LEFT'S GLOBAL FLAVOUR STRUCTURE

…and its running from *W* scale to *b* scale

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(based on WIP w/ S. Renner, B. Smith)

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THE LEFT HAS A LARGE (BROKEN) FLAVOUR SYMMETRY

$$\begin{aligned}\left(U(3)_{d_L} \times U(3)_{d_R} \rtimes \mathbb{Z}_{2,d}\right) \times \left(U(3)_{e_L} \times U(3)_{e_R} \rtimes \mathbb{Z}_{2,e}\right) \\ \times \left(U(2)_{u_L} \times U(2)_{u_R} \rtimes \mathbb{Z}_{2,u}\right) \times U(3)_{\nu_L}\end{aligned}$$

Kinetic terms invariant under $d_L^i \to U_{d_L}^{ij} d_L^j$, $d_R^i \to U_{d_R}^{ij} d_R^j$, $d_L \leftrightarrow d_R$, ...

Masses and other operators break this — their components are *charged* under the flavour group.

$$\begin{aligned}\mathcal{L} = i\overline{d}_L^i \not{D} d_L^i + i\overline{d}_R^i \not{D} d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ - [M^d]_{ij}\overline{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ + c_{ijkl}\left(\overline{d}_L^i \gamma d_L^j\right)\left(\overline{e}_L^k \gamma e_L^l\right) + [\text{other ops}]\end{aligned}$$

(Neglecting purely gluonic operators)

WE USE A SMALLER LARGE (BROKEN) FLAVOUR SYMMETRY

$$\boxed{U(3)_d \times U(3)_e \times U(2)_u \times \mathbb{Z}_2}$$

Kinetic terms invariant under $d_L^i \to U_d^{ij} d_L^j$, $d_R^i \to U_d^{ij} d_R^j$, ..., $(d_L, e_L, u_L) \leftrightarrow (d_R, e_R, u_R)$.

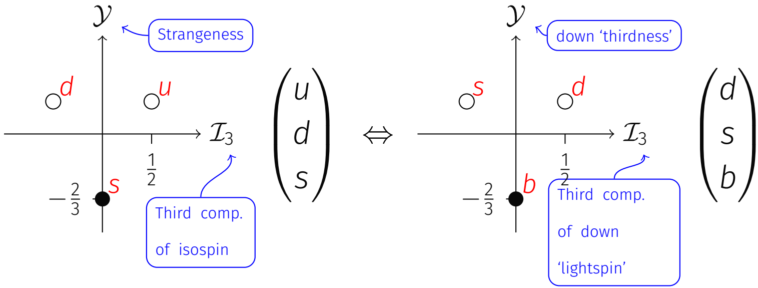
Masses and other operators break this — their components are *charged* under the flavour group.

$$\begin{aligned}\mathcal{L} = i\overline{d}_L^i \not{D} d_L^i + i\overline{d}_R^i \not{D} d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ - [M^d]_{ij} \overline{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ + c_{ijkl} \left(\overline{d}_L^i \gamma d_L^j\right) \left(\overline{e}_L^k \gamma e_L^l\right) + [\text{other ops}]\end{aligned}$$

(Neglecting purely gluonic operators)

FLAVOUR DECOMPOSITION

$U(3)_d \times U(3)_e \times U(2)_u$ is hierarchically broken. (cf. $SU(3)$ of *uds*, (Machado, Renner, and Sutherland 2023))



There are 11 flavour quantum numbers in total

$$\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_d \, , \{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_e \, , \{d, \mathcal{I}, \mathcal{I}_3\}_u$$

PARITY DECOMPOSITION (FOR VECTORIAL OPERATORS)

$$(\overline{\psi}\,\gamma^\mu P_{?}\,\psi)\,(\overline{\chi}\,\gamma^\mu P_{?}\,\chi) \qquad \psi, \chi \in \{d, e, u\}$$

	$+$	$-$
‘A’ type	$LL + RR$	$LL - RR$
‘B’ type	$LR + RL$	$LR - RL$

There is 1 parity quantum number

$\pm$

RUNNING w TO b SCALE, AT ONE LOOP

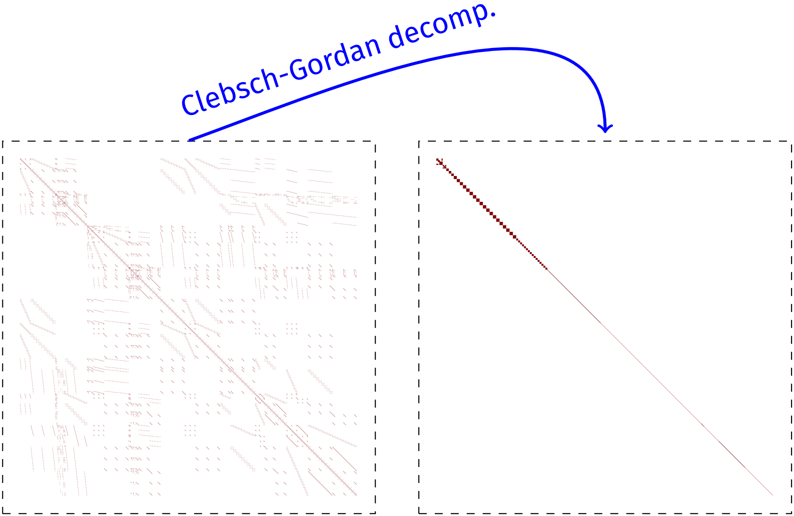
Schematically (Jenkins, Manohar, and Stoffer 2018)

