Physics at lepton colliders

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+ Goal: explore physics at least up to $M_{\rm NP} \approx 10 \,{\rm TeV}$





... and how is it related with the flavor problem?

+ Goal: explore physics at least up to $M_{\rm NP} \approx 10 \,{\rm TeV}$



- What causes EWSB? i.e. does the SM hold up to few TeV?
- What is dark matter? Is it a WIMP?



+ Goal: explore physics at least up to $M_{\rm NP} \approx 10 \,{\rm TeV}$



- What causes EWSB?
 i.e. does the SM hold up to few TeV?
- What is dark matter? Is it a WIMP?
- Observe restoration of EW symmetry (EW radiation)





Lepton colliders

- Lepton colliders are ideal probes of short-distance physics
 - elementary: no energy lost in PDFs,

all beam energy is available for hard scattering



no QCD background, high S/B

Electron-positron colliders

+ They could be built today, if approved and funded.

Our quickest way to multi-TeV energies (indirectly).



Muon colliders?

- + A muon collider is *not yet feasible* as of today!
- Several technical challenges that require major R&D effort



... it should not be compared with shovel-ready projects (like e+e- Higgs/EW factory)

Muon colliders!

- + A muon collider is not science-fiction either!
- Several technical challenges that require major R&D effort



High energy lepton collider (10 TeV or more) is a dream for particle physics...

... dedicated R&D program crucial to establish feasibility in the next years!

Lepton colliders

- Lepton colliders are ideal probes of short-distance physics
- Muons are elementary and heavy (207 x electrons)
 - negligible energy loss in synchrotron radiation
 - negligible beamstrahlung

But they decay...

- Luminosity increases with the square of beam energy
 - muon lifetime increases
 - transverse emittance decreases



✤ With CKM-like suppression (U(2)³ flavor symmetry):



Allwicher, Cornella, Isidori, Stefanek 2311.00020

- Where do we stand?
 - With CKM-like suppression (U(2)³ flavor symmetry):
 - + mild suppression of light gen. interactions
 - + some flavor alignment



Allwicher, Cornella, Isidori, Stefanek 2311.00020

 $\varepsilon_{\rm loop} = \frac{g_i}{16\pi^2}$

 $\varepsilon_Q = 0.16$

 $\varepsilon_L = 0.40$

 $\varepsilon_H = 0.31$

 $\varepsilon_F = 0.15$

Higgs factories

- All proposed future colliders will be able to produce millions of Higgses
 - → study single Higgs couplings with below percent precision!



Higgs factories

+ Low-energy e+e- factories: $e^+e^- \rightarrow Zh @ 240 \text{ GeV}$



- measure the recoil (missing mass) of h against Z
- + *direct* measurement of $gV \rightarrow$ other couplings + width
- + A high-energy lepton collider is a "vector boson collider"



- potentially huge single H production (10⁷-10⁸ at 10-30 TeV)
- hard neutrinos from W-fusion not seen 10²
 *ZZ fusion (forward lepton tagging) could still measure width



Higgs factories

<i>к</i> -0	HL-LHC	LHeC	HE	-LHC		ILC			CLIC	?	CEPC	FCC	C-ee	FCC-ee/	$\mu^+\mu^-$	
fit			S2	S2'	250	500	1000	380	1500	3000		240	365	$\rm eh/hh$	10000	
$\kappa_W \ [\%]$	1.7	0.75	1.4	0.98	1.8	0.29	0.24	0.86	0.16	0.11	1.3	1.3	0.43	0.14	0.1	dominant
$\kappa_Z \ [\%]$	1.5	1.2	1.3	0.9	0.29	0.23	0.22	0.5	0.26	0.23	0.14	0.20	0.17	0.12	0.4	 channels other Higgs factories
$\kappa_g \ [\%]$	2.3	3.6	1.9	1.2	2.3	0.97	0.66	2.5	1.3	0.9	1.5	1.7	1.0	0.49	0.7	
$\kappa_{\gamma} \ [\%]$	1.9	7.6	1.6	1.2	6.7	3.4	1.9	98*	5.0	2.2	3.7	4.7	3.9	0.29	0.8	
$\kappa_{Z\gamma}$ [%]	10.	—	5.7	3.8	$99\star$	$86\star$	$85\star$	$120\star$	15	6.9	8.2	$81\star$	$75\star$	0.69	7.2	
$\kappa_c \ [\%]$	—	4.1	-	—	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3	0.95	2.3	
$\kappa_t ~[\%]$	3.3	—	2.8	1.7	—	6.9	1.6	_	—	2.7	-		_	1.0	3.1	rare modes
$\kappa_b \ [\%]$	3.6	2.1	3.2	2.3	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67	0.43	0.4	veller (~ hadron
κ_{μ} [%]	4.6	-	2.5	1.7	15	9.4	6.2	$320\star$	13	5.8	8.9	10	8.9	0.41	3.4	collider)
κ_{τ} [%]	1.9	3.3	1.5	1.1	1.9	0.70	0.57	3.0	1.3	0.88	1.3	1.4	0.73	0.44	0.6	

