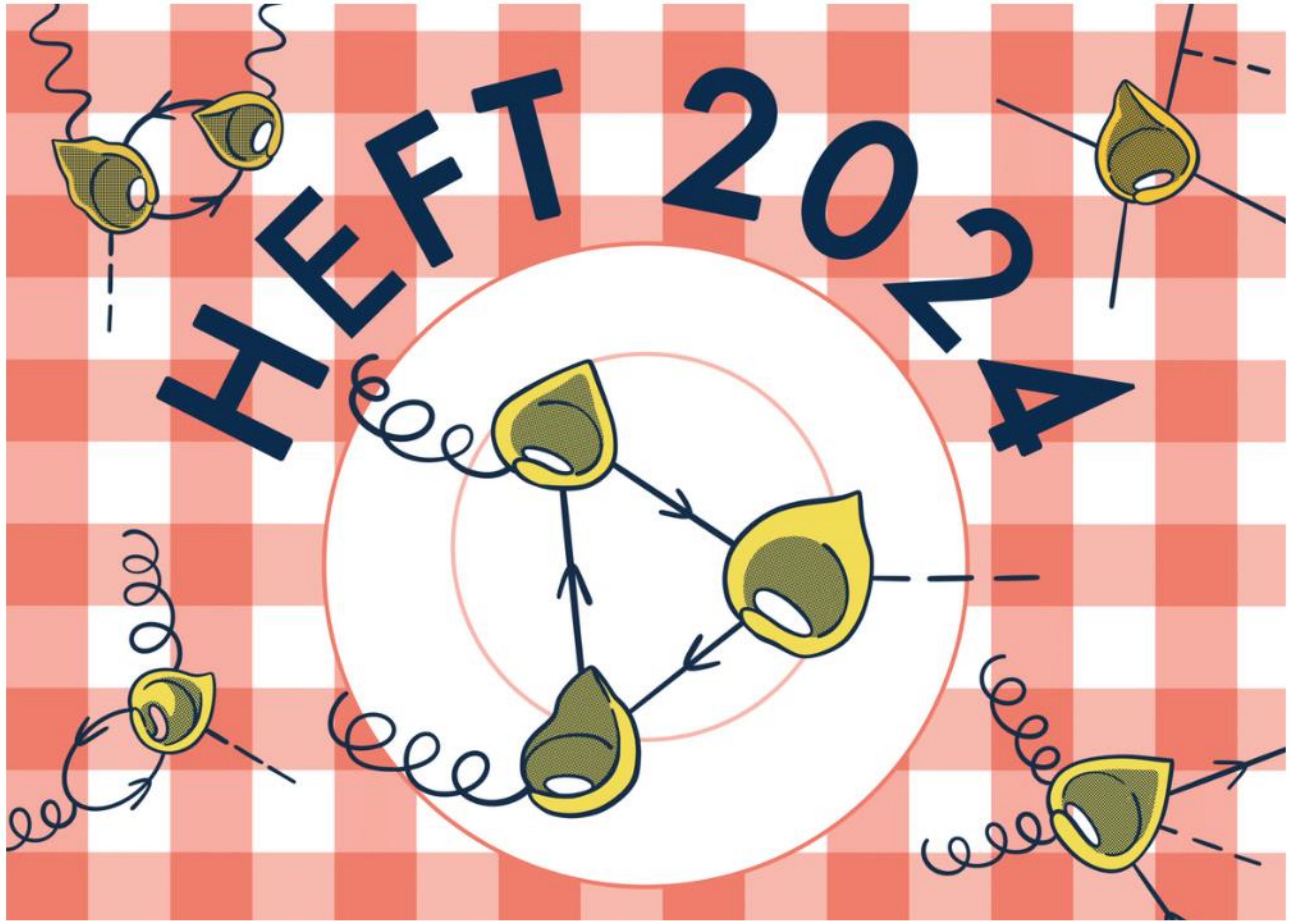
Universität Zürich[UZH]

Loop-corrected Fierz Identities and Evanescent shifts *

12.06.2024, HEFT 2024, Bologna



Marko Pesut

University of Zürich

*in collaboration with Jason Aebischer & Zach Polonsky

Outline of the talk

- Evanescent operators : definitions, prescription & scheme

- The shift approach :

  - *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P]),

  - *Dipole Operators in Fierz Identities* ([2211.01379 : J.Aebischer, M.P, Z.Polonsky])

- Shift approach in the context of change of bases of NLO ADMs:

  - *Renormalization Scheme Factorization of one-loop Fierz Identities* ([2306.16449 : J.Aebischer, M.P, Z.Polonsky])

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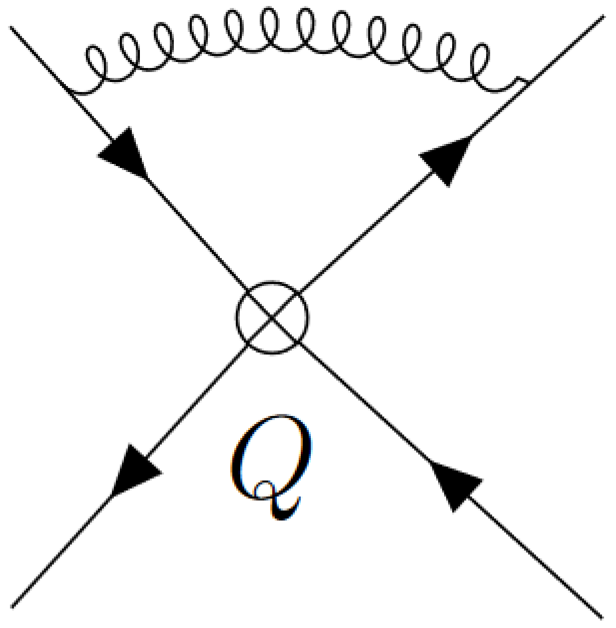
Plenty of Literature on ev. ops. / scheme / γ5 / LO and NLO ADMS & operator bases:

[Buras & Weisz (1990)], [Buras, Misiak, Urban (2000)], [Jenkins, Manohar, Stoffer (2018)], [Dugan & Grinstein (1991)], [Herrlich & Nierste (1995)], [Aebischer, Bobeth, Buras, Kumar (2020-2021)], [Bélusca-Maïto, Ilakovac, Mađor-Božinović, Stöckinger (2020)], ['t Hooft and Veltman (1972)], [Buras & Girrbach (2012)], [Chetyrkin, Misiak, Munz (1998)], [Dekens & Stoffer (2019)], [Grzadkowski, Iskrzynski, Misiak (2010)]...

General Context

Consider a set of physical operators $\{Q\}$, and compute one-loop matrix elements $\langle Q \rangle^{(1)}$:



$\longrightarrow$ Dirac Structure $\{\mathcal{E}\}$. In d=4, they map back to the physical basis :

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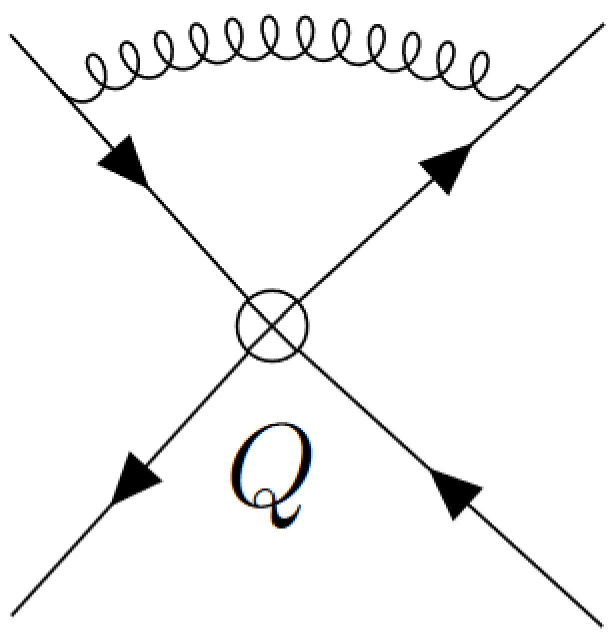

$\longrightarrow$ Dirac Structure $\{\mathcal{E}\}$. In d=4, they map back to the physical basis :

$$\mathcal{E} \overset{d=4}{=} \mathcal{F}_4 Q$$

➡ In dim. reg (d=4-2ε), no unambiguous way to continue Dirac Algebra !

