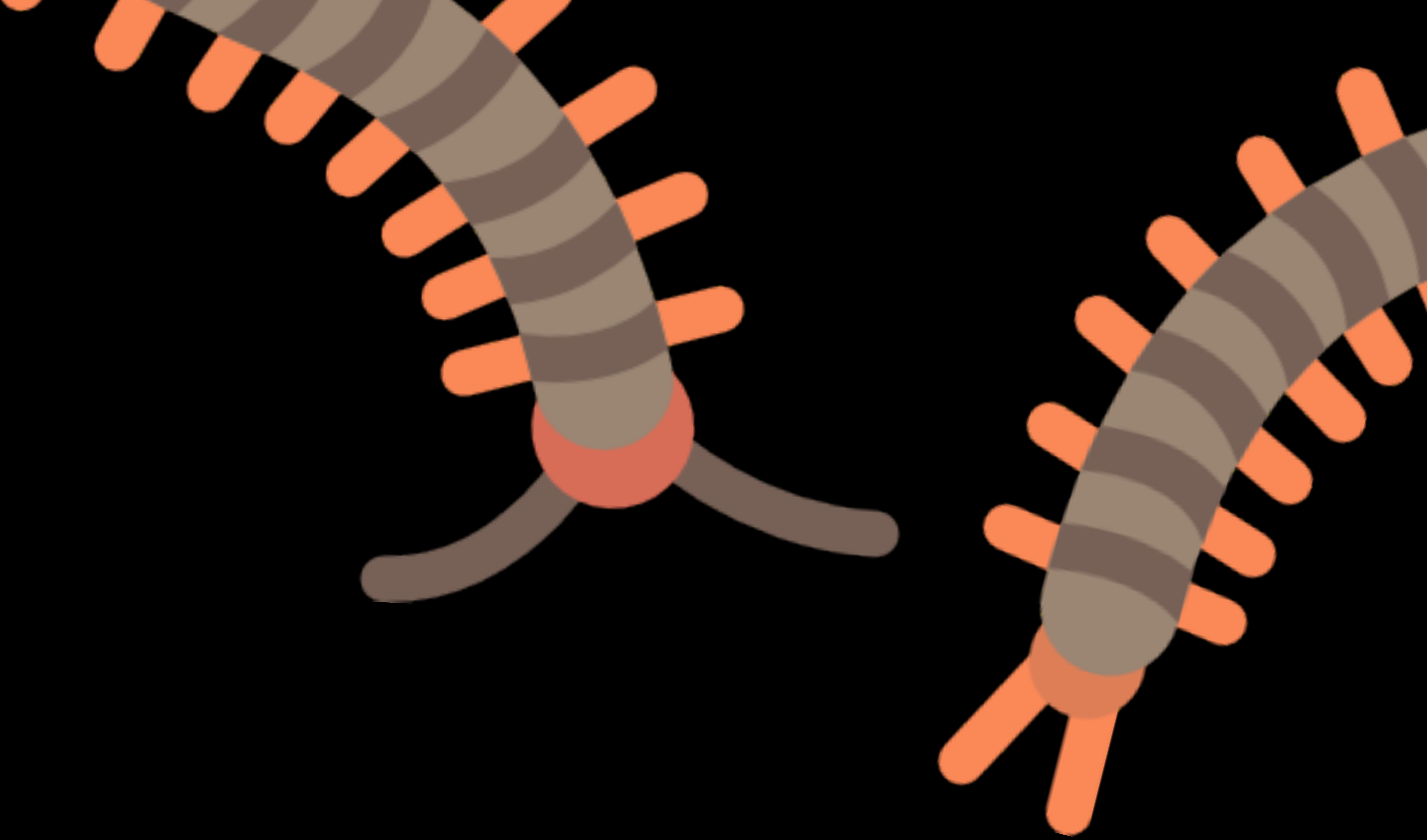
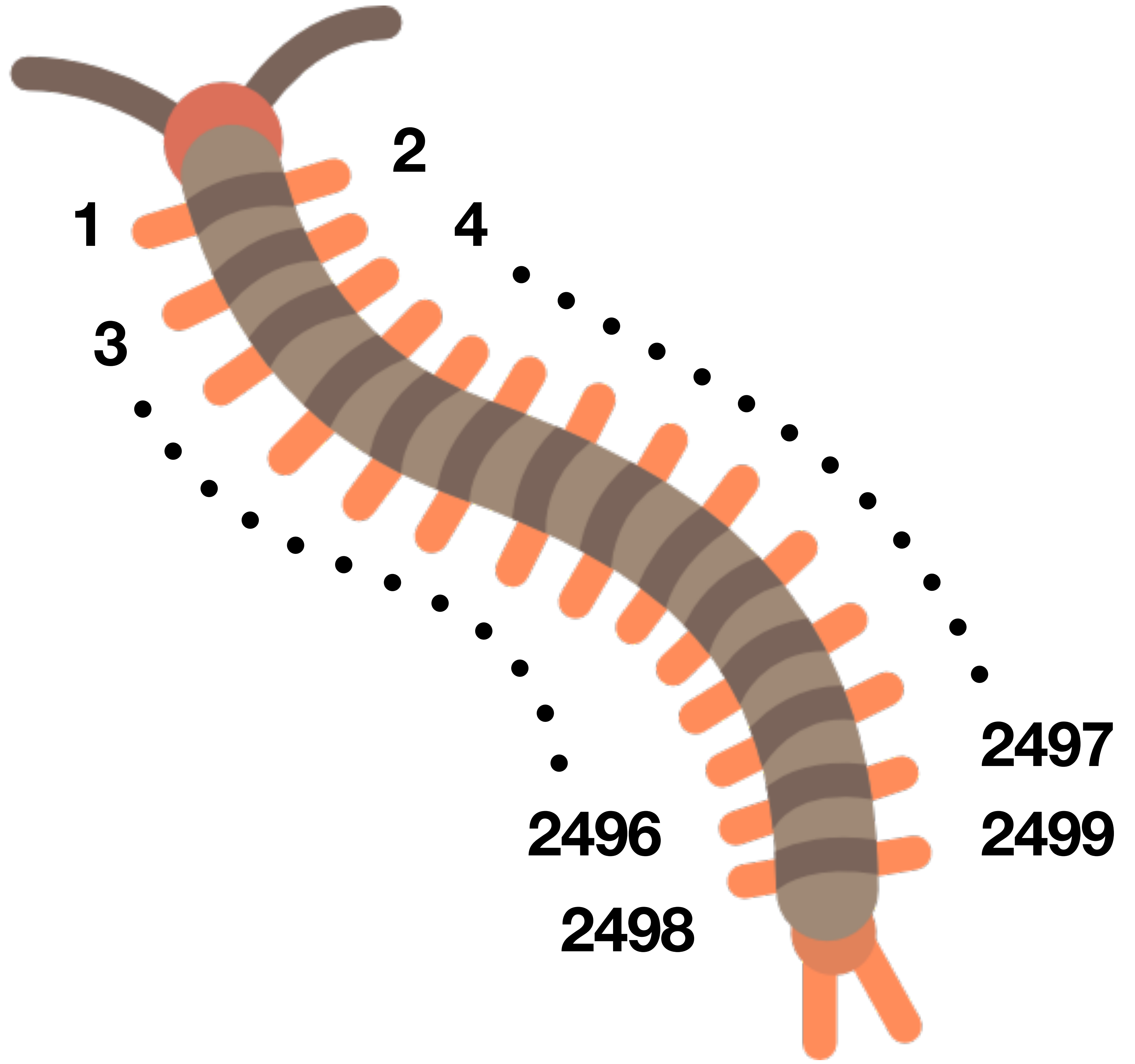
Uli Haisch, MPI Munich
HEFT 2024, 12.06.24



Precision tests of 3rd-generation four-quark SMEFT operators

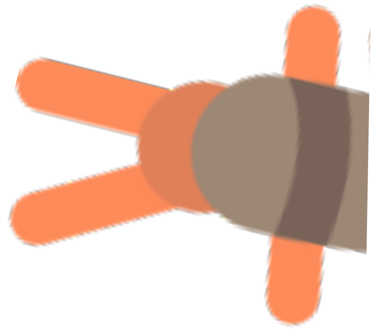
SM effective field theory

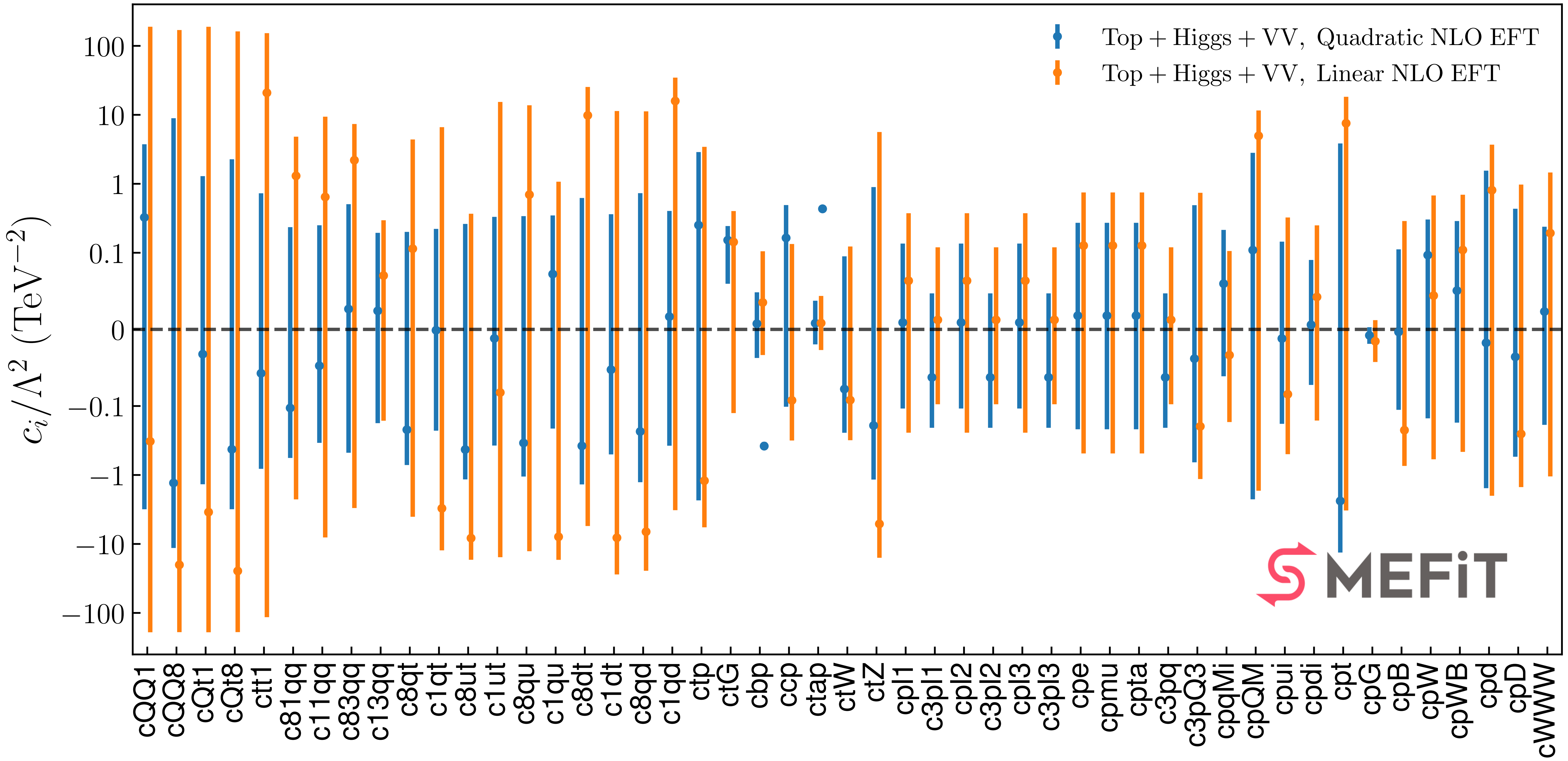
If electroweak (EW) symmetry is linearly realised, SM effective field theory aka SMEFT is higher-dimensional extension of SM. Already @ dimension 6, SMEFT is sort of a mutated millipede having 2499 independent baryon & lepton number conserving operators. In fact, large number mainly due to flavour indices



[see for instance Buchmüller & Wyler, NPB 268, 621 (1986); Grzadkowski et al., 1008.4884; Brivio & Trott, 1706.08945]

Status of global SMEFT fits



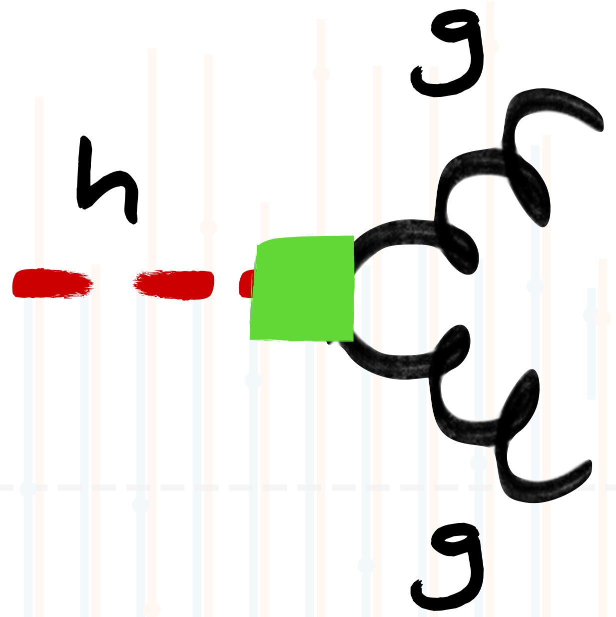


Most sophisticated fit considers 50 operators & employs Higgs, diboson & top data from LHC, including partially also EW precision observables (EWPOs)

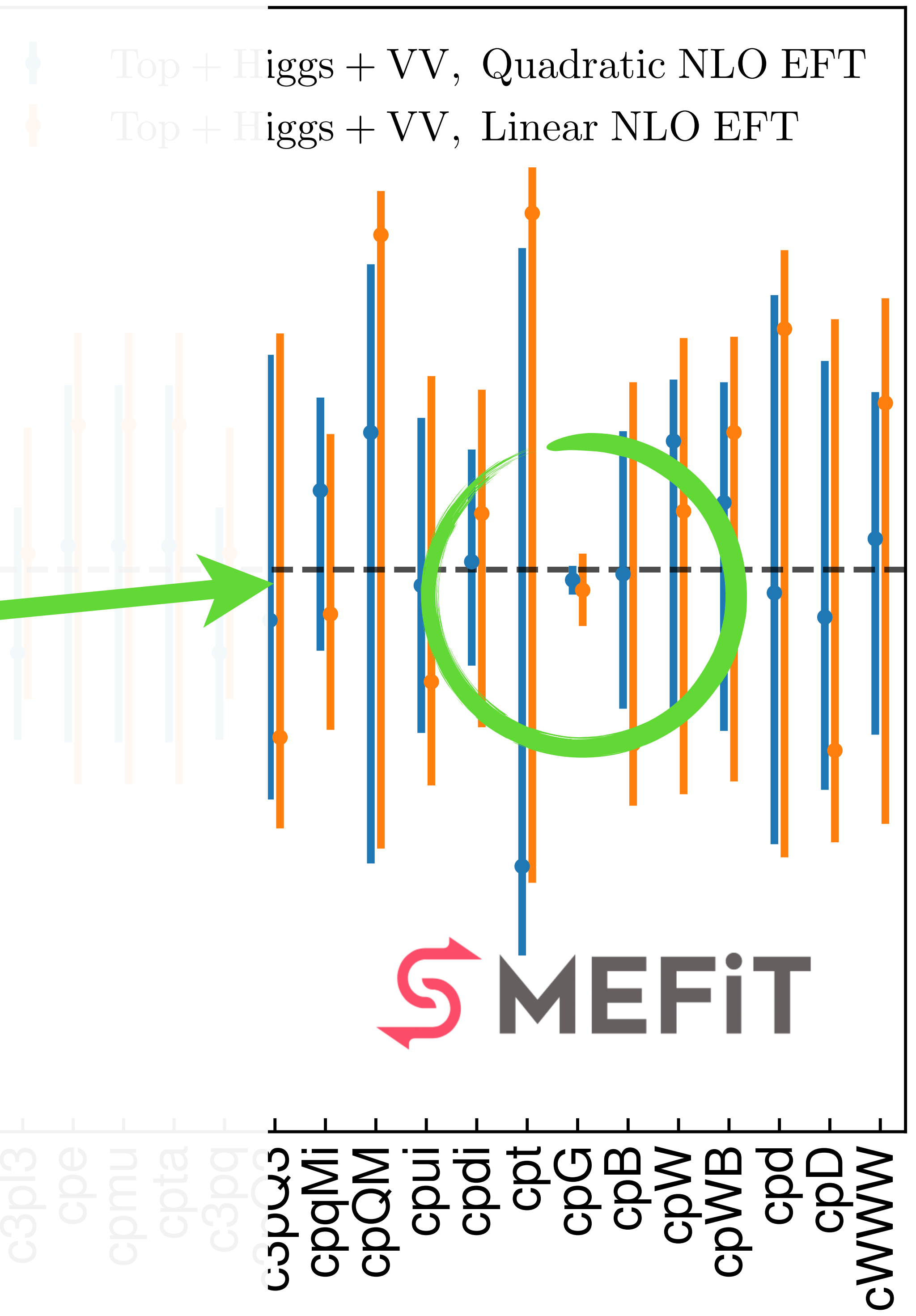
[see for instance Brivio et al., 1910.03606; Ellis et al., 2012.02779; Ethier et al., 2105.00006; Celada et al., 2404.12809]

