Gluon to $q\bar{q}$ antenna in anisotropic QCD matter: spin-polarized and azimuthal jet observables

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arXiv: 2407.04774 [hep-ph]

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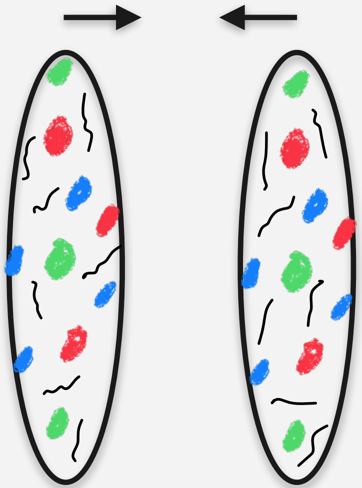
QCD states of matter in HICs

Initial nuclei

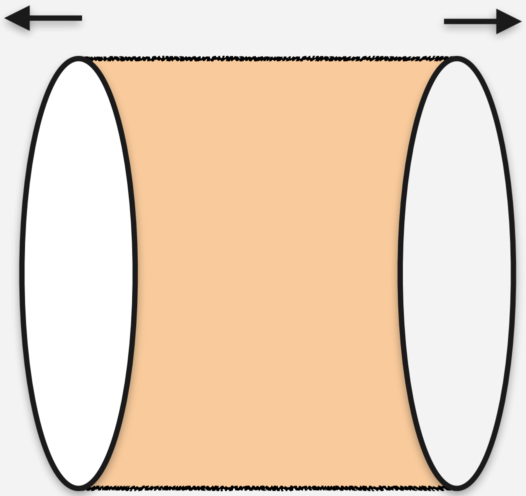


Gluon saturation
(~ CGC)

Driven to equilibrium

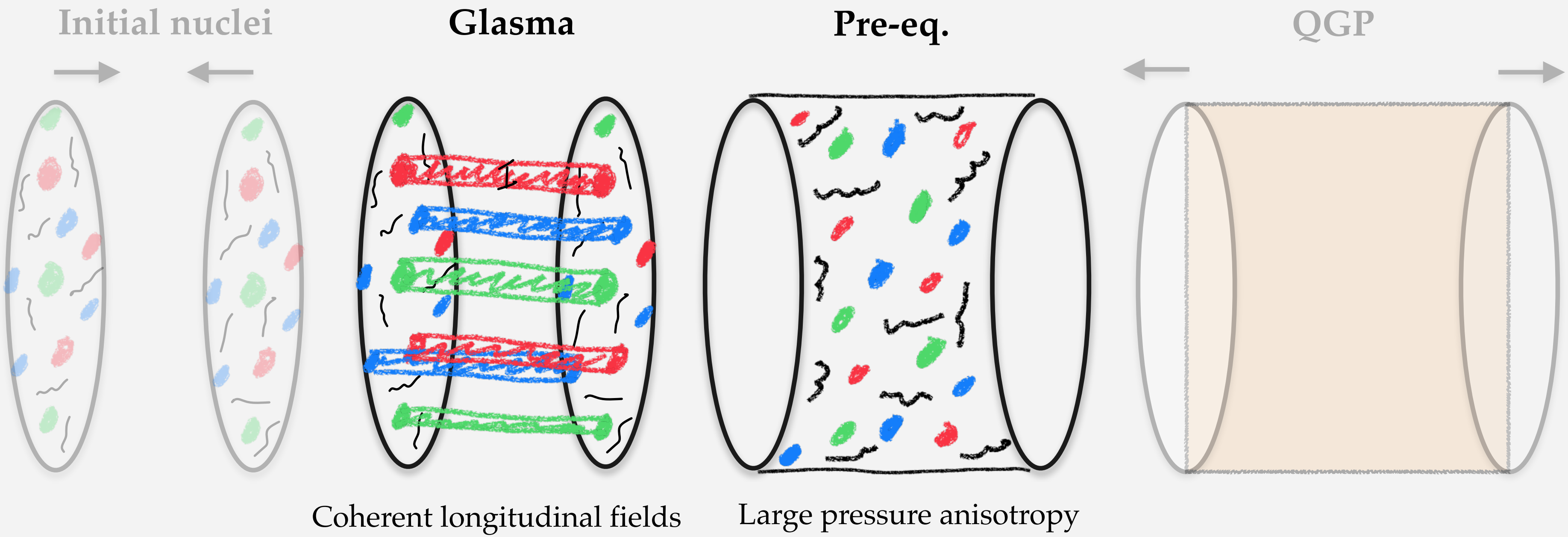
time

QGP



(Near) local thermal eq.
(~ (viscous) hydrodynamics)

QCD states of matter in HICs

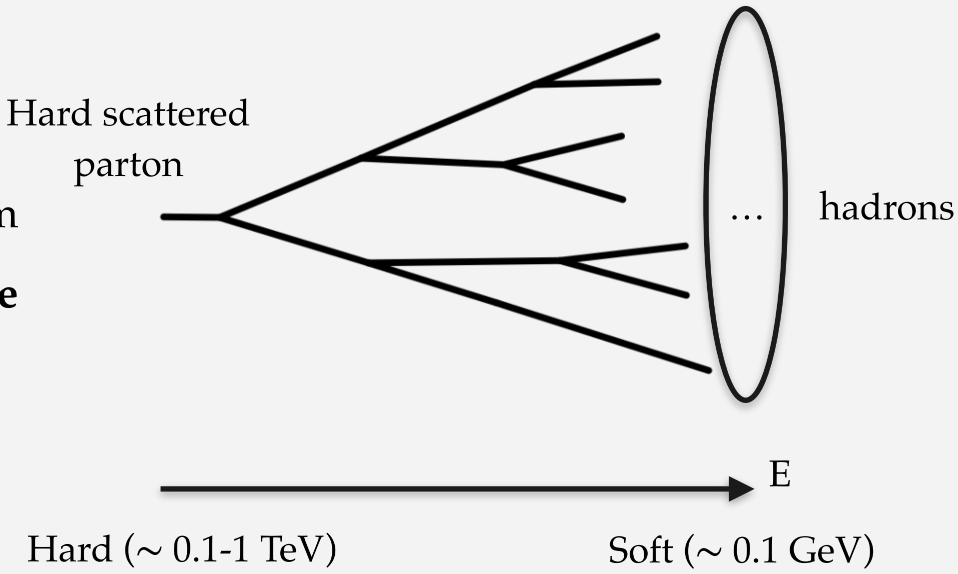


Out of equilibrium states of matter exhibiting **large anisotropies**

Probing QCD matter with jets

- ✦ We need a probe:
  - ❖ sensitive to multiple scales
  - ❖ concurrently produced with the medium
- ✦ **Jets are extended in space-time and evolve simultaneously with the produced matter!**

jets - collimated energetic QCD cascades



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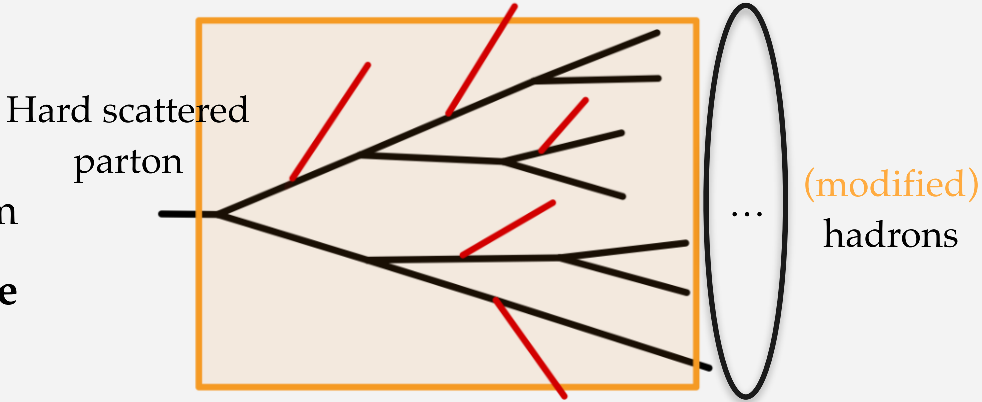


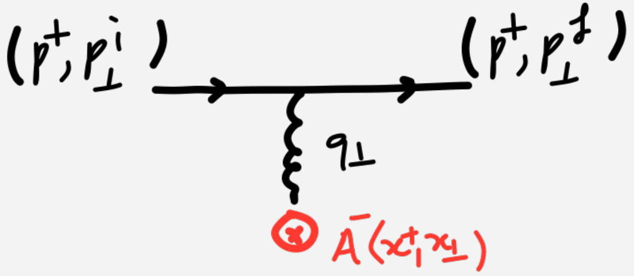
Jets in **heavy-ion** collisions have to **propagate through a medium and interact** with it

→ imprinted modifications tell a story (**jet quenching**)

Probing QCD matter with jets

How can one describe an energetic parton propagating in a dense QCD medium?

Perturbative QCD + **Medium model**



$$\boxed{\langle \mathscr{A}_a^-(x^+, \boldsymbol{x}), \mathscr{A}_b^{*-}(y^+, \boldsymbol{y}) \rangle}$$

soft single gluon exchanges with medium
$p^+ \gg |\boldsymbol{p}|, |\boldsymbol{q}|$

stochastic gauge field in light-cone gauge
gaussian white noise model

Probing QCD matter with jets: medium model

- Some assumptions reflected on the **two-point correlator** of the medium field:

$$\langle \mathcal{A}_a^-(x^+,\boldsymbol{x}), \mathcal{A}_b^{*-}(y^+,\boldsymbol{y})\rangle = \delta^{ab} n(x^+)\delta(x^+ - y^+)\gamma(\boldsymbol{x},\boldsymbol{y})$$

matter density

(directly related to) **in-medium elastic scattering rate**

- $n(x^+) = n$ (**static**)
- $\gamma(\boldsymbol{x},\boldsymbol{y}) = \gamma(\boldsymbol{y}-\boldsymbol{x})$ (**homogeneous**)
- $\tilde{\gamma}(\boldsymbol{q}) = \tilde{\gamma}(|\boldsymbol{q}|)$ (**isotropic**)

Probing QCD matter with jets: medium model

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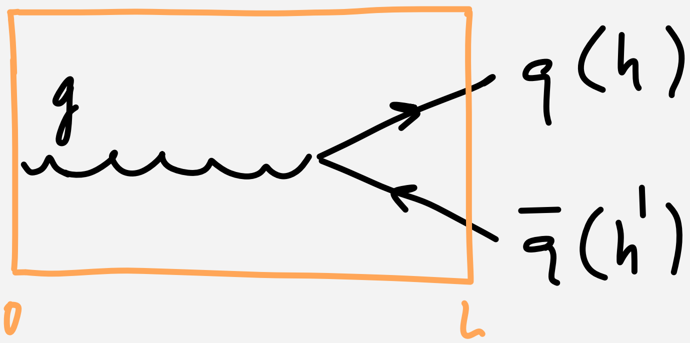
- $\tilde{\gamma}(\boldsymbol{q}) = \tilde{\gamma}(|\boldsymbol{q}|)$ **(isotropic)** ⟵ $\tilde{\gamma}(\boldsymbol{q}) \neq \tilde{\gamma}(|\boldsymbol{q}|)$ in this work; attempt to mimic the **anisotropic nature of early stages**

