

Lattice and Effective Field Theory for Hard Probes

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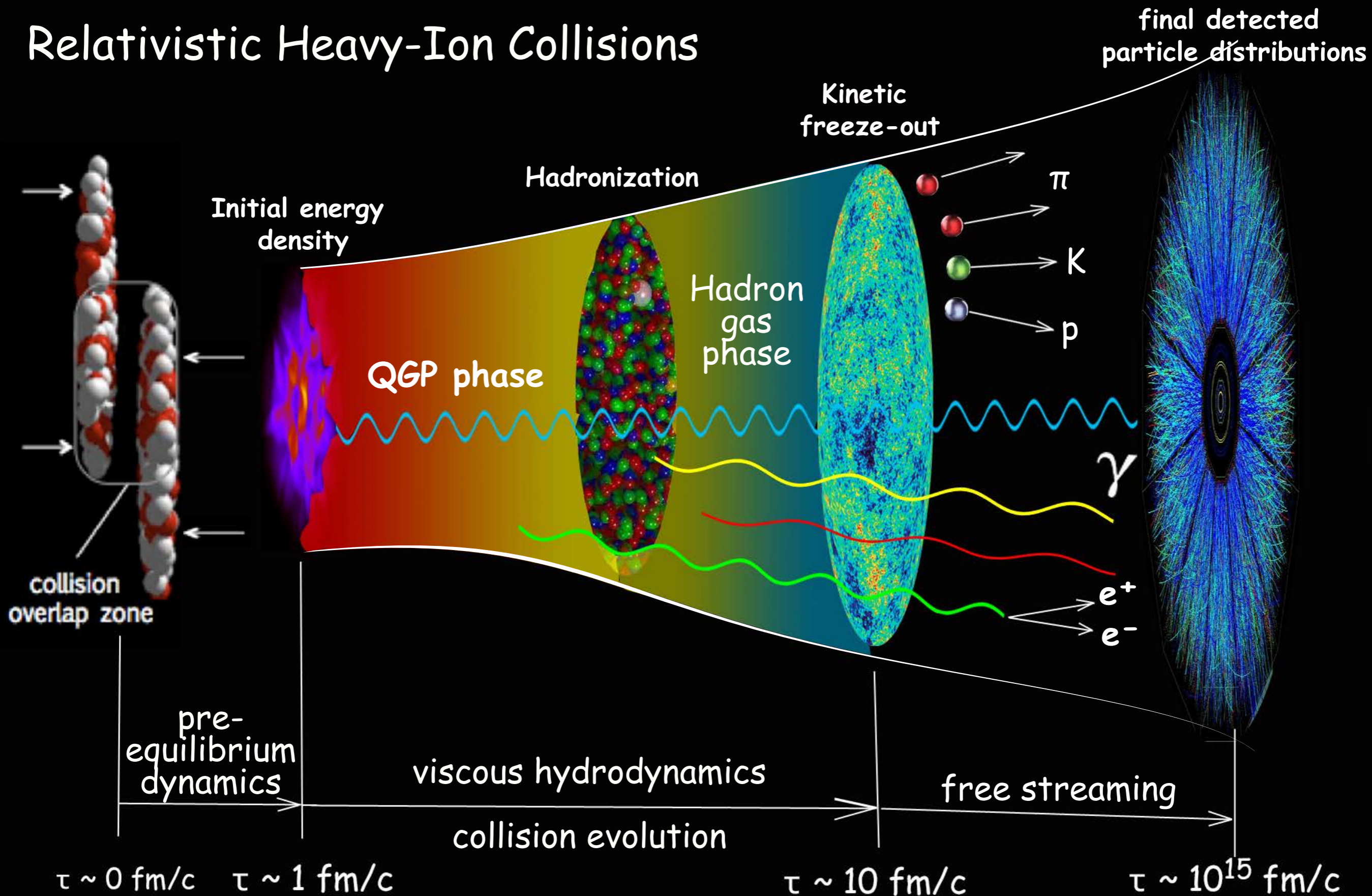


Hard Probe 2024

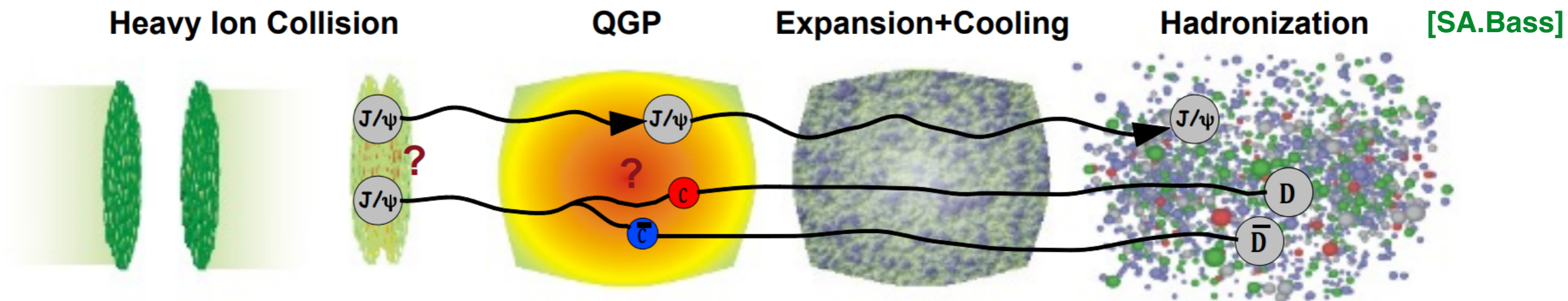
Sep. 22 - Sep. 27, 2024, Nagasaki, Japan

Heavy-ion collisions at LHC and RHIC

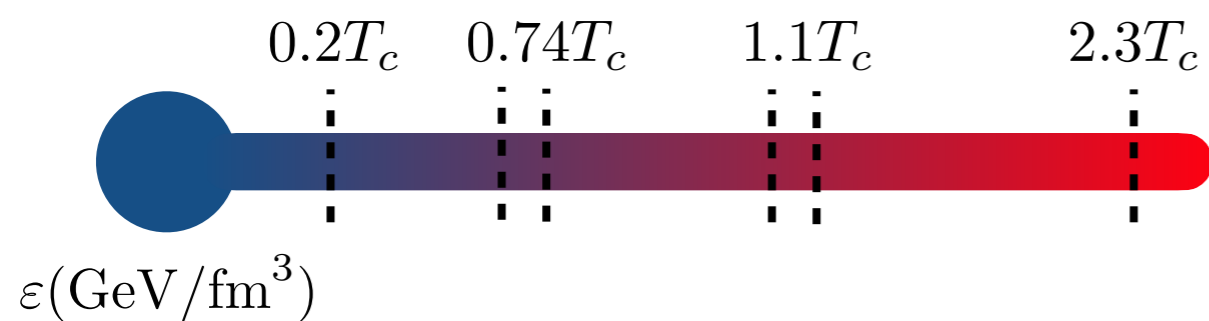
Relativistic Heavy-Ion Collisions



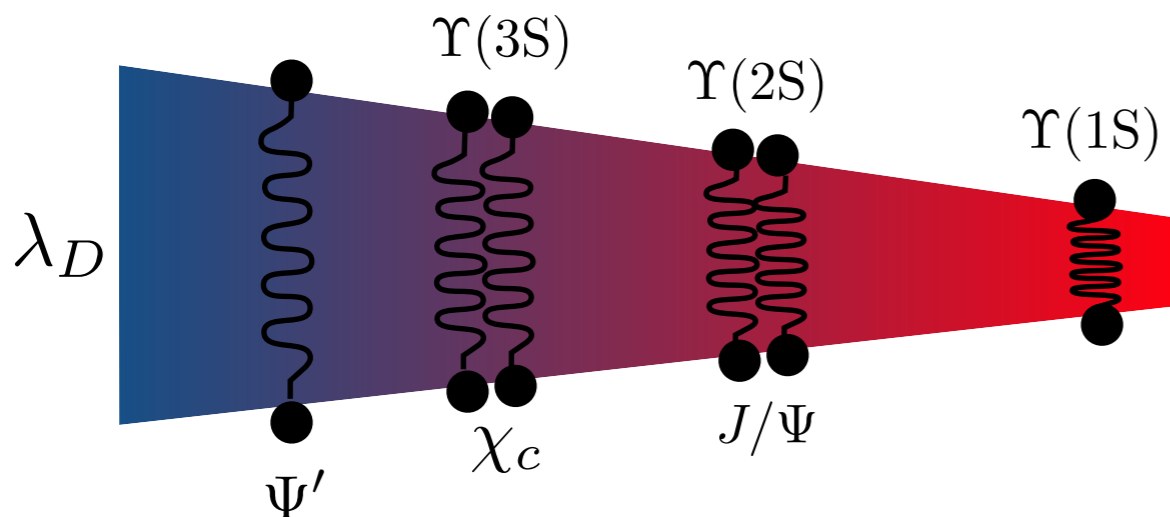
Heavy flavor probes



Probe the hot medium with **different length scales**



Where Lattice and EFT can help ...

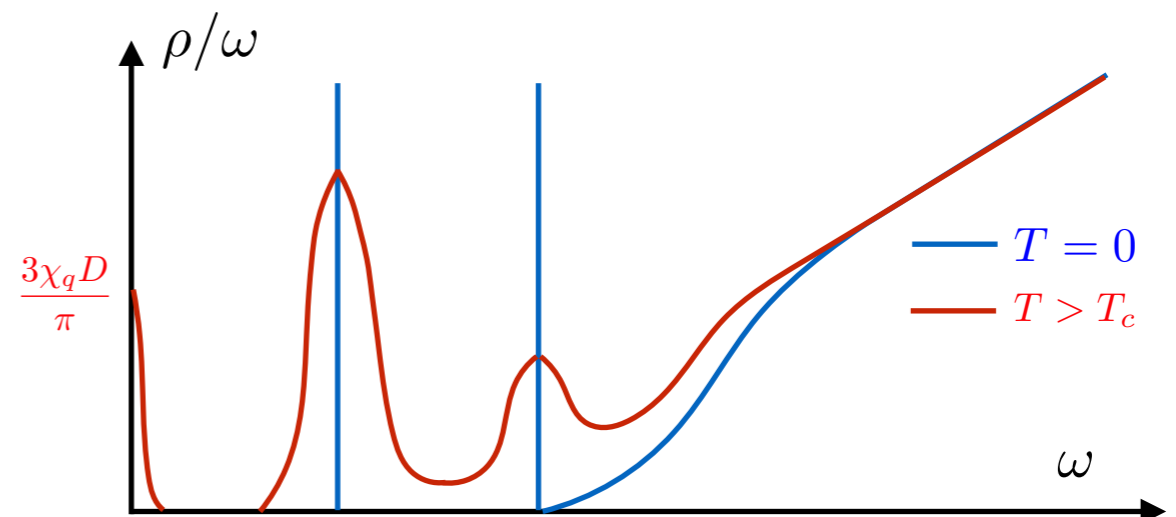
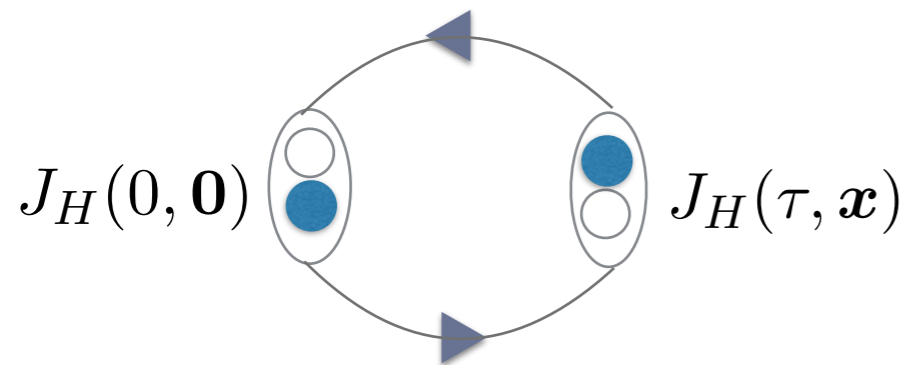


- In-medium quarkonium properties: masses, widths, melting T
- Complex quark-antiquark potential: $\text{Re}[V]$, $\text{Im}[V]$
- Heavy quark diffusion: D_s

2015 Long Run Plan Nuclear Science

From Imag.-time lattice to real-time physics

An example: meson spectral function



$$G_H(\tau, \vec{p}) = \sum_{x,y,z} \exp(-i \vec{p} \cdot \vec{x}) \langle J_H(0, \vec{0}) J_H^\dagger(\tau, \vec{x}) \rangle = \int \frac{d\omega}{2\pi} \frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/2T)} \rho_H(\omega, \vec{p}, T)$$

