Quarkonia Theory

From Open Quantum System to Classical Transport

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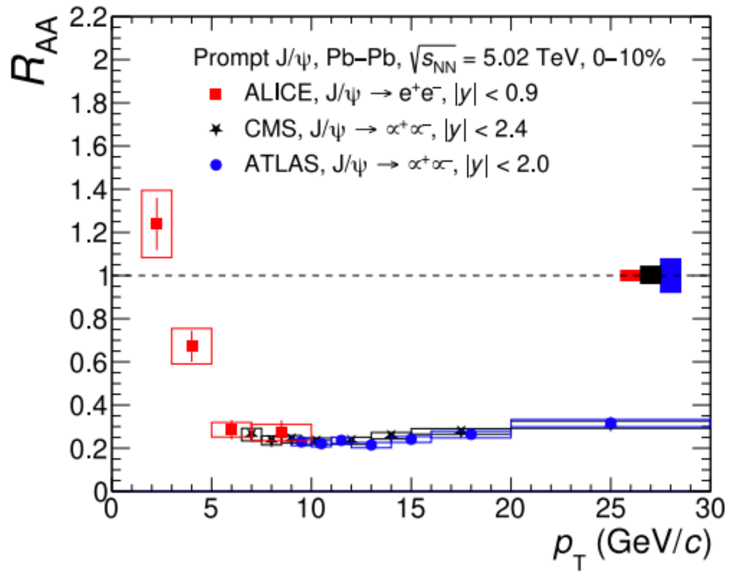
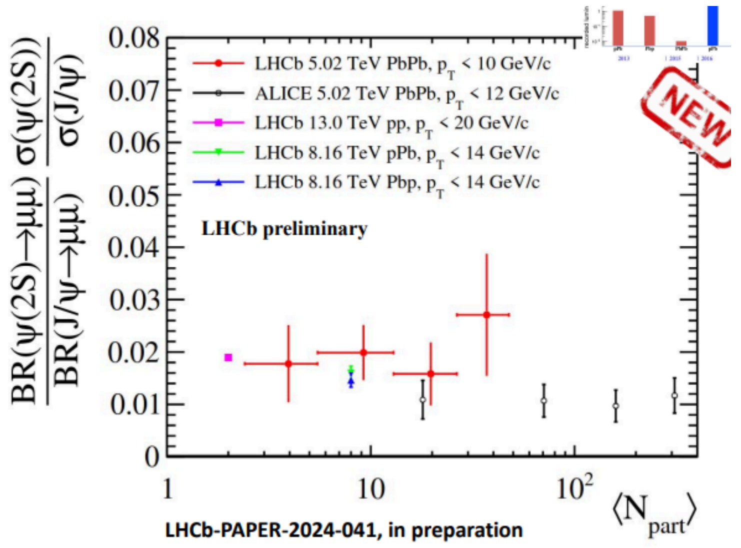
What unique properties of QGP are we probing via quarkonia?

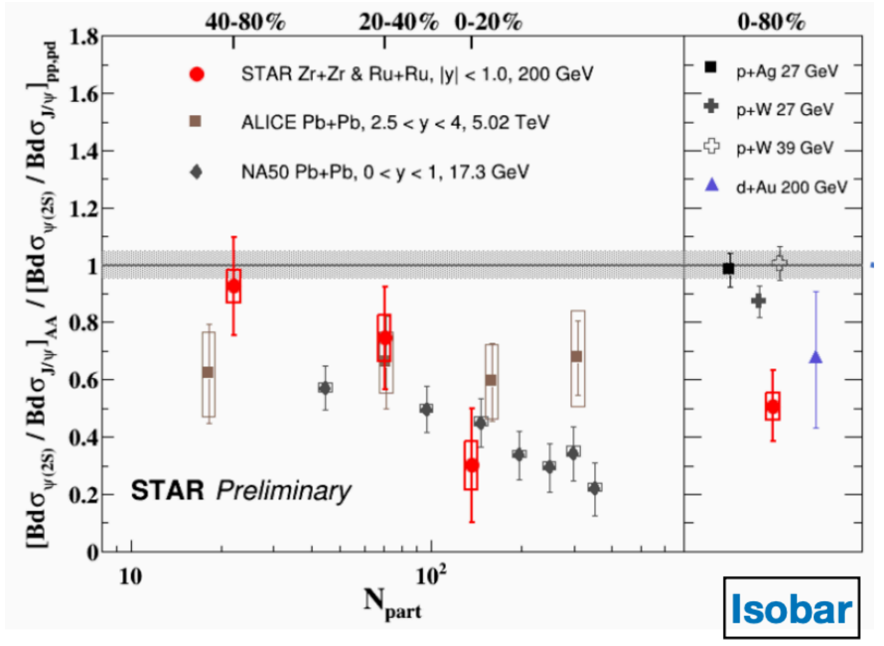
What Unique Properties of QGP Are We Probing?

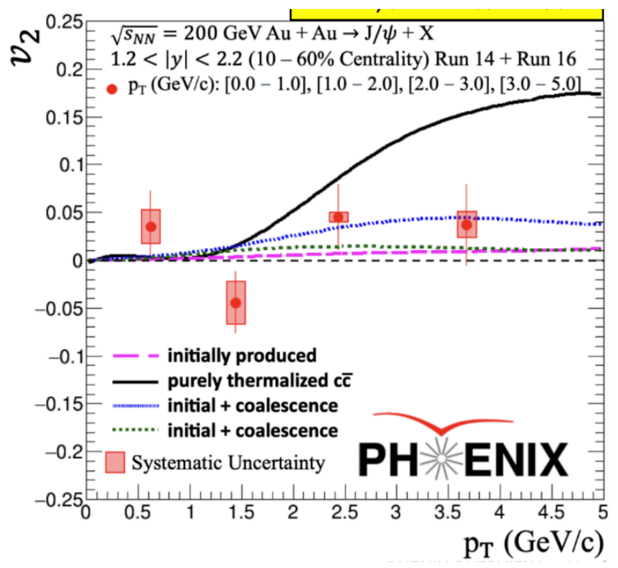
Experimental data on R_{AA}, v_n

(cold/hot medium effect, feeddown)









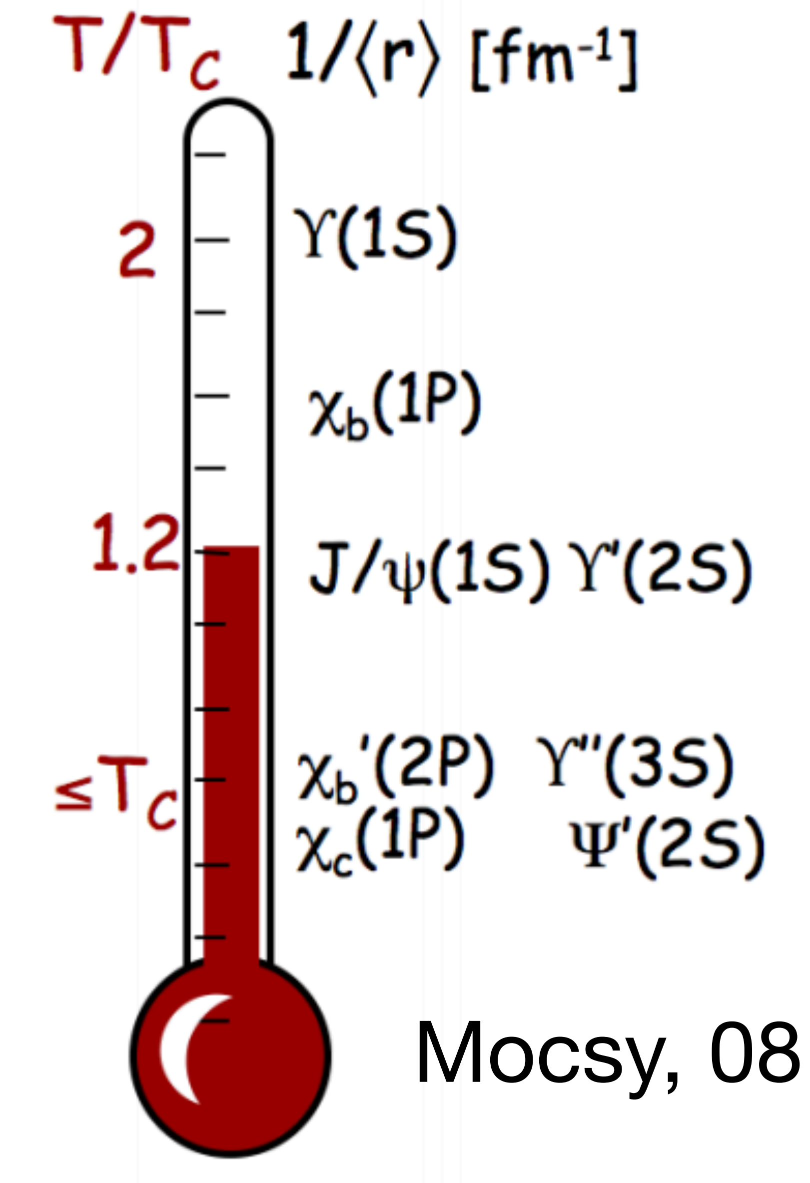
Focus on data at low p_T

Theory

Phenomenology

Computation

Properties of QGP



Mocsy, 0811.0337

- Nonrelativistic
- QGP affects $Q\bar{Q} \to$ quarkonia but not production of $Q\bar{Q}$