Relax assumptions → probe out of equilibrium properties

see also talks by Sadofyev (Mon **P7**), Mayo López, Salgado (Wed **P30**)

This work: $g \to q\bar{q}$ splitting in an anisotropic medium



$\hat{q}_y \neq \hat{q}_x$

$$\frac{dN^{hh'}}{dz d\phi}(L, \hat{q}_x, \hat{q}_y, \ldots)$$

anisotropy introduces non-trivial angular modulation that couples to spin

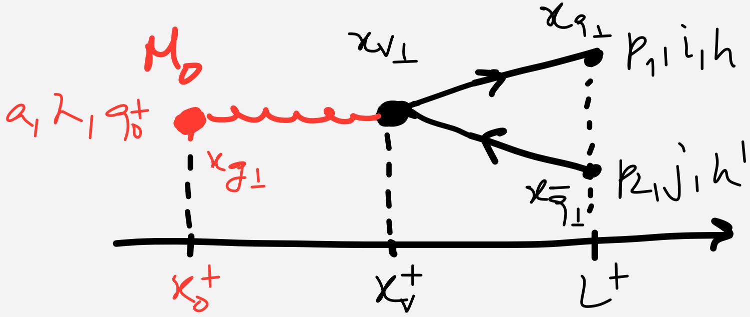
Other works, e.g:

- ✦ Attems et al, arXiv:2203.11241 ($g \to c\bar{c}$ splitting function, see talks by Brewer (Wed **P29**), Wiedemann (Tue **P22**))
- ✦ Hauksson et al, arXiv:2303.03914 (polarisation dependent $g \to gg$ energy spectrum with $\hat{q}_x \neq \hat{q}_y$)

! We do not attempt to discuss the physics of anisotropy and isotropization

see talks by Werthmann, Steinhorst, Lindenbauer, Barrera Cabodevila (Mon **P7**), Avramescu (Tue **P18**), Lamas (Wed **P30**)

Calculation: constructing $\mathscr{M}$



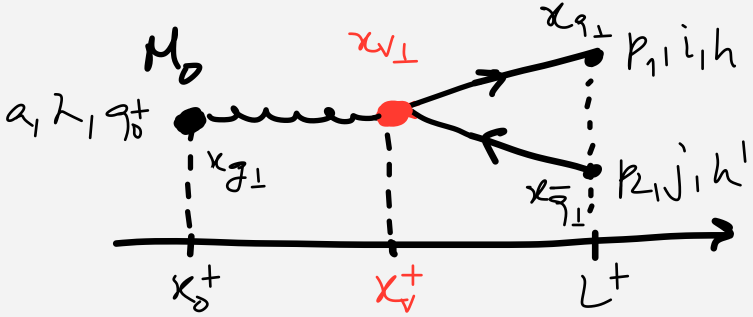
in-medium scalar propagator: resums multiple soft scatterings with the medium

$$i\mathscr{M} = \frac{e^{i(p_1^- + p_2^-)L^+}}{2q_0^+} \int_{\boldsymbol{p}} \mathscr{M}_0(\boldsymbol{p}) \int_{\boldsymbol{x}_g, \boldsymbol{x}_q, \boldsymbol{x}_{\bar{q}}, \boldsymbol{x}_v, \boldsymbol{x}, \boldsymbol{y}, x_v^+} e^{i\boldsymbol{p}\cdot\boldsymbol{x}_g} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}_q} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{x}_{\bar{q}}}$$

$$\overline{\mathscr{G}}^{jl}(L^+, \boldsymbol{x}_{\bar{q}}; x_v^+, \boldsymbol{x}_v \,|\, p_2^+) \mathscr{G}^{ik}(L^+, \boldsymbol{x}_q; x_v^+, \boldsymbol{x}_v - \boldsymbol{y} \,|\, p_1^+)$$

$$\left[i g t_{kl}^b V^{\lambda h h'}(z, \boldsymbol{x}, \boldsymbol{y}) \right] \mathscr{G}^{ba}(x_v^+, \boldsymbol{x}_v - \boldsymbol{x}; x_0^+, \boldsymbol{x}_g \,|\, q_0^+)$$

Calculation: constructing $\mathscr{M}$



unaltered by the medium; contains full **spin dependence**

$$i\mathscr{M} = \frac{e^{i(p_1^- + p_2^-)L^+}}{2q_0^+}\int_{\boldsymbol{p}} \mathscr{M}_0(\boldsymbol{p}) \int_{\boldsymbol{x_g},\boldsymbol{x_q},\boldsymbol{x_{\bar{q}}},\boldsymbol{x_v},\boldsymbol{x},\boldsymbol{y},x_v^+} e^{i\boldsymbol{p}\cdot\boldsymbol{x_g}} e^{-i\boldsymbol{p_1}\cdot\boldsymbol{x_q}} e^{-i\boldsymbol{p_2}\cdot\boldsymbol{x_{\bar{q}}}}$$

$$\overline{\mathscr{G}}^{jl}(L^+, \boldsymbol{x_{\bar{q}}}; x_v^+, \boldsymbol{x_v} \,|\, p_2^+)\mathscr{G}^{ik}(L^+, \boldsymbol{x_q}; x_v^+, \boldsymbol{x_v} - \boldsymbol{y} \,|\, p_1^+)$$

$$\left[igt^b_{kl}V^{\lambda hh'}(z, \boldsymbol{x}, \boldsymbol{y})\right] \mathscr{G}^{ba}(x_v^+, \boldsymbol{x_v} - \boldsymbol{x}; x_0^+, \boldsymbol{x_g} \,|\, q_0^+)$$

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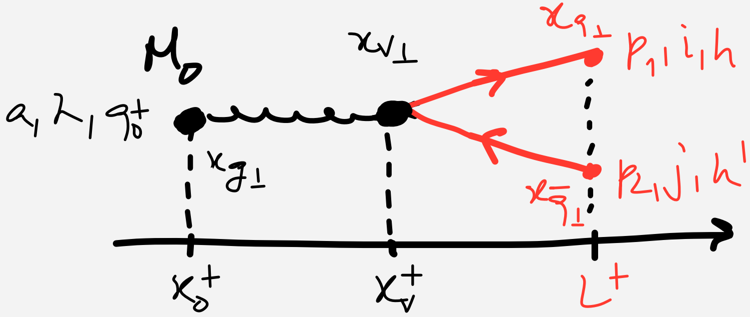

in-medium scalar propagators;
$(z = p_1^+/q_0^+)$

$$
i\mathscr{M} = \frac{e^{i(p_1^- + p_2^-)L^+}}{2q_0^+} \int_{\boldsymbol{p}} \mathscr{M}_0(\boldsymbol{p}) \int_{\boldsymbol{x_g},\boldsymbol{x_q},\boldsymbol{x_{\bar{q}}},\boldsymbol{x_v},\boldsymbol{x},\boldsymbol{y},x_v^+} e^{i\boldsymbol{p}\cdot\boldsymbol{x_g}} e^{-i\boldsymbol{p_1}\cdot\boldsymbol{x_q}} e^{-i\boldsymbol{p_2}\cdot\boldsymbol{x_{\bar{q}}}}
$$

$$
\overline{\mathscr{G}}^{jl}(L^+, \boldsymbol{x_{\bar{q}}}; x_v^+, \boldsymbol{x_v} \,|\, p_2^+)\mathscr{G}^{ik}(L^+, \boldsymbol{x_q}; x_v^+, \boldsymbol{x_v} - \boldsymbol{y} \,|\, p_1^+)
$$

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\left[igt^b_{kl} V^{\lambda hh'}(z, \boldsymbol{x}, \boldsymbol{y})\right] \mathscr{G}^{ba}(x_v^+, \boldsymbol{x_v} - \boldsymbol{x}; x_0^+, \boldsymbol{x_g} \,|\, q_0^+)
$$

Calculation: squaring $\mathscr{M}$ and medium averaging

$$4z(1-z)(2\pi)^5\sigma_0 \frac{dN^{hh'}}{dz\,d^2\boldsymbol{p_1}d^2\boldsymbol{p_2}} = 2\mathrm{Re}\left[\frac{1}{2}\sum_{\lambda}\frac{1}{N_c^2-1}\sum_{colors}\left\langle \mathscr{M}\mathscr{M}^{\dagger}\right\rangle\right]_{\bar{x}_1^+ > x_1^+}$$

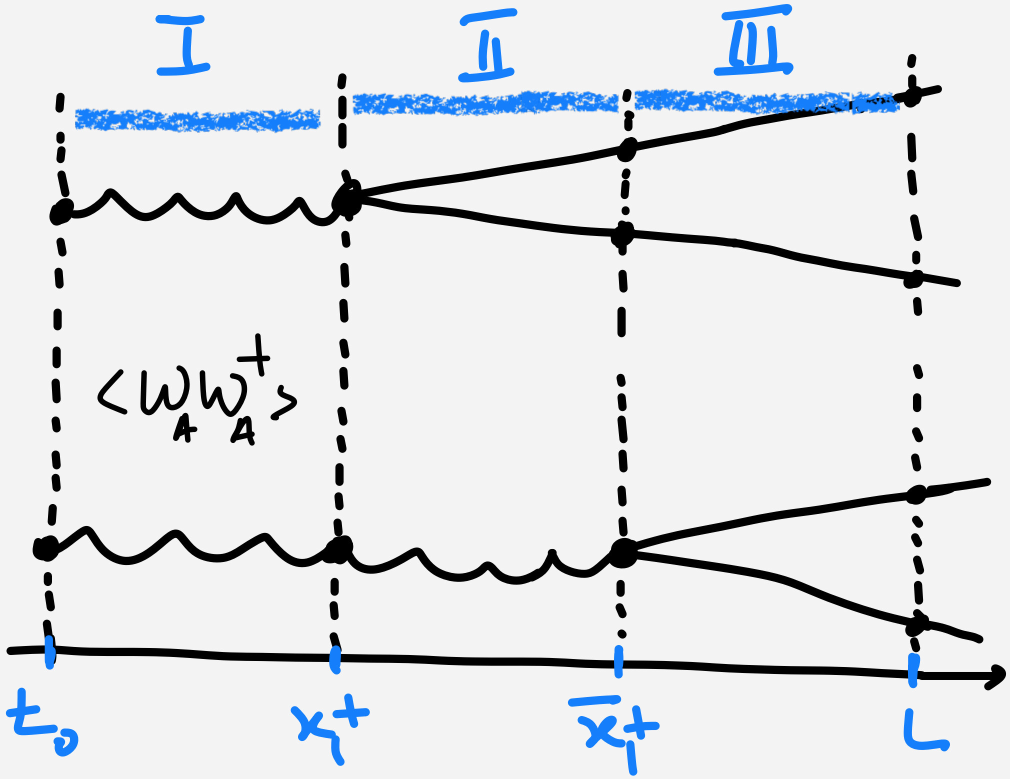
Use a **Gaussian white-noise model** for color charge distribution for the **average over medium ensembles:**