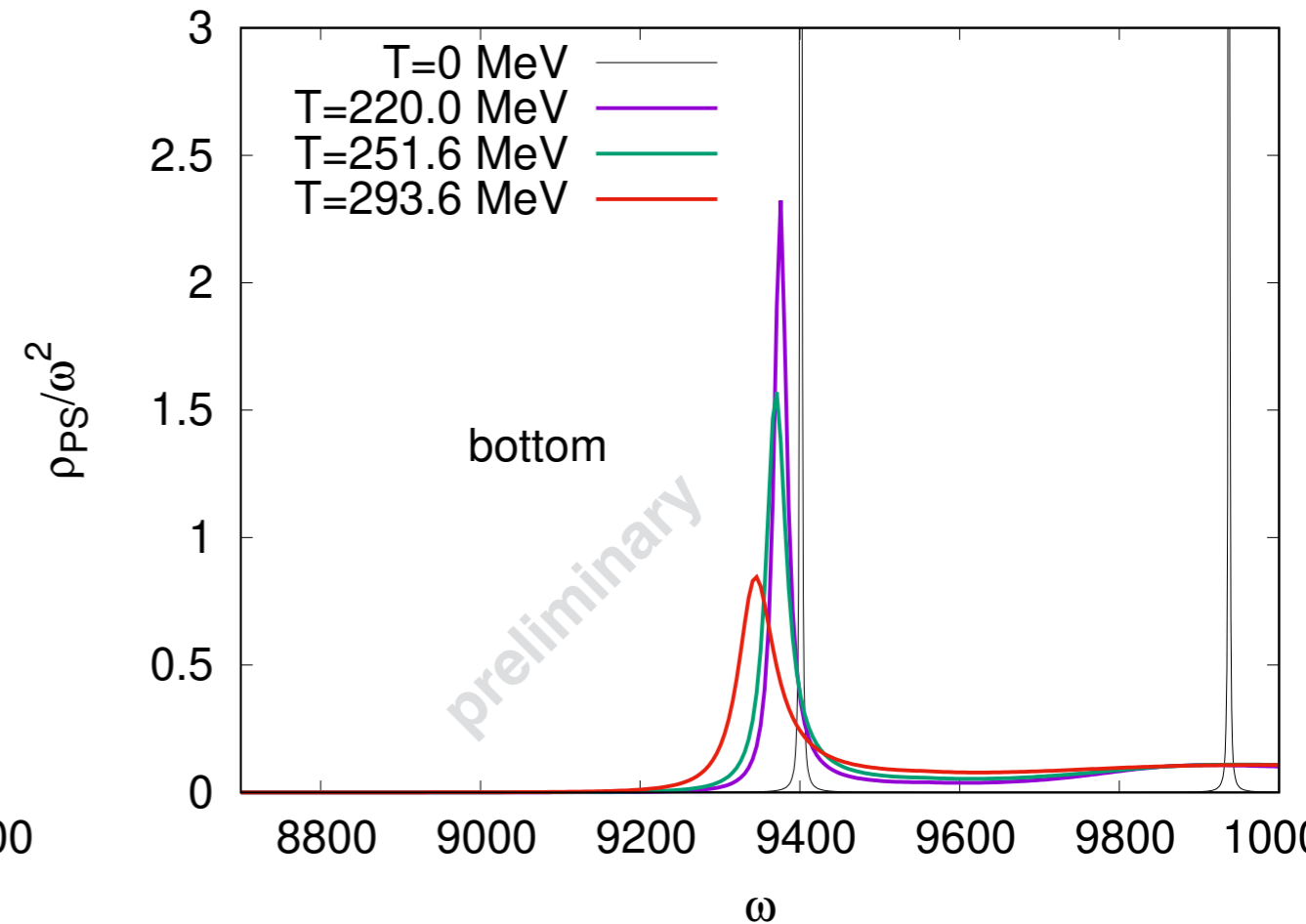
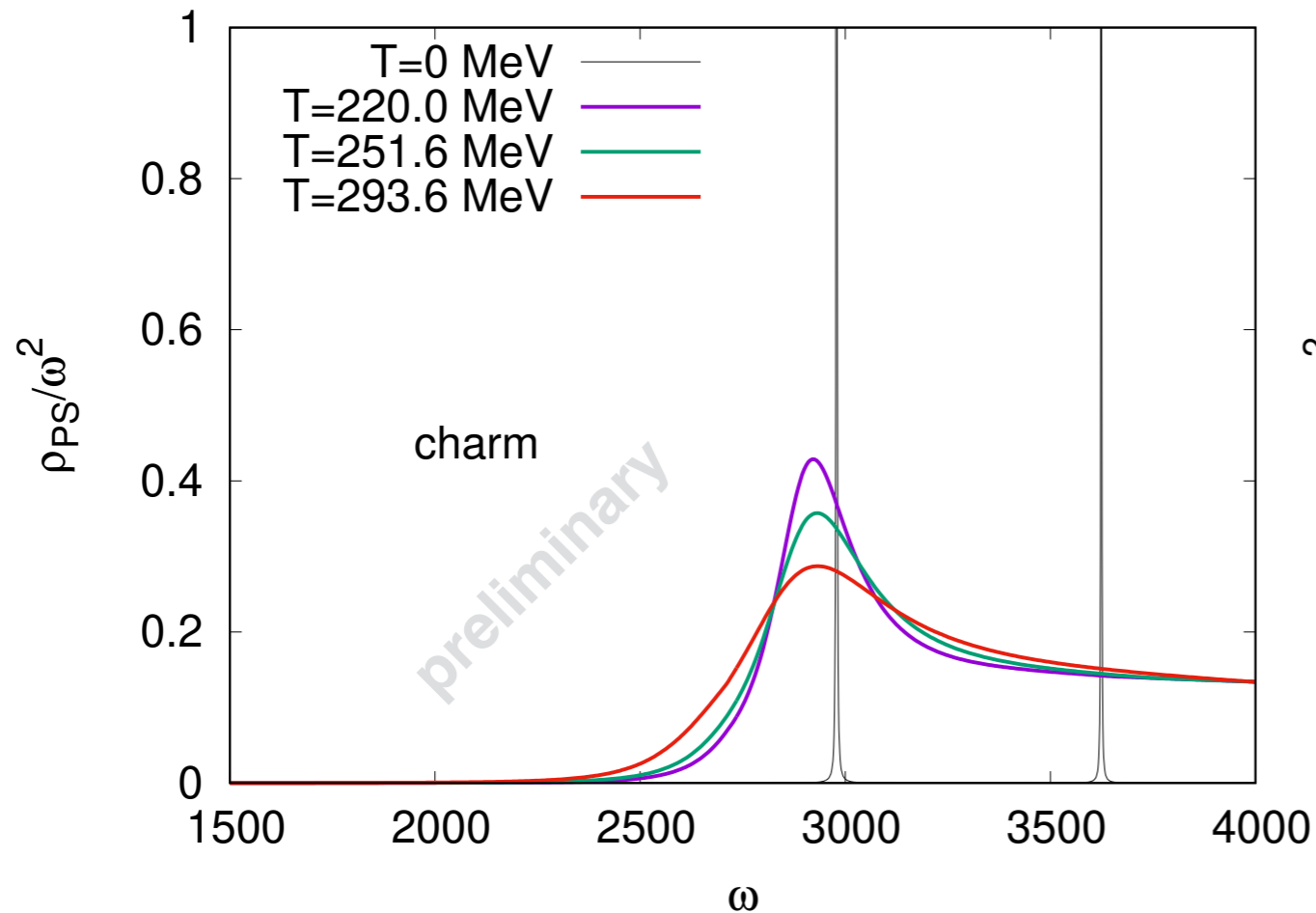
$K(\omega, \tau, T)$

- Meson spectral function tells **melting temperature** and **heavy quark diffusion**

Quarkonium spectral function (relativistic HQ)

Need very fine and large lattice for heavy quark $T = 1/(aN_\tau)$

First full QCD calculation with relativistic heavy quarks [hotQCD, Few Body Syst. 64(2023) 3, 52]
[S. Ali, D. Bala, et al., in prep.]



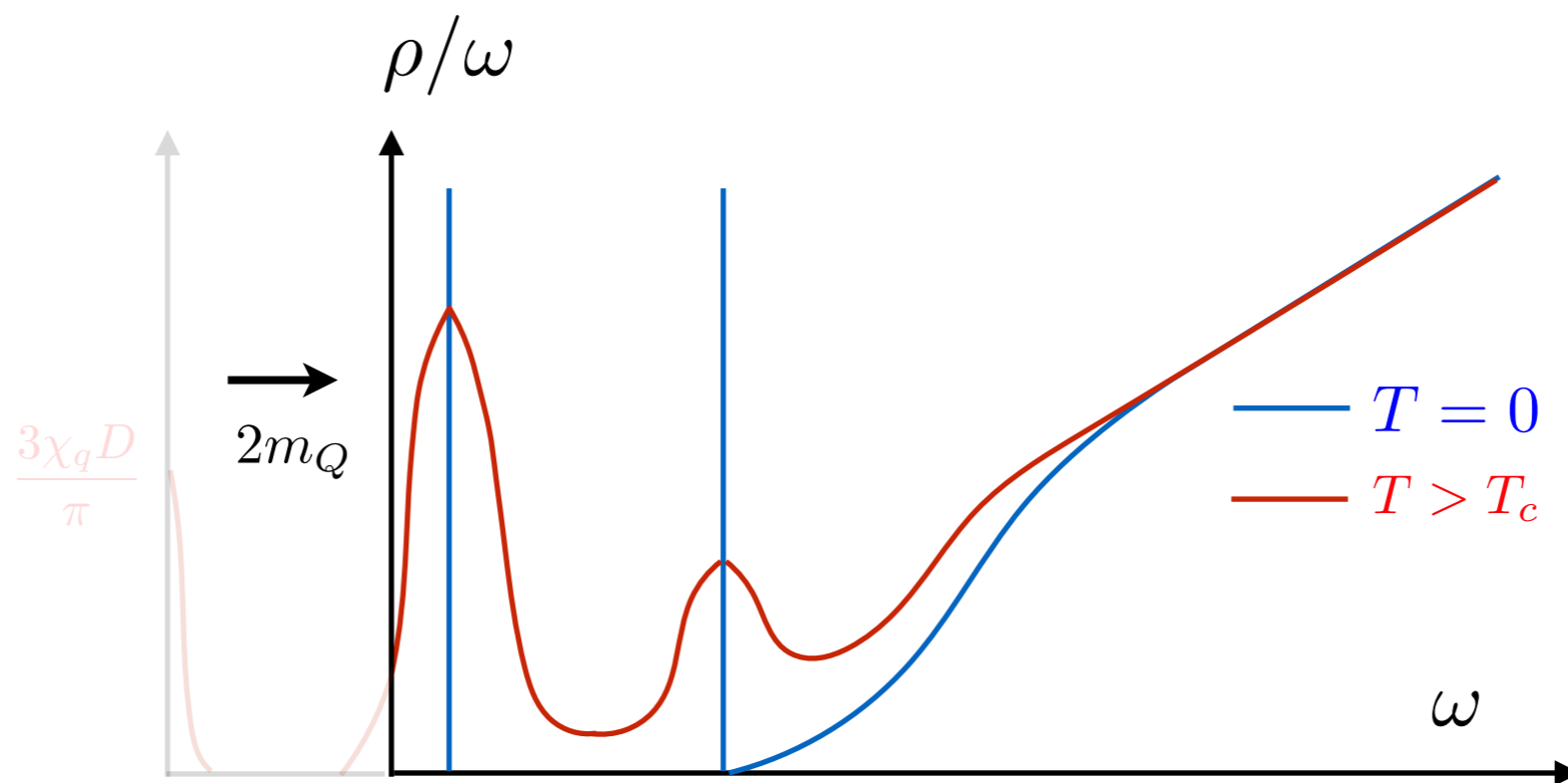
- Thermal broadening for both charm and bottom
- More broadening for charm than bottom
- Determination of dissociation temperature needs more investigation

Non-relativistic heavy quark

NRQCD becomes possible due to scale separation: $m_Q \gg m_{Qv} \gg m_{Qv}^2$

~~m_Q : hard scale, quark creation and annihilation~~
 m_{Qv} : soft scale, momentum exchange between $Q\bar{Q}$
 m_{Qv}^2 : ultrasoft scale, binding/melting of $Q\bar{Q}$

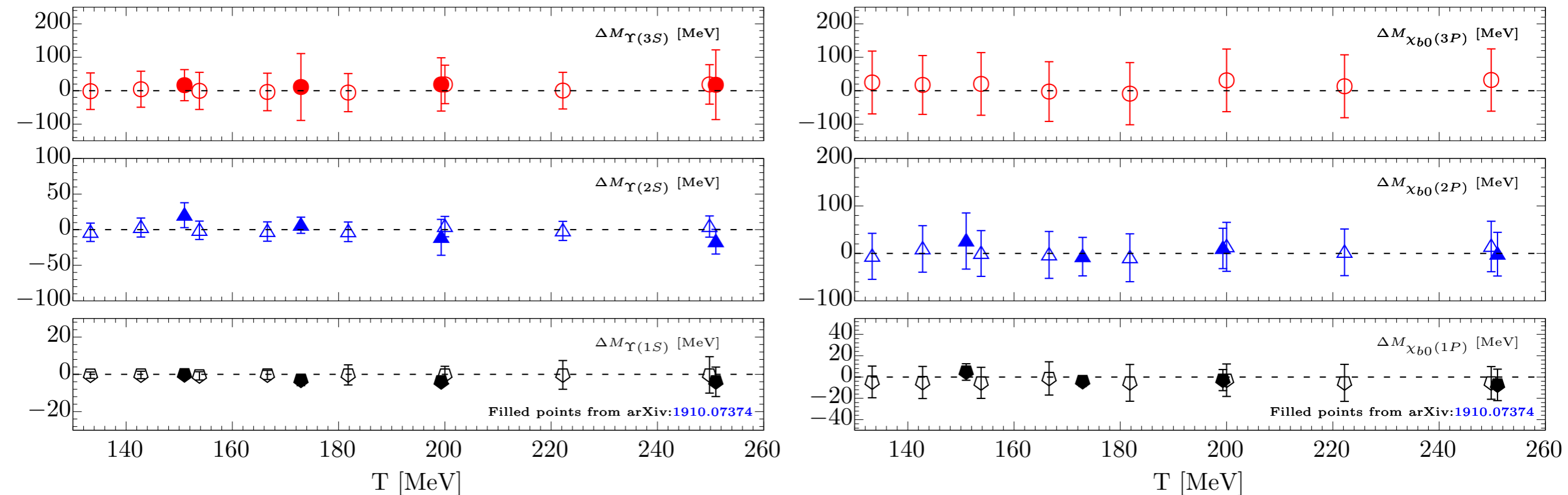
G. Aarts, et al., JHEP 07 (2014) 097
S. Kim, et al., JHEP 11 (2018) 088
R. Larsen, et al., PRD 100 (2019) 7, 074506
R. Larsen, et al., PLB 800 (2020) 135119



Quarkonium correlators can be computed with better resolution!

