Positivity and Real-world EFTs

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Real world EFTs can be highly non-trivial

Need all possible constraints in our disposal to reduce the complexity

SMEFT: $$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{n,i} \frac{1}{\Lambda^{n-4}} \mathcal{O}_{n,i}$$

Order	dim 4	dim 5	dim 6	dim 7	dim 8
# of interactions	Standard Model	*2*	*84*	*30*	*993*

Positivity bounds: surprising constraints on IR EFT from fundamental principles

How does it work for the SMEFT?

SMEFT: $$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{n,i} \frac{1}{\Lambda^{n-4}} \mathcal{O}_{n,i}$$

Order	dim 4	dim 5	dim 6	dim 7	dim 8
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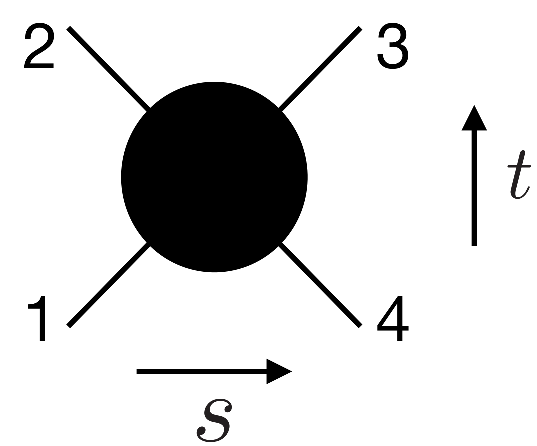
Review of Positivity Bounds

— surprising constraints on EFT from UV principles

- Simplest example: 2-to-2 scattering of a identical scalar

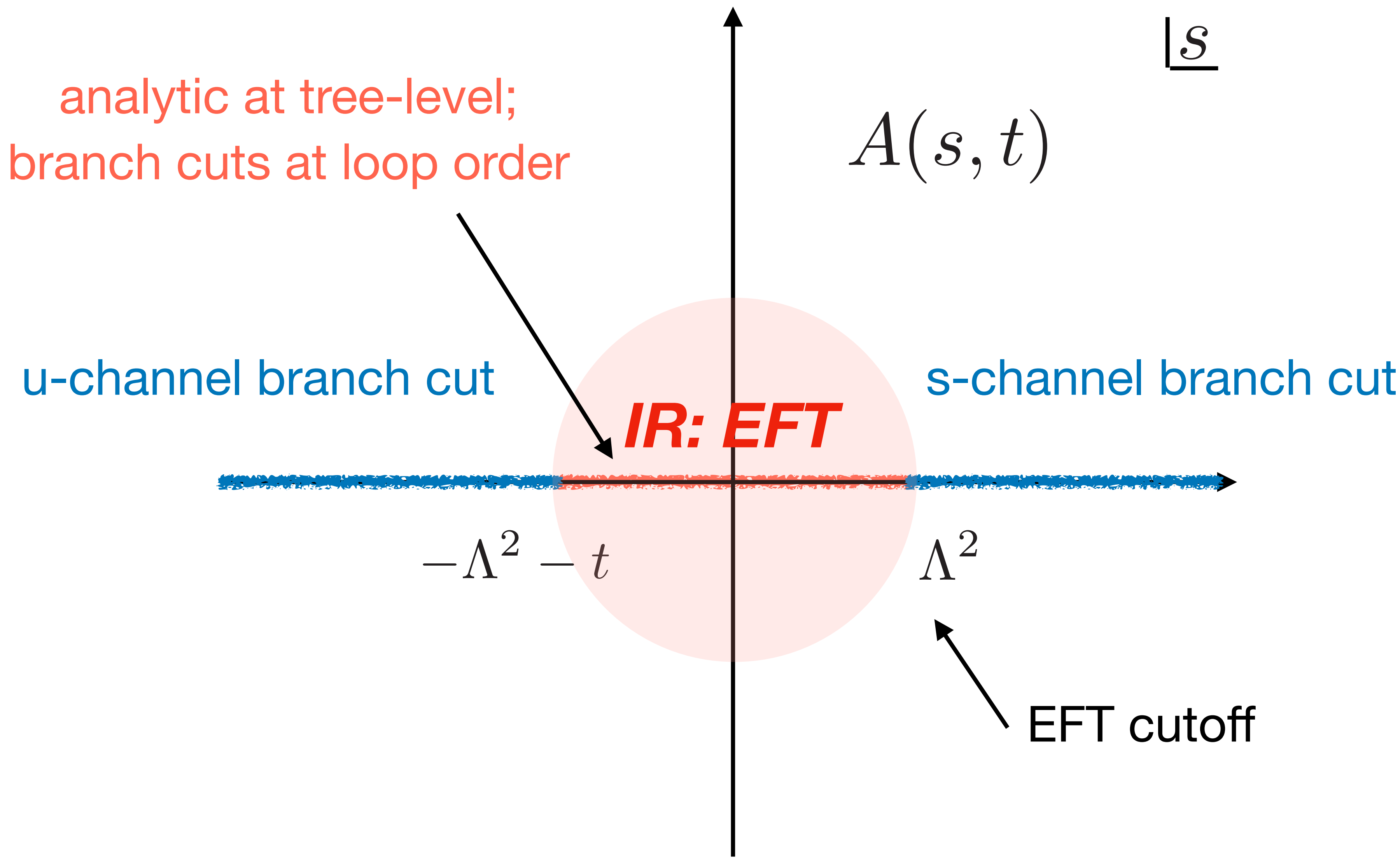
$$\mathcal{L}_{\rm EFT} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2\Lambda^4} c_8 (\partial\phi)^4 + \ldots$$

$$A_{\rm EFT}(s,t) = \frac{c_8}{\Lambda^4}(s^2+t^2+u^2) + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$