- Only relevant n-point function is the 2-point function:

$$\langle \mathscr{A}_a^-(x^+, \boldsymbol{x}), \mathscr{A}_b^{*-}(y^+, \boldsymbol{y})\rangle = \delta^{ab} n(x^+)\delta(x^+ - y^+)\gamma(\boldsymbol{x}-\boldsymbol{y})$$

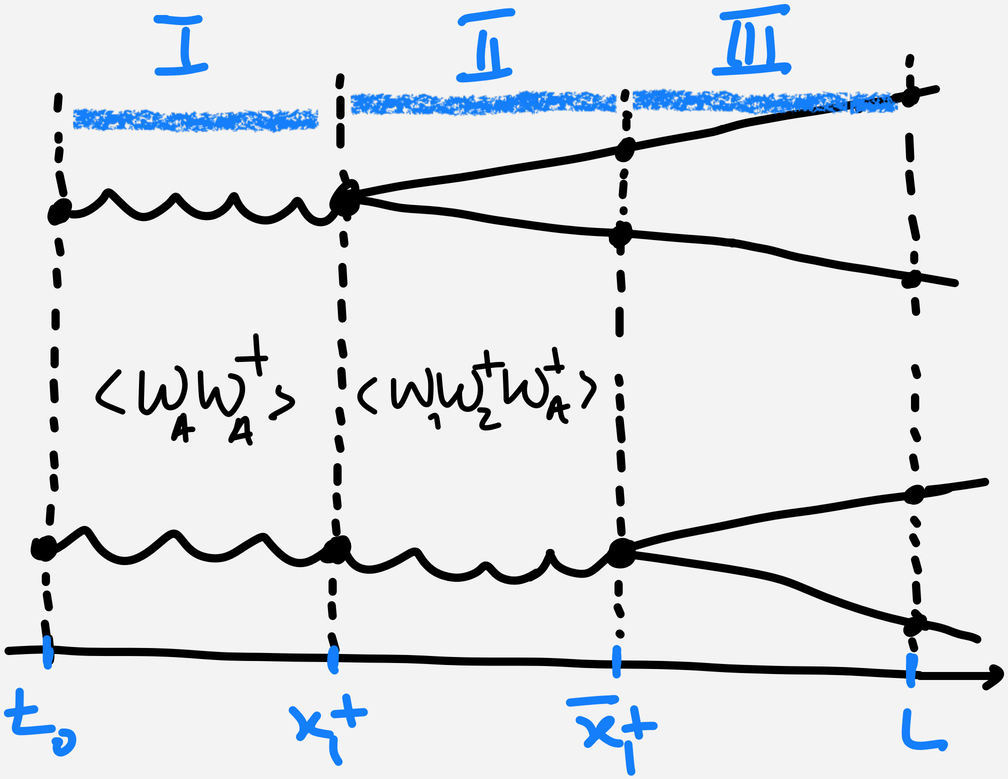
- Time locality of 2-point function + Wick's theorem: write **average of products** over whole time domain as **products of averages inside disjunct time regions.**

Calculation: squaring $\mathscr{M}$ and medium averaging



$\mathrm{I} =$ g transverse momentum broadening

Calculation: squaring $\mathscr{M}$ and medium averaging



I = g transverse momentum broadening

II = $q\bar{q}$ antenna formation

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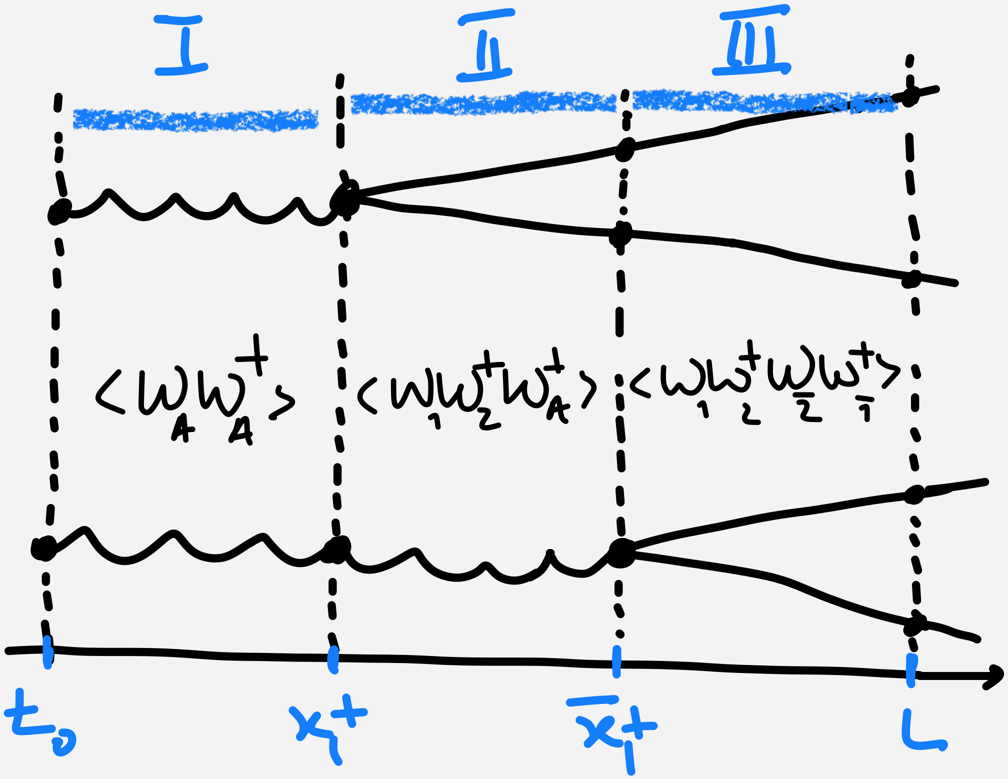
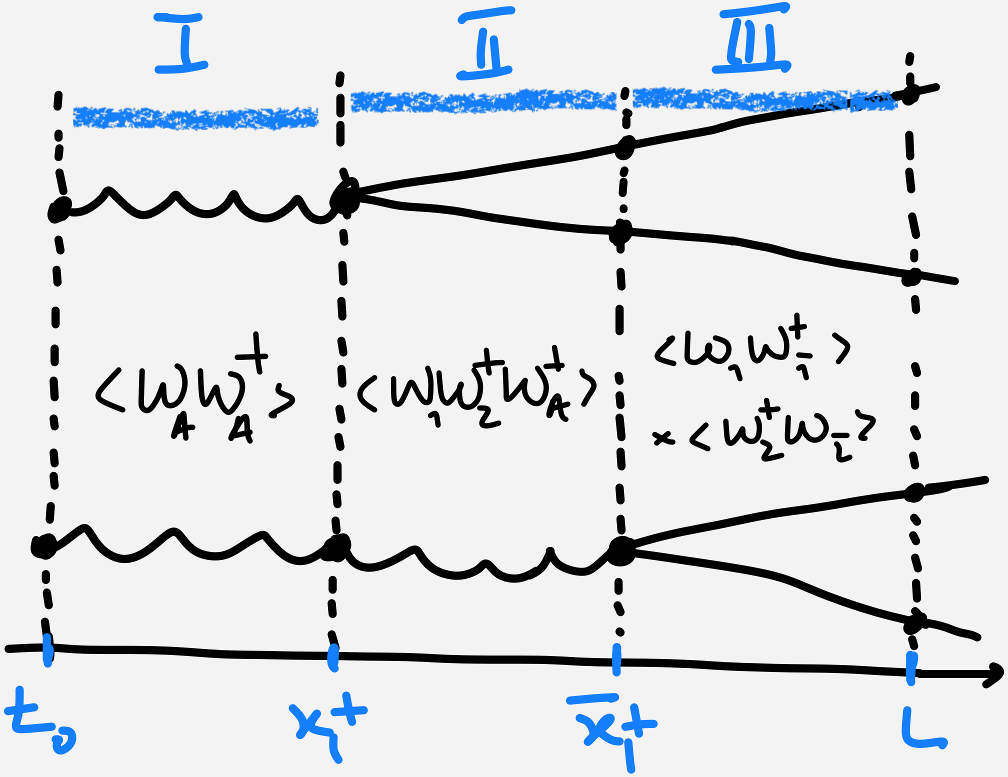


I = g transverse momentum broadening

II = $q\bar{q}$ antenna formation

III = 4-point function

Calculation: squaring $\mathscr{M}$ and medium averaging



I = $\boxed{\text{$g$ transverse momentum broadening}}$

II = $\boxed{\text{$q\bar{q}$ antenna formation}}$

Large-N_c

III $\simeq$ $\boxed{\text{$q$ and $\bar{q}$ independent transverse momentum broadening}}$

Apolinário et al, arXiv:1407.0599
Isaksen et al, arXiv:2107.02542

Introducing anisotropy and phase space integration

$$\sigma(\boldsymbol{x}_1 - \boldsymbol{x}_2) = 2g^2(\gamma(0) - \gamma(\boldsymbol{x}_1 - \boldsymbol{x}_2))$$ (dipole cross section)

Zakharov, arXiv: hep-ph/9807540

- **Harmonic approximation:** $N_c n \sigma(\boldsymbol{r}) \approx \frac{1}{2}\hat{q}\boldsymbol{r}^2$, $\hat{q} \sim \frac{\langle \boldsymbol{k}_\perp^2 \rangle}{\delta t} \sim$ average transverse mom. squared acquired by travelling parton in a given time interval

- Introducing an **anisotropy** via: $\hat{q}\boldsymbol{r}^2 \to \hat{q}_x r_x^2 + \hat{q}_y r_y^2$ $\hat{q}_y \neq \hat{q}_x$ more transverse mom. is transferred in one direction than the other

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- We want to keep some **directional information** to probe anisotropy:

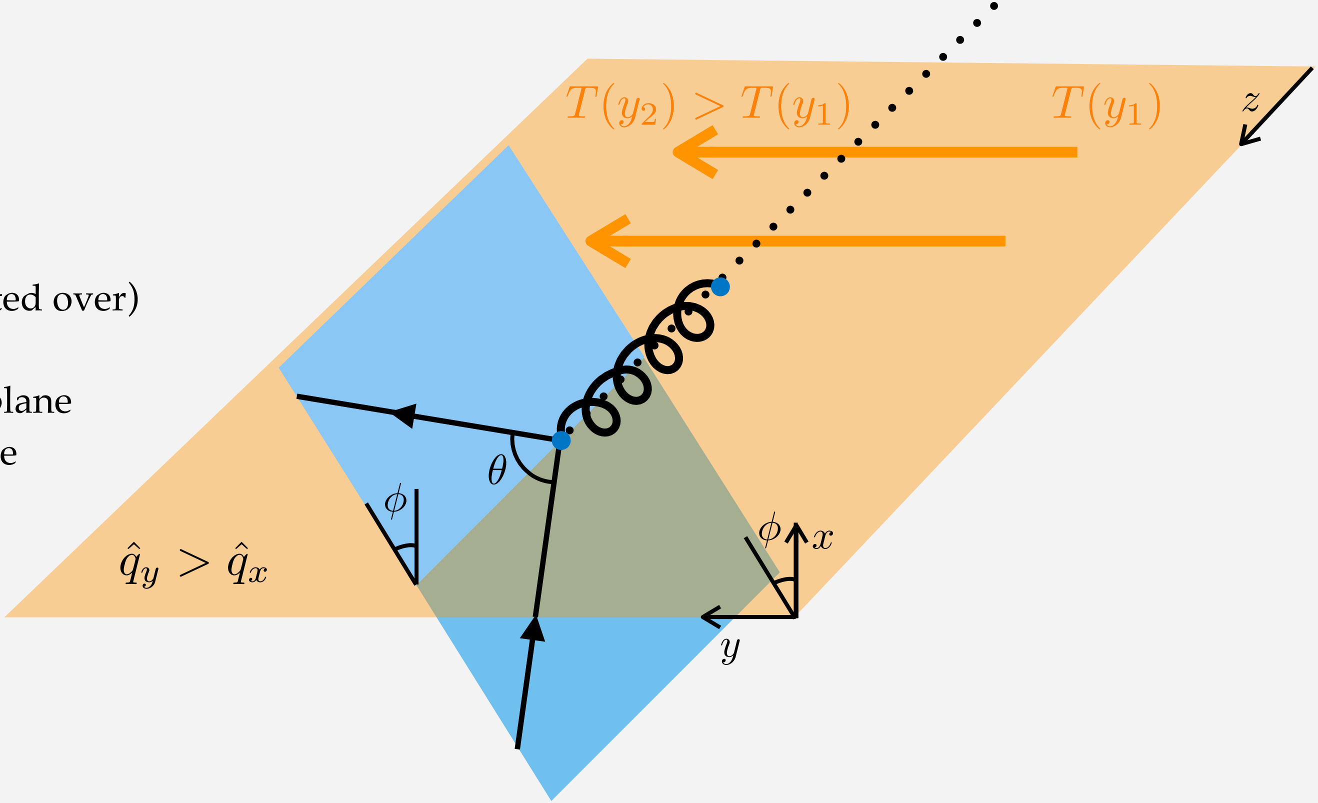
$$\frac{dN^{hh'}}{dz d^2\boldsymbol{p_1} d^2\boldsymbol{p_2}} \longrightarrow \boxed{\frac{dN^{hh'}}{dz d\phi}} \left(\boldsymbol{P}^{rel} = (1-z)\boldsymbol{p_1} - z\boldsymbol{p_2} \quad \tan\phi = \frac{\boldsymbol{P}_y^{rel}}{\boldsymbol{P}_x^{rel}} \right)$$

Geometry of the branching

$$\frac{dN^{hh'}}{dzd\theta d\phi}$$

θ - $q\bar{q}$ opening angle (integrated over)

ϕ - azimuthal orientation of plane spanned by $q\bar{q}$ in frame where $\boldsymbol{p_1} + \boldsymbol{p_2} = 0$



Parameter dependence

$$\boldsymbol{P}^{rel} = (1-z)\boldsymbol{p_1} - z\boldsymbol{p_2}, \quad \tan\phi = \boldsymbol{P}^{rel}_y \Big/ \boldsymbol{P}^{rel}_x$$

$$\boxed{2\pi \frac{dN^{hh'}}{dz\,d\phi} = \alpha_s\, \delta^{hh'}\, \mathrm{Re} \int_0^r d\Delta T \int_0^{r-\Delta T} d\Delta\bar{T}\; e^{-i\frac{\mu\Delta T}{z(1-z)}} \left(f_1(\Delta T, \Delta\bar{T}, \zeta, \phi, z) + h\, f_2(\Delta T, \Delta\bar{T}, \zeta, \phi, z)\right)}$$

$$\zeta \approx 1\ (-1) \quad \downarrow \quad \hat{q}_y \gg \hat{q}_x \quad (\hat{q}_x \gg \hat{q}_y)$$

(**anisotropy** param.)

$$\zeta \equiv \frac{\sqrt{\hat{q}_y} - \sqrt{\hat{q}_x}}{\sqrt{\hat{q}_y} + \sqrt{\hat{q}_x}}$$

(rescaled **medium length**)

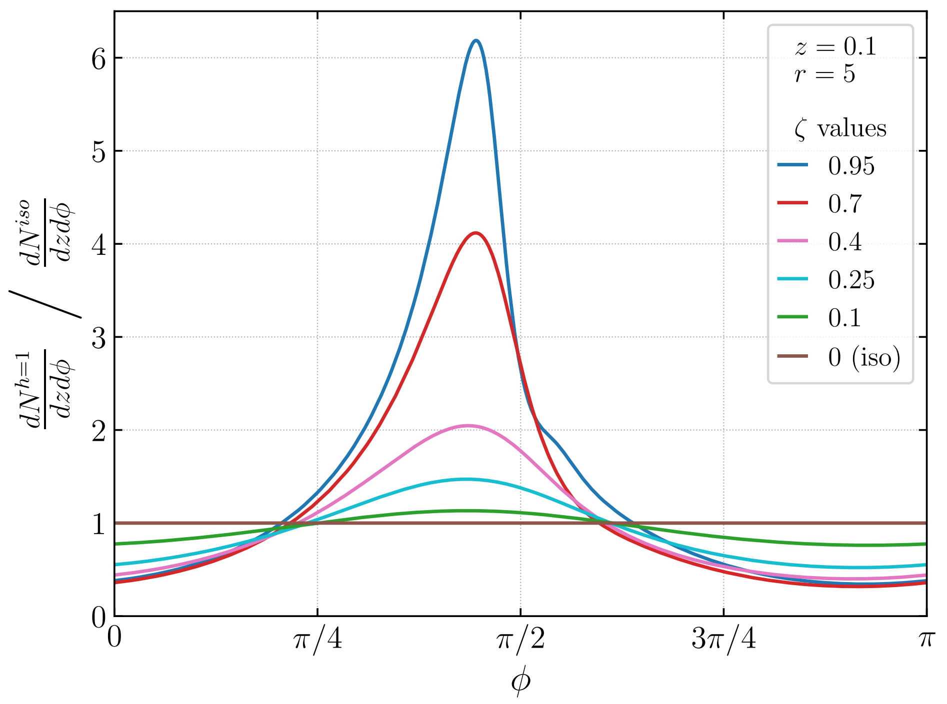
$$r \equiv L^+ \frac{\left(\sqrt{\hat{q}_y} + \sqrt{\hat{q}_x}\right)}{2\sqrt{2q_0^+}}$$

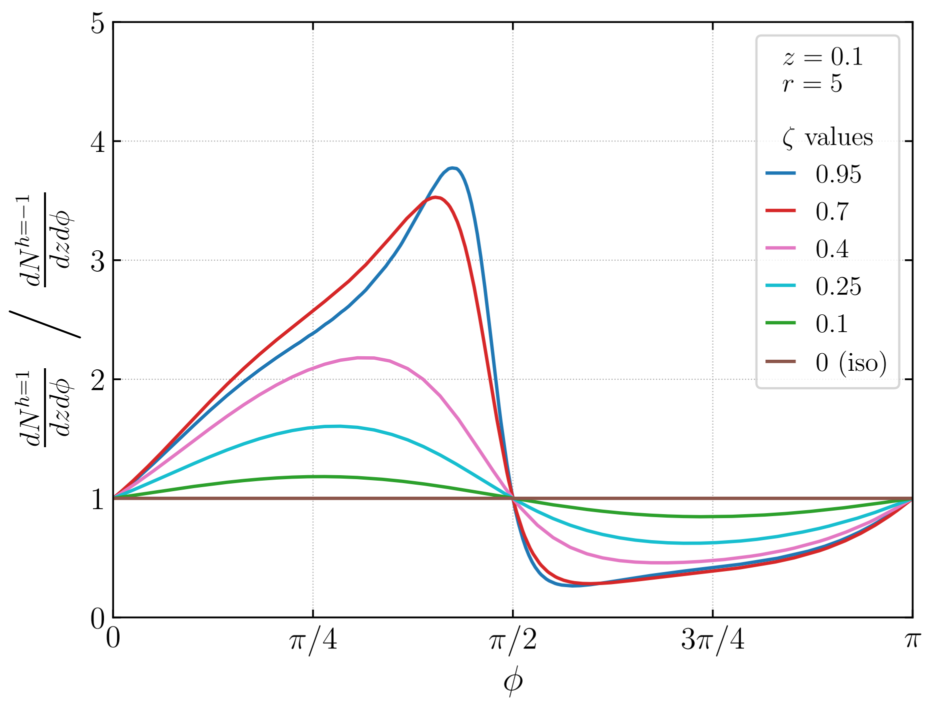
(rescaled **quark mass**)

$$\mu = \frac{\sqrt{2}m^2}{\left(\sqrt{\hat{q}_y} + \sqrt{\hat{q}_x}\right)\sqrt{q_0^+}}$$

$\zeta \to 0$ (**isotropy**) $\Longrightarrow$ $dN^{hh'}$ **independent of spin** ($f_2 \to 0$) and of ϕ

Azimuthal particle distribution (massless quarks)





Significant enhancement wrt. isotropic case near $\pi/2$
(would be near 0 if $\zeta < 0$, i.e., if $\hat{q}_x > \hat{q}_y$)

Significant helicity distinction near $\pi/2$
$dN^{h=+1}(\phi) = dN^{h=-1}(\pi - \phi)$

Harmonic decomposition

Let us do an **harmonic expansion** similar to what is usually done in the soft sector:

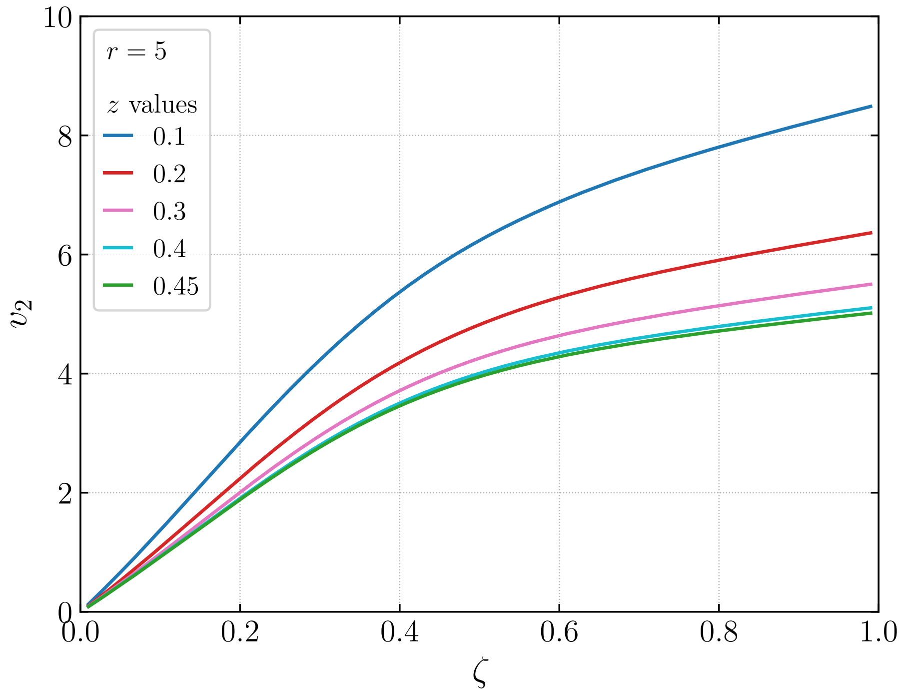
$$\frac{2\pi}{dN^h/dz}\frac{dN^h}{dzd\phi} = 1 + \sum_{n=1}^{\infty} v_{2n}^{jet}\cos\left(2n\phi\right) + \sum_{n=1}^{\infty} w_{2n}^{(h,jet)}\sin(2n\phi)$$