Status of global SMEFT fits

$$(H^\dagger H)\, G^a_{\mu\nu} G^{a,\mu\nu}$$



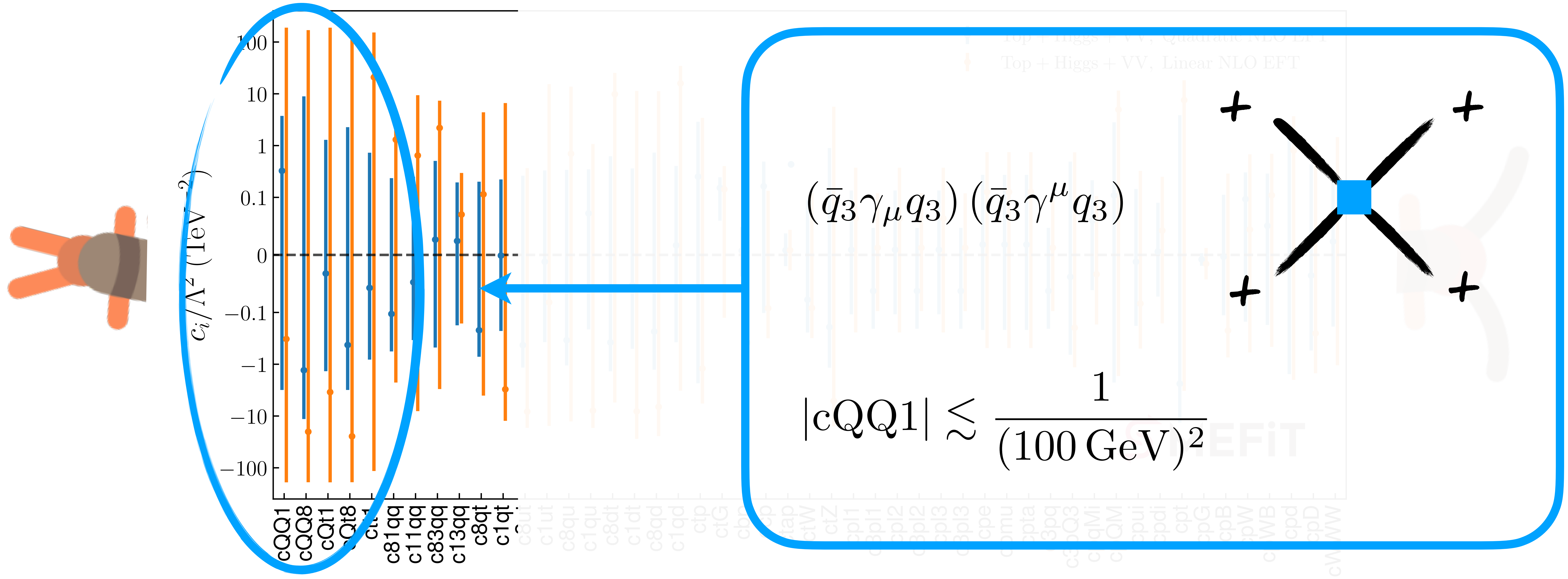
$$|\mathrm{cpG}| \lesssim \frac{1}{(13\,\mathrm{TeV})^2} \simeq \frac{\alpha_s}{4\pi}\,\frac{1}{(1\,\mathrm{TeV})^2}$$



Some operators like those altering leading Higgs couplings are well constrained, implying lower bounds on new weakly-coupled particle masses of about 1 TeV

[see for instance Brivio et al., 1910.03606; Ellis et al., 2012.02779; Ethier et al., 2105.00006; Celada et al., 2404.12809]

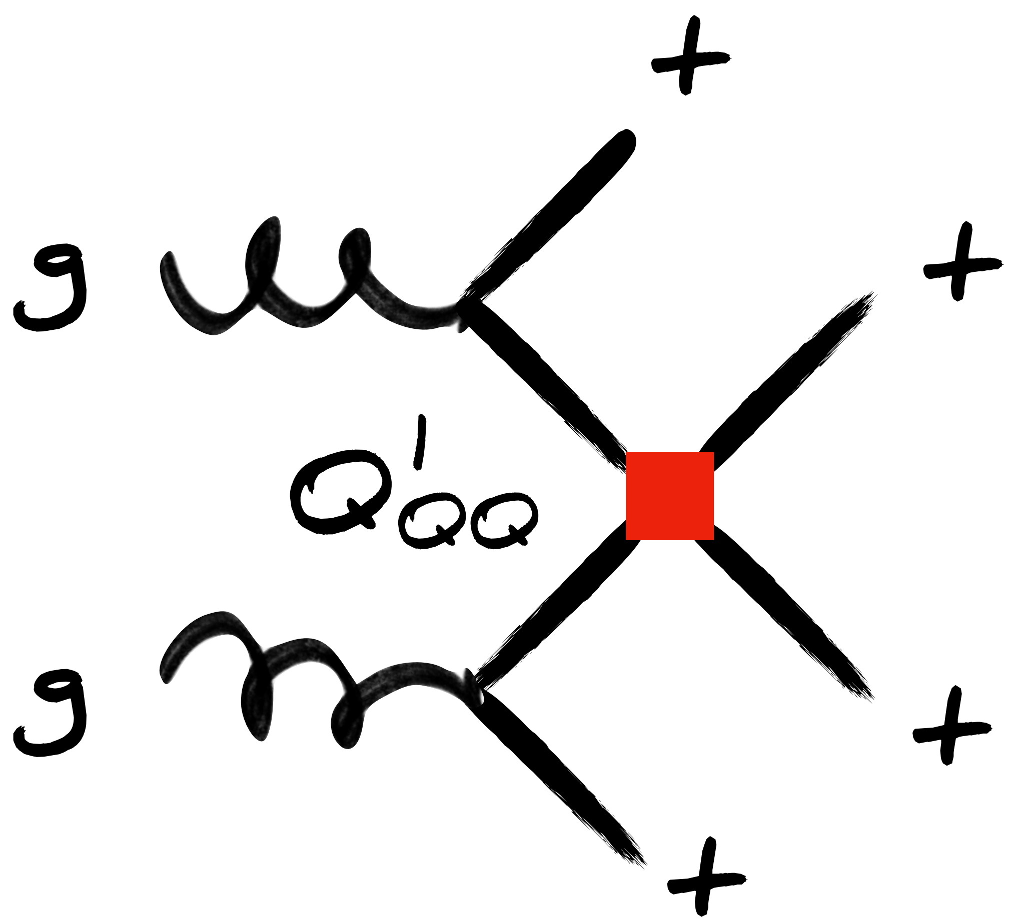
Status of global SMEFT fits

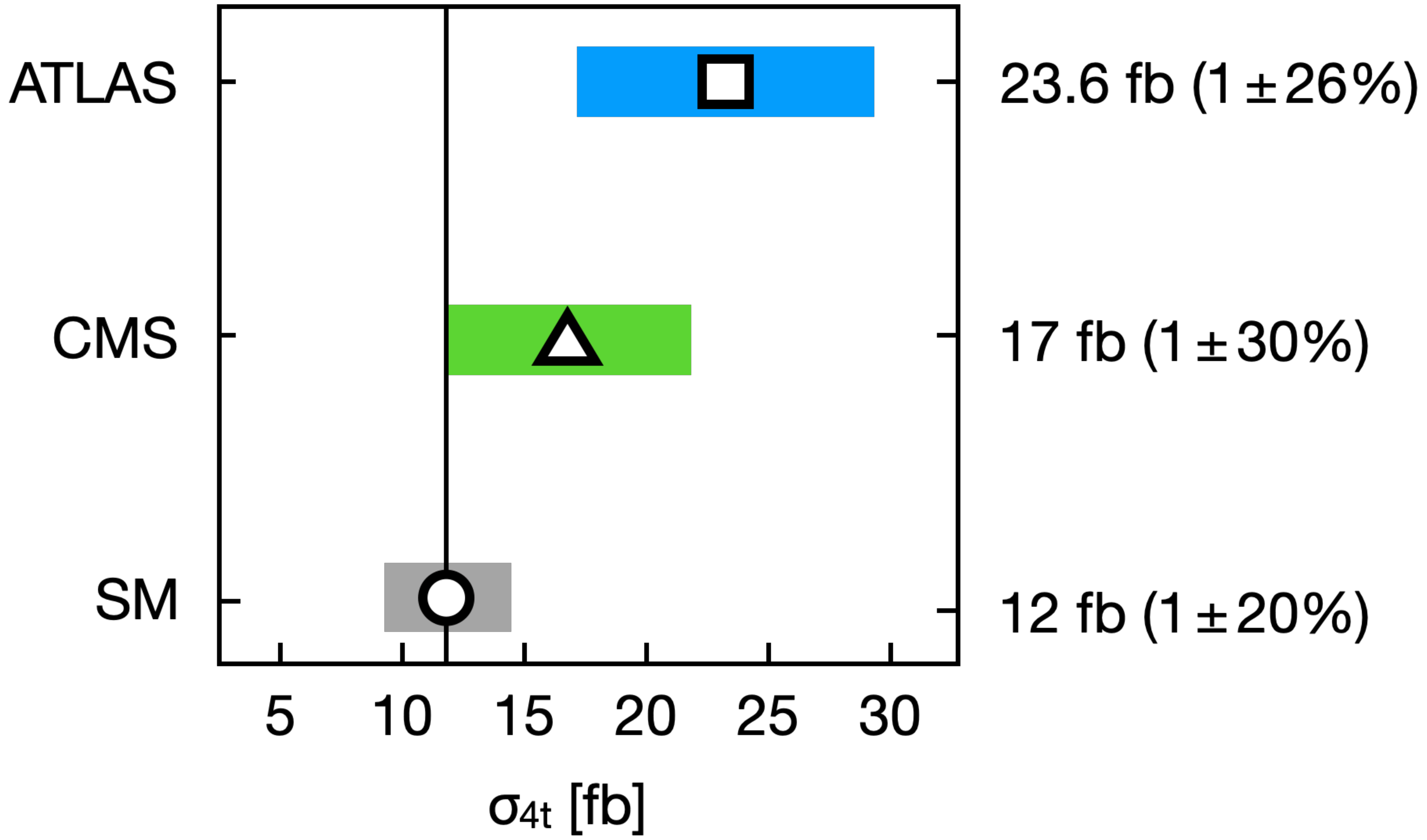


Other effective interactions remain poorly bounded. One such type are four-quark contact interactions that contain only 3rd generation fields

[see for instance Brivio et al., 1910.03606; Ellis et al., 2012.02779; Ethier et al., 2105.00006; Celada et al., 2404.12809]

Why are bounds on cQQ1, etc. so bad?

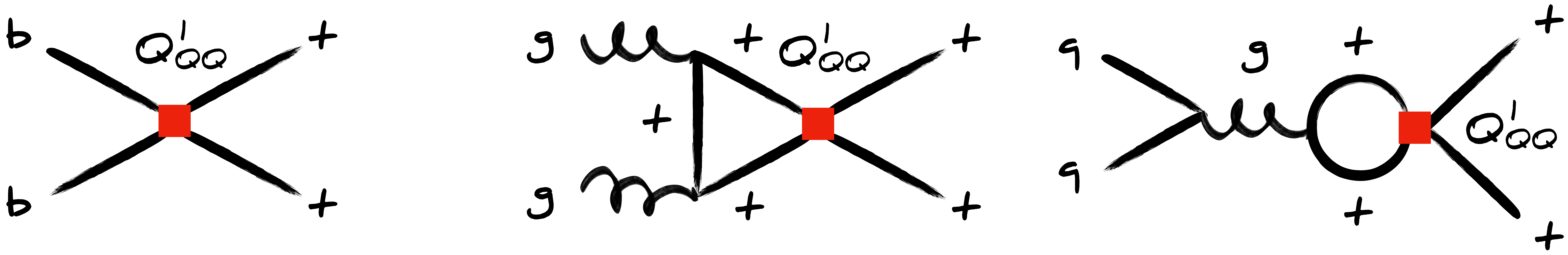




At LHC, 3rd generation four-quark operators can be probed @ tree level only in 4t, 4b || 2b2t production. Present measurements all have sizeable uncertainties

[see ATLAS, 2303.15061; CMS, 2303.03864 for latest 4t measurements]

Why are bounds on cQQ1, etc. so bad?



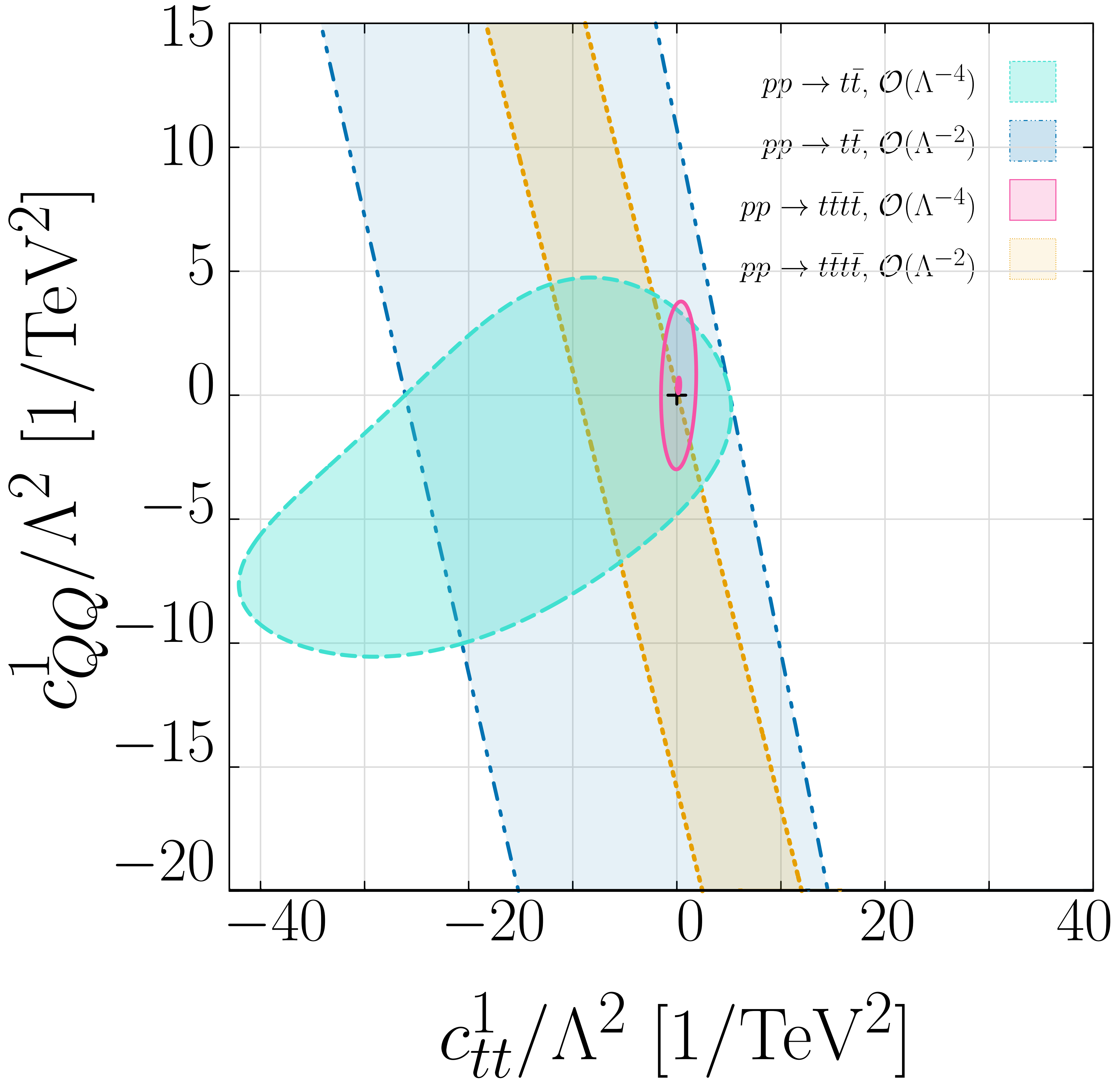
Since bottom-quark-initiated contributions are strongly PDF suppressed, leading effects from 3rd generation four-quark operators arise @ 1-loop in 2t production

[see for instance Brivio et al., 1910.03606; Degrande et al., 2008.11743; 2402.06528 & next talk by Andres]

Why are bounds on cQQ1, etc. so bad?

Limits depend on whether linear or quadratic terms are used & whether a single or several operators are studied. Raises questions about stability of fit under dimension-8 deformations & EFT applicability in general. Issue relevant, as limits arise from configurations with momentum transfer of around 0.4 TeV (1.3 TeV) in 2t (4t) production

[Degrande et al., 2402.06528]



[see also next talk by Andres]

Precision tests of cQQ1, etc.

