FTAE High Energy Theory

Junta de Andalucía

ugr Universidad de Granada

Recent developments in EFT matching and RG running

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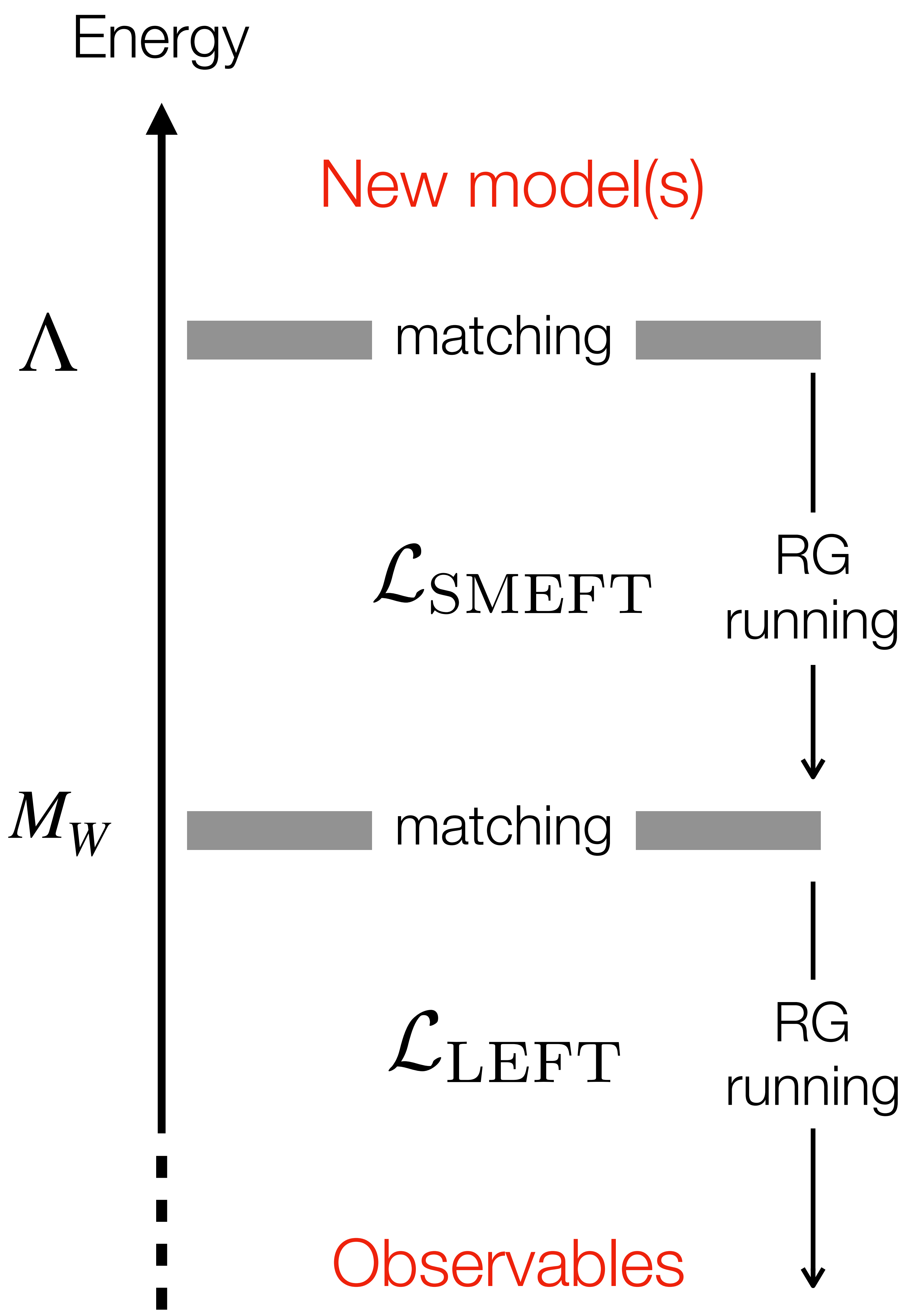
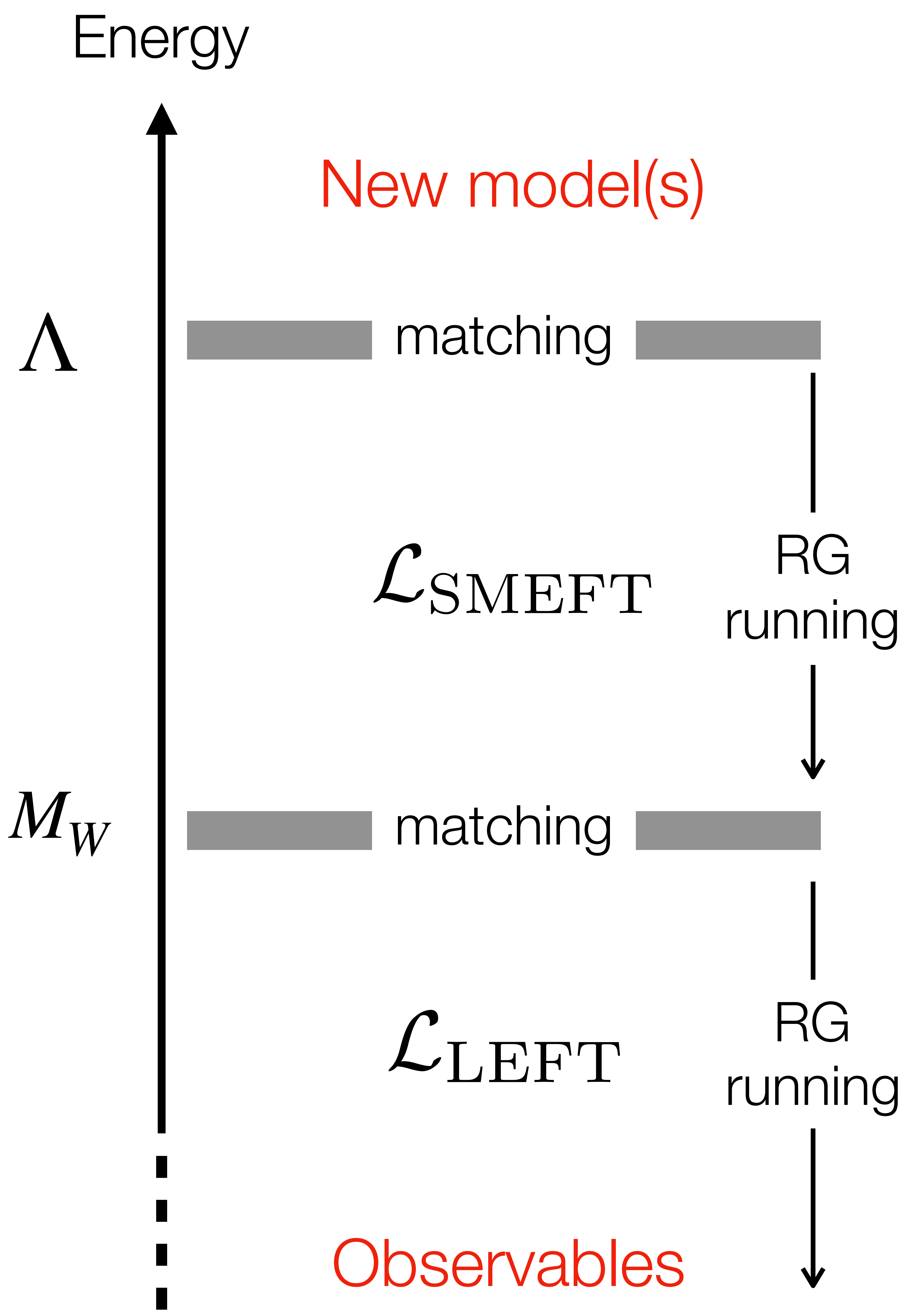
Higgs and Effective Field Theory — HEFT 2024, Bologna, 12-14 June 2024

The rise of automation



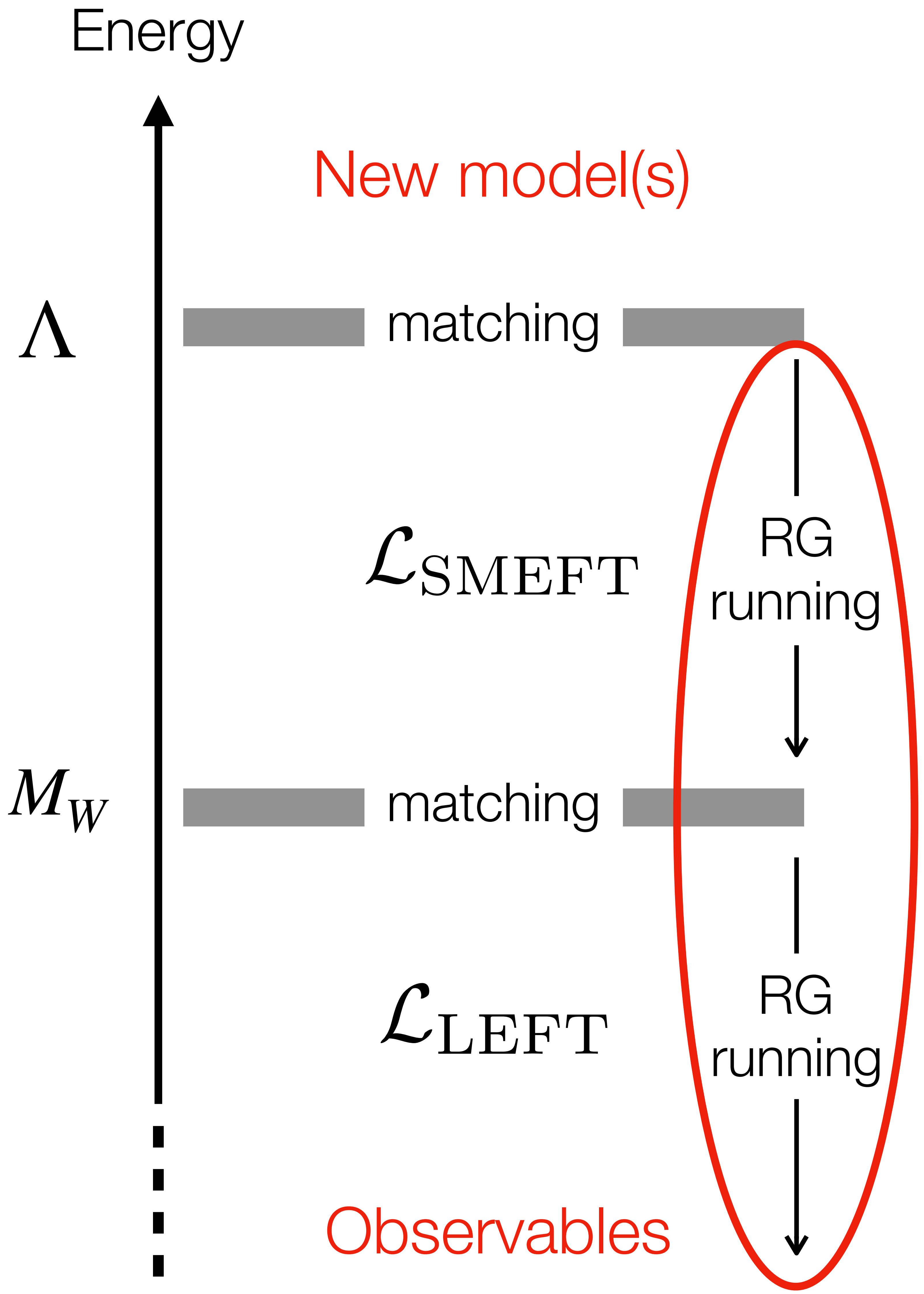
The rise of automation



Main motivation

The vast landscape of BSM models and the repetitive nature of EFT computations call for automated solutions

The rise of automation



DsixTools

JFM et al. '17 & '21

wilson

Aebischer et al. '18

"Hard-coded" one-loop results based on:

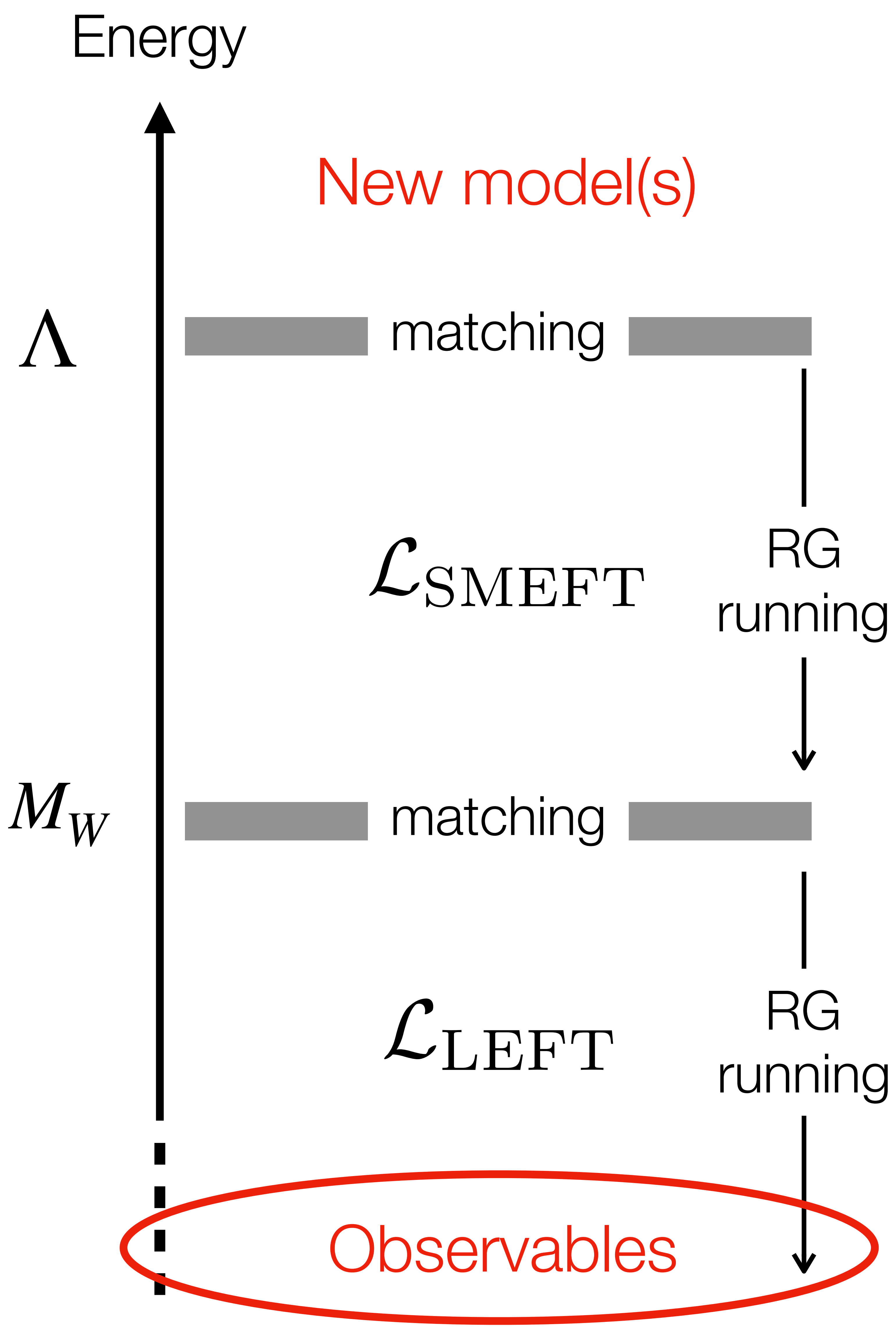
SMEFT running: Jenkins et al. '13, '14; Alonso et al. '14