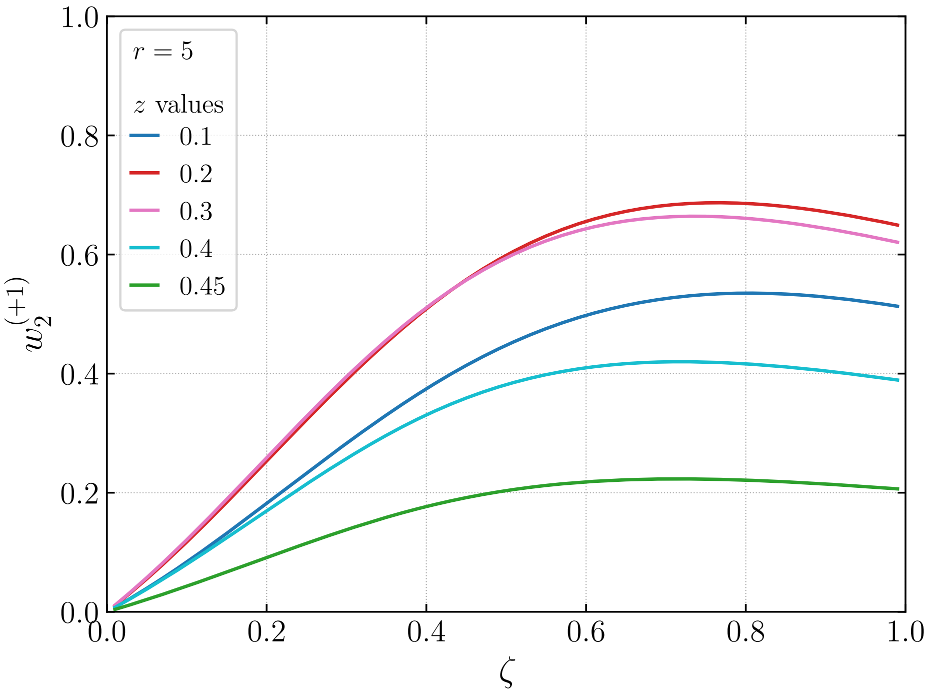
Helicity/spin dependence fully contained here

Small anisotropy $\longrightarrow$ $$\left.\frac{2\pi}{dN^h/dz}\frac{dN^h}{dzd\phi}\right|_{\zeta\ll1} = 1+\zeta\left[\tilde{v}_2^{jet}\cos(2\phi)+h\,\tilde{w}_2^{jet}\sin(2\phi)\right]$$

v_2 and $w_2^{(h)}$ are the leading harmonics in powers of the anisotropy (ζ)

Harmonic decomposition (massless quarks)





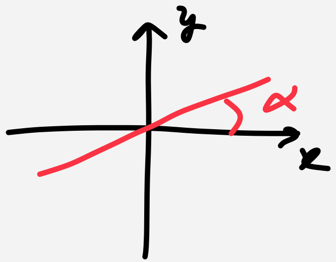
$v_2 \gg 1$ even for intermediate ζ

$w_2^{(+1)} = w_2^{h=1}$ non-monotonic with z

Spin measurement in the transverse plane

Vertex with $q/\bar{q}$ spins measured in the transverse plane along some **defined axis** (equal spins case):

$$\frac{\delta_{hh'}}{2}\sum_{\lambda}(V^{\lambda hh'})(z,\boldsymbol{P})\Big((V^{\lambda hh'})(z,\overline{\boldsymbol{P}})\Big)^{*}=\frac{\delta_{hh'}}{2z(1-z)}\left\{(\boldsymbol{P}\cdot\overline{\boldsymbol{P}})+m^{2}+imh\left[\left(\boldsymbol{P}^{x}-\overline{\boldsymbol{P}}^{x}\right)\sin\alpha-\left(\boldsymbol{P}^{y}-\overline{\boldsymbol{P}}^{y}\right)\cos\alpha\right]\right\}$$



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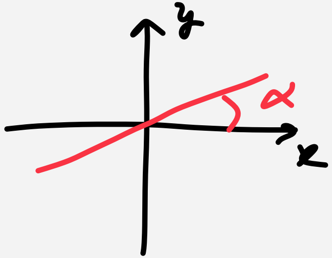


Very small $\hat{q}_x$ and finite $\hat{q}_y$

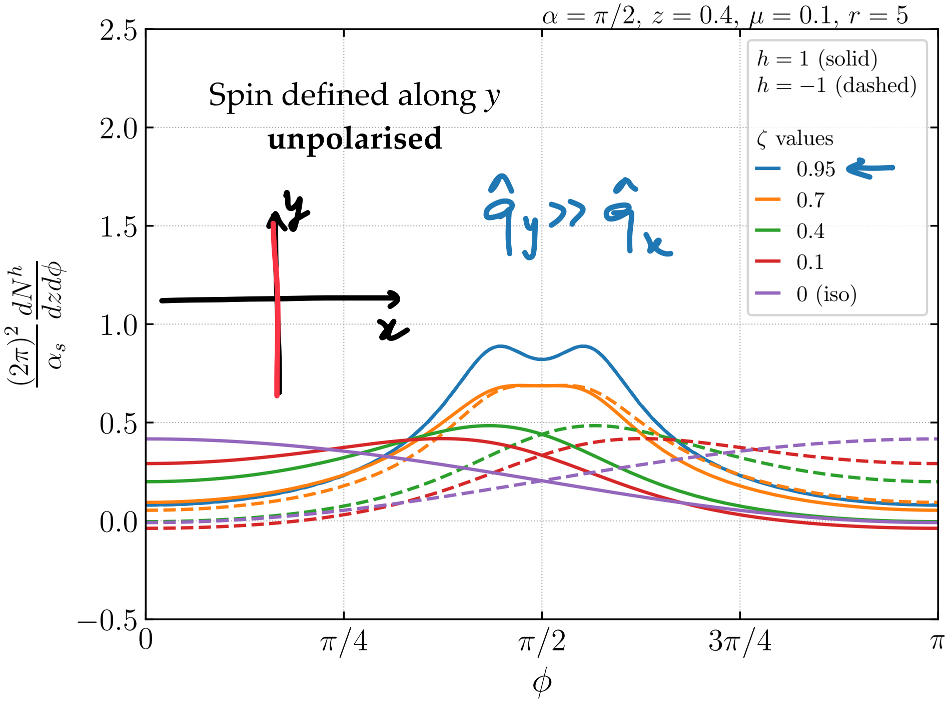
$$\Big\downarrow \quad (\boldsymbol{P}^i - \overline{\boldsymbol{P}}^i \sim \hat{q}_i)$$

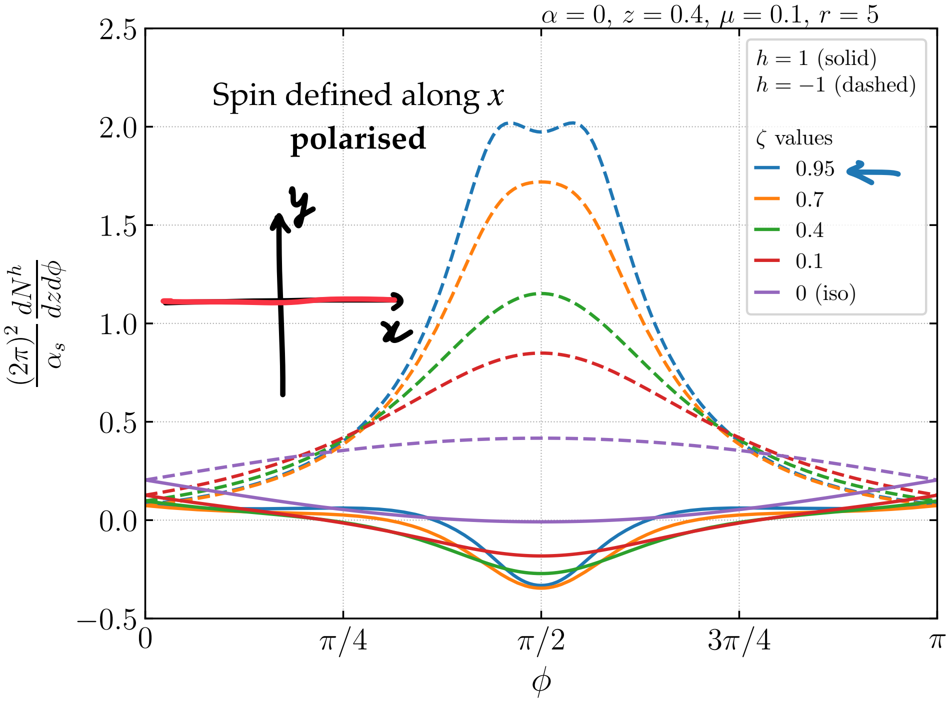
$$\frac{\delta_{hh'}}{2}\sum_{\lambda}(V^{\lambda hh'})(z,\boldsymbol{P})\left((V^{\lambda hh'})(z,\overline{\boldsymbol{P}})\right)^{*} \to \frac{\delta_{hh'}}{2z(1-z)}\left\{(\boldsymbol{P}\cdot\overline{\boldsymbol{P}}) + m^2 - imh\left(\boldsymbol{P}^y - \overline{\boldsymbol{P}}^y\right)\cos\alpha\right\}$$

$\alpha = \pi/2$ (spin measured along y) $\to$ **unpolarised** , i.e., $dN^{h=+1} = dN^{h=-1}$

$\alpha = 0$ (spin measured along x) $\to$ **polarised** , i.e., $dN^{h=+1} \neq dN^{h=-1}$

