Precision calculations in SMEFT

Ben Pecjak

IPPP Durham

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Precision SMEFT

- SMEFT Lagrangian

$$\mathcal{L}^{\rm SMEFT} = \mathcal{L}^{\rm SM} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu) + \text{(dim-8 and higher)}$$

- Observables O calculated as expansion in operator dimension and loops

$$O^{\rm SMEFT} = O^{\rm SM} + \frac{v^2}{\Lambda^2} O^{(6,0)} + \underbrace{\frac{v^2}{\Lambda^2} O^{(6,1)} + \frac{v^4}{\Lambda^4} O^{(8,0)}}_{\text{“Precision”}} + \dots$$

THIS TALK: focus on $O^{(6,1)}$ = NLO dim-6

SMEFT perspectives (at NLO)

$$\mathcal{L}^{\mathrm{SMEFT}} = \mathcal{L}^{\mathrm{SM}} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu) + \ldots$$

Two perspectives:

- bottom up (the present): determine C_i through global fits
- top down (the future): if NP is known, SMEFT is tool for calculating RG-improved cross sections at scales $\mu_{\mathrm{EW}} \ll \Lambda_{\mathrm{NP}}$

NLO calculations:

- better agreement with data (top-down), more robust fits (bottom up)
- reduce dependence on renormalization scheme (for instance μ-dependence in $\overline{\mathrm{MS}}$, but more generally on different EW input schemes)

THE NLO SMEFT LANDSCAPE

A rapidly expanding field:

- Current calculations done on case-by-case basis: [Giardino, Dawson, Maltoni, Zhang, Trott, Petriello, Duhr, Schulze, Passarino, Signer, Pruna, Shepherd, Hartmann, Baglio, Lewis, Zhang, Boughezal, Degrande, BP, Vryonidou, Mimasu, Deutschmann, Scott, Dedes, Suxho, Trifyllis, Gomez-Ambrosio, Durieux, Cullen, Gauld, Haisch, Zanderighi, Corbett . . .]
- Future is automated NLO as in SM
  (already available for QCD corrections [Degrande et al. arXiv:2008.11743])

Motivation for NLO SMEFT same as in SM, but there are important differences, which are the basis of this talk.

Outline

- Technical subtleties in NLO SMEFT calculations
- Scale uncertainties and proliferation of Wilson coefficients
- EW input schemes – absolute and relative convergence

Technical subtleties in NLO SMEFT

$h \to b\bar{b}$ at NLO illustrates many subtleties in "simple"[1] setting [Cullen, BP, Scott: '19]

- structure of tadpole contributions
- decoupling relations and hybrid renormalisation schemes (mix of $\overline{\text{MS}}$ and on-shell)
- EW Ward identities and electric charge renormalisation
- fermionic wave-function renormalisation and Higgs-Z mixing

Following few slides give a feel for how NLO calculations work...

[1] UV renormalisation procedure cancels poles in $\sim$50 Wilson coefficients

THE LO $h \to b\bar{b}$ AMPLITUDE IN SMEFT

$$Q_{bH} = H^\dagger H(\bar{q}_L H b_R + \mathrm{h.c.})\,,$$

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H)\,, \quad Q_{HD} = \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)\,, \quad Q_{HWB} = H^\dagger \sigma^I H W^I_{\mu\nu} B^{\mu\nu}$$

- LO decay amplitude

$$i\mathcal{M}^{(0)}(h \to b\bar{b}) = -i\bar{u}(p_b)\left(\mathcal{M}_L^{(0)} P_L + \mathcal{M}_L^{(0)*} P_R\right) v(p_{\bar{b}})$$

- Explicit results for $\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$:

$$\mathcal{M}_L^{(4,0)} = \frac{m_b}{\hat{v}_T}\,,$$

$$\mathcal{M}_L^{(6,0)} = m_b \hat{v}_T \left[-\frac{\hat{v}_T}{m_b}\frac{C_{bH}^*}{\sqrt{2}} + C_{H\Box} - \frac{C_{HD}}{4} - \frac{1}{2}\Delta v^{(6,0,\alpha)}\right]$$

