Anomalous Dimensions via On-Shell Methods

Operator Mixing and Leading Mass Effects

Higgs and Effective Field Theory – HEFT 2024

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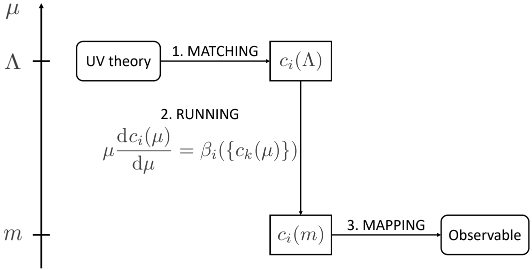
- EFT Approach: Standard Model as the low-energy description of a more fundamental theory emerging at a large energy scale Λ

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{c_i}{\Lambda^{[\mathcal{O}_i]-4}} \mathcal{O}_i \,.$$

- Running: The high-scale Wilson coefficients c_i to be evolved from the scale Λ down to the experimental scale.
- EFT anomalous dimensions: crucial for interpreting experimental results.



- Only (products of) operators with the same quantum numbers can mix.
- By focusing on the physical degrees of freedom, on-shell methods have been exploited to prove several nonrenormalization theorems (i.e. selection rules) based on:
  - HELICITY; [Cheung, Shen (15)]
  - LENGTH; [Bern, Parra-Martinez, Sawyer (20)]
  - ANGULAR MOMENTUM. [Jiang, Shu, Xiao, Zheng (21)]

S-Matrix & Dilatation Operator

- Form factor associated with a local, gauge-invariant operator $\mathcal{O}_i$:

$$F_i(\vec{n};q) = \frac{1}{\Lambda^{[\mathcal{O}_i]-4}} \langle \vec{n}|\mathcal{O}_i(q)|0\rangle \ .$$

- Exploiting the fundamental relations [Miró, Ingoldby, Riembau (20)]
  - ◇ Analyticity: $F_i^*(\{s_{ij}-i\epsilon\}) = F_i(\{s_{ij}+i\epsilon\})$
  - ◇ Unitarity: $\sum_{\vec{n}} \int d\Pi_n \ |\vec{n}\rangle\langle\vec{n}| = \mathbb{1}\,, \quad d\Pi_n = \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}$
  - ◇ CPT theorem: $\langle\vec{n};\text{out}|\mathcal{O}_i(x)|0\rangle \ = \ \langle 0|\mathcal{O}_i^\dagger(-x)|\vec{n};\text{in}\rangle$

it is possible to show that [Caron-Huot, Wilhelm (16)]

$$e^{-i\pi D}F_i^*(\vec{n}) = (SF_i^*)(\vec{n}) \left(= \sum_{\vec{m}} \int d\Pi_m \ \langle\vec{n}|S|\vec{m}\rangle \ F_i^*(\vec{m}) \right)$$

where $S = \mathbb{1} + i\mathcal{M}$ is the S-matrix and $D = \sum_i p_i \cdot \partial/\partial p_i$ is the dilatation operator.

$S\&D$ Relation

$$(e^{-i\pi D} - 1)F_i^* = i(\mathcal{M}F_i^*)$$

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- In dimensional regularization and in absence of masses, $D \simeq -\mu\,\partial/\partial\mu$, which implies

Callan-Symanzik Equation

$$DF_j = \left(\frac{\partial \beta_i}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}} + \delta_{ij}\beta_g \frac{\partial}{\partial g}\right) F_i$$

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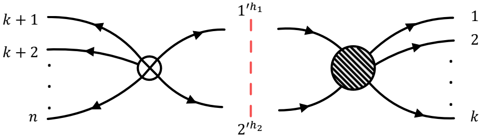
- Can be combined and expanded, *e.g.* at one-loop

$$\left(\frac{\partial\beta_i^{(1)}}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}}^{(1)} + \delta_{ij}\beta_g^{(1)}\frac{\partial}{\partial g}\right)F_i^{(0)} = -\frac{1}{\pi}(\mathcal{M}F_j)^{(1)}$$