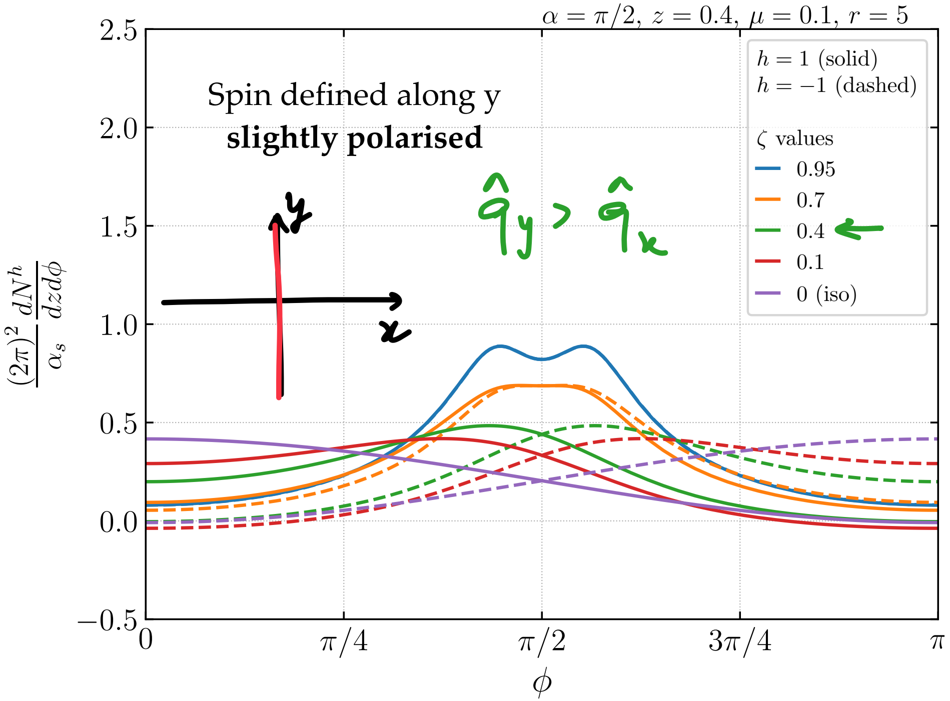
Spin measurement in the transverse plane



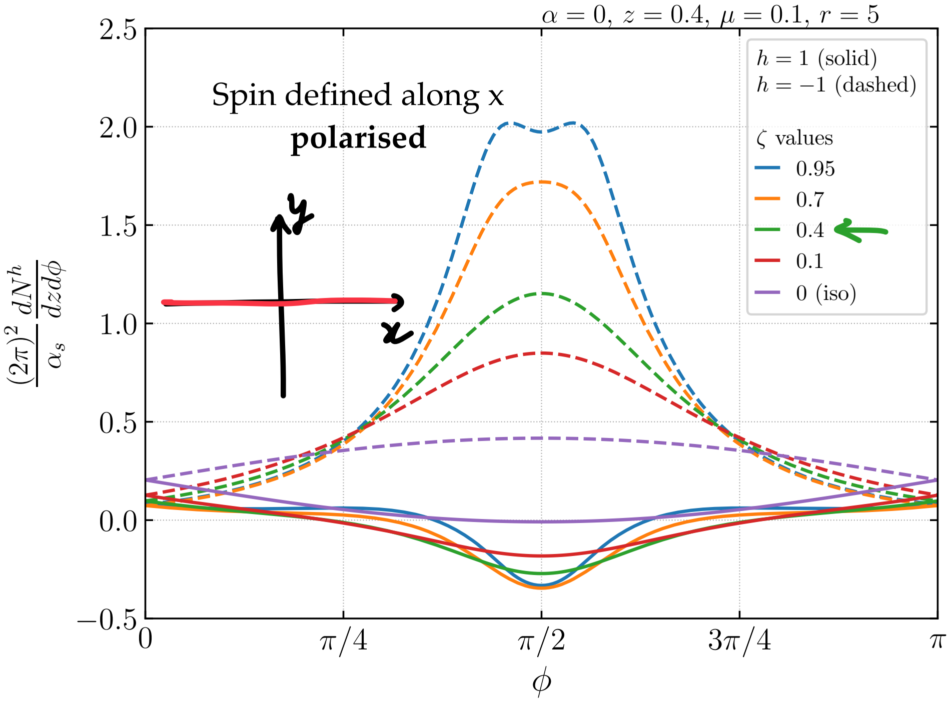


$\alpha = \pi/2$ $\qquad$ $\alpha = 0$

Spin measurement in the transverse plane



$\alpha = \pi/2$



$\alpha = 0$

Summary and outlook: anisotropic heavy EEC

$$\frac{1}{\sigma_0}\frac{d\Sigma^{(2)}}{d\theta d\phi} = \int_0^1 dz\, z(1-z)\frac{dN}{dz d\theta d\phi}$$

Small anisotropy

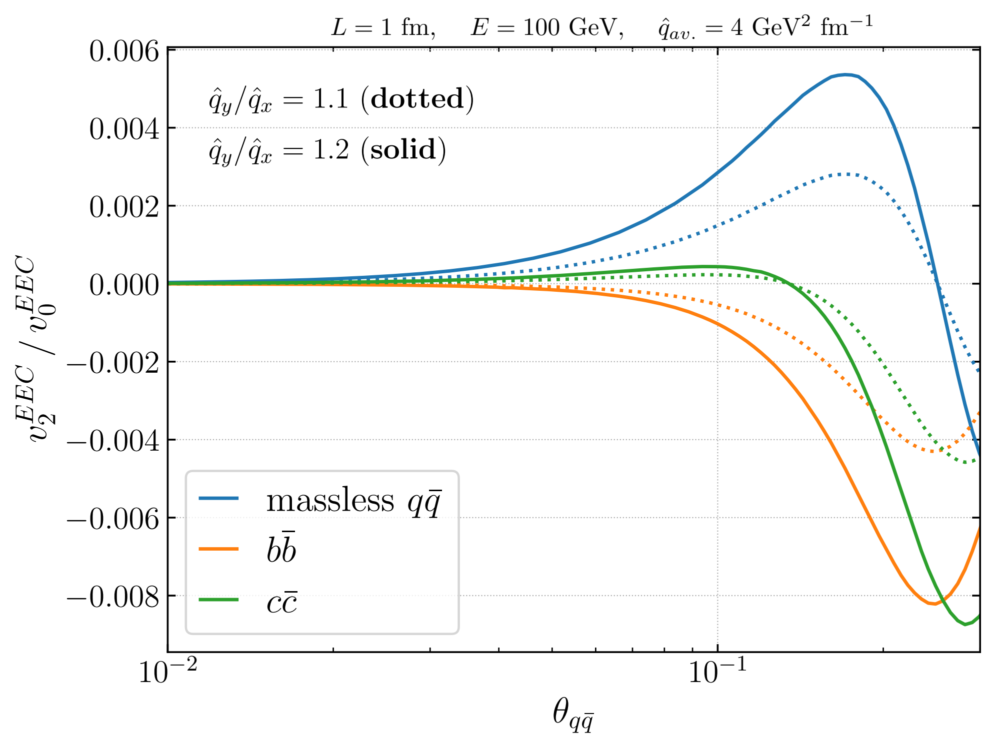
$$\frac{1}{\sigma_0}\frac{d\Sigma^{(2)}}{d\theta d\phi} \approx v_0^{EEC} + v_2^{EEC}\cos 2\phi$$

$v_2^{EEC} \propto \zeta$ - anisotropic contrib.

v_0^{EEC} - isotropic EEC

See talk by Brewer (Wed **P29**)

in prep. J. Barata, J. Brewer, K. Lee, JMS

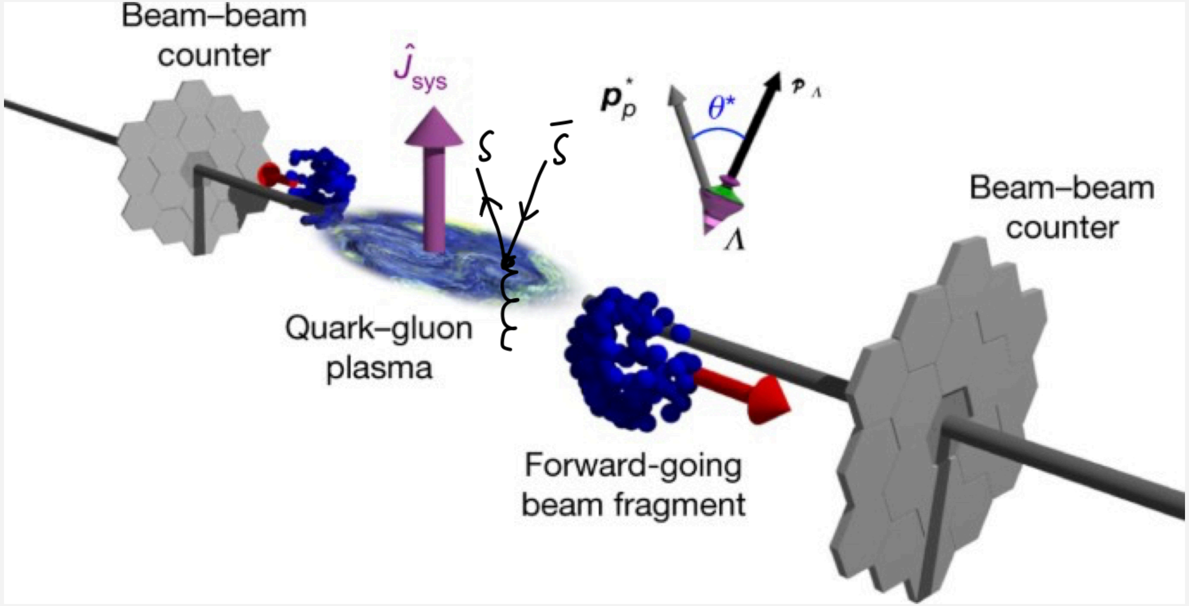


$\sim 1\,\%$ effect for anisotropy of $\sim 20\,\%$

Summary and outlook: high p_T Λ polarisation

High p_T lambda polarisation (in prep. J. Barata, JMS, E. Speranza)

How does the spin of an $s\bar{s}$ pair couple to the QGP angular momentum using a similar formalism?



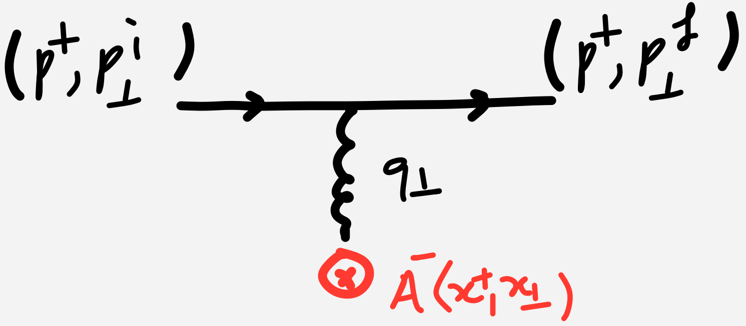
Summary and outlook

- We can probe anisotropy induced modifications by looking at **non-trivial angular modulation** of the helicity/spin-dependent $g \to q\bar{q}$ spectrum;
- For small anisotropies this modulation can be simply described by the **leading harmonics** v_2 **and** $w_2^{(h)}$
  - Anisotropic EEC with full angular dependence (in prep. J. Barata, J. Brewer, K. Lee, JMS)
- Measurements of **spin projections in the transverse plane** can probe the direction of stronger anisotropy
  - Spin polarisation of high p_T Λ (in prep. J. Barata, JMS, E. Speranza)

THANKS!