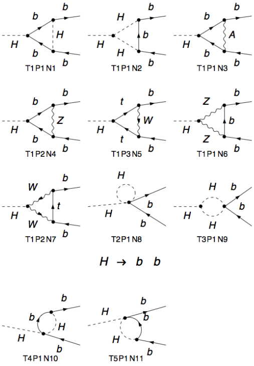
- result is particular to $\{\alpha, M_W, M_Z\}$ input scheme, where $v_T^2 \equiv \langle H^\dagger H\rangle/2$ replaced by

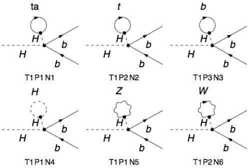
$$\frac{1}{v_{T,0}^2} = \frac{1}{\hat{v}_T^2}\left[1 - \hat{v}_T^2 \Delta v^{(6,0,\alpha)} + \dots\right]; \quad \Delta v^{(6,0,\alpha)} = -\frac{\hat{c}_w^2}{2\hat{s}_w^2} C_{HD} + 2\frac{\hat{c}_w}{\hat{s}_w} C_{HWB}$$

$$\hat{v}_T = \frac{2M_W \hat{s}_w}{\sqrt{4\pi\alpha}}, \quad \hat{c}_w = \frac{M_W}{M_Z}$$

NLO EW requires many diagrams...

Example in unitary gauge: SM and $Q_{bH} \sim H^\dagger H \bar{b}_L b_R H + \text{h.c.}$





- diagrams can be evaluated using automated (mathematica) tools
- subtleties mainly related to renormalisation procedure

UV counterterms in SMEFT

dimension-4 counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(4)}}{m_b} - \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)$$

dimension-6 counterterm is

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} =& \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right)\\ &+ \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)\\ &- \frac{\hat{v}_T^2}{\sqrt{2}}C_{bH}^*\left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w}C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)}\\ &+ m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)\end{aligned}$$

where

$$\frac{\delta\hat{v}_T}{\hat{v}_T} \equiv \frac{\delta M_W}{M_W} + \frac{\delta\hat{s}_w}{\hat{s}_w} - \frac{\delta e}{e}$$

and

$$\frac{\delta\hat{s}_w}{\hat{s}_w} = -\frac{\hat{c}_w^2}{\hat{s}_w^2}\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right), \quad \delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} = -\frac{1}{\hat{c}_w\hat{s}_w}\left(\frac{\delta\hat{s}_w^{(4)}}{\hat{s}_w}\right)$$

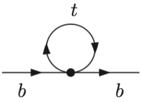
Complex C_i and renormalisation

Complex couplings beyond SM leads to subtleties in counterterms. Examples:

- Fermion w.f. renormalisation

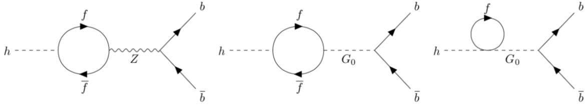
$$\delta Z_b^L = \left[-\widetilde{\mathrm{Re}}\, \Sigma_b^L(m_f^2) - m_f^2 \frac{\partial}{\partial p^2} \widetilde{\mathrm{Re}} \left[\Sigma_b^L(p^2) + \Sigma_b^R(p^2) + \Sigma_b^S(p^2) + \Sigma_b^{S*}(p^2) \right] |_{p^2=m_b^2} \right] + \Sigma_b^S(m_b^2) - \Sigma_b^{S*}(m_f^2)$$

