

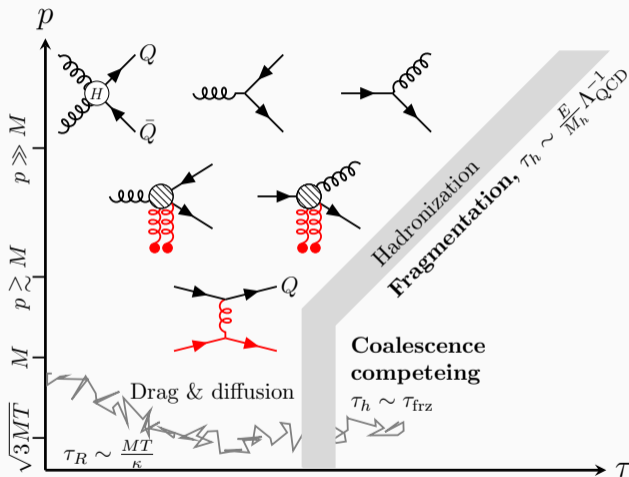
Open Heavy Flavor: Theory

The 12th International Conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions (HP2024), Nagasaki, Japan

Weiyao Ke
Central China Normal University
September 25, 2024



Physics of heavy quarks in a hot plasma



- $m_b = 4.2 \text{ GeV} \gg 3T$
 $m_c = 1.3 \text{ GeV} \gtrsim 3T$.
- Flavor-labeled energetic probes.
- Sensitive to non-eq. evolution.
- “Pull out” medium information at hadronization.

Heavy flavor hard production

The large mass allows perturbative calculation of HQ production. In practice, **not easy**.

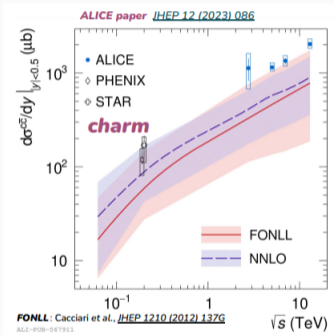
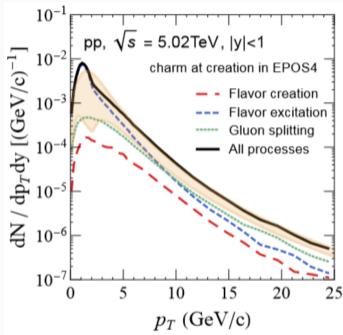
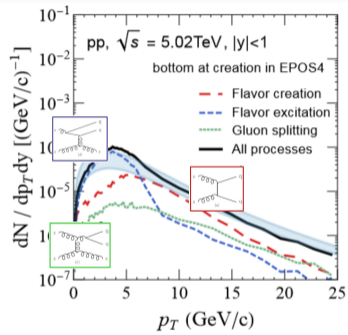


Fig. from M. Pennisi's talk [↗](#)
Better description for b .

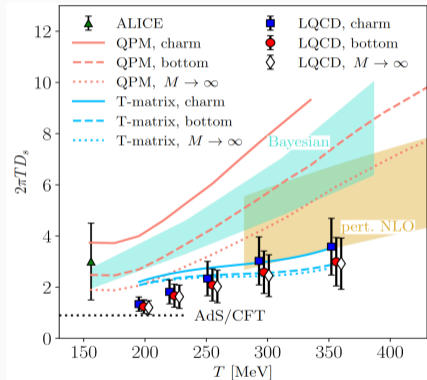


EPOS4HQ, talk by P. Gossiaux [↗](#) Reasonable agreement with FONLL. Also works for $D-\bar{D}$ correlations.