Back-up slides

In-medium parton propagation

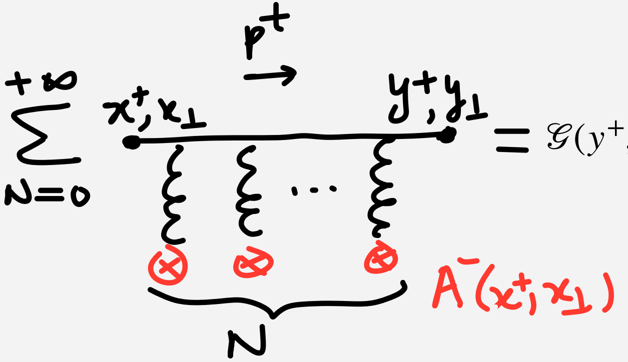


Leading correction (in $1/p^+$)

$$\bar{u}(p_f) i g \gamma^\mu \int_x e^{i(p_f - p_i)\cdot x} T_a \mathscr{A}^{\mu,a} u(p_i) \sim 2p^+ \delta(p_f^+ - p_i^+) \int_{\boldsymbol{x}} e^{i(\boldsymbol{p}_f - \boldsymbol{p}_i)\cdot \boldsymbol{x}} \int_{x^+} ig\, T_a \mathscr{A}^{-,a}(x^+, \boldsymbol{x})$$

Coordinate-space scalar propagator:

Re-summation of the $1/p^+$contributions in the **BDMPS-Z formalism** describing the effect of an arbitrary number of instantaneous soft scatterings with the medium $\mathscr{A}^-$



$$= \mathscr{G}(y^+, \boldsymbol{y}; x^+, \boldsymbol{x} \,|\, p^+) = \underbrace{\int_{\boldsymbol{r}(x^+)=\boldsymbol{x}}^{\boldsymbol{r}(y^+)=\boldsymbol{y}} \mathscr{D}\boldsymbol{r} \exp\left(\frac{ip^+}{2} \int_{x^+}^{y^+} d\xi \, \dot{\boldsymbol{r}}^2 \right)}_{\text{Transverse brownian motion}} \underbrace{\mathscr{P} \exp\left(ig \int_{x^+}^{y^+} d\xi \; T_a \mathscr{A}^{-,a}(\xi, \boldsymbol{r}(\xi)) \right)}_{\text{Color rotation matrix}}$$

Effective Feynman rules (massless)

$$\text{internal line} \quad \rightarrow \quad \mathcal{G}^{ij}(y^+, \boldsymbol{y}; x^+, \boldsymbol{x} \,|\, p^+)/2p^+$$

$$\text{outgoing external line} \quad \rightarrow \quad \mathcal{G}^{ij}(y^+, \boldsymbol{y}; x^+, \boldsymbol{x} \,|\, p^+) e^{i(\frac{\boldsymbol{p}^2}{2p^+} y^+ - \boldsymbol{p}\cdot\boldsymbol{y})}$$

$$\text{vertex (mom. space)} \quad \rightarrow \quad 2igt_a^{ij}\,\delta^{h,-h'}(\boldsymbol{\epsilon}^\lambda \cdot \boldsymbol{P}) \frac{1}{\sqrt{z(1-z)}} \left(z\delta^{\lambda h} - (1-z)\delta^{\lambda -h}\right)$$

$$\boldsymbol{P} = \boldsymbol{p}_g - z\boldsymbol{p}_q \,, \qquad z = p_q^+/p_g^+$$

Effective Feynman rules (massive)

$$\mathscr{G}^{ij}(y^+, \boldsymbol{y}; x^+, \boldsymbol{x} \,|\, p^+)\big|_{m\neq 0} \to \mathscr{G}^{ij}(y^+, \boldsymbol{y}; x^+, \boldsymbol{x} \,|\, p^+)\big|_{m=0} \exp\left\{ -i\frac{m^2}{2p^+}(y^+ - x^+) \right\}$$

vertex (mom. space) $\quad \to \quad 2igt_a^{ij}\left(\delta^{hh'}(\boldsymbol{\epsilon}^\lambda \cdot \boldsymbol{P})\frac{1}{\sqrt{z(1-z)}}\left(z\delta^{\lambda h} - (1-z)\delta^{\lambda -h}\right) - \delta^{\lambda h}\delta^{h,-h'}\frac{m}{\sqrt{2z(1-z)}} \right)$

$$\boldsymbol{P} = \boldsymbol{p}_q - z\boldsymbol{p}_g\,, \qquad\qquad z = p_q^+/p_g^+$$

Assumptions about initial production amplitude

$$\int_{p^-} e^{-ip^- x_g^+} \mathcal{M}_0^{\mu,a}(p) = \mathcal{M}_0^{\mu,a}(q_0^+, x_g^+, \boldsymbol{p})\delta(x_g^+ - x_0^+)$$

the gluon is produced at the initial time of the medium x_0^+ with a fixed energy q_0^+

$$\sum_{\lambda,\bar{\lambda},a,\bar{a}} \mathcal{M}_0^{\mu,a}(\boldsymbol{p})\epsilon^*_{\lambda,\mu} \left(\mathcal{M}_0^{\bar{\mu},\bar{a}}(\bar{\boldsymbol{p}})\epsilon^*_{\bar{\lambda},\bar{\mu}}\right)^\dagger \to \sum_{\lambda,\bar{\lambda},a,\bar{a}} \frac{\delta^{a\bar{a}}}{N_c^2 - 1} \frac{\delta^{\lambda\bar{\lambda}}}{2} \mathcal{M}_0(\boldsymbol{p})\mathcal{M}_0^*(\bar{\boldsymbol{p}})$$

the gluon is produced with a randomised polarization and color

$$\sum_{\lambda,a} \int_{x_g^+,\boldsymbol{x}_g} \mathcal{M}_0^{\lambda,a}(q_0^+, x_g^+, \boldsymbol{x}_g)\,(\ldots)^{\lambda,a,\ldots}(x_g^+, \ldots) \to \int_{\boldsymbol{p},\boldsymbol{x_g}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}_g} \mathcal{M}_0(\boldsymbol{p})\,(\ldots)^{\lambda,a,\ldots}(x_0^+, \ldots)$$

$$+$$

average over gluon polarisation and color after squaring

Calculation: vertex and helicity / spin dependence

$$\overline{\mathcal{G}}^{jl}(L^+, \boldsymbol{x_{\bar{q}}}; x_v^+, \boldsymbol{p_g} - \boldsymbol{p_q} \,|\, p_2^+)\mathcal{G}^{ik}(L^+, \boldsymbol{x_q}; x_v^+, \boldsymbol{p_q} \,|\, p_1^+)V^{\lambda h h'}(z, \boldsymbol{p_g}, \boldsymbol{p_q})\mathcal{G}^{ba}(x_v^+, \boldsymbol{p_g}; x_0^+, \boldsymbol{x_g} \,|\, q_0^+)$$

Momentum space

$$\big(f_1 \circledast f_2\big)(\boldsymbol{x_v}) = \int_{\boldsymbol{p}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x_v}}\tilde{f}_1(\boldsymbol{p})\tilde{f}_2(\boldsymbol{p})$$

Coordinate space

$$\overline{\mathcal{G}}^{jl}(L^+, \boldsymbol{x_{\bar{q}}}; x_v^+, \boldsymbol{x_v} \,|\, p_2^+)\mathcal{G}^{ik}(L^+, \boldsymbol{x_q}; x_v^+, \boldsymbol{x_v} - \boldsymbol{y} \,|\, p_1^+)V^{\lambda h h'}(z, \boldsymbol{x}, \boldsymbol{y})\mathcal{G}^{ba}(x_v^+, \boldsymbol{x_v} - \boldsymbol{x}; x_0^+, \boldsymbol{x_g} \,|\, q_0^+)$$

$$V^{\lambda h h'}(z, \boldsymbol{p_g}, \boldsymbol{p_q}) = 2\gamma^{\lambda h}(z)\delta_{h,-h'}\,\boldsymbol{\epsilon}^{\lambda}\cdot(\boldsymbol{p_g} - z\boldsymbol{p_q})$$

$$\gamma^{\lambda h}(z) = \frac{1}{\sqrt{z(1-z)}}\left(z\delta^{\lambda h} - (1-z)\delta^{\lambda -h}\right)$$

$$z = p_q^+/p_g^+$$

$$\longrightarrow \quad \frac{1}{2}\sum_{\lambda}(\ldots)\,|\,\mathcal{M}\,|^2 = X_1 + hX_2$$

$X_2 \neq 0$ iff. medium is anisotropic

helicity dependence is an **additional** handle on anisotropy sensitivity

Analytical result

$$4z(1-z)(2\pi)^3\frac{dN^h}{dz\,d^2\boldsymbol{P}^{rel}} = -\frac{2g^2}{(2q_0^+)^2 z(1-z)}\mathrm{Re}\int_{X_v^+}\int_{\boldsymbol{v}}\int_{\boldsymbol{t}_1,\boldsymbol{t}_2} e^{-i(\boldsymbol{P}^{rel}+\boldsymbol{t}_2)\cdot\boldsymbol{v}}\exp\left\{-\frac{C_F}{2}\int_{\bar{x}_v^+}^{L^+} ds^+\, n(s^+)(\sigma((1-z)\boldsymbol{v})+\sigma(z\boldsymbol{v}))\right\}$$

$$\times\left[P_{qg}^{vac}(z)\big(\boldsymbol{t}_1\cdot\boldsymbol{t}_2\big)+\frac{ih}{2}(1-2z)\big(\boldsymbol{t}_1\times\boldsymbol{t}_2\big)_z\right]\tilde{\mathscr{K}}(\boldsymbol{t}_1,\boldsymbol{t}_2)$$

$$\tilde{\mathscr{K}}(\boldsymbol{t}_1,\boldsymbol{t}_2) = \int_{\boldsymbol{a}_1,\boldsymbol{a}_2} e^{-i\boldsymbol{a}_1\cdot\boldsymbol{t}_1}e^{-i\boldsymbol{a}_2\cdot\boldsymbol{t}_2}\int_{\boldsymbol{a}_1}^{\boldsymbol{a}_2}\mathscr{D}\boldsymbol{u}\,\exp\left\{\frac{i\tilde{q}_0^+}{2}\int_{x_v^+}^{\bar{x}_v^+} ds^+\dot{\boldsymbol{u}}^2\right\}$$

$$\times\exp\left\{-\frac{1}{2}\int_{x_v^+}^{\bar{x}_v^+} ds^+\, n(s^+)(N_c(\sigma(z\boldsymbol{u})+\sigma(-(1-z)\boldsymbol{u}))-\frac{1}{N_c}\sigma(-\boldsymbol{u}))\right\}$$

Calculation: squaring $\mathscr{M}$ and medium averaging

- $\gamma \to q\bar{q}$ includes a quadrupole at leading N_c …

$$C^{(4)} = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[W_1 W_2^\dagger W_{\bar{2}} W_{\bar{1}}^\dagger\right]\right\rangle$$

leading N_c

- but $g \to q\bar{q}$ does not!

$$C^{(4)} = \frac{1}{N_c^2 - 1}\left\langle \mathrm{Tr}\left[W_1 W_{\bar{1}}^\dagger\right]\mathrm{Tr}\left[W_{\bar{2}} W_2^\dagger\right] - \frac{1}{N_c}\mathrm{Tr}\left[W_1 W_2^\dagger W_{\bar{2}} W_{\bar{1}}^\dagger\right]\right\rangle$$