Naively, c_8 can be arbitrary

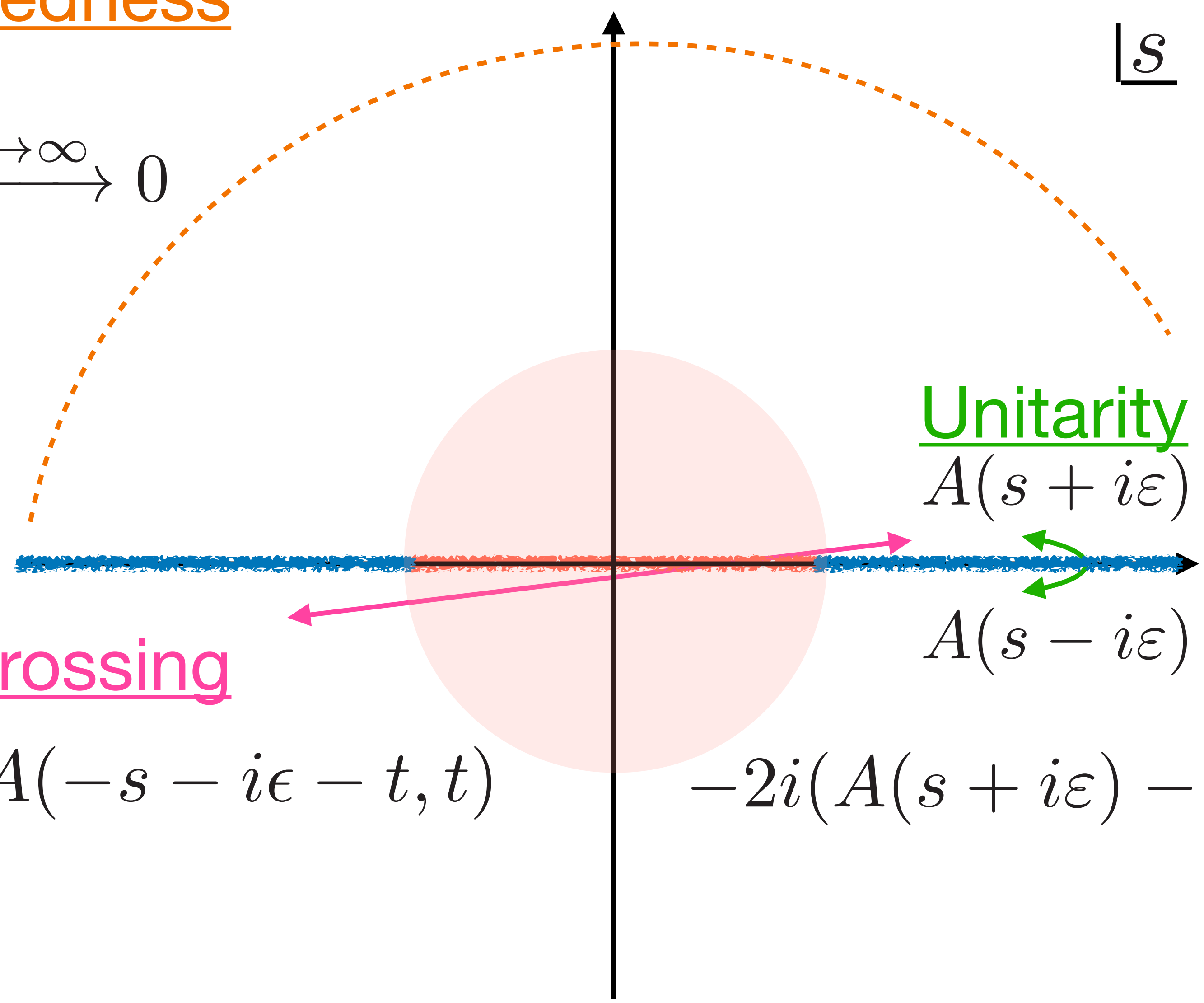
Analyticity



Crossing, Unitarity, Boundedness

Boundedness

$$\left|\frac{A(s,t)}{s^2}\right| \xrightarrow{s\to\infty} 0$$

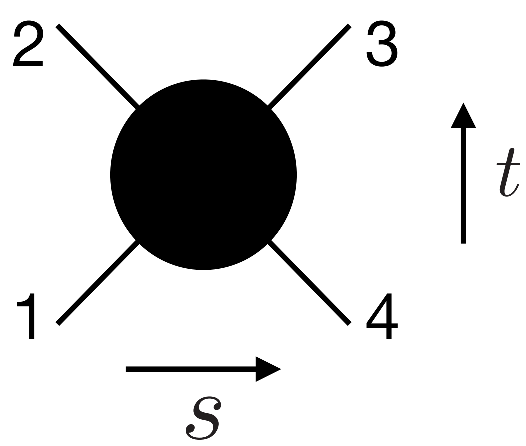


Crossing

$$A(s+i\varepsilon,t) = A(-s-i\epsilon-t,t)$$

$$-2i(A(s+i\varepsilon)-A(s-i\varepsilon)) = AA^\dagger$$

Positivity from Forward Scattering