Heavy quarks in deconfined environments

Heavy quarks in equilibrium



Progresses from lattice QCD in understanding HQ dynamics near equilibrium ($\frac{1}{2}M\langle v^2 \rangle = \frac{3}{2}T + \dots$)

$$\frac{1}{3} \frac{d}{dt} \langle \delta p^2 \rangle \equiv \kappa = \kappa_E + \frac{T}{M} \kappa_B + \mathcal{O}\left(\frac{T^2}{M^2}\right)$$

$$\frac{1}{6} \frac{d}{dt} \langle \delta x^2 \rangle \equiv D_s = \frac{2T^2}{\kappa} \frac{\langle p_Q^2 \rangle}{3MT}$$

2+1 flavor calculation at $m_\pi = 320$ MeV with mass dependence to first order in $\frac{T}{M}$. **H. Shu's talk Friday** ↗

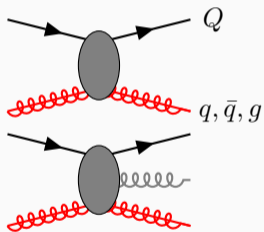
[HotQCD, PRL130(2023)231902 & PRL132(2024)051902]

Transport framework for HQ dynamics away from equilibrium.

For phenomenology, we need real-time tools for non-equilibrium evolution.

Consider $p_Q \sim M$ and try Boltzmann equation:

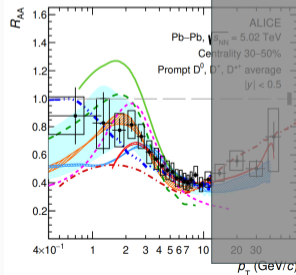
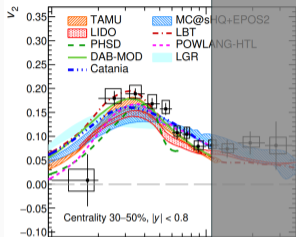
$$\underbrace{(\partial_t + v \cdot \nabla_x)}_{\text{free stream}} f_Q(t, x, p) = \underbrace{\mathcal{C}_{2 \leftrightarrow 2}[f_Q, f_{q,g}]}_{\text{elastic}} + \underbrace{\mathcal{C}_{2 \leftrightarrow 3}[f_Q, f_{q,g}]}_{\text{inelastic}} + \dots$$



A key assumption: good quasi-particles $\tau_{\text{int}} \ll \tau_{\text{life time}}$.
heavy quarks (\checkmark), light partons (??).

The Ads/CFT approach: heavy quarks motion under non-perturbative drags. **Extension to finite & high momentum, see JD Plesis's talk.** [↗](#).

Langevin dynamics of slow-moving HQ



Coarse-grain the Boltzmann equation in the small momentum-transfer limit ($q \ll p \sim M$)
 \Rightarrow Fokker-Planck equation \Leftrightarrow Stochastic Langevin dynamics

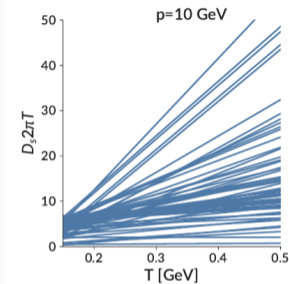
$$\vec{x}(t + \Delta t) = \vec{x}(t) + \vec{v}\Delta t$$

$$\vec{p}(t + \Delta t) = (1 - \eta_D \Delta t)\vec{p}(t) + \xi \Delta t.$$

$$\langle \xi_i \xi_j \rangle = \Delta t^{-1} (P_{T,ij} \kappa_T + P_{L,ij} \kappa_L).$$

- No longer requires medium quasi-particles.
- Flexible parametrization of the interaction, good for phenomenological extraction of κ , η_D from low- p_T data.

Langevin dynamics of slow-moving HQ



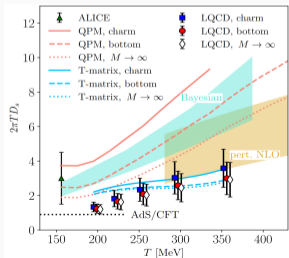
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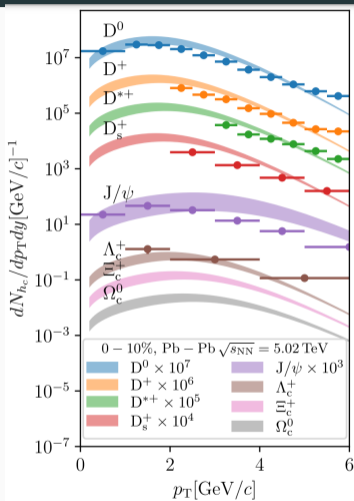
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An interesting development: from Fokker-Planck to hydrodynamics