2103.14043

What NP scales will we test with the Higgs?

$$\delta\kappa \sim \frac{v^2}{M_{\rm NP}^2} g_{\star}^2 \lesssim 0.2\%$$

 $\bullet \quad M_{\rm NP} \gtrsim g_{\star} \ \mathbf{6} \ \mathbf{TeV}$



Direct vs indirect

Compare single Higgs couplings measurements with reach of direct searches

• Example: singlet scalar $\mathscr{L}_{int} \sim \phi |H|^2$

 ϕ is like a heavy Higgs with narrow width + hh decay



High rate probes

High rate: more events = better precision



A High Energy Lepton Collider is a "vector boson collider"

For "soft" SM final state $\hat{s} \sim m_{\rm EW}^2$ cross-section is enhanced



High rate probes

High rate: more events = better precision



- Huge single Higgs rate
 in vector-boson-fusion:
 10⁷ Higgs bosons at 10 TeV
- Large double Higgs VBF rate
 - Higgs 3-linear coupling
- Triple Higgs production accessible
 - Higgs 4-linear coupling

Chiesa et al. 2003.13628

A High Energy Lepton Collider is a "vector boson collider"

For "soft" SM final state $\hat{s} \sim m_{\rm EW}^2$ cross-section is enhanced



+ Measurement of trilinear coupling: access to the Higgs potential



 Precise determination *only* possible at high-energy machines: FCC-hh or multi-TeV lepton collider

Mangano et al. 2004.03505 B, Franceschini, Wulzer 2012.11555 Costantini et al. 2005.10289 Han et al. 2008.12204 CLIC 1901.05897

- very poorly known today!
- HL-LHC will only reach 50% precision on SM value



+ Double Higgs production depends on trilinear coupling κ_3 but also on W-boson couplings κ_W , κ_{WW} that enter the production cross-section





large degeneracy in total cross-section:coefficients not determinedfrom hh production alone

- Double Higgs production depends on trilinear coupling κ_3 but also on W-boson couplings κ_W , κ_{WW} that enter the production cross-section
- Two dim. 6 operators:

wo dim. 6 operators:
$$\mathcal{O}_6 = -\lambda |H|^6$$
 $\mathcal{O}_H = \frac{1}{2} \left(\partial_\mu |H|^2 - \kappa_3 = 1 + v^2 \left(C_6 - \frac{3}{2} C_H \right) - \kappa_W = 1 - v^2 C_H / 2$



large degeneracy in total cross-section: coefficients not determined in general

 $\sqrt{2}$

- + Double Higgs production depends on trilinear coupling κ_3 but also on W-boson couplings κ_W , κ_{WW} that enter the production cross-section
- + Two dim. 6 operators:

 $\kappa_3 = 1 + v^2 \left(C_6 - \frac{3}{2} C_H \right)$

$$\mathcal{O}_6 = -\lambda |H|^6 \qquad \mathcal{O}_H = \frac{1}{2} \left(\partial_\mu |H|^2 \right)^2$$
$$\kappa_W = 1 - v^2 C_H/2 \qquad \kappa_{WW} = 1 - 2v^2 C_H$$



large degeneracy in total cross-section: coefficients not determined in general

O_H also affects all single Higgs couplings universally:

$$\kappa_{V,f} = 1 - v^2 C_H/2$$

C_H can be constrained from Higgs couplings $\Delta \kappa_V \sim C_H v^2 \lesssim \text{few} \times 10^{-3}$

Higgs at high-energy

 Higgs physics doesn't mean just couplings. There's much more information in the energy dependence of the interactions! (form factors)



+ NP effects are more important at high energies (\approx high-pT tails at LHC)



Double Higgs at high mass

• NP contribution from \mathcal{O}_H (equivalently κ_W, κ_{WW}) grows as E²: high mass tail gives a *direct* measurement of C_H

High-energy WW $\rightarrow hh$ more sensitive than Higgs pole physics at energies $\gtrsim 10$ TeV





 $\mu^+\mu^- \to hh\nu\bar{\nu}$

Double Higgs at high mass

- + SM Effective Theory: $\mathscr{L}_{EFT} = \mathscr{L}_{SM} + \sum C_i \mathscr{O}_i^{(6)} + \cdots$
- + Trilinear coupling is affected by two operators: $\kappa_3 = 1 + v^2 \left(C_6 \frac{3}{2} C_H \right)$

$$\mathcal{O}_6 = -\lambda |H|^6$$
 $\mathcal{O}_H = \frac{1}{2} \left(\partial_\mu |H|^2 \right)^2$