- Near the Gaussian fixed point $(*)$, where $c_i = 0\ \forall i$, the RGEs for the Wilson coefficients c_i can be Taylor expanded as

$$\mu \frac{\mathrm{d} c_i}{\mathrm{d}\mu} = \sum_{n>0} \frac{1}{n!} \gamma_{i\leftarrow j_1,\ldots,j_n} c_{j_1} \cdots c_{j_n} = \gamma_{i\leftarrow j} c_j + \frac{1}{2} \gamma_{i\leftarrow j,k} c_j c_k + \cdots \ ,$$

where

Anomalous Dimensions

$$\gamma_{i\leftarrow j_1,\ldots,j_n} = \left. \frac{\partial^n \beta_i}{\partial c_{j_1} \cdots \partial c_{j_n}} \right|_*$$

- They can be perturbatively expanded as functions of the couplings of the renormalizable Lagrangian

$$\gamma_{i\leftarrow j_1,\ldots,j_n} = \sum_{\ell>0} \gamma^{(\ell)}_{i\leftarrow j_1,\ldots,j_n} \ .$$

- In the case of a double operator insertion with $j,k\neq i$, we can neglect $\gamma_{i,\mathrm{IR}}$ and obtain the master formulae at the desired order:

One-loop order

$$\gamma^{(1)}_{i\leftarrow j,k} F_i\big|^{(0)}_{*} = -\frac{1}{\pi}\,\frac{\partial}{\partial c_k}\bigg|_{*}(\mathcal{M}F_j)^{(1)}$$

Two-loop order

$$\gamma^{(2)}_{i\leftarrow j,k} F_i\big|^{(0)}_{*} = -\frac{1}{\pi}\,\frac{\partial}{\partial c_k}\bigg|_{*}(\mathrm{Re}\,\mathcal{M}\,\mathrm{Re}\,F_j)^{(2)} - \gamma^{(1)}_{i\leftarrow j}\,\frac{\partial}{\partial c_k}\bigg|_{*}\mathrm{Re}\,F^{(1)}_i - \gamma^{(1)}_{i\leftarrow j,k}\,\mathrm{Re}\,F_i\big|^{(1)}_{*}$$

- The extension of these formulae for multiple operator insertions $\gamma_{i\leftarrow j_1,\dots,j_n}$ is straightforward.

Higgs Low-Energy Theorem

- Dimensions of the renormalizing operators and the renormalized one do not match $\Longrightarrow$ anomalous dimension must scale as a power of dimensionful couplings (*e.g.* fermion masses).
- $m_f \neq 0 \Longrightarrow$ chirality-violating and preserving operators can mix.

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- Fermionic Higgs interaction in the SM

$$\mathcal{L}_h^{\text{int}} = -\left(1 + \frac{h}{v}\right) \sum_f m_f \bar{f} f$$

implies [Ellis, Gaillard, Nanopoulos (76); Shifman et al. (79)]