Mass shift of bottomonium from NRQCD

Model spectra: $\rho_\alpha^{\text{med}}(\omega, T) = A_\alpha^{\text{cut}}(T) \delta(\omega - \omega_\alpha^{\text{cut}}(T)) + A_\alpha(T) \exp\left(-\frac{[\omega - M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$



Filled points

a [fm]	0.1088	0.0951	0.0825	0.0655	0.0493
T [MeV]	151	173	199	251	341

Open points

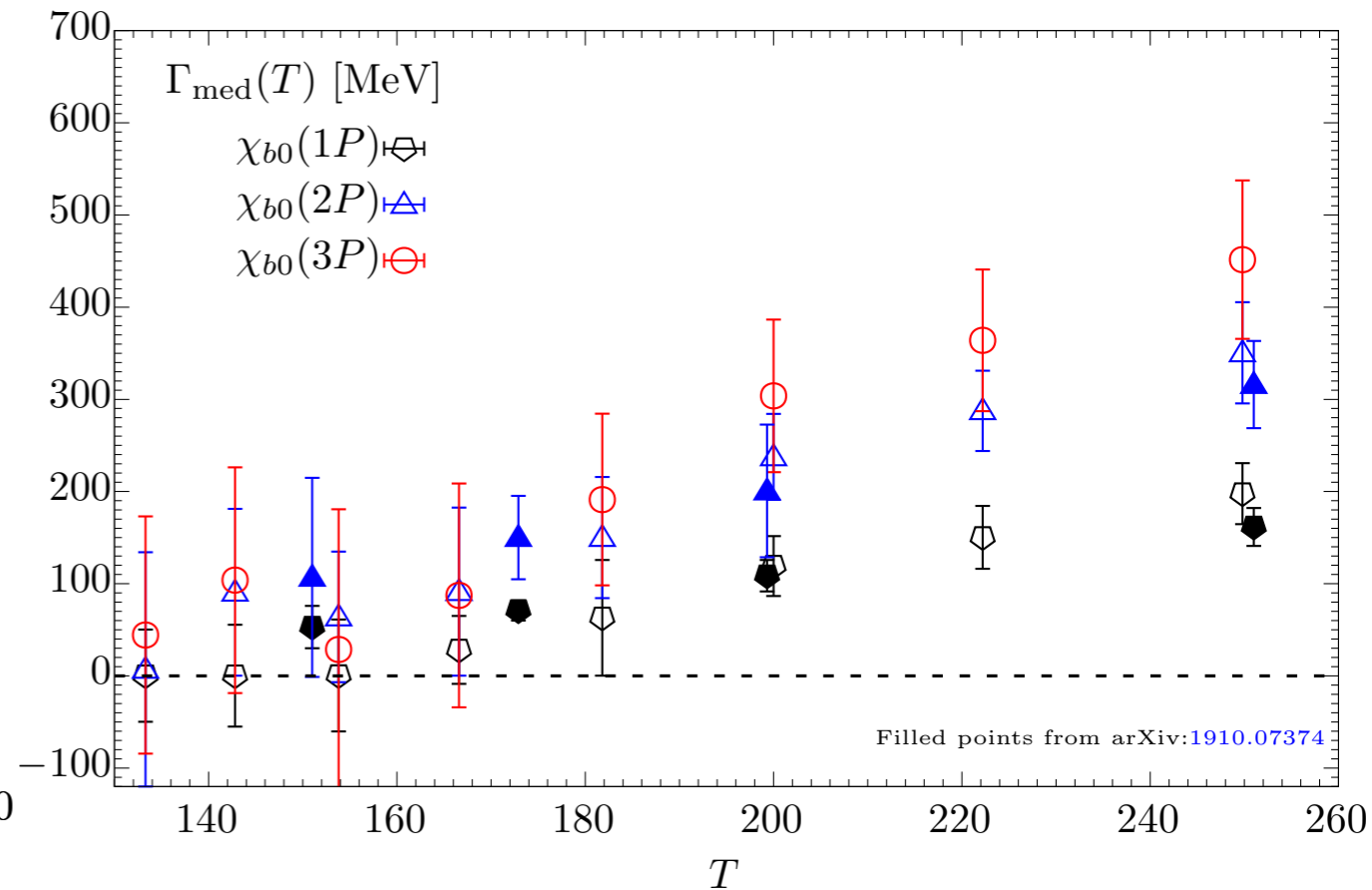
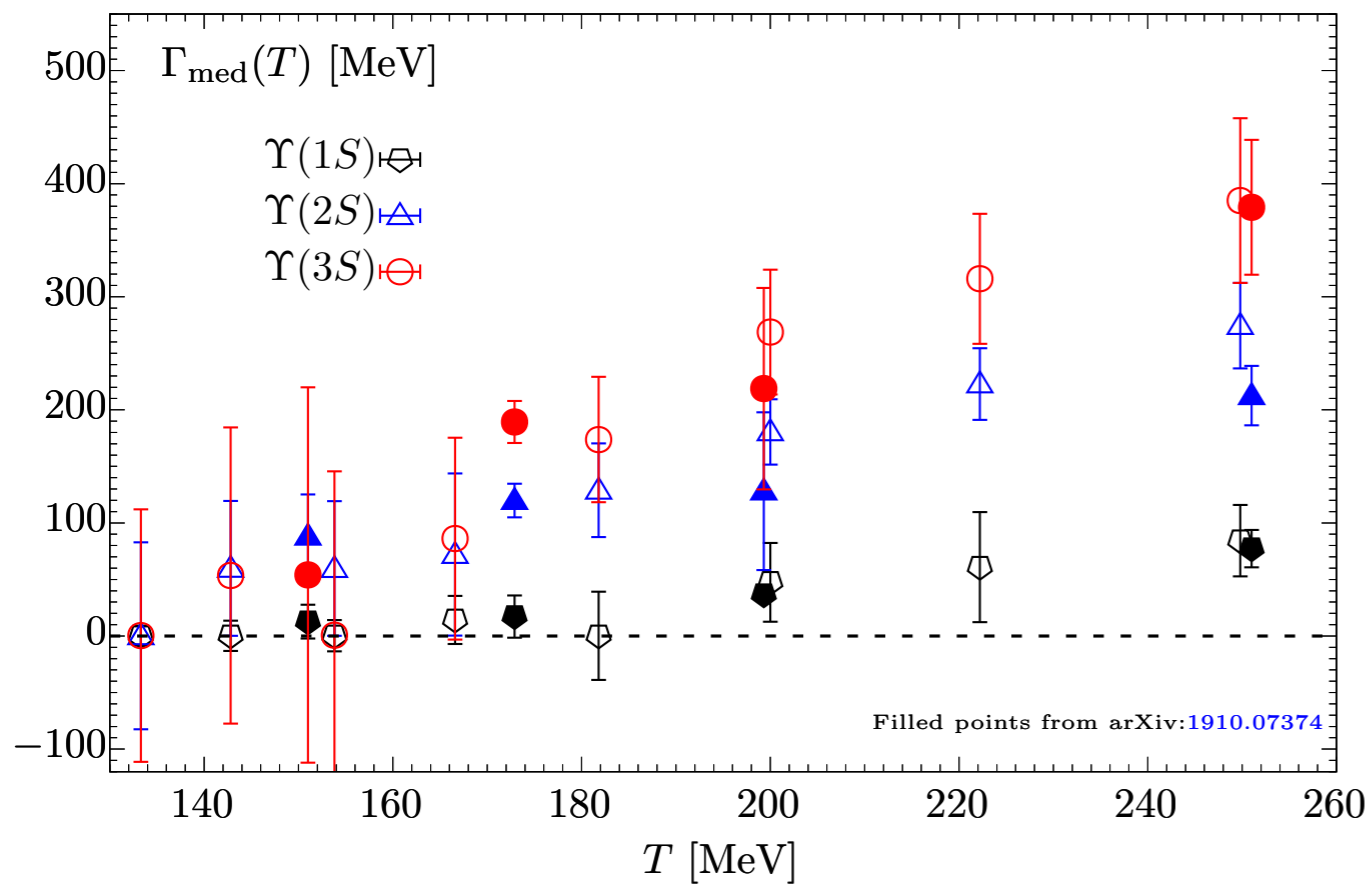
a [fm]	0.0493	0.0493	0.0493	0.0493	0.0493
T [MeV]	151	173	199	251	341

R. Larsen, et al., PLB 800 (2020) 135119

Wei-Ping Huang, talk @ Lattice 2024

- No mass shift for all states

Thermal width of bottomonium from NRQCD



R. Larsen, et al., PLB 800 (2020) 135119 & Wei-Ping Huang, talk @ Lattice 2024

- Increasing thermal width with temperature for all states
- Thermal broadening follows the hierarchical increasing pattern

Static quark potential

- Hard Thermal Loop resummed perturbation theory

M. Laine, JHEP0703,054 (2007)

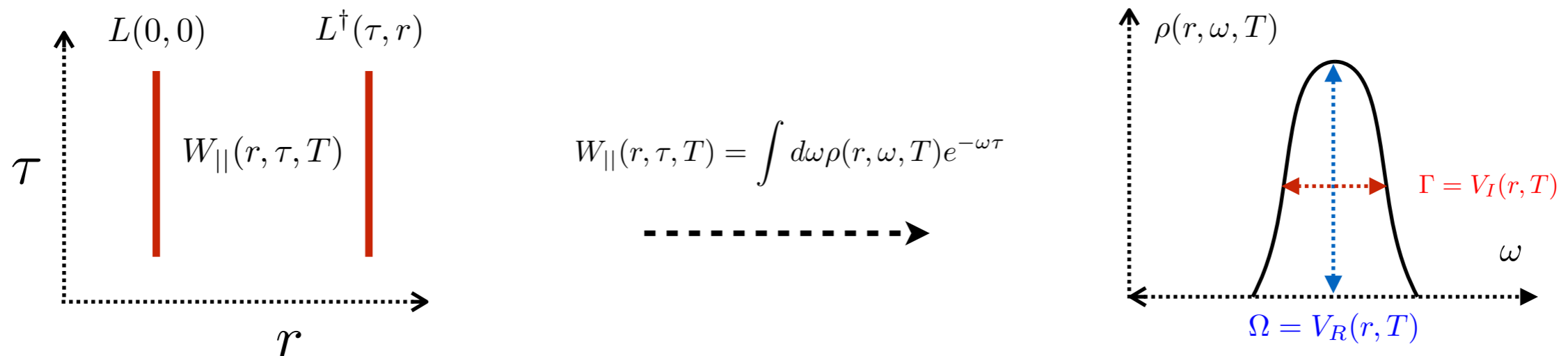
$$\lim_{t \rightarrow \infty} V_{>}^{(2)}(t, r) = -\frac{g^2 C_F}{4\pi} \left[m_D + \frac{\exp(-m_D r)}{r} \right] - \frac{ig^2 T C_F}{4\pi} \phi(m_D r)$$

Imaginary part becomes important for physical bottomonium at $T > 250$ MeV!

- Non-perturbative determination matters at around and above T_c

Wilson loop/thermal Wilson line correlators in Coulomb gauge

A. Rothkopf et al., PRL. 108 (2012) 162001

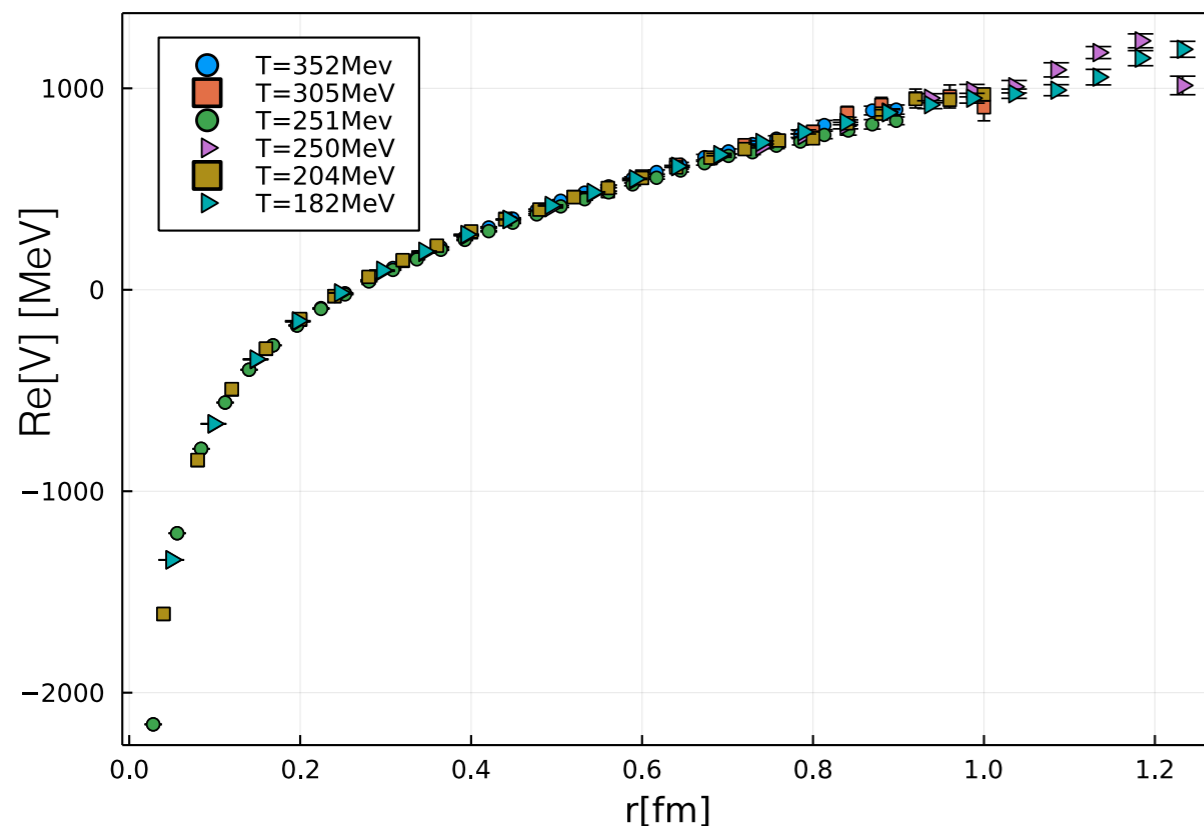


- Subtract continuum contribution from zero temperature correlators
- Model the potential as arguments of spectral function

