

Extracting the anomalous dimensions of Energy-Correlators in charged jets in pp collisions at 13 TeV with ALICE

Hard Probes 2024

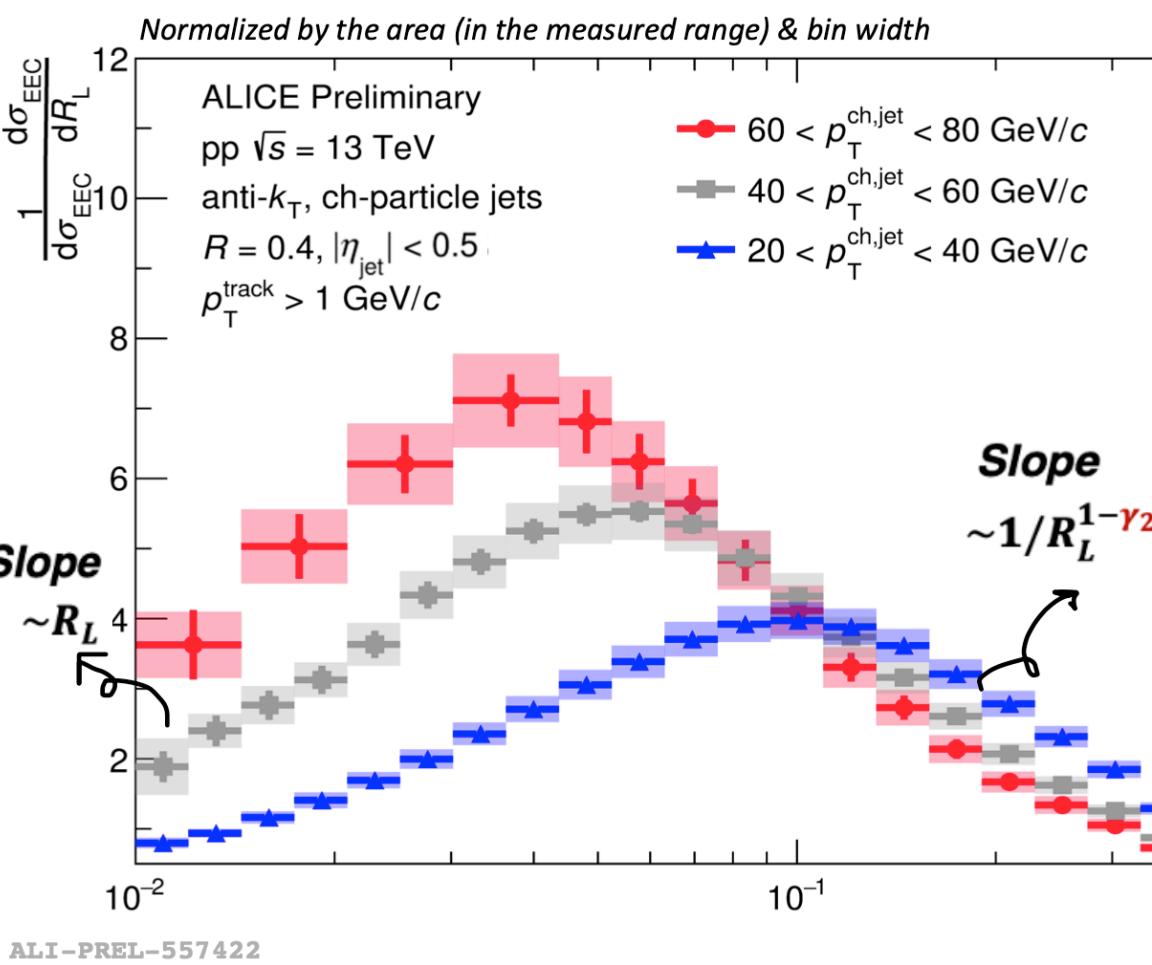
Ananya Rai (on behalf of ALICE), 24th September 2024

Supported in part by the

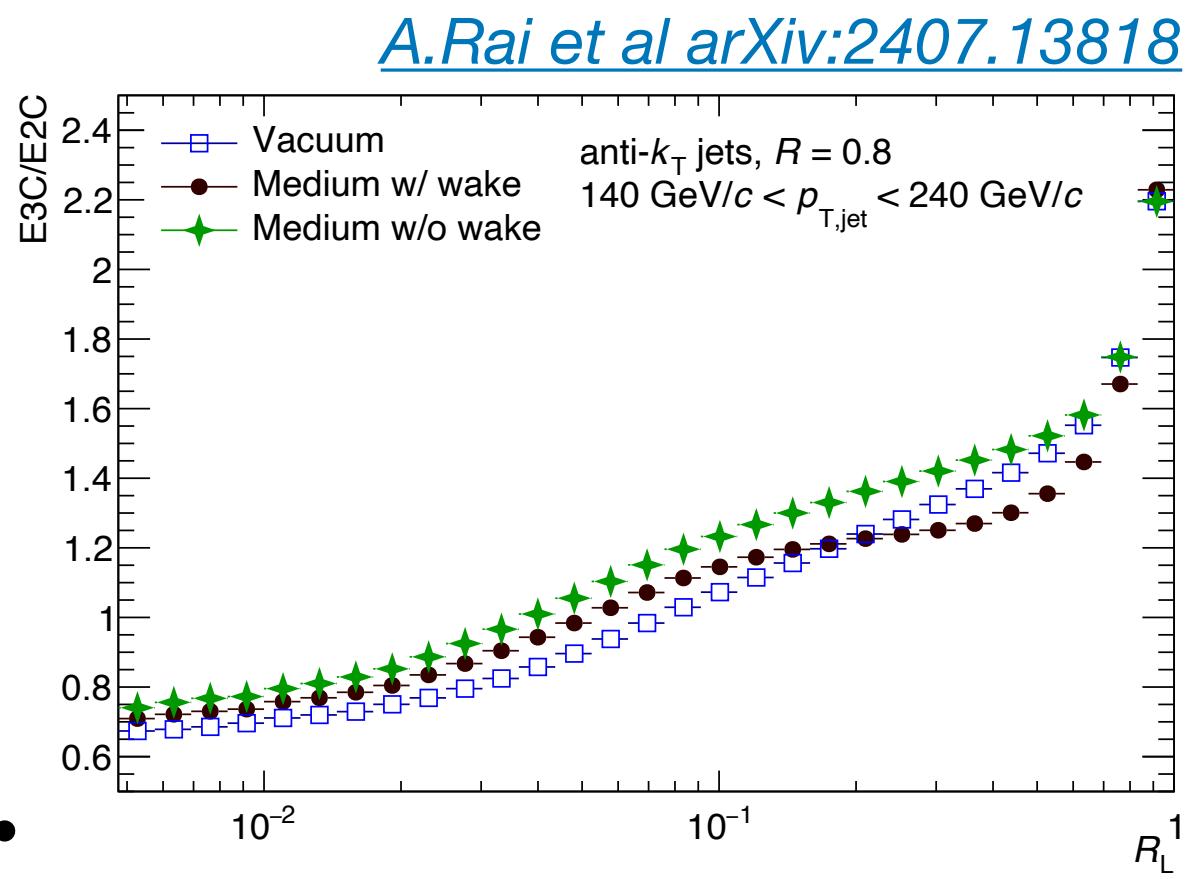
Talk outline



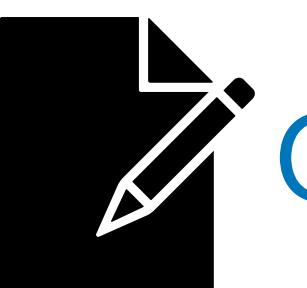
Why study jet substructure
with Energy Correlators?