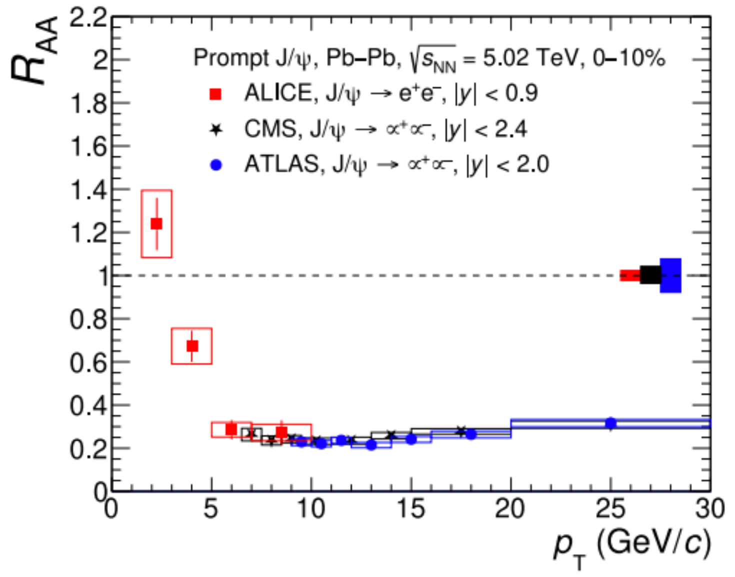
At high p_T: medium effect on $g \to c\bar{c}$: Wiedemann (Tu 3:55pm)
Brewer (Wed 11:10am)

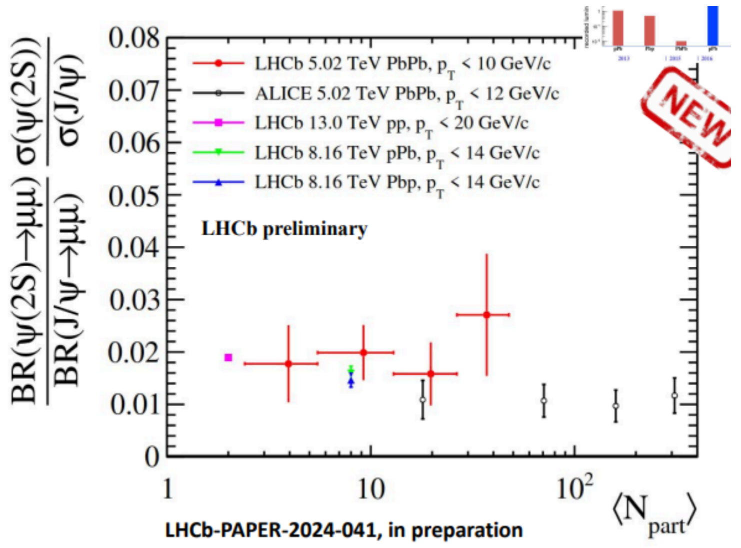
J/ψ production within jet: Zhang (Wed 10:50am)

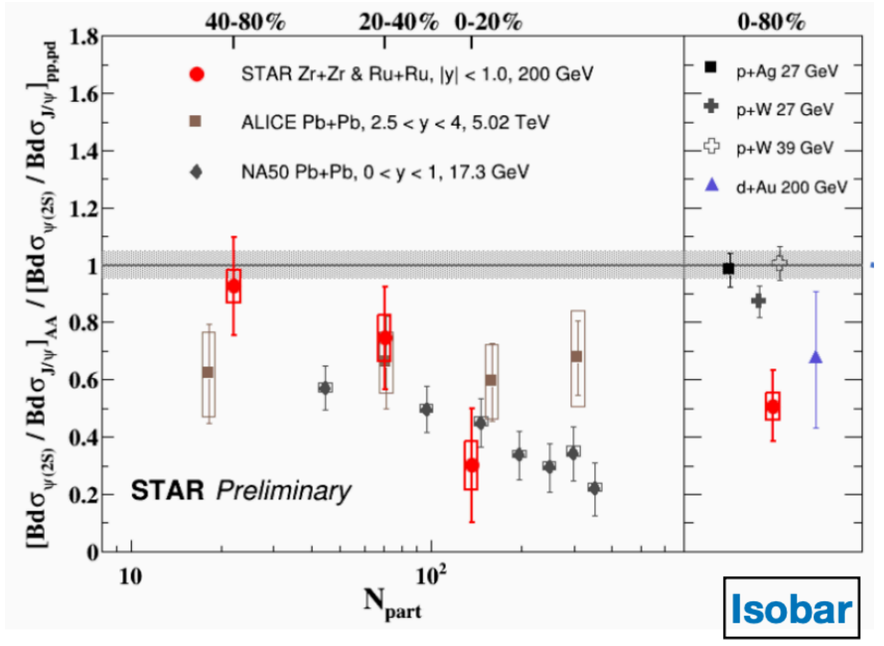
What Unique Properties of QGP Are We Probing?

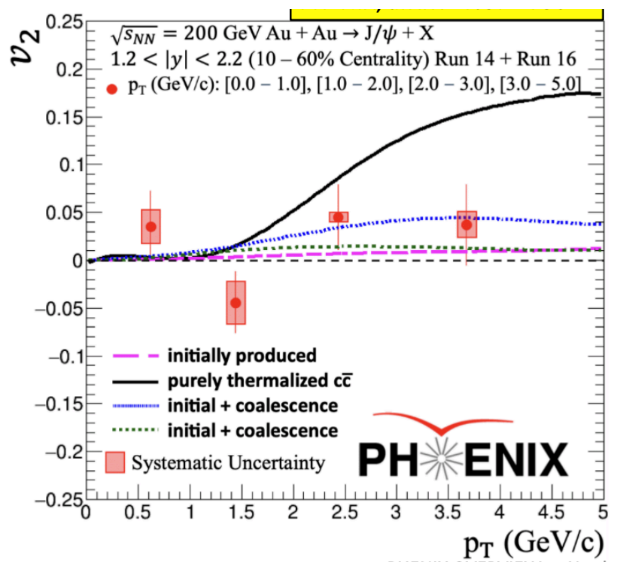
Experimental data on R_{AA}, v_n

(cold/hot medium effect, feeddown)

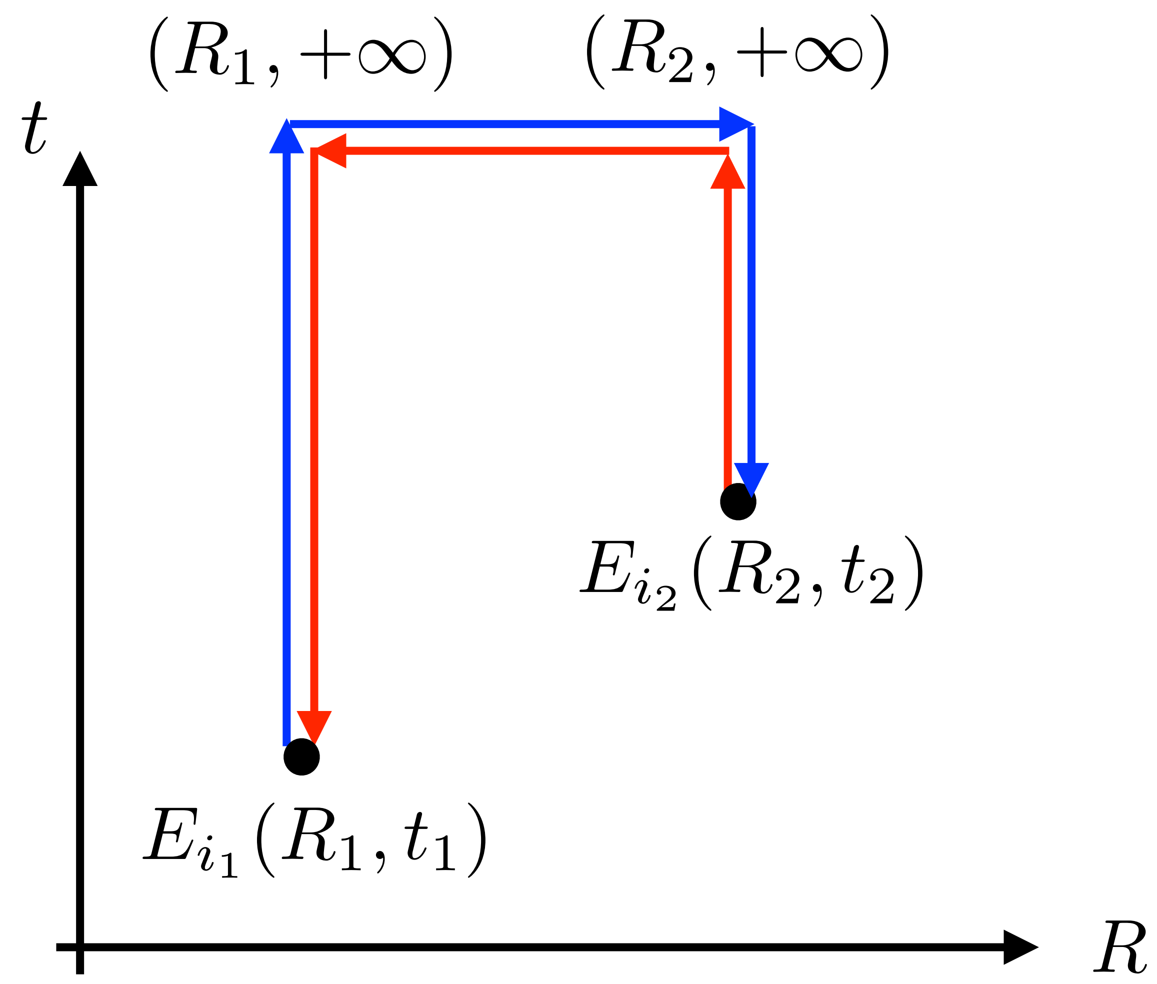








Properties of QGP



Yao, Mehen, 2009.02408

Theory
Phenomenology
Computation

Probe

Focus on data at low p_T → **Chromoelectric correlator**

- Nonrelativistic
- QGP affects $Q\bar{Q}$ → quarkonia but not production of $Q\bar{Q}$

Open quantum system
+
Effective field theory

- Novel transport coefficients
- New type gluon distribution

Open Quantum System

OQS: treat $Q\bar{Q}$ pairs as an open quantum system interacting with QGP

$$\rho_{Q\bar{Q}}(t) = \mathrm{Tr}_{\mathrm{QGP}}[U(t)\rho_{\mathrm{tot}}(0)U^{\dagger}(t)]$$

$$\rho_{\mathrm{tot}}(0) = \rho_{Q\bar{Q}} \otimes \rho_{\mathrm{QGP}}$$

Akamatsu, Rothkopf, 1110.1203; Akamatsu, 1403.5783

In certain limits, evolution equation is Lindblad equation (Markovian, non-unitary)