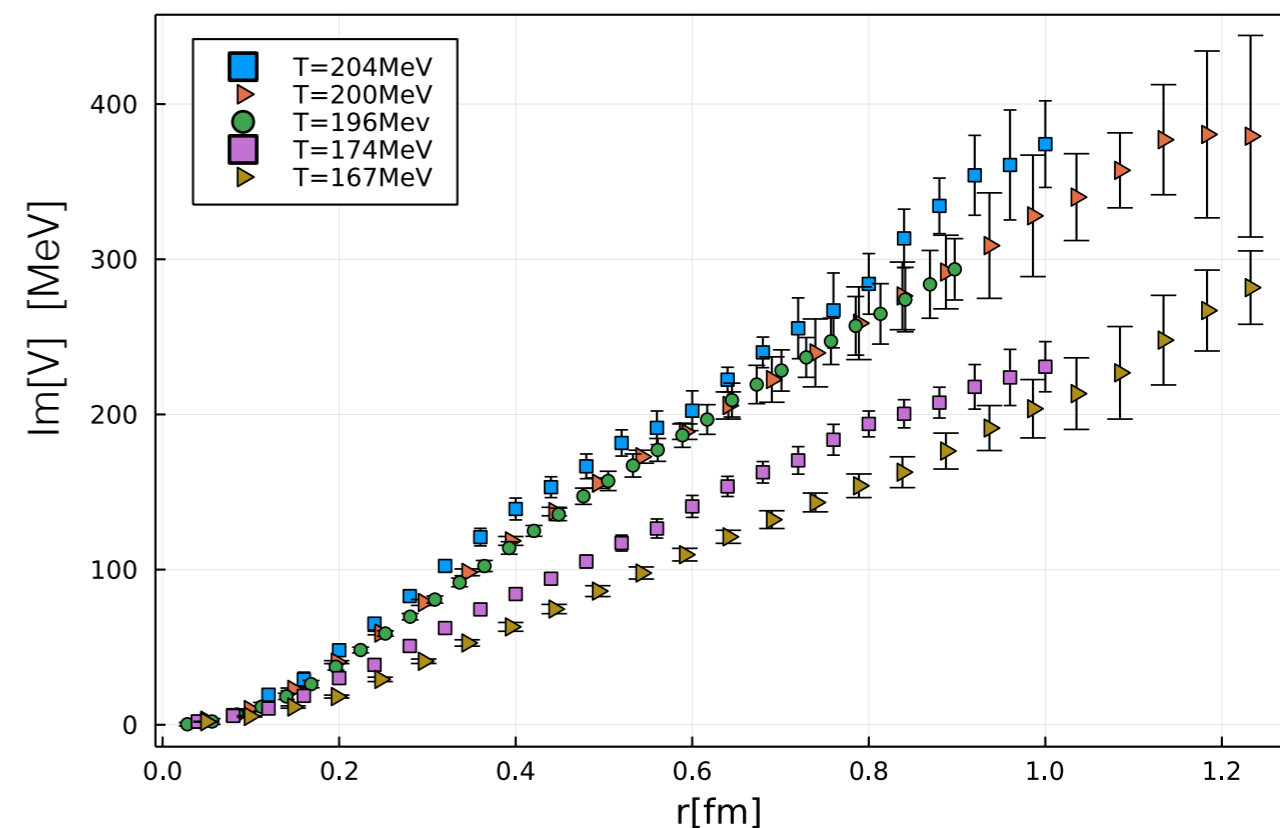
Static quark potential in hot QCD

$$N_f = 2 + 1, m_\pi = 160 \text{ MeV}, a \rightarrow 0$$

Real part: temperature insensitive



Imag. part: increasing with T & r



[HotQCD, PRD 109 (2024) 7, 074504]

$$\rho(r, \omega) = \frac{1}{\pi} \text{Im} \frac{A(T)}{\omega - \text{Re}V(r, T) - i\Gamma(\omega, r, T)}$$

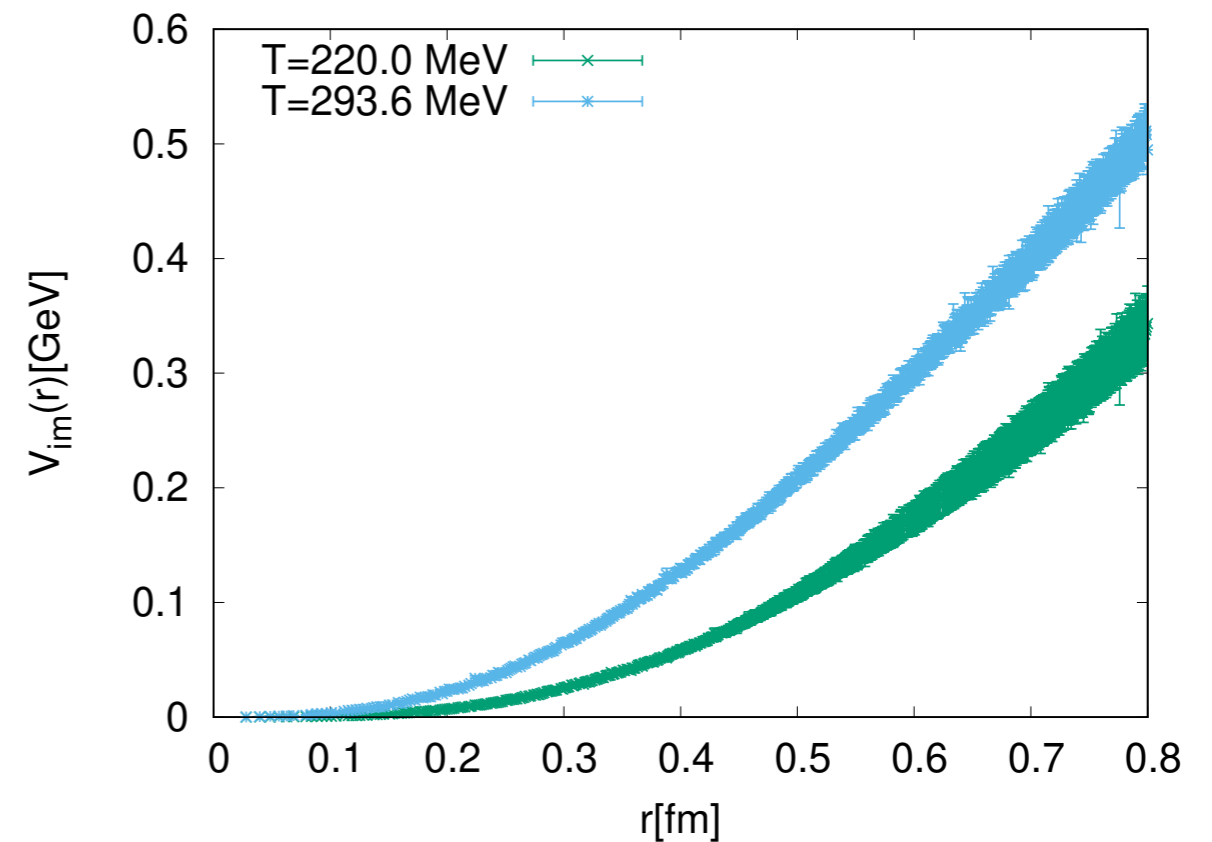
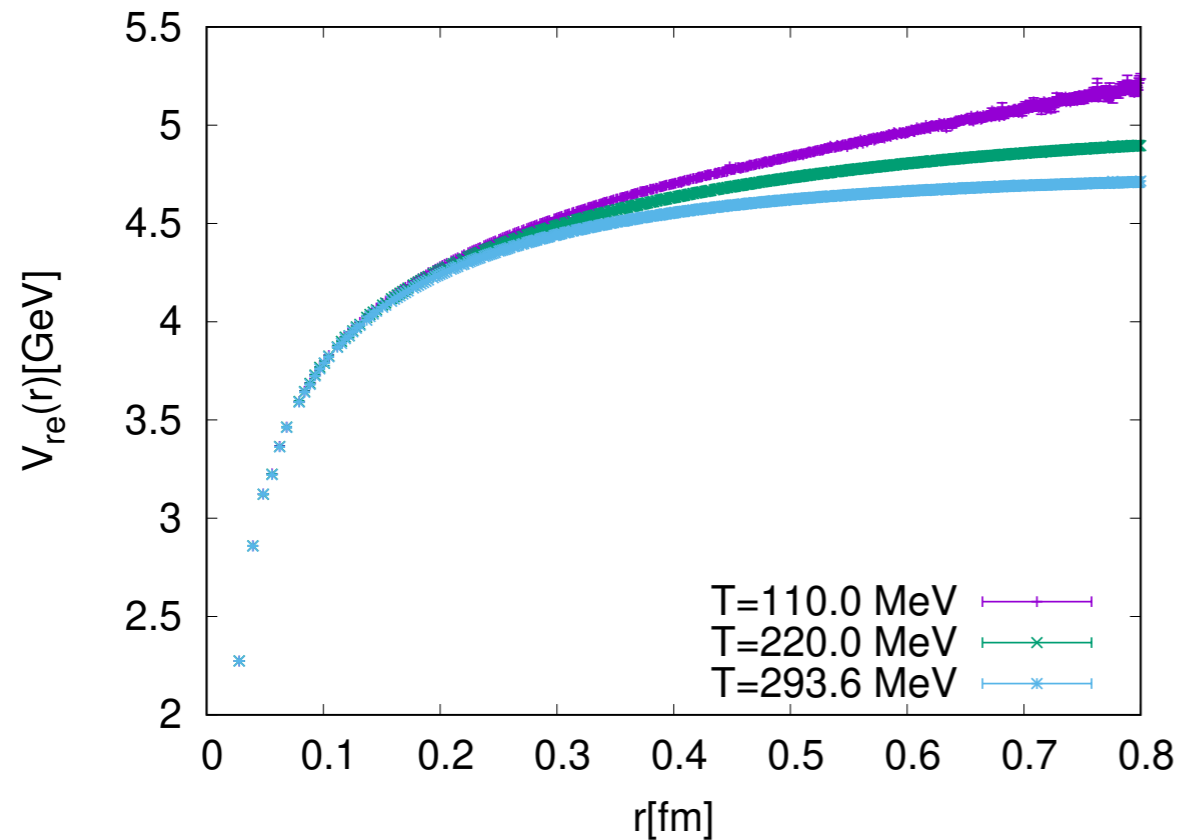
- No color screening from cut-Lorentzian model SPF (no mass shift for bottomonium)

Static quark potential in hot QCD

$$N_f = 2 + 1, m_\pi = 160 \text{ MeV}, a \rightarrow 0$$

Real part: temperature sensitive

Imag. part: increasing with T & r



[Dibyendu Bala, et al, in prep.]

$$\rho(r, \omega) = n_b(\omega) \left(c_{-1}/\omega + \sum_{l=0}^{\infty} c_{2l+1} \omega^{2l+1} \right)$$

- Color screening evident from HTL-inspired model SPF

Heavy quark diffusion via HQEFT

- Langevin equations of heavy quark motion

$$\partial_t p_i = -\eta_D p_i + \xi_i(t) \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

- Mass dependent **momentum** diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2 \omega^2}{3T \chi_q} \sum_i \frac{2T \rho_V^{ii}(\omega)}{\omega} \Big|_{\eta \ll |\omega| \lesssim \omega_{UV}}$$

- Large quark mass limit in HQ effective field theory

$$\kappa \equiv \frac{\beta}{3} \sum_{i=1}^3 \lim_{\omega \rightarrow 0} \left[\lim_{M \rightarrow \infty} \frac{M^2}{\chi_q} \int_{-\infty}^{\infty} dt e^{i\omega(t-t')} \int d^3 \vec{x} \left\langle \frac{1}{2} \{ \mathcal{F}^i(t, \vec{x}), \mathcal{F}^i(0, \vec{0}) \} \right\rangle \right]$$

J. Casalderrey-Solana and D. Teaney, PRD 74, 085012

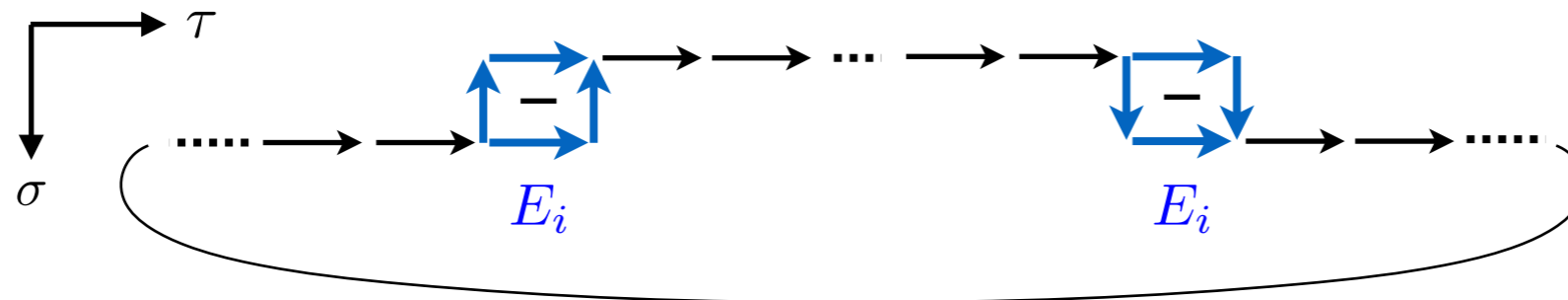
S. Caron-Huot et al., JHEP 0904 (2009) 053

A. Bouettefeux, M. Laine, JHEP 12 (2020) 150

$$\partial_t \mathbf{p} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \mathbf{F}$$

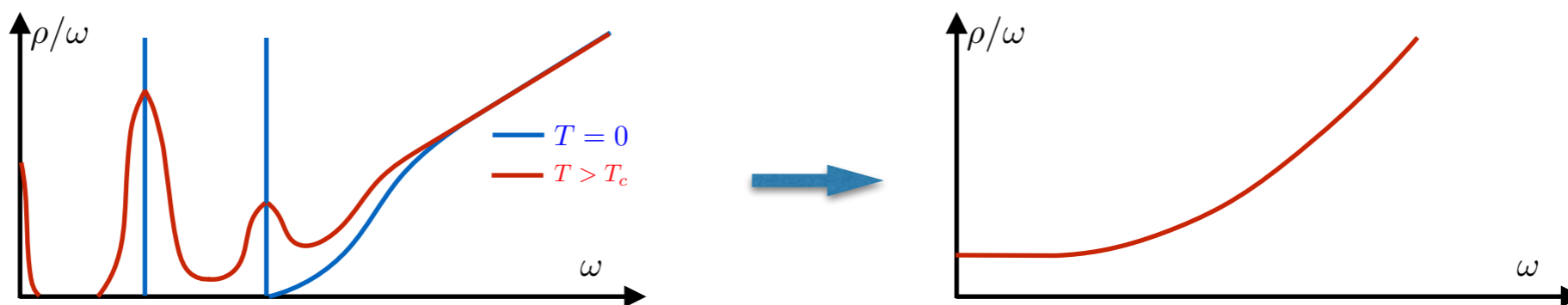
Momentum diffusion on the lattice

$$\kappa = \kappa_E + \frac{2}{3} \langle \mathbf{v}^2 \rangle \kappa_B \quad \langle \mathbf{v}^2 \rangle = \frac{3T}{M} \quad \frac{1}{2\pi T D} = \frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2} \lim_{\omega \rightarrow 0} \frac{\rho(\omega)}{\omega}$$