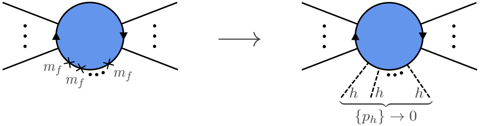
Higgs Low-Energy Theorem

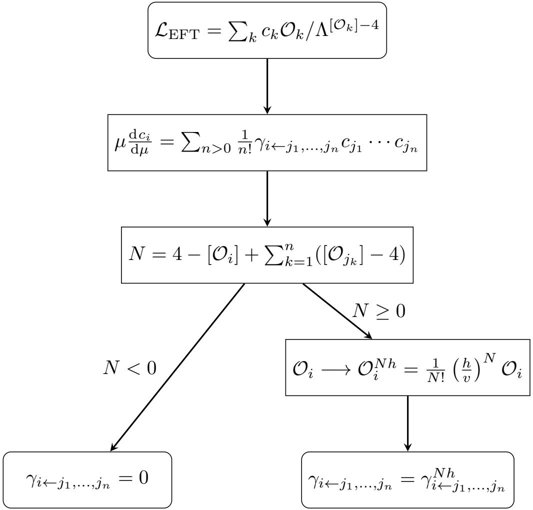
$$\lim_{\{p_h\}\to 0} \mathcal{M}(A \to B + Nh) = \sum_f \left(\frac{m_f}{v} \frac{\partial}{\partial m_f}\right)^N \mathcal{M}(A \to B)$$

The amplitude requires N fermion mass insertions not to vanish

$$\Downarrow$$

Consider an equivalent amplitude entailing N extra massless Higgs fields





LEFT

Low-energy EFT of the SM below the electroweak scale [Jenkins, Manohar, Stoffer (18); Jenkins, Manohar, Stoffer (18)]. The dimension-5 Lagrangian consists of dipole operators

$$\mathcal{L}^{(5)}_{\mathrm{LEFT}} \supset \frac{a_f}{\Lambda}\mathcal{O}_{\mathrm{CM}} + \frac{d_f}{\Lambda}\mathcal{O}_{\mathrm{CE}}\,, \qquad \mathcal{O}_{\mathrm{CM}} = \bar{f}\sigma^{\mu\nu}T^a f G^a_{\mu\nu}\,, \qquad \mathcal{O}_{\mathrm{CE}} = i\bar{f}\sigma^{\mu\nu}\gamma_5 T^a f G^a_{\mu\nu}\,.$$

- Consider the renormalization of $\mathcal{O}_\vartheta = G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu}$ as induced by $\mathcal{O}_{\mathrm{CM}}$ and $\mathcal{O}_{\mathrm{CE}}$.

Example: Running of θ_{QCD} in LEFT

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Low-energy EFT of the SM below the electroweak scale [Jenkins, Manohar, Stoffer (18); Jenkins, Manohar, Stoffer (18)]. The dimension-5 Lagrangian consists of dipole operators

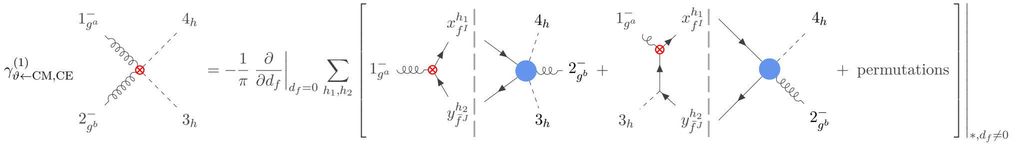
$$\mathcal{L}^{(5)}_{\text{LEFT}} \supset \frac{a_f}{\Lambda}\mathcal{O}_{\text{CM}} + \frac{d_f}{\Lambda}\mathcal{O}_{\text{CE}}\,, \qquad \mathcal{O}_{\text{CM}} = \bar{f}\sigma^{\mu\nu}T^a f G^a_{\mu\nu}\,, \qquad \mathcal{O}_{\text{CE}} = i\bar{f}\sigma^{\mu\nu}\gamma_5 T^a f G^a_{\mu\nu}\,.$$

- Consider the renormalization of $\mathcal{O}_\vartheta = G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu}$ as induced by $\mathcal{O}_{\text{CM}}$ and $\mathcal{O}_{\text{CE}}$.
- In the limit of massless fermions, $\gamma_{\vartheta\leftarrow\text{CM,CE}} = 0$, and possible mass effects must be of order $(m_f/\Lambda)^2$. Indeed $N = (4 - [\mathcal{O}_\vartheta]) + ([\mathcal{O}_{\text{CM}}] - 4) + ([\mathcal{O}_{\text{CE}}] - 4) = 2$.
- Thus, the required double Higgs insertion can be accounted for by introducing the operator $\mathcal{O}^{2h}_\vartheta = (h^2/2v^2)G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}$.

