EFT tools to probe CP-violating axion-like particles

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Axion-Like Particles: Motivations

Axion-Like Particles (ALPs)

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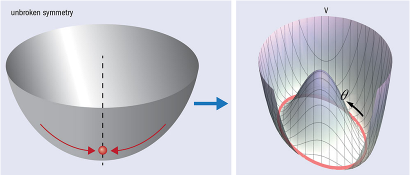

Differently from the QCD axion, ALPs:

- Need not solving the strong CP problem
- Have arbitrary masses and couplings ($f_\phi m_\phi \not\propto f_\pi m_\pi$)

Axion-Like Particles: Motivations

ALPs can address several open problems in particle physics:

- Strong CP problem (**QCD axion**)
- Hierarchy problem (**relaxion**)
- Flavour problem (**axiflavon/flaxion**)
- The observed **dark matter** abundance

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ALPs can be **probed experimentally** via:

- **Higgs and Z boson decay processes** ($h \to Z\phi$, $Z \to \gamma\phi$)
- **Flavour-changing neutral current** processes ($K^{\pm} \to \pi^{\pm}\phi$)
- **Electric Dipole Moments (EDMs)** of particles, nucleons, atoms, molecules (iff the ALP has CP-violating interactions)

Probing the CP violating ALP - I

Electric Dipole Moments (EDMs) are flavour-diagonal, CP-violating observables with (basically) **no SM background**

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Our idea: **probe CP-violating ALPs** at low energies. We started from the most general $SU(3)_c \times U(1)_{em}$ invariant

EFT for a CP-violating ALP ϕ at the EW scale ($\Lambda \gg M_W$)

$$\mathcal{L}_{\rm ALP}^{\text{dim-5}} \supset +e^2 \frac{C_\gamma}{\Lambda} \phi\, F^{\mu\nu} F_{\mu\nu} + e^2 \frac{\tilde{C}_\gamma}{\Lambda} \phi\, F^{\mu\nu} \tilde{F}_{\mu\nu} + g_s^2 \frac{C_g}{\Lambda} \phi\, G_a^{\mu\nu} G^a_{\mu\nu} + g_s^2 \frac{\tilde{C}_g}{\Lambda} \phi\, G_a^{\mu\nu} \tilde{G}^a_{\mu\nu} + \frac{v}{\Lambda} y_S^{ij} \phi\, \bar{f}_i f_j + i \frac{v}{\Lambda} y_P^{ij} \phi\, \bar{f}_i \gamma_5 f_j + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

[Di Luzio, Gröber, Paradisi,'20]

Jarlskog invariants: $C_a \tilde{C}_b,\ y_S^{ii} \tilde{C}_a,\ y_P^{ii} C_a,\ y_S^{ii} y_P^{jj},\ y_S^{ik} y_{\rm SM}^{kk} y_P^{ki}$

[Bonnefoy, Grojean, Kley,'22]

Probing the CP violating ALP - II

Three regimes:

- $m_\phi \gtrsim 3$ GeV: QCD is perturbative [Di Luzio, Gröber, Paradisi,'20]
- 1GeV $\lesssim m_\phi \lesssim 3$ GeV: QCD resonances. Dispersive approach?
- $m_\phi \lesssim 1$ GeV: QCD confines and χpt [Di Luzio, GL, Paradisi,'23]

Probing the CP violating ALP - II

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- $m_\phi \lesssim 1$ GeV: QCD confines and χpt [Di Luzio, GL, Paradisi,'23]

Different approaches are required, but common features are:

- **Renormalization** of $\mathcal{L}_{\text{ALP}}^{\text{dim-5}}$ + **running** of its Wilson coefficients (see Bresciani's talk!) [Chala, Guedes, Ramos, Santiago,'20],[Bakshi, Machado-Rodríguez, Ramos,'23],[Bauer, Neubert, Renner, Schnubel, Tamm,'20],[Bonilla, Brivio, Gavela, Sanz,'20]
- Lagrangian **Matching** on effective low-energy descriptions
- Classification of the **CPV Jarlskog invariants** of the theory
- **Experimental bounds** in terms of the Jarlskog invariants