Color-electric field correlation function

$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

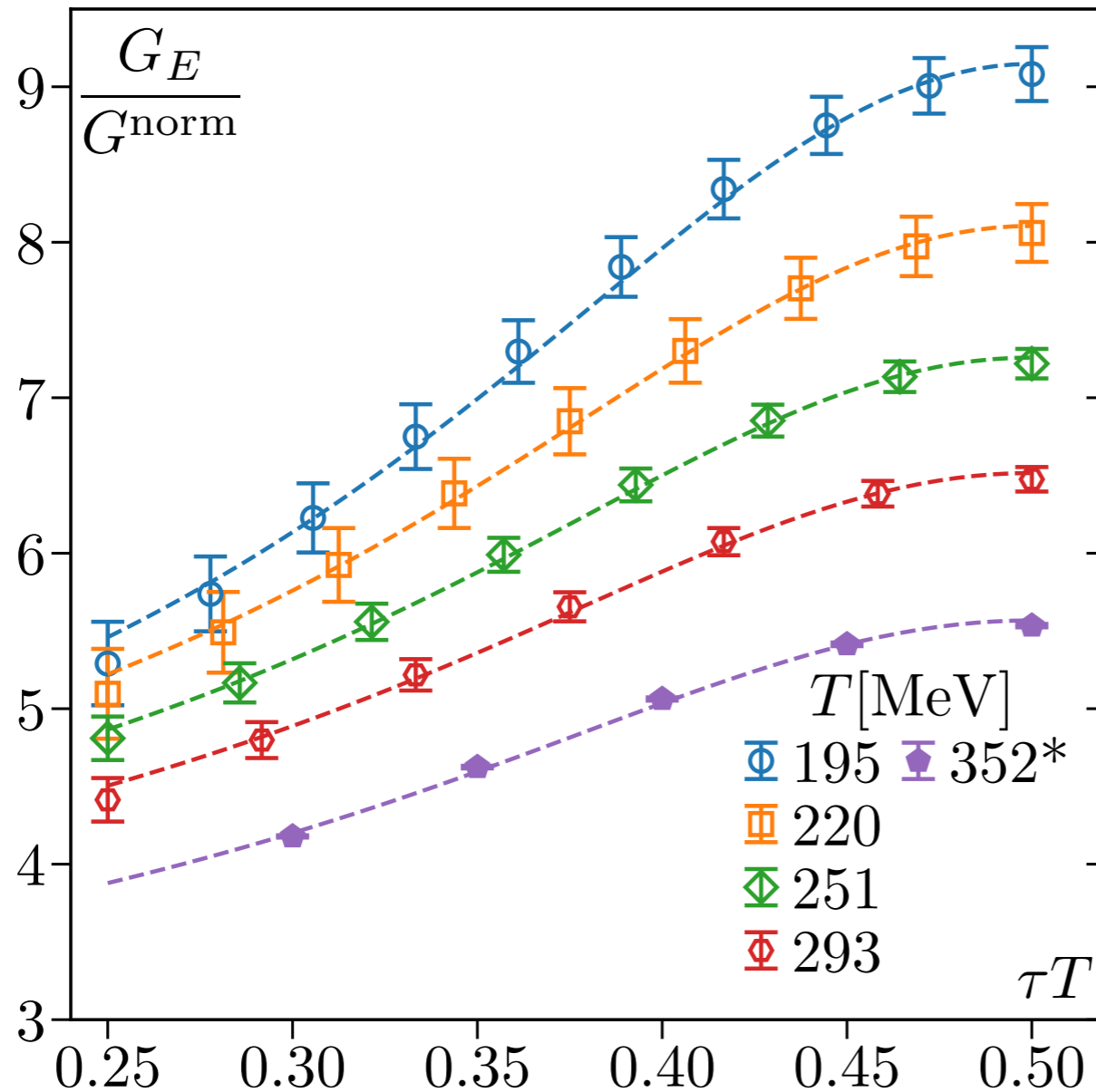


- Cheaper to measure on the lattice
- No peak structures in spectral functions
- Absence of transport peak

First full QCD calculation of EE correlators

First full QCD calculation of κ , only possible via Gradient Flow!

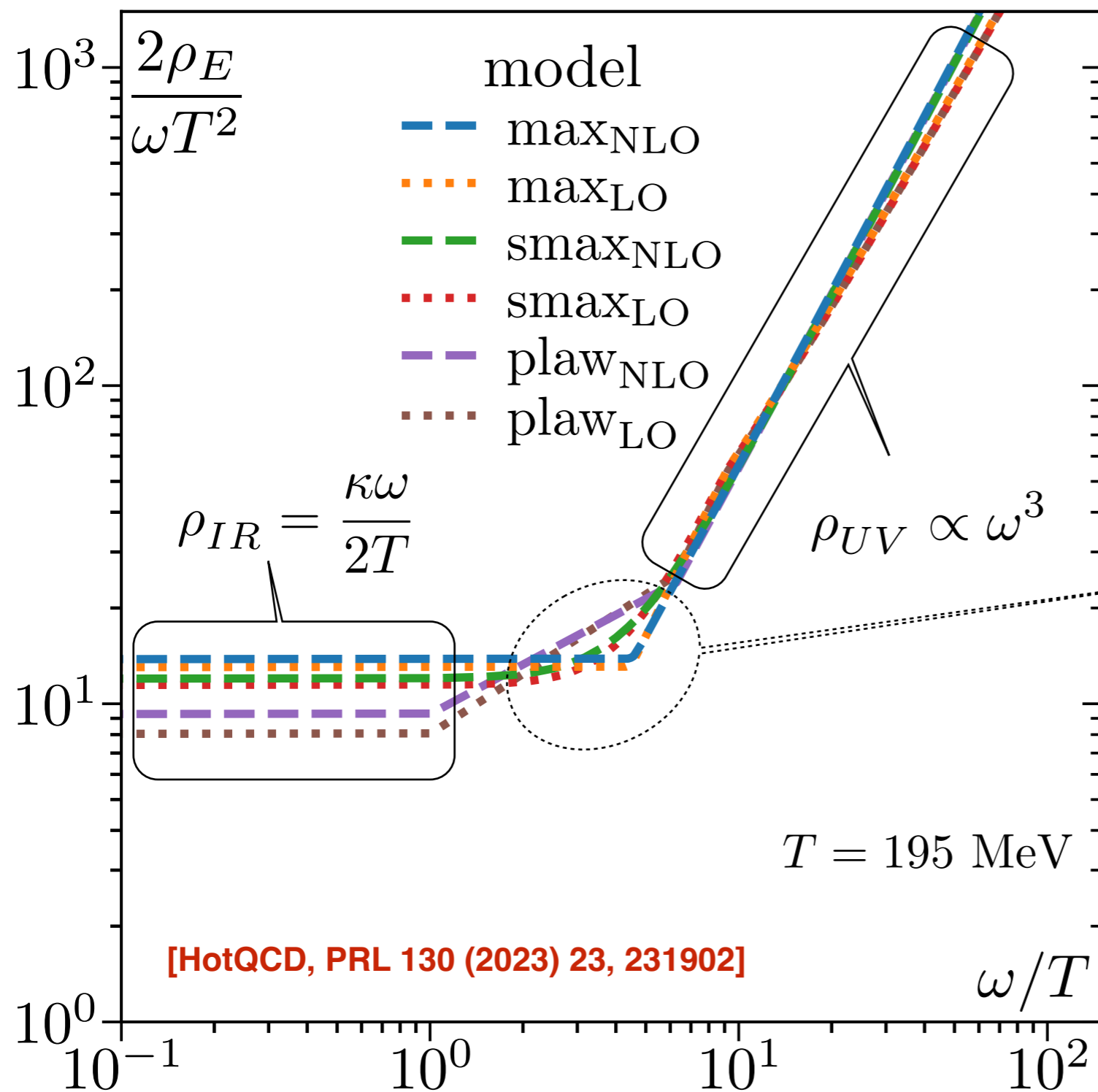
Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064



- $195 \text{ MeV} \leq T \leq 352 \text{ MeV}$
- 2+1 flavor in the sea
- pion mass 320 MeV

[HotQCD, PRL 130 (2023) 23, 231902]

Spectra analysis



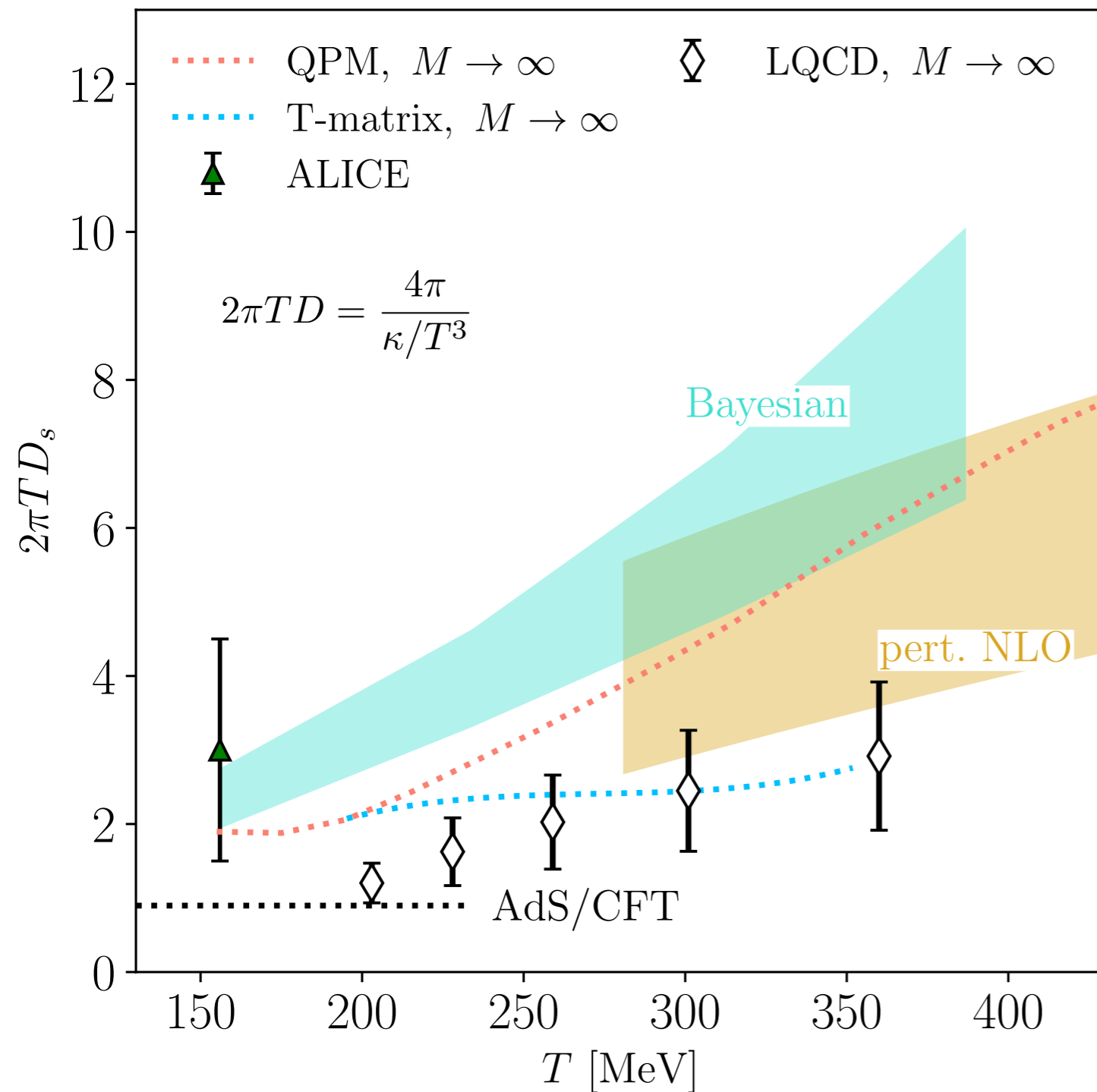
$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T) \rho(\omega, T)$$

$$\rho_{\max} \equiv \max(\phi_{IR}, \phi_{UV})$$

$$\rho_{\text{smax}} \equiv \sqrt{\phi_{IR}^2 + \phi_{UV}^2}$$

$$\rho_{\text{plaw}} \equiv \begin{cases} \phi_{IR} & \omega \leq \omega_{IR}, \\ a\omega^b & \text{for } \omega_{IR} < \omega < \omega_{UV}, \\ \phi_{UV} & \omega \geq \omega_{UV}, \end{cases}$$

HQ diffusion coefficient at HQ mass limit



- Agree with AdS/CFT at $\sim T_c$
- Agree with T-matrix estimate at moderate and high temperature
- Agree with NLO perturbative estimate at high temperature
- Mild temperature dependence
- Rapid equilibrium \longleftrightarrow QGP is near perfect fluid

$$\tau_{\text{kin}} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3} \left(\frac{T_c}{T}\right)^2 \left(\frac{M}{1.5 \text{ GeV}}\right) \frac{3 \text{ GeV}}{T_c^2}$$

[HotQCD, PRL 130 (2023) 23, 231902]

Finite mass correction

Physical charm & bottom quark not infinitely heavy!

$$M_c : \sim 1.3 \text{ GeV}$$

$$M_b : \sim 4.5 \text{ GeV}$$

D. Guazzini, et al., JHEP 10 (2007) 081

$$\kappa_E : M_Q \rightarrow \infty$$

→
$$\langle \mathcal{F}(t') \mathcal{F}(t) \rangle = q^2 \left\{ \langle E_i(t') E_j(t) \rangle + \frac{1}{3} \langle \mathbf{v}^2 \rangle \langle \delta_{ij} B_k(t') B_k(t) - B_j(t') B_i(t) \rangle \right\}$$

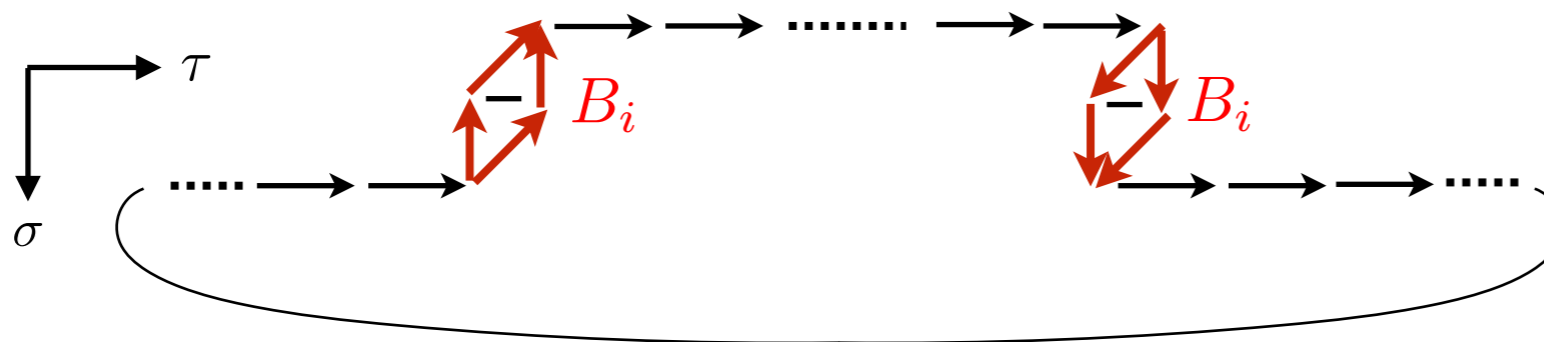
Infinite heavy limit
Finite mass correction

$$\kappa = \kappa_E + \frac{2}{3} \langle \mathbf{v}^2 \rangle \kappa_B$$

$$\frac{2}{3} \cdot \langle \mathbf{v}^2 \rangle_{\text{charm}} : 18\% \sim 30\%$$

$$\frac{2}{3} \cdot \langle \mathbf{v}^2 \rangle_{\text{bottom}} : 7\% \sim 13\%$$

$$D_s = \frac{2T^2}{\kappa} \implies D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$