Differential analysis in p_T and M_{hh} :



22

+ Higgs & EWSB physics \leftrightarrow Ew precision measurements



$$\begin{aligned} \mathscr{O}_W &= \left(H^{\dagger} \sigma^a D^{\mu} H \right) D^{\nu} W^a_{\mu\nu} & \sin^2 \theta_{\text{eff}} \\ \mathscr{O}_B &= \left(H^{\dagger} D^{\mu} H \right) \partial^{\nu} B_{\mu\nu} \end{aligned}$$

 FCC-ee: 6 x 10¹² Z bosons ultimate precision at the Z pole, limited by syst. and th. errors

$$\Delta \hat{S} \sim \frac{m_W^2}{M_{\rm NP}^2} \lesssim {\rm few} \times 10^{-5}$$

$$M_{\rm NP} \gtrsim 12 \,{\rm TeV}$$

	Current	HL-LHC	ILC ₂₅₀		CEPC	FCC-ee	CLIC ₃₈₀	
				(& ILC ₉₁)				(& CLIC ₉₁)
S	0.13	0.053	0.012	0.009	0.0068	0.0038	0.032	0.011
Т	0.08	0.041	0.014	0.013	0.0072	0.0022	0.023	0.012

EW precision

✤ U(2)³ flavor symmetry + suppression of light gen. + some flavor alignment



Allwicher, Cornella, Isidori, Stefanek 2311.00020

Flavor @ FCC-ee

- + Unique flavor physics program possible at FCC-ee!
 - ~ 10¹² b quark pairs (and 10¹¹ tau pairs) in a B-factory-like environment from Z boson decays

Decay mode/Experiment	Belle II $(50/ab)$	LHCb Run I	LHCb Upgr. $(50/fb)$	FCC-ee
EW/H penguins				
$B^0 \rightarrow K^*(892)e^+e^-$	~ 2000	~ 150	~ 5000	~ 200000
$\mathcal{B}(B^0 \to K^*(892)\tau^+\tau^-)$	~ 10	—	—	~ 1000
$B_s ightarrow \mu^+ \mu^-$	n/a	~ 15	~ 500	~ 800
$B^0 o \mu^+ \mu^-$	~ 5	—	~ 50	~ 100
$\mathcal{B}(B_s \to \tau^+ \tau^-)$				
Leptonic decays				
$B^+ o \mu^+ \nu$	5%	—	—	3%
$B^+ \to \tau^+ \nu$	7%	—	—	2%
$B_c^+ \to \tau^+ \nu$	n/a	_	_	5%

[Table from S. Monteil]

• can measure decay modes with missing energy (esp. τ 's and ν 's) with 100x more statistics than Belle II!

 $M_{\rm NP}\gtrsim\,$ several TeV, for NP coupled to 3rd family

complementary to other flavor probes

 Precision measurements need to be matched with theory predictions of comparable precision

 $\Delta \hat{S} \lesssim 10^{-5} \longrightarrow NNLO EW$ corrections required

- Already now, huge rates of b, c hadrons at LHC not always reflected in improvement of physics reach, due to QCD (e.g. hadronic channels, V_{cb} puzzle in semi-leptonic decays, K and D mixing, ...)
- High rate measurements eventually limited by systematics
 - Why 10¹² Z bosons?

Lepton asymmetries: $N_{\text{events}} = N_Z \times \text{BR}(Z \to \ell^+ \ell^-) \times A_\ell \sim 3 \times 10^{-4} N_Z$ $\implies N_Z \approx 10^{12}$ for 10⁻⁴ precision

• Eventually, we'll need to measure physics at higher energy to improve!

EW precision at high-energy

+ NP effects are more important at high energies $\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{\Lambda^2} \sum C_i \mathscr{O}_i$



+ Effective at LHC, FCC-hh, CLIC: "energy helps accuracy"...

Farina et al. 1609.08157, Franceschini et al. 1712.01310, ...

... taken to the extreme at a μ -collider with 10's of TeV!