Heavy ALPs ($m_\phi \gtrsim 3$ GeV) - I

Running from the EW scale to the ALP mass scale $m_\phi \gtrsim 5$ GeV, then one-loop matching onto [Pospelov, Ritz,'05]

$$\mathcal{L}_{\rm CPV} = \sum_{i,j=u,d,e} C_{ij}(\bar{f}_i f_i)(\bar{f}_j i\gamma_5 f_j) + \alpha_s C_{Ge}\, GG\, \bar{e}i\gamma_5 e + \alpha_s C_{\tilde{G}e}\, G\tilde{G}\, \bar{e}e$$
$$- \frac{i}{2} \sum_{i=u,d,e} d_i\, \bar{f}_i (F\cdot\sigma)\gamma_5 f_i - \frac{i}{2}\sum_{i=u,d} g_s\, d_i^C\, \bar{f}_i (G\cdot\sigma)\gamma_5 f_i + \frac{d_G}{3} f^{abc} G^a \tilde{G}^b G^c$$

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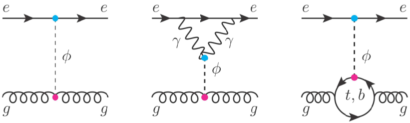

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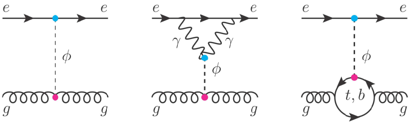

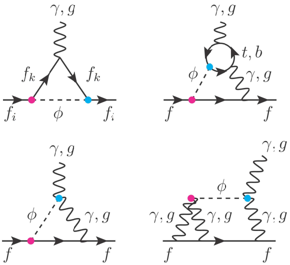

Heavy ALPs ($m_\phi \gtrsim 3$ GeV) - II

Bounds are set from:

- **Neutron EDM**: $d_n^{\rm exp} < 1.8 \cdot 10^{-26} e\, cm$,
  $d_n \simeq 0.8 d_u - 0.2 d_d - 0.6 e\, d_u^C - 1.1 e\, d_d^C - 50 \text{ MeV } e\, d_G + 30 \text{ MeV } e\, (C_{ud} - C_{du})$
- **Hg EDM**: $d_{Hg}^{\rm exp} < 6 \cdot 10^{-30} e\, cm$,
  $d_{Hg} \simeq 4 \times 10^{-4} d_n - [2.8 C_S - 2.1 C_P] \times 10^{-22}$
- **ThO electron precession frequency**: $\omega_{ThO}^{\rm exp} < 1.3$ mrad/s ,
  $\omega_{ThO} = 1.2 \text{ mrad/s } \left(\frac{d_e}{10^{-29} \text{e cm}}\right) + 1.8 \text{ mrad/s } \left(\frac{C_S}{10^{-9}}\right)$

with $C_S/v^2 \simeq -17(C_{ue} + C_{de}) + 5 \text{ GeV } C_{Ge},\ C_P/v^2 \simeq 350(C_{eu} + C_{ed}) + 1 \text{ GeV } C_{\tilde{G}e}$.

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For instance ($m_\phi = 5$ GeV, $\Lambda = 1$ TeV):

- $|C_g \tilde{C}_g| < 1.4 \cdot 10^{-6}$ from $d_n, d_{Hg} (d_G)$
- $|y_S^{uu} y_P^{ee}|, |y_S^{dd} y_P^{ee}| < 2.1 \cdot 10^{-13}$ from $\omega_{ThO}(C_S)$

Light ALPs ($m_\phi \lesssim 1$ GeV) - I

External gauge and scalar **fields** enter as sources in $\mathcal{L}_{\rm QCD}$:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(2r_\mu P_R + 2\ell_\mu P_L)q - \bar{q}(s - i\gamma_5 p)q$$