$N^\rho(x) = \int_p p^\rho / p^0 f_Q(x, p)$. Fokker-Planck \Rightarrow second-order viscous hydrodynamics for charm current

$$\nabla_\rho N^\rho = 0, \quad N^\rho = n_Q(T, \mu) u^\rho + \nu^\rho,$$

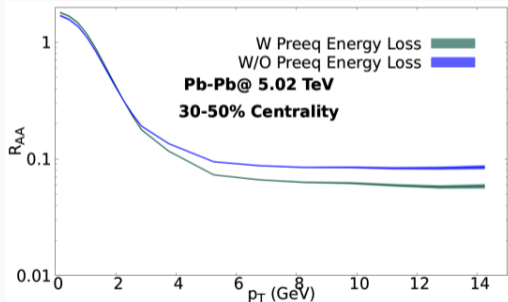
$$\frac{d\nu^\rho}{dt} = - \frac{\nu^\rho + \kappa \partial^\rho(\mu/T)}{\tau_R}$$

- Don't specify "which particle carries charmness"! Couple realistic charm initial production & evolution with statistical hadronization.
- **Assumption:** kinetic relaxation before hadronization.

[Capellino et al., PRD106(2022)034021 & PRD108(2023)116011] F. Capellino's talk Monday [↗](#)

Transport at the early stages

Early stage (first 1fm/c) is transient, but the medium has a larger energy scale. A simple estimates using Bjorken expansion ($\kappa \sim T_{\text{eff}}^3 \propto 1/\tau$) shows

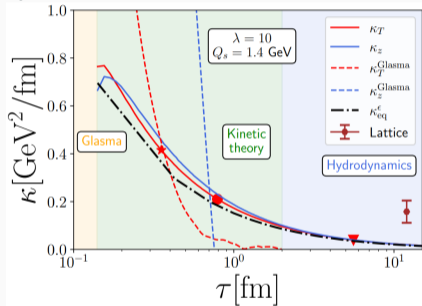


$$\langle \delta p^2 \rangle(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \kappa(\tau) d\tau \propto \ln \frac{\tau_2}{\tau_1}.$$

For example, a sensitivity study of early-stage heavy quark diffusion with an effective temperature **Mayank Singh's talk** ↗

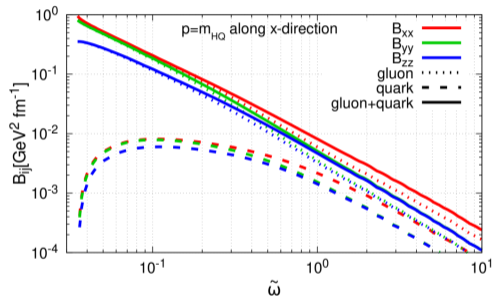
Transport at the early stages

At earlier stage, system is highly isotropic & away from kinetic and chemical equilibrium. Diffusion coefficients evaluated from QCD Effective Kinetic Theory.



QCD-EKT, pure gluon system

[K. Boguslavski et al. PRD109(2024)014025]




QCD-EKT with gluons & quarks

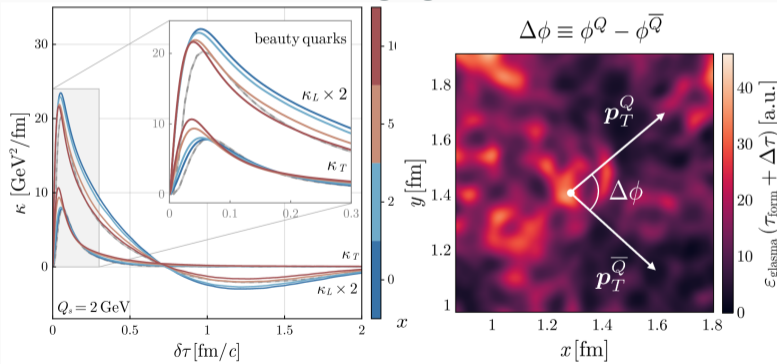
[X. Du arXiv:2306.02530]

Transport at the early stages

At even earlier stage, the system is still a colorful coherent field, i.e., glasma.
 κ has been estimated by simulating Wong's equations in a Glasma background.