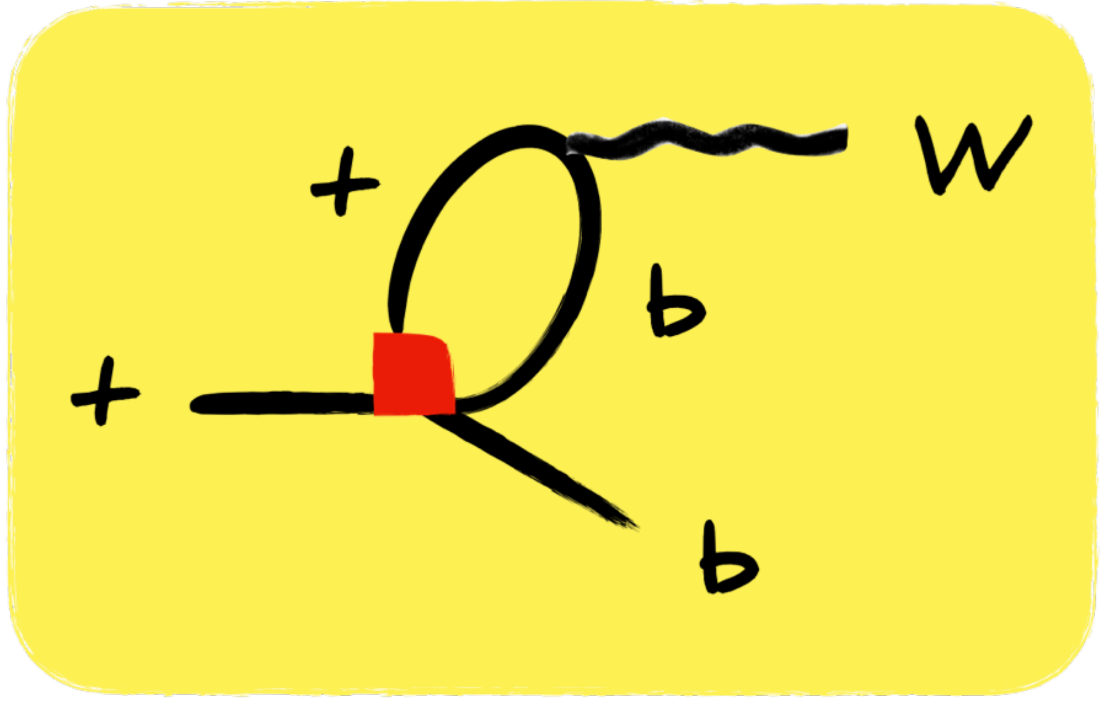
Given discussed limitations worthwhile to entertain alternative probes of 3rd generation four-quark operators by identify suitable low-energy precision tests

Clearly, relevant operators can modify low-energy observables first @ loop level. But loop suppression can be partially mitigated if a given observable receives corrections enhanced by top-quark Yukawa coupling & is precisely measured

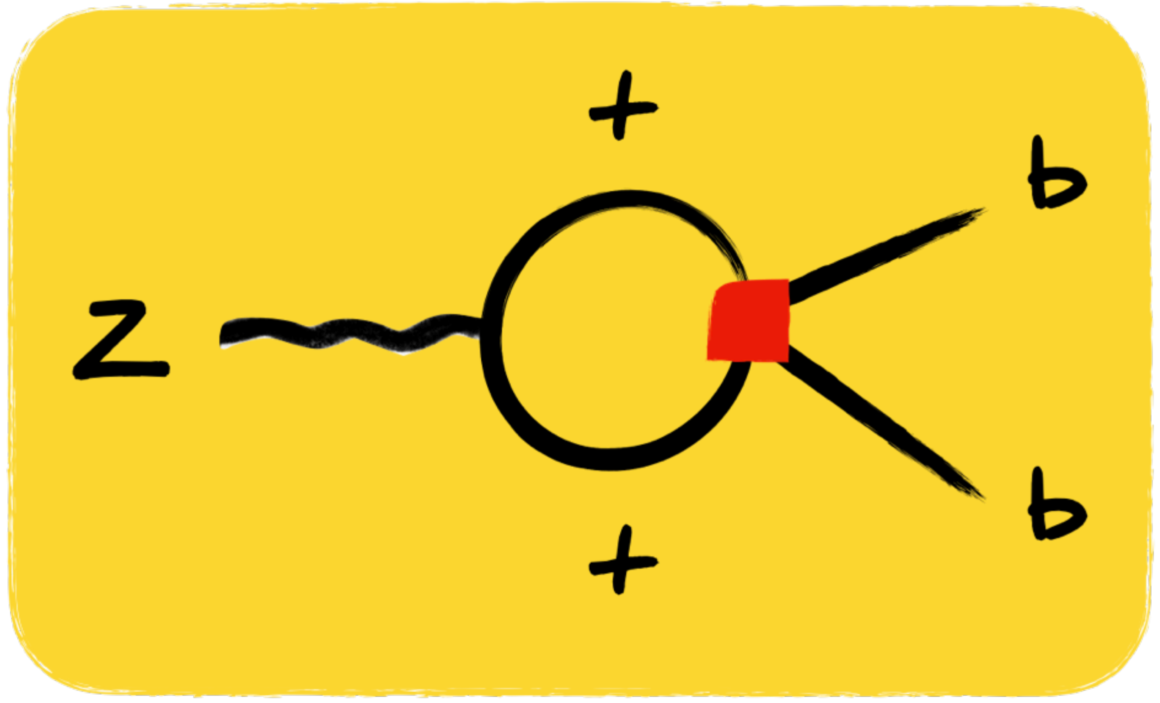
Thinking about SM, it is straightforward to figure out what relevant processes are

[see also Alasfar et al., 2202.02333; Di Noi et al., 2310.18221 & talk by Stefano for related work in Higgs sector]

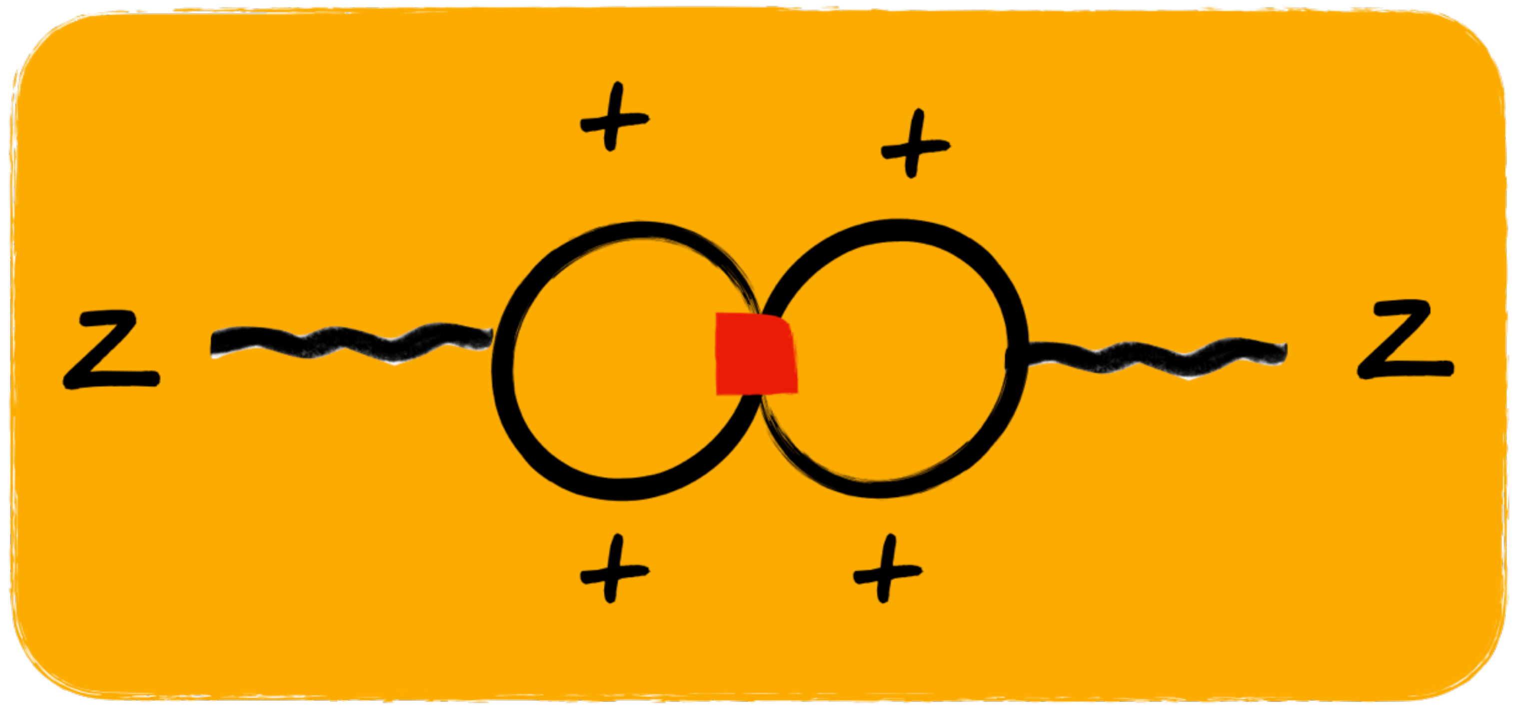
Precision tests of cQQ1, etc.



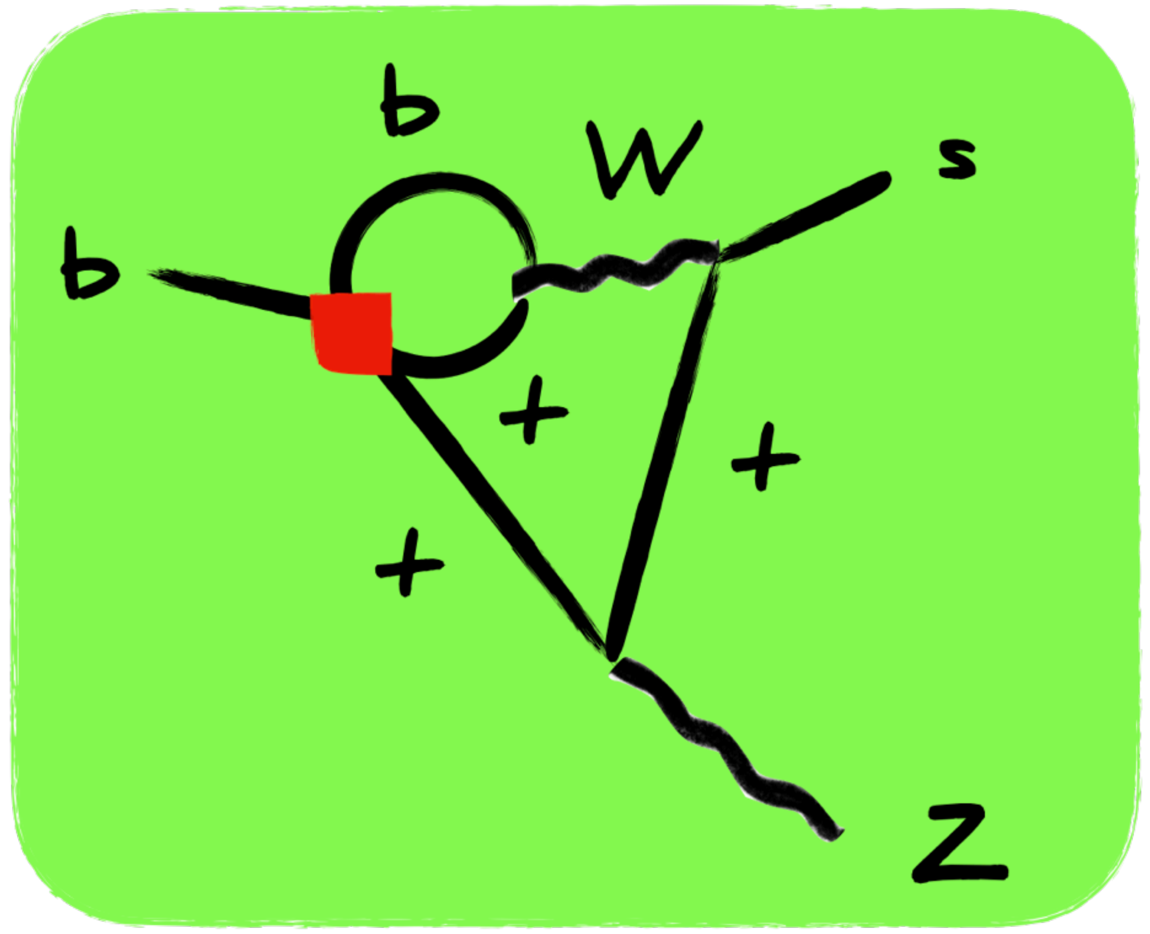
top decay



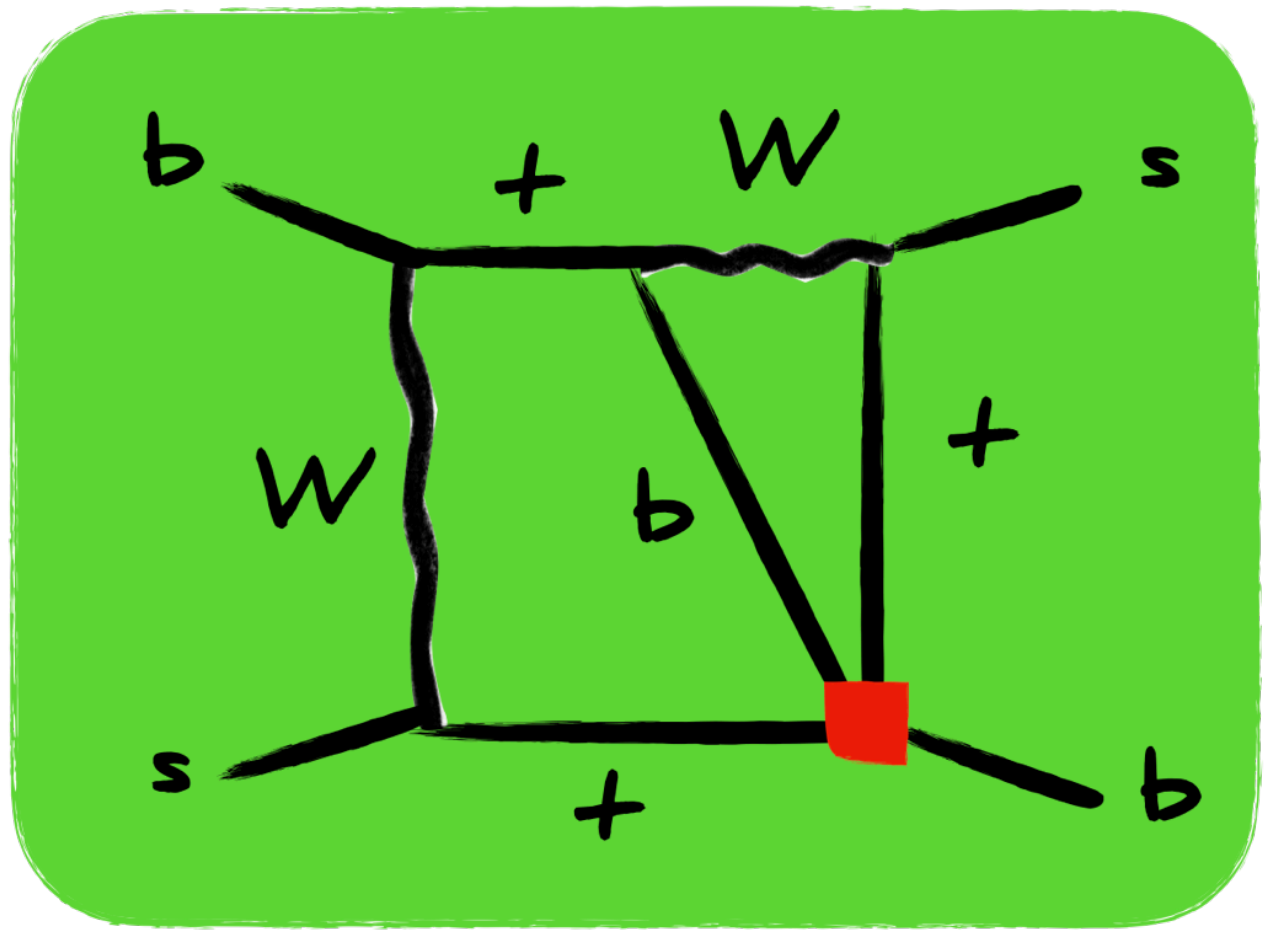
Z decay



Peskin-Takeuchi parameters



Z penguin



B_s mixing

[see also Boughezal et al., 1907.00997; Dawson & Giardino, 2201.09887 for calculation of top & Z decay, respectively]

B_s mixing in case of up-alignment

$$q = \begin{pmatrix} t_L \\ \sum_{\psi=d,s,b} V_{t\psi} \psi_L \end{pmatrix}$$

$$\mathcal{L} \supset \frac{c^1_{QQ}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2 \supset \frac{c^1_{QQ}}{\Lambda^2} (V^*_{ts} V_{tb})^2 (\bar{s}_L \gamma_\mu b_L)^2 + \text{h.c.}$$

[HFLAV, 2206.07501;
Albrecht et al., 2402.04224]

$$\frac{|c^1_{QQ}|}{\Lambda^2} \lesssim \frac{1}{(8\,\text{TeV})^2}$$

[see for instance Barducci et al., 1802.07237; Allwicher et al., 2311.00020]

Calculation in a nutshell

- Choice of Top WG operator basis avoids appearance of non-zero traces involving γ_5. Expressions for 1-loop observables differ from known Warsaw-basis results by finite terms related to a Fierz-evanescent operator
- 2-loop calculations performed off-shell using a background field gauge when necessary to maintain gauge invariance at level of Green's functions. Wilson coefficients renormalised in $\overline{\text{MS}}$, while pole mass* used for internal top quarks
- To avoid tree-level effects in B_s mixing, we consider down-alignment. Charm- & up-quark contributions still need to be included & lead to GIM mechanism in all 2-loop flavour-changing neutral current (FCNC) amplitudes

*scheme change to MS top-quark mass trivial & also computed

Anatomy of SMEFT corrections to EWPOs

$$\Delta\Gamma(Z \to b\bar{b}) \simeq -\frac{\alpha^2 m_Z}{192\pi c_w^4 s_w^4}\frac{m_t^2}{m_Z^2}\frac{v^2}{\Lambda^2}$$

$$\times \left\{\left[6\left(2s_w^2-3\right)\left(c_{QQ}^1-c_{Qt}^1\right)+12s_w^2\left(c_{Qb}^1-c_{tb}^1\right)\right]\ln\left(\frac{m_t^2}{m_Z^2}\right)+\left(2s_w^2-3\right)c_{QQ}^8\right\}$$

$$\Delta T \simeq -\frac{m_t^2}{256\pi^3 c_w^2 s_w^2 m_Z^2}\frac{m_t^2}{\Lambda^2}\left(42c_{QQ}^1+8c_{QQ}^8-72c_{Qt}^1+96c_{tt}^1\right)\ln^2\left(\frac{m_t^2}{m_Z^2}\right)$$

Dominant effects in Z decay observables are proportional to m_t^2 & have a single logarithm. Peskin-Takeuchi parameters such as T instead can develop double logarithms proportional to m_t^4. Can be understood in terms of operator mixing

[UH & Schnell, 2406.xxxx]