$$
\begin{aligned}
\text{masses} \to (4\pi)^2\,\dot{M} &= (e^2+g^2)M + (e+g)dM^2 + c_S M^3 + c_V M^3 + d^2 M^3\,,\\
\text{QED} \to (4\pi)^2\,\dot{e} &= e^3 + e^2 dM + d^2 M^2\,,\\
\text{QCD} \to (4\pi)^2\,\dot{g} &= g^3 + g^2 dM + d^2 M^2\,,\\
\text{dipoles} \to (4\pi)^2\,\dot{d} &= (e^2+g^2+eg)d + eM(c_S+c_T) + (e+g)d^2 M\,,\\
\text{4f scalar} \to (4\pi)^2\,\dot{c}_S &= (e^2+g^2)(c_S+c_T) + (e^2+g^2+eg)d^2\,,\\
\text{4f tensor} \to (4\pi)^2\,\dot{c}_T &= (e^2+g^2)(c_S+c_T)\,,\\
\text{4f vector} \to (4\pi)^2\,\dot{c}_V &= (e^2+g^2)c_V + (e^2+g^2+eg)d^2\,,
\end{aligned}
$$

Neglect operators in grey at $O\,(0.1\%)$ accuracy.

$\{c_S, c_T\}$ and c_V do not mix due to helicity selection rules. (Cheung and Shen 2015)

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)}\, c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$



DSixTools/San Diego basis

Flavour & parity basis

Few zeroes!

Flavour & parity basis (zoomed)

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \gamma(t) c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$

Solve running with integrating factor U

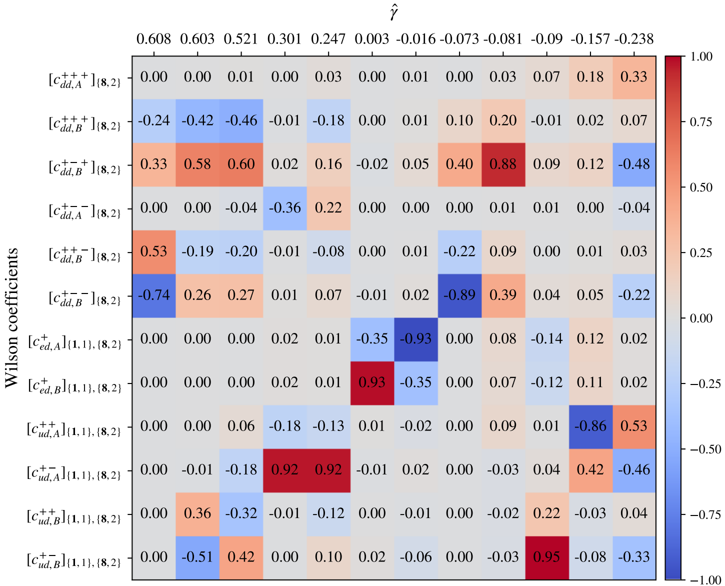
$$c_V(t_b) = U(t_b, t_W) c_V(t_W) + U(t_b, t_W) \int_{t_W}^{t_b} \mathrm{d}t \, U(t_W, t) s_V(t) \,.$$

Diagonalise U to understand RG flow basis-independently

$$(S^{-1} U S)_{ij} = \left(\frac{m_b}{m_W} \right)^{\frac{\hat{\gamma}_i}{(4\pi)^2}} \delta_{ij}$$

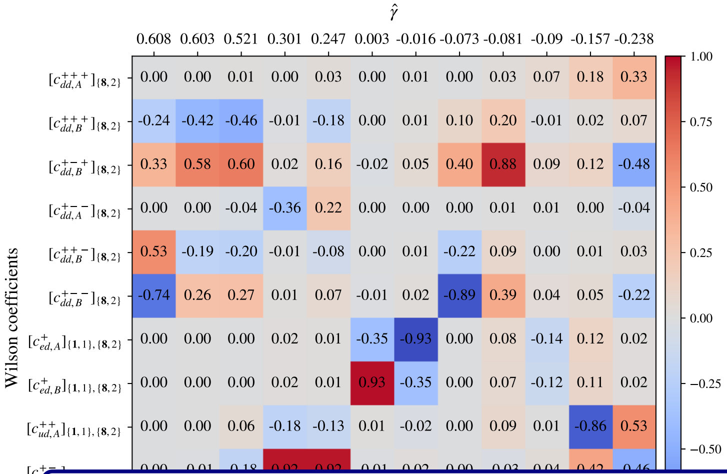
No mixing, directions with +ve $\hat{\gamma}$ shrink, -ve $\hat{\gamma}$ grow.