Calculation: squaring $\mathscr{M}$ and medium averaging

$$S^{(2)}S^{(3)}S^{(4)}$$

$$S^{(2)} = \int_{\boldsymbol{r}_1^i}^{\boldsymbol{r}_1^f} \mathscr{D}\boldsymbol{r}_1 \int_{\boldsymbol{r}_2^i}^{\boldsymbol{r}_2^f} \mathscr{D}\boldsymbol{r}_2 \exp\left\{ i\frac{q_0^+}{2}\int_{x_0^+}^{x_v^+} ds^+ \left(\dot{\boldsymbol{r}}_1^2 - \dot{\boldsymbol{r}}_2^2\right) \right\} C^{(2)}$$

$$S^{(3)} = \int_{\boldsymbol{r}_0^i}^{\boldsymbol{r}_0^f} \mathscr{D}\boldsymbol{r}_0 \int_{\boldsymbol{r}_1^i}^{\boldsymbol{r}_1^f} \mathscr{D}\boldsymbol{r}_1 \int_{\boldsymbol{r}_2^i}^{\boldsymbol{r}_2^f} \mathscr{D}\boldsymbol{r}_2 \exp\left\{ \frac{i}{2}\int_{x_v^+}^{\bar{x}_v^+} ds^+ \left(-q_0^+\dot{\boldsymbol{r}}_0^2 + p_1^+\dot{\boldsymbol{r}}_1^2 + p_2^+\dot{\boldsymbol{r}}_2^2\right) \right\} C^{(3)}$$

$$S^{(4)} = S_q^{(2)} S_{\bar{q}}^{(2)} + \mathcal{O}(1/N_c)$$

Path integrals

Medium average of n Wilson lines

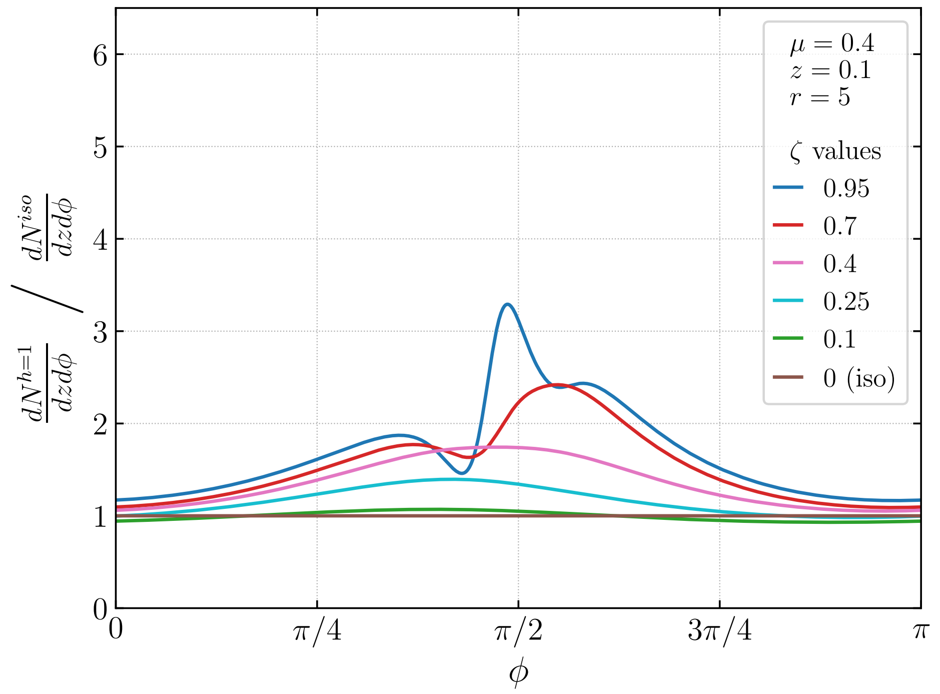
e.g.

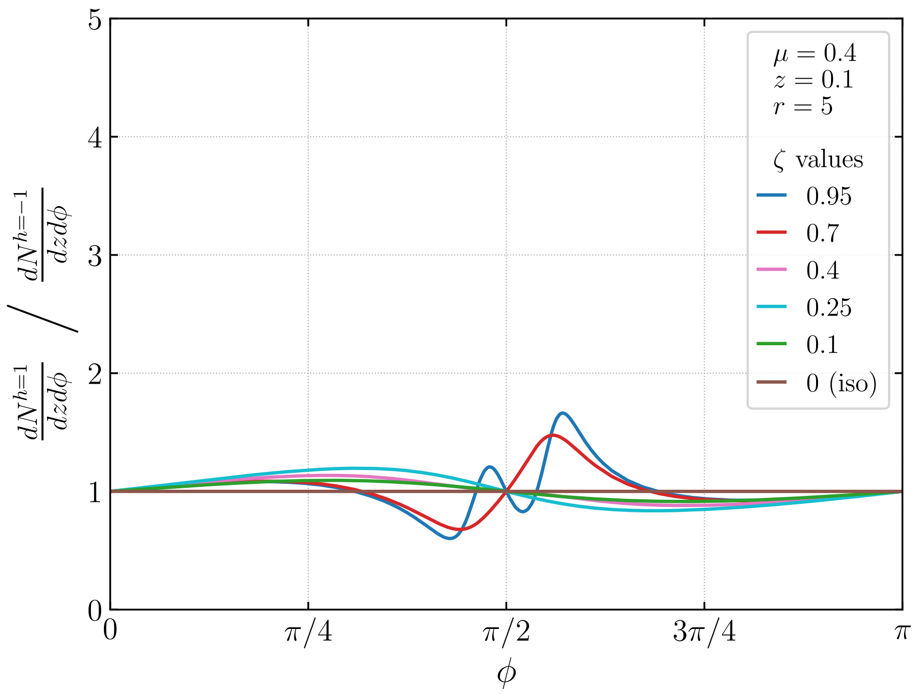
$$\langle \mathscr{A}_a^-(x^+, \boldsymbol{x}), \mathscr{A}_b^{*-}(y^+, \boldsymbol{y})\rangle = \delta^{ab} n(x^+)\delta(x^+ - y^+)\gamma(\boldsymbol{x} - \boldsymbol{y})$$

$$C^{(2)} = \frac{1}{N_c^2 - 1}\left\langle \mathrm{Tr}\left[W_A(\boldsymbol{r}_1)W_A^\dagger(\boldsymbol{r}_2)\right]\right\rangle = \exp\left\{-\frac{C_A}{2}\int ds^+ n(s^+)\sigma(\boldsymbol{r}_1 - \boldsymbol{r}_2)\right\}$$

$\sigma(\boldsymbol{x}_1 - \boldsymbol{x}_2) = 2g^2(\gamma(0) - \gamma(\boldsymbol{x}_1 - \boldsymbol{x}_2))$ **(dipole cross section)**

Azimuthal particle distribution (massive quarks)

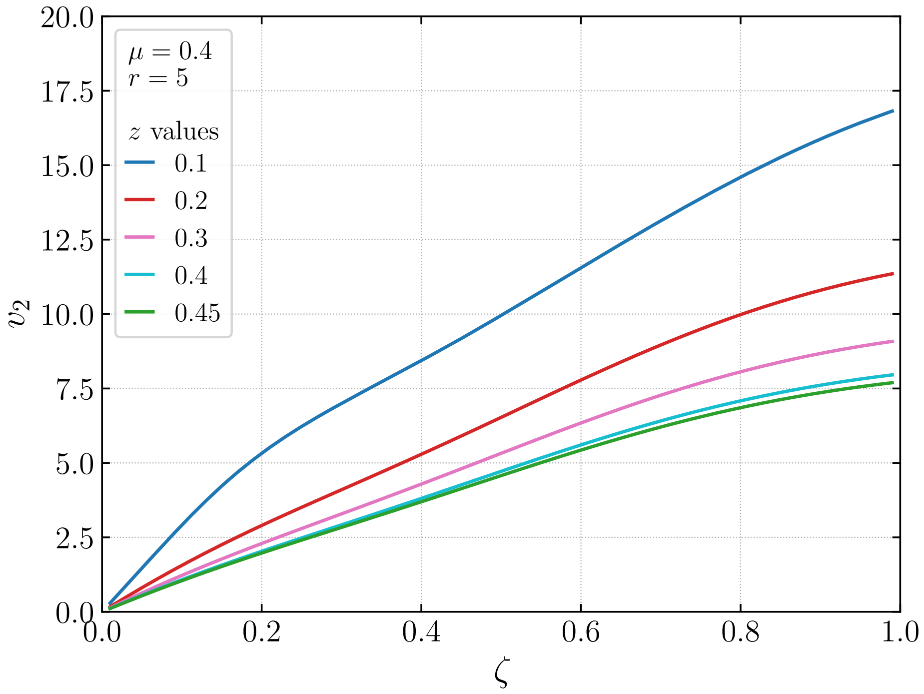


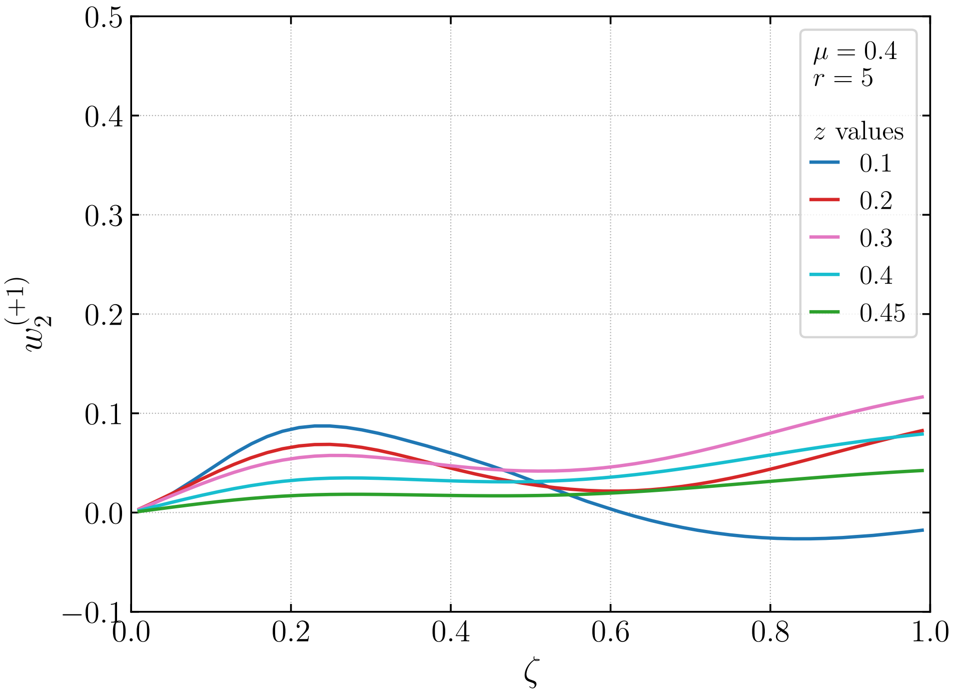


Sensitivity to anisotropy values is reduced and spin distinction is confounded

Symmetry properties are washed out

Harmonic decomposition (massive quarks)





$v_2 \gg 1$ even larger than the massless case

$w_2^{(+1)} = w_2^{h=1}$ non-trivial with z and ζ