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}(t)}{\mathrm{d}t} = -i[H, \rho_{Q\bar{Q}}(t)] + \sum_{ij} D_{ij}\Big(L_i\rho_{Q\bar{Q}}(t)L_j^{\dagger} - \frac{1}{2}\{L_j^{\dagger}L_i, \rho_{Q\bar{Q}}(t)\}\Big)$$

Semiclassical limits: Boltzmann equations (rate equation, Fokker-Planck/Langevin equation)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = C[f]$$

Reviews of OQS for quarkonia: Rothkopf, 1912.02253; Akamatsu, 2009.10559
Sharma, 2101.04268; Yao, 2102.01736

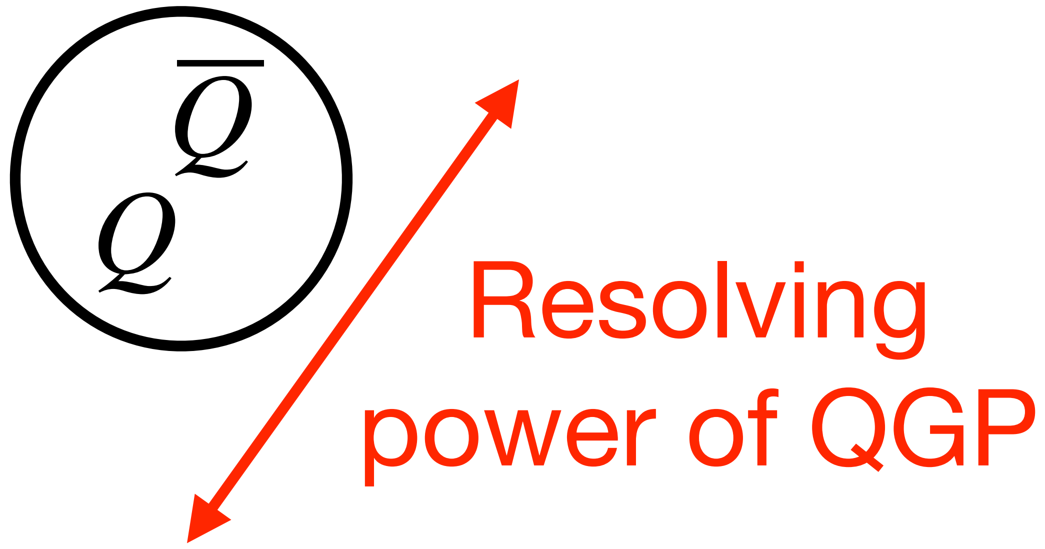
Separation of Energy Scales

In vacuum $M \gg Mv \gg Mv^2$

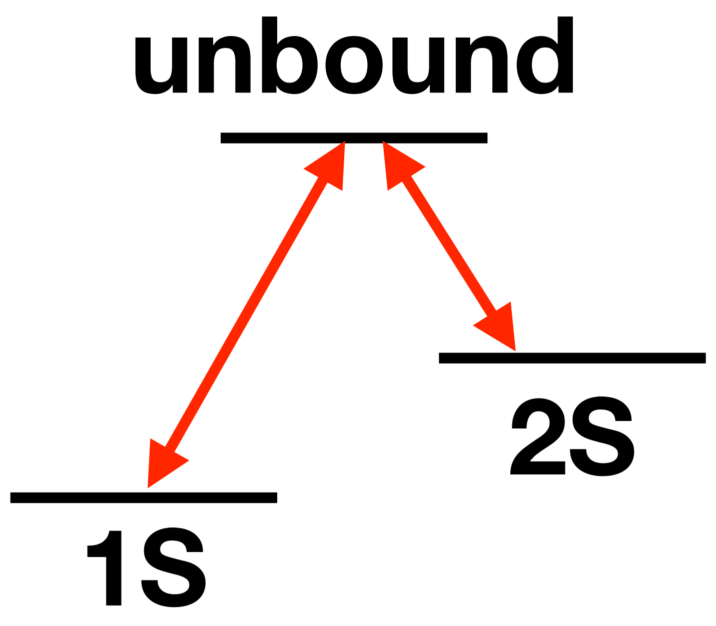
				$c\bar{c}$	$b\bar{b}$
QCD					
	M	heavy quark mass		1.3	4.2 GeV
NRQCD					
	Mv	inverse of quarkonium size r^{-1}		0.7	1.3 GeV
pNRQCD					
	Mv^2	quarkonium binding energy		0.5	0.5 GeV

Medium case depends on where T fits

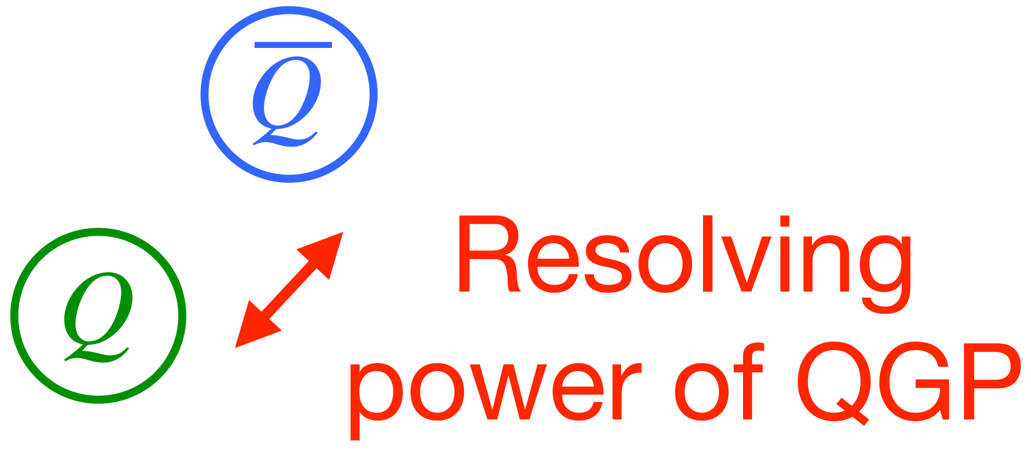
- **Quantum optical limit: low T**



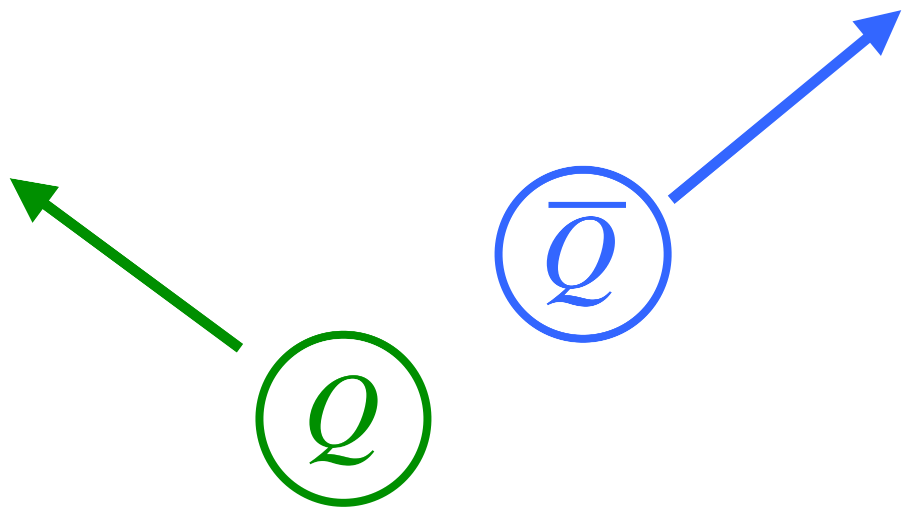
Transitions between levels



- **Quantum Brownian motion: high T**



Decoherence of $Q\bar{Q}$ pair, “diffusion”



Separation of Energy Scales and Physical Scenarios

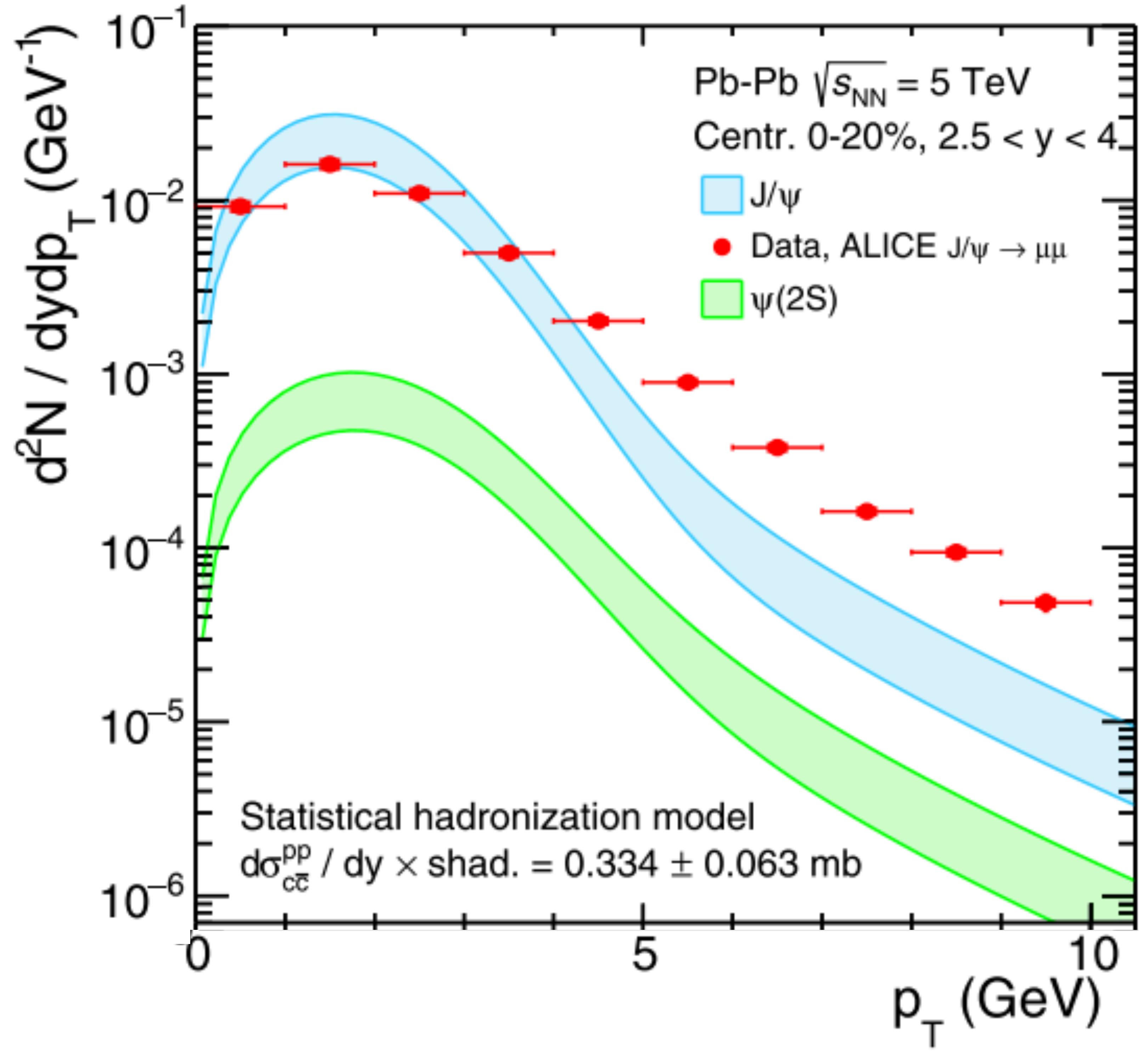
Hierarchy of energy scales	EFT	Quantum Description	Classical Description
$M \gg T \gg Mv^2, \Lambda_{QCD}$	NRQCD $\alpha_s(T)$ small	Lindblad (quantum Brownian motion)	Diffusion equation (semiclassical limit)
$M \gg Mv \gg T, \Lambda_{QCD}$ $T \gg Mv^2$, expand Mv^2/T	pNRQCD rT small	Lindblad (quantum Brownian motion)	
$M \gg Mv \gg T, \Lambda_{QCD}$ No expansion of Mv^2/T	pNRQCD rT small		Boltzmann equation (quantum optical and semiclassical limits)