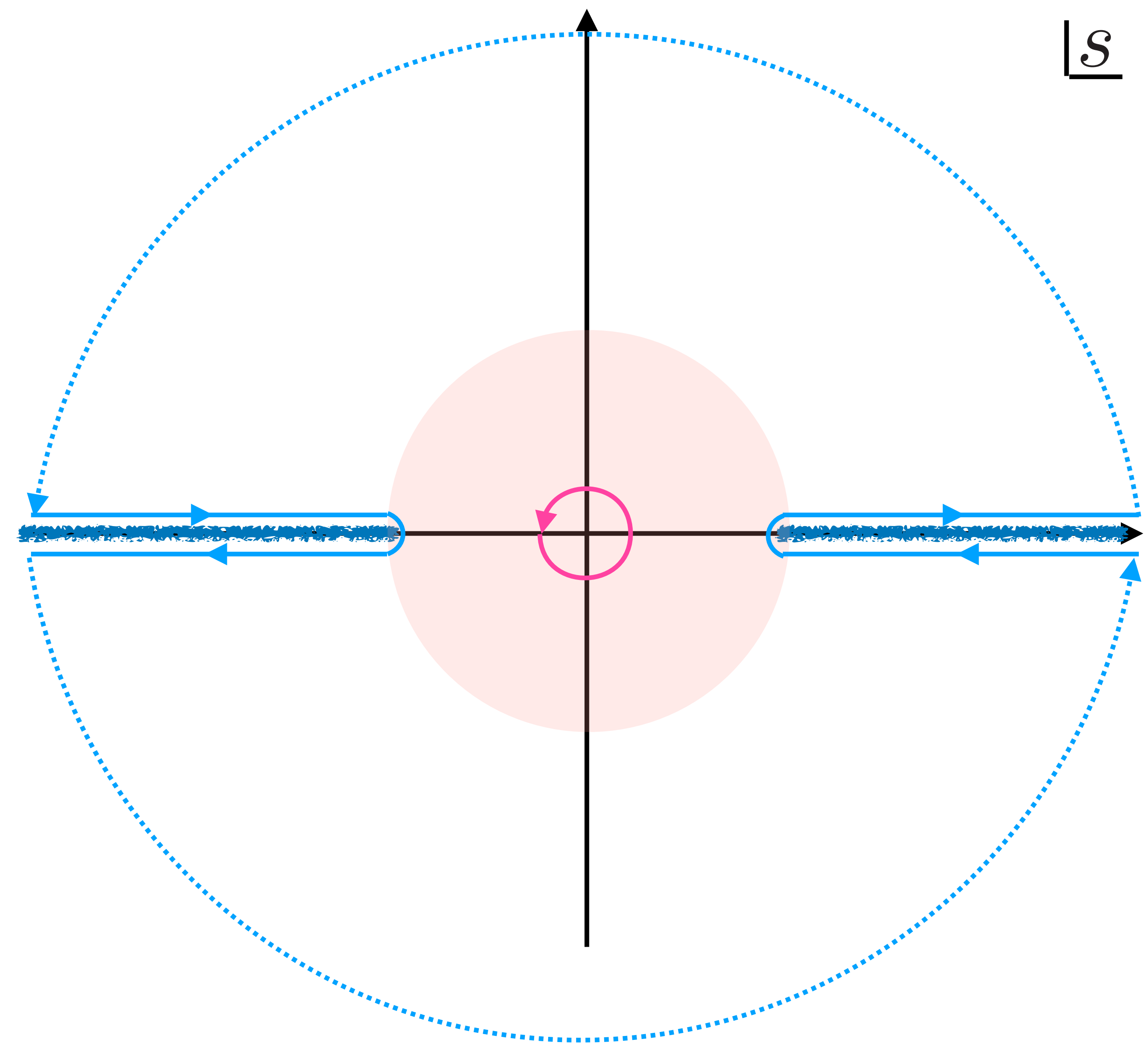


$$A(s) = A(s,t)|_{t\to 0}$$

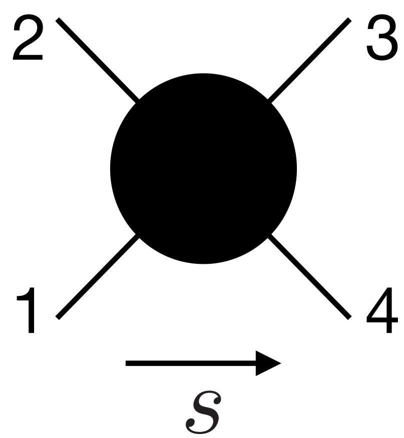
IR contour

leading EFT=tree

$$\frac{1}{2\pi i}\oint_{\rm IR}\frac{ds}{s^3}A(s) = \frac{c_8}{\Lambda^4} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$



Positivity from Forward Scattering



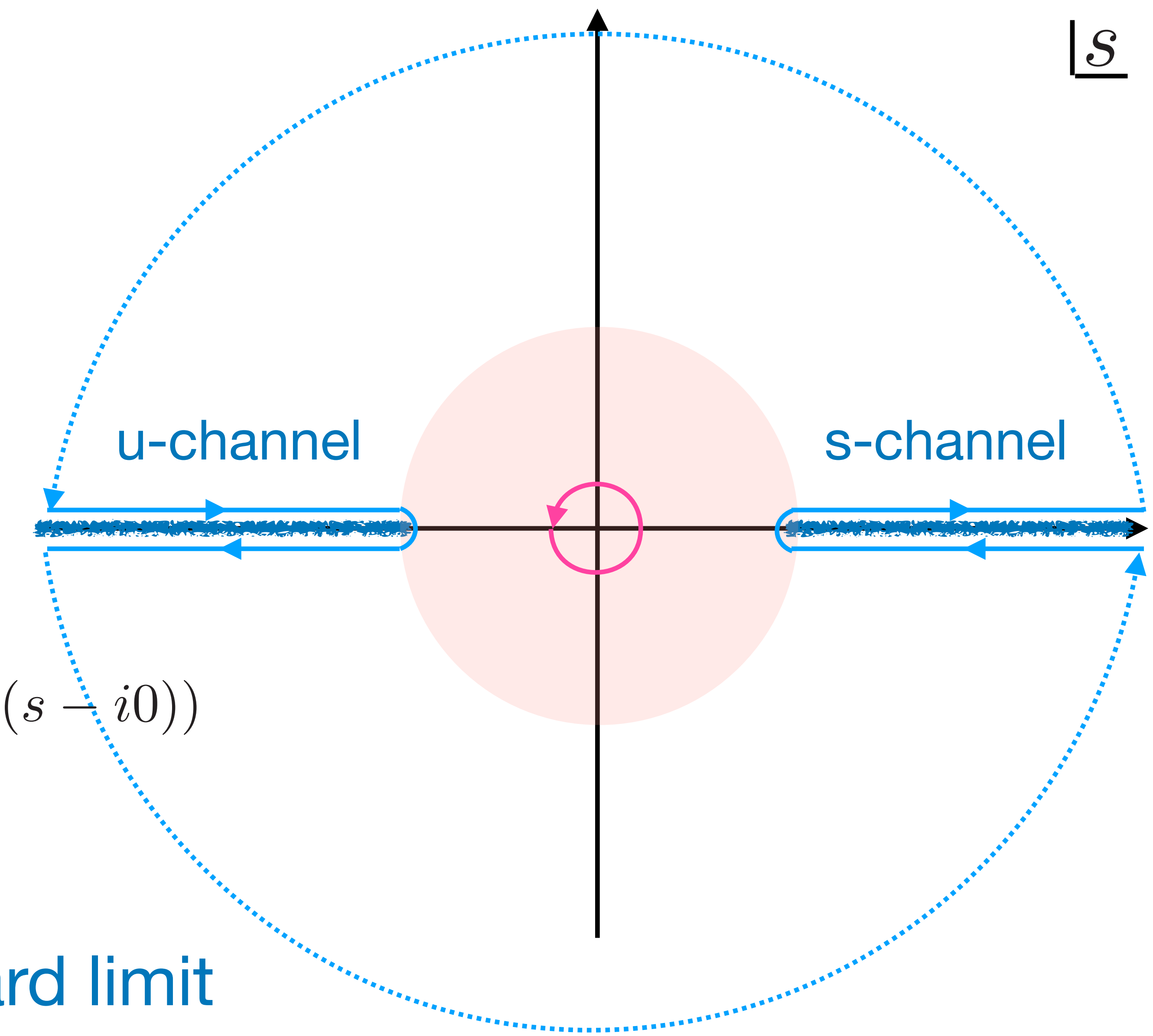
$\uparrow t \quad A(s) = A(s,t)|_{t\to 0}$