  - $\Sigma_b^S(m_b^2) - \Sigma_f^{S*}(m_b^2)$ vanishes in SM, but is proportional $\mathrm{Im}(C_i)$ in SMEFT.
  - appears in many places in renormalisation of amplitude – example:



$$Z_b^L = \frac{1}{\epsilon}\left[-\frac{m_t^3}{m_b}\left((2N_c+1)\left(C_{qtqb}^{(1)} - C_{qtqb}^{(1)*} \right) + c_{F,3}\left(C_{qtqb}^{(8)} - C_{qtqb}^{(8)*} \right) \right) \right] + \text{finite}$$

- Higgs-Goldstone mixing $\propto \eta_5 = \frac{\sqrt{2}}{\hat{v}_T} \mathrm{Im}\left[N_c m_b C_{bH} - N_c m_t C_{tH} + m_\tau C_{\tau H} + \dots \right]$



Outline

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- Scale uncertainties and proliferation of Wilson coefficients
- EW input schemes – relative and absolute convergence

Scale uncertainties in SMEFT

$$\Gamma = \Gamma_{SM} + v^2 \sum_i C_i(\mu)\Gamma_i(\mu)$$

- all-orders perturbative result for Γ is independent of μ
- μ dependence estimates missing higher orders in a fixed order calculation
- in SM, use i.e. $\alpha_s(\mu = M_Z) = 0.1$ and RG equation to get $\alpha_s(2M_Z)$ as a number
- in SMEFT, we don't know the $C_i(\mu)$. Therefore
  - pick a fixed reference scale: canonical choice is m_H
  - as long as $\mu \sim m_H$, solution to RG equation is approximately

$$C_i(\mu) = C_i(m_H) + \ln\left(\frac{\mu}{m_H}\right)\dot{C}_i(m_H)\,, \quad \dot{C}_i(m_H) = \frac{dC}{d\ln\mu}\bigg|_{\mu=m_H} = \sum_j \gamma_{ij} C_j(m_H)$$

  - can use to calculate scale uncertainties in Γ in terms of $C_i(m_H)$
  - LO scale uncertainties involve all Wilson coefficients in γ_{ij}

Scale uncertainties in $h \to b\bar{b}$

- result from varying μ_C, μ_R independently by factor of 2 around m_H and adding in quadrature:

$$\frac{\Gamma^{\rm LO}}{\Gamma^{\rm LO,SM}} = (1 \pm 0.08) + \frac{(\bar{v})^2}{\Lambda^2_{\rm NP}} \Big\{ (3.74 \pm 0.36)\tilde{C}_{HWB} + (2.00 \pm 0.21)\tilde{C}_{H\Box} - (1.41 \pm 0.07)\frac{\bar{v}}{\overline{m}_b}\tilde{C}_{bH} + (1.24 \pm 0.14)\tilde{C}_{HD}$$
$$\pm 0.35\tilde{C}_{HG} \pm 0.19\tilde{C}^{(1)}_{Hq} \pm 0.18\tilde{C}_{Ht} \pm 0.11\tilde{C}^{(3)}_{Hq} + [\text{roughly 30 more } C_i] \Big\}$$

$$\frac{\Gamma^{\rm NLO}}{\Gamma^{\rm LO,SM}} = 1.13^{+0.01}_{-0.04} + \frac{(\bar{v})^2}{\Lambda^2_{\rm NP}} \Big\{ \left(4.16^{+0.05}_{-0.14}\right) \tilde{C}_{HWB} + \left(2.40^{+0.04}_{-0.09}\right) \tilde{C}_{H\Box}$$
$$+ \left(-1.73^{+0.04}_{-0.03}\right) \frac{\bar{v}}{\overline{m}_b}\tilde{C}_{bH} + \left(1.33^{+0.01}_{-0.04}\right) \tilde{C}_{HD} + \left(2.75^{+0.49}_{-0.48}\right) \tilde{C}_{HG}$$
$$+ \left(-0.12^{+0.04}_{-0.01}\right) \tilde{C}^{(3)}_{Hq} + \left(-0.08^{+0.05}_{-0.01}\right) \tilde{C}_{Ht} + \left(0.06^{+0.00}_{-0.05}\right) \tilde{C}^{(1)}_{Hq} + \left(0.00^{+0.07}_{-0.04}\right) \frac{\tilde{C}_{tG}}{g_s}$$
$$+ [\text{ roughly 50 more } C_i] \Big\}$$