LEFT basis: Jenkins et al. '18

SMEFT-LEFT matching: Dekens, Stoffer '19

LEFT running: Jenkins et al. '18

The rise of automation



SMEFT likelihood (smelli)

Aebischer et al. '18

flavio

Straub '16

HighPT

Allwicher et al. '22

HEPfit

De Blas et al. '19

Fitmaker

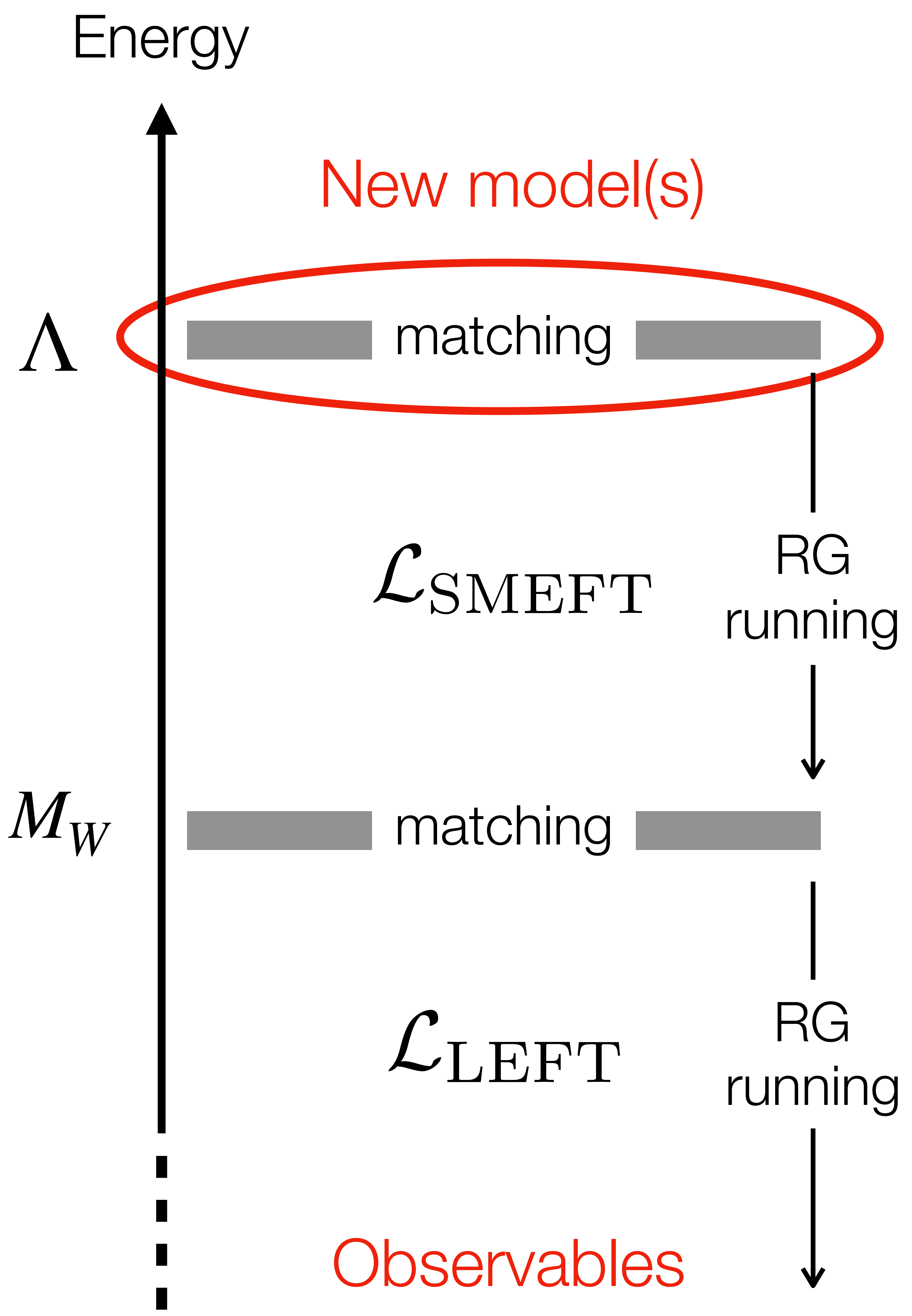
Ellis et al '20

SMEFiT

Giani et al. '23

+ others

The rise of automation

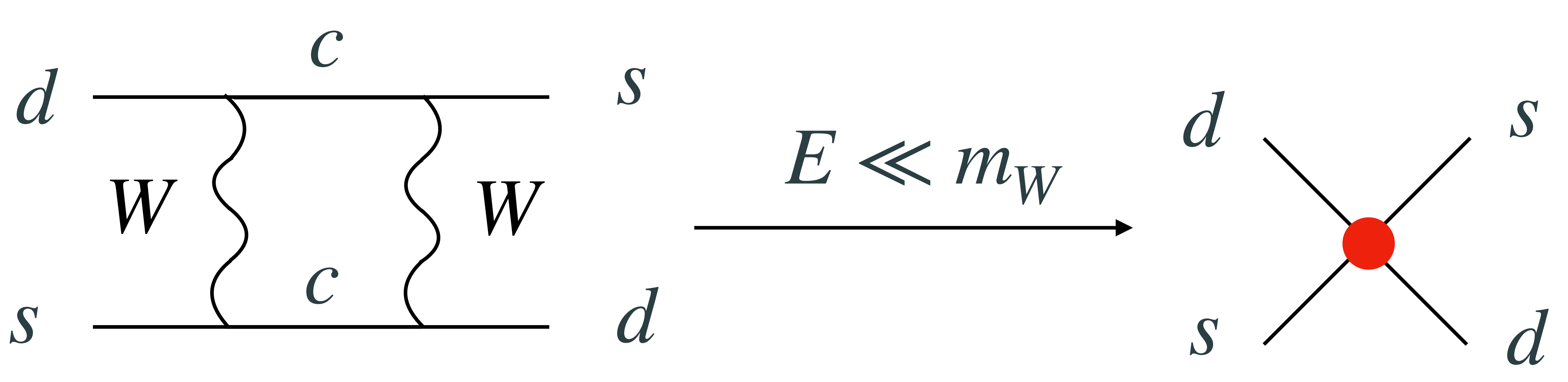


Much progress has been made:

- Tree-level matching to the SMEFT is a solved problem

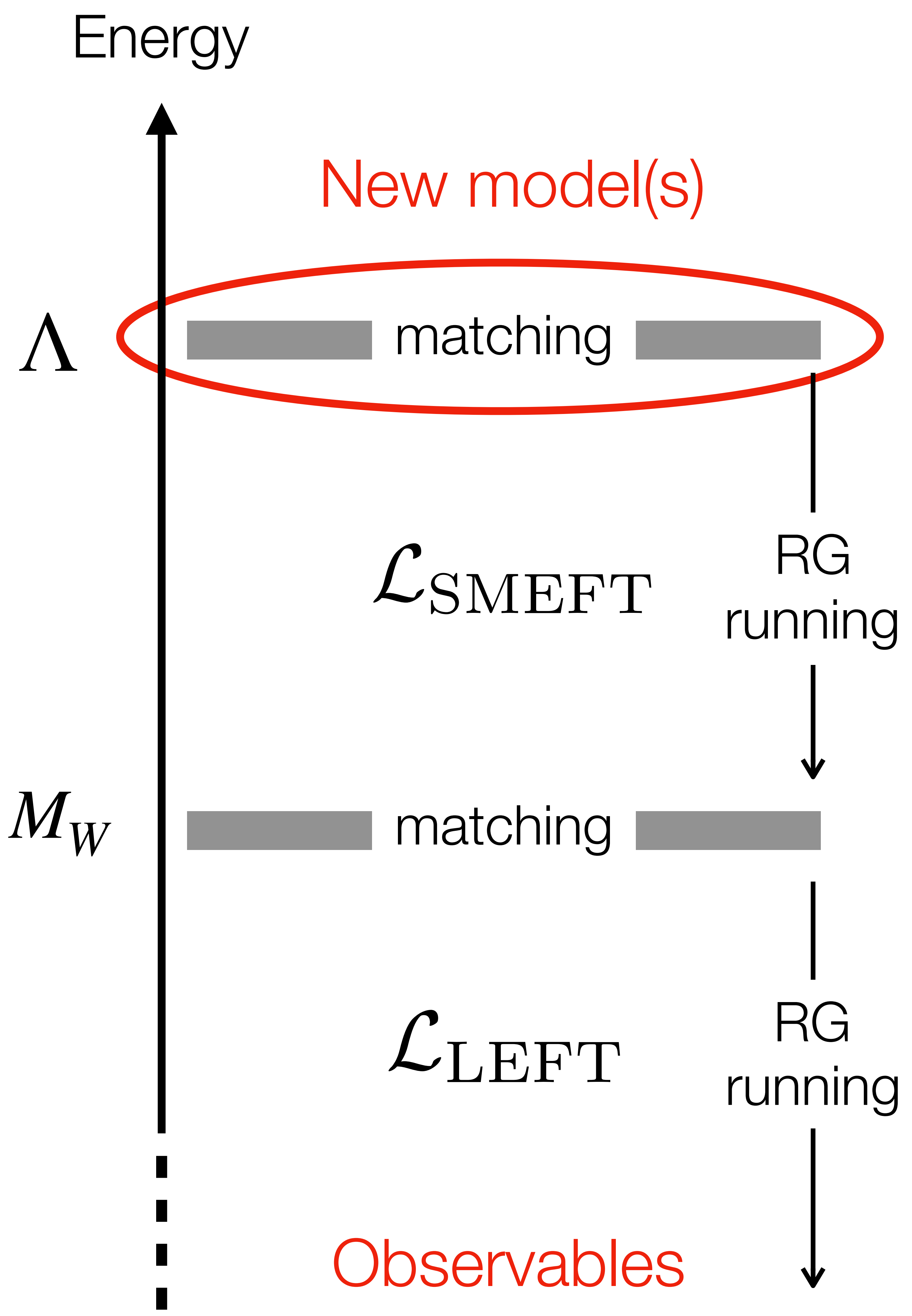
  [de Blas, Criado, Pérez-Victoria, Santiago, '17] MatchingTools: [Criado '17]

- One-loop can be the leading effect in important processes. E.g., in the SM



Similarly, in BSM models: dipoles, FCNCs, EW precision…

The rise of automation



matchmakereft
Carmona et al. '22

MATCHETE
JFM et al. '23

Automated one-loop matching of *many* models

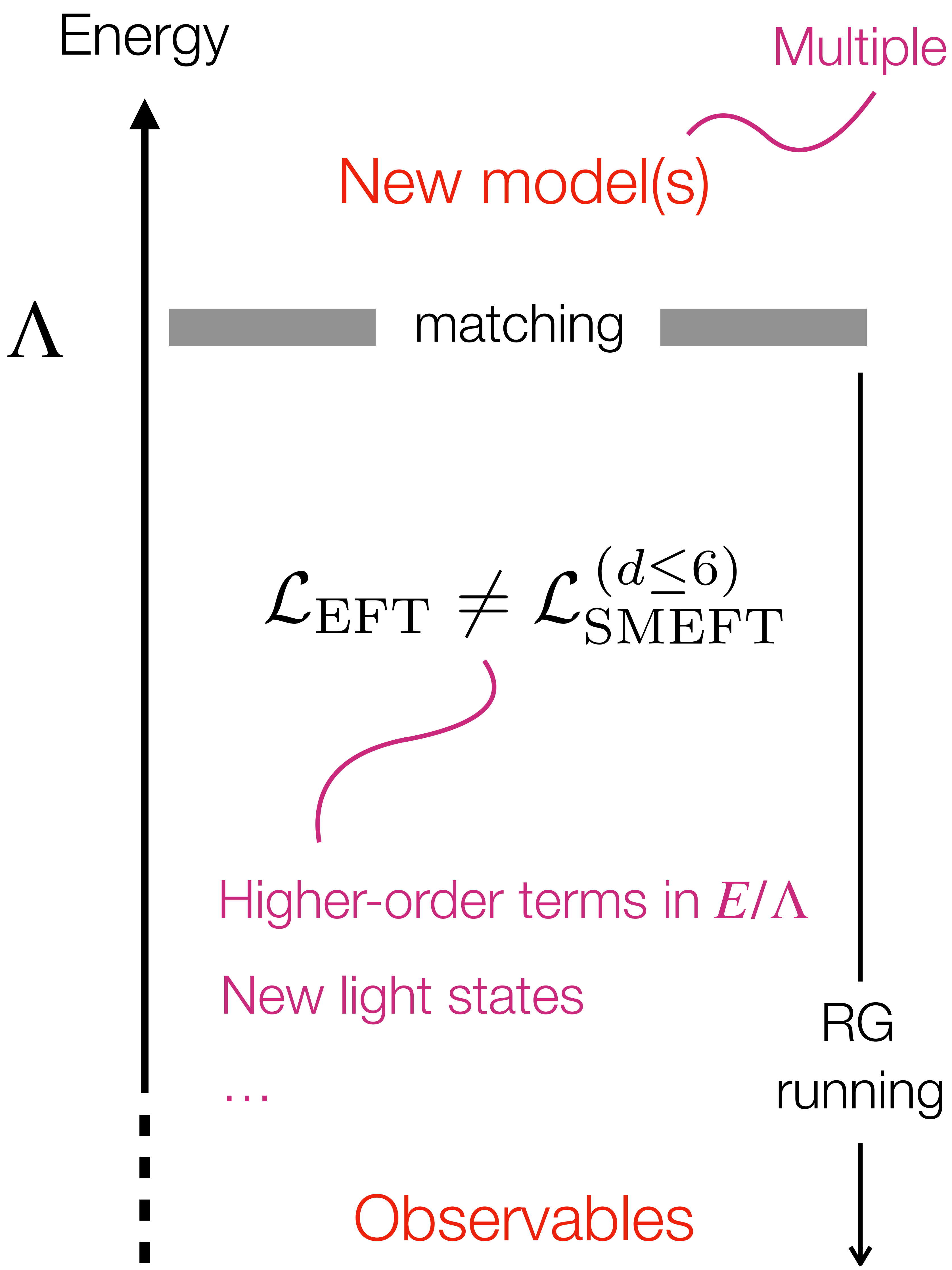
SOLD
Guedes et al. '23

UV-SMEFT dictionaries

"Breaking SMEFT operators"
UV-to-SMEFT mapping

Cepedello et al. '23

The rise of automation



Present limitations

Full automation only for simpler scenarios (no heavy vectors yet!)