Evanescent operators, Prescription and Scheme dependence

To account for this, we must :

- specify a ***prescription*** (i.e. how to *treat* Dirac Algebra: NDR, HV, Larin,…),

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$$E = \mathcal{E} - \mathcal{F}Q \qquad \mathcal{F} = \mathcal{F}_4 + \sum_{n=1}^{\infty} \epsilon^n \sigma_n$$

The d=4 part is fixed.

Constants fix the ***scheme dependence***.

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$$\Rightarrow \quad E\big|_{d=4} = 0$$

$$\Rightarrow \quad \boxed{\langle E \rangle^{(1)} \neq 0}$$

Finite pieces to be taken into account

One-Loop Fierz Identities [2208.10513]

- Evanescent operators : definitions, prescription & scheme ✓

- The shift approach :

  - *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P])*

  - *Dipole Operators in Fierz Identities* ([2211.01379 : J.Aebischer, M.P, Z.Polonsky])

Interpret the finite contributions of the Fierz-Ev. Ops. as one-loop corrections (*shifts*) to Fierz identities.

*Four-fermion operators in the LEFT

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$$\mathcal{F}_4 \circ \left(\bar{f}_1 \sigma_{\mu\nu} P_X f_2\right)\left(\bar{f}_3 \sigma^{\mu\nu} P_X f_4\right) \equiv -6\left(\bar{f}_1 P_X f_4\right)\left(\bar{f}_3 P_X f_2\right) + \frac{1}{2}\left(\bar{f}_1 \sigma_{\mu\nu} P_X f_4\right)\left(\bar{f}_3 \sigma^{\mu\nu} P_X f_2\right)$$

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One-Loop Fierz Identities [2208.10513]

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$$\langle \vec{\tilde{Q}} \rangle^{(1)} = \mathcal{F}_4 \langle \vec{Q} \rangle^{(1)} + \langle \vec{E} \rangle^{(1)} \Rightarrow \langle \vec{\tilde{Q}} \rangle^{(1)} = \mathcal{F}_4 \langle \vec{Q} \rangle^{(1)} + \frac{\alpha}{4\pi} R_1 \langle \vec{Q} \rangle^{(0)}$$

Shifts

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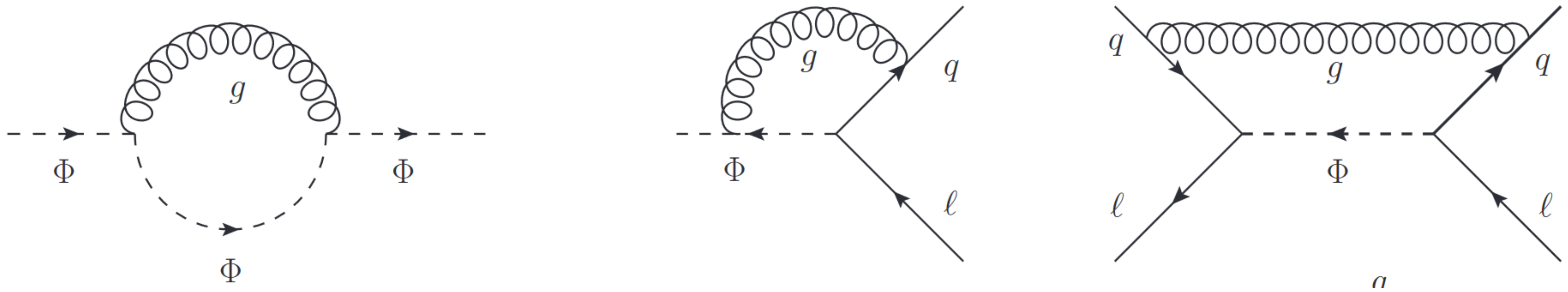
⇒ Only **relates physical operators** of the two bases !

- NLO Matching
- Change of bases for ADMs

One-Loop Fierz Identities [2208.10513]

Example: one-loop matching*

$$L_{q\ell}^{LQ} = \bar{q}\left(\Gamma_L^S P_L + \Gamma_R^S P_R\right)\ell\Phi^* + \text{h.c.}$$



$$\mathcal{L}\big|_{q\ell\ell q} \supset \widetilde{C}_S^{LL}\left(\bar{q}P_L\ell\right)\left(\bar{\ell}P_L q\right) + \widetilde{C}_T^{LL}\left(\bar{q}\sigma_{\mu\nu}P_L\ell\right)\left(\bar{\ell}\sigma^{\mu\nu}P_L q\right)$$

*Aebischer, Crivellin, Greub (2019)

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The two bases are related by Fierz Ids !

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One-Loop Fierz Identities [2208.10513]

Operator	Tree-level Fierz	QCD shift
$T^{LL}_{q_1q_2\ell_1\ell_2}$	$-6S^{LL}_{q_1\ell_2\ell_1q_2} + \frac{1}{2}T^{LL}_{q_1\ell_2\ell_1q_2}$	$\frac{7-7N_c^2}{N_c}S^{LL}_{q_1q_2\ell_1\ell_2}$
$S^{LL}_{q_1q_2\ell_1\ell_2}$	$-\frac{1}{2}S^{LL}_{q_1\ell_2\ell_1q_2} - \frac{1}{8}T^{LL}_{q_1\ell_2\ell_1q_2}$	$\frac{N_c^2-1}{16N_c}T^{LL}_{q_1q_2\ell_1\ell_2}$

$\mathcal{F}_4$

$$\begin{pmatrix} C_S^{LL} \\ C_T^{LL} \end{pmatrix} = \left[R_0 + \frac{\alpha_s}{4\pi} R_1 R_0\right]^{-T} \begin{pmatrix} \widetilde{C}_S^{LL} \\ \widetilde{C}_T^{LL} \end{pmatrix}$$

Example: one-loop matching*

$$\mathcal{L}\big|_{q\ell\ell q} \supset \widetilde{C}_S^{LL}\,(\bar{q}P_L\ell)\,(\bar{\ell}P_L q) + \widetilde{C}_T^{LL}\,(\bar{q}\sigma_{\mu\nu}P_L\ell)\,(\bar{\ell}\sigma^{\mu\nu}P_L q)$$

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$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ -6 & \frac{1}{2} \end{pmatrix} \qquad \begin{pmatrix} 0 & \frac{N_c^2-1}{16N_c} \\ \frac{7-7N_c^2}{N_c} & 0 \end{pmatrix}$$

$$\begin{pmatrix} C_S^{LL} \\ C_T^{LL} \end{pmatrix} = \left[\underset{\uparrow}{R_0} + \frac{\alpha_s}{4\pi}\underset{\uparrow}{R_1}R_0\right]^{-T} \begin{pmatrix} \widetilde{C}_S^{LL} \\ \widetilde{C}_T^{LL} \end{pmatrix}$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Another example* : ADM basis change

$$\vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E}), \quad \vec{\tilde{E}} = M(\epsilon U\vec{Q} + (\mathbb{1} + \epsilon UW)\vec{E})$$

*Gorbahn, Jäger, Nierste, Trine (2009)
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Change of bases formula for LO and NLO ADMs :

$$Z^{(1;0)}_{\tilde{Q}\tilde{Q}} = R_0 \left[W Z^{(1;0)}_{EQ} - \underbrace{\left(Z^{(1;1)}_{QE} + W Z^{(1;1)}_{EE} - Z^{(1;1)}_{QQ} W \right) U}_{\text{ev-to-ev}} \right] R_0^{-1}$$

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[Z^{(1,0)}_{\tilde{Q}\tilde{Q}}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$$

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Need to **relate** ev. ops. of the two bases (M and U matrices)

Need to **compute** 1-loop matrix elements ev. ops.