Example: high-energy di-bosons

+ Longitudinal $2 \rightarrow 2$ scattering amplitudes at high energy:



Determined by the same two operators that affect also EWPT (in flavor-universal theories):

$$\mathcal{O}_W = \left(H^{\dagger} \sigma^a D^{\mu} H \right) D^{\nu} W^a_{\mu\nu}$$
$$\mathcal{O}_B = \left(H^{\dagger} D^{\mu} H \right) \partial^{\nu} B_{\mu\nu}$$

related with Z-pole observables

$$\hat{S} = m_W^2 (C_W + C_B)$$

LEP: 10^{-3} , FCC: few 10^{-5} MuC: 10^{-6}

precision of measurement

+ All EW multiplets contribute to high-energy $2 \rightarrow 2$ fermion scattering: effects that grow with energy, can be tested at μ collider



can be WIMP dark matter if M ~ few TeV

High-energy probes: EW & Higgs physics

- High-energy processes at a 10–30 TeV lepton collider are able to probe EW new physics scales of ~100 TeV or more.
 - 10x higher than ultimate precision at Z pole



+ Example: new physics with mass m_{\star} and coupling g_{\star} to Higgs



Direct searches

Main motivation for a muon collider: ability to collide elementary particles at very high energies \implies directly explore physics at 10+ TeV



Produce pairs of EW particles up to kinematical threshold: no loss of energy due to parton distribution functions!





EW radiation

EW radiation becomes important at multi-TeV energies! Especially relevant for muon collider, but also FCC-hh...

- $m_{W,Z} \ll E: \gamma, W, Z$ are all similar!
- Multiple gauge boson emission is not suppressed

Sudakov factor
$$\frac{\alpha}{4\pi} \log^2 \left(\frac{E^2}{m_W^2}\right) \times \text{Casimir} \approx 1 \text{ for E} \sim 10 \text{ TeV}$$

 Which cross-section? Exclusive, (semi-)inclusive, depending on amount of radiation included see Chen, Glioti, Rattazzi, Ricci, Wulzer 2202.10509

Could one define EW jets? Neutrino "jet tagging"?



EW radiation



Gauge boson radiation important:

soft W emission allows to access charged processes $\ell \nu \to W^{\pm}Z, W^{\pm}H$





- contains new physical information!
- need to properly define inclusive observables, resummation of logs, ...

"effective neutrino approximation"

EW radiation

Resummation of large logarithms: lepton PDF


Summary

- One of the priorities for our field in the next decades will be to explore the 10+ TeV scale. Precision measurements might be the quickest way...
- Two complementary paths to precision measurements:



- Low-energy e+e⁻ collider: Higgs physics at 10⁻³, EW physics at 10⁻⁵, flavor. The easiest way to reach 10 TeV (indirectly)
- + **High-energy** $\mu^+\mu^-$ **collider:** collide elementary particles at the energy frontier.

VBF: Higgs physics at 10⁻³, Higgs self-coupling.

High-energy: EWPT at 10⁻⁷, i.e. scales > 100 TeV; EW particles at 10+ TeV.

- One of the priorities for our field in the next decades will be to explore the 10+ TeV scale. Precision measurements might be the quickest way...
- We need to start planning the next collider now, to ensure a physics program after LHC. Today e⁺e⁻ EW & Higgs factory is the only option.
- New technologies will be crucial to progress in high-energy physics!

Feasibility of a high-energy muon collider will be a game changer:

both energy and precision





Backup

Higgs couplings at muon collider

+ A full-fledged Higgs-physics program is possible at a μ C



Single Higgs: backgrounds

- Physics backgrounds (including the Higgs itself!)
- + Beam-induced background



- Detector performance
 - + "soft" and forward particles

Forslund, Meade 2203.09425

Production	Decay	$\Delta\sigma/\sigma~(\%)$		Signal Only
FIGUELIOI		$3\mathrm{TeV}$	$10 \mathrm{TeV}$	$10\mathrm{TeV}$
	bb	0.80	0.22	0.17
	сс	12	3.6	1.7
	gg	2.8	0.79	0.19
	$\tau^+\tau^-$	3.8	1.1	0.54
	$WW^*(jj\ell\nu)$	1.6	0.42	0.30
W^+W^- fusion	$WW^*(4j)$	5.4	1.2	0.49
	$ZZ^*(4\ell)$	48	13	12
	$ZZ^*(jj\ell\ell)$	12	3.4	2.3
	$ZZ^*(4j)$	65	15	1.4
	$\gamma\gamma$	6.4	1.7	1.3
	$Z(jj)\gamma$	45	12	2.0
	$\mu^+\mu^-$	28	5.7	3.9
	bb	2.6	0.77	0.49
	сс	72	17	-
ZZ fusion	gg	14	3.3	-
	$\tau^+\tau^-$	21	4.8	-
	$WW^*(jj\ell\nu)$	8.4	2.0	-
	$WW^*(4j)$	17	4.4	1.3
	$ZZ^*(jj\ell\ell)$	34	11	-
	$\gamma\gamma$	23	4.8	-
ttH	bb	61	53	12

Single Higgs at high mass (off-shell)