- in general, uncertainty band in LO result overlaps with NLO one, and decreases between LO and NLO
- exception is C_{HG}, which gives large corrections unrelated to RG eqns.
- large number of Wilson coefficients generated from scale variations already at LO

Relative convergence – corrections to LO results in $h \to b\bar{b}$

	SM	C_{HWB}	$C_{H\Box}$	C_{bH}	C_{HD}
NLO QCD-QED	18.2%	17.9%	18.2%	18.2%	18.2%
NLO EW	-5.3%	-6.5%	2.0%	4.1%	-11.0%
NLO correction	12.9%	11.4%	20.2%	22.3%	7.2%

Table: Size of NLO corrections to different terms in LO decay rate, split into QCD-QED and EW components.

- applying SM K-factor to dim.6 coefficients bad approximation for EW corrections
- this is generally the case, also for other decays such as $W \to \ell\nu$ and $Z \to \ell^+\ell^-$
- possible to decipher patterns across the C_i, EW input schemes, and decays [Biekötter, BP, Scott, Smith arXiv:2305.03763]

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EW input schemes and derived parameters

In SMEFT, EW input schemes related to

$$\{g_1, g_2, v_T\} \to \{p_1, p_2, M_Z, C_i\}; \qquad p_1, p_2 \in \{\alpha, G_F, M_W, \sin\theta^\ell_{\rm eff}\}$$

- each set of 3 input parameters defines an EW input scheme
- 5 EW input schemes considered to NLO in [Biekötter, BP, Smith '24]

scheme	inputs	derived parameters
$v_\mu^{\rm eff}$	$G_F,\ \sin\theta^\ell_{\rm eff},\ M_Z$	M_W, α
$v_\alpha^{\rm eff}$	$\alpha,\ \sin\theta^\ell_{\rm eff},\ M_Z$	M_W, G_F
α	$\alpha,\ M_W,\ M_Z$	$s_w^{\rm eff}, G_F$
α_μ	$G_F,\ M_W,\ M_Z$	$s_w^{\rm eff}, \alpha$
LEP	$G_F,\ \alpha,\ M_Z$	$s_w^{\rm eff}, M_W$

- In, e.g., LEP scheme, M_W is a "derived parameter", i.e. has SMEFT expansion

$$M_W^{\rm LEP}(\alpha, G_F, M_Z, C_i) = M_W^{{\rm LEP}(4,0)} + M_W^{{\rm LEP}(4,1)} + M_W^{{\rm LEP}(6,0)} + M_W^{{\rm LEP}(6,1)} + \ldots$$

- derived parameters not only precision observables, but also key quantities for converting between schemes

Absolute convergence across schemes in SM

- in SM, good agreement of "derived parameters" implies that more complicated observables such as $\Gamma^s_{Z\ell\ell}$ converge to a common numerical value at NLO

EW input scheme		$\frac{M_W^s}{M_W} - 1$	$\frac{\alpha^s}{\alpha} - 1$	$\frac{G_F^s}{G_F} - 1$	$\frac{(s_w^{\text{eff},s})^2}{(s_w^{\text{eff}})^2} - 1$	$\frac{\Gamma^s_{Z\ell\ell}}{\Gamma_{Z\ell\ell}} - 1$
$\{G_F, s_w^{\text{eff}}, M_Z\}$	LO	−0.56%	0.21%	–	–	−0.70%
	NLO	0.05%	0.23%	–	–	0.12%
$\{\alpha, s_w^{\text{eff}}, M_Z\}$	LO	−0.56%	–	−0.21%	–	−0.91%
	NLO	0.04%	–	−0.23%	–	−0.11%
$\{G_F, M_W, M_Z\}$	LO	–	−2.44%	–	−3.72%	−0.08%
	NLO	–	0.51%	–	0.34%	−0.07%
$\{\alpha, M_W, M_Z\}$	LO	–	–	2.50%	−3.72%	2.41%
	NLO	–	–	−0.67%	0.45%	−0.74%
$\{G_F, \alpha, M_Z\}$	LO	−0.51%	–	–	−0.30%	−0.66%
	NLO	0.09%	–	–	−0.32%	0.16%