[D. Avramescu et al. 2409.10564 & 2409.10565] **D. Avramescu's talk on Tuesday** 

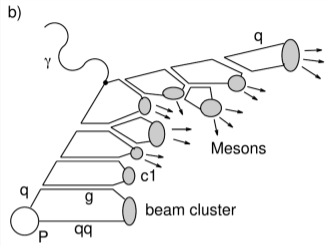
No e-loss, HQ ride on the gauge field with color rotation.



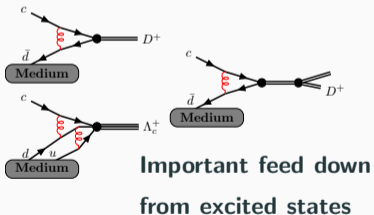
Would be interesting to see a combined calculation with Glasma+EKT+Hydrodynamics in the future on $C(\Delta\phi)$.

HF hadronization in an environment

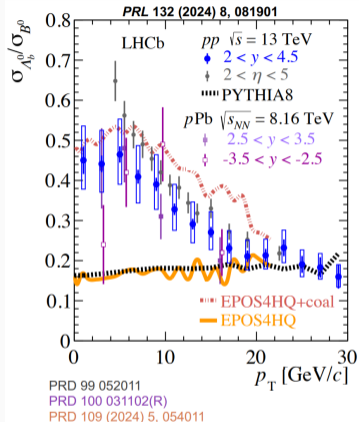
Heavy quark hadronization mechanisms



- Energetic system mainly undergoes fragmentation
 $(p_Q + p_{q \text{ med}})^2 \sim 6E_Q T_{\text{eff}} + M^2 + m^2 \gg (M_h + m_\pi)^2$.
 A large phase space for string breaking dynamics.
- $(p_Q + p_{q \text{ med}})^2 \gtrsim M_h^2$, no phase-space for fragments, recombine. HF picks up **medium chemistry**.



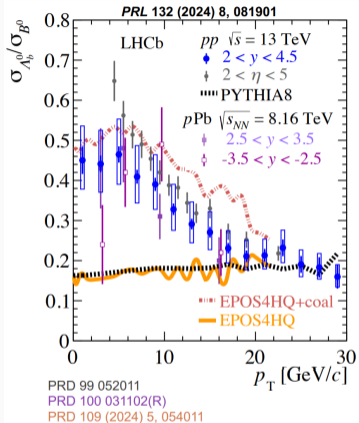
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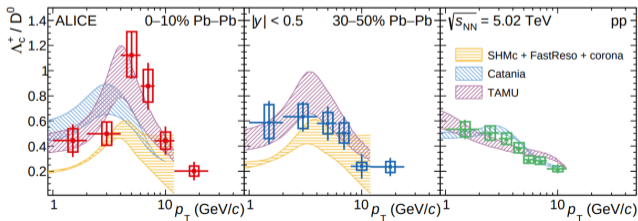
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Taken from J. Berkey's talk [↗](#)

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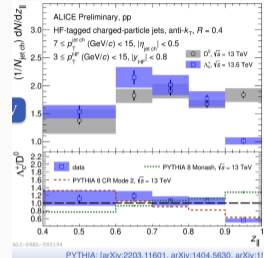
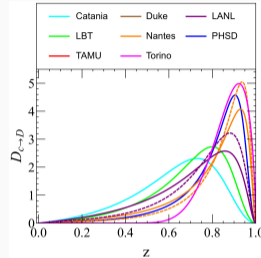
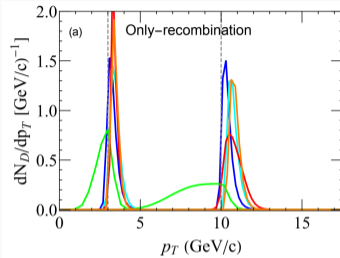


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- In AA, recombination also picks up **radial flow**.