Some steps/approaches require prior knowledge of the target EFT

(Automated) inclusion of higher-loop orders is (so far) non-trivial

EFT matching

The path-integral approach in a nutshell

Functional matching

- **Lagrangian:** $\mathscr{L}_{\mathrm{UV}}$ with fields $\eta = (\eta_H \; \eta_L)^{\intercal}$ and hierarchy $m_H \gg m_L$

- **Background field method:** shift *all* fields $\eta \to \hat{\eta} + \eta$

  $\hat{\eta}$: background fields (satisfy the quantum EOM) [Tree lines in Feynman graphs]

  $\hat{\eta}$: quantum fluctuations [Loop lines in Feynman graphs]

- **Quantum effective action:**

$$e^{i\Gamma_{\mathrm{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta \;\exp\left(i\int \mathrm{d}^d x \,\mathscr{L}_{\mathrm{UV}}(\eta + \hat{\eta})\right)$$

Goal: Evaluate the path integral
("integrate out" the quantum fluctuations)
and isolate the EFT contribution

Functional matching

- **Expanding the Lagrangian in η:**

$$\mathscr{L}_{\mathrm{UV}}(\hat{\eta}+\eta) = \mathscr{L}_{\mathrm{UV}}(\hat{\eta}) + \left.\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_a}\right|_{\eta=\hat{\eta}} \eta_a + \frac{1}{2}\,\bar{\eta}_a(x) \left.\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \eta_b(x')\,\delta \bar{\eta}_a(x)}\right|_{\eta=\hat{\eta}} \eta_b(x') + \mathcal{O}(\eta^3)$$

Functional matching

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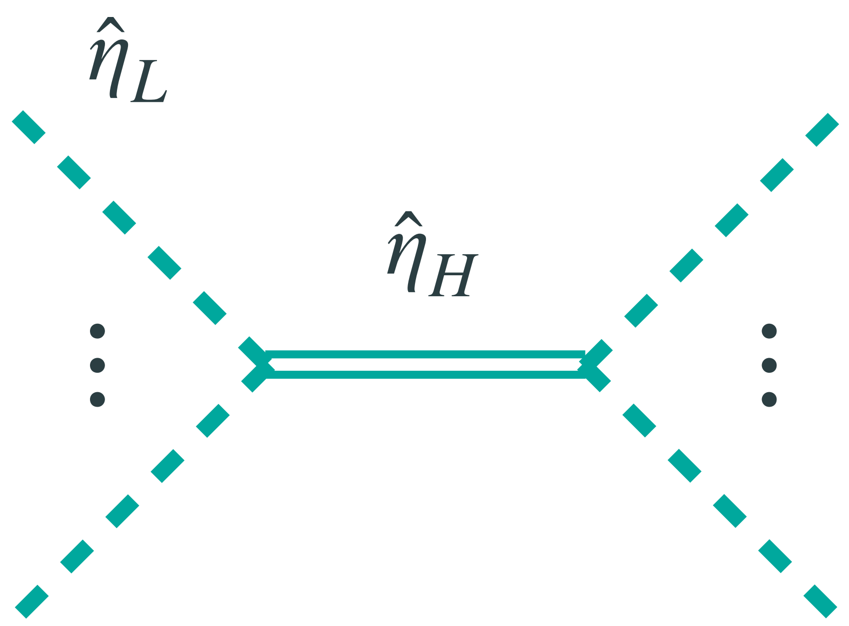
- **Tree-level:** $\mathscr{L}^{(0)}_{\mathrm{EFT}} = \mathscr{L}_{\mathrm{UV}}\big(\hat\eta_L, \hat\eta_H(\hat\eta_L)\big)$

— Substitute $\hat\eta_H$ by its EOM expanded in m_H^{-1}

[Simpler than computing Feynman graphs]

$$\left.\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_H}\right|_{\eta=\hat\eta} = 0$$



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Functional matching

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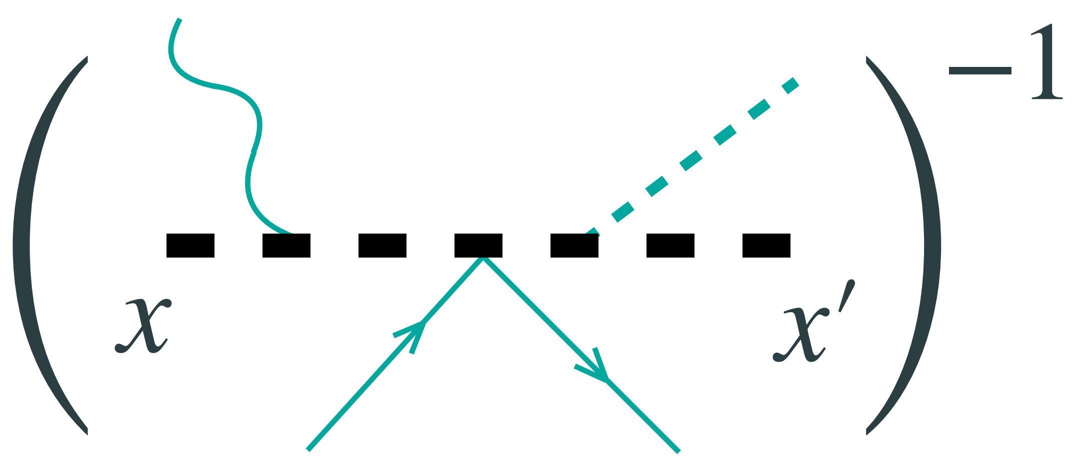
- **Inverse quantum-field propagator:**

Wilson line

[parallel transport $x \leftrightarrow x'$]

New!!

$$\mathcal{Q}_{ab}(x,x') = Q_{ac}\Big(\hat\eta(x), \hat D_x^{\mu_1}\cdots\hat D_x^{\mu_n}\hat\eta(x), \hat D_x^{\mu}\Big)\, \boldsymbol{U_{cb}(x,x')}\,\delta(x-x')$$



$$\left.\hat D_x^{\mu_1}\cdots\hat D_x^{\mu_n}\,U(x,x')\right|_{x=x'} = p_n\left(G^{\mu\nu}, D^\mu G^{\nu\rho}, \ldots\right)$$

[Kuzenko, McArthur, ’03]

[JFM, Moreno, Palavrić, Thomsen, w.i.p]

Functional matching

- **Expanding the Lagrangian in η:**

$$\mathcal{L}_{\rm UV}(\hat\eta+\eta) = \boxed{\mathcal{L}_{\rm UV}(\hat\eta)} + \cancelto{0}{\frac{\delta \mathcal{L}_{\rm UV}}{\delta\eta_a}\bigg|_{\eta=\hat\eta}} \eta_a + \frac{1}{2}\, \bar\eta_a(x) \overset{\displaystyle\int_{x'} \mathcal{Q}_{ab}(x,x') \;\equiv}{\boxed{\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\eta_b(x')\,\delta\bar\eta_a(x)}\bigg|_{\eta=\hat\eta}}} \eta_b(x') + \boxed{\mathcal{O}(\eta^3)}$$

Higher-loop orders

(more later)

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[parallel transport $x \leftrightarrow x'$]

New!!

$$\mathcal{Q}_{ab}(x,x') = Q_{ac}\Big(\hat\eta(x), \hat D_x^{\mu_1}\cdots\hat D_x^{\mu_n}\hat\eta(x), \hat D_x^{\mu}\Big)\, \boldsymbol{U_{cb}(x,x')}\, \delta(x-x')$$

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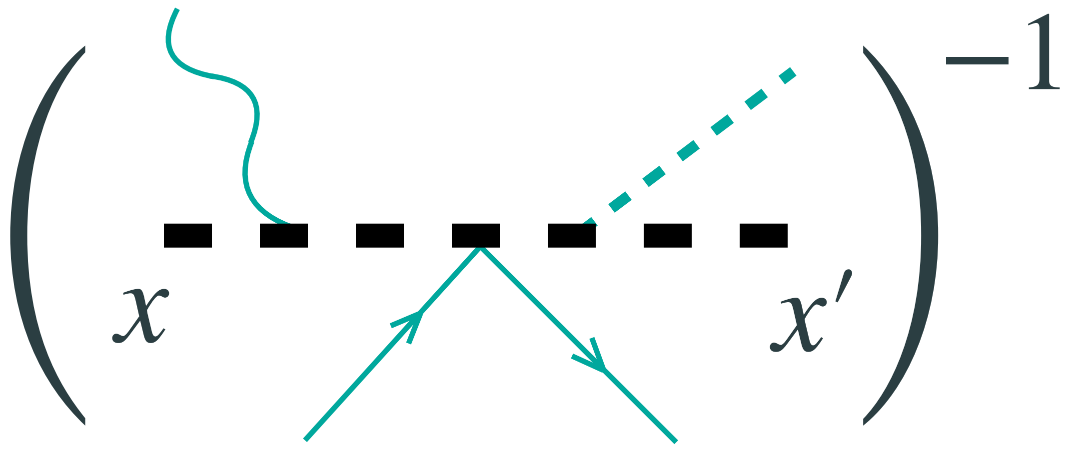

[Kuzenko, McArthur, ’03]

[JFM, Moreno, Palavrić, Thomsen, w.i.p]

Functional matching

- **Expanding the Lagrangian in η:**

$$\int_{x'} \mathcal{Q}_{ab}(x,x') \equiv$$

$$\mathscr{L}_{\mathrm{UV}}(\hat{\eta}+\eta) = \boxed{\mathscr{L}_{\mathrm{UV}}(\hat{\eta})} + \cancelto{0}{\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_a}}\Bigg|_{\eta=\hat{\eta}} \eta_a + \frac{1}{2}\,\bar{\eta}_a(x) \boxed{\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \eta_b(x')\,\delta \bar{\eta}_a(x)}\Bigg|_{\eta=\hat{\eta}}} \eta_b(x') + \boxed{\mathcal{O}(\eta^3)}$$

Higher-loop orders
(more later)

- **1-loop effective action:**

$$e^{i\,\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta\ \exp\left(i \int_{x,x'} \bar{\eta}_a\, \mathcal{Q}_{ab}\, \eta_b \right) \xrightarrow{\text{Gaussian integration}} \Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = -\,i \ln \mathrm{SDet}\, \mathcal{Q}^{-1/2} = \frac{i}{2}\, \mathrm{STr} \ln \mathcal{Q}$$

How to evaluate supertraces

$$\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = \frac{i}{2}\,\mathrm{STr}\ln\mathcal{Q} = \pm\frac{i}{2}\int_{x,x'}\delta(x-x')\,\overbrace{[\,\ln Q(x,iD^{\mu}_{x})\,]_{ab}\,U_{ba}(x,x')\,\delta(x-x')}^{[\ln \mathcal{Q}(x,x')]_{aa}}$$

How to evaluate supertraces

$$[\ln \mathcal{Q}(x,x')]_{aa}$$

$$\int_k e^{ik(x-x')}$$

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$$= \pm\frac{i}{2}\int_{x,k} [\ln Q\big(x, i\hat{D}^\mu_x - k\big)]_{ab}\,U_{ba}(x,x')\Big|_{x=x'}$$

How to evaluate supertraces

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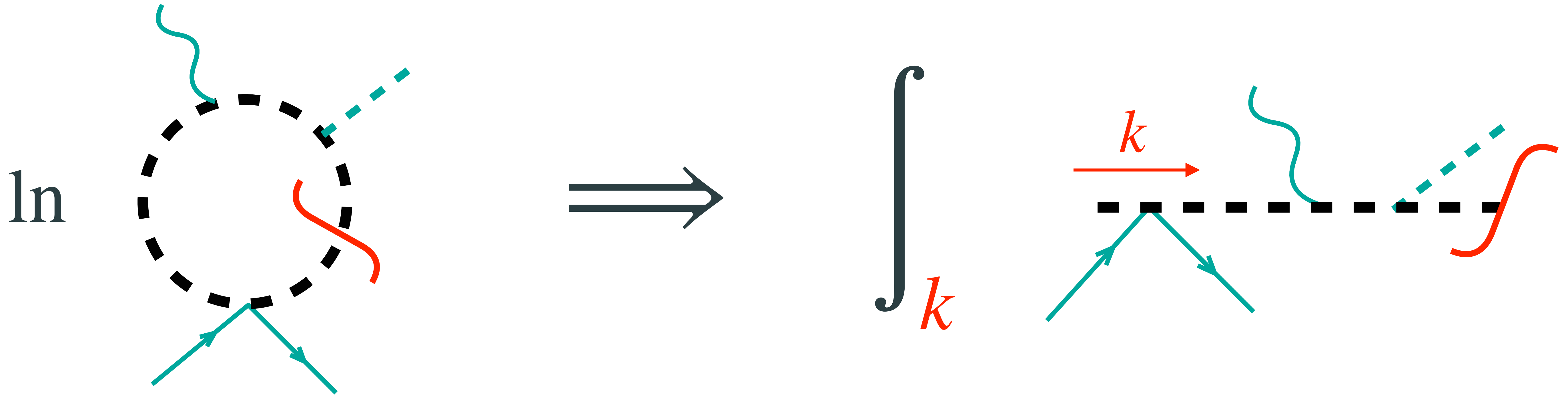

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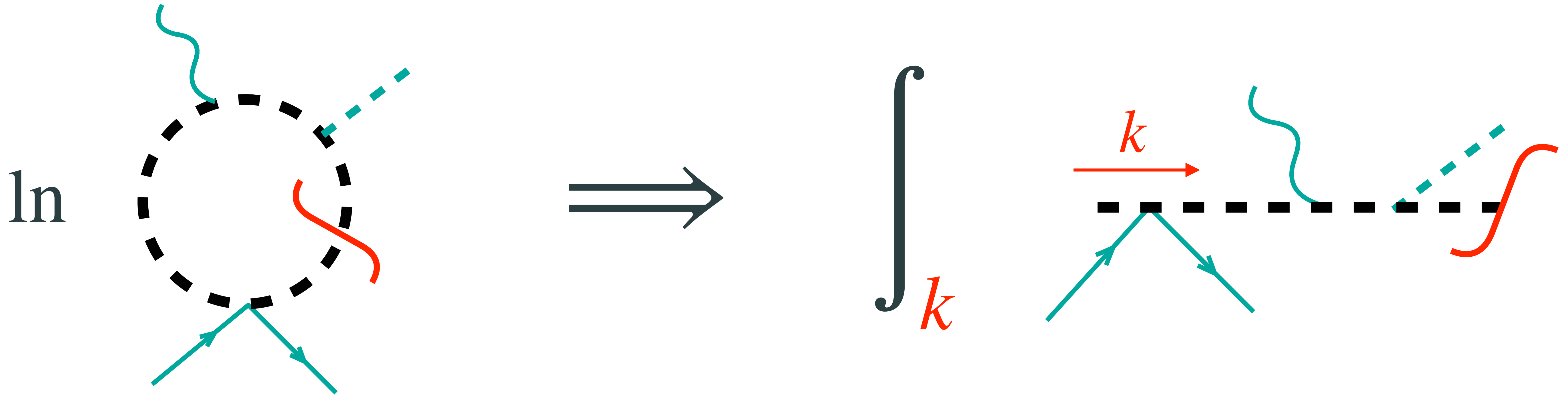


The EFT action (to arbitrary EFT order) is obtained *directly* from the hard-momentum expansion: $k \gtrsim m_H$

[JFM, Portolés, Ruiz-Femenía, 1607.02142; Z. Zhang 1610.00710]