+ Off-shell single Higgs production: independent of width





Forslund, Meade 2308.02633



precision limited (~ 3%) due to backgrounds: not possible to determine κ_W precisely through WW scattering

 \rightarrow correlation width vs. coupling

Single Higgs at high mass (off-shell)

+ Off-shell single Higgs production: independent of width





Forslund, Meade 2308.02633



precision limited (~ 3%) due to backgrounds: not possible to determine κ_W precisely through WW scattering

 \rightarrow correlation width vs. coupling

Inclusive Higgs search

Caveat: single Higgs at µC can access only

$$\mu_f = \sigma_h \times \mathrm{BR}_{h \to f} \sim \frac{g_W^2 \times g_f^2}{\Gamma_h} \quad \text{(similar to LHC)}$$



 $s = (p_h + p_Z)^2$

Inclusive measurement, $\sigma_h \sim g_Z^2$

Hard neutrinos not seen, $WW \rightarrow h \rightarrow WW \text{ depends}$ on g_W and Γ

cannot disentangle deviations in the couplings from modifications of total width 41

Inclusive Higgs search

+ Try to do an inclusive single Higgs measurement with ZZ \rightarrow h



- Untagged: % sensitivity if muons detected at $\eta \gtrsim 6$ P. Li, Z. Liu, K. Lyu 2401.08756
- Invisible: 10^{-3} sensitivity if muons detected at $\eta \gtrsim 5$

Ruhdorfer, Salvioni, Wulzer 2303.14202 Forslund, Meade 2308.02633

- cross-section ~ 10x lower than WW
- needs forward muon detection!

$$s = (p_h + p_{\mu 1} + p_{\mu 2})^2$$



Invisible Higgs @ muon collider

- Invisible BSM Higgs Branching Ratio can be one of the contributions to total width Γ.
- Can also be studied in ZZ-fusion:

10⁻³ sensitivity *if muons detected at* $\eta \gtrsim 5$



Ruhdorfer, Salvioni, Wulzer 2303.14202 Forslund, Meade 2308.02633



EW precision

+ In general, several more operators enter the EW fit



Several 4-fermion interactions enter through one loop RGE

2311.00020, 1704.04504

$$\begin{array}{c} {}^{H} & \cdots & {}^{l_{L}^{a}} \\ & t \\ {}^{H} & \cdots & {}^{l_{L}^{a}} \\ {}^{H} & \cdots & {}^{l_{L}^{a}} \\ {}^{H} & {}^{l_{L}^{a}} \\ {}^{I}_{l_{q}}^{(1,3)}]_{aa33} \end{array}$$
 $[C_{Hl}^{(1,3)}]_{aa} \qquad M_{NP}^{4f} \gtrsim 10 \, \text{TeV} \times g_{\star}$

Example: WIMP Dark Matter

- Weakly Interacting Massive Particle: most general EW multiplet with DM candidate that is
 similar to Minimal DM:
 - (a) stable,
 - (b) without coupling to $\gamma \& Z$,
 - (c) calculable (perturbative).



similar to Minimal DM: Cirelli, Fornengo, Strumia hep-ph/0512090

$$\chi_n = (\cdots, \chi^{-} \chi^{0} \chi^{+}, \cdots)$$

Bottaro, DB, Costa, Franceschini, Panci, Redigolo, Vittorio 2107.09688, 2205.04486



EW n-plet	Mass [TeV]
2 _{1/2}	1.08
3 0	2.86
4 _{1/2}	4.8
5 ₀	13.6
5 ₁	9.9
61/2	31.8
70	48.8
9 0	113

Example: WIMP Dark Matter

• Mono- γ /W/Z signals: $\mu \bar{\mu} \rightarrow \chi \bar{\chi} + X$ DM pair production + EW radiation

> Han et al. 2009.11287 Bottaro et al. 2107.09688, 2205.04486

• Disappearing tracks: charged components of χ can be long-lived $\chi^{\pm} \rightarrow \chi^0 \pi^{\pm}$

Capdevilla et al. 2102.11292







µC can probe all relevant WIMP candidates!