Absolute convergence across schemes in SMEFT

Selected SMEFT contributions to the $Z \to \tau\tau$ decay rate with scale variations:

scheme		C_{HD}	C_{HWB}	$C_{\underset{33}{He}}$	$C_{\underset{33}{Hu}}$	$C^{(3)}_{\underset{33}{Hq}}$
$\{G_F, s_w^{\text{eff}}, M_Z\}$	LO	$-0.500^{+0.033}_{-0.033}$	$0.000^{+0.000}_{-0.000}$	$-1.843^{+0.048}_{-0.048}$	$0.000^{+0.052}_{-0.052}$	$0.000^{+0.000}_{-0.000}$
	NLO	$-0.527^{+0.005}_{-0.000}$	$0.004^{+0.000}_{-0.000}$	$-1.905^{+0.004}_{-0.000}$	$0.048^{+0.000}_{-0.013}$	$0.022^{+0.000}_{-0.004}$
$\{\alpha, s_w^{\text{eff}}, M_Z\}$	LO	$0.000^{+0.000}_{-0.000}$	$2.370^{+0.081}_{-0.081}$	$-1.843^{+0.050}_{-0.050}$	$0.000^{+0.003}_{-0.003}$	$0.000^{+0.005}_{-0.005}$
	NLO	$-0.001^{+0.000}_{-0.000}$	$2.439^{+0.000}_{-0.006}$	$-1.903^{+0.004}_{-0.000}$	$0.005^{+0.000}_{-0.001}$	$0.002^{+0.000}_{-0.000}$
$\{G_F, M_W, M_Z\}$	LO	$-0.169^{+0.011}_{-0.011}$	$0.355^{+0.012}_{-0.012}$	$-1.764^{+0.046}_{-0.046}$	$0.000^{+0.018}_{-0.018}$	$0.000^{+0.001}_{-0.001}$
	NLO	$-0.289^{+0.009}_{-0.007}$	$0.258^{+0.003}_{-0.004}$	$-1.897^{+0.006}_{-0.002}$	$0.018^{+0.011}_{-0.016}$	$0.006^{+0.000}_{-0.002}$
$\{\alpha, M_W, M_Z\}$	LO	$1.573^{+0.108}_{-0.108}$	$4.088^{+0.143}_{-0.143}$	$-1.764^{+0.050}_{-0.050}$	$0.000^{+0.162}_{-0.162}$	$0.000^{+0.008}_{-0.008}$
	NLO	$1.408^{+0.002}_{-0.019}$	$3.869^{+0.002}_{-0.013}$	$-1.898^{+0.006}_{-0.002}$	$-0.142^{+0.030}_{-0.000}$	$-0.073^{+0.014}_{-0.000}$
$\{G_F, \alpha, M_Z\}$	LO	$-0.600^{+0.040}_{-0.040}$	$-0.474^{+0.016}_{-0.016}$	$-1.837^{+0.048}_{-0.048}$	$0.000^{+0.062}_{-0.062}$	$0.000^{+0.001}_{-0.001}$
	NLO	$-0.631^{+0.005}_{-0.000}$	$-0.475^{+0.001}_{-0.000}$	$-1.899^{+0.004}_{-0.000}$	$0.057^{+0.000}_{-0.015}$	$0.025^{+0.000}_{-0.005}$

- in SMEFT, dependence of "derived parameters" on C_i obfuscates absolute convergence of more complicated observables in different schemes
- fits of C_i in different schemes sensitive to different sets of dim-8 and loop corrections, can provide sanity checks on fits