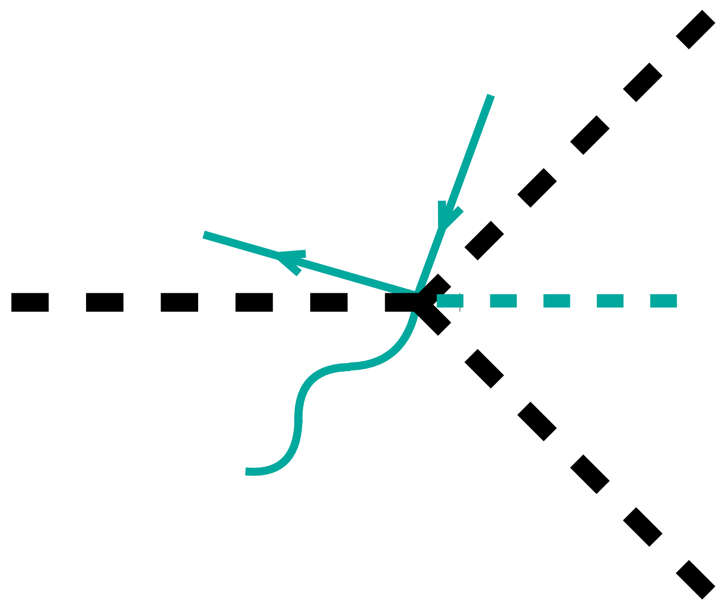
Going beyond one loop

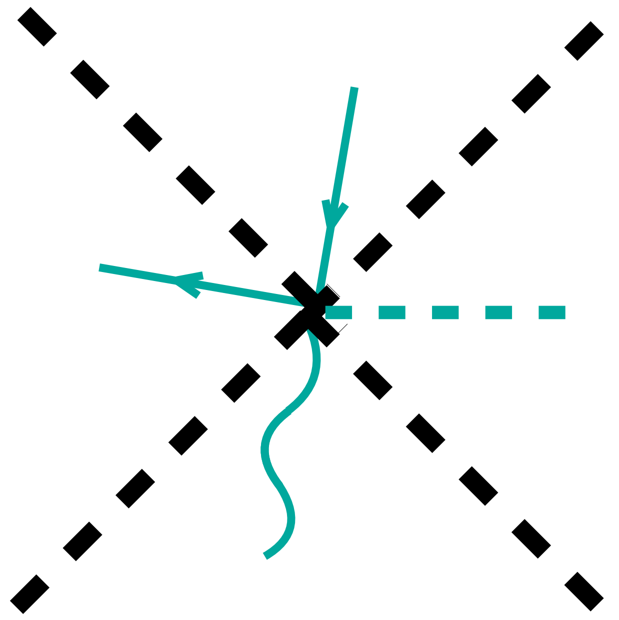
[JFM, Palavrić, Thomsen, 2311.13630]

$$\Gamma_{\mathrm{UV}}[\hat{\eta}] = S_{\mathrm{UV}}[\hat{\eta}] - i \ln \int \mathscr{D}\eta \exp\left[i \left(\frac{1}{2}\, \bar{\eta}_I \mathscr{Q}_{IJ} \eta_J + \frac{1}{3!}\, \eta_K \eta_J \eta_I \mathscr{C}_{KJI} + \frac{1}{4!}\, \eta_L \eta_K \eta_J \eta_I \mathscr{D}_{IJKL} + \dots \right) \right]$$

$$\mathscr{C}_{IJK} \equiv \left. \frac{\delta^3 \mathscr{L}_{\mathrm{UV}}}{\delta\eta_I\, \delta\eta_J\, \delta\eta_K} \right|_{\eta=\hat{\eta}}$$

$$\mathscr{D}_{IJKL} \equiv \left. \frac{\delta^4 \mathscr{L}_{\mathrm{UV}}}{\delta\eta_I\, \delta\eta_J\, \delta\eta_K\, \delta\eta_L} \right|_{\eta=\hat{\eta}}$$





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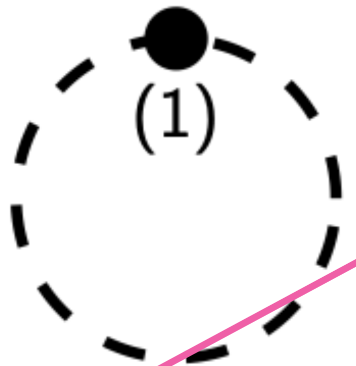
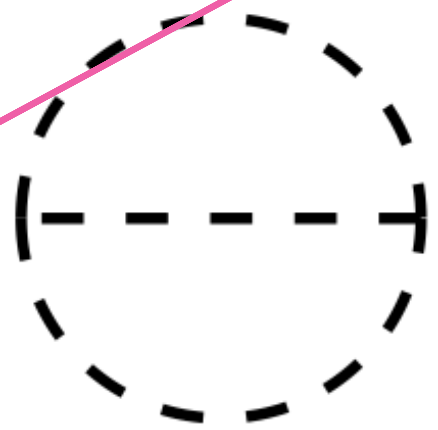
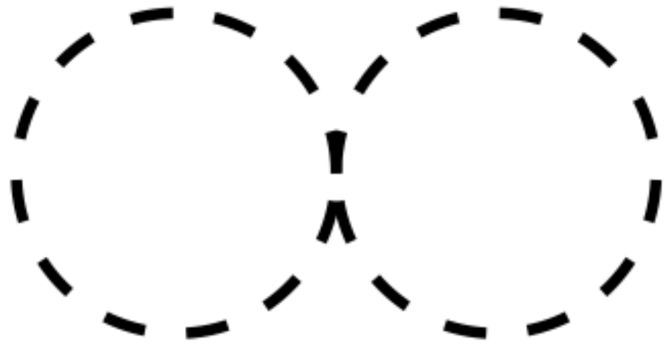
$$\Gamma_{\mathrm{UV}}[\hat{\eta}] = S_{\mathrm{UV}}[\hat{\eta}] - i \ln \int \mathscr{D}\eta \exp\left[i \left(\frac{1}{2}\,\bar{\eta}_I \mathcal{Q}_{IJ} \eta_J + \frac{1}{3!}\,\eta_K \eta_J \eta_I \mathscr{C}_{KJI} + \frac{1}{4!}\,\eta_L \eta_K \eta_J \eta_I \mathscr{D}_{IJKL} + \dots \right) \right]$$

$$= S_{\mathrm{UV}}[\hat{\eta}] + \frac{i\hbar}{2} \mathrm{STr} \ln \mathcal{Q} + \frac{i\hbar^2}{2} \mathcal{Q}^{-1}_{IJ} \mathcal{Q}^{(1)}_{JI} + \frac{\hbar^2}{12} \mathscr{C}_{IJK}\, \mathcal{Q}^{-1}_{IL} \mathcal{Q}^{-1}_{JM} \mathcal{Q}^{-1}_{KN}\, \mathscr{C}_{LMN} - \frac{\hbar^2}{8} \mathcal{Q}^{-1}_{IJ}\, \mathscr{D}_{IJKL}\, \mathcal{Q}^{-1}_{KL} + \mathcal{O}(\hbar^3)$$

$= S_{\mathrm{UV}}[\hat{\eta}] + \frac{i}{2} \log$  $+ \frac{i}{2}$ 

$+ \frac{1}{12}$  $- \frac{1}{8}$  $+ \mathcal{O}(\hbar^3)$

Tree level

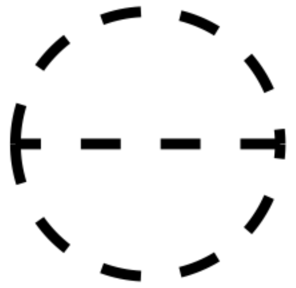
One loop

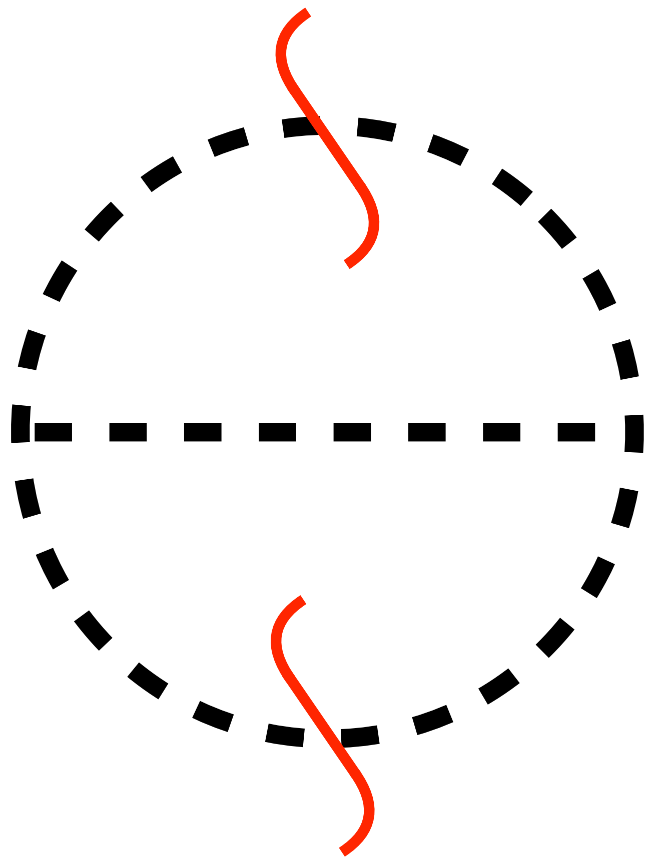
Two loops

Every two-loop contribution is included here!

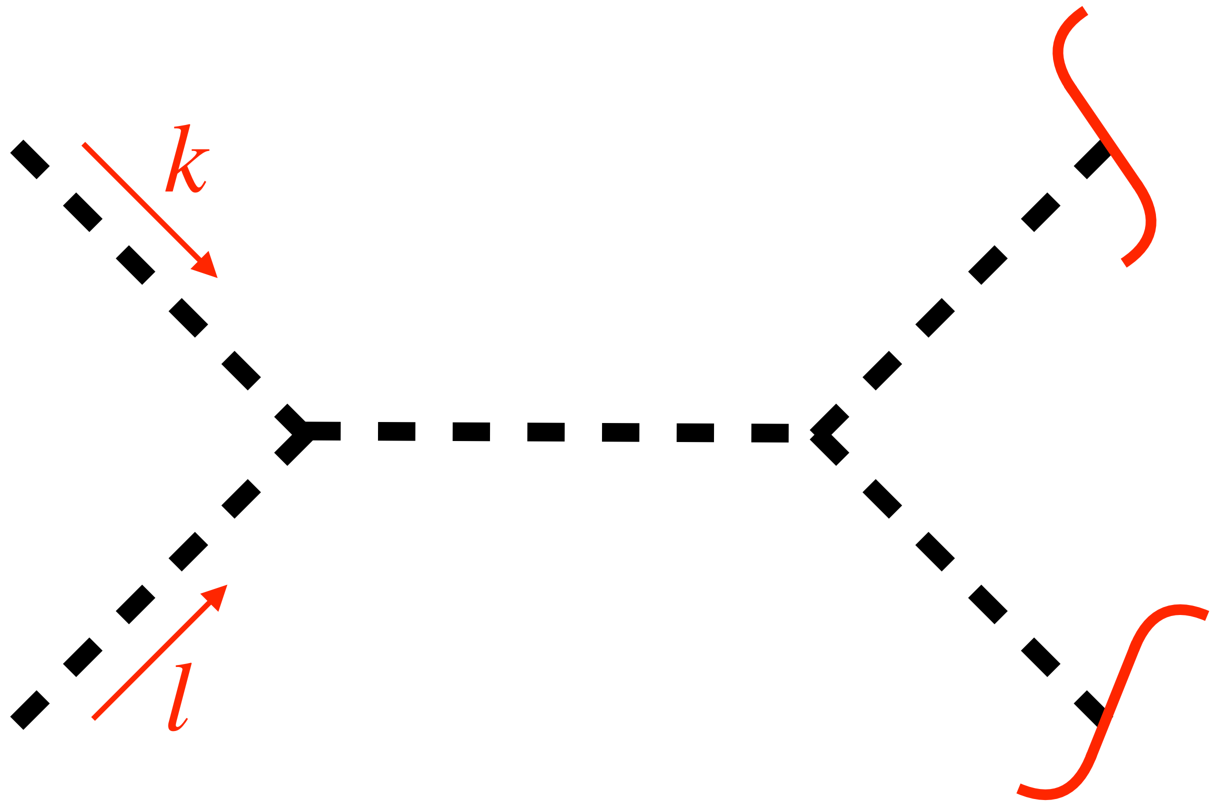
Two-loop functional evaluation

[JFM, Moreno, Palavrić, Thomsen, w.i.p]

How to evaluate  ?



$$\Longrightarrow \int_{k,l}$$



Two-loop functional evaluation

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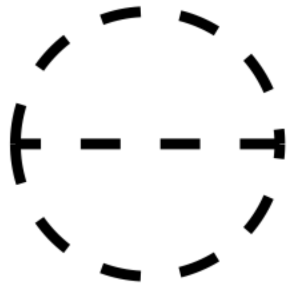
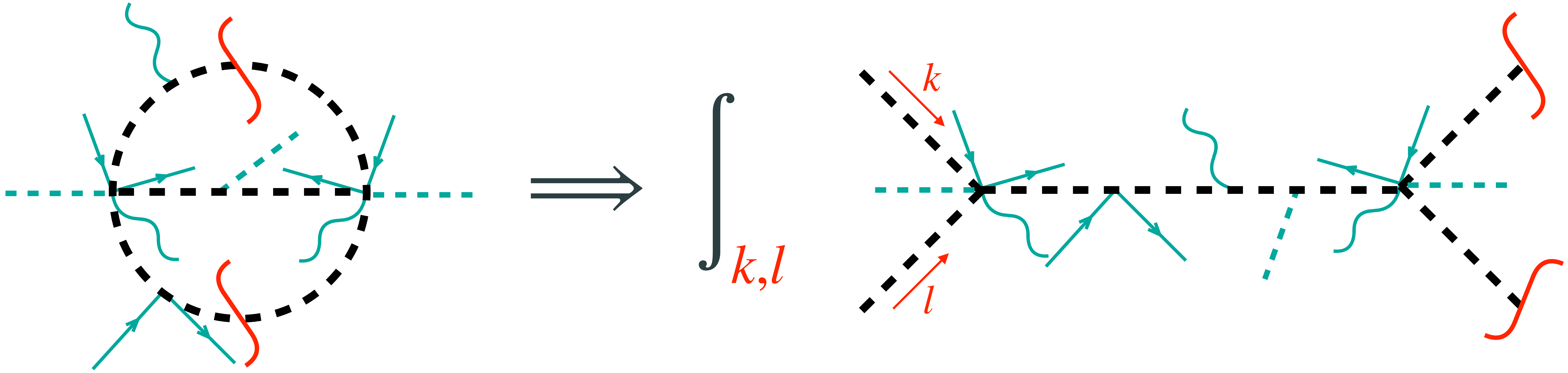
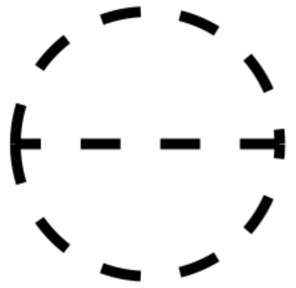
How to evaluate  ?

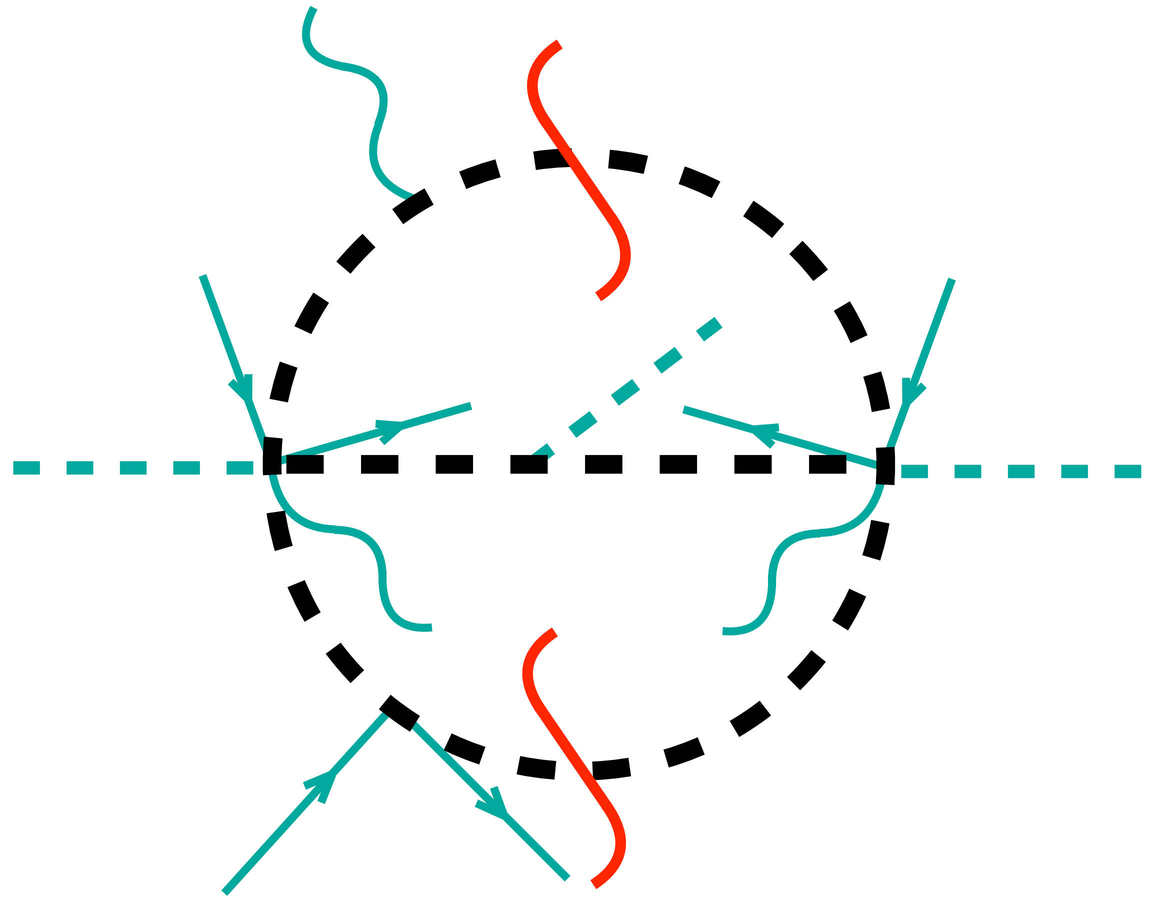


Every (non-factorizable) two-loop contribution is included here!