Anatomy of SMEFT corrections to EWPOs

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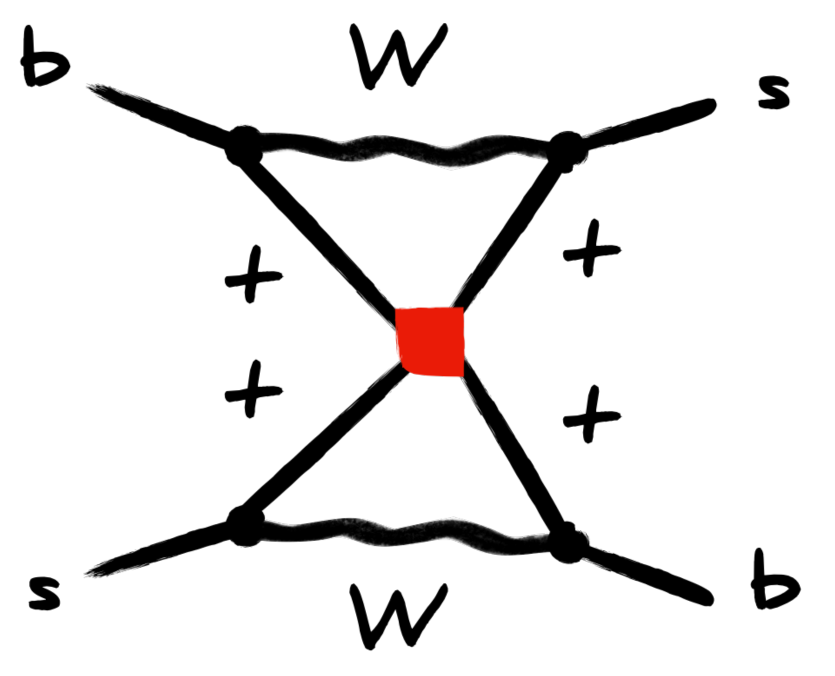
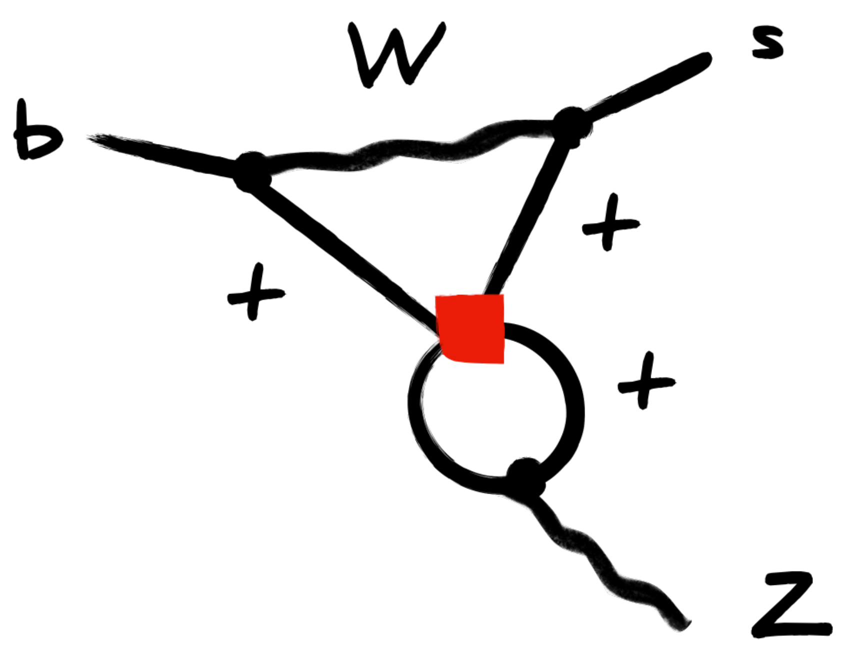
Observables depend differently on various SMEFT operators. In particular, Z-pole observables not sensitive to purely right-handed top-quark coefficient c_{tt}^1

[UH & Schnell, 2406.xxxx]

Anatomy of SMEFT corrections to FCNCs



$$\Delta F \simeq -\frac{\alpha}{192\pi s_w^2}\frac{m_t^4}{m_W^4}\frac{v^2}{\Lambda^2}\left[-10.7c_{QQ}^1 + 2.4c_{QQ}^8 - 4.1c_{Qt}^1 + 3.4c_{Qt}^8 - 3c_{tt}^1\right]$$

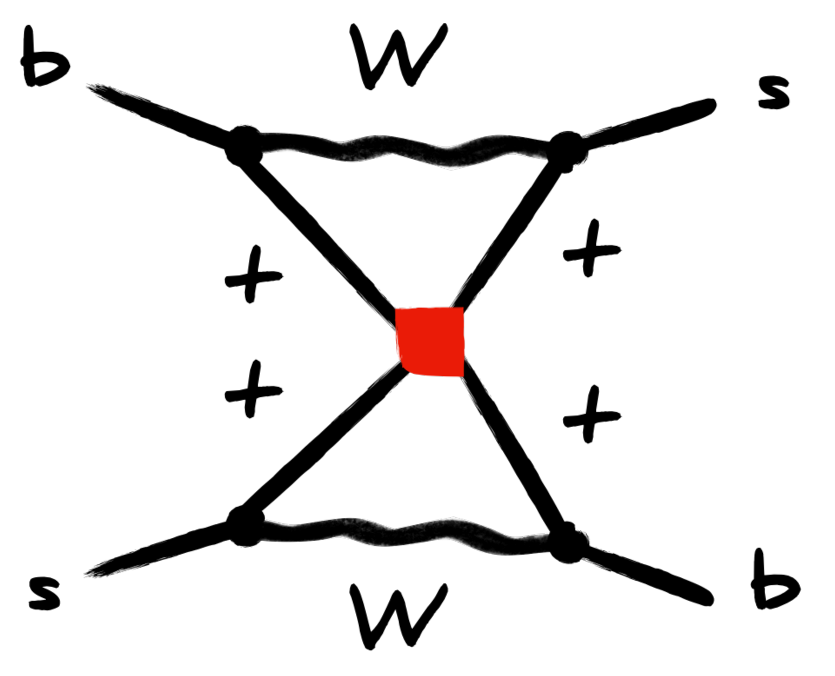


$$\Delta C \simeq -\frac{\alpha}{1536\pi s_w^2}\frac{m_t^4}{m_W^4}\frac{v^2}{\Lambda^2}\left[-8.0\left(c_{QQ}^1 + \frac{4}{3}c_{QQ}^8\right) - 8.8\left(c_{Qt}^1 + \frac{4}{3}c_{Qt}^8\right) - 12c_{tt}^1\right]$$

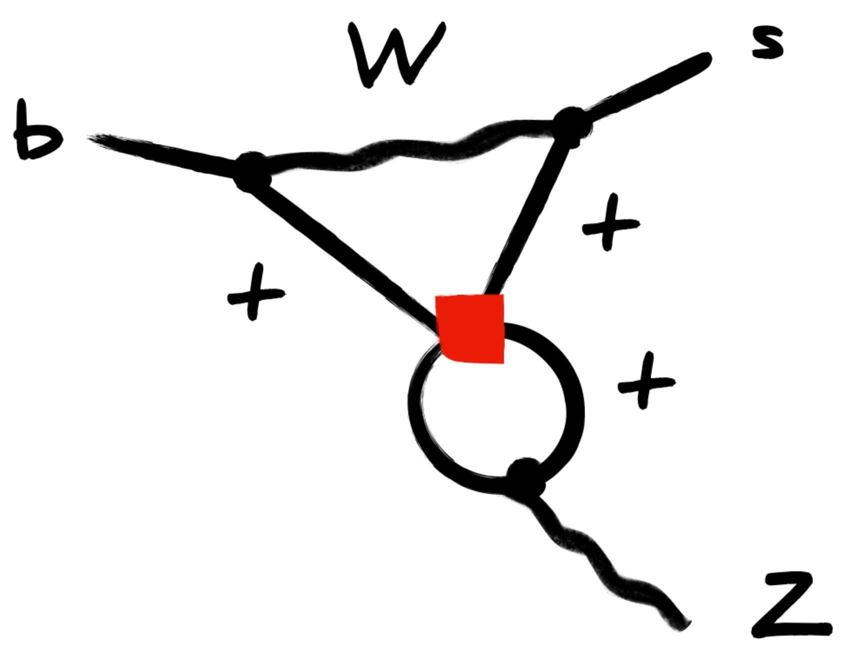
B_s-mixing & Z-penguin amplitudes scale as m_t^4. $b \to s l^+ l^-$ box & photon penguin only scale as m_t^2, but need to be included to obtain gauge-independent C_9 & C_{10}

[UH & Schnell, 2406.xxxx]

Anatomy of SMEFT corrections to FCNCs



$$\Delta F \simeq -\frac{\alpha}{192\pi s_w^2}\frac{m_t^4}{m_W^4}\frac{v^2}{\Lambda^2}\left[-10.7c_{QQ}^1 + 2.4c_{QQ}^8 - 4.1c_{Qt}^1 + 3.4c_{Qt}^8 - 3c_{tt}^1\right]$$



$$\Delta C \simeq -\frac{\alpha}{1536\pi s_w^2}\frac{m_t^4}{m_W^4}\frac{v^2}{\Lambda^2}\left[-8.0\left(c_{QQ}^1 + \frac{4}{3}c_{QQ}^8\right) - 8.8\left(c_{Qt}^1 + \frac{4}{3}c_{Qt}^8\right) - 12c_{tt}^1\right]$$

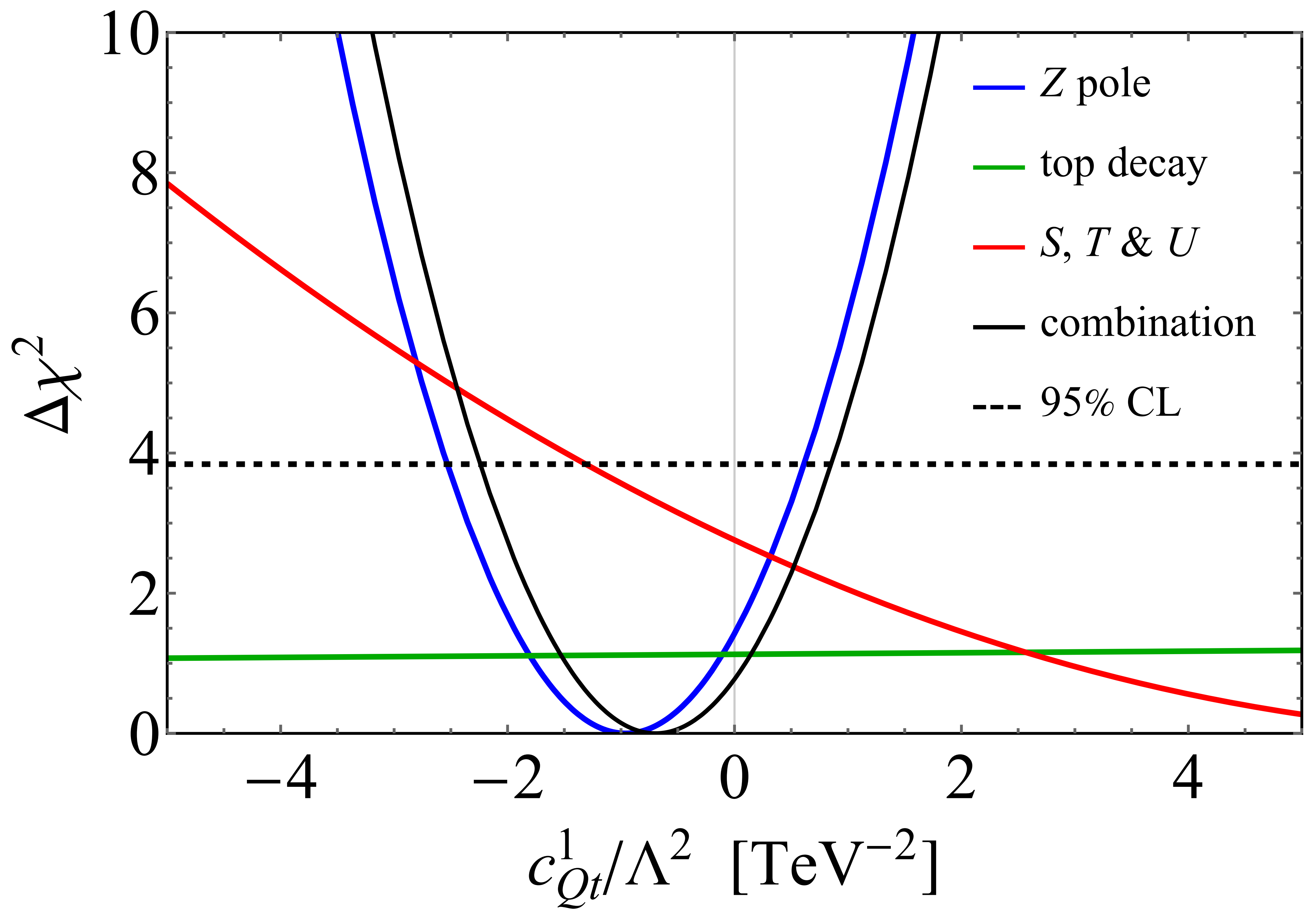
For calculated 1- & 2-loop observables, higher-order terms in heavy top-quark mass expansion important & therefore should be included in numerical analysis

[UH & Schnell, 2406.xxxx]

Some preliminary fit results

Class	DoF	95% CL bounds, $\mathcal{O}(\Lambda^{-2})$ Individual	95% CL bounds, $\mathcal{O}(\Lambda^{-2})$ Marginalised	95% CL bounds, $\mathcal{O}(\Lambda^{-4})$, Individual	95% CL bounds, $\mathcal{O}(\Lambda^{-4})$, Marginalised
	cQt1	[-195,159]	[-190,189]	[-1.830,1.862]	[-1.391,1.251]

[Ethier et al., 2105.00006]



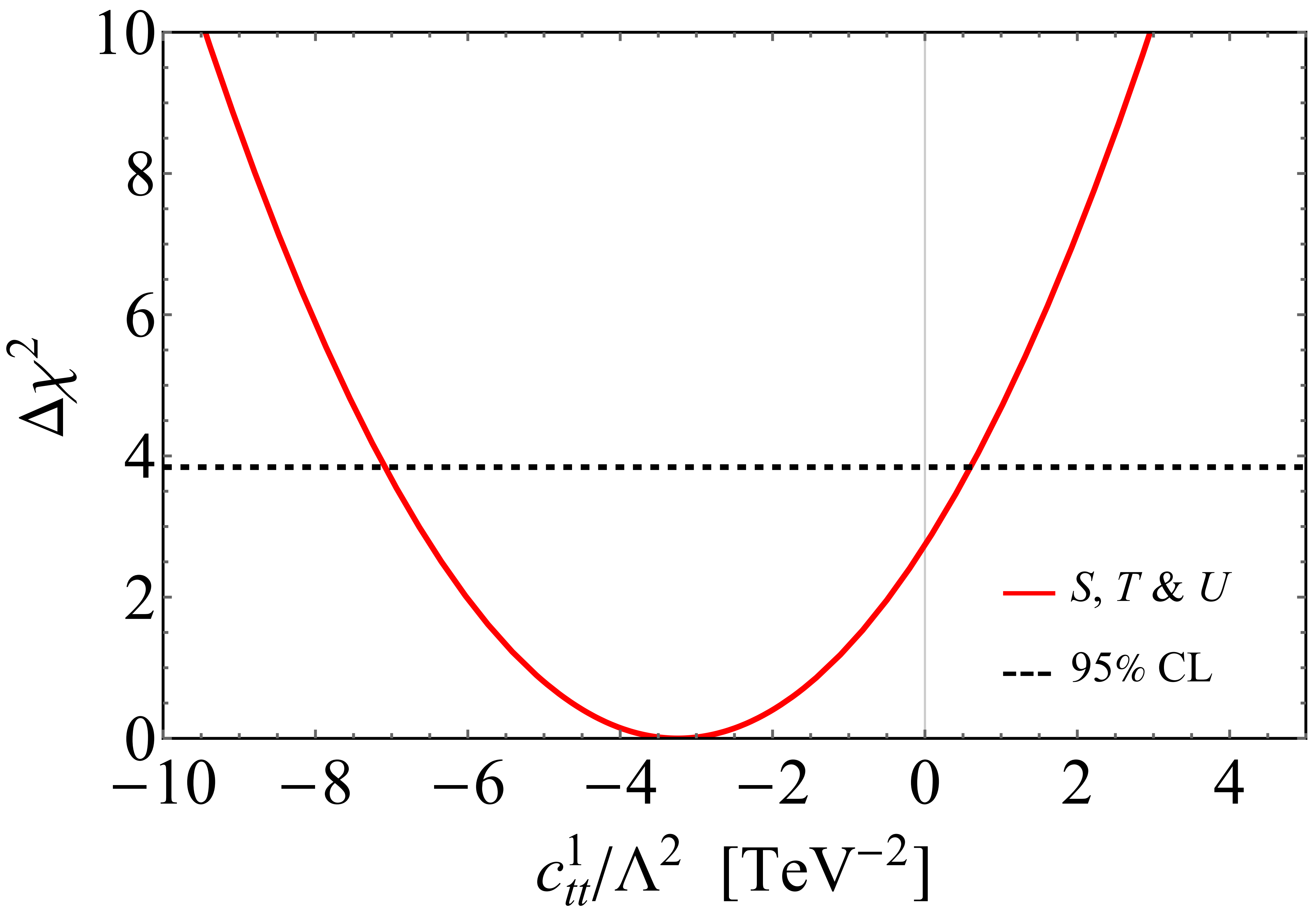
$$\frac{c^1_{Qt}}{\Lambda^2} \in \frac{[-2.2, 0.9]}{\mathrm{TeV}^2}$$

[UH & Schnell, 2406.xxxx]

Some preliminary fit results

Class	DoF	95% CL bounds, $\mathcal{O}(\Lambda^{-2})$ Individual	95% CL bounds, $\mathcal{O}(\Lambda^{-2})$ Marginalised	95% CL bounds, $\mathcal{O}(\Lambda^{-4})$, Individual	95% CL bounds, $\mathcal{O}(\Lambda^{-4})$, Marginalised
	ctt1	[-2.782,12.114]	[-115,153]	[-1.151,1.025]	[-0.791,0.714]