Energy Correlator
measurements at ALICE



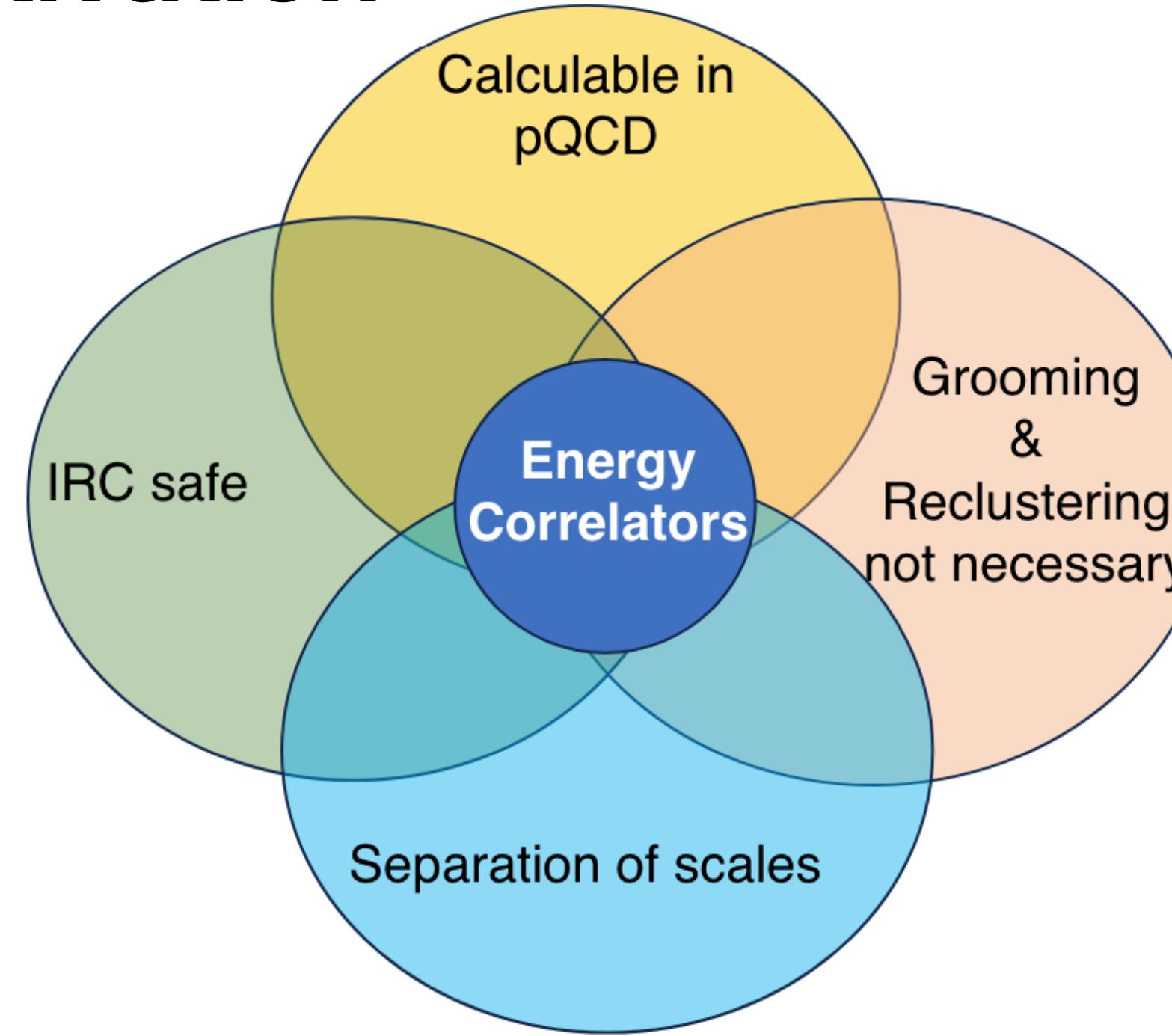
Extending Energy Correlator
measurements to
Heavy Ions