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*Chetyrkin, Misiak, Münz (1998)
*Gorbahn, Haisch (2005)
*Brod, Gorbahn (2010)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

$$\underbrace{\langle \vec{\tilde{Q}} \rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}}_{\text{Scheme \& Prescription}} = R_0 \langle \vec{Q} \rangle^{(1)}_{\Sigma;S} + \Delta \langle \vec{Q} \rangle^{(0)}$$

Shifts (« generalized » R_1)

$$\Delta \langle \vec{Q} \rangle^{(0)} = P_{Q;\Sigma} \left(\langle \vec{\tilde{Q}} \rangle^{(1)}_{\tilde{\Sigma};\tilde{S}} \right) - (R_0 + \epsilon \Sigma) \langle \vec{Q} \rangle^{(1)}_{\Sigma;S}$$

Projects the matrix elements on the Q-basis using the Σ-scheme

Scheme dependence

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$
$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[\Delta R_0^{-1}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} \Delta R_0^{-1}$$

- **No need to relate** evanescent operators accross different bases,
- 1-loop matrix elements of **physical operators only**,
- the shift **factorises** the schemes : erases S - dependence & restores $\tilde{S}$ - dependence

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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Two Loop ADMs in the LEFT, work in progress with J.Aebischer, Pol Morell, MP, J.Virto & 2402.00249 (Morell & Virto)

Conclusion

Interpreting the Fierz-evanescent contribution as shifts allows to express finite, ev. scheme-dependent contributions purely in terms of physical operators -> computationally easier :

- *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P]),
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The « shift approach » provides a useful and more transparent picture on how to relate different evanescent schemes and different bases of operators :

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- Novel method that allows simplified computations + discard evanescent-to-physical mixing contributions in NLO calculations :

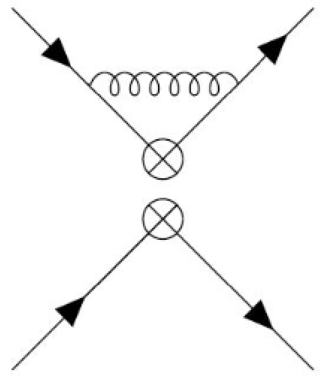
  - *A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices* ([2401.16904 : J.Aebischer, M.P, Z.Polonsky])

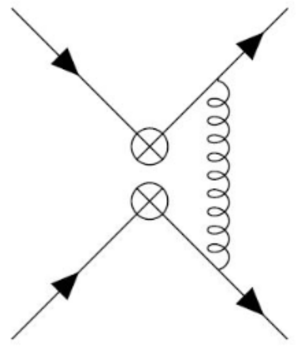
Thank you for your attention !

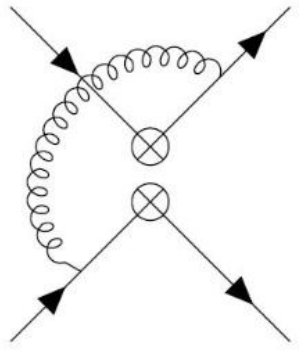
Backup Slides

One-Loop Fierz Identities [2208.10513]

Greek Identities in the (generalized) BMU scheme* :







VLR

$$\gamma_\alpha\gamma_\beta\gamma_\mu\,(1\pm\gamma_5)\,\gamma^\beta\gamma^\alpha \otimes \gamma^\mu\,(1\mp\gamma_5) = 4(1-2\underline{b_1}\epsilon)\gamma_\mu\,(1\pm\gamma_5)\otimes\gamma^\mu\,(1\mp\gamma_5)\,, \tag{A.4}$$

$$\gamma_\mu\,(1\pm\gamma_5)\,\gamma_\alpha\gamma_\beta \otimes \gamma^\mu\,(1\mp\gamma_5)\,\gamma^\alpha\gamma^\beta = 4(1+\underline{b_2}\epsilon)\gamma_\mu\,(1\pm\gamma_5)\otimes\gamma^\mu\,(1\mp\gamma_5)\,, \tag{A.5}$$

$$\gamma_\mu\,(1\pm\gamma_5)\,\gamma_\alpha\gamma_\beta \otimes \gamma^\beta\gamma^\alpha\gamma^\mu\,(1\mp\gamma_5) = 16(1-\underline{b_3}\epsilon)\gamma_\mu\,(1\pm\gamma_5)\otimes\gamma_\mu\,(1\mp\gamma_5)\,. \tag{A.6}$$

SLR

$$\gamma_\nu\gamma_\mu\,(1\mp\gamma_5)\,\gamma^\mu\gamma^\nu \otimes (1\pm\gamma_5) = 16(1-\underline{c_1}\epsilon)\,(1\mp\gamma_5)\otimes(1\pm\gamma_5)\,, \tag{A.7}$$

$$(1\mp\gamma_5)\,\gamma_\mu\gamma_\nu \otimes (1\pm\gamma_5)\,\gamma^\mu\gamma^\nu = 4(1+\underline{c_2}\epsilon)\,(1\mp\gamma_5)\otimes(1\pm\gamma_5)\,, \tag{A.8}$$

$$(1\mp\gamma_5)\,\gamma_\nu\gamma_\mu \otimes \gamma^\mu\gamma^\nu\,(1\pm\gamma_5) = 4(1-2\underline{c_3}\epsilon)\,(1\mp\gamma_5)\otimes(1\pm\gamma_5)\,. \tag{A.9}$$

SLL

$$\gamma_\nu\gamma_\mu\,(1\pm\gamma_5)\,\gamma^\mu\gamma^\nu \otimes (1\pm\gamma_5) = 16(1-\underline{d_1}\epsilon)\,(1\pm\gamma_5)\otimes(1\pm\gamma_5)\,, \tag{A.10}$$

$$\begin{aligned}&(1\pm\gamma_5)\,\gamma_\mu\gamma_\nu \otimes (1\pm\gamma_5)\,\gamma^\mu\gamma^\nu\\ &= (4-2\underline{d_2}\epsilon)\,(1\pm\gamma_5)\otimes(1\pm\gamma_5) - \sigma_{\mu\nu}\,(1\pm\gamma_5)\otimes\sigma^{\mu\nu}\,(1\pm\gamma_5)\,,\end{aligned} \tag{A.11}$$

$$\begin{aligned}&(1\pm\gamma_5)\,\gamma_\mu\gamma_\nu \otimes \gamma^\nu\gamma^\mu\,(1\pm\gamma_5)\\ &= (4-2\underline{d_3}\epsilon)\,(1\pm\gamma_5)\otimes(1\pm\gamma_5) + \sigma_{\mu\nu}\,(1\pm\gamma_5)\otimes\sigma^{\mu\nu}\,(1\pm\gamma_5)\,.\end{aligned} \tag{A.12}$$

*The **Greek Method** was used to reduce the Dirac Algebra in D-dim :
Tracas & Vlachos (1982),
Buras, Misiak, Urban (2000)

One-Loop Fierz Identities [2208.10513]

Fierz Identities* are relations, valid in d=4, between four-fermion operators :

$$\mathcal{F}_4 \circ \left(\bar{f}_1 \gamma_\mu P_X f_2\right)\left(\bar{f}_3 \gamma^\mu P_X f_4\right) \equiv \left(\bar{f}_1 \gamma_\mu P_X f_4\right)\left(\bar{f}_3 \gamma^\mu P_X f_2\right)$$

*Fierz (1939)

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$$\mathcal{F}_4 \circ \left(\bar{f}_1 \sigma_{\mu\nu} P_X f_2\right)\left(\bar{f}_3 \sigma^{\mu\nu} P_X f_4\right) \equiv -6\left(\bar{f}_1 P_X f_4\right)\left(\bar{f}_3 P_X f_2\right) + \frac{1}{2}\left(\bar{f}_1 \sigma_{\mu\nu} P_X f_4\right)\left(\bar{f}_3 \sigma^{\mu\nu} P_X f_2\right)$$

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Fierz Identities are relations, valid in d=4, between four-fermion operators :

$$\mathcal{F}_4 \circ (\bar{f}_1 \gamma_\mu P_X f_2)(\bar{f}_3 \gamma^\mu P_X f_4) \equiv (\bar{f}_1 \gamma_\mu P_X f_4)(\bar{f}_3 \gamma^\mu P_X f_2)$$

$$\mathcal{F}_4 \circ (\bar{f}_1 \sigma_{\mu\nu} P_X f_2)(\bar{f}_3 \sigma^{\mu\nu} P_X f_4) \equiv -6\,(\bar{f}_1 P_X f_4)(\bar{f}_3 P_X f_2) + \frac{1}{2}(\bar{f}_1 \sigma_{\mu\nu} P_X f_4)(\bar{f}_3 \sigma^{\mu\nu} P_X f_2)$$

$$\vdots$$

$$\mathcal{O} - \mathcal{F}_4 \circ \mathcal{O} = 0 \quad (d = 4)$$

One-Loop Fierz Identities [2208.10513]

These relations do not hold at the loop-level and lead to the introduction of Evanescent Operators.

$$\langle \mathcal{O} \rangle^{(1)} - \langle \mathcal{F}_4 \circ \mathcal{O} \rangle^{(1)} = \langle E \rangle^{(1)} \quad (d = 4 - 2\epsilon)$$