Various Calculation Approaches Differ in Treatments of Temperature Regimes

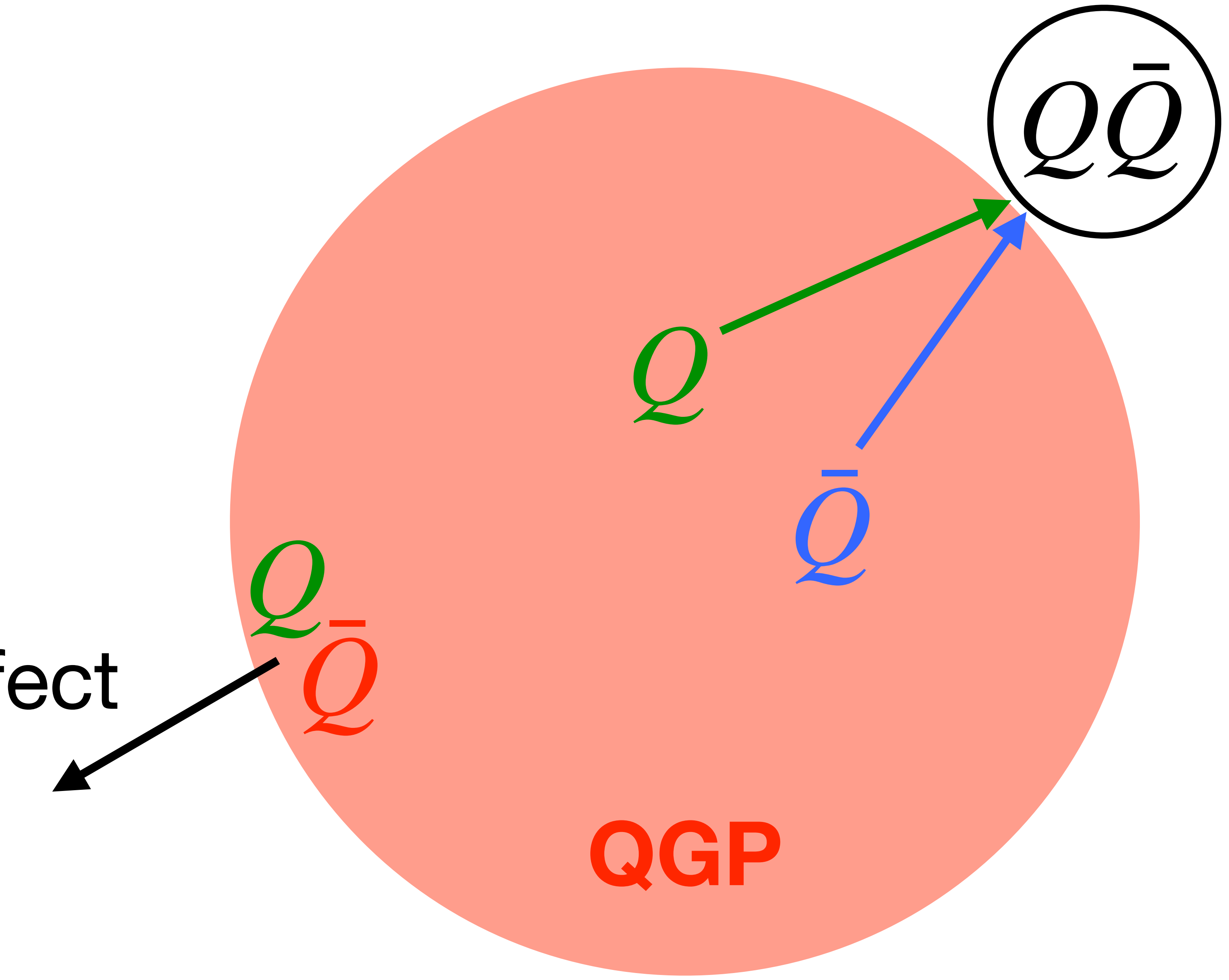
Nomenclature from Andronic, et al, 2402.04366, summarizing efforts of EMMI Rapid Reaction Task Force

Example 1: Statistical Hadronization Model

- No dynamical evolution by assuming heavy quarks **unbound** in medium and reach **thermal equilibrium** (**kinetic only, large** M)
- Instantaneous hadronization at freezeout



Andronic, Braun-Munzinger, Köhler, Redlich, Stachel, 1901.09200



- Using Wigner function in hadronization: **scale** Mv **enters, but not** Mv^2

See e.g. Instantaneous Coalescence Model, Parton-Hadron-String Dynamics

Application in pp: Gossiaux (Tu 2pm)

Example 2: Lindblad Equation in Osaka and Nantes Approaches

- Focus on $M \gg T \gg Mv^2, \Lambda_{\rm QCD}$ —> Lindblad equation (quantum Brownian motion), 1D numerics

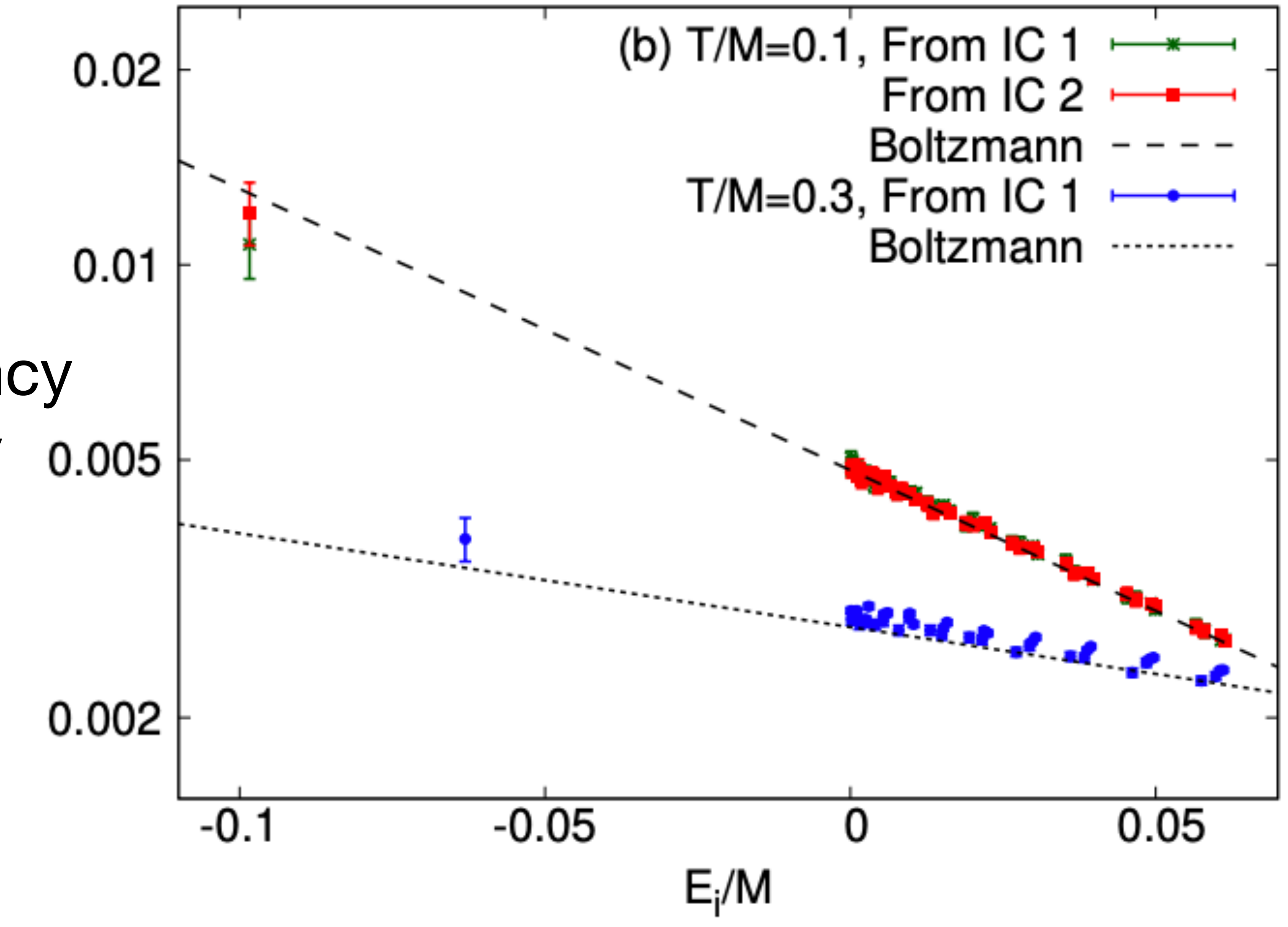
$$\frac{\mathrm{d}\rho_{Q\bar{Q}}(t)}{\mathrm{d}t} = -i[H, \rho_{Q\bar{Q}}(t)] + \sum_{ij} D_{ij}\left(L_i \rho_{Q\bar{Q}}(t) L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_i, \rho_{Q\bar{Q}}(t)\}\right)$$