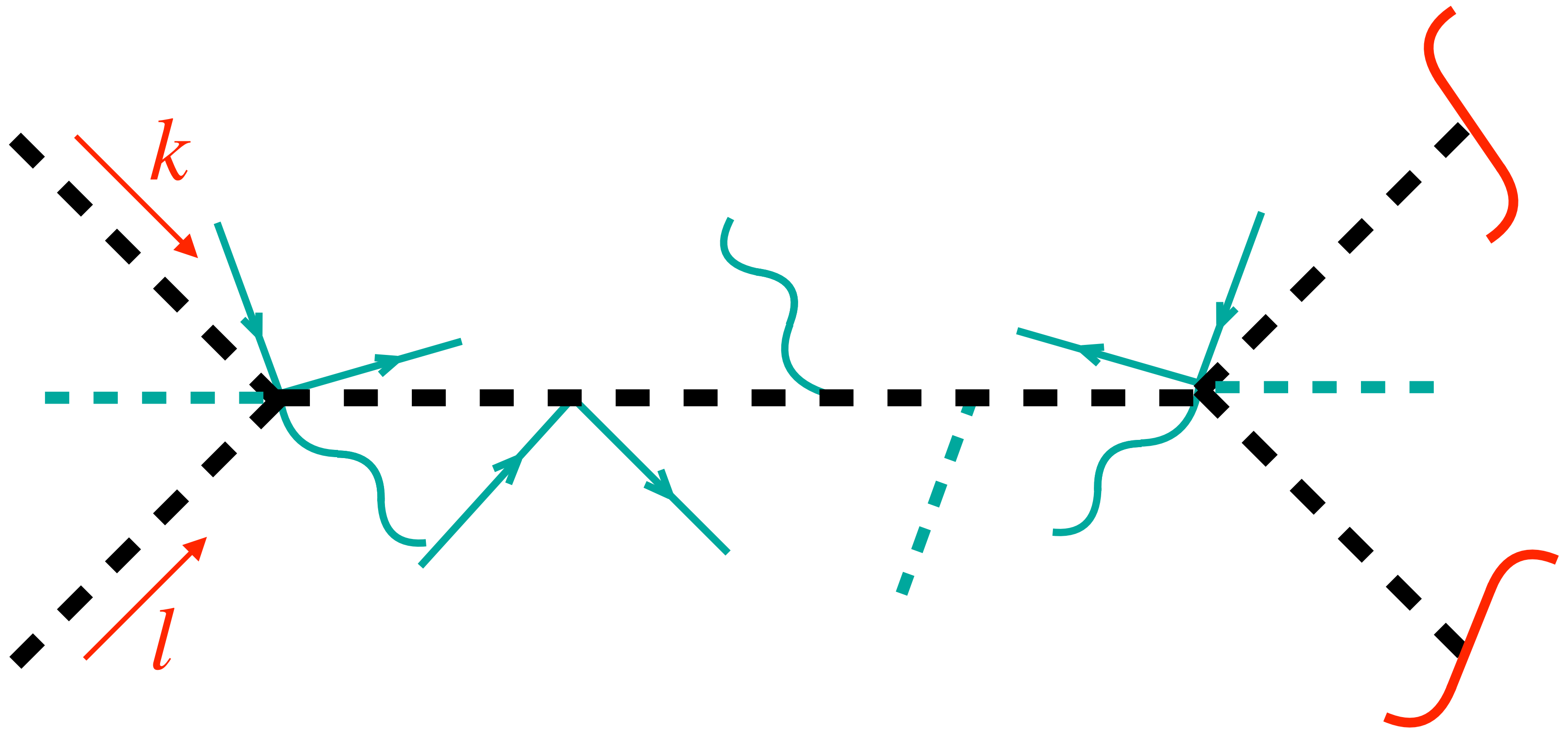
Two-loop functional evaluation

[JFM, Moreno, Palavrić, Thomsen, w.i.p]

How to evaluate  ?



$\Longrightarrow \int_{k,l}$



$$G_{\ominus} = \int_x \int_{k,l} C_{abc}(x)\, Q^{-1}_{aa'}(x', P_{x'} + k + l)\, C_{a'b'c'}(x') \left[Q^{-1}_{bd}(x, P_x - k) U_{db'}(x, x')\right] \left[Q^{-1}_{ce}(x, P_x - l) U_{ec'}(x, x')\right]$$

Valid to *all orders* in the EFT expansion!

General EFT matching formula

The EFT action is given by

$$S_{\rm EFT}[\phi] = \Gamma_{\rm UV}[\hat{\Phi},\phi]\Big|_{\rm hard} \qquad\qquad \frac{\delta\Gamma_{\rm UV}\big|_{\rm hard}}{\delta\Phi}[\hat{\Phi},\phi] = 0$$

Φ : Heavy

ϕ : Light

"hard" denotes the loop region where all loop momenta are $p \gtrsim \Lambda$ (incl. tree-level contributions) $^{(*)}$

- Explicit proof to two-loop order and (constructive) proof to any loop order in progress

  [JFM, Palavrić, Thomsen, 2311.13630] [JFM, Moreno, Palavrić, Thomsen, w.i.p]

- The hard region is by far the easiest to compute (only vacuum integrals at zero external momenta)
- The method can be trivially adapted to extract UV divergences
- Enables functional matching and RG running at any loop order

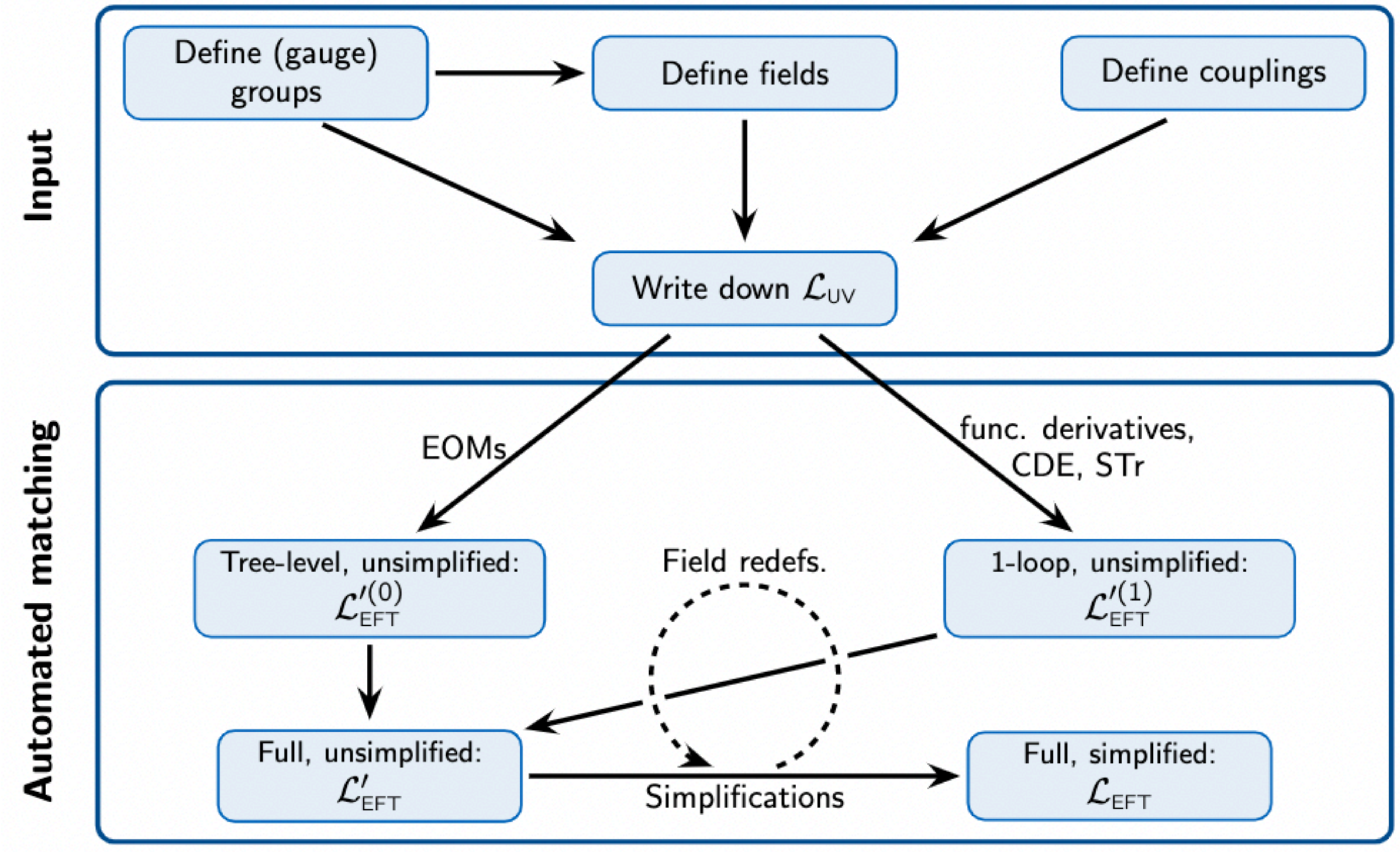
$^{(*)}$ Method of regions: Beneke, Smirnov, '97; Jantzen, '11

MATCHETE

To make your way through the BSM jungle

The Matchete package

MATCHETE is a Mathematica package aimed at fully automating EFT matching and RG evolution of arbitrary weakly-coupled UV theories using functional methods



[JFM, König, Pagès, Thomsen, Wilsch, 2212.04510]

Matchete v0.2 now publicly available:

- One-loop matching of *any* model with heavy scalars and/or fermions
- Simple and intuitive input/output
- Handles *all* group theory (any group and reps)
- Fully automated simplifications to EFT basis (IBP, field redefinitions/EOMs,…)

An example

SM + Vector-like lepton

$$E \sim (\mathbf{1},\mathbf{1})_{-1}$$

Setup

SM Lagrangian

In[3]:=
```
LSM = LoadModel["SM"];
```

Define new field

In[4]:=
```
DefineField[EE, Fermion, Charges → {U1Y[-1]}, Mass → {Heavy, ME}]
```

Define new coupling

In[5]:=
```
DefineCoupling[yE, EFTOrder → 0, Indices → {Flavor}]
```

Write interactions

In[6]:=
```
Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\overline{yE}^{p}\,\bar{H}_{i}\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Define full UV Lagrangian

In[8]:=
```
LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A 2}-\frac{1}{4}W^{\mu\nu I 2}+D_{\mu}\bar{H}_{i}\,D_{\mu}H^{i}+\mu^{2}\,\bar{H}_{i}\,H^{i}+i\left(\bar{d}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}d^{ap}\right)+i\left(\bar{e}^{p}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}e^{p}\right)+$$
$$i\left(\overline{EE}\cdot\gamma_{\mu}\cdot D_{\mu}EE\right)-ME\left(\overline{EE}\cdot EE\right)+i\left(\bar{l}^{p}_{i}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}l^{ip}\right)+i\left(\bar{q}^{p}_{ai}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}q^{aip}\right)+i\left(\bar{u}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}u^{ap}\right)-$$
$$\frac{1}{2}\lambda\,\bar{H}_{i}\,\bar{H}_{j}\,H^{i}\,H^{j}-\overline{Yd}^{pr}\,\bar{H}_{i}\left(\bar{d}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)-\overline{Ye}^{pr}\,\bar{H}_{i}\left(\bar{e}^{r}\cdot P_{L}\cdot l^{ip}\right)-Ye^{pr}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot e^{r}\right)-Yd^{pr}\,H^{i}\left(\bar{q}^{p}_{ai}\cdot P_{R}\cdot d^{ar}\right)-$$
$$Yu^{pr}\,\bar{H}_{i}\left(\bar{q}^{p}_{aj}\cdot P_{R}\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}\,H^{j}\left(\bar{u}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)\bar{\varepsilon}_{ij}-\overline{yE}^{p}\,\bar{H}_{i}\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

An example: SM + Vector-like lepton

Main matching routine

In[9]:=
```
LEFT = Match[LUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 → 0;
```