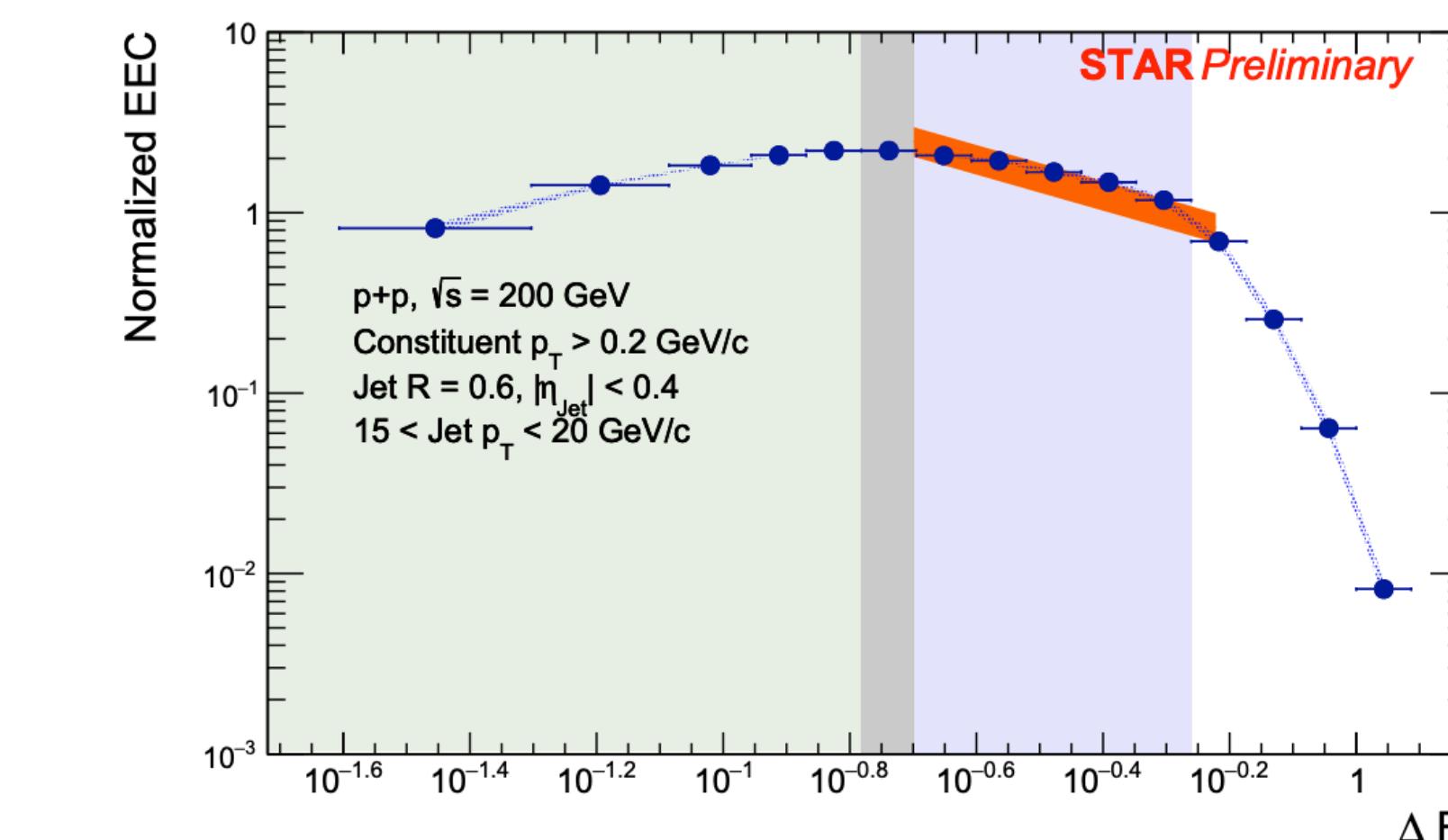
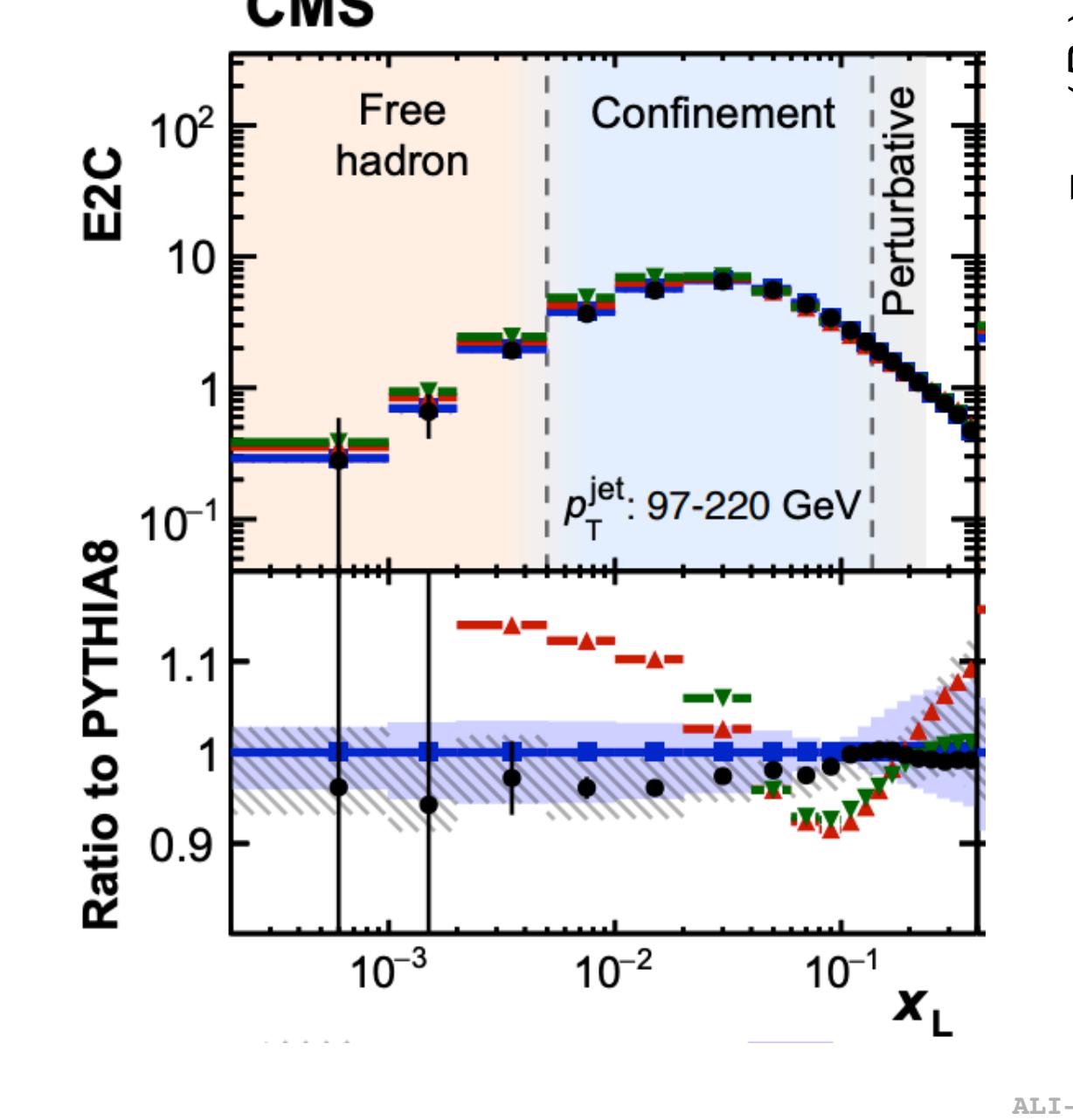
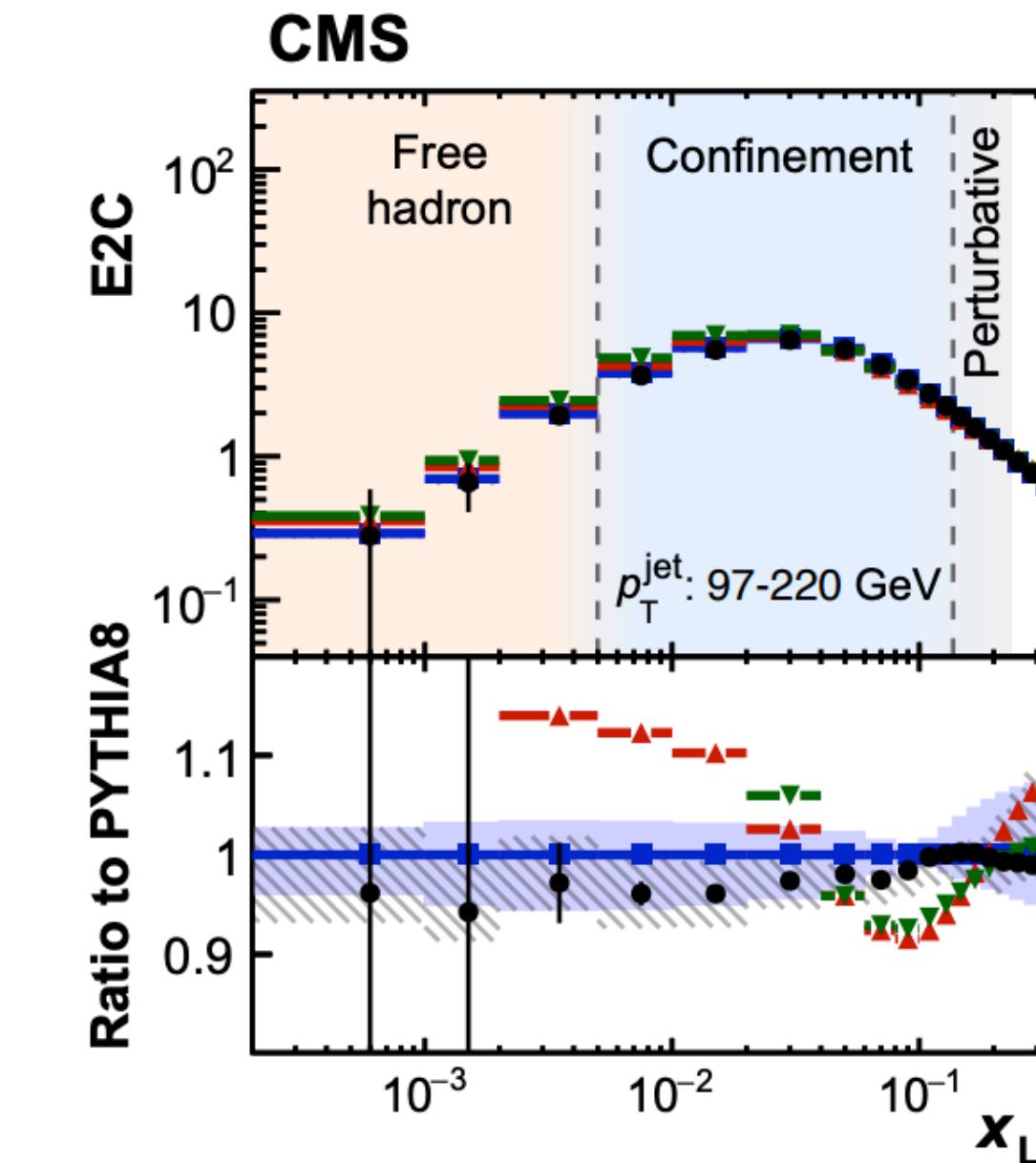
Outlook and summary