$H =$ kinetic + potential + …

L_i : singlet-octet, octet-octet transitions

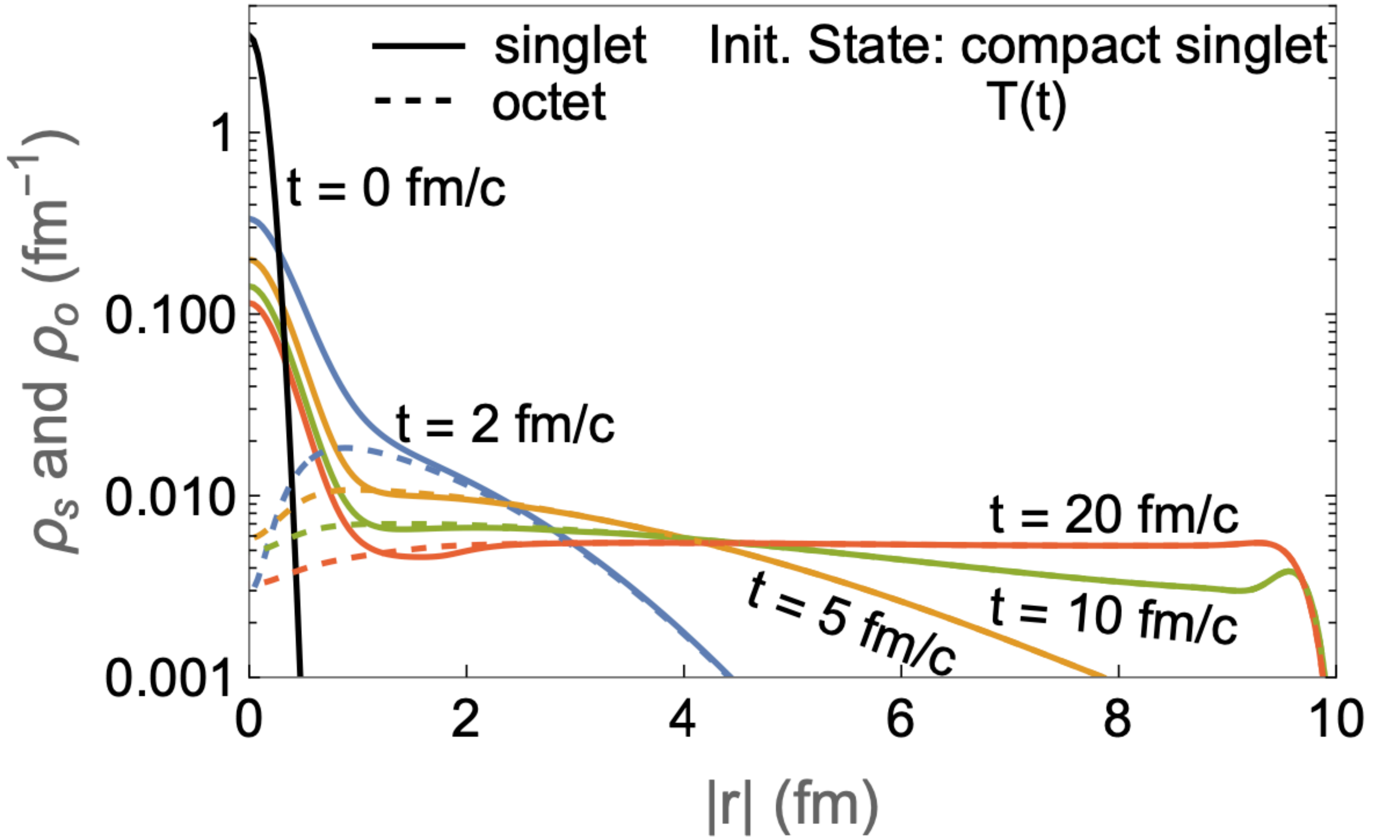
- **Osaka: momentum basis $D_{ij} \propto \delta_{ij}$**

Occupancy in steady state



Miura, Akamatsu, Asakawa, Kaida, 2205.15551

- **Nantes: position basis D_{ij} has off-diagonal part**



Delorme, Katz, Gousset, Gossiaux, Blaizot, 2402.04488

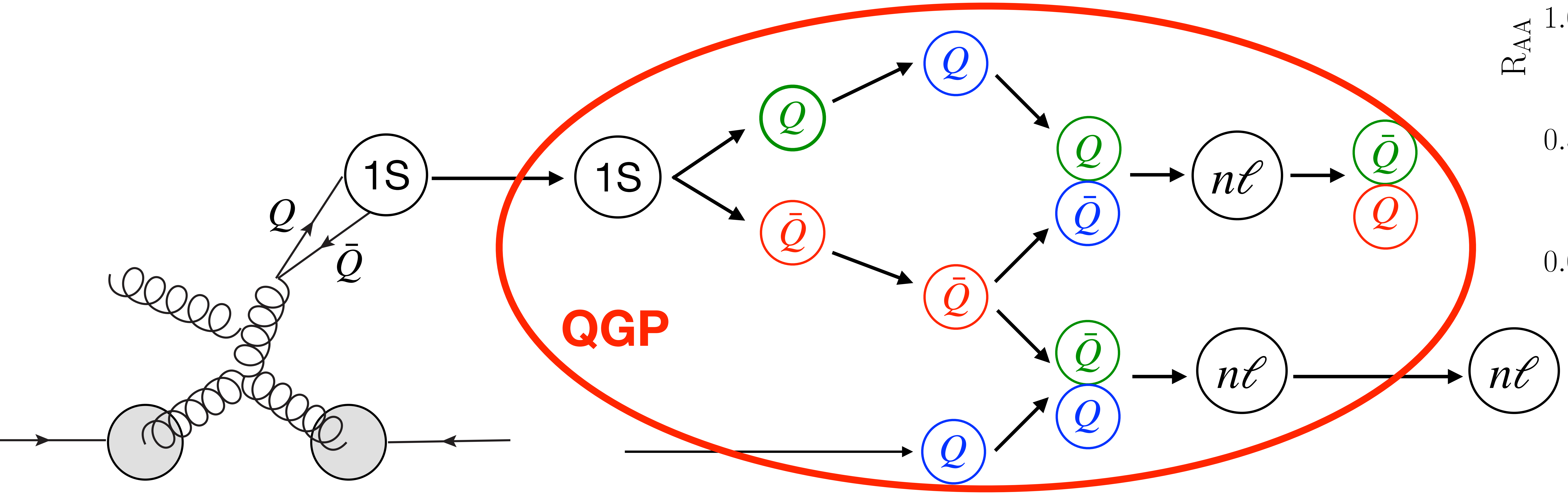
Example 3: Coupled Boltzmann Equations in Duke/MIT Approach

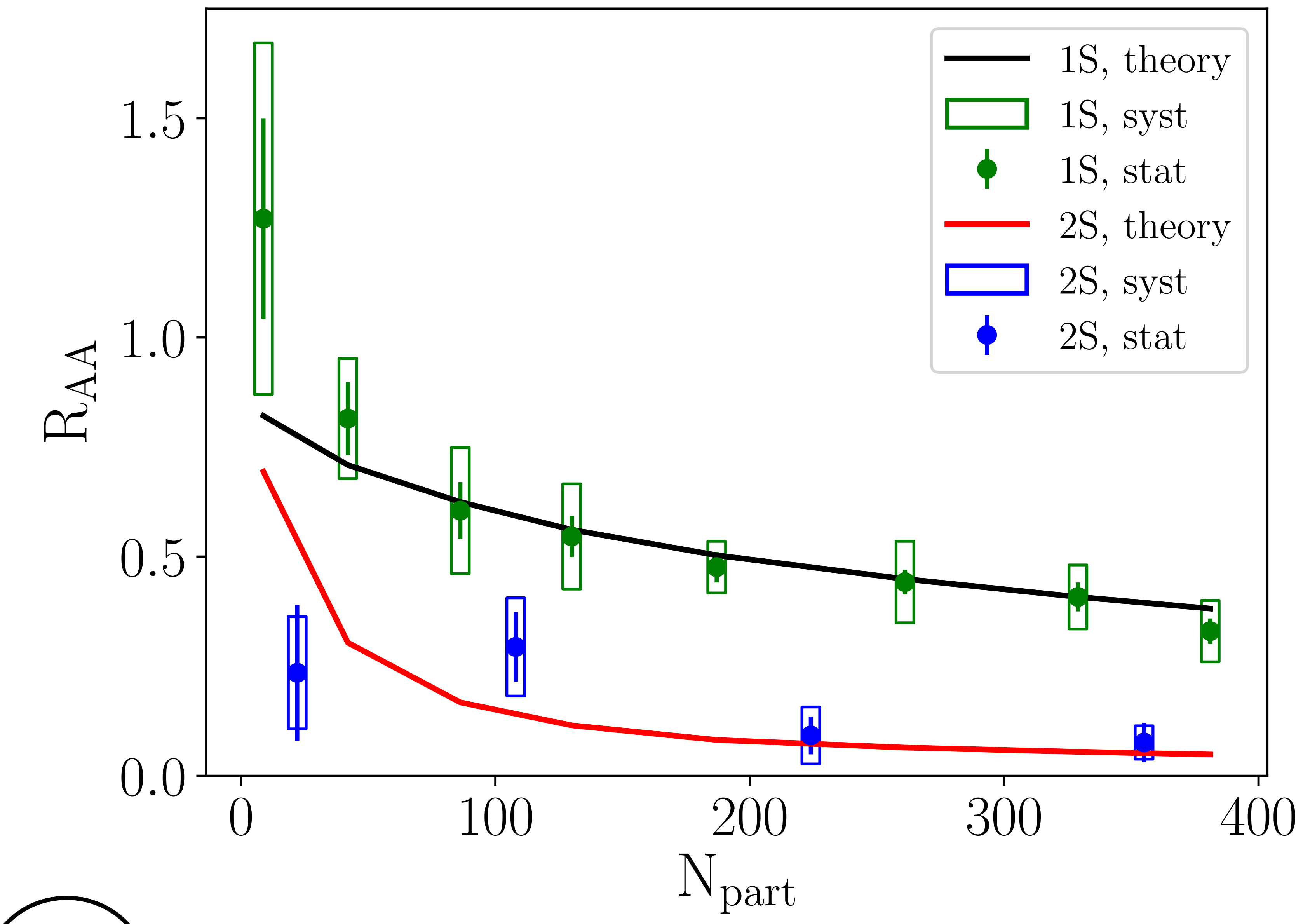
- Open heavy quarks & antiquarks for $T \gg Mv^2$, unbound pair