Simplification to on-shell basis

In[10]:=
```
LEFTOnShell = LEFT // EOMSimplify;
Length@%
```

» The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

Out[11]= 66

Select Higgs-lepton current operator

In[12]:=
```
SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

Out[12]//NiceForm=

$$\frac{i}{360}\,\hbar\,\frac{1}{\mathrm{ME}^2}\left(48\,\mathrm{gY}^4\,\delta^{pr} + 5\,\overline{\mathrm{yE}}^{s}\left(3\,\mathrm{yE}^{t}\,\overline{\mathrm{Ye}}^{tr}\,\mathrm{Ye}^{sp}\left(1 + 6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right]\right) - 2\,\mathrm{yE}^{s}\,\mathrm{gY}^2\left(13 + 6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right]\right)\delta^{pr}\right)\right)$$
$$\left(-D_\mu\bar{H}_i\,H^i\,\left(\bar{e}^r\cdot\gamma_\mu\,P_R\cdot e^p\right) + \bar{H}_i\,D_\mu H^i\,\left(\bar{e}^r\cdot\gamma_\mu\,P_R\cdot e^p\right)\right)$$

$$Q_{He}^{pr} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$$

What's new since December 2022?

`v0.1.0` $\longrightarrow$ `v0.2.0`

- More robust simplification routines: flavor, symmetry-vanishing operators…
- Changed evaluation of supertraces: from Covariant Derivative Expansion (CDE) to Wilson lines
- Significant performance improvements!

Version	`Match` [s]	`EOMSimplify` [s]
`v0.1.0`	74	281
`v0.2.0`	12	81

Dimension-six one-loop matching

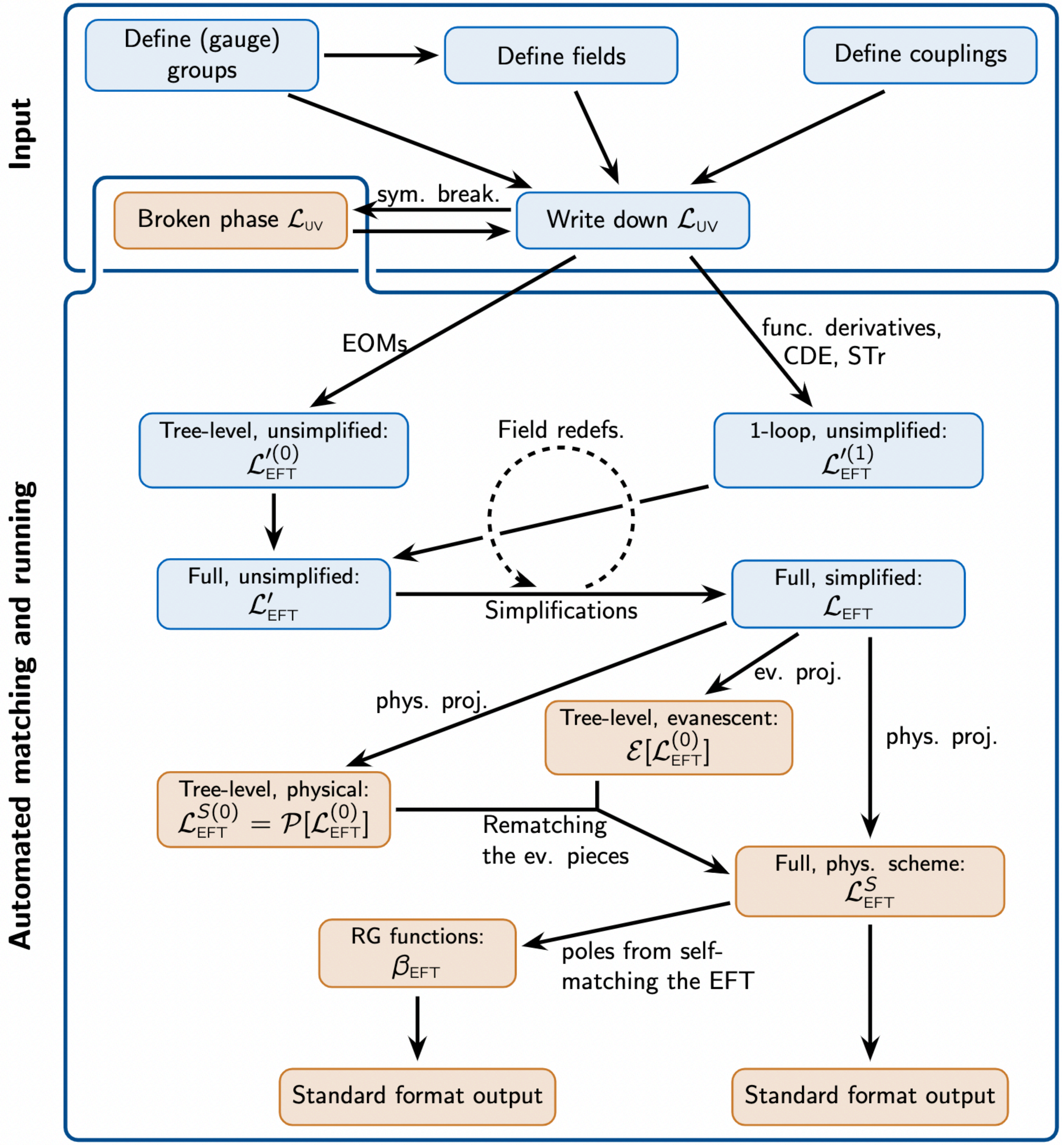
Model: SM + S1 + S3 (LQs)

CPU: Apple M3 (single core)

- Bug fixing: matching, group theory, simplifications…

The community has been a tremendous help bringing issues to our attention!

Work in progress and future plans



Upcoming!

- One-loop RG computations
- Handling of evanescent contributions
- Interface with other EFT tools

Longer term:

- Heavy vectors and symmetry breaking
- Matching and running beyond one loop

Work in progress: Counterterm evaluation and RG equations

- A taste of what it will look like using **Matchete** for the SM

```
LSM // NiceForm
```

eForm=

$$-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A2} - \frac{1}{4} W^{\mu\nu I2} + D_\mu \bar{H}_i D_\mu H^i + \mu^2 \bar{H}_i H^i + i\left(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}\right) + i\left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p\right) + i\left(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}\right) + i\left(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}\right) + i\left(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}\right) - \frac{1}{2}\lambda \bar{H}_i \bar{H}_j H^i H^j - \overline{Yd}^{pr} \bar{H}_i \left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) - \overline{Ye}^{pr} \bar{H}_i \left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) - Ye^{pr} H^i \left(\bar{l}^p_i \cdot P_R \cdot e^r\right) - Yd^{pr} H^i \left(\bar{q}^p_{ai} \cdot P_R \cdot d^{ar}\right) - Yu^{pr} \bar{H}_i \left(\bar{q}^p_{aj} \cdot P_R \cdot u^{ar}\right) \varepsilon^{ji} - \overline{Yu}^{pr} H^j \left(\bar{u}^r_a \cdot P_L \cdot q^{aip}\right) \varepsilon_{ij}$$

```
UVDivergentAction[LSM, EFTOrder → 4] // NiceForm
```

eForm=

$$\frac{1}{\epsilon}\Bigg(-\frac{41}{24}\hbar\, gY^2 B^{\mu\nu 2} + \frac{5}{2}\hbar\, gs^2 G^{\mu\nu A2} + \frac{31}{24}\hbar\, gL^2 W^{\mu\nu I2} + \frac{1}{8}\hbar\mu^2\left(15\, gL^2 + 5\, gY^2 - 24\,\overline{Yd}^{pr} Yd^{pr} - 8\,\overline{Ye}^{pr} Ye^{pr} - 24\,\overline{Yu}^{pr} Yu^{pr} - 24\lambda\right)\bar{H}_i H^i +$$
$$\hbar\left(\frac{9}{16} gL^4 - \frac{1}{16} gY^4 - 3\,\overline{Yd}^{pr}\,\overline{Yd}^{st}\, Yd^{pt} Yd^{sr} - \overline{Ye}^{pr}\,\overline{Ye}^{st}\, Ye^{pt} Ye^{sr} - 3\,\overline{Yu}^{pr}\,\overline{Yu}^{st}\, Yu^{pt} Yu^{sr} - \frac{5}{8}\lambda\, gY^2 + 3\,\overline{Yd}^{pr} Yd^{pr}\lambda + \overline{Ye}^{pr} Ye^{pr}\lambda + 3\,\overline{Yu}^{pr} Yu^{pr}\lambda + 3\lambda^2 - \frac{1}{8} gL^2\left(gY^2 + 15\lambda\right)\right)$$
$$\bar{H}_i \bar{H}_j H^i H^j + \hbar\left(\frac{3}{4}\left(\overline{Yd}^{ps}\,\overline{Yd}^{tr}\, Yd^{ts} - \overline{Yd}^{sr}\,\overline{Yu}^{pt}\, Yu^{st}\right) + \frac{1}{144}\overline{Yd}^{pr}\left(-27\, gL^2 + 192\, gs^2 - 17\, gY^2 + 216\,\overline{Yd}^{st} Yd^{st} + 72\,\overline{Ye}^{st} Ye^{st} + 216\,\overline{Yu}^{st} Yu^{st}\right)\right)\bar{H}_i\left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) +$$
$$\frac{1}{16}\hbar\left(12\,\overline{Ye}^{ps}\,\overline{Ye}^{tr}\, Ye^{ts} + \overline{Ye}^{pr}\left(-3\, gL^2 + 7\, gY^2 + 24\,\overline{Yd}^{st} Yd^{st} + 8\,\overline{Ye}^{st} Ye^{st} + 24\,\overline{Yu}^{st} Yu^{st}\right)\right)\bar{H}_i\left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) -$$
$$\frac{1}{16}\hbar\left(3\, Ye^{rp} gL^2 - 7\, Ye^{rp} gY^2 - 24\,\overline{Yd}^{st} Yd^{st} Ye^{rp} - 12\,\overline{Ye}^{st} Ye^{rt} Ye^{sp} - 8\,\overline{Ye}^{st} Ye^{rp} Ye^{st} - 24\, Ye^{rp}\,\overline{Yu}^{st} Yu^{st}\right) H^i\left(\bar{l}^r_i \cdot P_R \cdot e^p\right) +$$
$$\frac{1}{144}\hbar\left(-27\, Yd^{rp} gL^2 + 192\, Yd^{rp} gs^2 - 17\, Yd^{rp} gY^2 + 108\,\overline{Yd}^{st} Yd^{rt} Yd^{sp} + 216\,\overline{Yd}^{st} Yd^{rp} Yd^{st} + 72\, Yd^{rp}\,\overline{Ye}^{st} Ye^{st} - 108\, Yd^{sp}\,\overline{Yu}^{st} Yu^{rt} + 216\, Yd^{rp}\,\overline{Yu}^{st} Yu^{st}\right) H^i\left(\bar{q}^r_{ai} \cdot P_R \cdot d^{ap}\right) +$$
$$\frac{1}{144}\hbar\left(-27\, Yu^{rp} gL^2 + 192\, Yu^{rp} gs^2 + 7\, Yu^{rp} gY^2 + 216\,\overline{Yd}^{st} Yd^{st} Yu^{rp} + 72\,\overline{Ye}^{st} Ye^{st} Yu^{rp} - 108\,\overline{Yd}^{st} Yd^{rt} Yu^{sp} + 108\,\overline{Yu}^{st} Yu^{rt} Yu^{sp} + 216\,\overline{Yu}^{st} Yu^{rp} Yu^{st}\right)\bar{H}_i\left(\bar{q}^r_{aj} \cdot P_R \cdot u^{ap}\right)\varepsilon^{ji} +$$
$$\hbar\left(\frac{1}{144}\overline{Yu}^{pr}\left(-27\, gL^2 + 192\, gs^2 + 7\, gY^2 + 216\,\overline{Yd}^{st} Yd^{st} + 72\,\overline{Ye}^{st} Ye^{st} + 216\,\overline{Yu}^{st} Yu^{st}\right) + \frac{3}{4}\overline{Yu}^{tr}\left(-\overline{Yd}^{ps} Yd^{ts} + \overline{Yu}^{ps} Yu^{ts}\right)\right) H^j\left(\bar{u}^r_a \cdot P_L \cdot q^{aip}\right)\varepsilon_{ij}\Bigg)$$

*Ghost loops are not yet included

Work in progress: Evanescent contributions

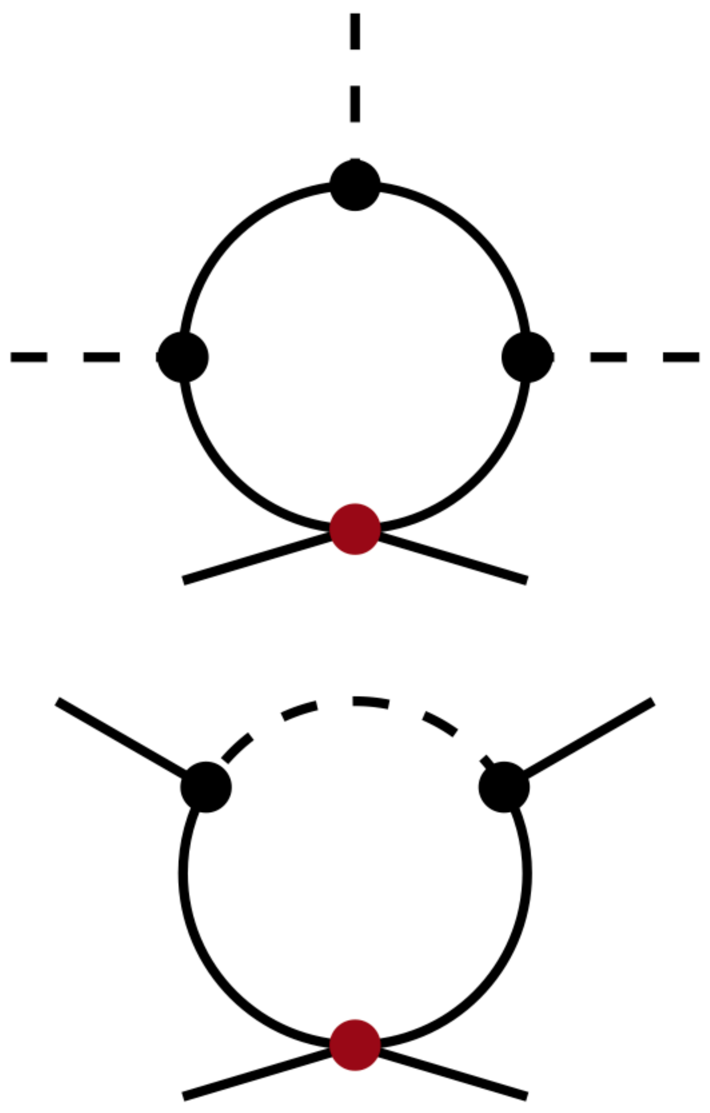
Some operator identities (like Fierz) are only valid in strictly $d = 4$ dimensions

Application to the SMEFT: JFM, König, Pagès, Thomsen, Wilsch, 2211.09144

$$\underbrace{\mathcal{P}\,O_d}_{\text{phys. part}}$$

$$O_d = \underbrace{\mathcal{P}\,O_d}_{\text{phys. part}} + \underbrace{\overbrace{\mathcal{E}_{\mathcal{P}}}^{\text{id}-\mathcal{P}}\,O_d}_{\text{ev. part}}$$

$$\mathcal{L}_{\text{EFT}} + \text{ev.} \longrightarrow \mathcal{L}_{\text{EFT}} + \Delta\mathcal{L}$$



*Sample diagrams

- Initial step: automatic identification of evanescent operators!

```
redOp = CRqe[p, r, s, t] (Bar@q[c, i, p] ** e[r]) (Bar@e[s] ** q[c, i, t]);
% // NiceForm
```

:eForm=

$\text{CRqe}^{prst}\,(\bar{e}^{s}\cdot P_L\cdot q^{cit})\,(\bar{q}^{p}_{ci}\cdot P_R\cdot e^{r})$

```
GreensSimplify[redOp, Basis4D → Evanescent] // NiceForm
```

:eForm=

$\text{CRqe}^{prst}\,E_1^{stpr} - \frac{1}{2}\,\text{CRqe}^{tpsr}\,(\bar{e}^{s}\cdot\gamma_\mu P_R\cdot e^{p})\,(\bar{q}^{t}_{ai}\cdot\gamma_\mu P_L\cdot q^{air})$

Work in progress: Matching to a particular SMEFT basis

Compute on-shell EFT Lagrangian from UV model

```
LEFTOnShell = Match[LUV, LoopOrder → 1, EFTOrder → 6] // EOMSimplify // AdjustWIP;
```

```
The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these
  terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.
```

Load generic SMEFT Lagrangian

```
LSMEFT = LoadModel["SMEFT"] // EOMSimplify // ShiftRenCouplings;
```

Equate the two, to solve for the SMEFT coefficients