Energy Correlators in Jets

Motivation



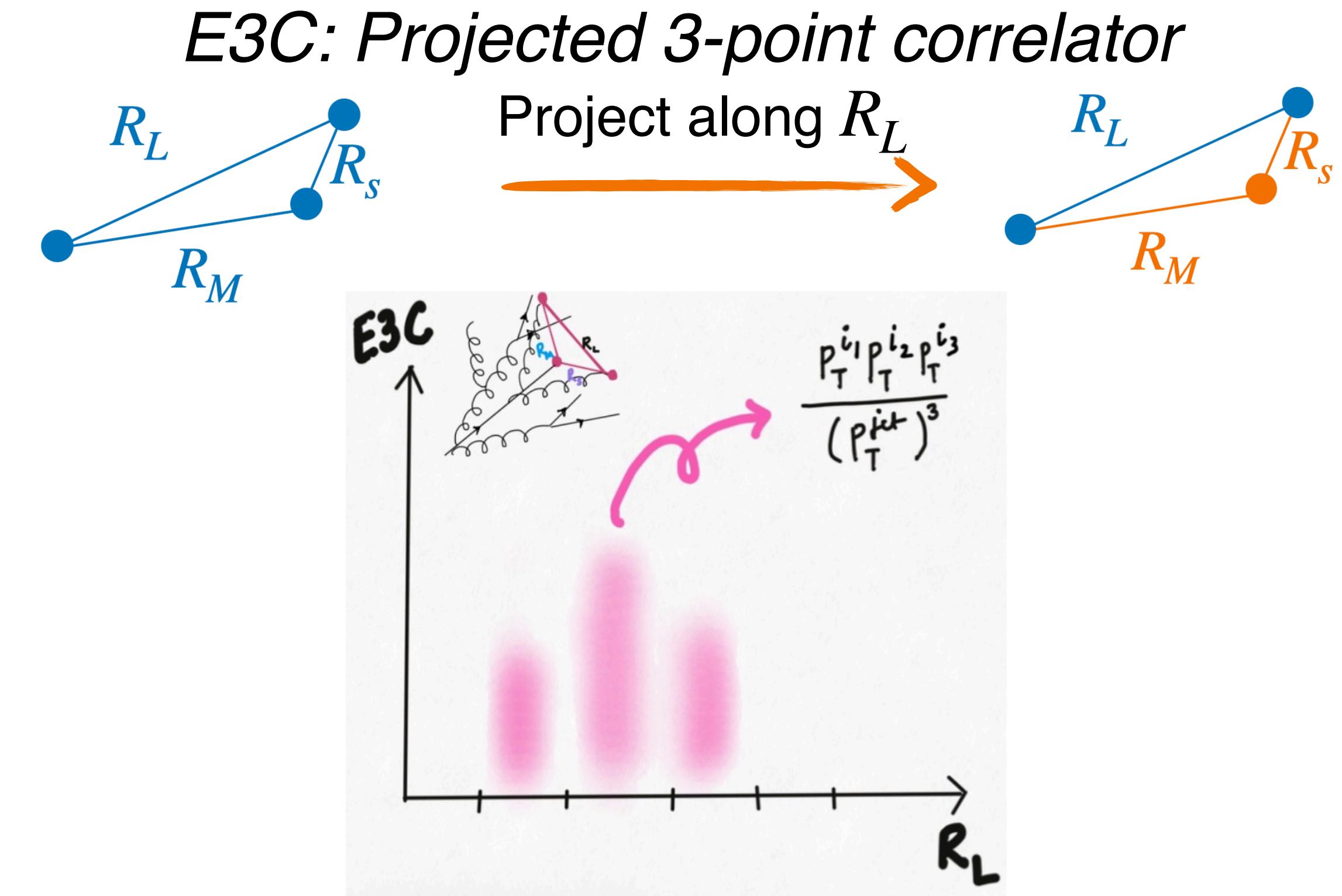
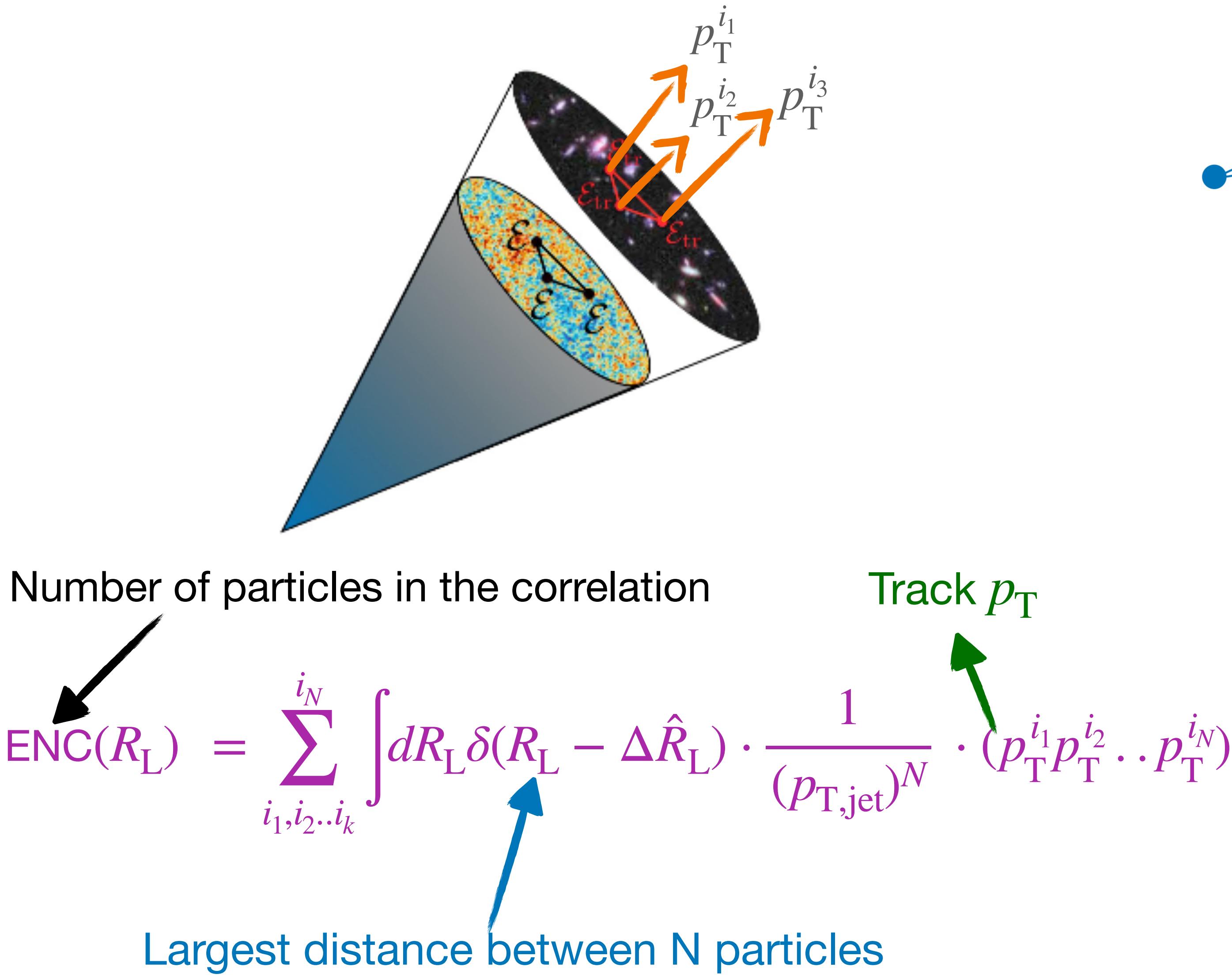
- * Active area of research
- * Recent results from ALEPH, ALICE, CMS, STAR — wide energy range!
- * Extension to QCD matter in experiment!