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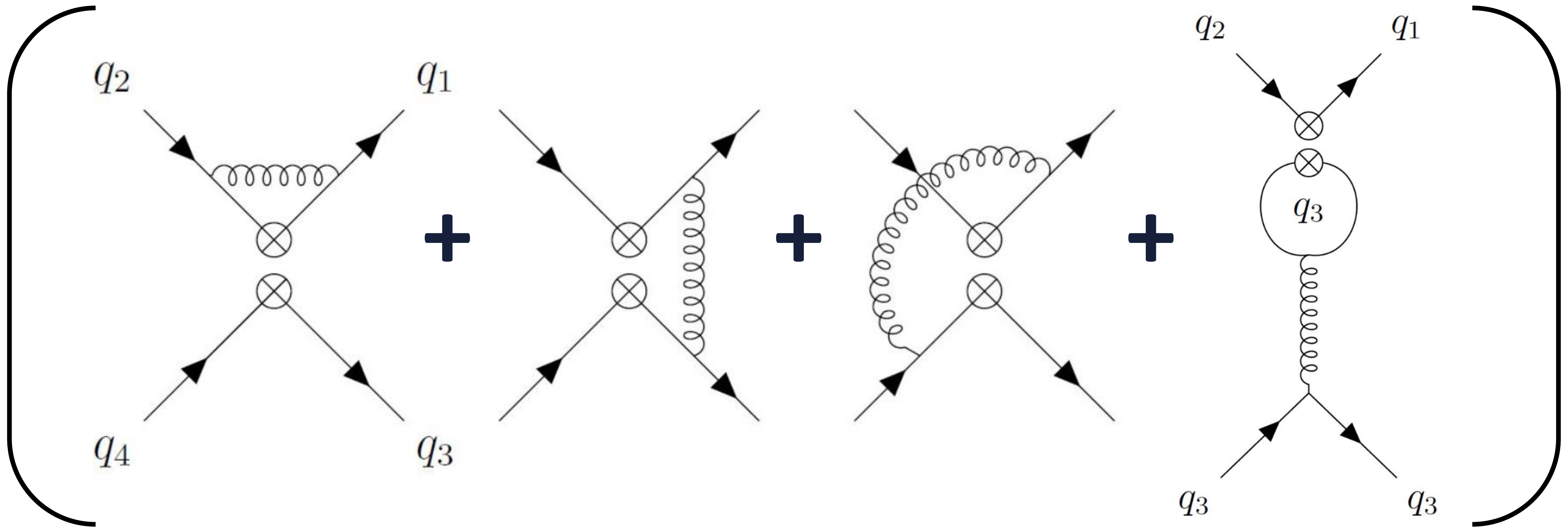
(QCD)



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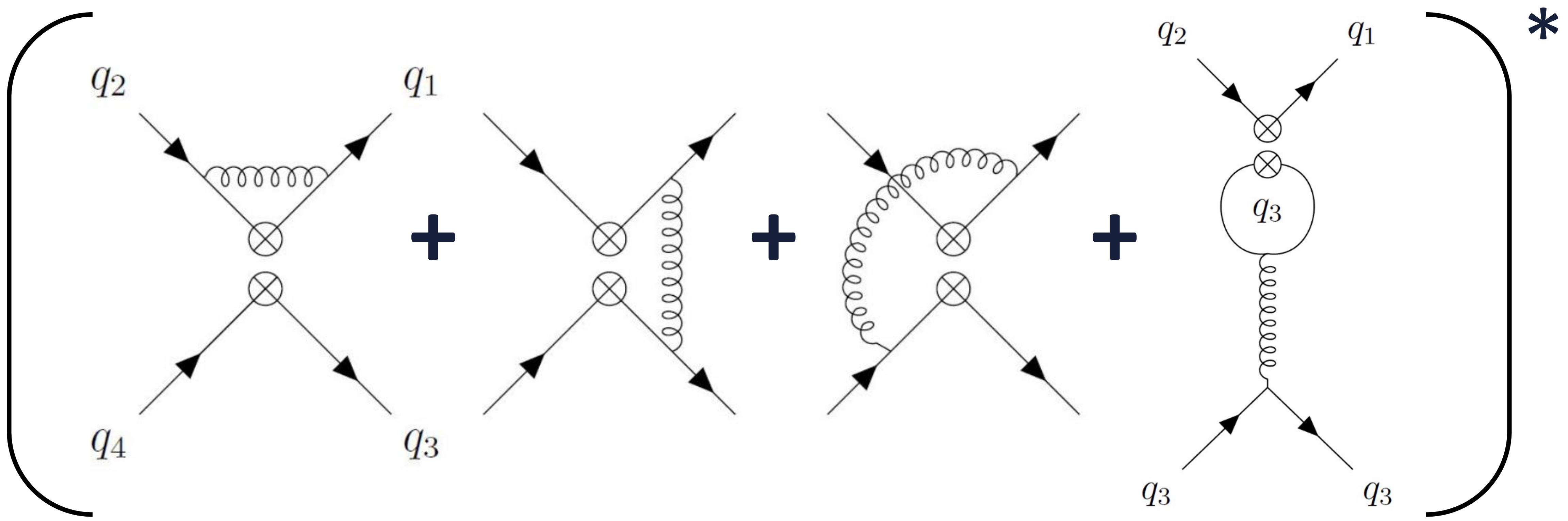

At one-loop, **finite, local, scheme-dependent** shifts are generated !

*Only UV poles need to be computed

One-Loop Fierz Identities [2208.10513]

One-loop QED and QCD shifts to four-fermion ops. in the (generalized) BMU scheme*

$$\mathcal{O} = \left(\bar{f}_1 \Gamma_A f_2\right) \left(\bar{f}_3 \Gamma_B f_4\right) \text{ with } f_i = \{q, \ell\} \text{ and } \Gamma_X = \{P_X,\, \gamma^\mu P_X,\, \sigma^{\mu\nu} P_X\} \quad X = L \text{ or } R$$

*The **Greek Method** was used to reduce the Dirac Algebra :
Tracas & Vlachos (1982),
Buras, Misiak, Urban (2000)

One-Loop Fierz Identities [2208.10513]

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Operator	Tree-level Fierz
$S^{LL}_{q_1 q_2 q_3 q_4}$	$-\frac{1}{2}\widetilde{S}^{LL}_{q_1 q_4 q_3 q_2} - \frac{1}{8}\widetilde{T}^{LL}_{q_1 q_4 q_3 q_2}$

$$\tilde{S}^{AB}_{f_1 f_2 f_3 f_4} \equiv \left(\bar{f}_1^\alpha P_A f_2^\beta\right)\left(\bar{f}_3^\beta P_B f_4^\alpha\right)$$

$$S^{AB}_{f_1 f_2 f_3 f_4} \equiv \left(\bar{f}_1^\alpha P_A f_2^\alpha\right)\left(\bar{f}_3^\beta P_B f_4^\beta\right)$$

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Operator	Tree-level Fierz	QCD shift	QED shift
$S^{LL}_{q_1q_2q_3q_4}$	$-\frac{1}{2}\widetilde{S}^{LL}_{q_1q_4q_3q_2} - \frac{1}{8}\widetilde{T}^{LL}_{q_1q_4q_3q_2}$	$-\frac{1}{N_c}S^{LL}_{q_1q_2q_3q_4} + \widetilde{S}^{LL}_{q_1q_2q_3q_4}$ $+\frac{N_c^2-6}{8N_c}T^{LL}_{q_1q_2q_3q_4} + \frac{5}{8}\widetilde{T}^{LL}_{q_1q_2q_3q_4}$	$\frac{1}{2}(Q_1+Q_2)(Q_3+Q_4)S^{LL}_{q_1q_2q_3q_4}$ $+\frac{1}{8}\left(Q_{1234}+2Q_{1423}+3Q_{1324}\right)T^{LL}_{q_1q_2q_3q_4}$

(QCD)

$$\boxed{\langle S^{LL}_{q_1q_2q_3q_4}\rangle^{(1)} - \langle \mathcal{F}_4 \circ S^{LL}_{q_1q_2q_3q_4}\rangle^{(1)}}$$

(QED)