Light ALPs ($m_\phi \lesssim 1$ GeV) - I

External gauge and scalar **fields** enter as sources in $\mathcal{L}_{\text{QCD}}$:

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}^0_{\text{QCD}} + \bar{q}\gamma^\mu(2r_\mu P_R + 2\ell_\mu P_L)q - \bar{q}(s - i\gamma_5 p)q$$

These enter $\mathcal{L}_{\chi\text{pt}}$ via

$$\mathcal{L}_{\chi\text{PT}} = \frac{f^2}{4}\text{Tr}\left[D_\mu\Sigma^\dagger D^\mu\Sigma + \Sigma^\dagger\chi + \chi^\dagger\Sigma\right]$$

$$D_\mu\Sigma = \partial_\mu\Sigma + i\Sigma\ell_\mu - ir_\mu\Sigma, \qquad \chi = 2B_0(s + ip)$$

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Chiral counterparts to quark-containing operators are found exploiting the low-energy path-integral **duality**

$$\int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}G_\mu\,\exp\left(i\int d^4x\,\mathcal{L}^{\rm ext}_{\rm QCD}\right) = \int \mathcal{D}\Sigma\,\exp\left(i\int d^4x\,\mathcal{L}^{\rm ext}_{\chi\rm pt}\right)\;(*)$$

Light ALPs ($m_\phi \lesssim 1$ GeV) - II

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{c_\gamma}{\Lambda}\phi FF + e^2 \frac{\tilde{c}_\gamma}{\Lambda}\phi F\tilde{F} + \frac{\partial_\mu \phi}{\Lambda} \bar{q}\gamma^\mu (Y_V + Y_A\gamma_5) q$$
$$- \kappa \frac{\phi}{\Lambda} T^\mu{}_\mu + \frac{v}{\Lambda}\phi \bar{q}\mathcal{Z}q + \bar{q}_L M_q^\phi q_R + \text{h.c.} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Its counterpart is found by using the **duality** in (*)

low-energy CP-violating ALP Lagrangian

$$\mathcal{L}_{\phi\chi} = -\frac{1}{3}\frac{m_\pi^2}{m_\pi^2 - M_\phi^2}\frac{\Delta_{ud}}{f_\pi \Lambda}\Big[-2\partial\phi\left(2\pi^+\pi^-\partial\pi_0 + \pi_0\pi^+\partial\pi^- + \pi_0\pi^-\partial\pi^+\right)$$
$$+ M_\phi^2 \phi\left(\pi_0^3 + 2\pi^+\pi^-\pi_0\right)\Big] + 2\kappa\frac{\phi}{\Lambda}[\partial_\mu\pi^+\partial^\mu\pi^- + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0]$$
$$- m_\pi^2 \omega \frac{\phi}{\Lambda}[\pi^+\pi^- + \frac{1}{2}\pi_0^2] + C_N^S\frac{\phi}{\Lambda}\bar{N}_v N_v + C_N^A \frac{\partial_\mu\phi}{\Lambda}\bar{N}_v\gamma^\mu\gamma_5 N_v$$
$$+ e^2 \widetilde{C}'_\gamma \frac{\phi}{\Lambda} F\widetilde{F} + e^2 C'_\gamma \frac{\phi}{\Lambda}FF + + i\frac{v}{\Lambda}y_{P,\ell}^{ij}\,\phi\bar{\ell}_i\gamma_5\ell_j + \frac{v}{\Lambda}y_{S,\ell}^{ij}\,\phi\bar{\ell}_i\ell_j$$