Parity-even lepton universal operators mediating $b \to s$



	0.608	0.603	0.521	0.301	0.247	0.003	-0.016	-0.073	-0.081	-0.09	-0.157	-0.238
$[c_{dd,A}^{+++}]_{\{\mathbf{8},2\}}$	0.00	0.00	0.01	0.00	0.03	0.00	0.01	0.00	0.03	0.07	0.18	0.33
$[c_{dd,B}^{+++}]_{\{\mathbf{8},2\}}$	-0.24	-0.42	-0.46	-0.01	-0.18	0.00	0.01	0.10	0.20	-0.01	0.02	0.07
$[c_{dd,B}^{+-+}]_{\{\mathbf{8},2\}}$	0.33	0.58	0.60	0.02	0.16	-0.02	0.05	0.40	0.88	0.09	0.12	-0.48
$[c_{dd,A}^{+--}]_{\{\mathbf{8},2\}}$	0.00	0.00	-0.04	-0.36	0.22	0.00	0.00	0.00	0.01	0.01	0.00	-0.04
$[c_{dd,B}^{++-}]_{\{\mathbf{8},2\}}$	0.53	-0.19	-0.20	-0.01	-0.08	0.00	0.01	-0.22	0.09	0.00	0.01	0.03
$[c_{dd,B}^{+--}]_{\{\mathbf{8},2\}}$	-0.74	0.26	0.27	0.01	0.07	-0.01	0.02	-0.89	0.39	0.04	0.05	-0.22
$[c_{ed,A}^{+}]_{\{\mathbf{1},1\},\{\mathbf{8},2\}}$	0.00	0.00	0.00	0.02	0.01	-0.35	-0.93	0.00	0.08	-0.14	0.12	0.02
$[c_{ed,B}^{+}]_{\{\mathbf{1},1\},\{\mathbf{8},2\}}$	0.00	0.00	0.00	0.02	0.01	0.93	-0.35	0.00	0.07	-0.12	0.11	0.02
$[c_{ud,A}^{++}]_{\{\mathbf{1},1\},\{\mathbf{8},2\}}$	0.00	0.00	0.06	-0.18	-0.13	0.01	-0.02	0.00	0.09	0.01	-0.86	0.53
$[c_{ud,A}^{+-}]_{\{\mathbf{1},1\},\{\mathbf{8},2\}}$	0.00	-0.01	-0.18	0.92	0.92	-0.01	0.02	0.00	-0.03	0.04	0.42	-0.46
$[c_{ud,B}^{++}]_{\{\mathbf{1},1\},\{\mathbf{8},2\}}$	0.00	0.36	-0.32	-0.01	-0.12	0.00	-0.01	0.00	-0.02	0.22	-0.03	0.04
$[c_{ud,B}^{+-}]_{\{\mathbf{1},1\},\{\mathbf{8},2\}}$	0.00	-0.51	0.42	0.00	0.10	0.02	-0.06	0.00	-0.03	0.95	-0.08	-0.33

Parity-even lepton universal operators mediating $b \to s$



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$$\delta E_n = \left\langle n \middle| \hat{H}_1 \middle| n \right\rangle$$

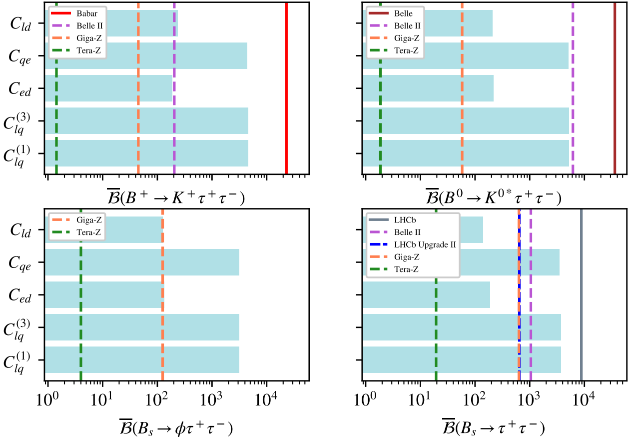
$$\delta \left| n \right\rangle = \sum_{k \neq n} \frac{\left\langle k \middle| \hat{H}_1 \middle| n \right\rangle}{E_n - E_k} \left| k \right\rangle \quad \Longrightarrow$$

two-loop can significantly mix nearly degenerate eigenvectors

τ only at W scale $\implies$ τ, and some e and μ, at b scale

$$3\bar{b}\gamma s\,\bar{\tau}\gamma\tau = \bar{b}\gamma s\,\underbrace{(\bar{e}\gamma e+\bar{\mu}\gamma\mu+\bar{\tau}\gamma\tau)}_{\text{LFU}} - \bar{b}\gamma s\,\underbrace{(\bar{e}\gamma e+\bar{\mu}\gamma\mu-2\bar{\tau}\gamma\tau)}_{\text{LFNU}}$$

$bs\mu\mu$ (teal bars) may be better than $bs\tau\tau$ (lines)



Also e.g. (Cornella, Faroughy, Fuentes-Martin, Isidori, and Neubert 2021)

SUMMARY

- Flavour charges organise the LEFT.
- They block-diagonalise γ to all orders.
- They make the map $m_W \to m_b$ understandable!