UV contour

s- and u-channel
add up

$$\frac{1}{2\pi i}\oint_{\text{UV}} \frac{ds}{s^3} A(s) = 2\int_{\Lambda^2}^{\infty} \frac{ds}{2\pi i s^3}\left(A(s+i0) - A(s-i0)\right)$$
$$= \int_{\Lambda^2}^{\infty} \frac{ds}{\pi s^3} \langle i| |A^\dagger A|i\rangle > 0$$

unitarity in the forward limit



no contribution from infinity

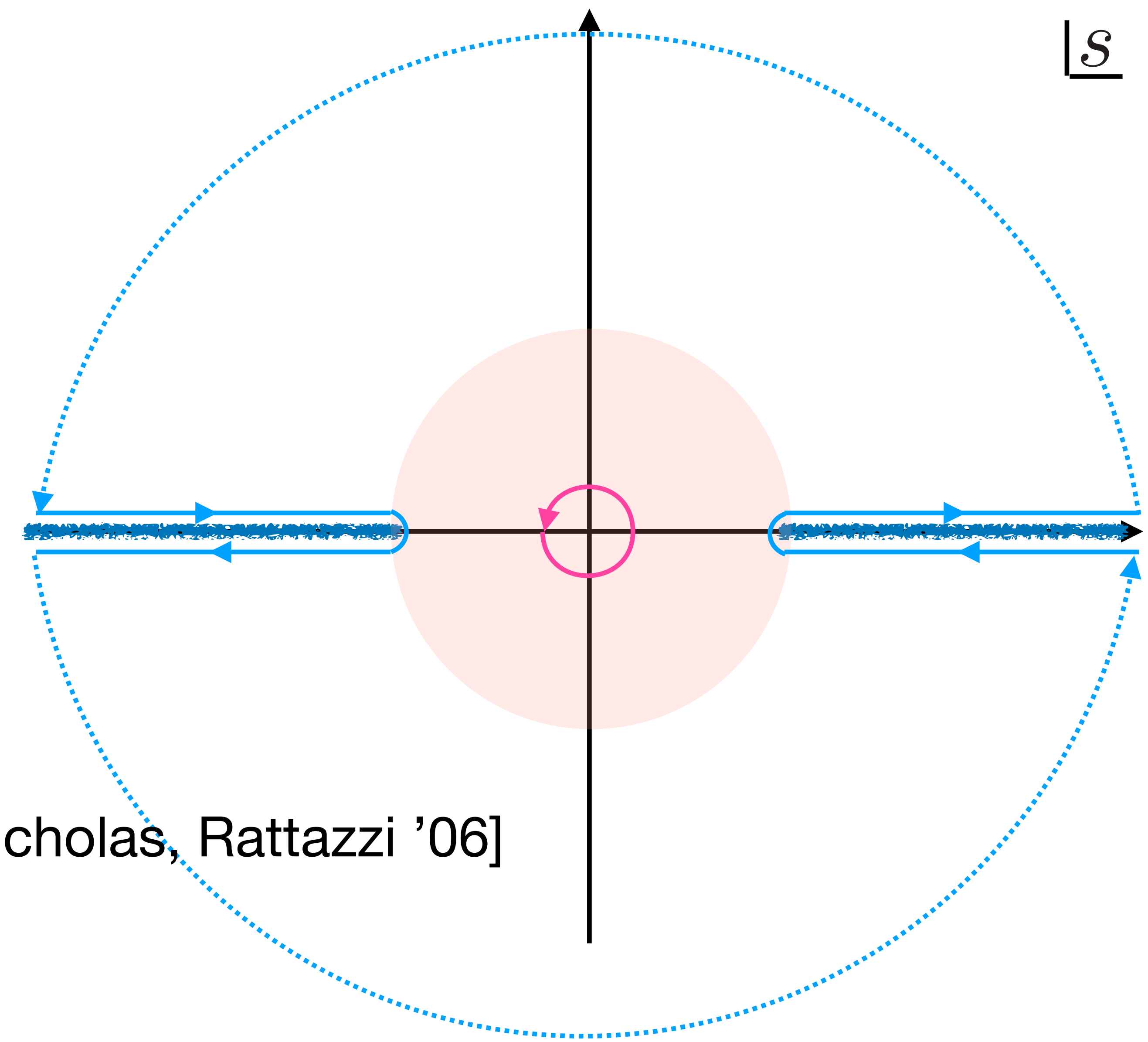
Positivity from Forward Scattering

IR contour = UV contour

$$c_8 \geq 0$$

[Pham, Truong, '85]