Relative convergence: 3 decays in 3 input schemes

NLO EW corrections ($\alpha_s = 0$ in $h \to bb$)

$h \to b\bar{b}$	SM	$C_{H\Box}$	C_{HD}	$C_{\substack{dH\\33}}$	C_{HWB}	$C^{(3)}_{\substack{Hl\\jj}}$	$C_{\substack{ll\\1221}}$
α-scheme: $\{M_W, M_Z, \alpha\}$	-5.2 %	2.1%	-11.0%	4.2%	-6.7%	-	-
α_μ-scheme: $\{M_W, M_Z, G_F\}$	-0.8 %	2.1%	2.0%	1.9%	-	0.9%	-0.8%
LEP scheme: $\{\alpha, M_Z, G_F\}$	-0.7 %	2.1%	1.6%	1.9%	-	0.7%	-0.9%

$Z \to \tau\tau$	SM	C_{HD}	C_{HWB}	$C_{\substack{He\\33}}$	$C^{(1)}_{\substack{Hl\\33}}$	$C^{(3)}_{\substack{Hl\\33}}$	$C^{(3)}_{\substack{Hl\\jj}}$	$C_{\substack{ll\\1221}}$
α	-4.0%	-10.6%	-5.4%	7.7%	0.3%	-0.5%	—	—
α_μ	$< 0.1\%$	71.1%	-27.2%	7.6%	0.1%	-0.4%	2.9%	0.6%
LEP	1.0%	7.8%	17.4%	2.0%	4.7%	4.2%	6.9%	4.5%

$W \to \tau\nu$	SM	C_{HD}	C_{HWB}	$C^{(3)}_{\substack{Hl\\jj}}$	$C_{\substack{ll\\1221}}$	$C^{(3)}_{\substack{Hl\\33}}$
α	-4.2%	-1.7%	-3.0%	—	—	2.2%
α_μ	-0.3%	—	—	2.5%	-0.2%	2.2%
LEP	2.0%	8.1%	3.2%	5.1%	2.5%	4.6%

Is there any rhyme or reason to the pattern across C_i?

Connecting schemes

Start with $\mathcal{L}_{\rm bare}(M_W, M_Z, v_T, \dots)$, and renormalise v_T as

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\sigma^2}\left[1 - v_\sigma^2 \Delta v_\sigma^{(6,0,\sigma)} - \frac{1}{v_\sigma^2}\Delta v_\sigma^{(4,1,\sigma)} - \Delta v_\sigma^{(6,1,\sigma)}\right]; \quad \sigma \in \{\alpha, \mu\}$$

$$v_\alpha \equiv \frac{2 M_W s_w}{\sqrt{4\pi\alpha}}, \qquad v_\mu \equiv \left(\sqrt{2} G_F\right)^{-\frac{1}{2}}$$

- for α scheme $\{M_W, M_Z, \alpha\}$: use $\sigma = \alpha$ and determine Δv_α from charge ren.
- for α_μ scheme $\{M_W, M_Z, G_F\}$: use $\sigma = \mu$ and determine Δv_μ from muon decay
- for LEP scheme $\{\alpha, M_Z, G_F\}$: start with α_μ scheme, and then eliminate M_W using

$$\frac{v_\alpha^2}{v_\mu^2} - 1 \equiv \Delta r = v_\mu^2 \Delta r^{(6,0)} + \frac{1}{v_\mu^2}\Delta r^{(4,1)} + \Delta r^{(6,1)}$$

where $\boldsymbol{\Delta r^{(i,j)}}$ **are finite** and related to $\Delta v_{\mu\alpha} = \Delta v_\mu - \Delta v_\alpha$

$$\Delta r^{(6,0)} = \Delta v_{\mu\alpha}^{(6,0)}, \quad \Delta r^{(4,1)} = \Delta v_{\mu\alpha}^{(4,1)}, \quad \Delta r^{(6,1)} = \Delta v_{\mu\alpha}^{(6,1)} + 2\Delta v_\mu^{(4,1,\mu)} \Delta v_{\mu\alpha}^{(6,0)}$$