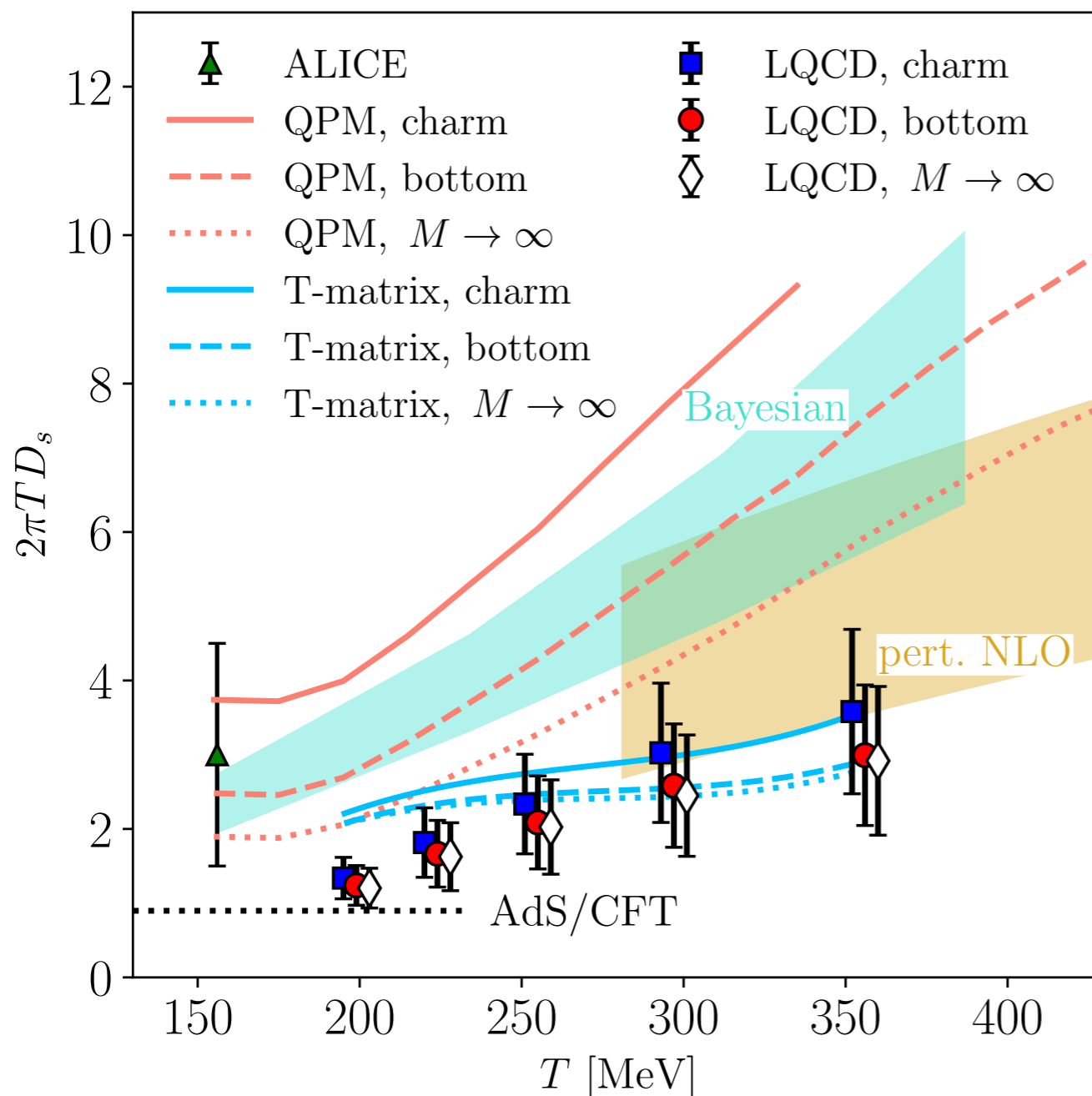
Color-magnetic field correlation function

$$G_B^{\text{phys.}} = Z_{\text{match}} G_B^{\text{flow}}(\tau_F)$$

A. Bouteffaux, M. Laine, JHEP 12 (2020) 150

Charm and Bottom quark diffusion

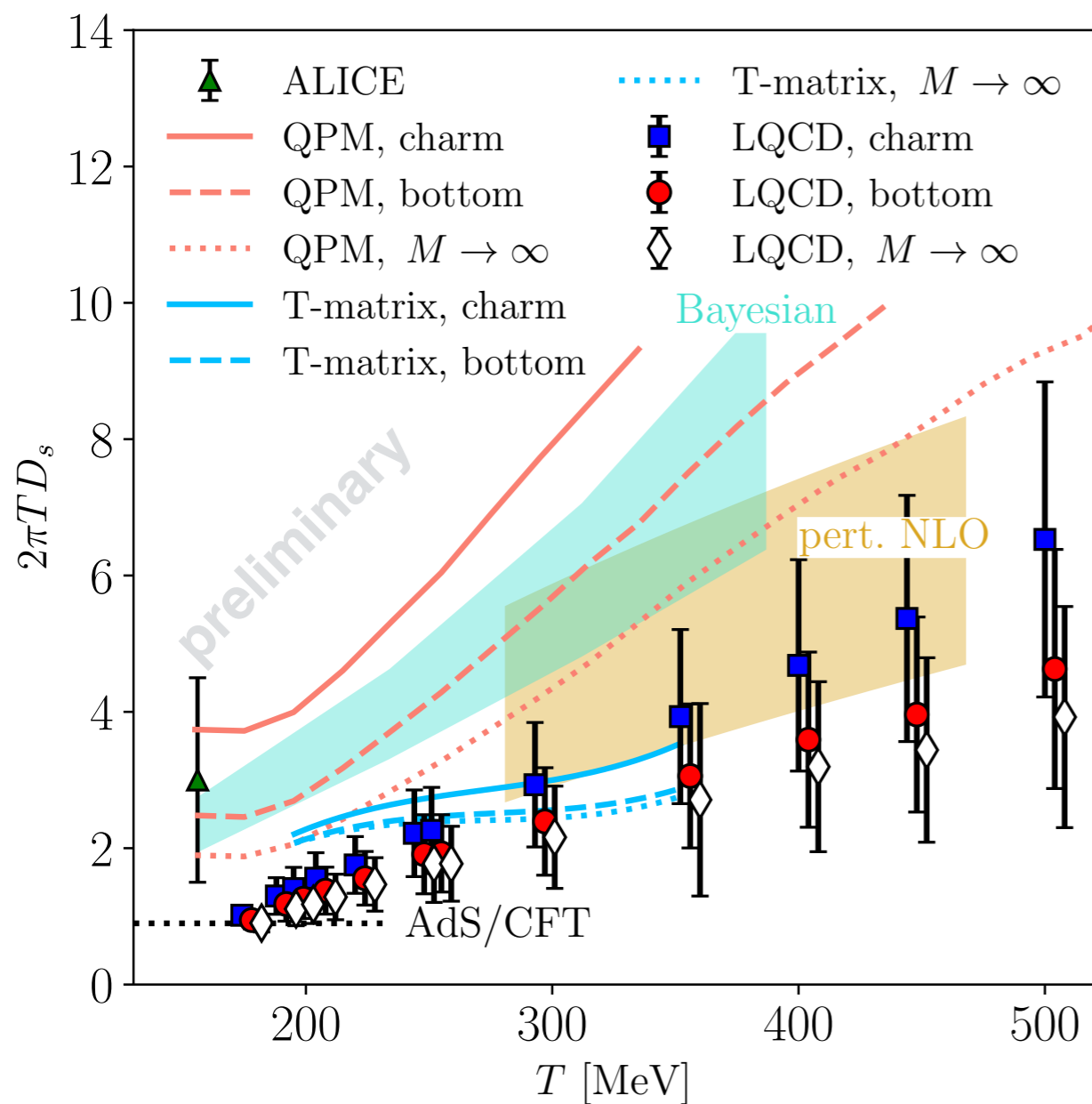
First full QCD determination of charm&bottom quark diffusion!



- HQ mass dependence of HQ diffusion: mild
- Universal change pattern with quark mass
- Weak quark mass dependence in LQCD & T-matrix
- Weaker than quasi-particle model (QPM) calculations

[HotQCD, PRL 132 (2024) 5, 051902]

HQ diffusion at the physical point



[J.D. Golan, Swagato Mukherjee, P. Petreczky, HTS, J.H. Webber, work in progress]

$$195 \text{ MeV} \leq T \leq 352 \text{ MeV}$$

$$m_\pi \sim 320 \text{ MeV}$$



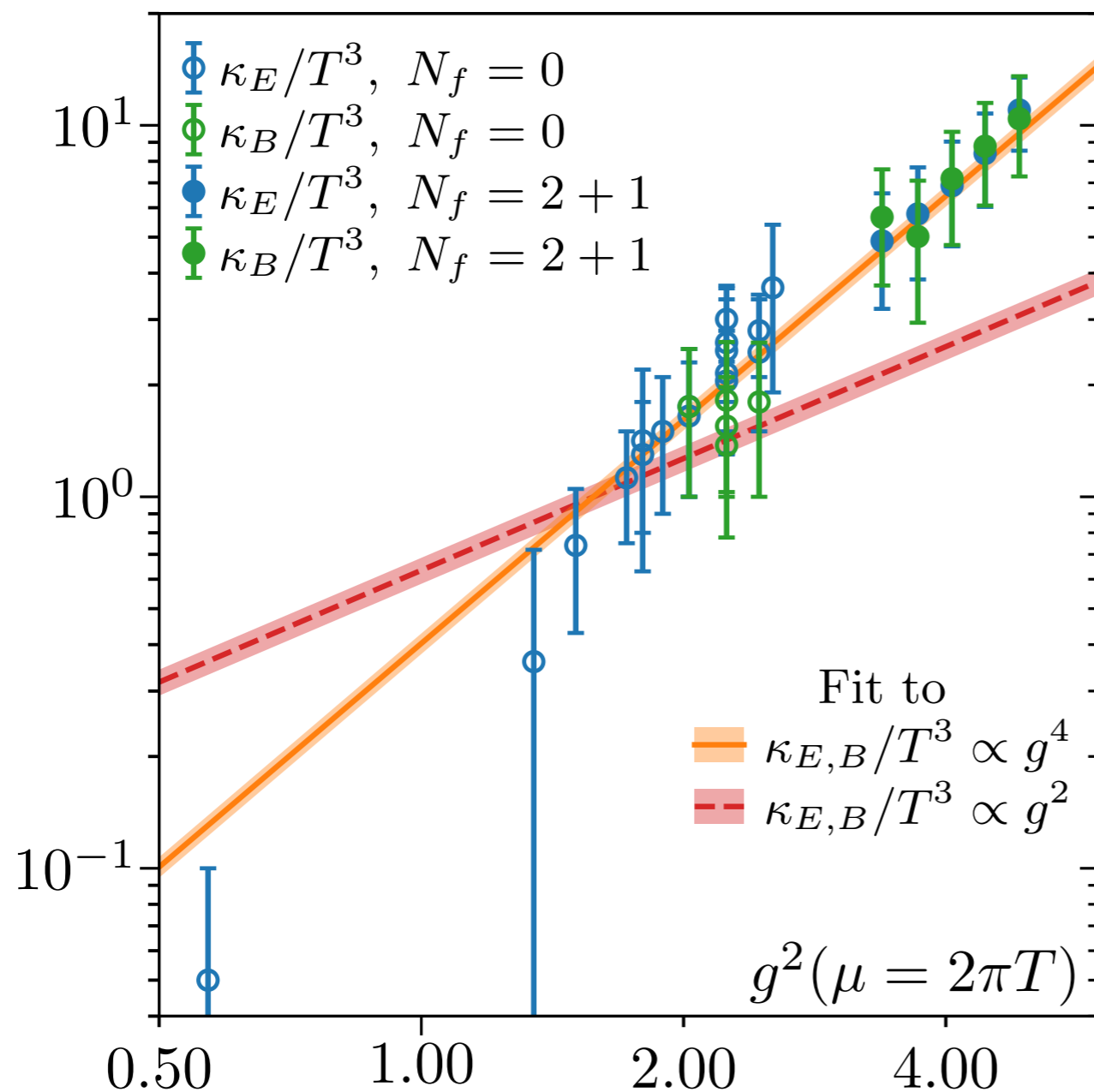
$$174 \text{ MeV} \leq T \leq 500 \text{ MeV}$$

$$m_\pi \sim 160 \text{ MeV}$$

Increasing stat. for $T = 153 \text{ MeV}, 164 \text{ MeV}$

- Physical pion mass
- Wide temperature range down to 174 MeV and up to 500 MeV
- Consistent observation as previous studies
- Almost invisible light quark mass dependence

κ_E V.S. κ_B



[L. Altenkort, D. Cruz, O. Kaczmarek, G.D. Moore, HTS, PRD 109 (2024) 11, 114505]