More difficult at hadron colliders, due to PDF suppression

FCC physics study Cirelli, Sala, Taoso 1407.7058

High-energy probes: EW & Higgs physics

- High-energy processes at a 10–30 TeV lepton collider are able to probe EW new physics scales of 100 TeV or more.
 - 10x higher than ultimate precision at Z pole



Example: heavy resonance with mass m_z, and coupling g_z, to fermions



Flavour: muons vs. electrons

- New Physics (especially if related to the Higgs sector) could distinguish the different families of fermions.
- EW interactions are flavour-universal: an accidental property of the gauge lagrangian, *not* a fundamental symmetry of nature!
 - Example: Yukawa couplings, the only non-gauge interactions in the SM, violate flavour universality maximally!

$$m_u \sim (\cdot \cdot)$$

$$m_d \sim (\cdot \cdot \bullet) \qquad m_\ell \sim (\cdot \cdot \bullet)$$



A muon collider collides 2nd generation particles: could test flavour structure

➡ High-energy probes can be even more powerful in this case: enhancement wrt. low energy observables can be as large as (E/m_µ)² + Flavor processes: rare decays & tiny effects

 $BR(B_s \to \mu\mu) \sim 10^{-9}, \quad BR(\tau \to 3\mu) \lesssim 10^{-8}, \quad \Delta a_\mu \approx 10^{-9}$

- need billions of events, usually probed by means of high-intensity experiments
- Muon-collider: very large number of (clean) EW particles, but overall event rate not comparable to flavor factories



Quark flavor violation



Four-fermion interactions: muon current coupled to flavor-violating bilinear

$$\frac{c_{bs}}{\Lambda^2}(\bar{b}_{L,R}\gamma^{\rho}s_{L,R})(\bar{\mu}_{L,R}\gamma_{\rho}\mu_{L,R})$$

Contributes to (semi-)leptonic rare B decays b → sµµ: branching ratios
 & angular observables of various hadronic processes

$$B_s \to \mu\mu, \qquad B \to K^{(*)}\mu\mu, \qquad B_s \to \phi\mu\mu, \qquad \Lambda_b \to \Lambda\mu\mu$$

 Theory uncertainties: cannot improve indefinitely with rare decays

$$BR(B \to K\mu\mu) \sim \frac{m_W^4}{\Lambda^4}, \quad \sigma(\mu\bar{\mu} \to jj) \sim \frac{E^2}{\Lambda^4}$$

Azatov, Garosi, Greljo, Marzocca, Salko, Trifinopoulos 2205.13552



Flavour @ muon collider: the muon g-2

+ Example: muon g-2. Can it be tested at high energies at a muon collider?



$$\Delta a_{\mu} = ???$$

Flavour @ muon collider: the muon g-2

- + Example: muon g-2. Can it be tested at high energies at a muon collider?
- + If new physics is heavy: EFT!

One dim. 6 operator contributes at tree-level: $\mathscr{L}_{g-2} = \frac{C_{e\gamma}}{\Lambda^2} H(\bar{\ell}_L \sigma_{\mu\nu} e_R) eF^{\mu\nu} + h.c.$



Dipole operator generates both Δa_{μ} and $\mu\mu \rightarrow h\gamma$

B, Paradisi 2012.02769

At high energy

$$\sigma_{\mu^+\mu^- \to h\gamma} = \frac{s}{48\pi} \frac{|C_{e\gamma}|^2}{\Lambda^4} \approx 0.7 \text{ ab} \left(\frac{\sqrt{s}}{30 \text{ TeV}}\right)^2 \left(\frac{\Delta a_{\mu}}{3 \times 10^{-9}}\right)^2$$

$$N_{h\gamma} = \sigma \cdot \mathscr{L} \approx \left(\frac{\sqrt{s}}{10 \text{ TeV}}\right)^4 \left(\frac{\Delta a_{\mu}}{3 \times 10^{-9}}\right)^2 \quad \text{need E} > 10 \text{ TeV}$$

Muon g-2 @ muon collider

- + SM irreducible bakground is small: $\sigma_{\mu^+\mu^- \to h\gamma}^{(SM)} \approx 10^{-2} \operatorname{ab} \left(\frac{30 \text{ TeV}}{\sqrt{s}}\right)^2$ tree-level is suppressed by muon mass; loop contribution dominant
- Main background from $\mu\mu \rightarrow Z\gamma$ (where Z is mistaken for H) (large due to transverse Z polarizations)

$$\frac{d\sigma_{\mu\mu\to h\gamma}}{d\cos\theta} = \frac{|C^{\mu}_{e\gamma}(\Lambda)|^2}{\Lambda^4} \frac{s}{64\pi} (1 - \cos^2\theta)$$

$$\frac{d\sigma_{\mu\mu\to Z\gamma}}{d\cos\theta} = \frac{\pi\alpha^2}{4s} \frac{1+\cos^2\theta}{\sin^2\theta} \frac{1-4s_W^2+8s_W^4}{s_W^2c_W^2}$$

- Search in h \rightarrow bb channel: $\epsilon_b \approx 80 \%$ $|\cos \theta_{\rm cut}| < 0.6$ ${\rm BR}_{h \rightarrow b\bar{b}} = 58 \%$ At 30 TeV, 90 ab⁻¹, for $\Delta a_\mu = 3 \times 10^{-9}$: $N_S = 22$, $N_B = 886 \times p_{Z \rightarrow h}$