Example: Running of θ_{QCD} in LEFT

- Then, $\gamma^{(1)}_{\vartheta\leftarrow\text{CM,CE}}$ can be extracted from

$$\gamma^{(1)}_{\vartheta\leftarrow\text{CM,CE}} F^{2h}_{\vartheta}\big|^{(0)}_{*} = -\frac{1}{\pi}\,\frac{\partial}{\partial d_f}\bigg|_{*} (\mathcal{M}F_{\text{CM}})^{(1)}\,.$$



- From the calculation of tree-level form factors and amplitudes, we obtain

$$F^{2h}_{\vartheta}\big|^{(0)}_{*}(1^-_{g^a}, 2^-_{g^b}, 3_h, 4_h) = -\frac{2i}{v^2}\delta^{ab}\langle 1\,2\rangle^2\,,$$

$$(\mathcal{M}F_{\text{CM}})\big|^{(1)}_{*,d_f\neq 0}(1^-_{g^a}, 2^-_{g^b}, 3_h, 4_h) = \frac{i d_f}{\pi\Lambda^2}\,\frac{m_f^2}{v^2}\delta^{ab}\langle 1\,2\rangle^2\,,$$

implying $\gamma^{(1)}_{\vartheta\leftarrow\text{CM,CE}} = m_f^2/(2\pi^2\Lambda^2)$.

- We derived a master formula accounting for the general operator mixings of operators at one- and two-loop orders.
- Leading mass effects can be included within this framework, still working in the massless limit and relying on the Higgs low-energy theorem.
- These findings have been validated by reproducing well-established results of the literature, relative to popular EFTs.

Future Prospects

- Future experimental advances will improve the limits on low-energy observables (*e.g.* flavor violating processes, electric and magnetic dipole moments) by orders of magnitude.
- The computation of higher-order contributions will be crucial for the precise assessment of new physics effects.
- While this is a very challenging task when approached with standard techniques, on-shell and unitarity-based methods offer a simpler, more efficient and elegant way to reach this goal.

Thank you for your attention!

Q&A

Selection Rules: Dimension-6 Operators

	F^3	$\phi^2 F^2$	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		$\times_1$	(2)	$\times_2$	$\times_2$	$\times_2$	$\times_3$	$\times_3$
$\phi^2 F^2$							(2)	$\times_2$
$F\phi\psi^2$							$\times_1$	$\times_3$
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$D\phi^2\psi^2$							$\times_1$	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								

Table: From [Bern, Parra-Martinez, Sawyer (20)]. Dimension-6 operator mixing pattern. Operators labeling the rows are renormalized by the operators labeling the columns.

- $\times_L$: length selection rules apply at L-loop order
- (L): no diagrams before L loops, but renormalization is possible at that order
- Light-gray: zero at one loop due to helicity selection rules

The 4-momentum of an on-shell state is mapped onto a 2×2 matrix

$$p^\mu = (p^0, \vec{p}) \qquad \longrightarrow \qquad p^{\dot{\alpha}\alpha} = \bar{\sigma}_\mu^{\dot{\alpha}\alpha} p^\mu = \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix} ,$$

where $\bar{\sigma}^{\mu\,\dot{\alpha}\alpha} = (\mathbb{1}, -\vec{\sigma})^{\dot{\alpha}\alpha}$. If the particle is massless then

$$p^2 = \det\left(p^{\dot{\alpha}\alpha}\right) = m^2 = 0 \qquad \Longrightarrow \qquad p^{\dot{\alpha}\alpha} = \tilde{\lambda}^{\dot{\alpha}} \lambda^\alpha ,$$