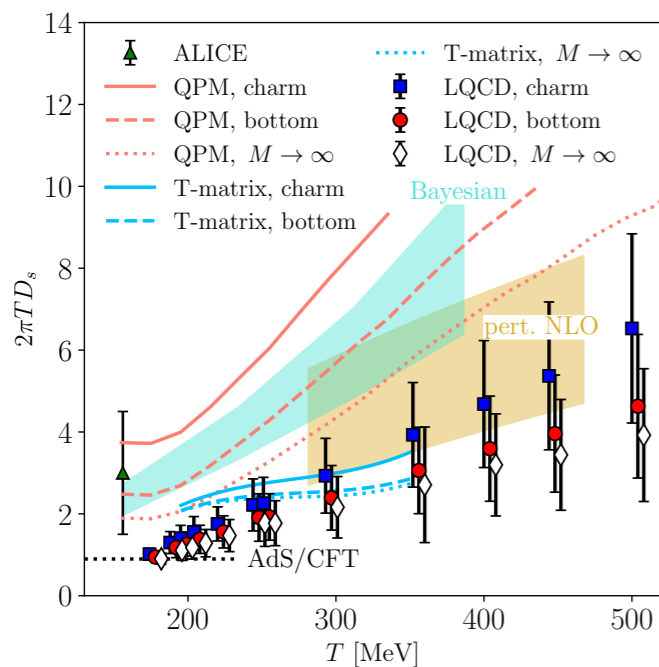
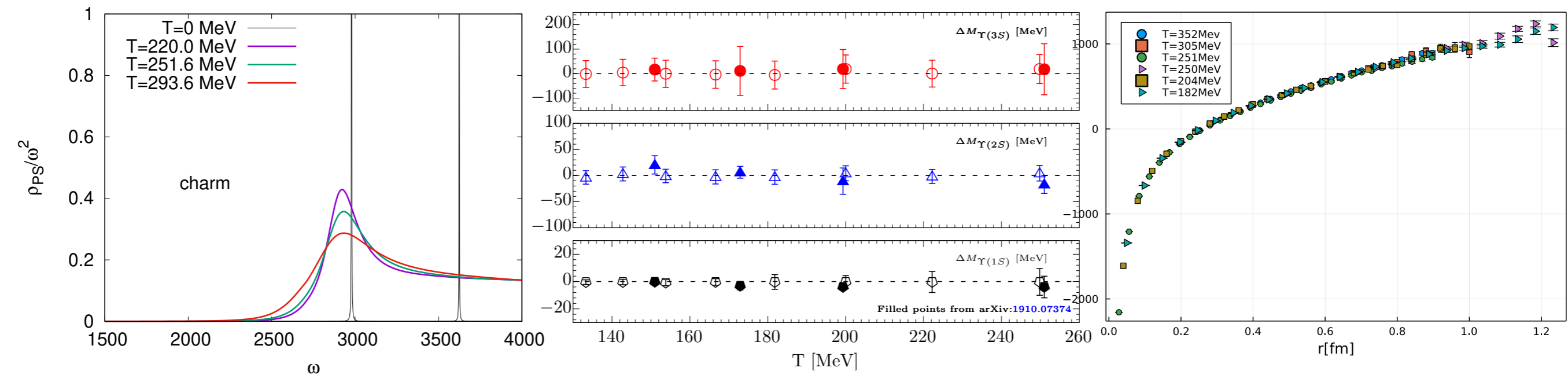
Quenched results from:

- A. Francis, et al., PRD92, 116003
- B. L. Altenkort, et al., PRD103,014511
- D. Banerjee, et al., Nucl.Phys.A.2023.122721
- D. Banerjee, et al., JHEP 08 (2022) 128
- N. Brambilla, et al., PRD107, 054508

==> S. Caron-Huot and G. D. Moore, PRL. 100, 052301 (2008)

- Similar magnitude for κ_E and κ_B in full QCD & quenched
- Smooth connection between quenched and full QCD in temperature
- Lattice results confirms the form suggested by pert. computations

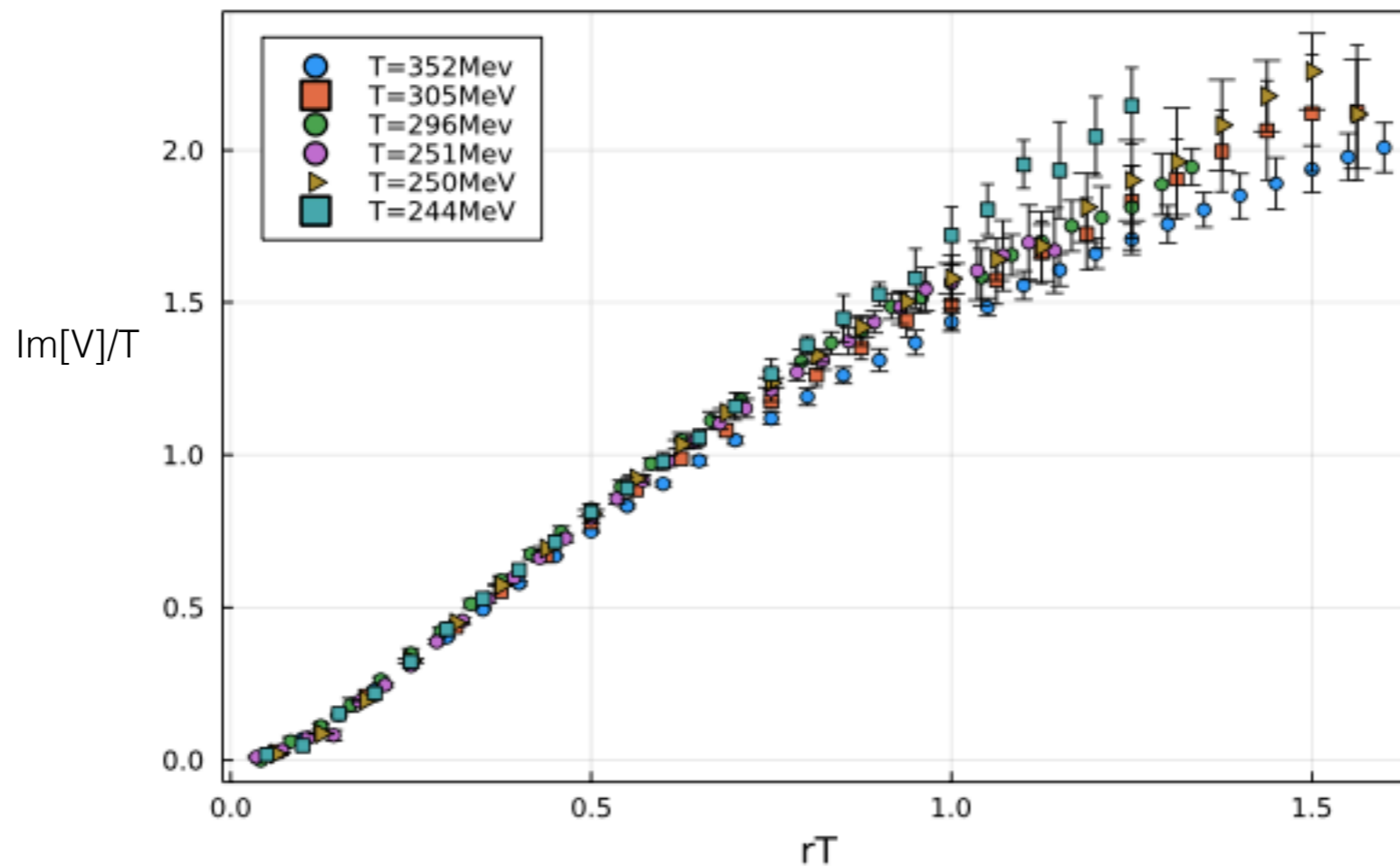
Major updates since HP 2023



- First full QCD study of quarkonium using relativistic heavy quarks
- No mass shift for bottomonium & increasing thermal width
- Screening (or not?) for $\text{Re}[V]$ & increasing $\text{Im}[V]$ with T & r
- First full QCD calculation of HQ diffusion coefficient at the physical point in $174 \text{ MeV} \leq T \leq 500 \text{ MeV}$

Lattice & Effective theory are powerful tools to provide accurate and realistic inputs for HIC phenomenology

Backup: why no screening



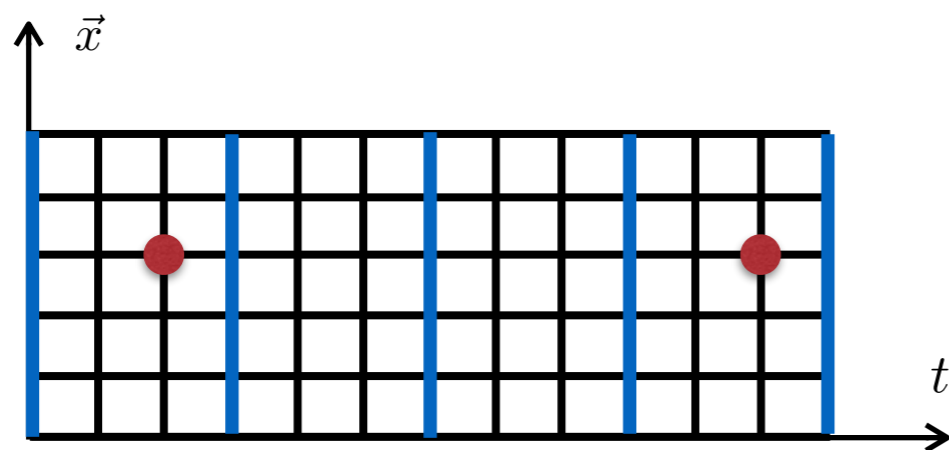
$t \gg 1/T$ for free energy, dissociation & recombination
 $1/\text{Im}[V] < 1/T$ decays within time scale

$$\langle P(\mathbf{m})P(\mathbf{n})^\dagger \rangle = e^{-a N_T F_{\bar{q}q}(a|\mathbf{m}-\mathbf{n}|)} = e^{-F_{\bar{q}q}(r)/T}$$

$$W(\tau, r, T) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_r(\omega, T)$$

Multi-level v.s. gradient flow can go!!!!

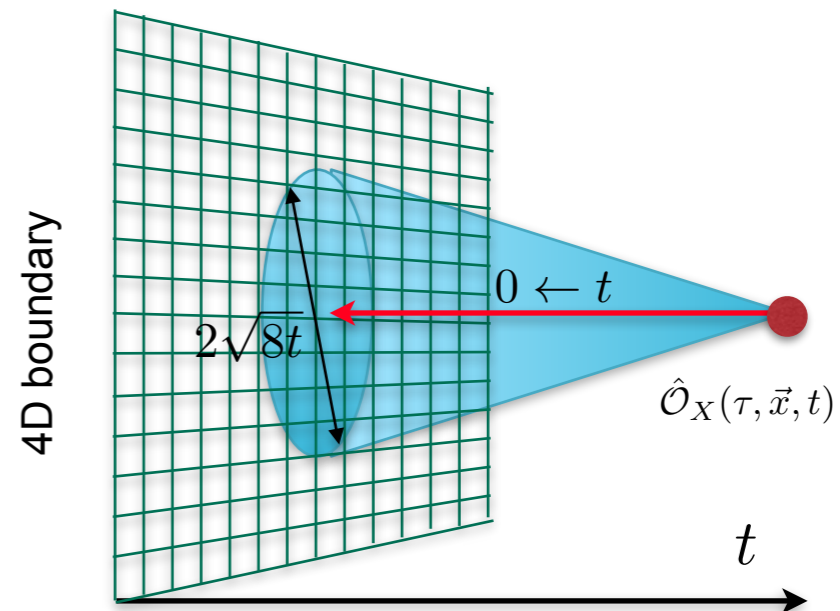
multi-level



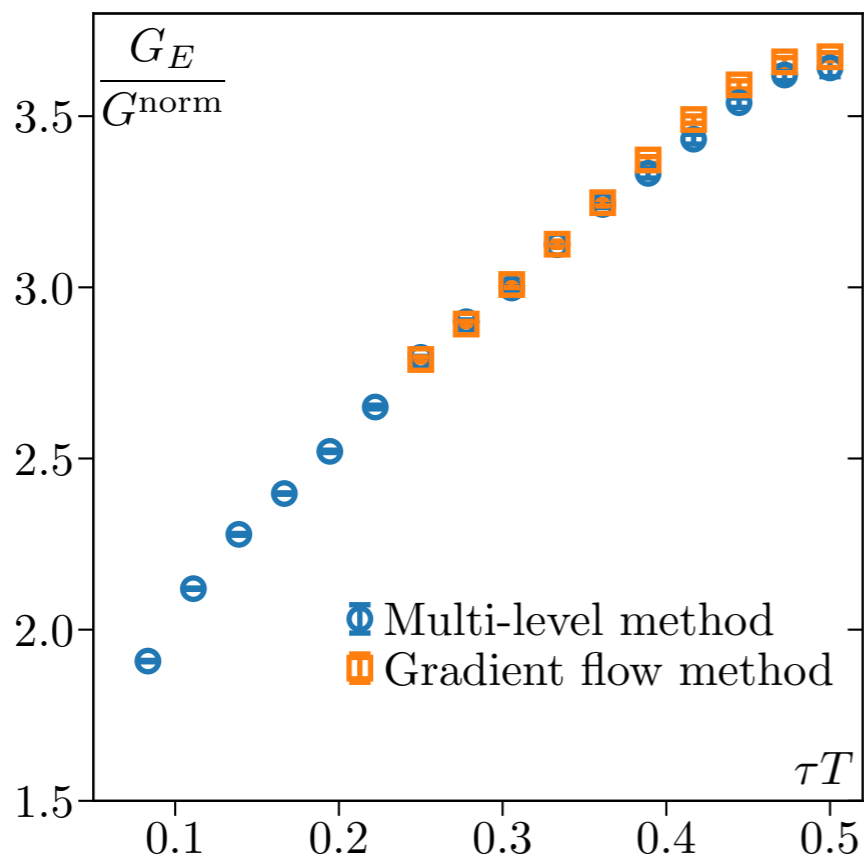
M. Luscher and P. Weisz, JHEP 09 (2001) 010

v.s.