 Δa_{μ} can be tested at 95% CL at a 30 TeV collider if Z→h mistag probability < 10-15%





Muon g-2 @ muon collider



• Other operators enter g-2 at 1 loop:

$$\Delta a_{\mu} \approx \left(\frac{250 \,\mathrm{TeV}}{\Lambda^2}\right)^2 \left(C_{e\gamma} - \frac{C_{Tt}}{5} - \frac{C_{Tc}}{1000} - \frac{C_{eZ}}{20}\right)$$

Full set of operators with Λ ≥ 100 TeV
 can be probed at a high-energy
 muon collider





Muon g-2 @ muon collider



Lepton g-2 from rare Higgs decays

Tau magnetic dipole moment: enhanced due to the larger mass

$$\Delta a_{\tau} = \frac{4v \, m_{\tau}}{\Lambda^2} C_{e\gamma}^{\tau} \approx \Delta a_{\mu} \frac{m_{\tau}^2}{m_{\mu}^2} \approx 10^{-6}$$

if $C_{e\gamma}^{\ell}$ scales as y_{ℓ}

Present bound: $\Delta a_{\tau} \lesssim 10^{-2}$ from LEP $e^+e^- \rightarrow e^+e^-\tau^+\tau^$ hep-ex/0406010 Can be improved to few 10-3 at HL-LHC 1908.05180

• Contribution to $h \rightarrow \tau \tau \gamma$ decays:

 $\mathrm{BR}^{(\mathrm{SM})}_{h \to \tau^+ \tau^- \gamma} \approx 5 \times 10^{-4}$ (with cut on soft collinear photon)

could be measured at few % level by Higgs factory

$$\mathsf{BR}_{h \to \tau^+ \tau^- \gamma}^{(\mathsf{NP})} \approx 0.2 \times \Delta a_{\tau}$$



Further possibilities to measure Δa_{τ} precisely from high-energy probes

+ $H\tau\tau$ associated production



work in progress with Levati, Paradisi, Maltoni, Wang

• Main background from $\mu\mu \rightarrow Z\gamma$ (where Z is mistaken for H)

Could probe $\Delta a_{\tau} \sim 10^{-5}$ @ 10 TeV



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• Reach on Higgs trilinear coupling: $hh \rightarrow 4b$

E [TeV]	£ [ab-1]	N _{rec}	δκ3
3	5	170	~ 10%
10	10	620	~ 4%
14	20	1340	~ 2.5%
30	90	6'300	~ 1.2%

B, Franceschini, Wulzer 2012.11555,Han et al. 2008.12204, Costantini et al. 2005.10289





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Number of events ~ $s \log(s/m_h^2) \approx 10^5$ at 14 TeV

Naïve estimate of the reach: $\delta \sigma \sim (N \times \epsilon)^{-1/2} \approx 1 \%$ reconstruction eff. $\sim 30 \%$ BR $(hh \rightarrow 4b) = 34 \%$ $\epsilon \sim 10 \%$



hh signal is strongly peaked in forward region









 Contribution from trilinear coupling is more central: loss due to angular cut is less important

- Backgrounds are important and cannot be neglected (see also CLIC study 1901.05897)
 - Mainly VBF di-boson production: Zh & ZZ, but also WW, Wh, WZ...
 - Precise invariant mass reconstruction is crucial to isolate signal





NB: (Very!) simplified background analysis *(at parton level!)*

All this should be done properly with a detector simulation

However, perfect agreement with 1901.05897! (3 TeV CLIC)

Double Higgs at high mass

- Fully differential analysis in p_T and M_{hh} to optimize combined sensitivity to C_H and C₆
- Very boosted Higgs bosons: treat them as a single h-jet, without reconstructing the 4 b's.
 We assumed a boosted-H tagging efficiency ~ 50%





 $C_H \times \nu^2$

+ Longitudinal $2 \rightarrow 2$ scattering amplitudes at high energy:

Process	BSM Amplitude
$ \begin{array}{c} \ell_L^+ \ell_L^- \to Z_0 h \\ \bar{\nu}_L \nu_L \to W_0^+ W_0^- \end{array} $	$s\left(G_{3L}+G_{1L}\right)\sin\theta_{\star}$
$ \begin{pmatrix} \ell_L^+ \ell_L^- \to W_0^+ W_0^- \\ \bar{\nu}_L \nu_L \to Z_0 h \end{pmatrix} $	$s\left(G_{3L}-G_{1L}\right)\sin\theta_{\star}$
$\ell_R^+ \ell_R^- \to W_0^+ W_0^-, Z_0 h$	$s G_{lR} \sin \theta_{\star}$
$ \bar{\nu}_L \ell_L^- \to W_0^- Z_0 / W_0^- h \\ \nu_L \ell_L^+ \to W_0^+ Z_0 / W_0^+ h $	$\sqrt{2}sG_{3L}\sin\theta_{\star}$