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}\right) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

- Each quarkonium state $n\ell s$ for $Mv^2 \gtrsim T$

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$





Yao, Ke, Xu, Bass, Müller, 2004.06746

$f_{Q\bar{Q}} \neq f_Q f_{\bar{Q}}$ to handle **correlated** and **uncorrelated** (re)combination

Boltzmann Equation for Quarkonium

- Boltzmann/rate equation also used in **Tsinghua, Saclay, Santiago, TAMU** approaches

Density of bound state: $$\frac{\mathrm{d}n_b(t,\mathbf{x})}{\mathrm{d}t} = -\Gamma\, n_b(t,\mathbf{x}) + F(t,\mathbf{x})$$ Modeling reaction rates

- **Rigorous derivation** from OQS + pNRQCD in quantum optical and semiclassical limits
  Dissociation/recombination terms depend on **chromoelectric correlator, a new gluon distribution**

$$\Gamma = \int \frac{\mathrm{d}^3 p_{\rm rel}}{(2\pi)^3} |\langle \psi_b | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle|^2 [g_{\rm adj}^{++}]^{>}(-\Delta E)$$

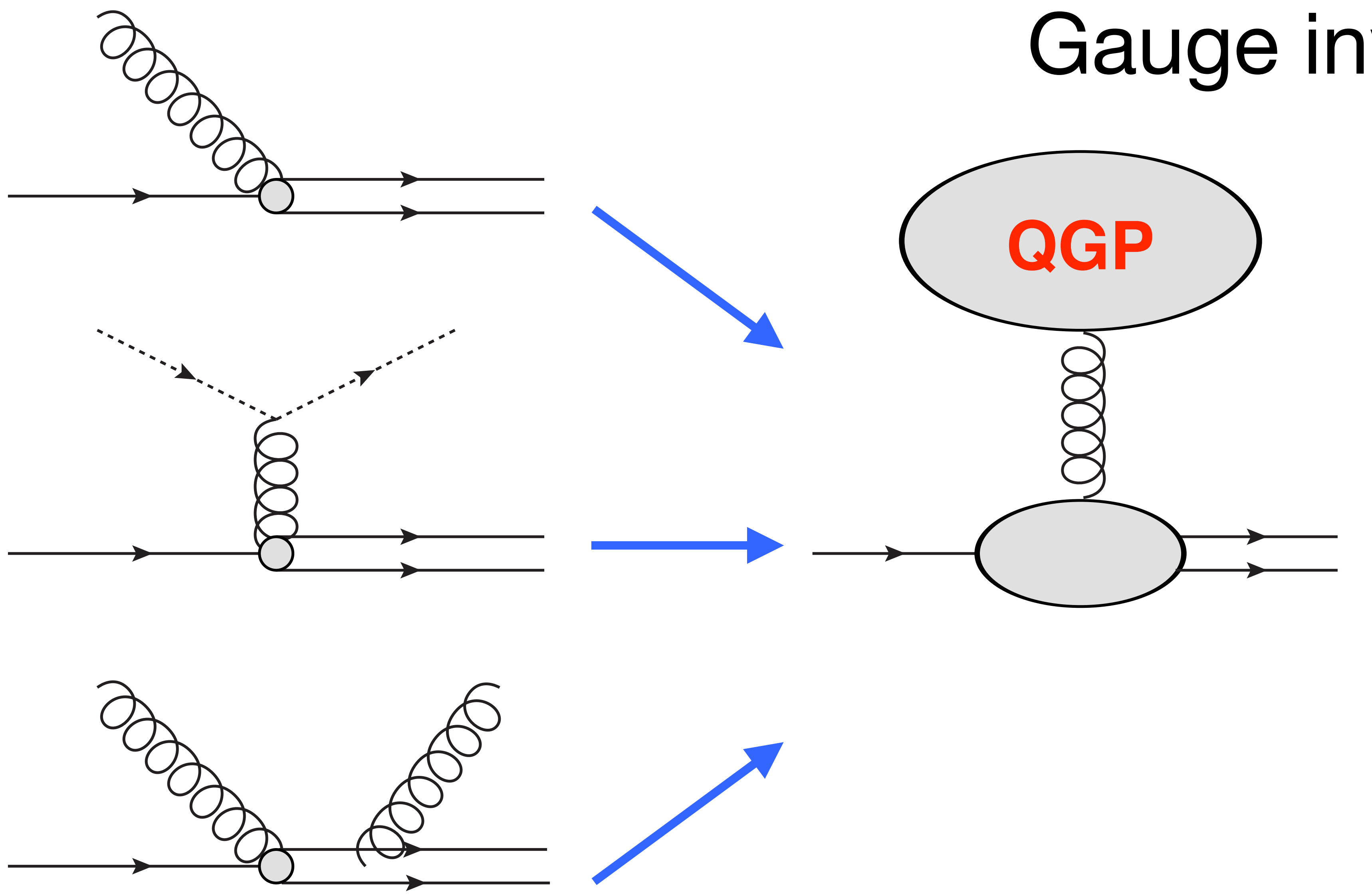
$$F = \int \frac{\mathrm{d}^3 p_{\rm cm}}{(2\pi)^3} \frac{\mathrm{d}^3 p_{\rm rel}}{(2\pi)^3} f_{Q\bar{Q}} |\langle \psi_b | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle|^2 [g_{\rm adj}^{--}]^{>}(\Delta E)$$

Yao, Mehen, 1811.07027, 2009.02408

$$[g_{\rm adj}^{++}]^{>}(\omega) = e^{\omega/T} [g_{\rm adj}^{--}]^{>}(-\omega)$$

$$[g_{\rm adj}^{++}]^{>}(t) \propto \langle E_i^a(t) \mathcal{W}^{ab}(t,0) E_i^b(0) \rangle_T$$

Gauge invariant



Nijs, Scheihing, Yao, 2310.09325

Example 4: Lindblad Equation in Munich-KSU Approach

- Focus on $M \gg Mv \gg T, \Lambda_{\text{QCD}}$ and $T \gg Mv^2$ —> Lindblad equation (quantum Brownian motion)

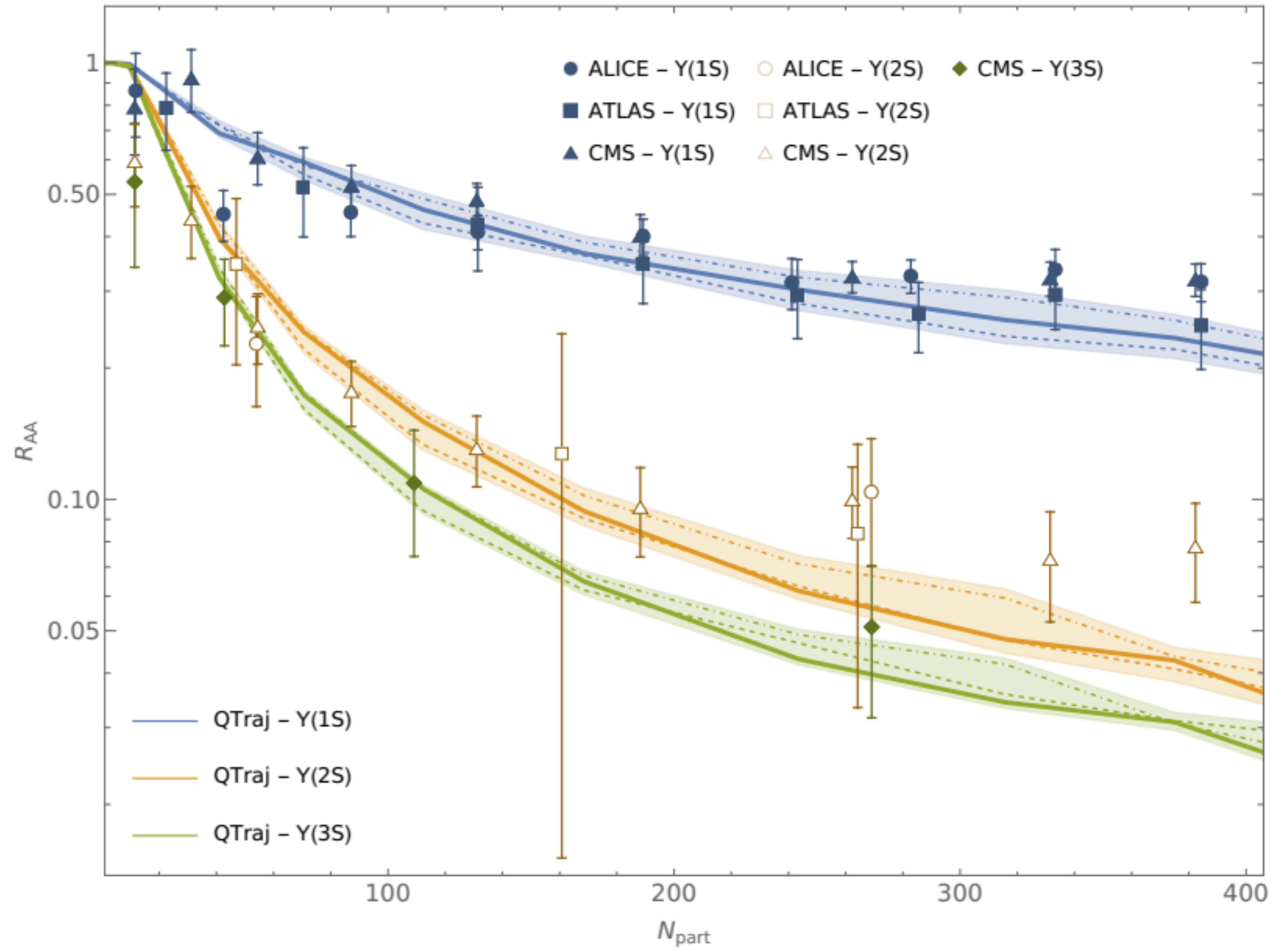
$$\frac{\mathrm{d}\rho_{Q\bar{Q}}(t)}{\mathrm{d}t} = -i\left[H + \gamma_{\text{adj}}\Delta h,\ \rho_{Q\bar{Q}}(t)\right] + \kappa_{\text{adj}}\left(L_{\alpha i}\rho_{Q\bar{Q}}(t)L_{\alpha i}^{\dagger} - \frac{1}{2}\{L_{\alpha i}^{\dagger}L_{\alpha i},\ \rho_{Q\bar{Q}}(t)\}\right)$$