Top loops and universal corrections

- Δr is physical, Δv_σ is not. However, in large-m_t limit in SM:

$$\frac{1}{v_{T,0}^2}\bigg|_{m_t\to\infty} = \frac{1}{v_\sigma^2}\left[1 + \frac{1}{v_\sigma^2}\left(\Delta r_t^{(4,1)}\delta_{\alpha\sigma} - 2\Delta M_{W,t}^{(4,1)}\right)\right] \; ; \quad \sigma \in \{\alpha, \mu\}$$

$$\frac{\Delta r_t^{(4,1)}}{v_\alpha^2} = -\frac{c_w^2}{s_w^2}\frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \approx -3.5\% \,, \qquad \frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \equiv \frac{3}{16\pi^2}\frac{m_t^2}{v_\alpha^2} \approx 1\%$$

- universal correction $\Delta r_t^{(4,1)}$ in α-scheme comes along with LO (can resum!)
- we generalised this to include universal scheme-dependent corrections in SMEFT through a substitution procedure on LO [arXiv:2305.03763], for example

$$\frac{1}{v_T^2} \to \frac{1}{v_\sigma^2}\left[\underbrace{1 + v_\sigma^2 K_t^{(6,0,\sigma)} + \frac{K_t^{(4,1,\sigma)}}{v_\sigma^2} + K_t^{(6,1,\sigma)}}_{\mathrm{LO}_K} + (\text{divergent and unphysical stuff})\right]$$

- the K_t are physical top-loop corrections that always come along with LO
- $\Rightarrow$ re-organise pert. theory. to include them already in "LO_K" approximation

NLO corrections to LO_K results

$W \to \tau\nu$	SM	C_{HD}	C_{HWB}	$C^{(3)}_{\underset{jj}{Hl}}$	$C_{\underset{1221}{ll}}$	$C^{(3)}_{\underset{33}{Hl}}$
α	−0.9%	1.1%	0.6%	—	—	2.2%
α_μ	−0.3%	—	—	0.6%	−0.2%	2.2%
LEP	0.0 %	1.9%	0.9 %	0.1%	0.2%	2.5%

$Z \to \tau\tau$	SM	C_{HD}	C_{HWB}	$C_{\underset{33}{He}}$	$C^{(1)}_{\underset{33}{Hl}}$	$C^{(3)}_{\underset{33}{Hl}}$	$C^{(3)}_{\underset{jj}{Hl}}$	$C_{\underset{1221}{ll}}$
α	−0.9%	−1.4%	−0.1%	3.3%	2.0%	1.3%	—	—
α_μ	0.0%	11.2%	−3.4%	3.2%	1.8%	1.3%	0.8%	0.0%
LEP	0.0%	2.3%	−3.0%	2.5%	2.5%	2.0%	0.8%	0.0%

$h \to b\bar{b}$	SM	$C_{H\Box}$	C_{HD}	$C_{\underset{33}{dH}}$	C_{HWB}	$C^{(3)}_{\underset{jj}{Hl}}$	$C_{\underset{1221}{ll}}$
α	-1.9 %	2.1%	2.5%	2.5%	-1.5%	-	-
α_μ	-0.8 %	2.1%	2.0%	1.9%	-	0.9%	-0.8%
LEP	-0.8 %	2.1%	1.6%	1.9%	-	0.7%	-0.9%

Corrections smaller and less scheme dependent compared to pure fixed order

Summary

- NLO dim-6 SMEFT calculations involve many subtleties compared to SM
- Precision SMEFT is a game of many Wilson coefficients and requires global fits
- Global fits in different EW input schemes can provide important consistency checks
- Longer-term future is automated NLO SMEFT