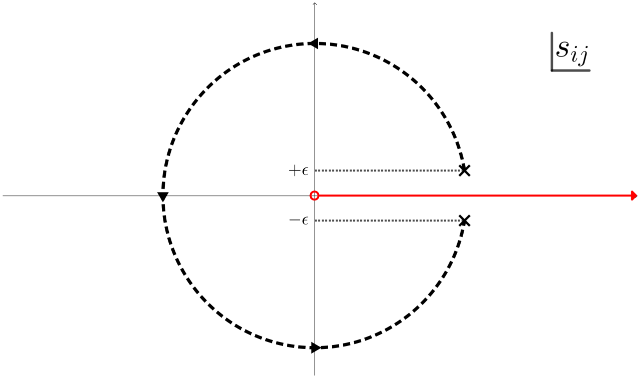
where $\lambda, \tilde{\lambda}$ are commuting Weyl spinors known as helicity spinors.
The angle and square inner products are Lorentz invariant

$$\langle i\, j \rangle \equiv \lambda_i^\alpha \lambda_{j\,\alpha} = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta = -\langle j\, i \rangle , \qquad [i\, j] \equiv \tilde{\lambda}_{i\,\dot{\alpha}} \tilde{\lambda}_j^{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\lambda}_i^{\dot{\alpha}} \tilde{\lambda}_j^{\dot{\beta}} = -[j\, i] .$$

The Mandelstam invariants can thus be written as

$$s_{ij} \equiv (p_i + p_j)^2 = 2 p_i \cdot p_j = \langle i\, j \rangle [j\, i] .$$

D as Generator of Complex Rotations



Dilatation operator

$$D = \sum_i p_i \cdot \frac{\partial}{\partial p_i} \,.$$

Transformation of the analytically continued Mandelstam invariants s_{ij} under the action of the D

$$p_i \to e^{i\alpha} p_i \,, \qquad F_{\mathcal{O}} \to e^{i\alpha D} F_{\mathcal{O}} \,.$$

For $\alpha = \pi$ their infinitesimal imaginary part ϵ changes sign

$$F_{\mathcal{O}}(\{s_{ij} - i\epsilon\}) = e^{i\pi D} F_{\mathcal{O}}(\{s_{ij} + i\epsilon\}) \,.$$

Phase Space Integration

$$(\mathcal{M}F_{\mathcal{O}})^{(1)}(1,2,3) = \sum_{\{x,y\}} \int d\Pi_2 \, F_{\mathcal{O}}^{(0)}(3,x,y)\mathcal{M}^{(0)}(x,y \to 1,2) + \text{permutations}$$

Angular integration

$$\begin{pmatrix} \lambda_x \\ \lambda_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & \cos\theta \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

$$\int d\Pi_2 = \frac{1}{8\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\pi/2} 2\cos\theta \sin\theta \, d\theta$$

Stokes' integration

$$\begin{pmatrix} \lambda_x \\ \lambda_y \end{pmatrix} = \frac{1}{\sqrt{1+z\bar{z}}} \begin{pmatrix} 1 & \bar{z} \\ -z & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

$$\int d\Pi_2 = -\frac{1}{8\pi} \oint dz \int d\bar{z} \, \frac{1}{(1+z\bar{z})^2}$$

IR Anomalous Dimensions

- In theories with massless fields, IR singularities originate from configurations where loop momenta become soft or collinear.
- The IR anomalous dimension only depends on the external state $\langle\vec{n}|$

$$\gamma^{(1)}_{\mathrm{IR}}(\{s_{ij}\};\mu) = \frac{g^2}{4\pi^2}\sum_{i<j} T^a_i T^a_j \log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^{(1)}_{i,\mathrm{coll.}}\,.$$

- Since the stress-energy tensor $T_{\mu\nu}$ is UV protected, it can be computed as

$$\gamma^{(1)}_{\mathrm{IR}} F^{(0)}_{T_{\mu\nu}}(\vec{n}) = \frac{1}{\pi}(\mathcal{M}F_{T_{\mu\nu}})^{(1)}(\vec{n})\,.$$