See talk by A. Nambrath!
See poster by P. Dhanker!
See poster by B. Liangman!

Energy Correlators in Jets

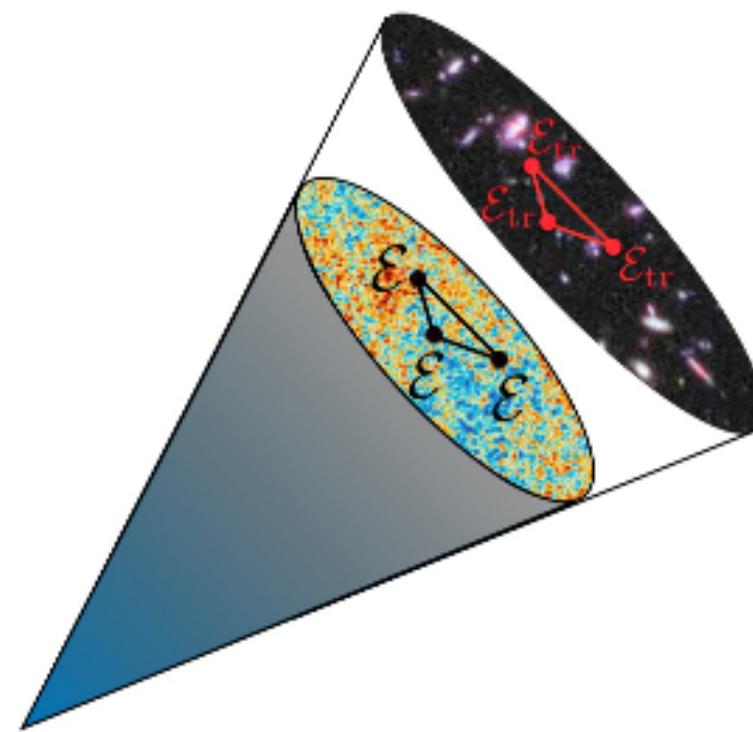
Definition and construction



- Step 1. Find a jet
- Step 2. Create triplets from jet constituents
- Step 3. Find the largest distance between any pair in the triplet
- Step 4. Fill an energy-weighted histogram