```
MatchLagrangians[LEFTOnShell, LSMEFT] // CleanUpFlavor // TableForm // NiceForm
```

iceForm=

$\mu \to \sqrt{-2\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{ME}^2 - 2\hbar\,\frac{1}{\epsilon}\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{ME}^2 + \mu^2 - 2\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{ME}^2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right]}$

$\lambda \to \lambda$

$\mathrm{cHBox} \to -\frac{1}{30}\hbar\,\mathrm{gY}^4\,\frac{1}{\mathrm{ME}^2} - \frac{5}{24}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gL}^2\,\frac{1}{\mathrm{ME}^2} + \frac{13}{72}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gY}^2\,\frac{1}{\mathrm{ME}^2} - \frac{1}{3}\hbar\,\overline{\mathrm{yE}}^{p}\,\overline{\mathrm{yE}}^{r}\,\mathrm{yE}^{p}\,\mathrm{yE}^{r}\,\frac{1}{\mathrm{ME}^2} + \frac{3}{2}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{r}\,\overline{\mathrm{Ye}}^{rs}\,\mathrm{Ye}^{ps}\,\frac{1}{\mathrm{ME}^2} - \frac{1}{4}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gL}^2\,\frac{1}{\mathrm{ME}^2}\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right] + \frac{1}{12}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}$

$\mathrm{cHD} \to -\frac{2}{15}\hbar\,\mathrm{gY}^4\,\frac{1}{\mathrm{ME}^2} + \frac{13}{18}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gY}^2\,\frac{1}{\mathrm{ME}^2} - \frac{1}{2}\hbar\,\overline{\mathrm{yE}}^{p}\,\overline{\mathrm{yE}}^{r}\,\mathrm{yE}^{p}\,\mathrm{yE}^{r}\,\frac{1}{\mathrm{ME}^2} + \frac{1}{2}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{r}\,\overline{\mathrm{Ye}}^{rs}\,\mathrm{Ye}^{ps}\,\frac{1}{\mathrm{ME}^2} + \frac{1}{3}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gY}^2\,\frac{1}{\mathrm{ME}^2}\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right] + \hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{r}\,\overline{\mathrm{Ye}}^{rs}\,\mathrm{Ye}^{ps}\,\frac{1}{\mathrm{ME}^2}\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{\mathrm{ME}^2}\right]$

$\mathrm{cH} \to \frac{1}{3}\hbar\,\overline{\mathrm{yE}}^{p}\,\overline{\mathrm{yE}}^{r}\,\overline{\mathrm{yE}}^{s}\,\mathrm{yE}^{p}\,\mathrm{yE}^{r}\,\mathrm{yE}^{s}\,\frac{1}{\mathrm{ME}^2} + 2\hbar\,\overline{\mathrm{yE}}^{p}\,\overline{\mathrm{yE}}^{r}\,\mathrm{yE}^{p}\,\mathrm{yE}^{s}\,\overline{\mathrm{Ye}}^{st}\,\mathrm{Ye}^{rt}\,\frac{1}{\mathrm{ME}^2} - 2\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{r}\,\overline{\mathrm{Ye}}^{rs}\,\overline{\mathrm{Ye}}^{tu}\,\mathrm{Ye}^{pu}\,\mathrm{Ye}^{ts}\,\frac{1}{\mathrm{ME}^2} - \frac{5}{18}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\lambda\,\mathrm{gL}^2\,\frac{1}{\mathrm{ME}^2} - \hbar\,\overline{\mathrm{yE}}^{p}\,\overline{\mathrm{yE}}^{r}\,\mathrm{yE}^{p}\,\mathrm{yE}^{r}\,\lambda\,\frac{1}{\mathrm{ME}^2} + \hbar\,\overline{\mathrm{yE}}$

$\mathrm{cHB} \to \frac{1}{8}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gY}^2\,\frac{1}{\mathrm{ME}^2}$

$\mathrm{cHG} \to 0$

$\mathrm{cG} \to 0$

$\mathrm{cHWB} \to -\frac{1}{6}\hbar\,\mathrm{gL}\,\mathrm{gY}\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\frac{1}{\mathrm{ME}^2}$

$\mathrm{cHW} \to \frac{1}{24}\hbar\,\overline{\mathrm{yE}}^{p}\,\mathrm{yE}^{p}\,\mathrm{gL}^2\,\frac{1}{\mathrm{ME}^2}$

$\mathrm{cW} \to 0$

$\mathrm{cHBt} \to 0$ SMEFT coefficients

$\mathrm{cHGt} \to 0$

$\mathrm{cHWtB} \to 0$

$\mathrm{cHWt} \to 0$

$\mathrm{cGt} \to 0$

The aim is to use the SMEFT Warsaw basis to interface with smelli and HighPT!

Aebischer et al., 1810.07698

Allwicher et al., 2207.10756

*Example model: SM + vector-like lepton $E \sim (\mathbf{1}, \mathbf{1})_{-1}$

Summary and conclusions

- (Automated) EFT matching and RG evolution is crucial to BSM phenomenology
- Functional matching is ideal for automation (also useful for pen-and-paper computations!)
- Huge progress towards complete (one-loop) automation: Lagrangian in, fully simplified EFT Lagrangian out
- The ultimate goal is a tool (or chain of tools) that fully automates
  - Matching
  - RG evolution

  Multi-step matching

  - Connection to observables / fit to data **Interface with other EFT pheno codes**

  streamlining future BSM analyses

https://gitlab.com/matchete/matchete

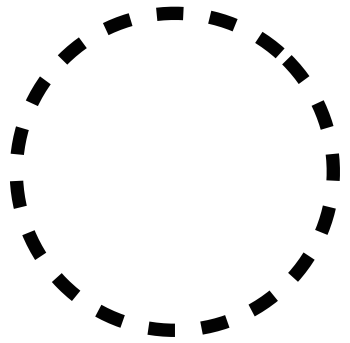
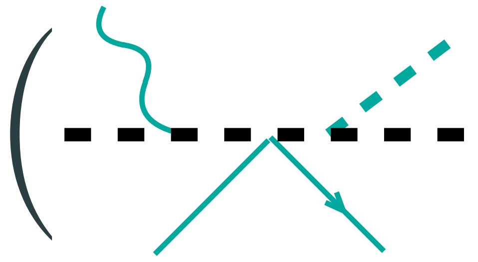
MATCHETE

Thank you

Matching models is about to become easy!

Backup

More on evaluating supertraces

- **Supertraces:** $\displaystyle\int d^dx\,\mathscr{L}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \frac{i}{2}\,\mathrm{STr}\ln\mathcal{Q}\Big|_{\rm hard} = \pm\frac{i}{2}\ln$  $\Big|_{\rm hard}$ $Q\,[\,\hat{\eta}\,] = \Big($  $\Big)^{-1}$

interaction terms

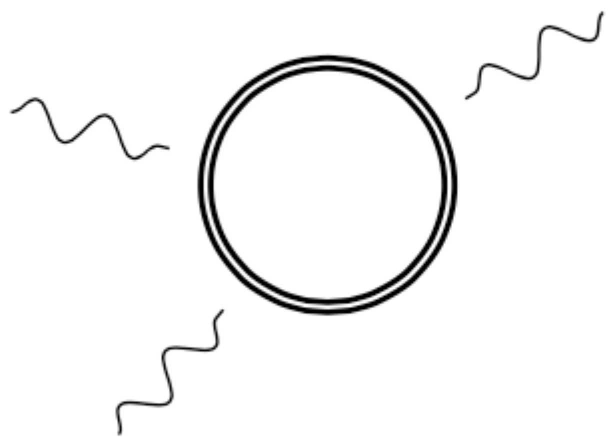
- **Fluctuation operator:** $Q_{ij} \equiv \delta_{ij}\Delta_i^{-1} - X_{ij} = \Delta_i^{-1}(\delta_{ij} - \Delta_i X_{ij})$

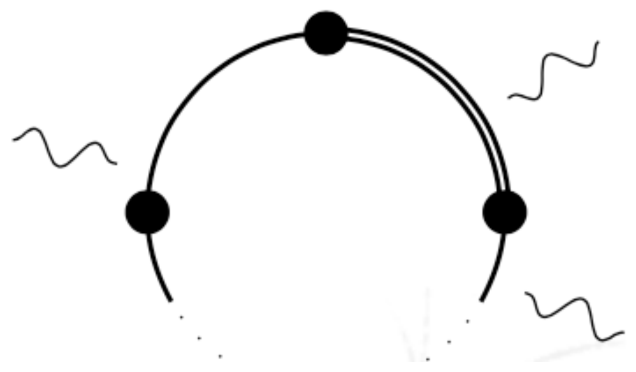