[Ethier et al., 2105.00006]

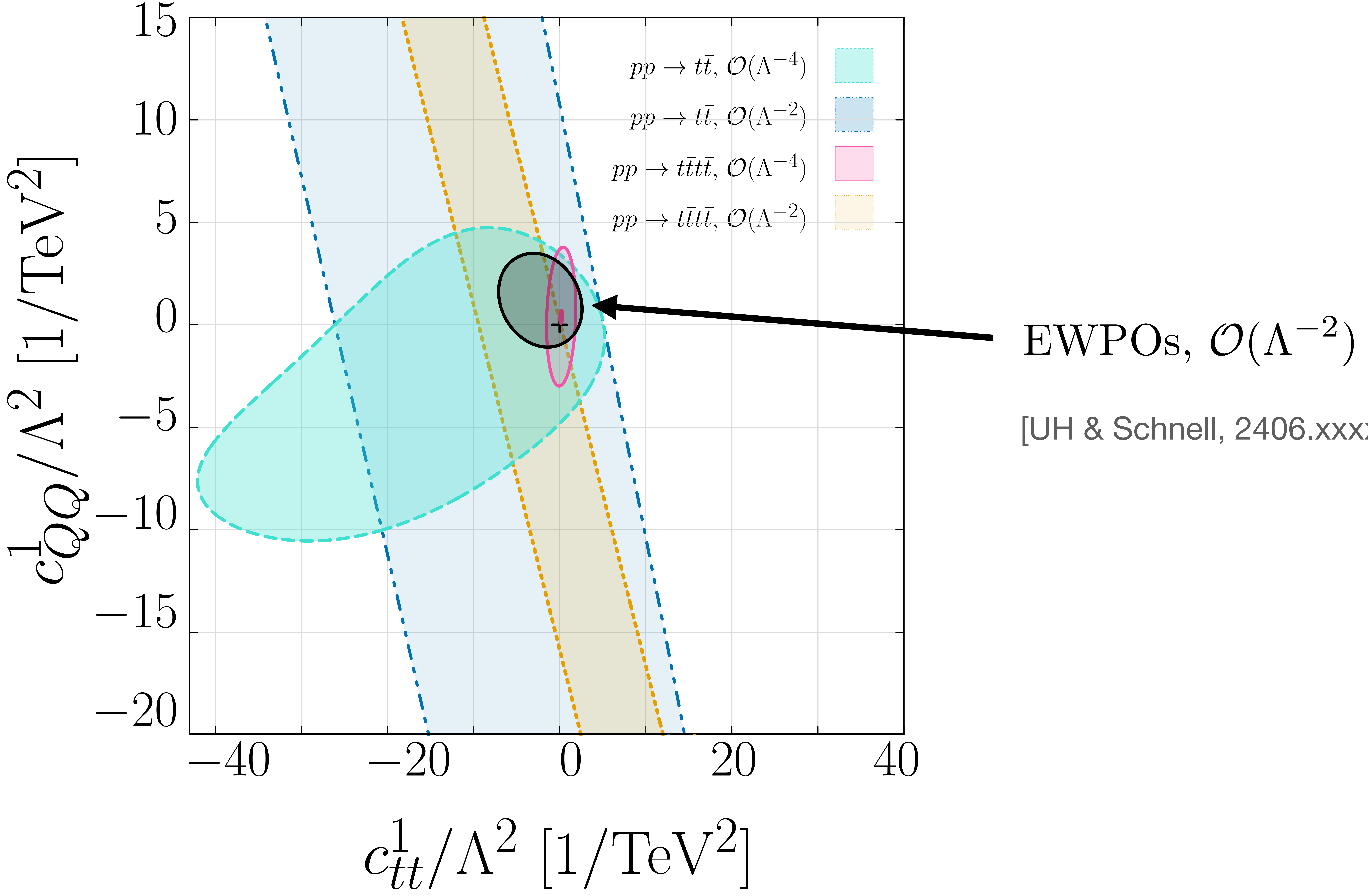


$$\frac{c^1_{tt}}{\Lambda^2} \in \frac{[-7.1, 0.6]}{\text{TeV}^2}$$

[UH & Schnell, 2406.xxxx]

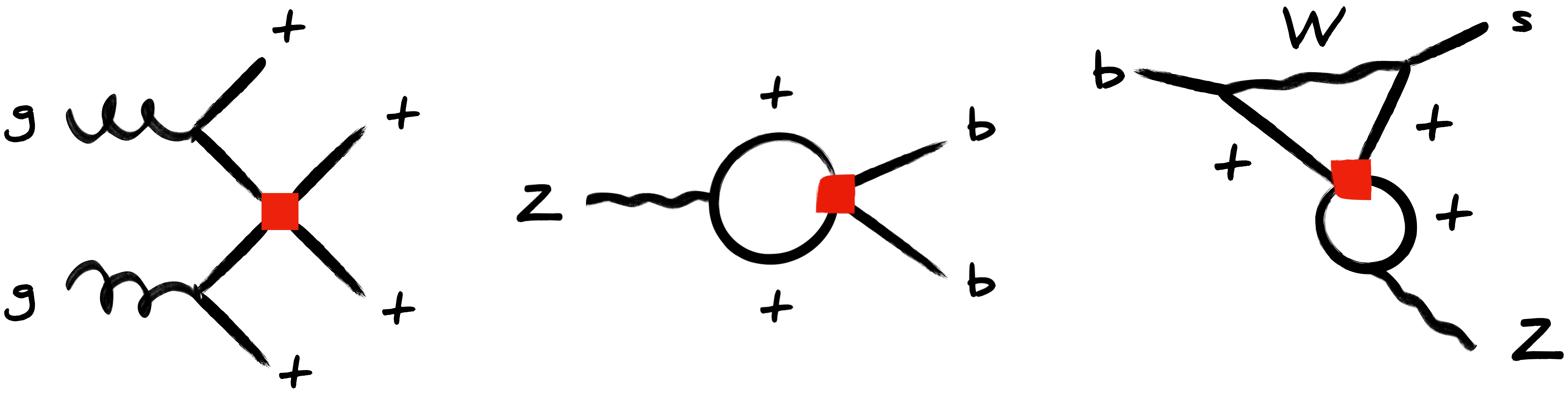
Some preliminary fit results

[Degrande et al., 2402.06528]



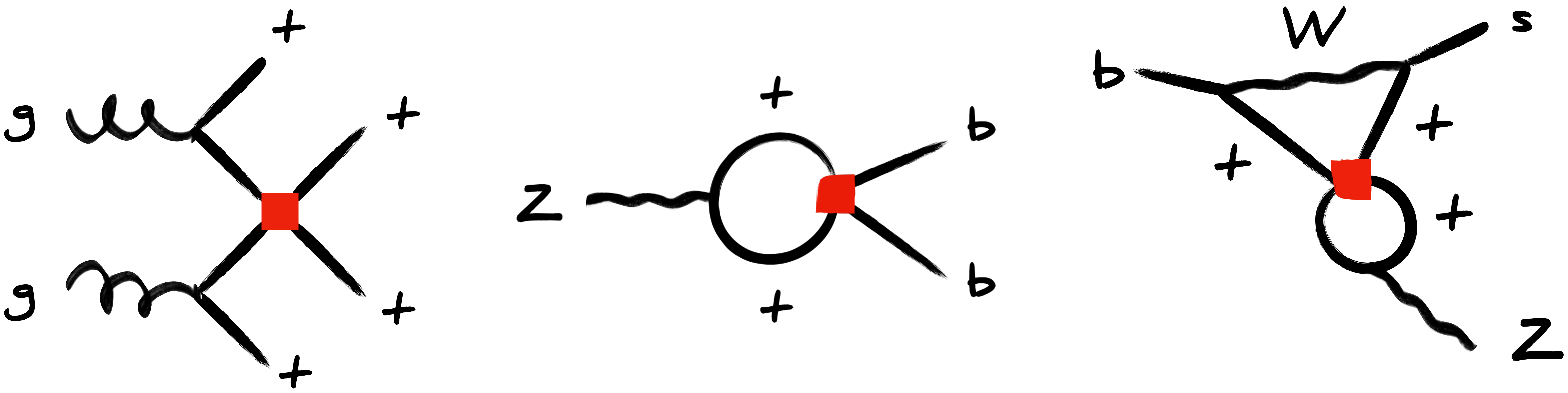
[UH & Schnell, 2406.xxxx]

Summary & outlook



To obtain best possible & most robust constraints on Wilson coefficients of 3rd generation four-quark operators crucial to combine information from 4t, 4b, 2b2t & 2t production @ LHC, top decays, EWPOs & flavour physics

Summary & outlook



Presented calculation of 1- & 2-loop SMEFT corrections displays some interesting QFT features such as evanescent operators, anomalous terms, etc. In case you are interested ask or look at backup slides

Backup

Top WG operator basis

$$\mathcal{O}^1_{QQ} = \frac{1}{2}\,(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)\,, \qquad \mathcal{O}^8_{QQ} = \frac{1}{2}\,(\bar{q}\gamma_\mu T^a q)(\bar{q}\gamma^\mu T^a q)\,,$$

$$\mathcal{O}^1_{Qt} = (\bar{q}\gamma_\mu q)(\bar{t}\gamma^\mu t)\,, \qquad \mathcal{O}^8_{Qt} = (\bar{q}\gamma_\mu T^a q)(\bar{t}\gamma^\mu T^a t)\,,$$

$$\mathcal{O}^1_{Qb} = (\bar{q}\gamma_\mu q)(\bar{b}\gamma^\mu b)\,, \qquad \mathcal{O}^8_{Qb} = (\bar{q}\gamma_\mu T^a q)(\bar{b}\gamma^\mu T^a b)\,,$$

$$\mathcal{O}^1_{tt} = (\bar{t}\gamma_\mu t)(\bar{t}\gamma^\mu t)\,, \qquad \mathcal{O}^1_{tb} = (\bar{t}\gamma_\mu t)(\bar{b}\gamma^\mu b)\,, \qquad \mathcal{O}^8_{tb} = (\bar{t}\gamma_\mu T^a t)(\bar{b}\gamma^\mu T^a b)\,,$$

$$\mathcal{O}^1_{QtQb} = (\bar{q}t)\,\varepsilon\,(\bar{q}b)\,, \qquad \mathcal{O}^8_{QtQb} = (\bar{q}T^a t)\,\varepsilon\,(\bar{q}T^a b)$$

[Barducci et al., 1802.07237]

D mixing in case of down-alignment

$$q = \begin{pmatrix} \sum_{\psi=u,c,t} V^*_{\psi b} \psi_L \\ b_L \end{pmatrix}$$

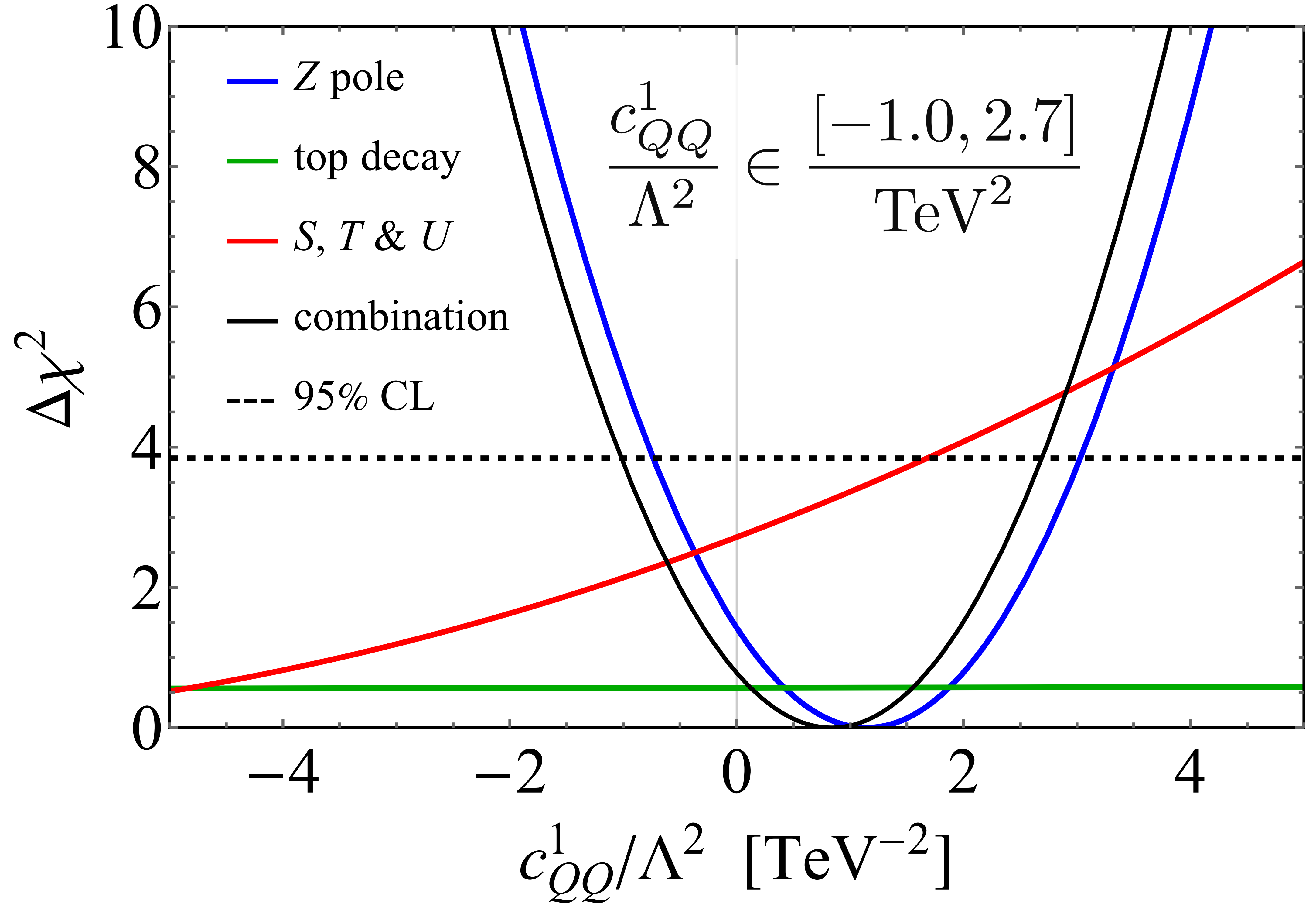
$$\mathcal{L} \supset \frac{c^1_{QQ}}{\Lambda^2} (\bar{q}\gamma_\mu q)^2 \supset \frac{c^1_{QQ}}{\Lambda^2} (V^*_{cb} V_{ub})^2 (\bar{u}_L \gamma_\mu c_L)^2 + \text{h.c.}$$

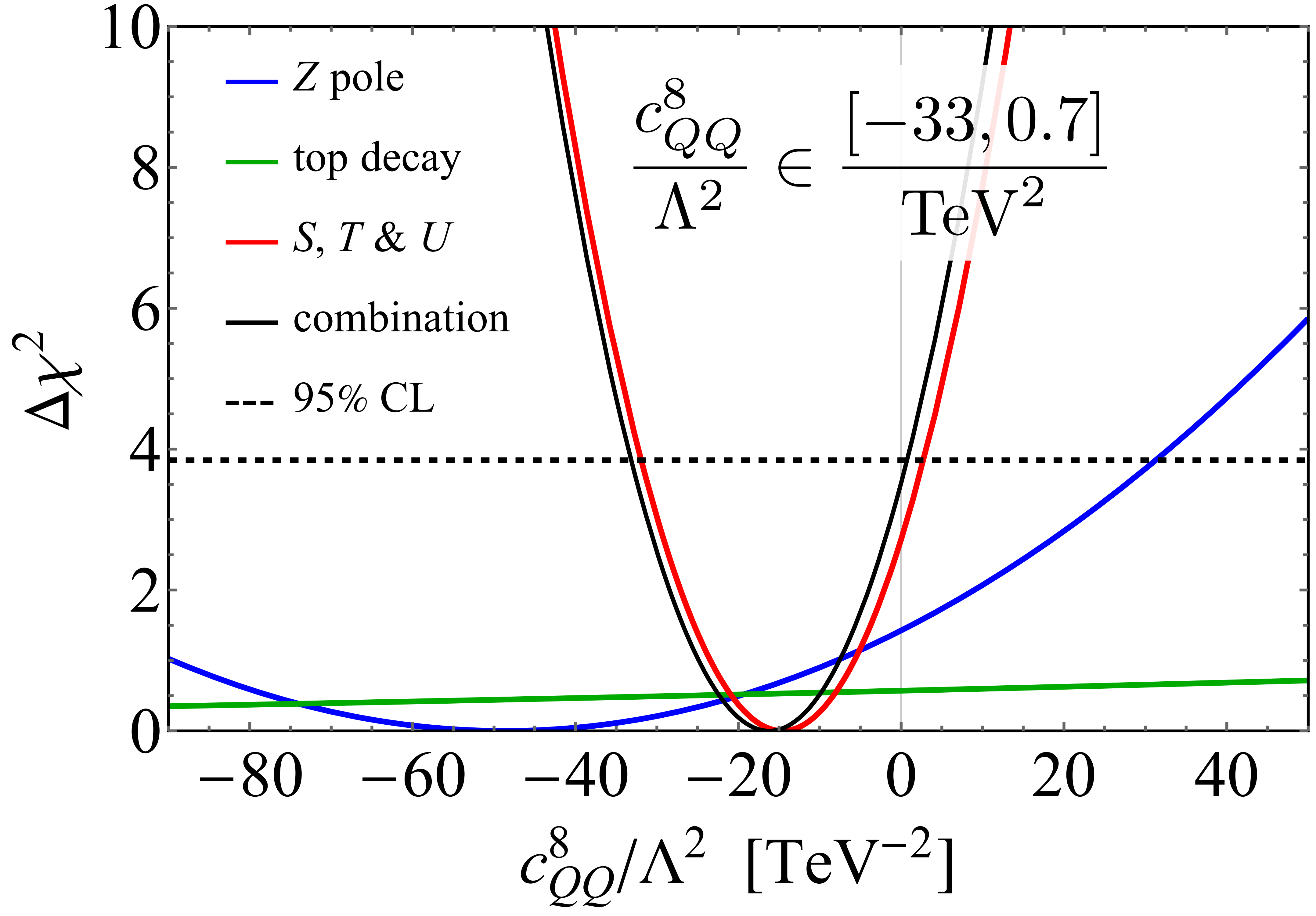
[Silvestrini, 18]

$$\frac{|c^1_{QQ}|}{\Lambda^2} \lesssim \frac{1}{(300\,\text{GeV})^2}$$

[see for instance Barducci et al., 1802.07237; Allwicher et al., 2311.00020]