Light ALPs ($m_\phi \lesssim 1$ GeV) - III

	c_γ	$y_{\ell,S}$	κ	$\mathcal{Z}$	$C_{\phi NN}$
$\tilde{c}_\gamma$	$\tilde{c}_\gamma\, c_\gamma$	$\tilde{c}_\gamma\, y_{\ell,S}$	$\tilde{c}_\gamma\, \kappa$	$\tilde{c}_\gamma\, \mathcal{Z}$	$\tilde{c}_\gamma\, C_{\phi NN}$
$y_{\ell,P}$	$y_{\ell,P}\, c_\gamma$	$y_{\ell,P}\, y_{\ell,S}$	$y_{\ell,P}\, \kappa$	$y_{\ell,P}\, \mathcal{Z}$	$y_{\ell,P}\, C_{\phi NN}$
Δ^A_{ud}	$\Delta^A_{ud}\, c_\gamma$	$\Delta^A_{ud}\, y_{\ell,S}$	$\Delta^A_{ud}\, \kappa$	$\Delta^A_{ud}\, \mathcal{Z}$	$\Delta^A_{ud}\, C_{\phi NN}$
$\tilde{C}_{\phi N}$	$\tilde{C}_{\phi N}\, c_\gamma$	$\tilde{C}_{\phi N}\, y_{\ell,S}$	$\tilde{C}_{\phi N}\, \kappa$	$\tilde{C}_{\phi N}\, \mathcal{Z}$	$\tilde{C}_{\phi N}\, C_{\phi NN}$

Table: Jarlskog invariants of the low-energy chiral Lagrangian $\mathcal{L}_{\phi\chi}$

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$y_{\ell,P}$	$y_{\ell,P}\, c_\gamma$	$y_{\ell,P}\, y_{\ell,S}$	$y_{\ell,P}\, \kappa$	$y_{\ell,P}\, \mathcal{Z}$	$y_{\ell,P}\, C_{\phi NN}$
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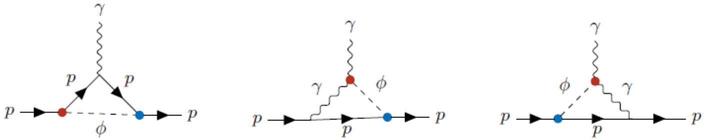
Table: Jarlskog invariants of the low-energy chiral Lagrangian $\mathcal{L}_{\phi\chi}$

EDMs of protons, neutrons, atoms, molecules ...



$$d_p \simeq -\frac{e\, Q_p}{4\pi^2\Lambda^2}\left[C_{\phi pp}\tilde{C}_{\phi p} + 6e^2 m_p c_\gamma \tilde{C}_{\phi p} + 2e^2 \tilde{c}_\gamma C_{\phi pp}\right]$$

$$\longrightarrow |C_g \tilde{C}_g| < 4.4 \times 10^{-8}$$

Summary

We have:

- Shown that **EDMs** are flavour-diagonal probes of CP violation and offer huge potentialities for discoveries (here ALPs)
- Provided the **matching dictionary** relating the IR couplings in low-energy Lagrangians to the UV couplings at the EW scale
- Classified the **Jarlskog invariants** of the theory (at high and low energies)
- **Explored the parameter space** for light and heavy ALPs
- `FeynRules` **model** available for both the 2- and the 3-flavors setting in χpt $\rightarrow$ extensive, automatized pheno analyses

Thanks for your attention!

Backup slides

From quarks to mesons

We want to find the **chiral counterpart** to our Lagrangian

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{C_\gamma}{\Lambda} \phi\, F\, F + e^2 \frac{\tilde{C}'_\gamma}{\Lambda} \phi\, F \tilde{F} + g_s^2 \frac{C_g}{\Lambda} \phi\, G\, G + g_s^2 \frac{\tilde{C}'_g}{\Lambda} \phi\, G \tilde{G} + \frac{\partial_\mu \phi}{\Lambda} \bar{q}\, \gamma^\mu (Y_S + Y_P \gamma_5)\, q + \frac{v}{\Lambda} \phi\, \bar{q}\, y_{q,S}\, q + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Chiral counterparts to quark-containing operators are found exploiting the low-energy path-integral **duality** (*). For instance:

Example

$$\bar{q}_i y_{ij}^S q_j = -y_{ij}^S \frac{\partial \mathcal{L}_{\text{QCD}}}{\partial y_{ij}^S} \longrightarrow -y_{ij}^S \frac{\partial \mathcal{L}_{\chi\text{pt}}}{\partial y_{ij}^S} = -\frac{f_\pi^2}{2} B_0 \text{Tr}\left[y^S (\Sigma + \Sigma^\dagger) \right]$$

Getting rid of gluons

- Eliminate ϕGG thanks to the **trace anomaly** equation [Leutwyler, Shifman,'89]:

$$T^{\mu}{}_{\mu} = \sum_q m_q \bar{q} q - \frac{\alpha_s}{8\pi} \beta^0_{\rm QCD} \, G_a^{\mu\nu} G^a_{\mu\nu} - \frac{\alpha_{\rm em}}{8\pi} \beta^0_{\rm QED} \, F^{\mu\nu} F_{\mu\nu}$$

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- Eliminate $\phi G \tilde{G}$ via an **ALP-dependent quark field redefinition**[Georgi, Kaplan, Randall,'86]:

$$q \to q = \exp\left[i \frac{\phi}{\Lambda} \left(Q_V + \lambda^*_g Q_A \gamma_5 \right) \right] q'$$

with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal, Tr(Q_A)$= 1/2$, $\lambda^*_g = 32\pi^2 \tilde{C}'_g$).

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with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal, ${\rm Tr}(Q_A) = 1/2$, $\lambda_g^* = 32\pi^2 \tilde{C}'_g$).

- Other **couplings** are **modified** (currents, masses, ...)!

Chiral Lagrangian for the CPV ALP

All of the previous modifications lead to the following

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{c_\gamma}{\Lambda} \phi FF + e^2 \frac{\tilde{c}_\gamma}{\Lambda} \phi F\tilde{F} + \frac{\partial_\mu \phi}{\Lambda} \bar{q}\gamma^\mu (Y_V + Y_A \gamma_5) q$$
$$- \kappa \frac{\phi}{\Lambda} T^\mu_{\ \mu} + \frac{v}{\Lambda} \phi \bar{q} \mathcal{Z} q + \bar{q}_L M_q^\phi q_R + \text{h.c.} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Its counterpart is found by using the **duality** in (*)

Mesonic Chiral Lagrangian for a CP-violating ALP ϕ at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\chi\text{pt}} = \frac{\partial_\mu \phi}{\Lambda} \left[2\,\text{Tr}(Y_V T_a) j_V^{\mu,a} + 2\,\text{Tr}(Y_A T_a) j_A^{\mu,a}\right] + \frac{f_\pi^2}{2} B_0 \text{Tr}\left[M_\phi \Sigma^\dagger + \Sigma M_\phi{}^\dagger\right]$$
$$+ \kappa \frac{f_\pi^2}{2} \frac{\phi}{\Lambda} \left[\text{Tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + 4 B_0 \text{Tr}\left[M_q (\Sigma + \Sigma^\dagger)\right]\right]$$
$$- \frac{f_\pi^2}{2} \frac{v}{\Lambda} B_0 \phi \text{Tr}\left[\mathcal{Z}(\Sigma + \Sigma^\dagger)\right] + e^2 \frac{c_\gamma}{\Lambda} \phi FF + e^2 \frac{\tilde{c}_\gamma}{\Lambda} \phi F\tilde{F} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Matching onto the low-energy Lagrangian ($n_f = 2$)