Determined by 3 fermion/scalar current-current interactions (Warsaw):

$$\begin{aligned} \mathcal{O}_{3L} &= \left(\bar{\mathrm{L}}_L \gamma^{\mu} \sigma^a \mathrm{L}_L \right) \left(i H^{\dagger} \sigma^a \overset{\leftrightarrow}{D}_{\mu} H \right), \\ \mathcal{O}_{1L} &= \left(\bar{\mathrm{L}}_L \gamma^{\mu} \mathrm{L}_L \right) \left(i H^{\dagger} \overset{\leftrightarrow}{D}_{\mu} H \right), \\ \mathcal{O}_{lR} &= \left(\bar{l}_R \gamma^{\mu} l_R \right) \left(i H^{\dagger} \overset{\leftrightarrow}{D}_{\mu} H \right). \end{aligned}$$

"high-energy primary effects"



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"high-energy primary effects"

$$\mathcal{O}_{W} = \frac{ig}{2} \left(H^{\dagger} \sigma^{a} \overset{\leftrightarrow}{D^{\mu}} H \right) D^{\nu} W^{a}_{\mu\nu}$$
$$\mathcal{O}_{B} = \frac{ig'}{2} \left(H^{\dagger} \overset{\leftrightarrow}{D^{\mu}} H \right) \partial^{\nu} B_{\mu\nu}$$
$$\mathcal{O}_{HW} = ig(D^{\mu}H)^{\dagger} \sigma^{a} (D^{\nu}H) W^{a}_{\mu\nu}$$
$$\mathcal{O}_{HB} = ig'(D^{\mu}H)^{\dagger} (D^{\nu}H) B_{\mu\nu}$$

 In flavor-universal theories, they are generated by SILH operators (via e.o.m.):

$$G_{1L} = \frac{1}{2}G_{lR} = \frac{{g'}^2}{4}(C_B + C_{HB})$$

$$G_{3L} = \frac{g^2}{4}(C_W + C_{HW})$$

High-energy di-bosons

• C_W and C_B determined from high-energy $\mu^+\mu^- \rightarrow ZH$, W^+W^- total cross-sections

0.0075 WW 10 TeV 0.0050 0.0025 ZΗ TeV^2 0.0000 -0.0025 ^Š_0__0.0050) ⊢ −0.01 -0.0075 -0.02 -0.0100-0.03 -0.04-0.04 -0.03 -0.02 -0.01 0.00 0.01 $C_B \cdot \text{TeV}^2$ -0.0125 -0.010-0.0050.000 0.005 $C_B \cdot \text{TeV}^2$

 In universal theories, C_{W,B} related with Z-pole and other EW observables

$$\hat{S} = m_W^2 (C_W + C_B)$$

- Muon collider:

10 TeV :
$$C_W \lesssim (40 \text{ TeV})^{-2}$$
, $\hat{S} \lesssim 10^{-6}$
30 TeV : $C_W \lesssim (120 \text{ TeV})^{-2}$, $\hat{S} \lesssim 10^{-7}$

Limits on $C_{W,B}$ scale as E^2

$$\sigma_{\mu\mu\to ZH} \approx 122 \text{ ab} \left(\frac{10 \text{ TeV}}{E_{\text{cm}}}\right)^2 \left[1 + \# E_{\text{cm}}^2 C_W + \# E_{\text{cm}}^4 C_W^2\right]$$



High-energy WW: angular analysis

- O_{W,B} contribute to longitudinal scattering amplitudes:
- In the SM, large contribution to $\mu^+\mu^- \rightarrow W^+W^$ from transverse polarizations.

$$\mathcal{A}_{00}^{(\text{NP})} = s \left(G_{1L} - G_{3L}\right) \sin \theta_{\star}$$
$$\mathcal{A}_{-+} = -\frac{g^2}{2} \sin \theta_{\star}$$
$$\mathcal{A}_{+-} = g^2 \cos^2 \frac{\theta_{\star}}{2} \cot^2 \frac{\theta_{\star}}{2}$$

Interference between $\pm \mp$ and 00 helicity amplitudes cancels in the total cross-section \Rightarrow signal suppressed! see also Panico et al. 1708.07823, 2007.10356



Can exploit the SM/BSM interference by looking at fully differential WW crosssection in scattering and decay angles!

B, Franceschini, Wulzer 2012.11555



 $(\theta_{\pm}, \varphi_{\pm} \text{ polar and azimuthal angle of } W^{\pm} \text{ decay products})$

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Top quark Yukawa