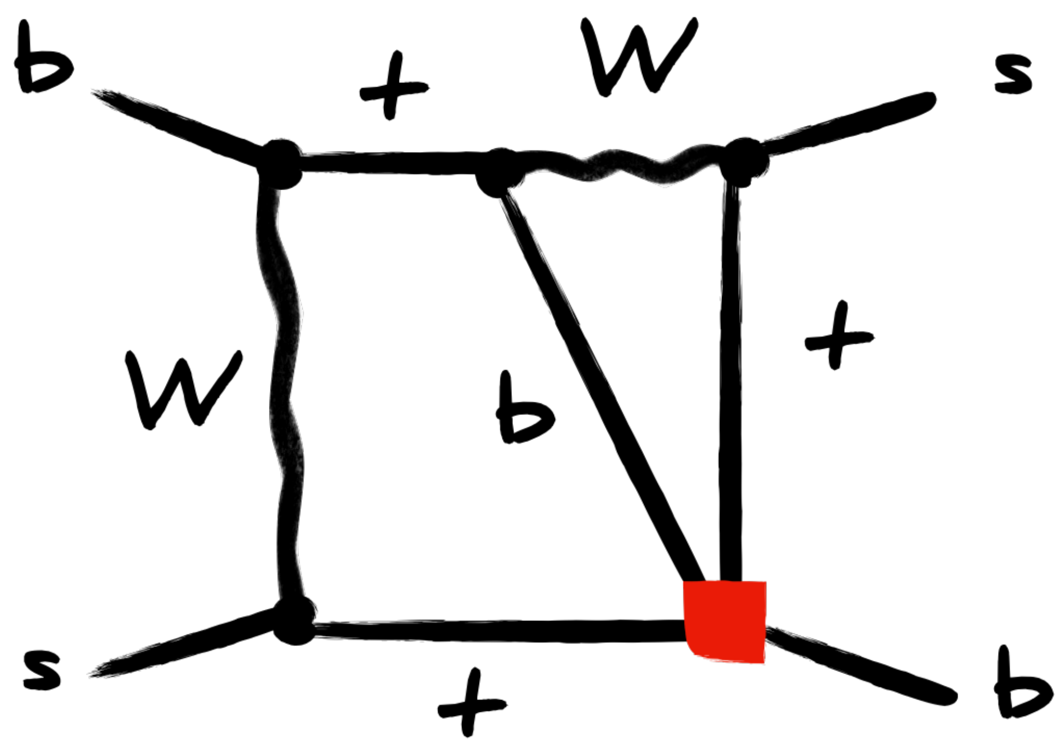
Some more preliminary fit results





[UH & Schnell, 2406.xxxx]

FCNC constraints: B_s mixing



$$\Delta F = \left(26.3\, c^1_{QQ} - 0.3\, c^8_{QQ} + 2.4\, c^1_{Qt} - 3.6\, c^8_{Qt} + 4.3\, c^1_{tt}\right) \cdot 10^{-4}$$

$$R_\Delta = \frac{\Delta M_s}{\Delta M_s^{\rm SM}} = (1 + 0.43\,\Delta F)^2$$

$$R_\Delta = 0.974 \pm 0.034$$

[HFLAV, 2206.07501;
Albrecht et al., 2402.04224]

$$\frac{c^1_{QQ}}{\Lambda^2} \in \frac{[-41, 18]}{\rm TeV^2}$$

[UH & Schnell, 2406.xxxx]

FCNC constraints: $B_s \to \mu^+\mu^-$

$$\Delta C_{10} = -\left(4.77\,c^{1}_{QQ} + 6.35\,c^{8}_{QQ} + 6.40\,c^{1}_{Qt} + 8.53\,c^{8}_{Qt} + 9.53\,c^{1}_{tt}\right)\cdot 10^{-4}$$

$$R_{\mu^+\mu^-} = \frac{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)}{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{SM}}} = \left(1 - 0.24\,\Delta C_{10}\right)^2$$

$$R_{\mu^+\mu^-} = 0.94 \pm 0.09$$

[HFLAV, 2206.07501;
Beneke et al.,1708.09152]

$$\frac{c^{1}_{tt}}{\Lambda^2} \in \frac{[-533, 242]}{\mathrm{TeV}^2}$$

[UH & Schnell, 2406.xxxx]

Fit results based on latest ATLAS 4t analysis

Operators	Expected C_i/Λ^2 [TeV $^{-2}$]	Observed C_i/Λ^2 [TeV $^{-2}$]
$\mathcal{O}^1_{QQ}$	[-2.4, 3.0]	[-3.5, 4.1]
$\mathcal{O}^1_{Qt}$	[-2.5, 2.0]	[-3.5, 3.0]
$\mathcal{O}^1_{tt}$	[-1.1, 1.3]	[-1.7, 1.9]
$\mathcal{O}^8_{Qt}$	[-4.2, 4.8]	[-6.2, 6.9]

From a quadratic fit to each single operator, ATLAS obtains limits on suppression scales in range [380, 770] GeV. Earlier fits by theorists find very similar bounds

[ATLAS, 2303.15061]

Change of operator basis

Top WG:

$$\mathcal{O}^1_{QQ} = \frac{1}{2}\,(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)\,,$$

$$\mathcal{O}^8_{QQ} = \frac{1}{2}\,(\bar{q}\gamma_\mu T^a q)(\bar{q}\gamma^\mu T^a q)\,,$$

Warsaw:

$$\mathcal{O}^1_{qq} = (\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)\,,$$

$$\mathcal{O}^3_{qq} = (\bar{q}\gamma_\mu \tau^i q)(\bar{q}\gamma^\mu \tau^i q)\,,$$

$$(\gamma_\mu P_L)_{ij} \otimes (\gamma^\mu P_L)_{kl} = -(\gamma_\mu P_L)_{il} \otimes (\gamma^\mu P_L)_{kj}$$ Fierz identity

$d = 4$

$$Q^8_{QQ} = \frac{1}{24} Q^1_{qq} + \frac{1}{8} Q^3_{qq}$$

[Barducci et al., 1802.07237; Grzadkowski et al., 1008.4884]

Change of operator basis

Top WG:

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$$(\gamma_\mu P_L)_{ij} \otimes (\gamma^\mu P_L)_{kl} = -(\gamma_\mu P_L)_{il} \otimes (\gamma^\mu P_L)_{kj}$$

Fierz identity

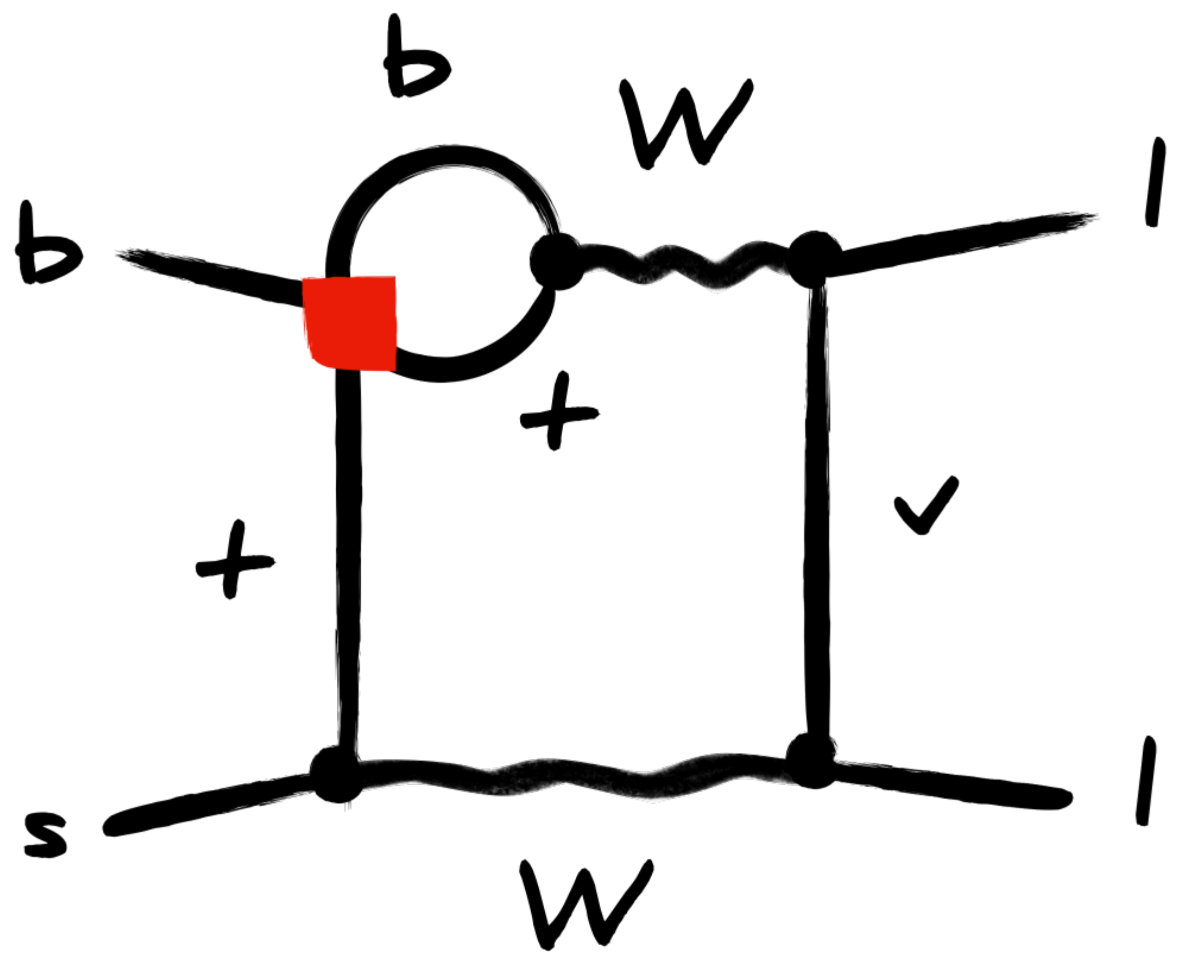
$d = 4 - 2\epsilon$

evanescent operator

$$Q^8_{QQ} = \frac{1}{24} Q^1_{qq} + \frac{1}{8} Q^3_{qq} + E_{qq}$$

[Barducci et al., 1802.07237; Grzadkowski et al., 1008.4884]

Top WG operator basis: b→sl+l- box



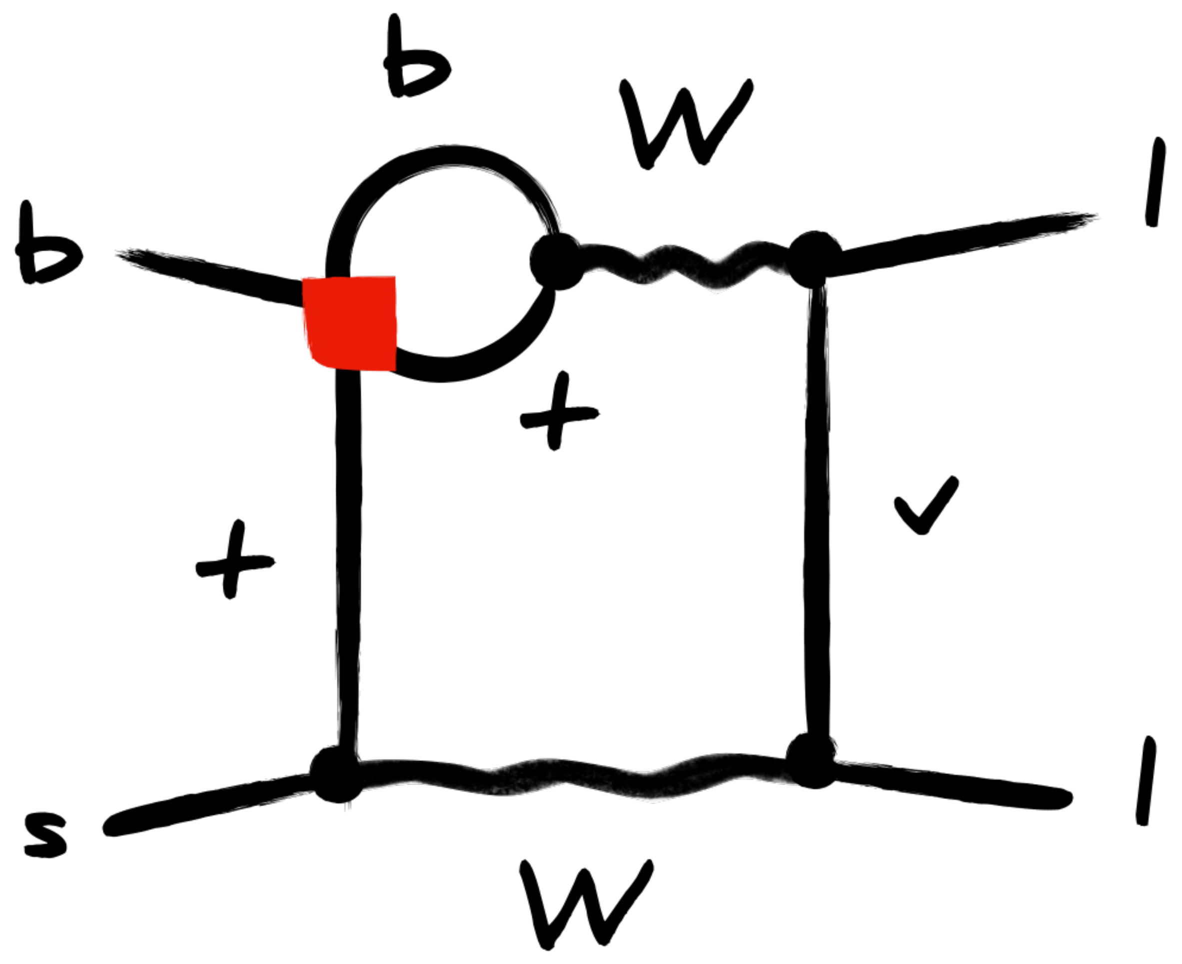
$$\Delta B = -\frac{\alpha}{384\pi s_w^2}\frac{v^2}{\Lambda^2}\sum_{i={}^1_{QQ},{}^8_{QQ},{}^1_{Qt},{}^8_{Qt}} \xi_i c_i$$

$$\xi^1_{QQ} = -12x^2_{t/W} - 12x^2_{t/W}\ln x_{t/W} + 3x^2_{t/W}\left(2x_{t/W}-1\right)\ln^2 x_{t/W}$$
$$+ 6x^2_{t/W}\left(2x_{t/W}-1\right)\mathrm{Li}_2\left(1-x_{t/W}\right) + \pi^2 x^2_{t/W}\left(2x_{t/W}-1\right) + 6f(x_{t/W})\ln x_{\mu/t}\,,$$

$$\xi^8_{QQ} = \frac{4}{3}\xi^1_{QQ} \qquad f(x) = \frac{x^2}{x-1} - \frac{x^2}{(x-1)^2}\ln x \qquad (T^aT^a)_{ij} = C_F\delta_{ij} = \frac{4}{3}\delta_{ij}$$

[UH & Schnell, 2406.xxxx]

Warsaw operator basis: b→sl+l- box



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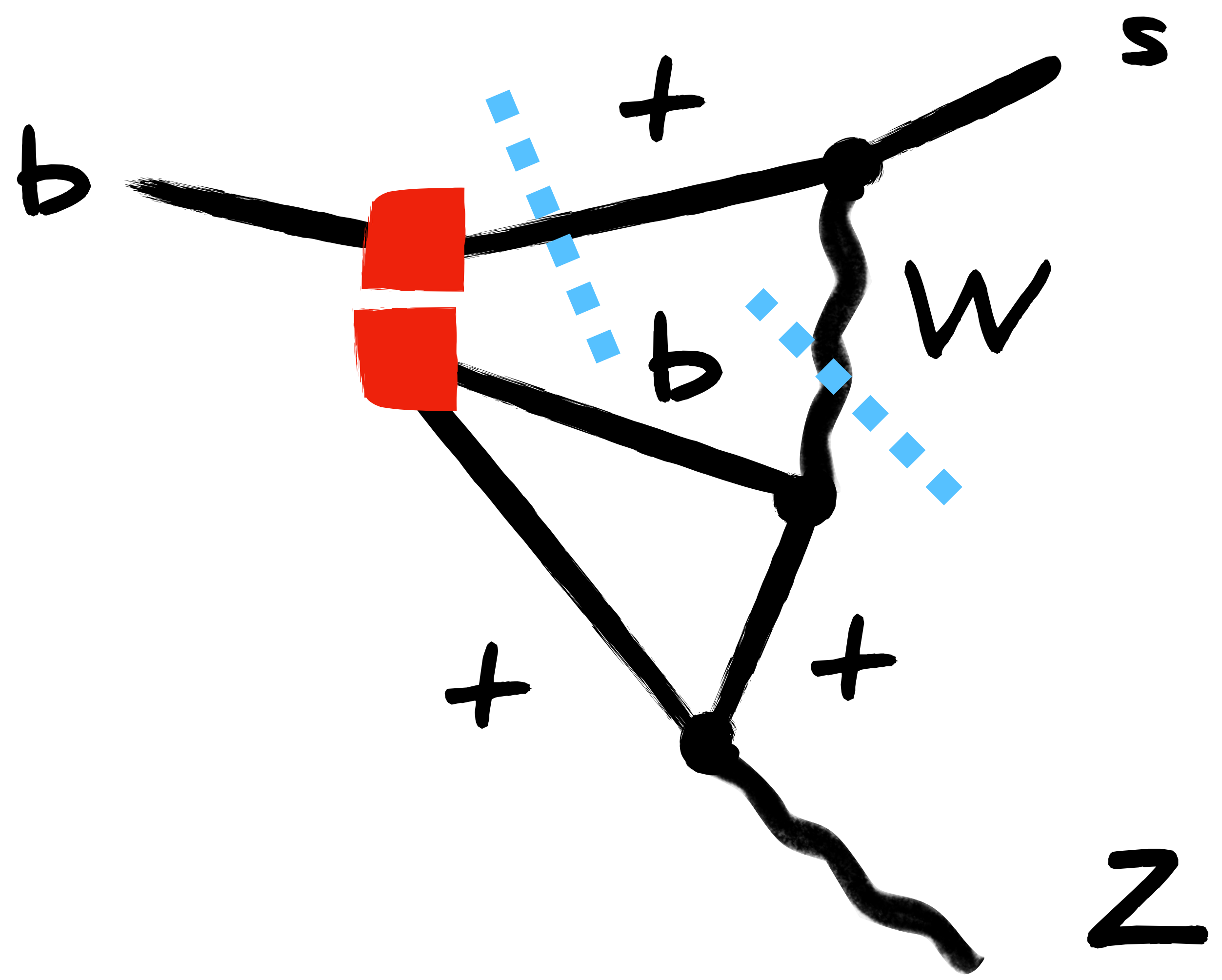
$$+\,6x^2_{t/W}\left(2x_{t/W}-1\right)\mathrm{Li}_2\left(1-x_{t/W}\right) + \pi^2 x^2_{t/W}\left(2x_{t/W}-1\right) + 6f(x_{t/W})\ln x_{\mu/t}\,,$$