[Adams, Arkani-Hamed, Dubovsky, Nicholas, Rattazzi '06]

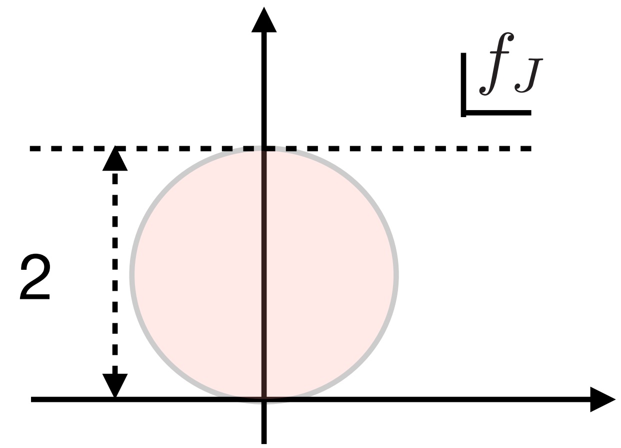


Modern Extensions

Partial wave expansion

$$A(s,t) = \sum_J n_J f_J(s) P_J(\cos\theta)$$

Unitarity: $\quad |S_J(s)|^2 = |1 + i f_J(s)|^2 \leq 1$



Cauchy Theorem

IR EFT coefficients = $\displaystyle \sum_J \int_{\Lambda^2}^{\infty} ds\, \underbrace{\tilde{n}(s) |f_J(s)|^2}_{\text{positive measure}}\, \underbrace{F(s, J^2)}_{\text{UV kernel}} \equiv \langle F(s, J^2) \rangle$

positive measure UV kernel

Example:

$$A_{\mathrm{EFT}}(s,t) = \frac{c_8}{\Lambda^4}(s^2+t^2+u^2) + \frac{c_{10}}{\Lambda^6}(stu) + \frac{c_{12}}{\Lambda^8}(s^2+t^2+u^2)^2 + \dots$$

$\mathcal{J}^2 = J(J+1)$

$$\frac{c_8}{\Lambda^4} = \left\langle \frac{1}{x^4} \right\rangle \geq 0 \qquad \frac{c_{10}}{\Lambda^6} = \left\langle \frac{3 - 2\mathcal{J}^2}{x^6} \right\rangle \qquad \frac{c_{12}}{\Lambda^8} = \left\langle \frac{1}{2x^8} \right\rangle \geq 0$$

Modern Extensions

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$\mathcal{J}^2 = J(J+1)$

- Cauchy-Schwartz inequality: *EFT-Hedron* [Arkani-Hamed, Huang, Huang '20] [Bellazzini et al. '20]

$$\langle ab \rangle^2 \leq \langle a^2 \rangle \langle b^2 \rangle \longrightarrow \left\langle \frac{1}{2x^8} \right\rangle^2 \leq \left\langle \frac{1}{x^4} \right\rangle \left\langle \frac{1}{4x^{12}} \right\rangle \longrightarrow c_{12}^2 \leq c_8 c_{16}$$

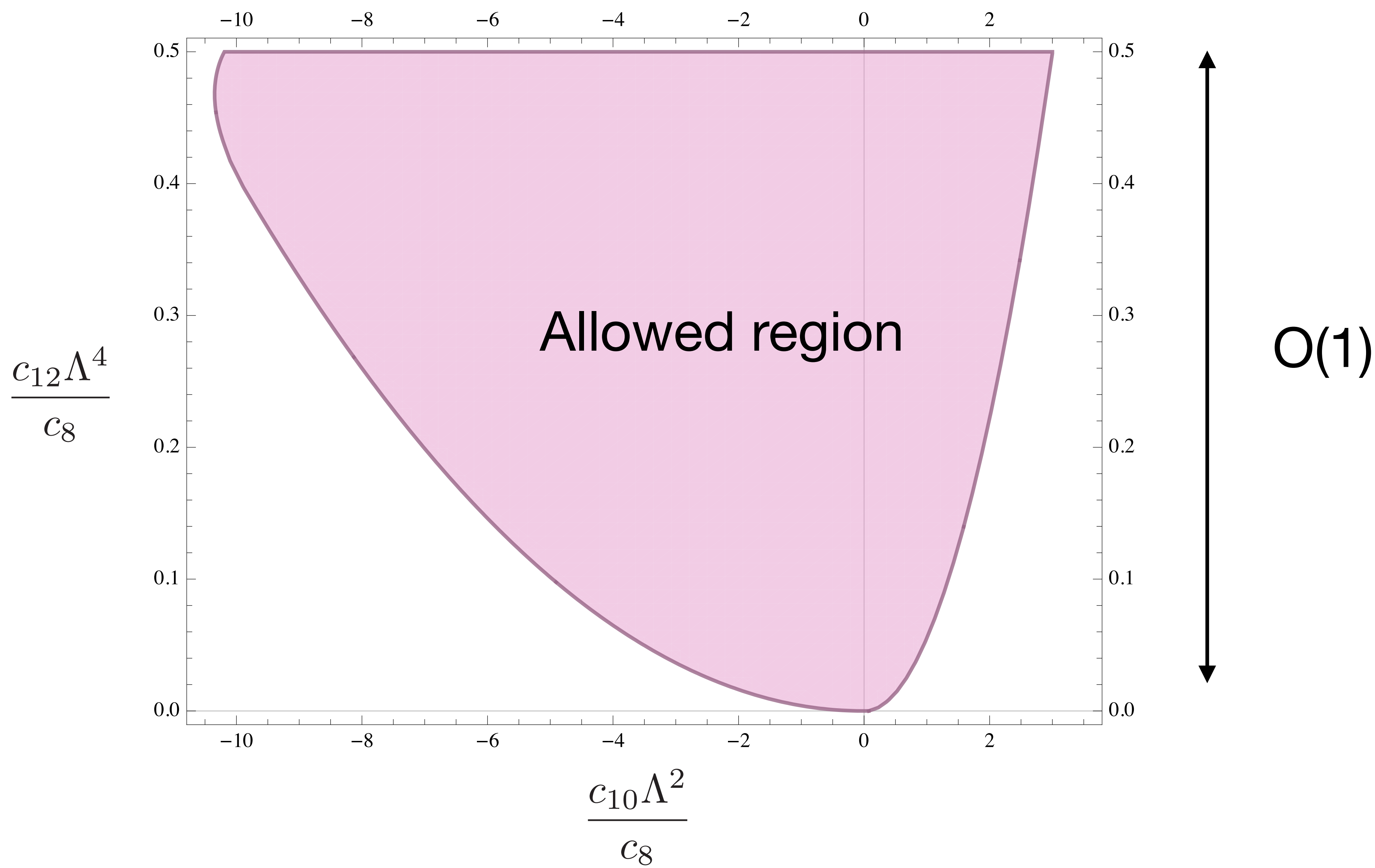
- Crossing: *Null constraints* [Caron-Huot, Duong '20]