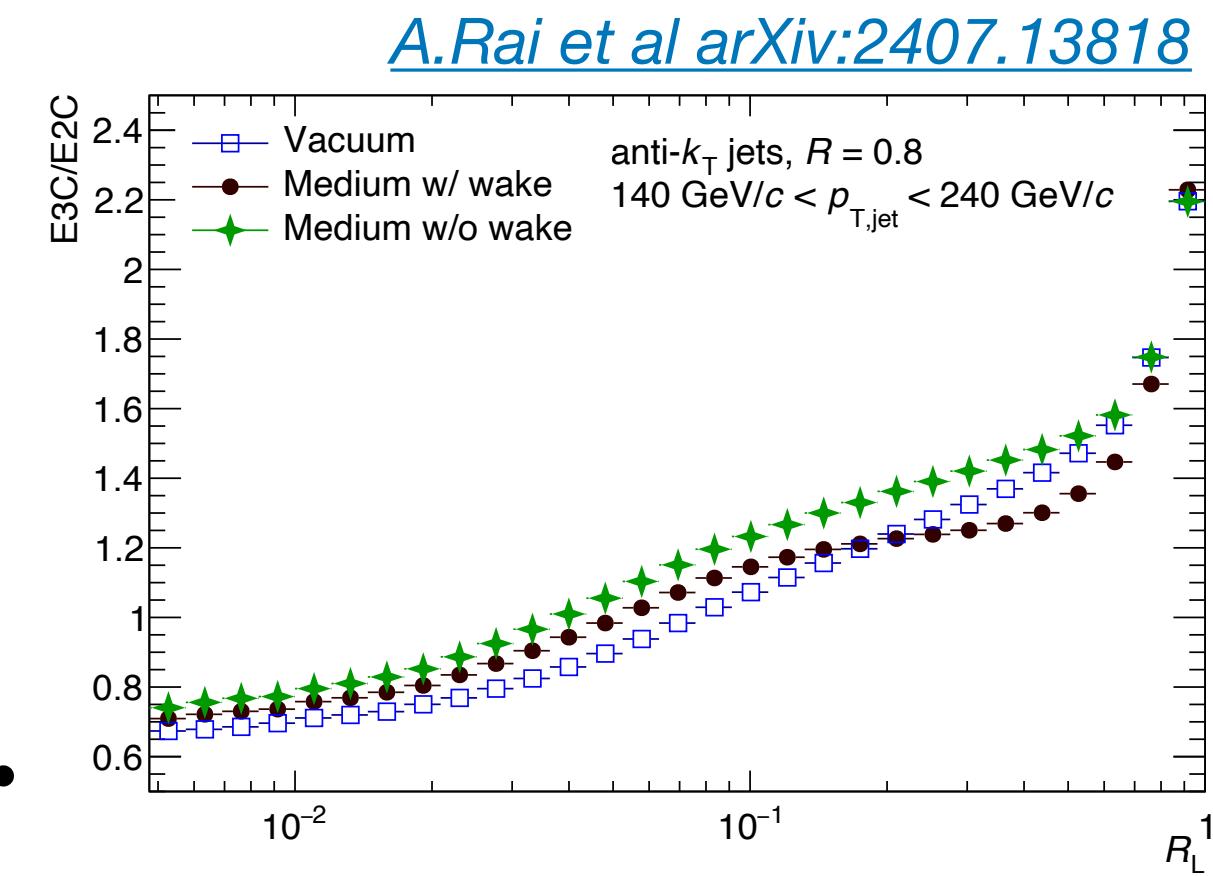
Talk outline

Why study jet substructure
with Energy Correlators?

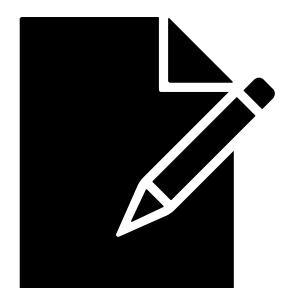


YOU
ARE
HERE

Energy Correlator
measurements at ALICE



Extending Energy Correlator
measurements to
Heavy Ions



Outlook and summary

The ALICE Detector

Constructing jets

* Inner Tracking System

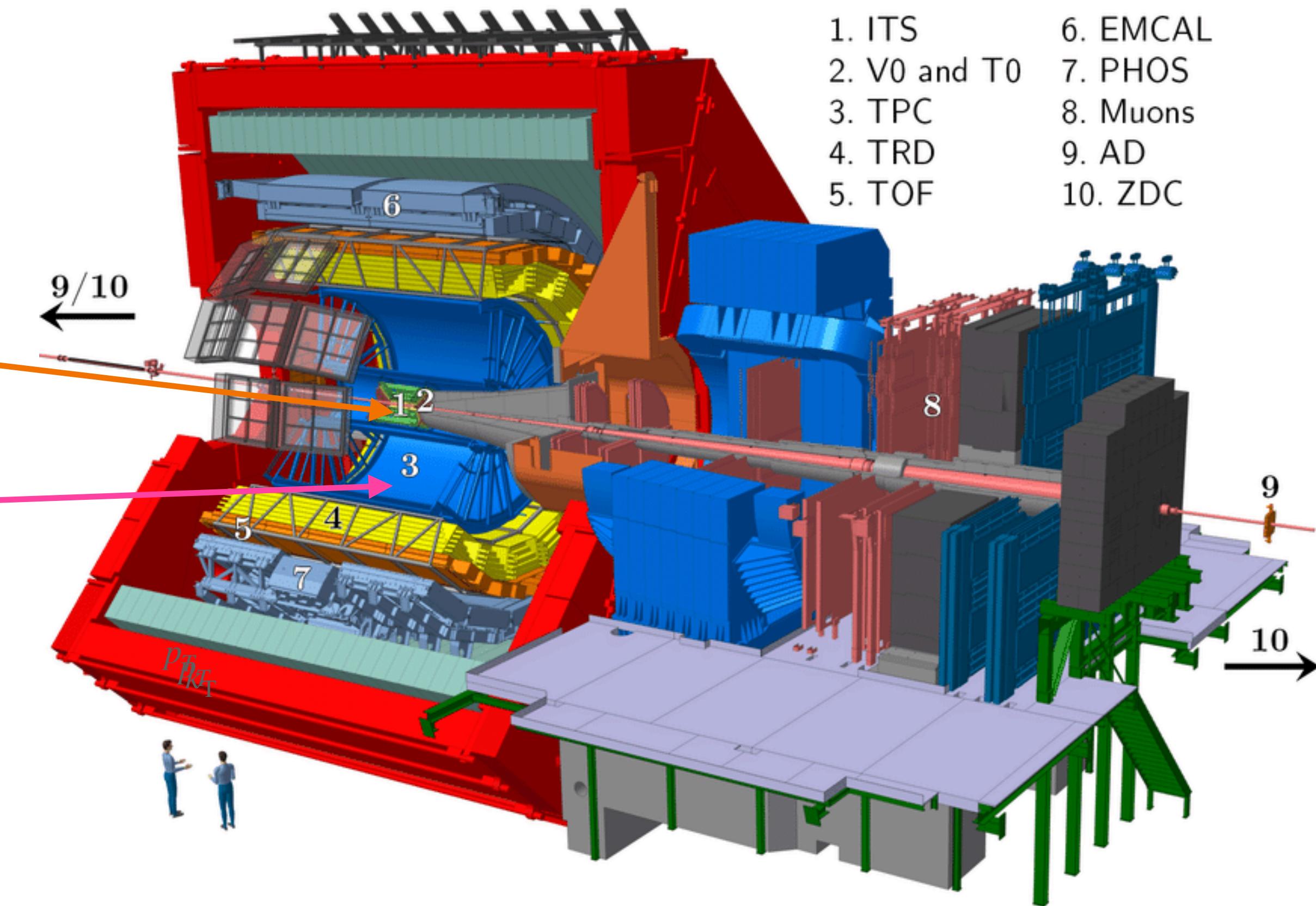
* Time Projection Chamber

Charged-particle jet p_T bins

20 - 40 GeV/c

40 - 60 GeV/c

60 - 80 GeV/c



- * ALICE tracking detectors offer great angular resolution ($\sim 1\text{mrad}$ for $p_T^{\text{track}} = 1 \text{ GeV}/c$)
- * This analysis: Run 2 data at $\sqrt{s} = 13 \text{ TeV}$, **Charged-particle**, anti- k_T jets, $R = 0.4$, $|\eta_{\text{jet}}| < 0.5$, $p_T^{\text{track}} > 1 \text{ GeV}/c$ to build correlators