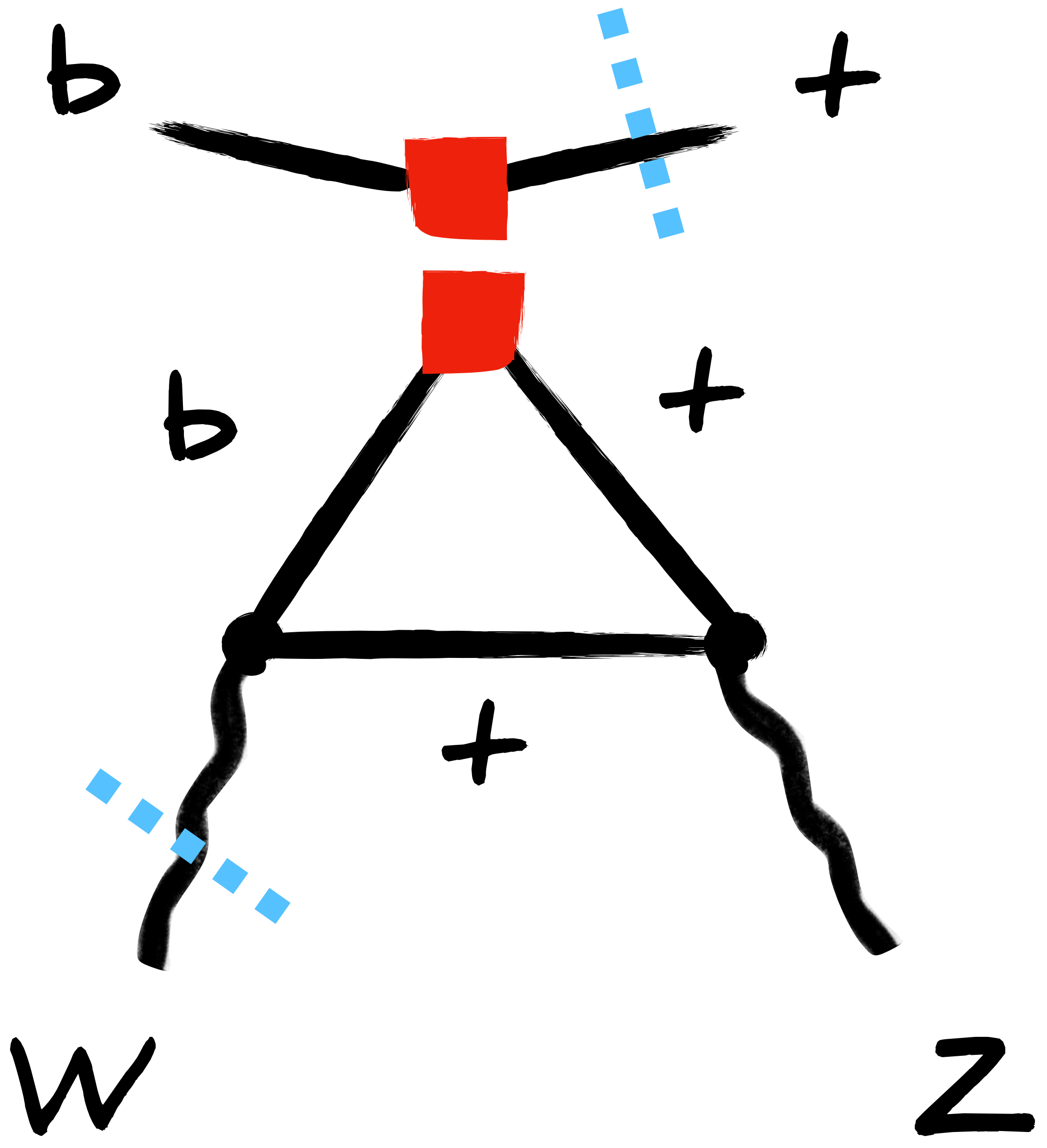
$$\xi^8_{QQ} = \frac{4}{3}\xi^1_{QQ} + 9f(x_{t/W})$$

contribution from Fierz-evanescent operator E_{qq}

[UH & Schnell, 2406.xxxx]

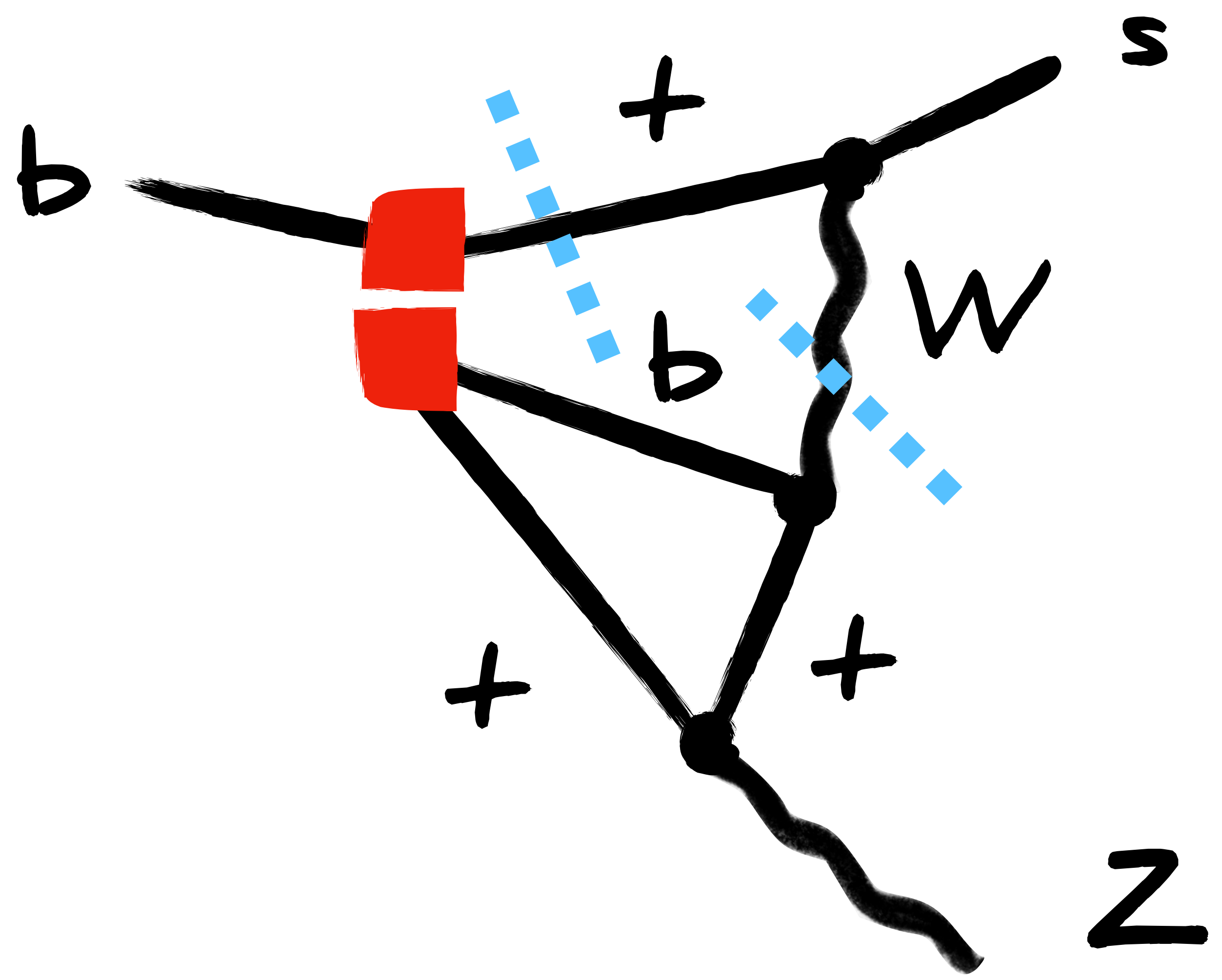
Warsaw operator basis: anomalous terms

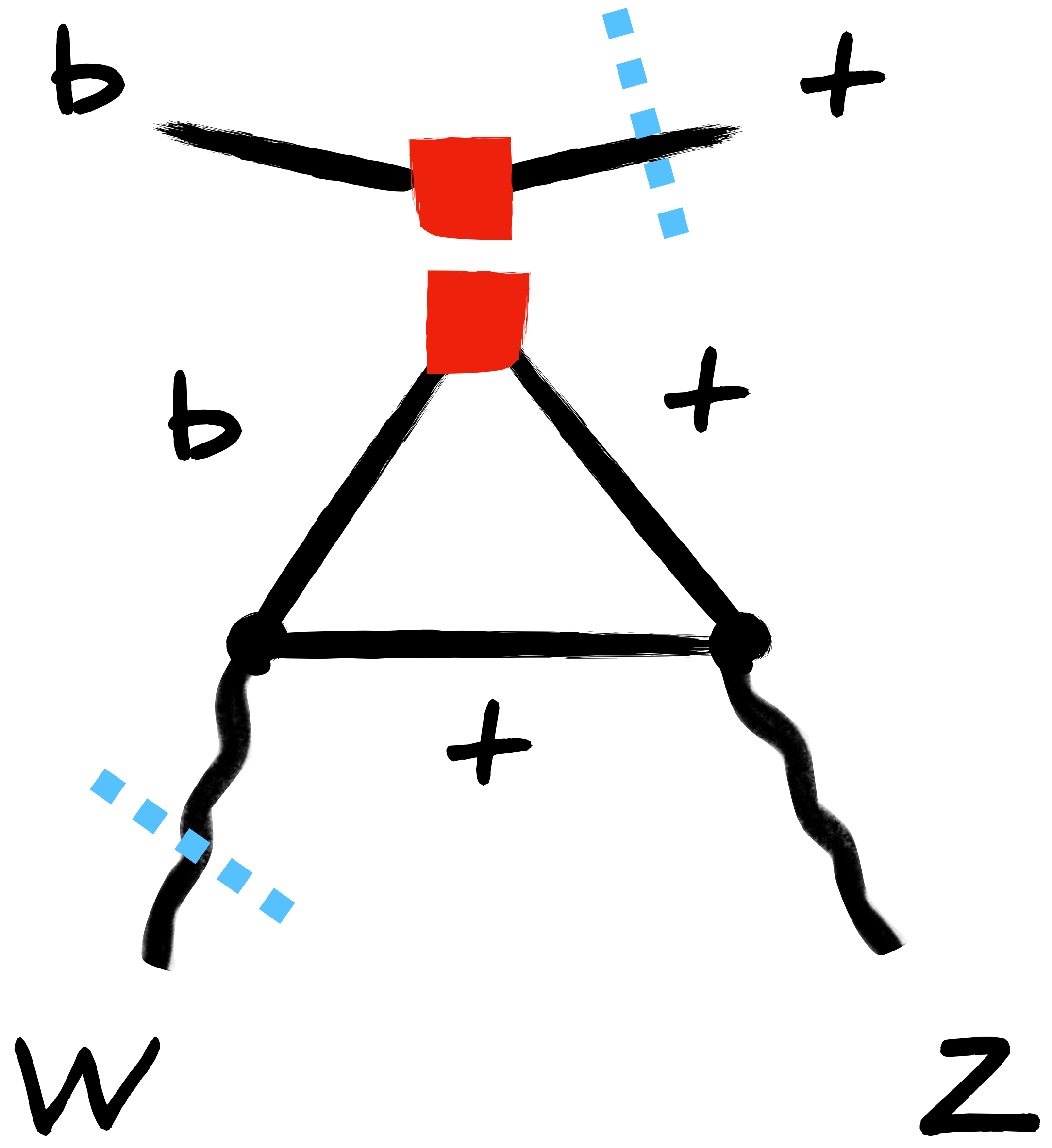




In Warsaw operator basis, insertions of operator Q^{3}_{qq} lead to non-zero traces involving γ_5. Does this mean that SMEFT has new anomalies?

Warsaw operator basis: anomalous terms

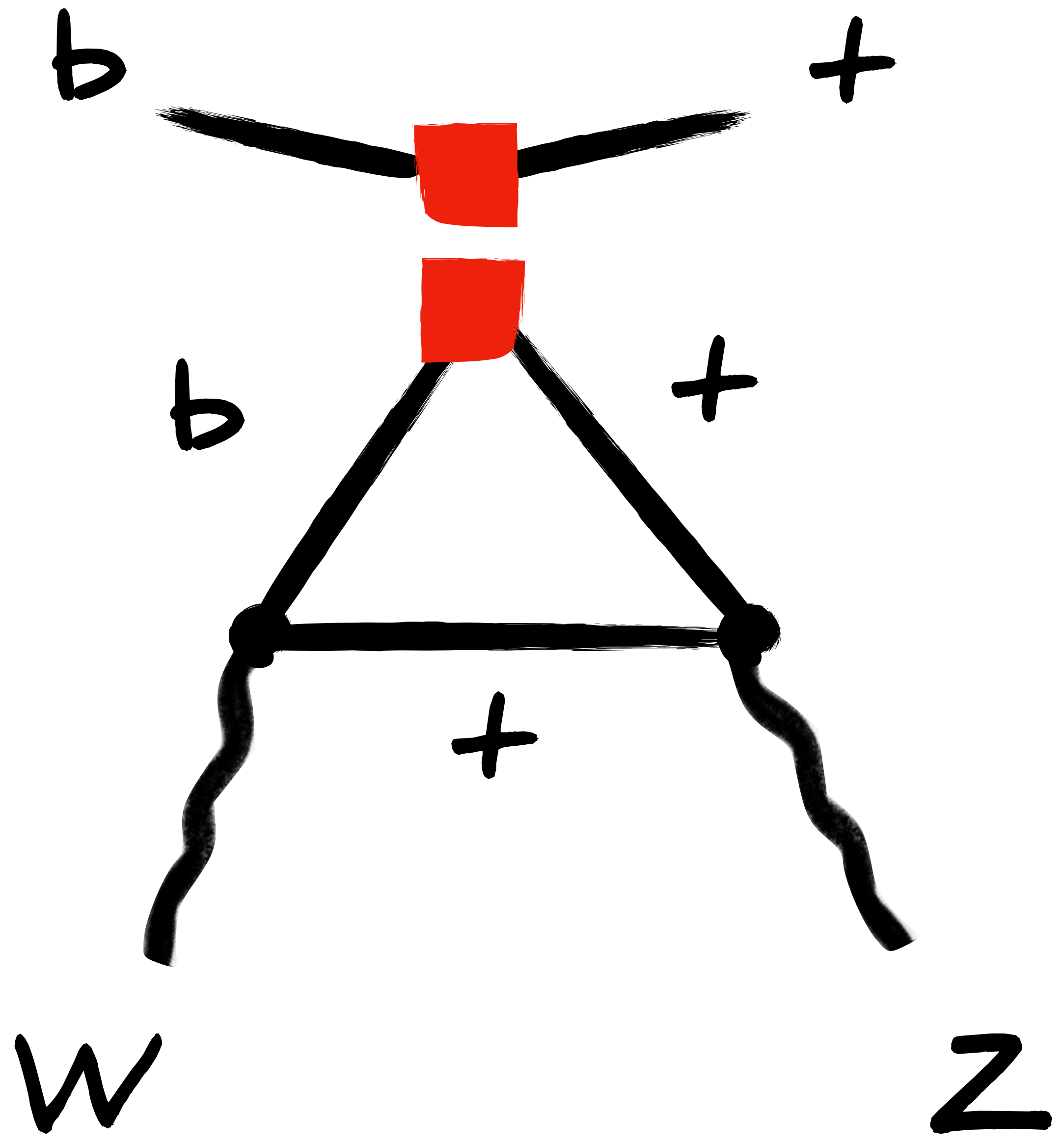


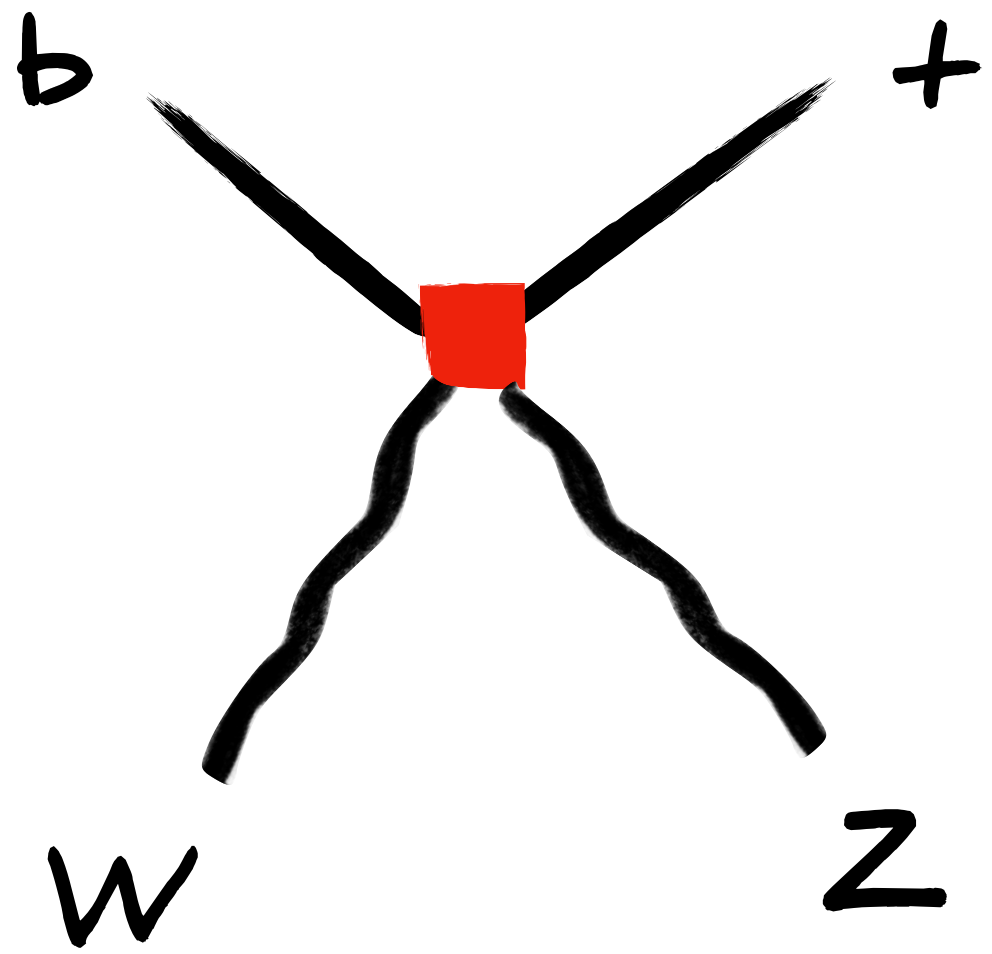


Well-known that this is not case in general — also clear in case at hand because anomalous terms absent in Top WG operator basis