Novel transport coefficients in terms of **chromoelectric correlator**

$$\kappa_{\text{adj}} + i\gamma_{\text{adj}} \equiv \frac{g^2 T_F}{3N_c}\int \mathrm{d}t \langle \mathcal{T} E_i^a(t)\mathcal{W}^{ab}(t,0)E_i^b(0)\rangle_T \qquad \kappa_{\text{adj}} = [g_{\text{adj}}^{++}]^{>}(\omega = 0)$$

- New development: **3-loop QCD potential, beyond Coulomb**

	PDG	V_s^c	$V_s^{3\text{L+np}}$
$M(1S)$/ GeV	9.445	9.445	9.445
$M(2S)$/ GeV	10.017	9.635	10.042
$M(3S)$/ GeV	10.355	9.670	10.395
$M(1P)$/ GeV	9.888	9.635	9.887
$M(2P)$/ GeV	10.251	9.670	10.279

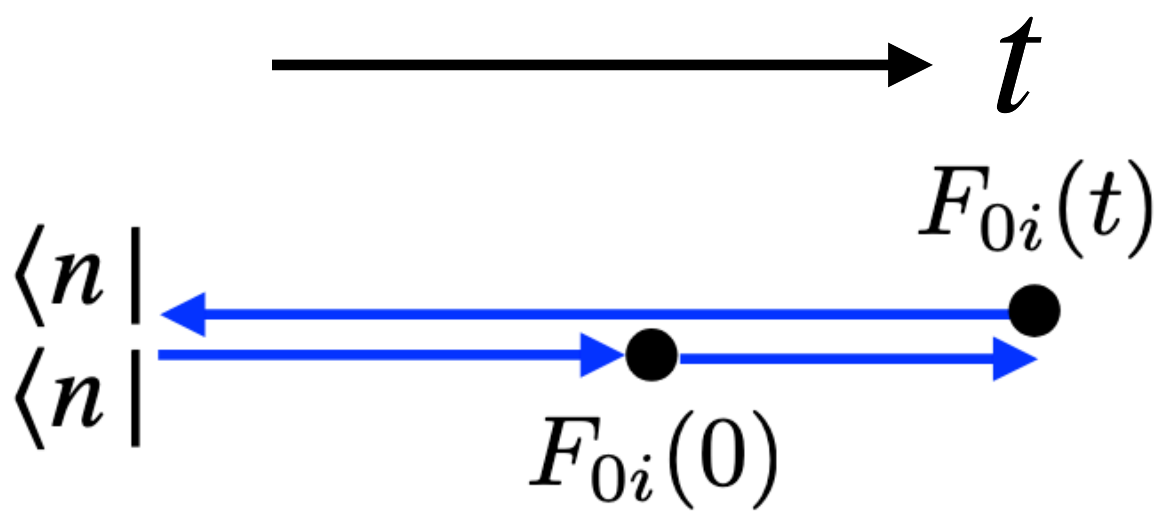


Brambilla, Magorsch, Strickland, Vairo, Griend, 2403.15545

Chromoelectric Correlators

- Similar to but different from heavy quark diffusion coefficient, in terms of operator ordering

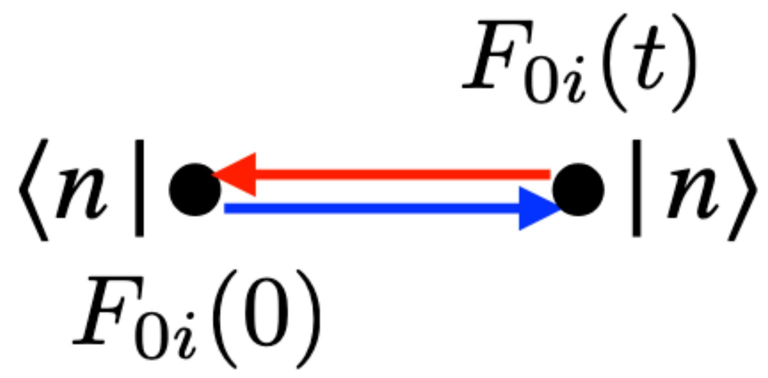
HQ diffusion



$$\kappa_{\rm fund} = \frac{g^2}{3N_c}\mathrm{Re}\int \mathrm{d}t \langle \mathrm{Tr_c}[U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)]\rangle_{T,Q}$$

Quarkonium

$$\kappa_{\rm adj} = [g^{++}_{\rm adj}]^{>}(\omega=0) = \frac{g^2T_F}{3N_c}\int \mathrm{d}t \langle E_i^a(t)\mathcal{W}^{ab}(t,0)E_i^b(0)\rangle_T$$



- **Axial gauge puzzle**: Feynman gauge calculations show the two correlators differ, but would **be same in (temporal) axial gauge**

Eller, Ghiglieri, Moore, 1903.08064; Binder, Mukaida, Scheihing, Yao, 2107.03945

Bruno Scheihing: obstruction in defining (temporal) axial gauge for $\kappa_{\rm fund}$

puzzle solved Scheihing, Yao, PRL 130, 052302 (2023)

Chromoelectric Correlators in Weakly and Strongly Coupled Plasmas

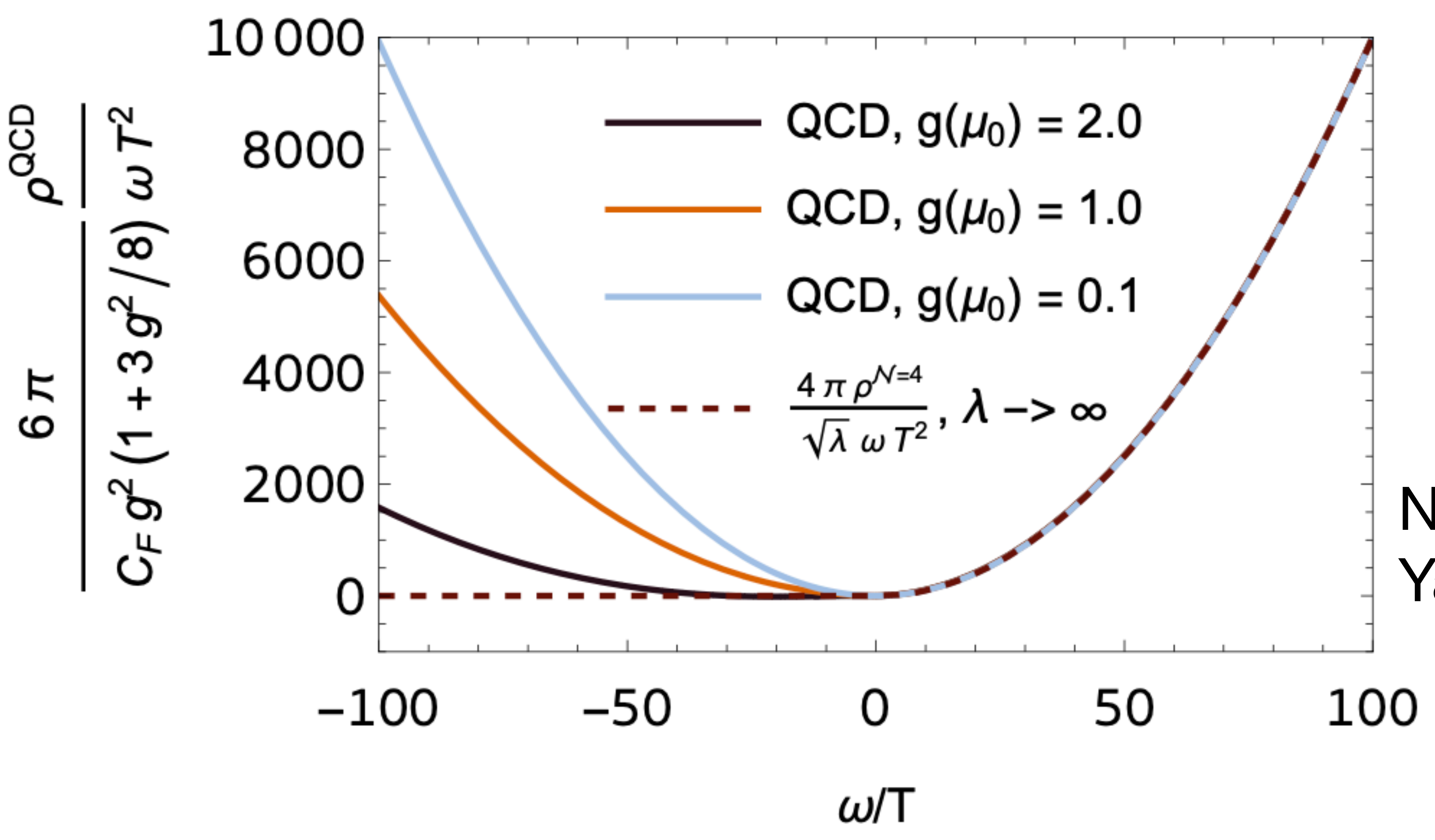
- NLO calculation in QCD compared with strong coupling results in $\mathcal{N} = 4$ SYM