Taken from J. Berkey's talk [↗](#)

Collaborative progresses in modelling

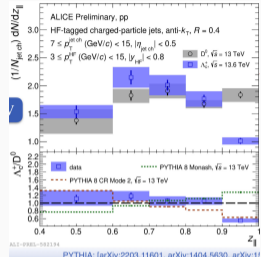
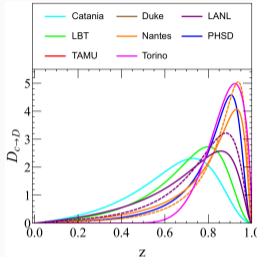
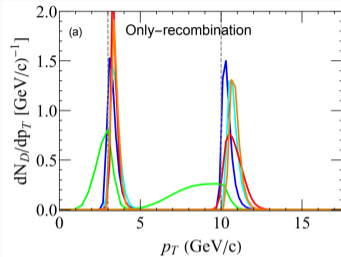


Left & middle: Jiaxing Zhao et al. 2311.10621.

Right: HF-in-jet FF, talks by K. Klein & D. Roy

- Theoretical community aims for better constraints on fragmentation function $c \rightarrow HF$ & $g \rightarrow HF$ and understanding differences in coalescence schemes.

Collaborative progresses in modelling

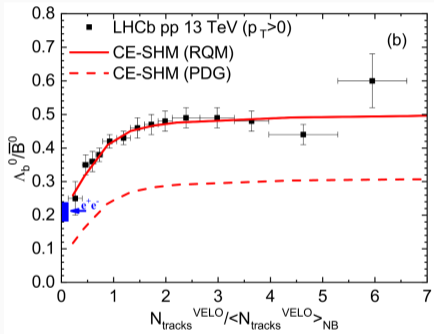


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Right: HF-in-jet FF, talks by K. Klein & D. Roy

- Theoretical community aims for better constraints on fragmentation function $c \rightarrow HF$ & $g \rightarrow HF$ and understanding differences in coalescence schemes.
- Progress from the HEFTY collaboration: combine resonance recombination model [Ravagli, Rapp PLB655(2007)126, Ravagli, van Hees, Rapp PRC79(2009)064902] with radiation-improved partonic transport [LIDO, Ke, Xu, Bass PRC100(2019)064911].

HF chemistry's correlation with underlying events in pp



[Y. Dai et al. 2402.03692] M. He's talk on Tuesday [↗](#)

Statistical Hadronization + Conservation of all U(1).

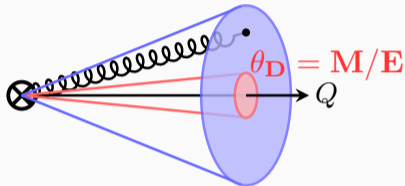
Grand canonical ensemble \Rightarrow canonical ensemble.

- Charges produced in pairs. Heavy charged pairs disfavored in neutral system \Rightarrow canonical suppression $\frac{Z(Q_{\text{tot}} - q_h, V)}{Z(Q_{\text{tot}}, V)}$, stronger for smaller system.
- Importance of unknown b -hadron states!

**Mass effects of energetic heavy
quarks (where radiation dominants)**

Mass-dependent energy loss at large p_T

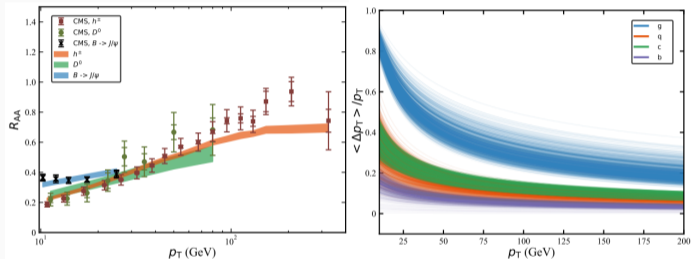
HF is a natural tool for mass & color-charge dependence of ΔE .



Mass in the quark propagator suppresses collinear emission

$$\frac{1}{\mathbf{k}^2} \rightarrow \frac{1}{\mathbf{k}^2 + x^2 M^2} = \frac{1}{\mathbf{k}^2} \frac{\theta_g^2}{\theta_g^2 + \theta_D^2}$$

Dead cone physics is more complicated in the medium.