QFT Detour: Anomalous dimensions

What they are and why they matter

QFT operators have a **scaling/mass**

dimension $\Delta_{\mathcal{O}}$. E.g., in 3+1D, scalar field ϕ

has $\Delta_{\phi} = 1$, fermion field ψ has $\Delta_{\psi} = 3/2$

Quantum mechanical effects $\rightarrow \Delta_{\mathcal{O}}$

gets shifted by “anomalous dimensions”,

$\gamma_{\mathcal{O}}$:

$$\Delta_{\mathcal{O}} = \Delta_{\mathcal{O}} + \gamma_{\mathcal{O}}$$



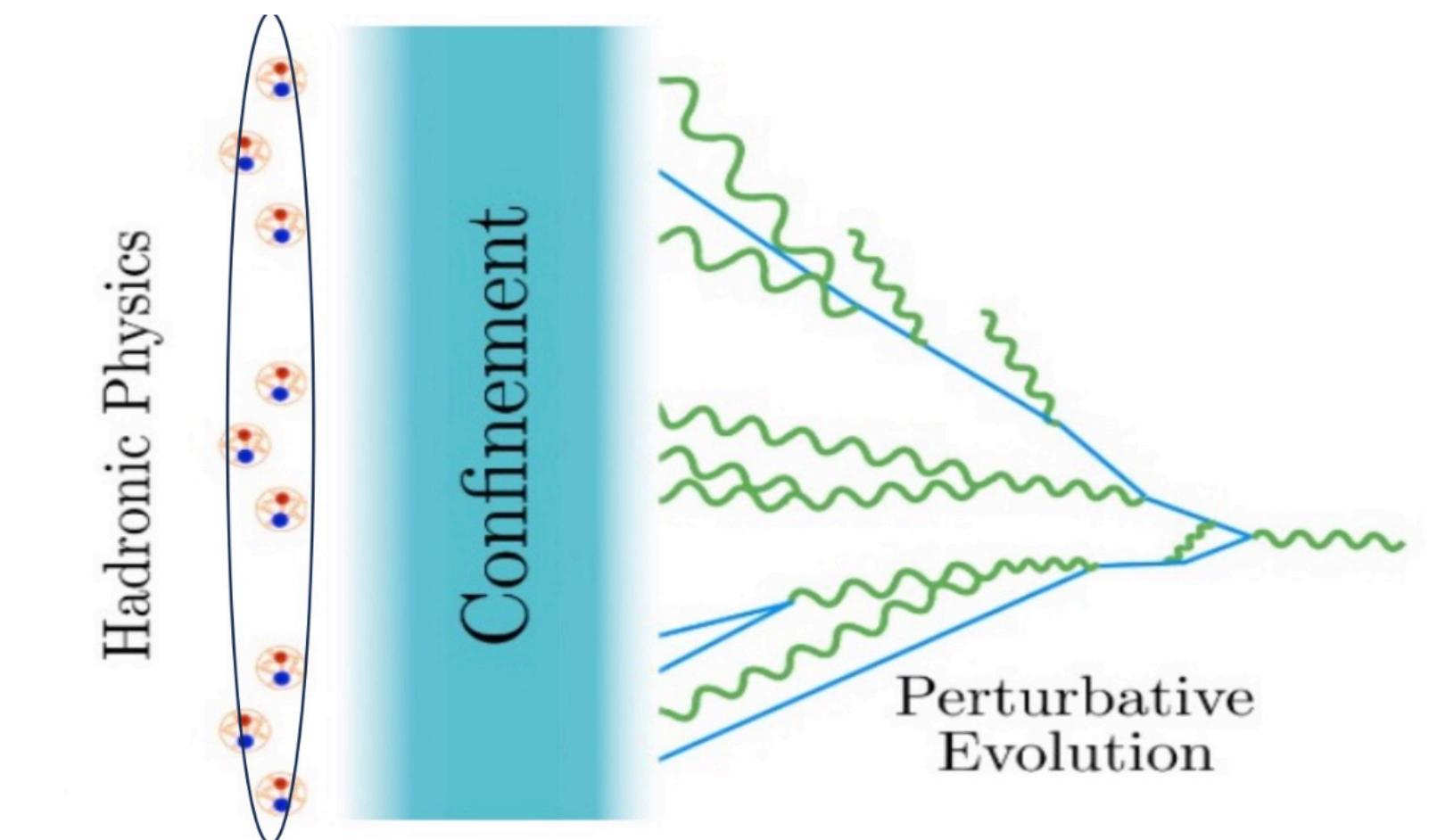
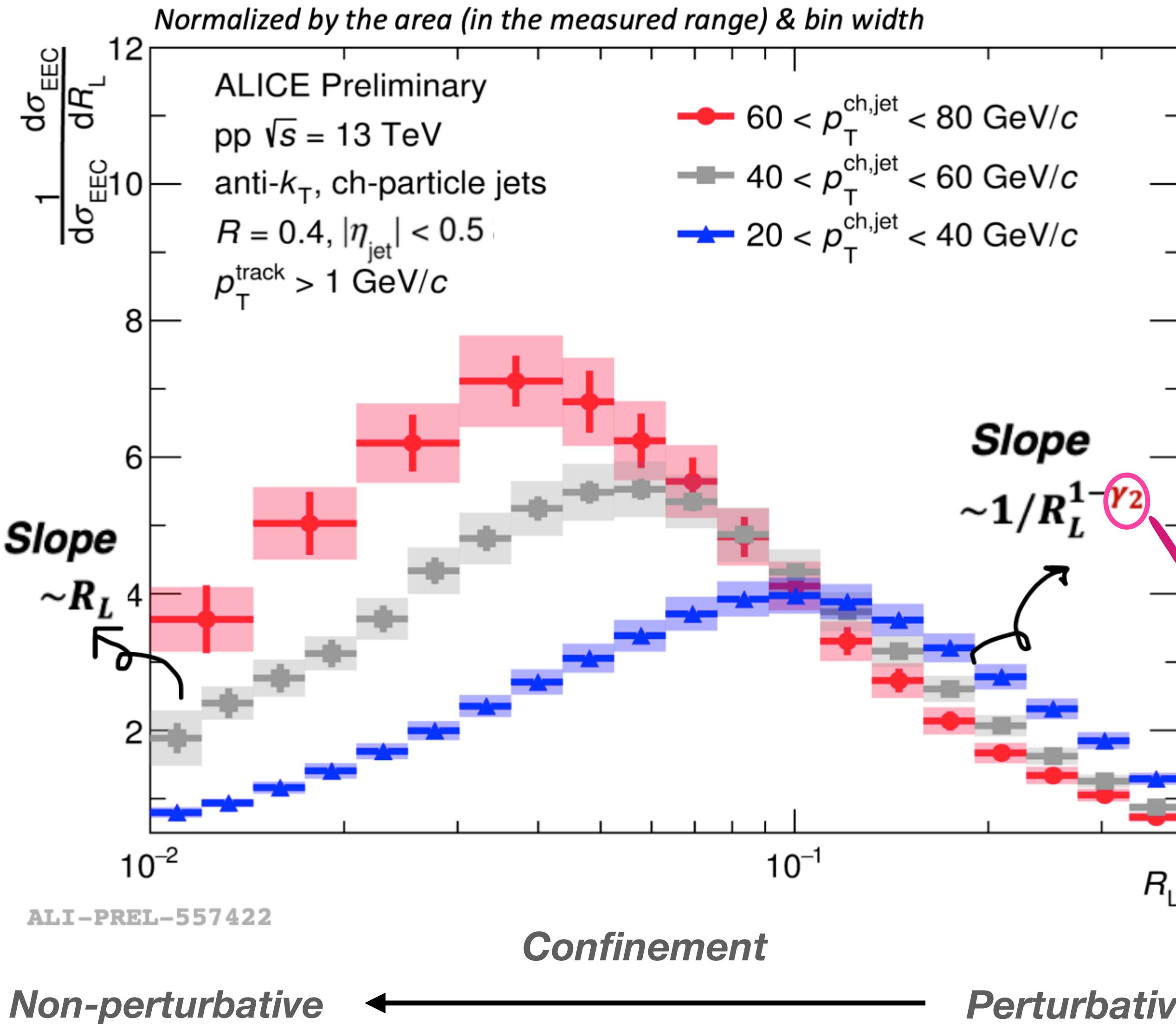
Probes **quantum mechanical corrections!**