In strong coupling limit:

$[g_{\rm adj}^{++}]^{>}(\omega)$ **vanishes** at $\omega < 0$

$\kappa_{\rm adj} = 0$

Lindblad and Boltzmann equations (Markovian) become trivial (no dynamics)

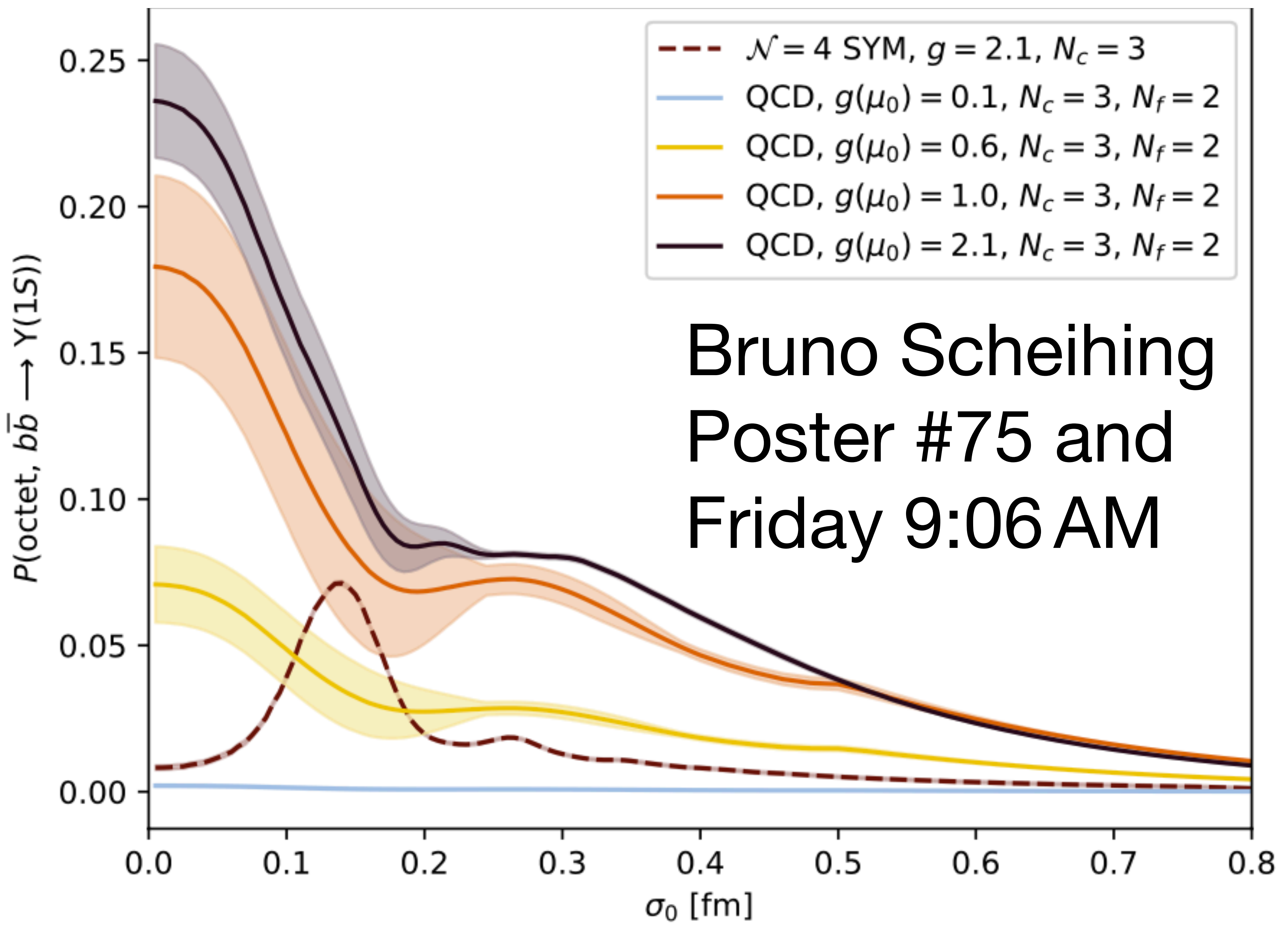


Nijs, Scheihing, Yao, 2310.09325

- Implication for phenomenology

Need **non-Markovian** description

$$\rho_{Q\bar{Q}}(t) = \mathrm{Tr}_{\rm QGP}[U(t)\rho_{\rm tot}(0)U^{\dagger}(t)]$$



Summary and Path Forward

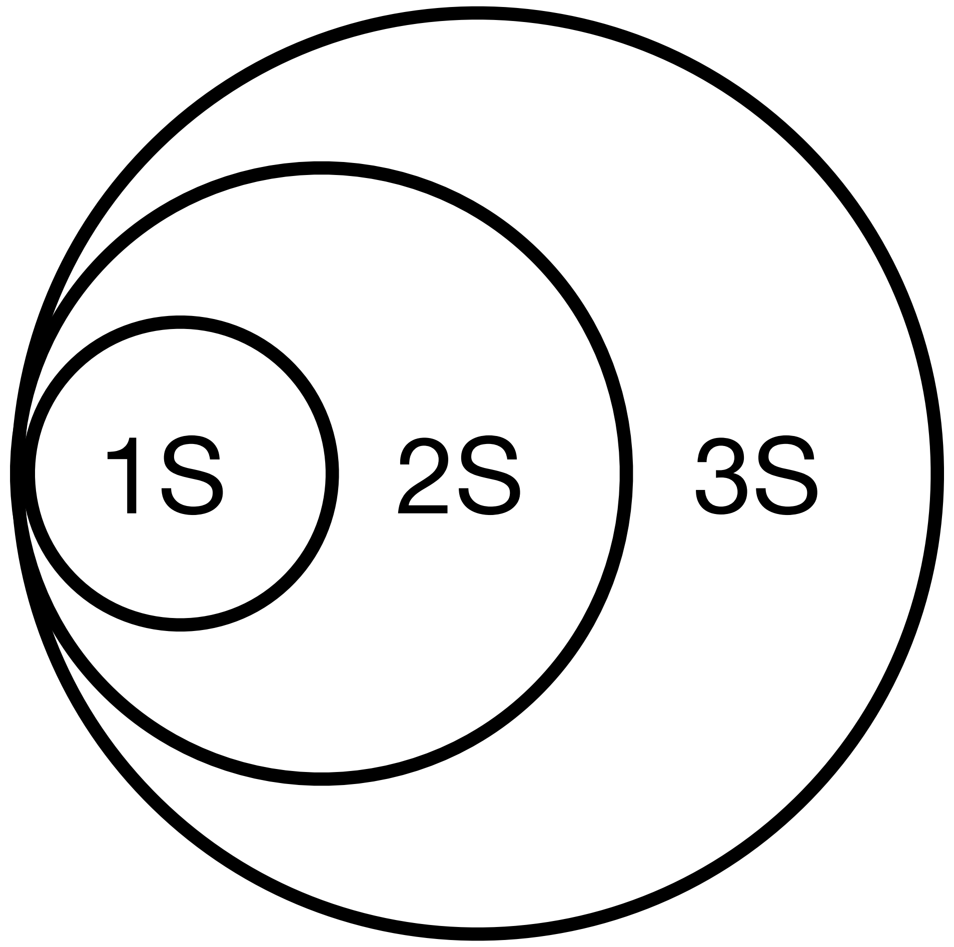
Quarkonia data at low p_T $\xrightarrow[\text{Theory, Phenomenology, Computation}]{\text{Probe}}$ **Chromoelectric correlator**

Probe

Theory
Phenomenology
Computation

- **Future question 1: What is microscopic structure of QGP probed by different quarkonia, as reflected by chromoelectric correlator?**

Is the QGP a weakly coupled gas of quarks/gluons or a strongly coupled fluid, probed by $\Upsilon(nS)$?



Constrain chromoelectric correlator from data

We need both RHIC and LHC data for this

Data from RHIC important to constrain $[g_{\rm adj}^{++}]^{>}(\omega)$ at **finite ω** **(sPHENIX & STAR)**

Summary and Path Forward

- **Future question 2: lattice calculation of chromoelectric correlator**

Analytically continue to Euclidean spacetime, evaluate and invert

$$G_{\rm adj}(\tau) = \int_{-\infty}^{+\infty} \frac{{\rm d}\omega}{2\pi} \frac{\exp\left(\omega(\frac{1}{2T}-\tau)\right)}{2\sinh\left(\frac{\omega}{2T}\right)} \rho^{++}_{\rm adj}(\omega)$$

Spectral function is non-odd Scheihing, Yao, 2306.13127

$$\kappa_{\rm adj} = \lim_{\omega\to 0} \frac{T}{2\omega}\left[\rho^{++}_{\rm adj}(\omega) - \rho^{++}_{\rm adj}(-\omega)\right]$$

Renormalization Leino, 2401.06733; Brambilla, Wang, 2312.05032

- **Future question 3: spin alignment/polarization of quarkonium probes chromomagnetic correlator**

Cheung, Vogt, 2203.10154; Zhao, Chen, 2312.01799

$$\langle B_i^a(t)\mathcal{W}^{ab}(t,0)B_i^b(0)\rangle_T$$

Boltzmann/Lindblad equations derived from OQS + pNRQCD Yang, Yao, 2405.20280

Theoretical calculations and experimental constrain

- **Future question 4: solve Lindblad equation for multiple $Q\bar{Q}$ pairs**

Important for charmonium phenomenology, but computationally expensive

Machine learning? Quantum computing?

Lin, Luo, Yao, Shanahan, 2402.06607

de Jong, Metcalf, Lee, Mulligan, Płoskoń, Ringer, Yao, 2010.03571
2106.08394