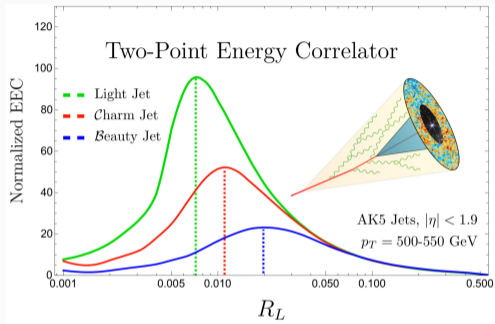


One expects $\Delta E_g > \Delta E_q \gtrsim \Delta E_c > \Delta E_b$.

Verifying the hierarchy from a data driven approach

[W. Xing et al. PLB850(2024)138523] **G.-Y. Qin, Wednesday**

Dead cone effects in jet substructure



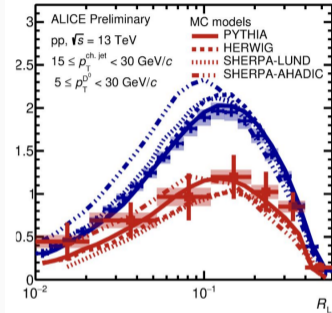
[E. Craft et al. 2210.09311]

EEC provides another way to scan through energy scales in jets

$$\Sigma_n(\theta) = \frac{\sum_{\text{jet}}}{N_{\text{jet}}} \sum_{ij} \delta(\theta - R_{ij}) \left[\frac{E_i}{E_{\text{ref}}} \right]^n \left[\frac{E_j}{E_{\text{ref}}} \right]^n$$

Dead cone angles $\theta_D = M/E$ are expected to manifest in the θ scan.

Dead cone effects in jet substructure



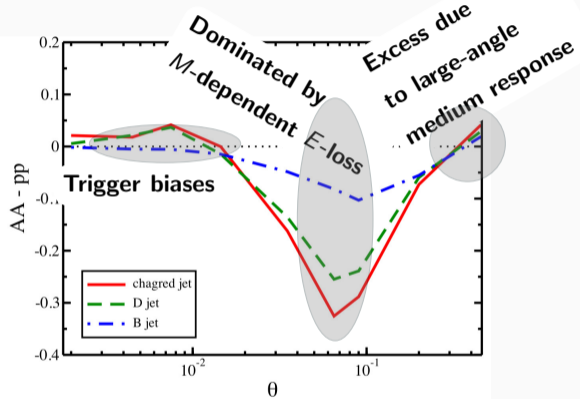
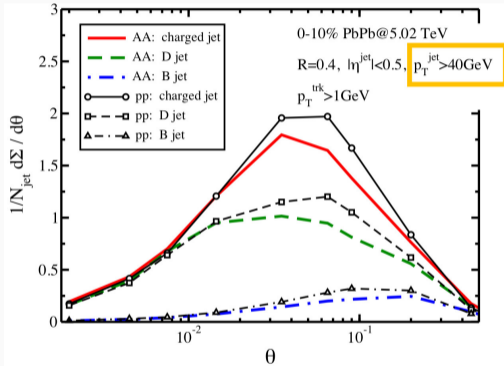
See [A. Nambrath's talk](#)

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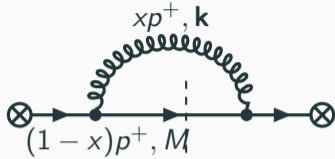
HF EEC in central Pb+Pb collisions



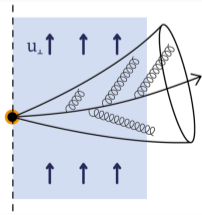
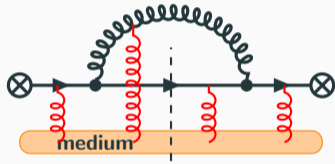
Heavy flavor jet EEC from Linear-Boltzmann Transport (LBT) simulation