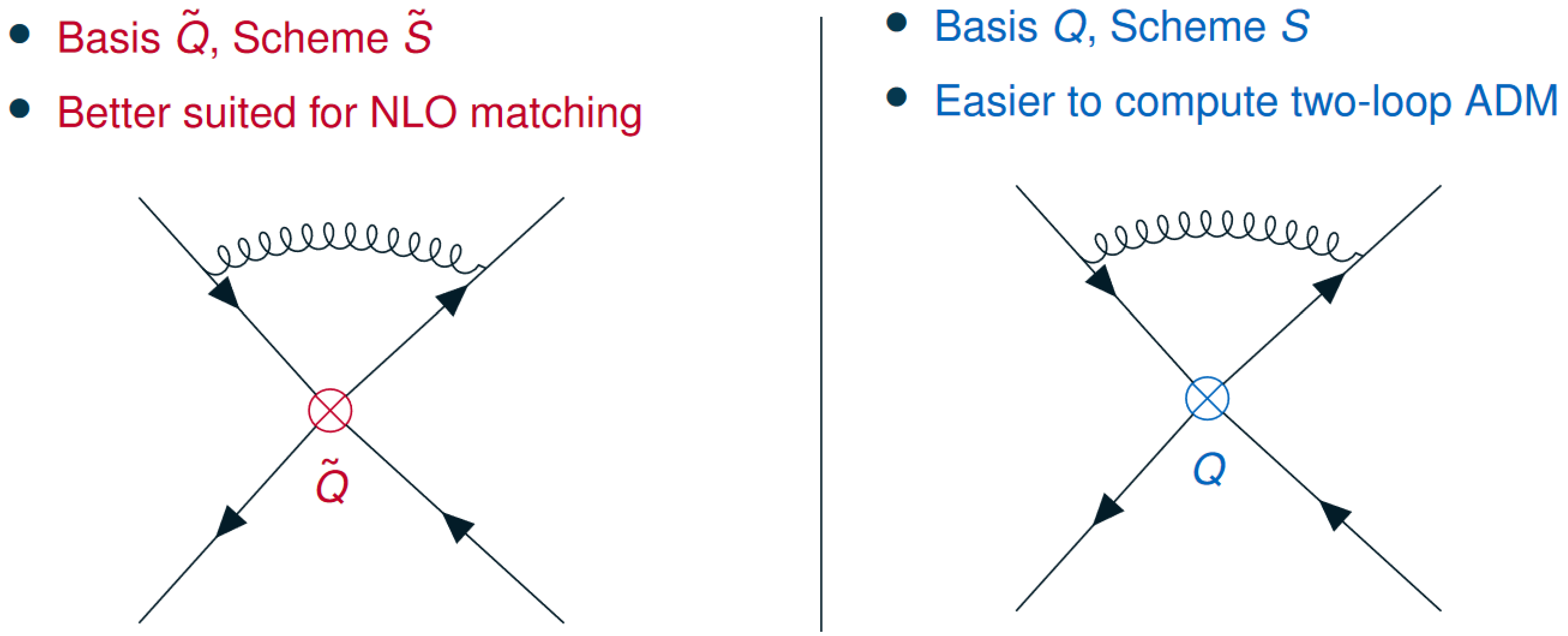
Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

- Evanescent operators : definitions, prescription & scheme ✓

- The shift paradigm ✓

- Shift paradigm in the context of change of bases of NLO ADMs:

  - ***Renormalization Scheme Factorization of one-loop Fierz Identities*** (**[2306.16449 : J.Aebischer, M.P, Z.Polonsky]**)



[Zach's slides]

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- Shift paradigm in the context of change of bases of NLO ADMs:

  - ***Renormalization Scheme Factorization of one-loop Fierz Identities*** **([2306.16449 : J.Aebischer, M.P, Z.Polonsky])**

- Shift « method » allows to simultaneously change basis and scheme in a simple way !
- The <u>double scheme-dependence</u> appearing in the shifts <u>factorizes</u>.

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

In our paper, we checked :

➡ The <u>full equivalence</u> with the « traditional » method a.k.a $Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$

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- The shift « method » in an explicit example* :

*Brod, Polonsky, and Stamou (2023)

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$$\mathcal{L}_{\mathrm{Lar}} = -\sqrt{2}G_F\left(C_1^{eb}\mathcal{O}_1^{eb} + C_1^{be}\mathcal{O}_1^{be} + C_2\mathcal{O}_2 + C_3\mathcal{O}_3\right)$$

Larin's scheme [Larin, 1993]

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

CP-odd operators: induce electron EDM.
(Explicit form in backup slides)

*Brod, Polonsky, and Stamou (2023)

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$$\Big\downarrow \mathcal{F}_4$$

$$\mathcal{L}_{\mathrm{NDR}} = -\sqrt{2}G_F\Big(\tilde{C}_s\tilde{\mathcal{O}}_s + \tilde{C}_v\tilde{\mathcal{O}}_v + \tilde{C}_t\tilde{\mathcal{O}}_t + \tilde{C}_3\tilde{\mathcal{O}}_3\Big)$$

- Computed** LO and NLO ADMs + Shifts using NDR

*Brod, Polonsky, and Stamou (2023)
**Chetyrkin, Misiak, Munz (1998)

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$$\tilde{\gamma}^{(1)} = \tilde{\gamma}^{(1),\mathrm{SI}} + \underset{\uparrow}{a_s}\tilde{\gamma}^{(1),s} + \underset{\uparrow}{a_v}\tilde{\gamma}^{(1),v} + \underset{\uparrow}{a_t}\tilde{\gamma}^{(1),t}$$

NDR scheme-dependence

- Computed** LO and NLO ADMs + Shifts using NDR

*Brod, Polonsky, and Stamou (2023)
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Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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- Computed** LO and NLO ADMs + Shifts using NDR

Explicitely checked the ADMs and the scheme factorization via the shifts Δ
(+ NDR is much simpler…)

*Brod, Polonsky, and Stamou (2023)
**Chetyrkin, Misiak, Munz (1998)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

$$\langle \vec{\tilde{Q}} \rangle^{(1)}_{\tilde{\Sigma};\tilde{S}} = R_0 \langle \vec{Q} \rangle^{(1)}_{\Sigma;S} + \Delta \langle \vec{Q} \rangle^{(0)} \qquad \Delta \langle \vec{Q} \rangle^{(0)} = P_{Q;\Sigma}\left(\langle \vec{\tilde{Q}} \rangle^{(1)}_{\tilde{\Sigma};\tilde{S}} \right) - (R_0 + \epsilon \Sigma) \langle \vec{Q} \rangle^{(1)}_{\Sigma;S}$$

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$
$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[\Delta R_0^{-1}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} \Delta R_0^{-1}$$

Example* (Explicit form in backup slides) :

$$\mathcal{L}_{\text{Lar}} = -\sqrt{2} G_F \left(C_1^{eb} \mathcal{O}_1^{eb} + C_1^{be} \mathcal{O}_1^{be} + C_2 \mathcal{O}_2 + C_3 \mathcal{O}_3 \right) \xrightarrow{\mathcal{F}_4} \mathcal{L}_{\text{NDR}} = -\sqrt{2} G_F \left(\tilde{C}_s \tilde{\mathcal{O}}_s + \tilde{C}_v \tilde{\mathcal{O}}_v + \tilde{C}_t \tilde{\mathcal{O}}_t + \tilde{C}_3 \tilde{\mathcal{O}}_3 \right)$$