[see for instance Bonnefoy et al., 2012.07740; Feruglio, 2012.13989]

Warsaw operator basis: anomalous terms

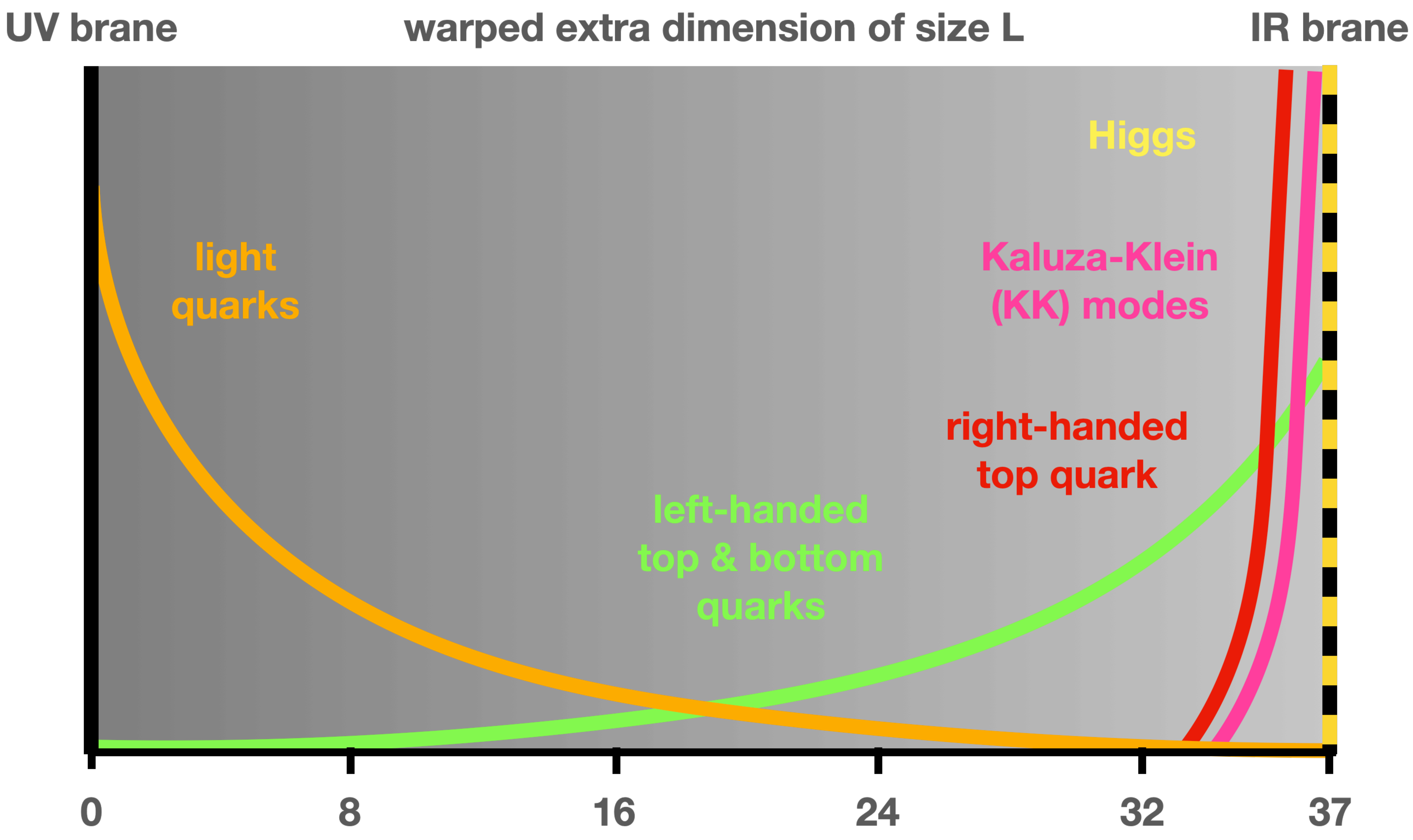




In order to proof statement need to show that anomalous 2-loop terms can be removed by local counterterms, i.e. Wess-Zumino terms. Work in progress

[UH & Schnell, 2406.xxxx]

How big can Wilson coefficients be?



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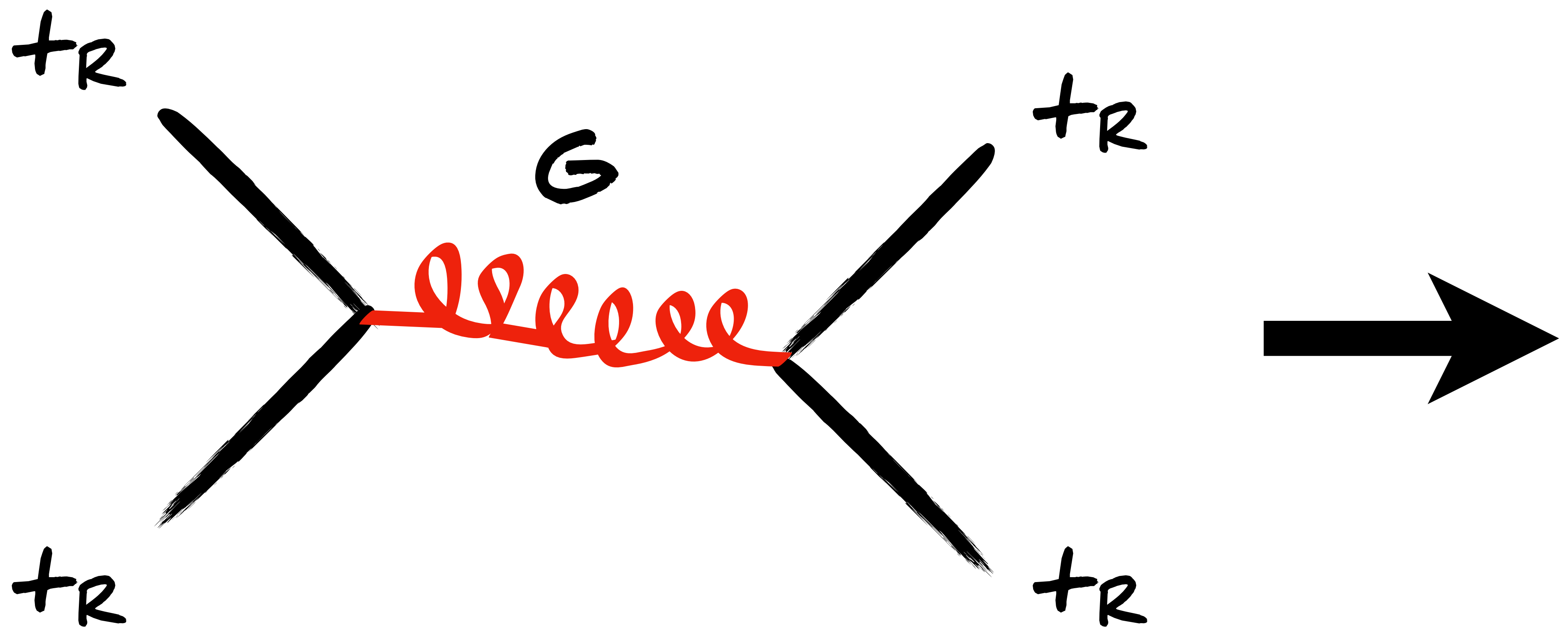
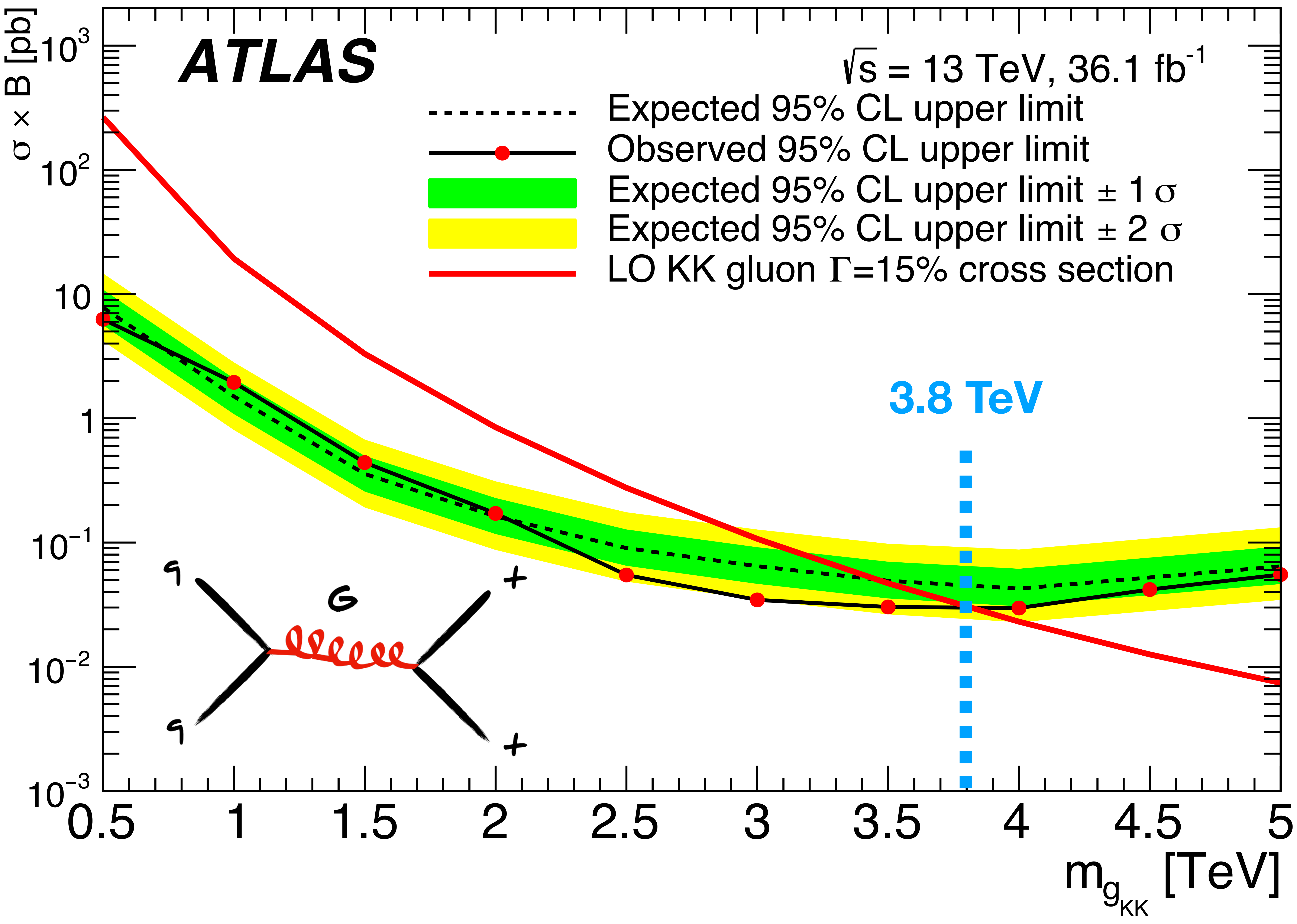


$$\frac{c^1_{tt}}{\Lambda^2} \simeq \frac{4\pi\alpha_s L}{3M^2_{\mathrm{KK}}} \simeq \frac{65}{M^2_G}$$

In flavour-anarchic Randall-Sundrum model with fully IR localised right-handed top quark, KK gluon exchange leads to sizeable tree-level contribution to Q^1_{tt}. Left-right (left-left) contributions smaller by a factor of around 5 (50)

How big can Wilson coefficients be?

[ATLAS, 1804.10823]



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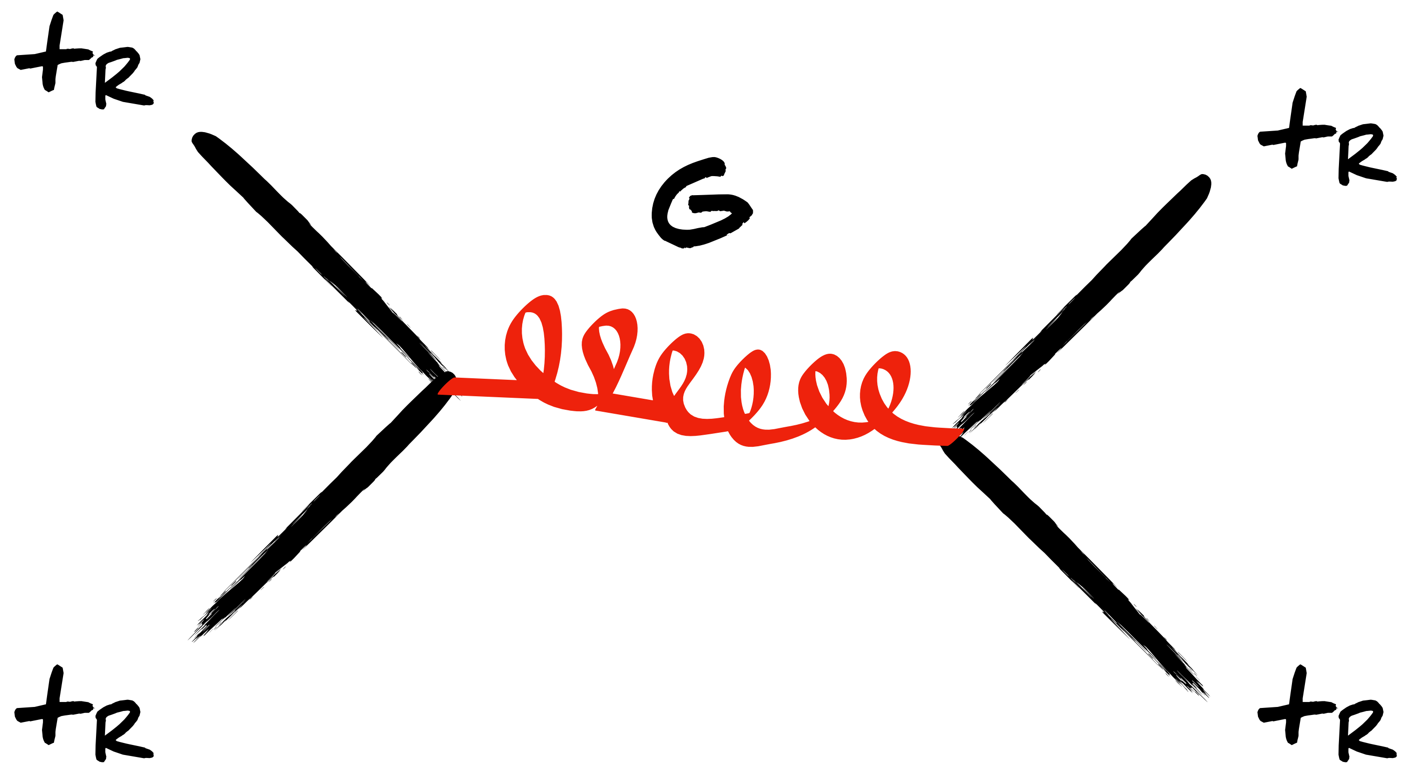

$$\frac{c^1_{tt}}{\Lambda^2} \simeq \frac{4\pi\alpha_s L}{3M^2_{\rm KK}} \simeq \frac{65}{M^2_G}$$

[ATLAS, 1804.10823]

$$\frac{|c^1_{tt}|}{\Lambda^2} \lesssim \frac{4.5}{\rm TeV^2}$$