The $\mathcal{O}(\Lambda^{-2})$ low-energy Lagrangian $\mathcal{L}_{\phi\chi}$ valid for $E < 1-2$ GeV is:

low-energy CP-violating ALP Lagrangian

$$\mathcal{L}_{\phi\chi} = -\frac{1}{3}\frac{m_\pi^2}{m_\pi^2 - M_\phi^2}\frac{\Delta_{ud}}{f_\pi \Lambda}\Big[-2\partial\phi\left(2\pi^+\pi^-\partial\pi_0 + \pi_0\pi^+\partial\pi^- + \pi_0\pi^-\partial\pi^+\right)$$
$$+ M_\phi^2\phi\left(\pi_0^3 + 2\pi^+\pi^-\pi_0\right)\Big] + 2\kappa\frac{\phi}{\Lambda}[\partial_\mu\pi^+\partial^\mu\pi^- + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0]$$
$$- m_\pi^2\omega\frac{\phi}{\Lambda}[\pi^+\pi^- + \frac{1}{2}\pi_0^2] + C_N^S\frac{\phi}{\Lambda}\bar{N}_v N_v + C_N^A\frac{\partial_\mu\phi}{\Lambda}\bar{N}_v\gamma^\mu\gamma_5 N_v$$
$$+ e^2\widetilde{C}'_\gamma\frac{\phi}{\Lambda}F\widetilde{F} + e^2 C'_\gamma\frac{\phi}{\Lambda}FF + +i\frac{v}{\Lambda}y_{P,\ell}^{ij}\,\phi\bar{\ell}_i\gamma_5\ell_j + \frac{v}{\Lambda}y_{S,\ell}^{ij}\,\phi\bar{\ell}_i\ell_j$$

All the couplings in $\mathcal{L}_{\phi\chi}$ can be expressed in terms of those in $\mathcal{L}_{\rm ALP}^{\text{dim-5}}$ or at most of **measurable/computable** quantities.

Example: $Y_A^{ij} = -y_{q,P}^{ij}\frac{v}{m_i+m_j} - 32\pi^2\, Q_A^{ij}\,\tilde{C}_g$

CPV Jarlskog invariants ($n_f = 2$)

The **low-energy Jarlskog invariants** are found from $\mathcal{L}_{\phi\chi}$ by multiplying the Wilson coefficients of operators possessing **opposite CP** transformation properties

Example

$$c_\gamma FF \xrightarrow{CP} c_\gamma FF$$
$$\tilde{c}_\gamma F\widetilde{F} \xrightarrow{CP} -\tilde{c}_\gamma F\widetilde{F}$$

$\longrightarrow$ $c_\gamma \tilde{c}_\gamma$ is a Jarlskog invariant!

	c_γ	$y_{\ell,S}$	κ	$\mathcal{Z}$	$C_{\phi NN}$
$\tilde{c}_\gamma$	$\tilde{c}_\gamma\ c_\gamma$	$\tilde{c}_\gamma\ y_{\ell,S}$	$\tilde{c}_\gamma\ \kappa$	$\tilde{c}_\gamma\ \mathcal{Z}$	$\tilde{c}_\gamma\ C_{\phi NN}$
$y_{\ell,P}$	$y_{\ell,P}\ c_\gamma$	$y_{\ell,P}\ y_{\ell,S}$	$y_{\ell,P}\ \kappa$	$y_{\ell,P}\ \mathcal{Z}$	$y_{\ell,P}\ C_{\phi NN}$
Δ_{ud}^A	$\Delta_{ud}^A\ c_\gamma$	$\Delta_{ud}^A\ y_{\ell,S}$	$\Delta_{ud}^A\ \kappa$	$\Delta_{ud}^A\ \mathcal{Z}$	$\Delta_{ud}^A\ C_{\phi NN}$
$\tilde{C}_{\phi N}$	$\tilde{C}_{\phi N}\ c_\gamma$	$\tilde{C}_{\phi N}\ y_{\ell,S}$	$\tilde{C}_{\phi N}\ \kappa$	$\tilde{C}_{\phi N}\ \mathcal{Z}$	$\tilde{C}_{\phi N}\ C_{\phi NN}$

Table: Jarlskog invariants of the low-energy chiral Lagrangian $\mathcal{L}_{\phi\chi}$