Larin’s scheme [Larin, 1993]

$$\gamma_5 = \frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma$$

Computed LO and NLO ADMs + Shifts using NDR

*Brod, Polonsky, and Stamou (2023)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

$$\mathcal{L}_{\rm Lar} = -\sqrt{2}G_F\Big(C_1^{eb}\mathcal{O}_1^{eb} + C_1^{be}\mathcal{O}_1^{be} + C_2\mathcal{O}_2 + C_3\mathcal{O}_3\Big)$$

$$\mathcal{O}_1^{ij} = \big(\bar{\psi}_i\psi_i\big)\big(\bar{\psi}_j\, i\gamma_5\,\psi_j\big), \quad \mathcal{O}_2 = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\big(\bar{e}\sigma_{\mu\nu}e\big)\big(\bar{b}\sigma_{\rho\sigma}b\big), \quad \mathcal{O}_3 = \frac{Q_e m_b}{2e}\big(\bar{e}\sigma^{\mu\nu}e\big)\tilde{F}_{\mu\nu}$$

$$\Big\downarrow \quad \mathcal{F}_{ij} = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{8} & 0 \\ 1 & -1 & 0 & 0 \\ -3 & -3 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{\mathcal{O}}_i = \left[\mathcal{F}_{ij} + \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n \Delta_{ij}^{(n)}\right]\mathcal{O}_j$$

$$\mathcal{L}_{\rm NDR} = -\sqrt{2}G_F\Big(\tilde{C}_s\tilde{\mathcal{O}}_s + \tilde{C}_v\tilde{\mathcal{O}}_v + \tilde{C}_t\tilde{\mathcal{O}}_t + \tilde{C}_3\tilde{\mathcal{O}}_3\Big)$$

$$\tilde{\mathcal{O}}_s = \frac{1}{2}\Big[\big(\bar{b}i\gamma_5 e\big)\big(\bar{e}b\big) + \big(\bar{b}e\big)\big(\bar{e}i\gamma_5 b\big)\Big], \quad \tilde{\mathcal{O}}_v = \frac{1}{2}\Big[\big(\bar{b}i\gamma^\mu\gamma_5 e\big)\big(\bar{e}\gamma_\mu b\big) - \big(\bar{b}\gamma^\mu e\big)\big(\bar{e}i\gamma_\mu\gamma_5 b\big)\Big]$$

$$\tilde{\mathcal{O}}_t = \frac{1}{2}\Big[\big(\bar{b}i\sigma_{\mu\nu}\gamma_5 e\big)\big(\bar{e}\sigma^{\mu\nu}b\big) + \big(\bar{b}\sigma_{\mu\nu}e\big)\big(\bar{e}i\sigma^{\mu\nu}\gamma_5 b\big)\Big], \quad \tilde{\mathcal{O}}_3 = \frac{Q_e m_b}{2e}\big(\bar{e}i\sigma_{\mu\nu}\gamma_5 e\big)F^{\mu\nu}.$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Relating two bases with different schemes :

$$\underbrace{\langle \vec{\tilde{Q}} \rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}}_{\text{Scheme \& Prescription}} = R_0 \langle \vec{Q} \rangle^{(1)}_{\Sigma;S} + \Delta \langle \vec{Q} \rangle^{(0)}$$

Shifts (« Generalized » R_1)

$$P_{Q;\Sigma}\,\mathcal{M}\left[\langle \vec{\tilde{Q}} \rangle^{(0)}\right] = \mathcal{M}\left[(\mathcal{F} - \epsilon\Sigma)\langle \vec{Q} \rangle^{(0)}\right]$$

$$\vec{E} = K\left(\vec{\tilde{Q}} - (\mathcal{F} + \epsilon\Sigma)\vec{Q}\right)$$

$$\Delta \langle \vec{Q} \rangle^{(0)} = P_{Q;\Sigma}\left(\langle \vec{\tilde{Q}} \rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) - (R_0 + \epsilon\Sigma)\langle \vec{Q} \rangle^{(1)}_{\Sigma;S}$$

Projects the matrix elements on the Q-basis using the Σ-scheme

$$\tilde{\mathcal{O}}_i = \left[\mathcal{F}_{ij} + \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n \Delta^{(n)}_{ij}\right]\mathcal{O}_j$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

- NDR: $(\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\sigma d_L)(\bar{s}_L \gamma^\mu \gamma^\nu \gamma^\sigma d_L) - (16 - a\epsilon) Q_{S2}$ (Herrlich, Nierste, 1996)

- Larin:
$$[(\bar{e}\gamma_{[\mu}\gamma_{\nu]}\gamma^{[\rho}\gamma^{\sigma]}e)(\bar{q}\gamma^{[\mu}\gamma^{\nu]}\gamma^{[\tau}\gamma^{\zeta]}q) + (\bar{e}\gamma^{[\rho}\gamma^{\sigma]}\gamma_{[\mu}\gamma_{\nu]}e)(\bar{q}\gamma^{[\tau}\gamma^{\zeta]}\gamma^{[\mu}\gamma^{\nu]}q)]\epsilon_{\rho\sigma\tau\zeta}$$
$$-48(Q_1^{eq} + Q_1^{qe}) + 16 Q_2^{eq}$$
(Brod, Stamou, ZP, 2023)

- HV: $(\bar{q}_p \hat{\gamma}^\mu \hat{\gamma}^\nu \tilde{\sigma}^{\lambda\sigma} q_p)(\bar{q}_r \hat{\gamma}_\mu \hat{\gamma}_\nu \sigma_{\lambda\sigma} q_r)$ (Bühler, Stoffer, 2023)

[Zach's slides]

A Simple Dirac Prescription for Two-Loop Anomalous Dimension Matrices ([2401.16904 : J.Aebischer, M.P, Z.Polonsky])

- Any physical observable must be *independent* of the choice of both the prescription and scheme (O(ε))

- In one prescription, a structure may be reducible, but in another prescription, the same structure will require an evanescent operator and hence a finite subtraction, which is not *does not* correspond to a choice of renormalization scheme

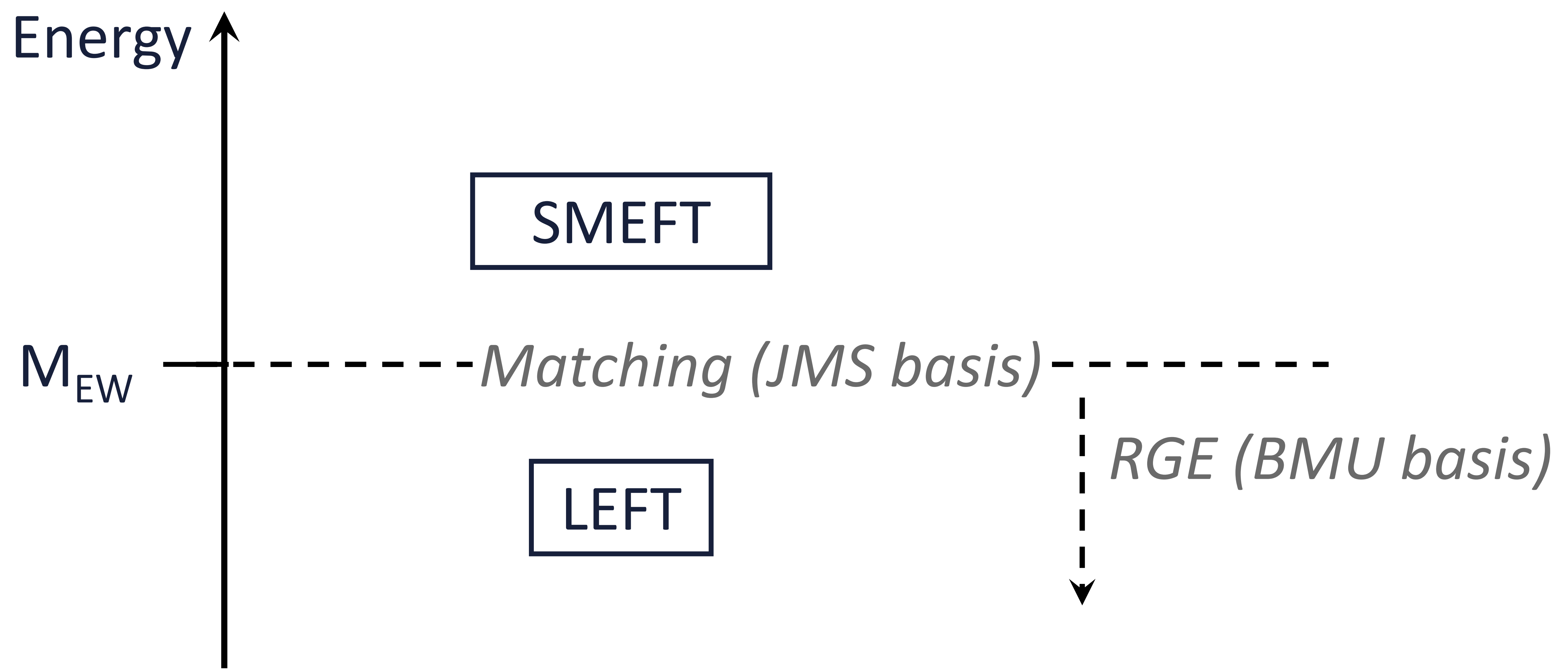
➡ remove evanescent-to-physical mixing contributions in NLO calculations:

$$\gamma^{(1)} = 4Z_{QQ}^{(2;1)} - 2Z_{QE}^{(1;1)}\underline{Z_{EQ}^{(1;0)}}$$

- Considered the EFT of scalar-mediated muon decay and treated it with 3 prescriptions: NDR, NP and GP

$$\text{"}\langle E^i\rangle^{(1)} \propto \langle E^j\rangle^{(0)} - C_{i\ell}\langle E^\ell\rangle^{(0)} + \underbrace{(K_{im} - C_{i\ell}C_{\ell m})}_{\text{The choice of prescription must set this piece to 0}}\langle Q^m\rangle^{(0)}\text{"}$$