$$A_{\text{EFT}} = \dots + \frac{4c_{12}}{\Lambda^8}(s^4 + s^3 t + \dots) + \dots \longrightarrow \left\langle \frac{\mathcal{J}^2(\mathcal{J}^2-8)}{x^8} \right\rangle = 0$$

measure the same coefficient in two channels

Modern Extensions

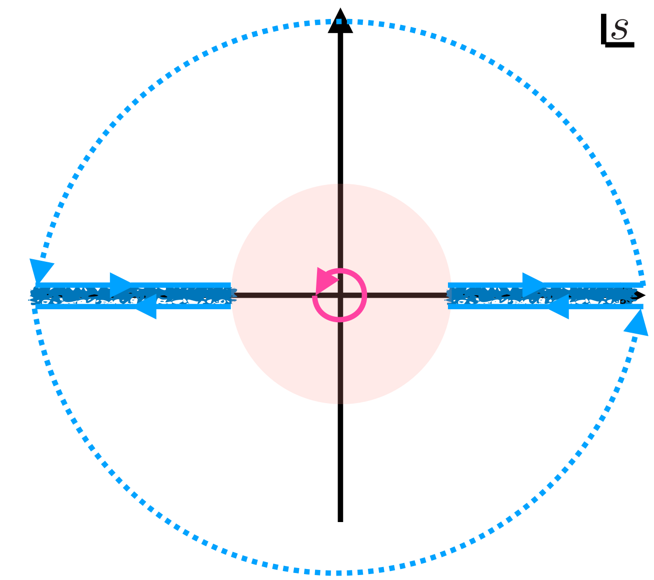
[Caron-Huot, Duong '20]



Quantitative Proof of Naive Dimension Analysis

Do they apply to the SMEFT?

From Toy model to SMEFT



- No analogous bounds for dim-6 operators
    - contribution from infinity
    - s- and u-channel cancellation
- Tree-level bounds for dim-8 SMEFT by the same arguments
  [Remmen, Rodd;….]
- [Caveat] IR side is not analytic: loops in the SMEFT at dim-8 level
- Violation of positivity known in gravity (divergent forward limit) and inflation (analyticity broken by the breaking of Lorentz boost)
  [Caron-Huot, Mazac, Rastelli, Simmons-Duffin; Caron-Huot, Li, Parra-Martinez, Simmons-Duffin]
  [Hui, Nicolis, Podo, Zhou; Creminelli, Delladio, Janssen, Longo, Senatore]