[W. Xing et al. 2303.12485] See Wenjing Xing's talk on Tuesday ↗

Mass effects is much more than the “dead cone”



$$\frac{dP_{gQ}^{\text{vac}}}{dx d^2\mathbf{k}} = \frac{\alpha_s C_F}{2\pi^2} \frac{p_{q \rightarrow qg}(x) \mathbf{k}^2 + x^3 M^2}{(\mathbf{k}^2 + x^2 M^2)^2}$$

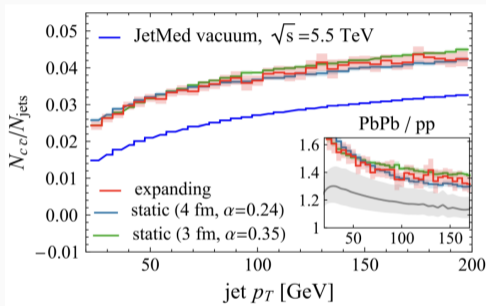


- Terms proportional to M^2 from **helicity-flipping**.
- Medium corrections to the two terms are different! New insights to medium effects.
- Mass also provides novel coupling to flows u^μ , manifest in calculating soft-gluon emission spectra. **Talk by C. Salgado** [↗](#).

$$\hat{q} \rightarrow \hat{q}_{ij} = \hat{q} \left(1 - \frac{M^2}{2E^2} u_\perp^2 \right) \delta_{ij} - \frac{M^2}{E^2} u_{\perp,i} u_{\perp,j}$$

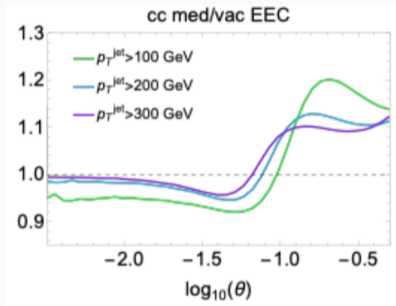
Not all charms are produced in the early stage

Energetic $g \rightarrow c\bar{c}$ delayed by Lorentz boost, comes at characteristic large scale $\frac{\mathbf{k}^2 + m^2}{x(1-x)}$.



Med.-induced $g \rightarrow c\bar{c}$ brings more c in jet.

See **U. Wiedemann's talk** [↗](#).



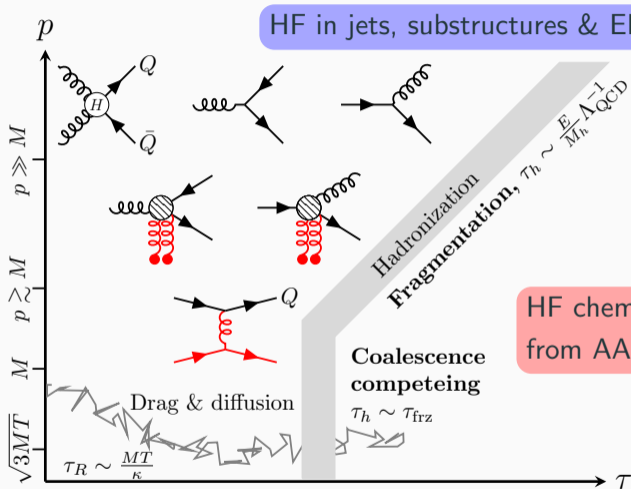
Investigate medium effects at a more controllable energy scales, which imprints on EEC. **J. Brewer's talk** [↗](#).

Summary and Prospectives

- Open heavy flavors continue to deepen the understanding of QGP: charge diffusion, chemical composition, spatial and temporal evolution of fireball, etc.
- Its mass/multi-scale nature requires different strategies for specific problems
 - Early-time vs late time.
 - Relativistic vs non-relativistic.
 - Fragmentation vs coalescence.
 - Near & far from equilibrium.
 - Collisions vs radiation, strong/weak coupling, etc
- Very rich physics, high complexity. Need theory collaborations to make progress.

Summary and Prospectives

Early stage: HQ in coherent fields, and over-occupied system.



Toward a hydro EFT for charm.

HF chemistry from AA to pp.

Thank you!
Questions?