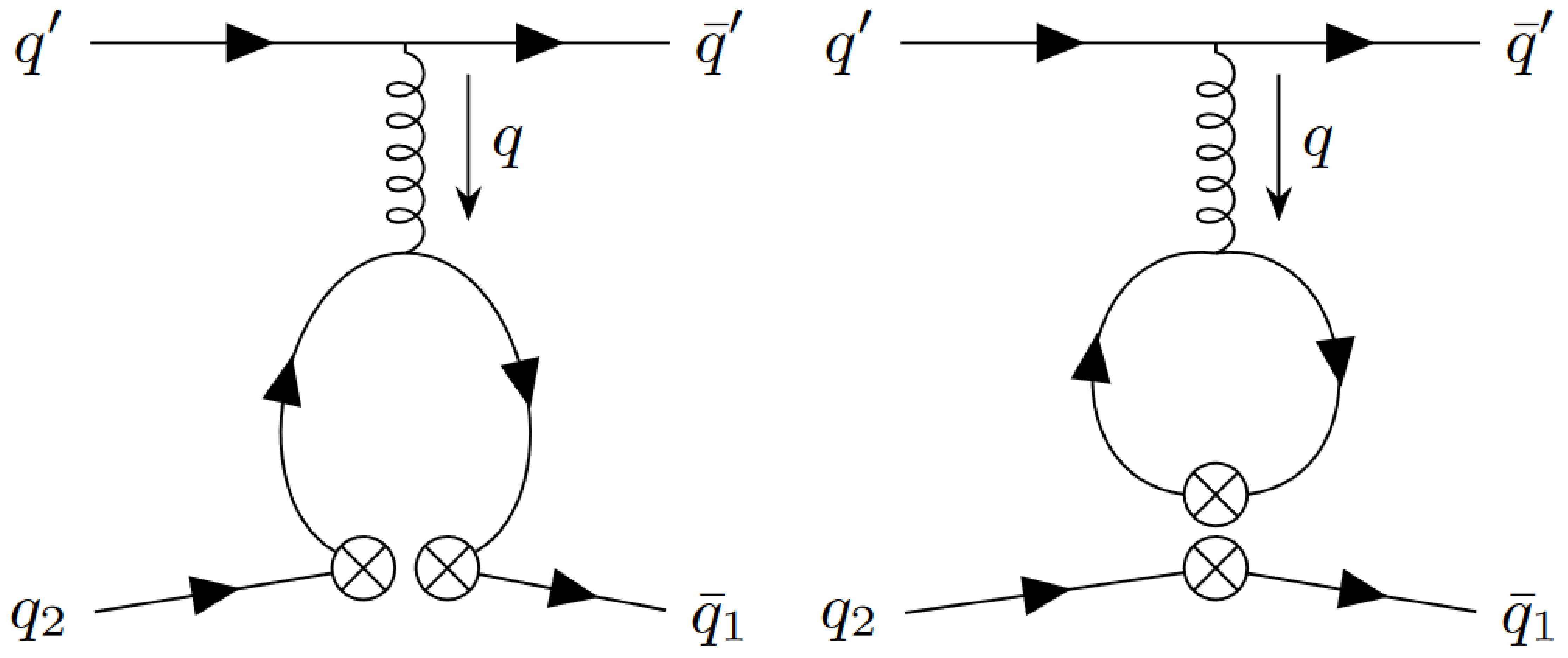
Two Loop ADMs in the LEFT ([J.Aebischer, Pol Morell, M.P, J.Virto])



Compute and express all the ADMs in the JMS ev. scheme.

Dipole Operators in Fierz Identities [2211.01379]

- Only Penguin contributions to four-fermion operators



open closed

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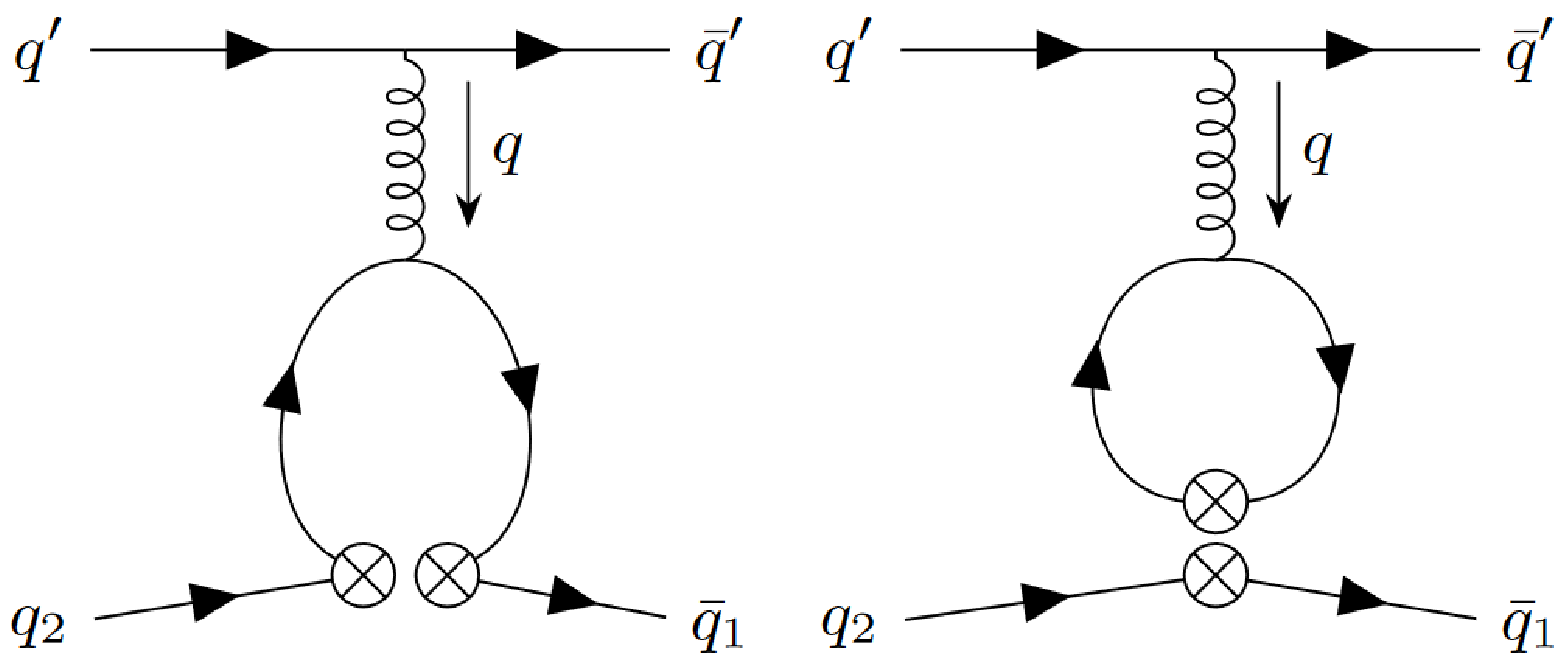

open closed

- One complication* : $\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho\gamma_5\right] \longrightarrow$ inconsistent in NDR !