Need to scrutinize the arguments before applying on the SMEFT

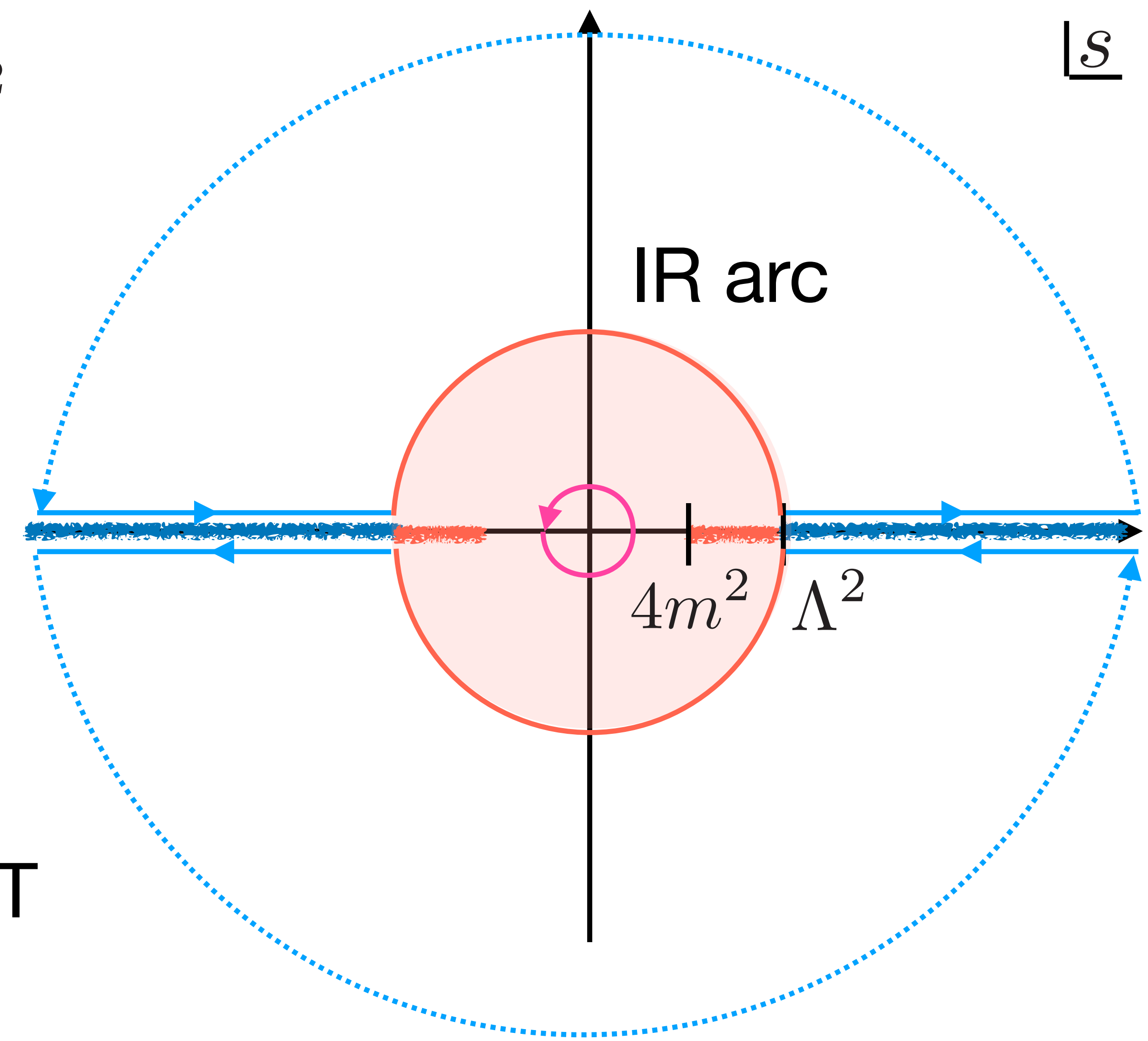
Mass deformation?

Run the same argument for $\mu^2 \leq 4m^2$ instead of $\mu^2 \leq \Lambda^2$

1. positivity for $c_8(\mu^2 = 4m^2)$
2. Still, massless photon/fermions are still around in the SMEFT

Need to analyze the IR contour in EFT to find valid bounds

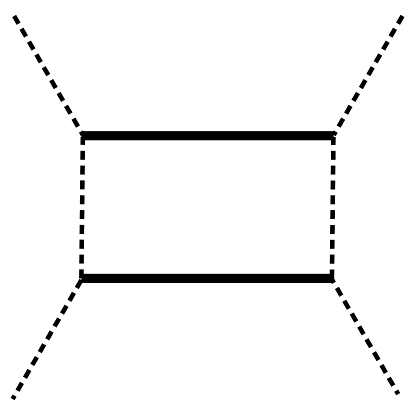
[Bellazinni, Riva, Riembau]



Analyze the EFT coefficients

Chala et al. examine the tree-level positivity below the cutoff
[Chala, Santiago '21]

$$c_i^{(8)}(\mu) = \underline{c_i^{(8)}(\Lambda)} + \underline{\beta_i^{(8)}} \ln\left(\frac{\mu}{\Lambda}\right)$$



>0? <0?

$$\gamma_{ij} c_j^{(8)} + \gamma_{ijk} c_j^{(6)} c_k^{(6)}$$

no no yes

For certain operators from weakly-coupled UV models, Chala argues

$$\beta_i^{(8)} < 0$$ [Chala; Chala Li '23]

which leads to very interesting non-renormalization theorem.

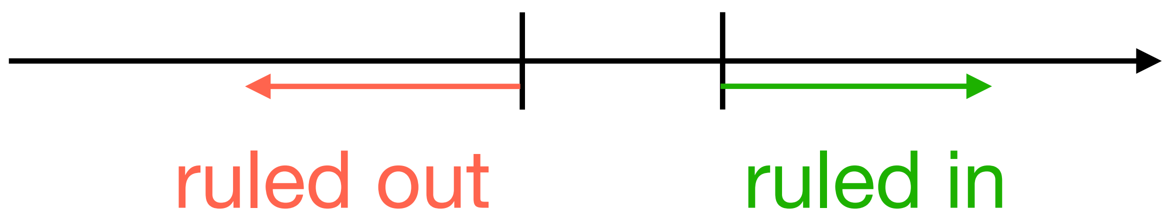
[See talk by Chala in HEFT last year]

Rule In Parameters

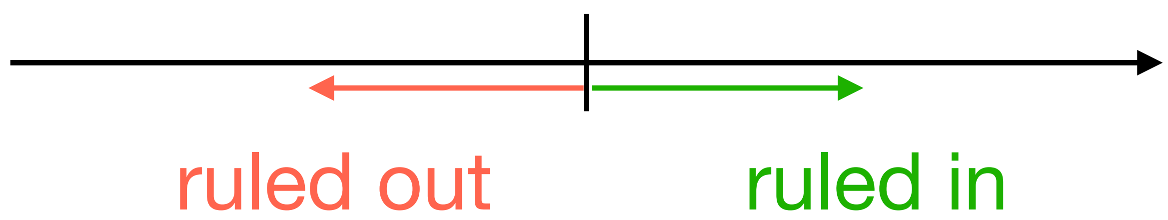
[Guerrieri, Paulos, Penedones, Toledo, van Rees, Vieira,…]

- S-matrix bootstrap: rule in parameter space by writing all ansatze consistent with unitarity, crossing, analyticity

Non-optimal:



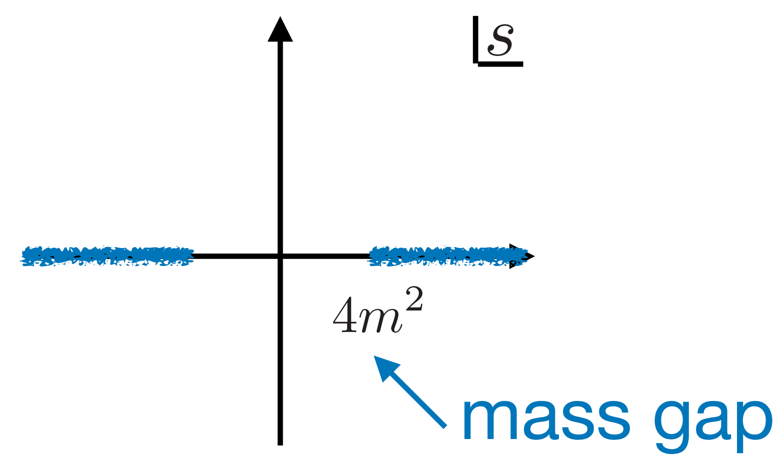
Optimal:



- Ansatze:

$$A(s,t)=\sum_{a,b,c}\alpha_{abc}\rho(s)^a\rho(t)^b\rho(u)^c$$

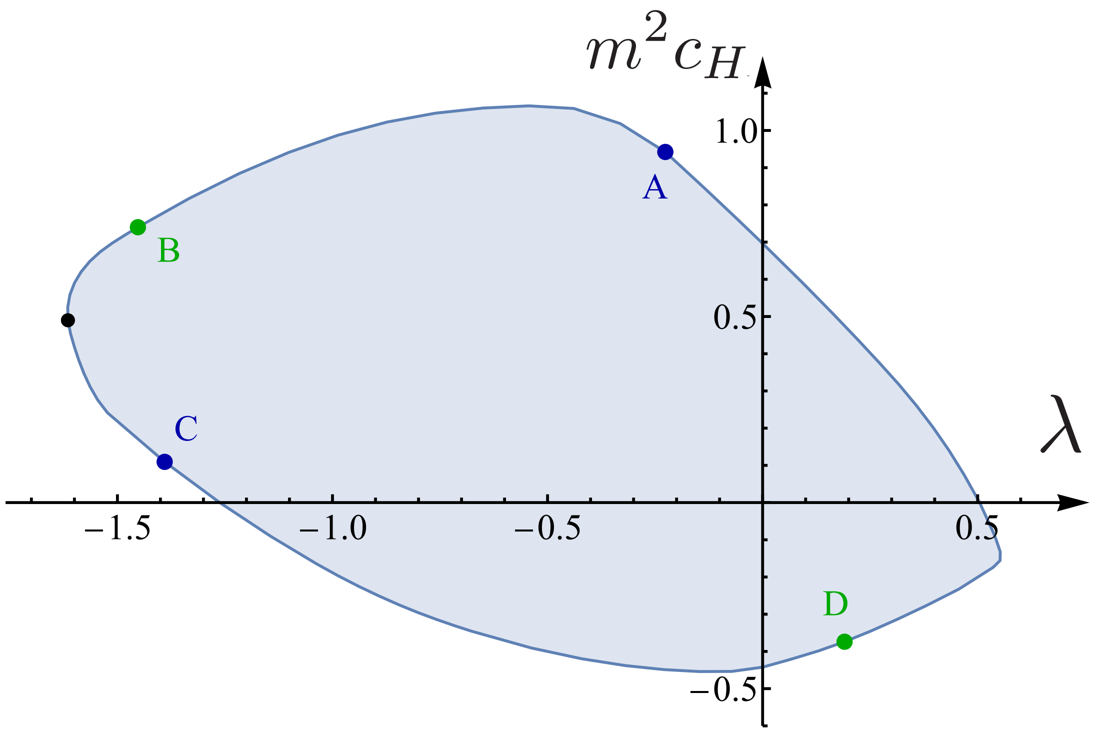
Manifest crossing and analyticity,
impose unitarity to constrain α_{abc}



The full ansatze for amplitudes contain dim-4, 6, etc.

Rule In Parameters—Higgs Application

- Proof of concept for Higgs: constraints on ***dim-6*** coefficient c_H



Generic theories

$$-0.31 < c_H \times \Lambda^2 < 0.35$$

Theories w/ weakly-coupled IR EFT

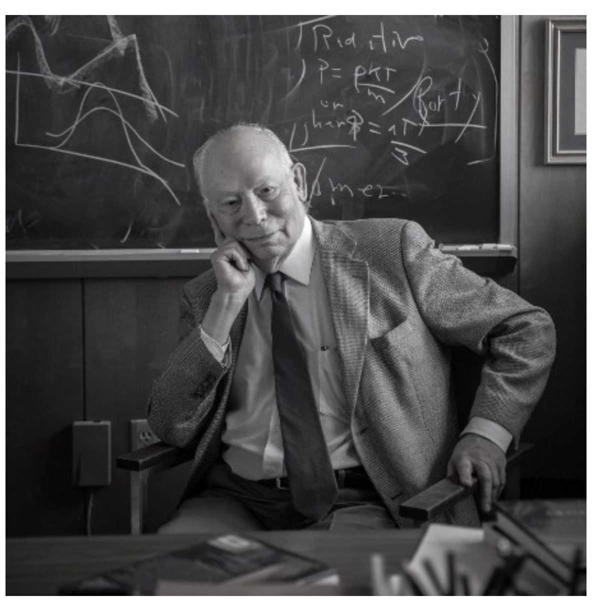
[Elias-Miro, Guerrieri, Gumus 2311.09283]

Conclusion

- Positivity bounds are surprising constraints for a wide range of UV theories
- However, applying to the SMEFT requires scrutinizing the assumptions
    - Branch cut in the IR from EFT loops
    - Counter examples have been found against tree-level bounds
- It would be great to have full understanding of the violation
    - Calculation of the IR contour in the SMEFT
    - What UV models give these violation from IR loops?
- Promising developments from S-matrix bootstrap

"My advice is to go for the messes. That is where the action is."

—Steven Weinberg, Four Golden Lessons



Thank you.