propagators

$$\Delta_i^{-1} = \begin{cases} -(D^2 + M_i^2) \\ i\gamma^\mu D_\mu - M_i \\ g^{\mu\nu}(D^2 + M_i^2) \end{cases}$$

Expanding the logarithm and taking ΔX at most $\mathcal{O}(m_H^{-1})$

$$\int d^dx\,\mathscr{L}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \quad \frac{i}{2}\,\mathrm{STr}\,\ln\,\Delta^{-1}\Big|_{\rm hard} \quad -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\left[(\Delta X)^n\right]\Big|_{\rm hard}$$





Evanescent operators

In $d = 4$, we can use the Fierz identity $R_{\ell e} = -\frac{1}{2} Q_{\ell e}$

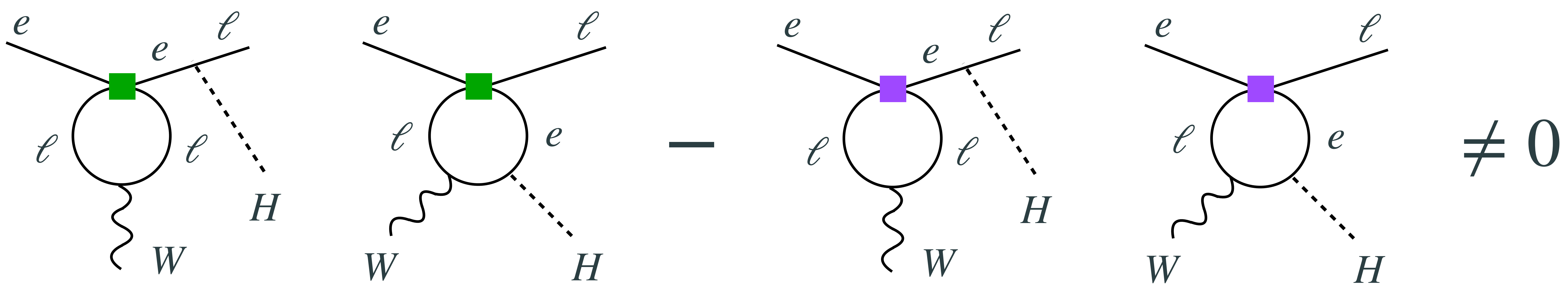
$$\mathscr{L}_{\text{EFT}} \supset C_{\ell e}^{prst} R_{\ell e}^{prst}$$

$$\mathscr{L}'_{\text{EFT}} \supset -\frac{1}{2} C_{\ell e}^{prst} Q_{\ell e}^{prst}$$

$$R_{\ell e}^{prst} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

$$Q_{\ell e}^{prst} = (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r)$$

so that $\mathscr{L}_{\text{EFT}} = \mathscr{L}'_{\text{EFT}}$ at tree level. However,



Evanescent operators

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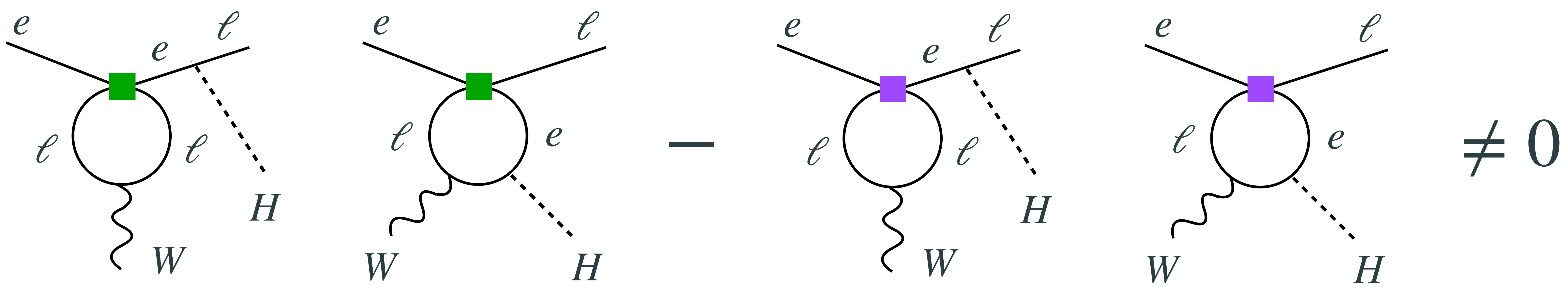
$$R_{\ell e}^{prst} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

$$Q_{\ell e}^{prst} = (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r)$$

so that $\mathscr{L}_{\text{EFT}} = \mathscr{L}'_{\text{EFT}}$ at <u>tree level</u>. However,



In $d = 4 - 2\epsilon$, there is an evanescent operator that also contributes to the amplitude

$$R_{\ell e}^{prst} = -\frac{1}{2} Q_{\ell e}^{prst} + E_{\ell e}^{prst} \qquad E_{\ell e}^{prst} \xrightarrow{\epsilon \to 0} 0 \qquad E_{\ell e}^{prst} \to -\frac{g_L\, y_e^{ts}}{128\pi^2} Q_{eW}^{pr} + \text{[many other contributions]}$$

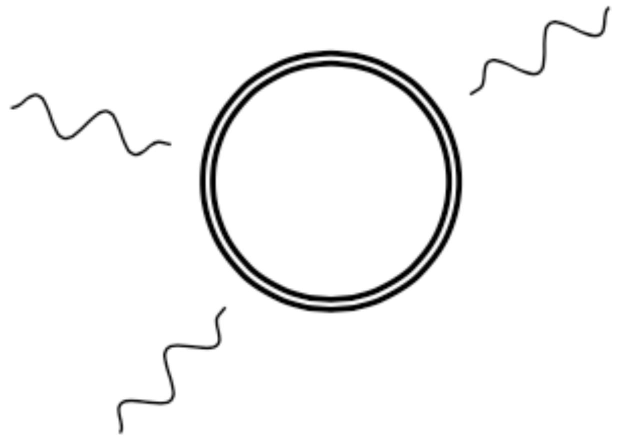
Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

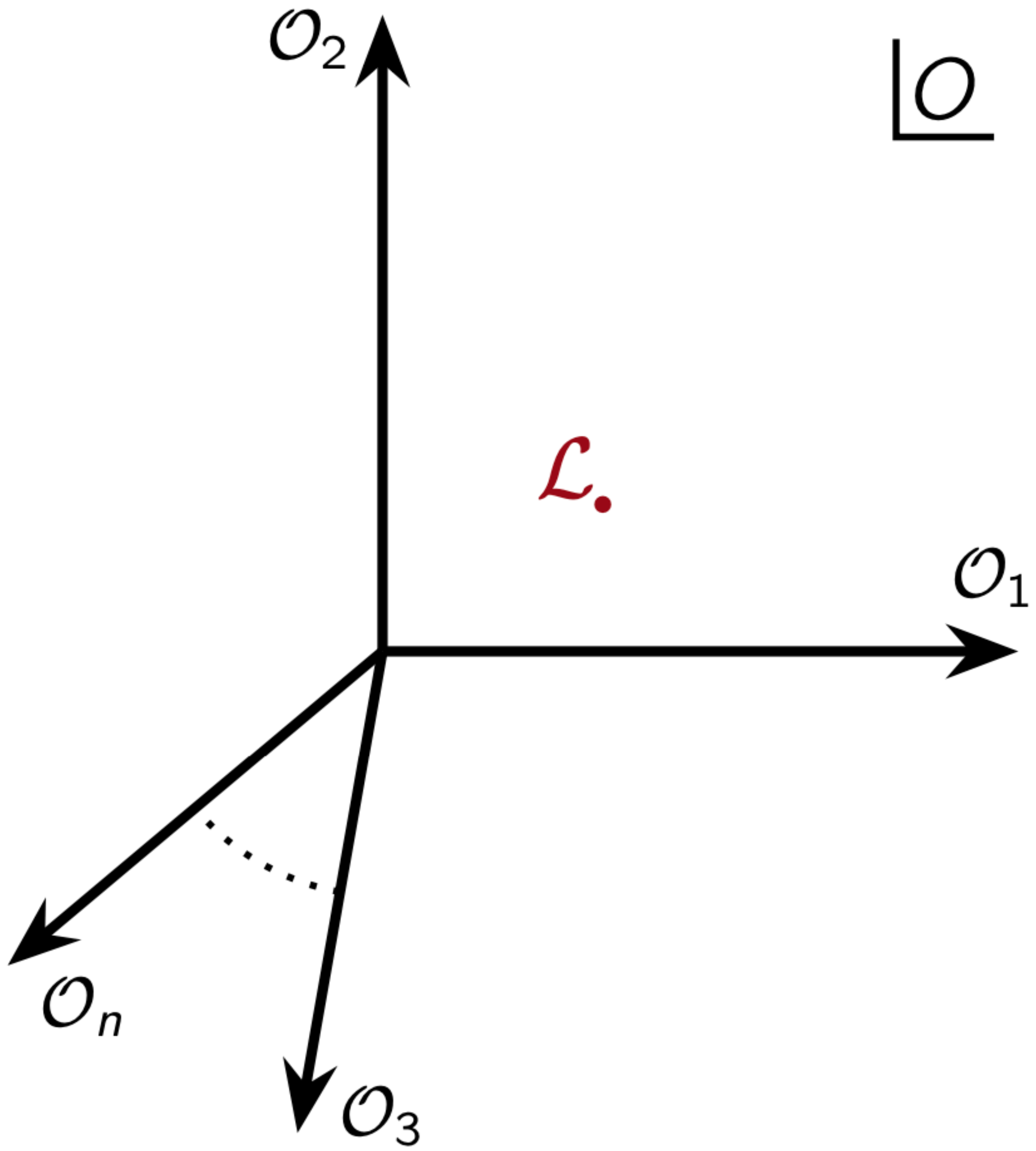
```
In[12]:= LEFT // NiceForm
```

Out[12]//NiceForm=

$$\begin{aligned}&\frac{7}{540}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \\ &\frac{1}{40}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D^2 G^{\mu\nu A} + \frac{7}{540}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,D_\rho G^{\mu\nu A}\,D_\nu G^{\mu\rho A} - \\ &\frac{1}{180}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,D_\nu G^{\mu\nu A}\,D_\rho G^{\mu\rho A} + \frac{1}{40}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D_\nu D_\rho G^{\mu\rho A} + \\ &\frac{1}{40}\,\hbar\, g^2\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\,\hbar\, g^3\,\frac{1}{M\Psi^2}\,G^{\mu\nu A}\,G^{\mu\rho B}\,G^{\nu\rho C}\,f^{ABC}\end{aligned}$$



(log supertrace)



$$\mathscr{L}_{\mathrm{EFT}} = \sum_i C_i \mathscr{O}_i \in O$$

Linear simplifications

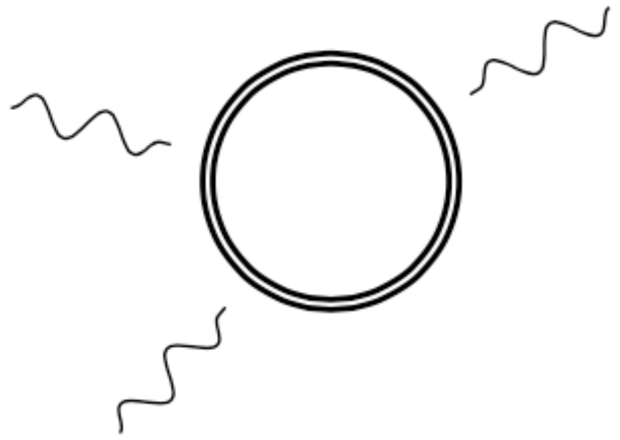
Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

In[12]:= `LEFT // NiceForm`

Out[12]//NiceForm=

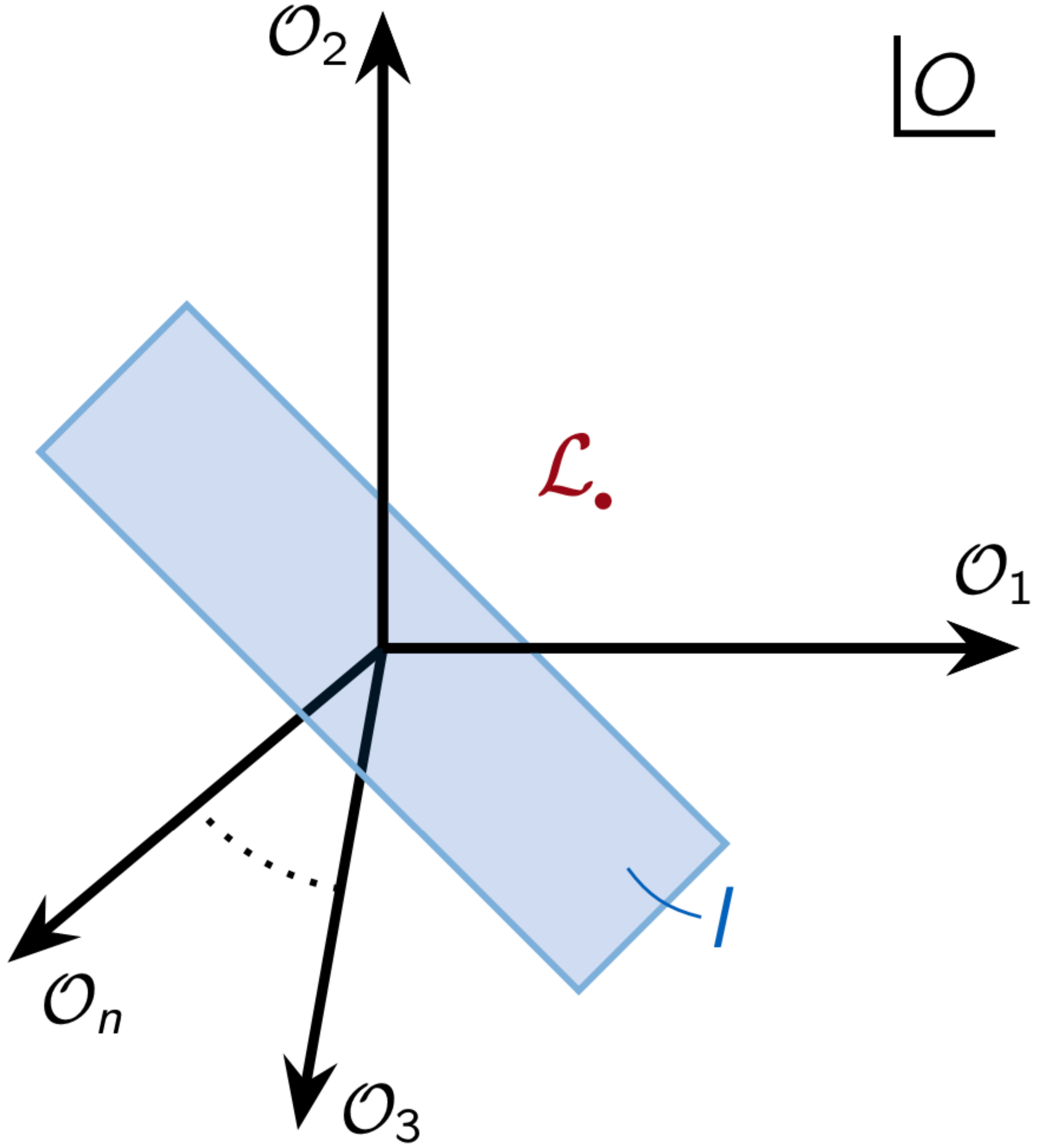
$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

$$\mathscr{L}_{\mathrm{EFT}} = \sum_i C_i \mathscr{O}_i \in O$$



(log supertrace)

O



$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathscr{O}_1 + 2\,\mathscr{O}_3 = 0$$

interpreted as

$$\mathscr{O}_1 + 2\,\mathscr{O}_3 \in I$$

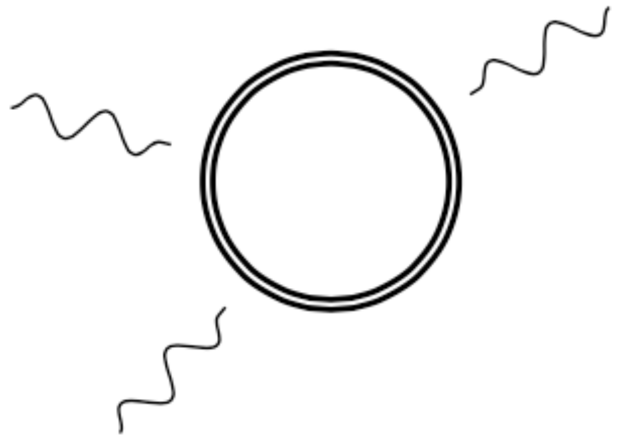
Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)

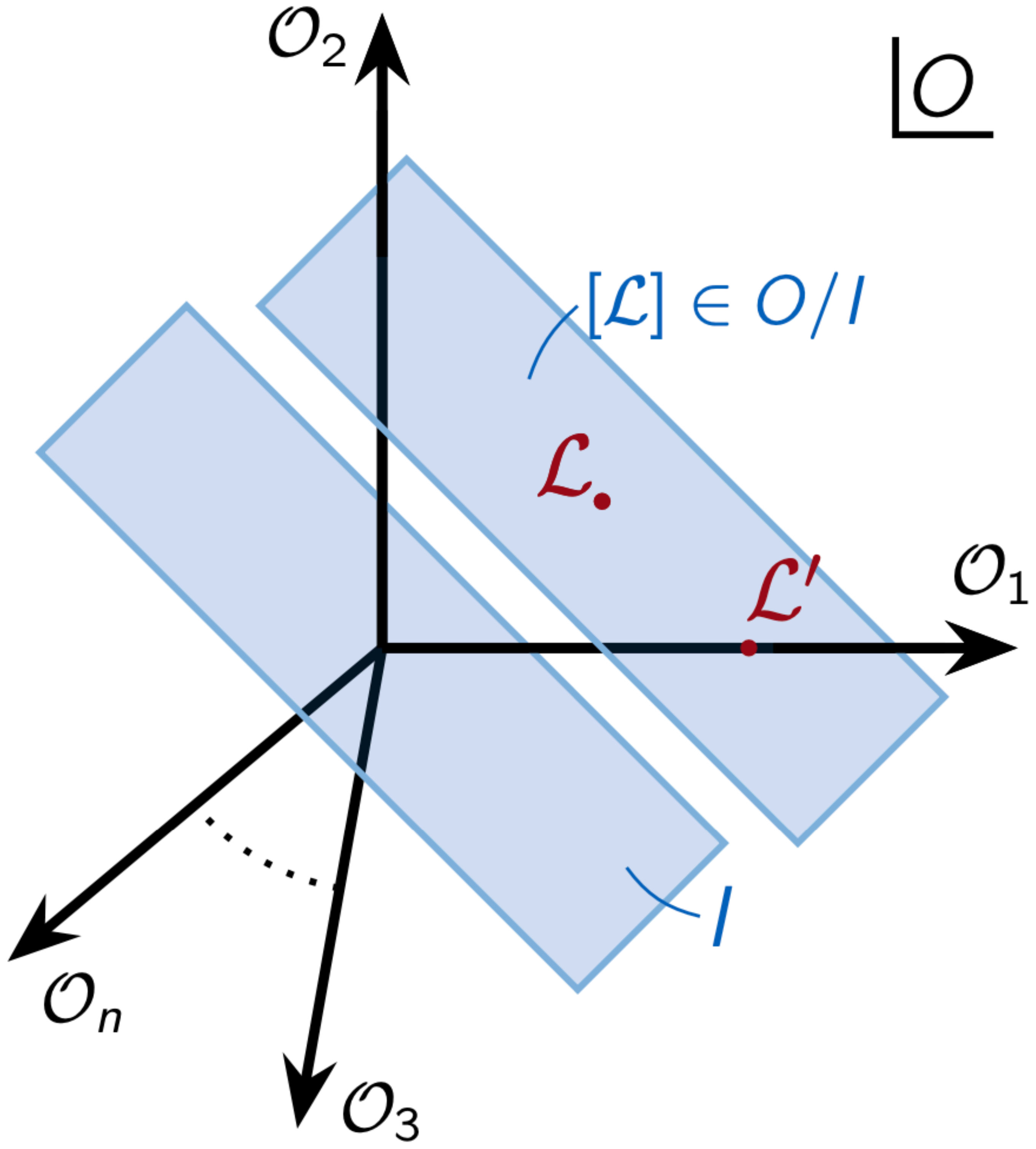
In[12]:= `LEFT // NiceForm`

Out[12]//NiceForm=

$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$



(log supertrace)



$$\mathscr{L}_{\mathrm{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$

By gaussian elimination, we can *choose* a representative element for $[\mathscr{L}_{\mathrm{EFT}}] \in O/I$ to get an EFT basis

In[13]:= `LEFT // GreensSimplify // NiceForm`

Out[13]//NiceForm=

$$-\frac{1}{15}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 = 0$$

interpreted as

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 \in I$$

Linear simplifications with evanescent operators

Evanescent operators appear from a special type of linear simplification (valid only for $d = 4$)

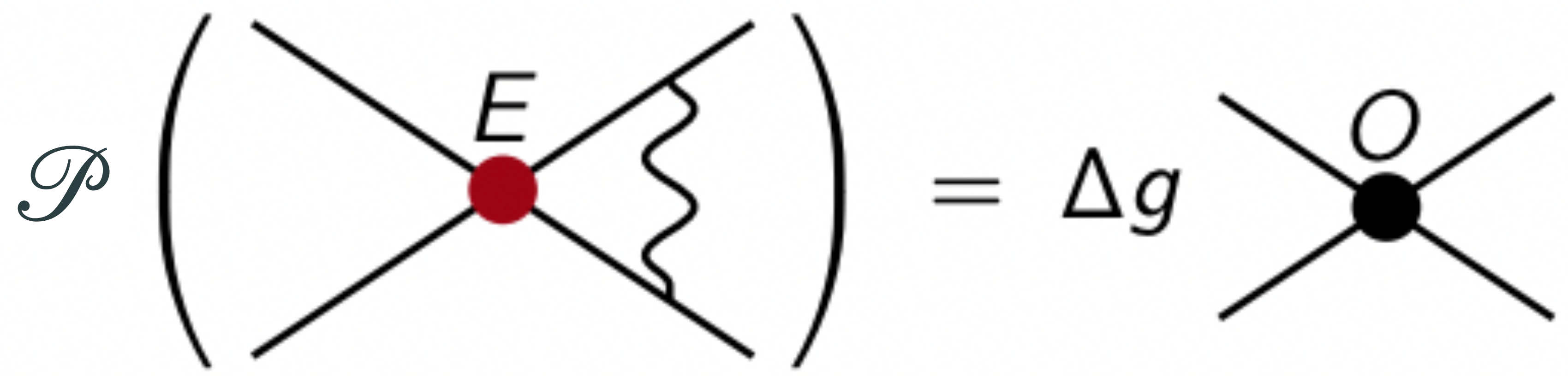
$$O_d = \underbrace{\mathscr{P}\, O_d}_{\text{Physical part}} + \underbrace{\overset{Id - \mathscr{P}}{\mathscr{E}}\, O_d}_{\text{Evanescent part}}$$

$\mathscr{P} \equiv$ Projection to the physical ($d = 4$) basis

E.g. Fierz identities

$$(\bar{\ell}_p e_r)(\bar{e}_s \ell_t) = -\frac{1}{2}(\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) + \overbrace{E^{prst}_{\ell e}}^{\text{rank}\,(\,d-4\,)} \longrightarrow (\bar{\ell}_p e_r)(\bar{e}_s \ell_t) + \frac{1}{2}(\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) - E^{prst}_{\ell e} \in I$$

Representative elements are chosen so evanescent operators are retained. Afterwards, these are removed by shifting the coefficients of physical operators



e.g. $E^{prst}_{\ell e} \to -\dfrac{g_L\, y_e^{ts}}{128\pi^2}\, Q^{pr}_{eW} +$ [other contributions]