*Only closed penguins with tensors insertions

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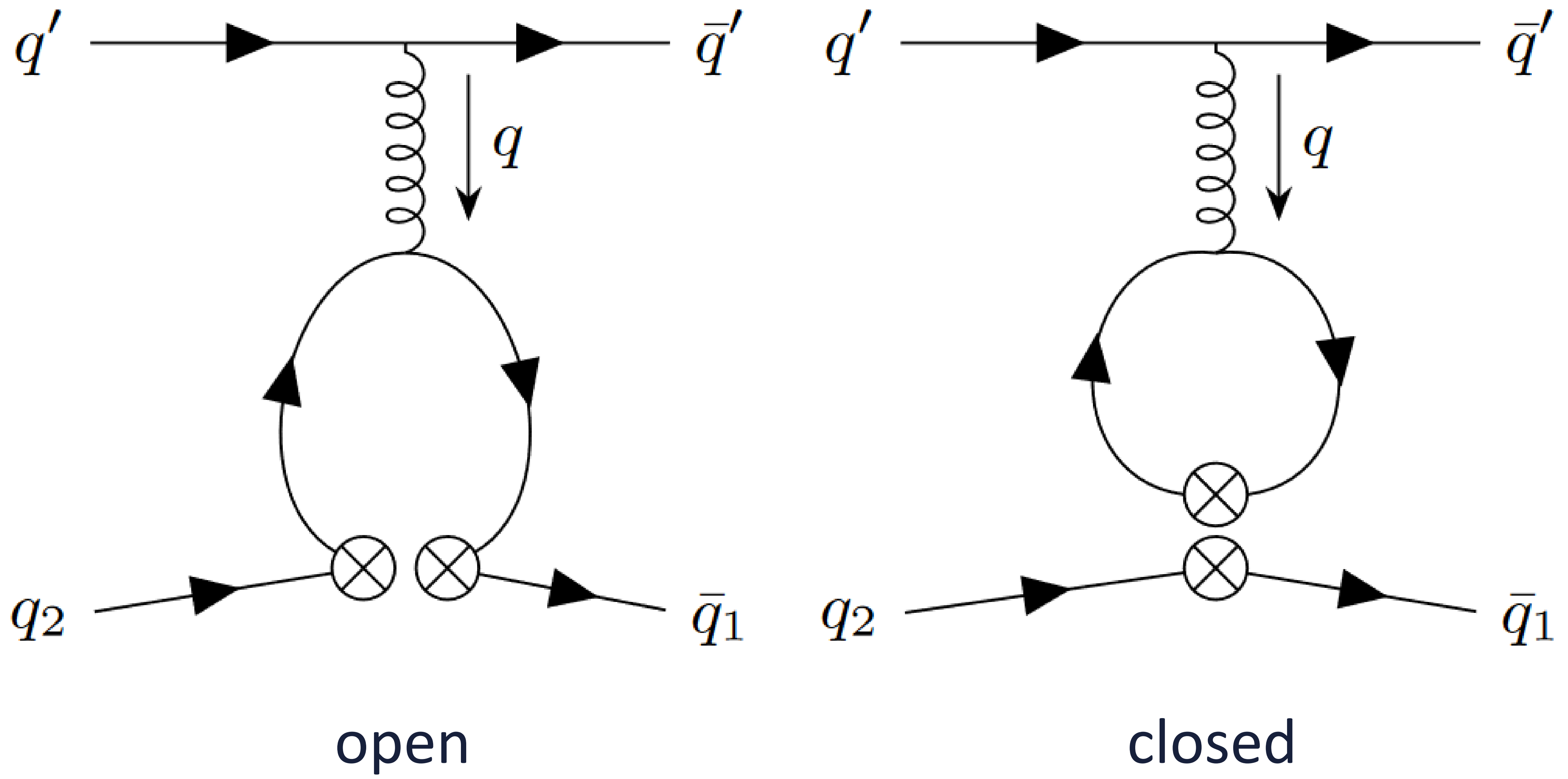

- One complication* : $\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho\gamma_5\right] \longrightarrow$ inconsistent in NDR !

- Prescription : **modified** Naive Dimensional Regularization (NDR)**

*Only closed penguins with tensors insertions
**Misiak (1993)
**Chetyrkin, Zoller (2012)
**Mihaila, Salomon, Steinhauser (2012)

Dipole Operators in Fierz Identities [2211.01379]

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- Prescription : **modified** Naive Dimensional Regularization (NDR)*

Operator	QCD shift	QED shift
$V^{LR}_{q_1 q_3 q_3 q_2}$	$\frac{m_{q_3}}{m_q}\mathcal{D}^R_{q_1 q_2 G}$	$A_{q_3}\mathcal{D}^R_{q_1 q_2 \gamma}$

$$V^{AB}_{f_1 f_2 f_3 f_4} \equiv (\bar{f}_1^\alpha \gamma^\mu P_A f_2^\alpha)\,(\bar{f}_3^\beta \gamma_\mu P_B f_4^\beta)$$

$$A_{f'} \equiv \frac{m_{f'}}{m_f} Q_{f'}$$

$$D^B_{q_1 q_2 G} = \frac{1}{g_s} m_q (\bar{q}_1 \sigma^{\mu\nu} P_B T^A q_2) G^A_{\mu\nu}$$

$$D^B_{f_1 f_2 \gamma} = \frac{1}{e} m_f (\bar{f}_1 \sigma^{\mu\nu} P_B f_2) F_{\mu\nu}$$

*Misiak (1993)
*Chetyrkin, Zoller (2012)
*Mihaila, Salomon, Steinhauser (2012)

Dipole Operators in Fierz Identities [2211.01379]

Example* : One-loop contributions to the muon magnetic moment in the LEFT

$$a_\ell = \frac{\alpha \mathsf{q}_e^2}{2\pi} - 4\frac{m_\ell}{e\mathsf{q}_e}\mathrm{Re}\, L_{\substack{e\gamma\\ \ell\ell}}(\mu)\left\{1 - \frac{\alpha\mathsf{q}_e^2}{4\pi}\left[2 + 5\log\left(\frac{\mu^2}{m_\ell^2}\right)\right]\right\} + a_\ell^{4\ell} + \boxed{a_\ell^{2\ell 2q}} + \mathcal{O}(L_{e\gamma}^2)$$

Semileptonic tensor contribution

$$O^{T,RR}_{\substack{eu\\ ijkl}} = (\overline{e}^i \sigma_{\mu\nu} P_R e^j)(\overline{u}^k \sigma^{\mu\nu} P_R u^l)$$

*Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer (2021),
*Dekens, Stoffer (2022)

Dipole Operators in Fierz Identities [2211.01379]

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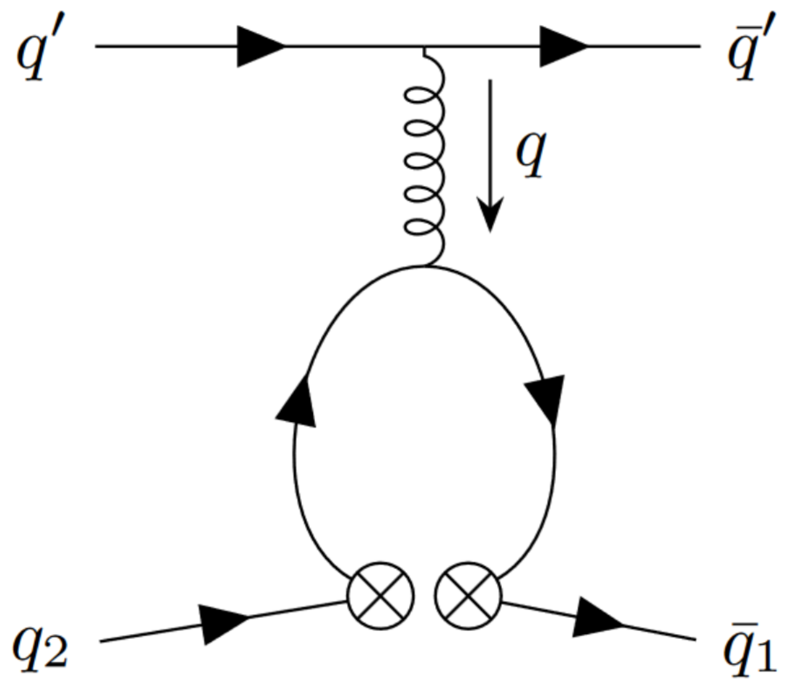
$$O^{T,RR}_{\substack{eu\\ ijkl}} = (\bar{e}^i \sigma_{\mu\nu} P_R e^j)(\bar{u}^k \sigma^{\mu\nu} P_R u^l) \xrightarrow{\mathcal{F}_4}$$



$$\cancel{\mathrm{Tr}\left[\gamma_\mu \gamma_\nu \gamma_\sigma \gamma_\rho \gamma_5\right]}$$

Use NDR + our shifts to recover the result

$$\boxed{\text{Illustration of how Fierz + shifts allow to go to a simpler basis to compute and convert the result back into the original basis}}$$

*Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer (2021),
*Dekens, Stoffer (2022)