gradient flow



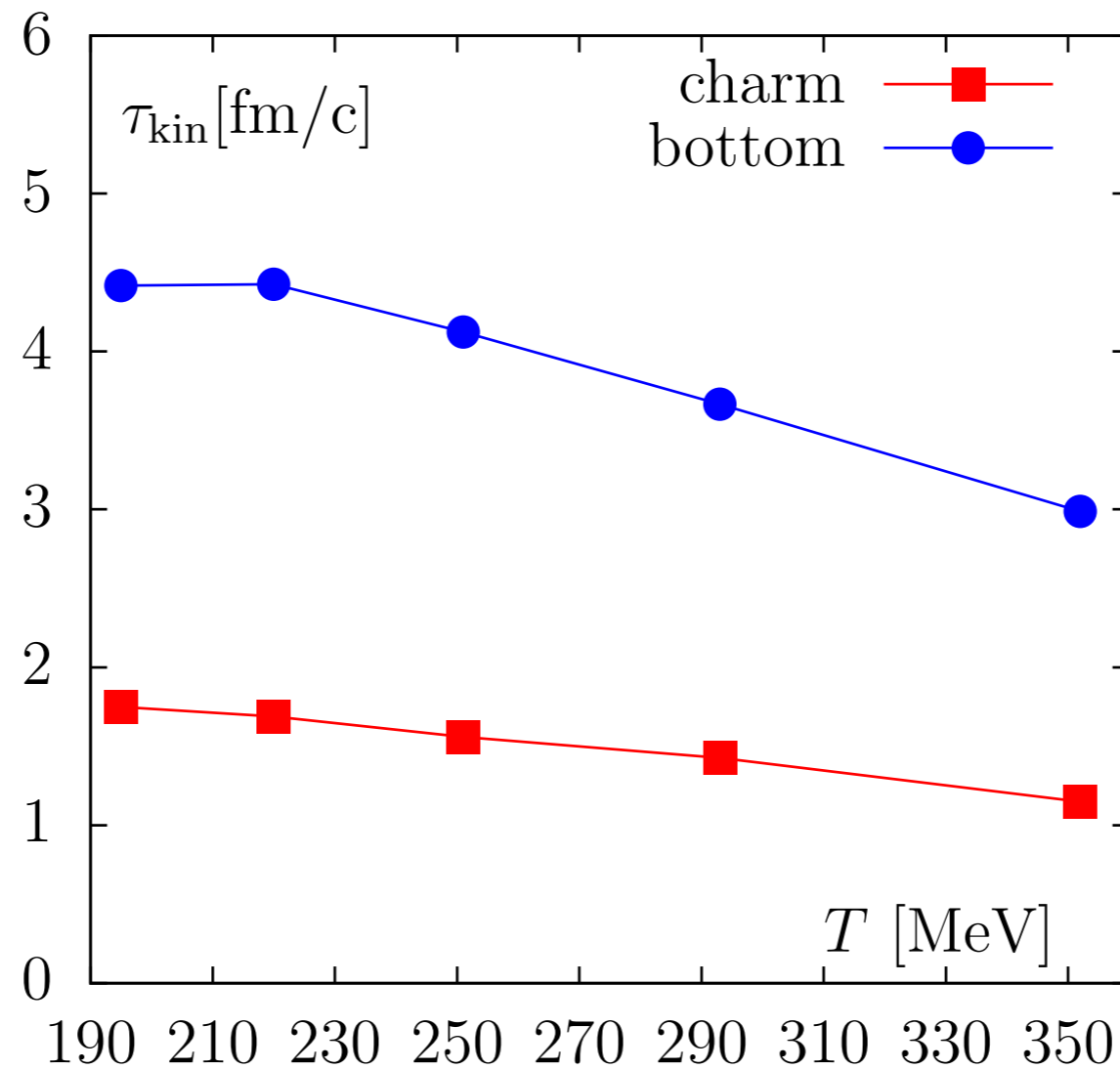
Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064



L. Altenkort, HTS, et al., PRD 103 (2021) 1, 014511

- Consistent quenched results from ML & GF
- Gradient flow paves the way to full QCD

Backup: equilibration time



$$\tau_{\text{kin}} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3} \left(\frac{T_c}{T}\right)^2 \left(\frac{M}{1.5 \text{ GeV}}\right)^3 \frac{3 \text{ GeV}}{T_c^2}$$

- Equilibration time of charm quark favors the experimental estimate (~ 1 fm/c for all)

Backup: full QCD setup

$$N_f = 2 + 1, \text{ HISQ}, m_\pi = 320 \text{ MeV}$$

T [MeV]	β	am_s	am_l	N_σ	N_τ	# conf.
195	7.570	0.01973	0.003946	64	20	5899
	7.777	0.01601	0.003202	64	24	3435
	8.249	0.01011	0.002022	96	36	2256
220	7.704	0.01723	0.003446	64	20	7923
	7.913	0.01400	0.002800	64	24	2715
	8.249	0.01011	0.002022	96	32	912
251	7.857	0.01479	0.002958	64	20	6786
	8.068	0.01204	0.002408	64	24	5325
	8.249	0.01011	0.002022	96	28	1680
293	8.036	0.01241	0.002482	64	20	6534
	8.147	0.01115	0.002230	64	22	9101
	8.249	0.01011	0.002022	96	24	688
352	8.249	0.01011	0.002022	96	20	2488

- Wide temperature range
- Different lattice spacings
- Large lattices towards thermodynamic limit

Backup: anomalous dimension of B-field

- Anomalous dimension in MSbar-scheme $Z_B = 1 + \frac{g^2 C_A}{(4\pi)^2} \left[\frac{1}{\epsilon} + 2 \ln \left(\frac{\bar{\mu} e^{\gamma_E}}{4\pi T} \right) - 2 \right] + \mathcal{O}(g^4)$

- Gradient flow-scheme \rightarrow MSbar-scheme \rightarrow physical values

- Scale dependence must go for “WeWant” and $\langle BB \rangle_{\tau_F}$

$$\text{WeWant} = Z_B^2 \langle BB \rangle_{\text{MS}}$$

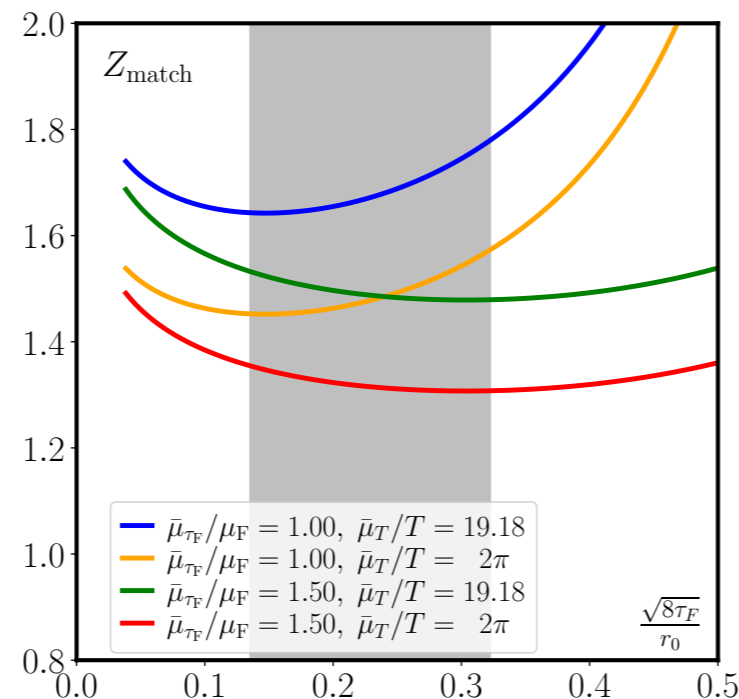
$$Z^2 = \left(1 - 2 \frac{g^2 C_A}{16\pi^2} \ln(\mu^2 \tau_F) \right) \left(1 + 2K \frac{g^2 C_A}{16\pi^2} \right) \equiv Z_f^2 Z_K^2$$

$$\langle BB \rangle_{\tau_F} \equiv Z^{-2} \langle BB \rangle_{\text{MS}}$$

$$\text{WeWant} = Z_B^2 Z^2 \langle BB \rangle_{\tau_F}$$

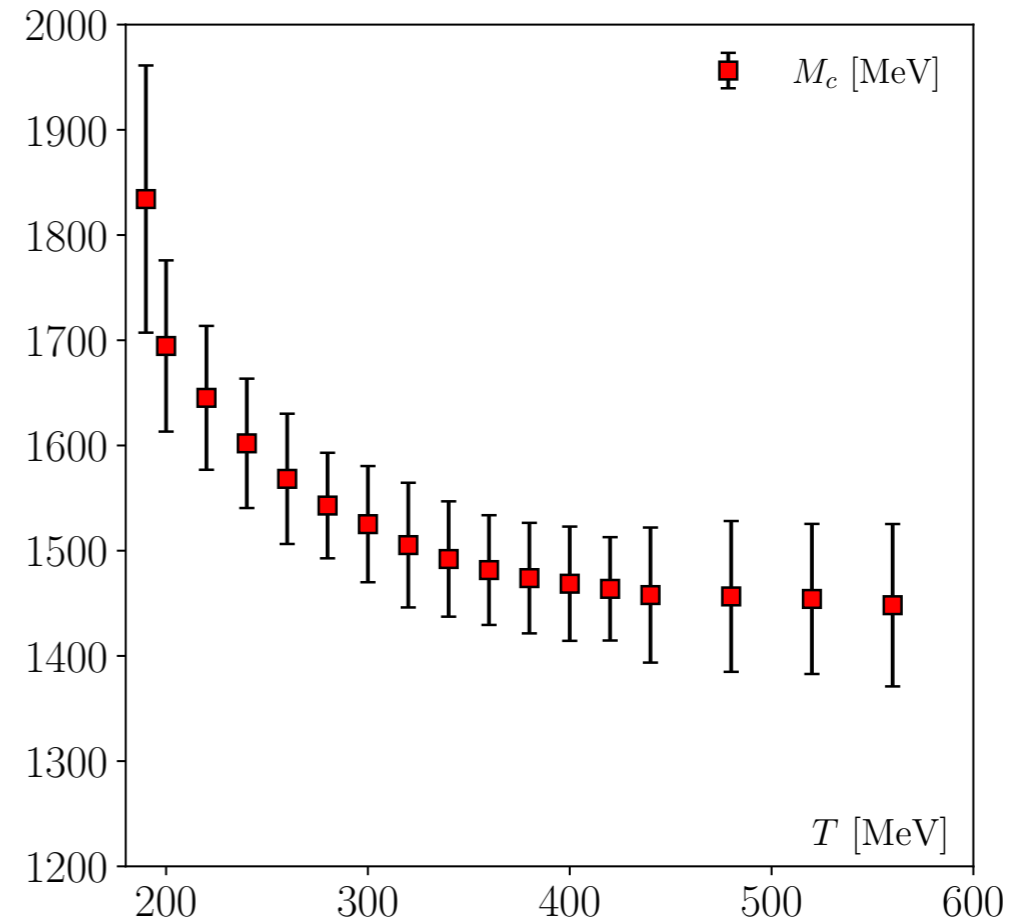
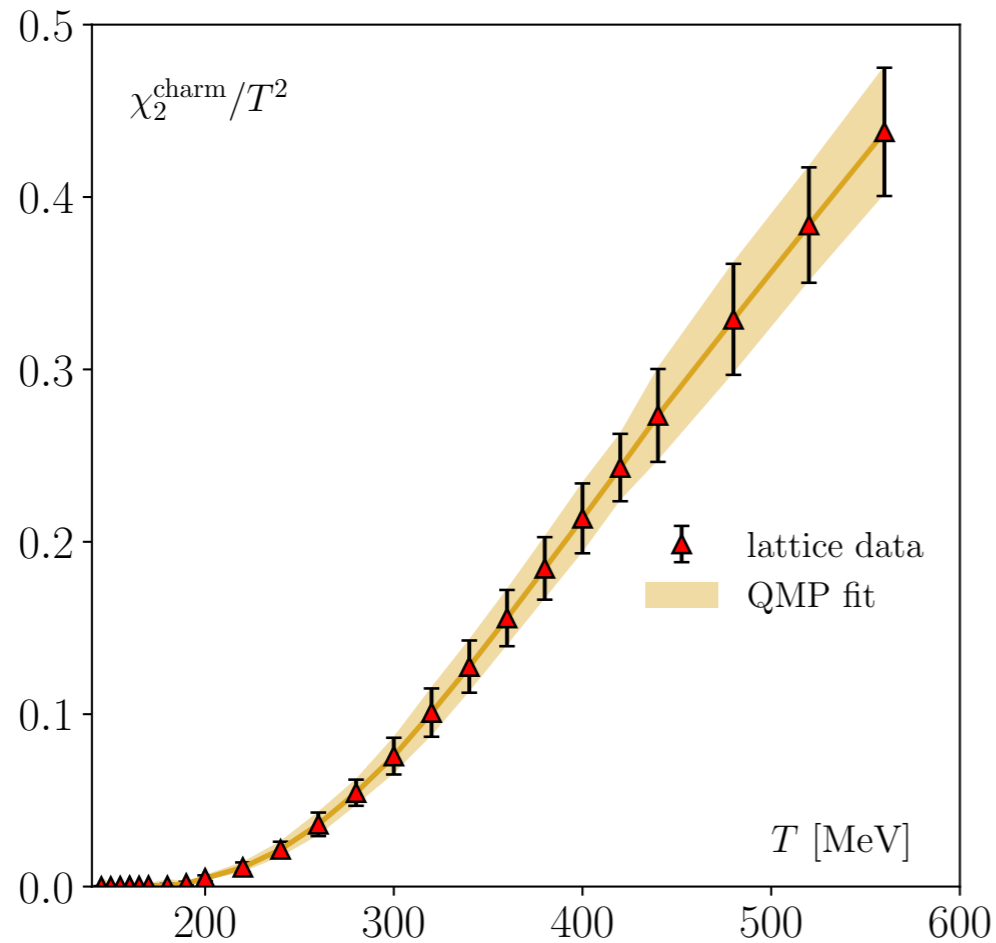
- Determination of the matching factor

$$\ln Z_{\text{match}} = \int_{\bar{\mu}_T^2}^{\bar{\mu}_{\tau_F}^2} \gamma_0 g_{\text{MS}}^2(\bar{\mu}) \frac{d\bar{\mu}^2}{\bar{\mu}^2} + \gamma_0 g_{\text{MS}}^2(\bar{\mu}_T) \left[\ln \frac{\bar{\mu}_T^2}{(4\pi T)^2} - 2 + 2\gamma_E \right] - \gamma_0 g_{\text{MS}}^2(\bar{\mu}_{\tau_F}) \left[\ln \frac{\bar{\mu}_{\tau_F}^2}{4\mu_F^2} + \gamma_E \right]$$



[PRL 132 (2024) 5, 051902]

Backup: T-dependent charm quark mass



[PRL 132 (2024) 5, 051902]

$$\frac{\chi_2^{\text{charm}}}{T^2} = \frac{4N_c}{(2\pi T)^3} \int d^3p e^{-E_p/T}$$

$$E_p^2(T) = m^2(T) + p^2$$

$m(T)$

$$\kappa = \kappa_E + \frac{2}{3} \langle \mathbf{v}^2 \rangle \kappa_B$$

$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2 \rangle}{3MT}$$

$$\langle v^2 \rangle = \left(\int d^3p \frac{p^2}{E_p^2} e^{-E_p/T} \right) / \left(\int d^3p e^{-E_p/T} \right)$$

$$\langle p^2 \rangle = \left(\int d^3p p^2 e^{-E_p/T} \right) / \left(\int d^3p e^{-E_p/T} \right)$$