Measuring and extracting these is a great way to probe QCD dynamics

In the case of energy correlators, extracting these probes the strong coupling constant, α_S !

Results: EEC

pp $\sqrt{s} = 13 \text{ TeV}$

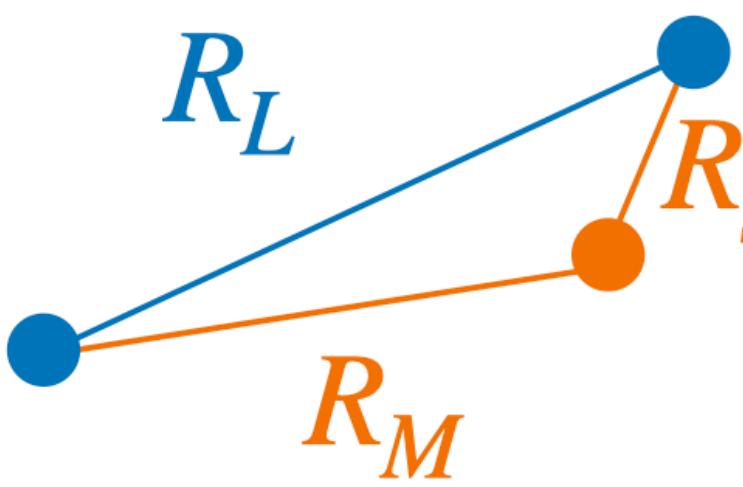


- * QCD evolution from right (perturbative partons) to left (non-perturbative, free streaming hadrons), with peak region representing confinement transition
- * Curve shifts left with increasing jet p_T – elongating the perturbative regime
- * γ_2 – anomalous dimension of EEC operator - quantum correction!
- * Powers of the slope:
“Scaling behavior”

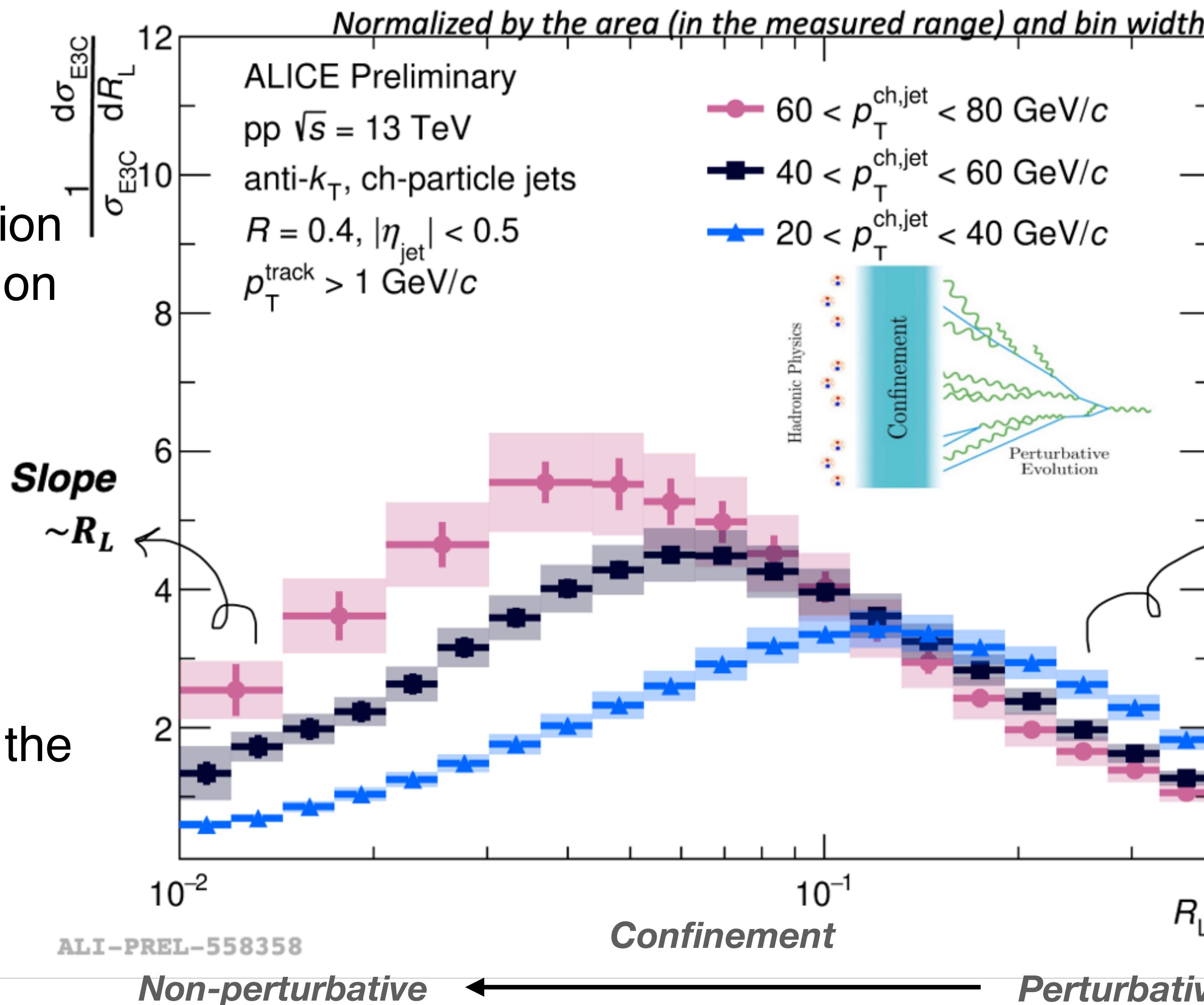
Results: E3C

$\text{pp } \sqrt{s} = 13 \text{ TeV}$

- * Preserve the overall “size” of the correlation by keeping information about largest angle



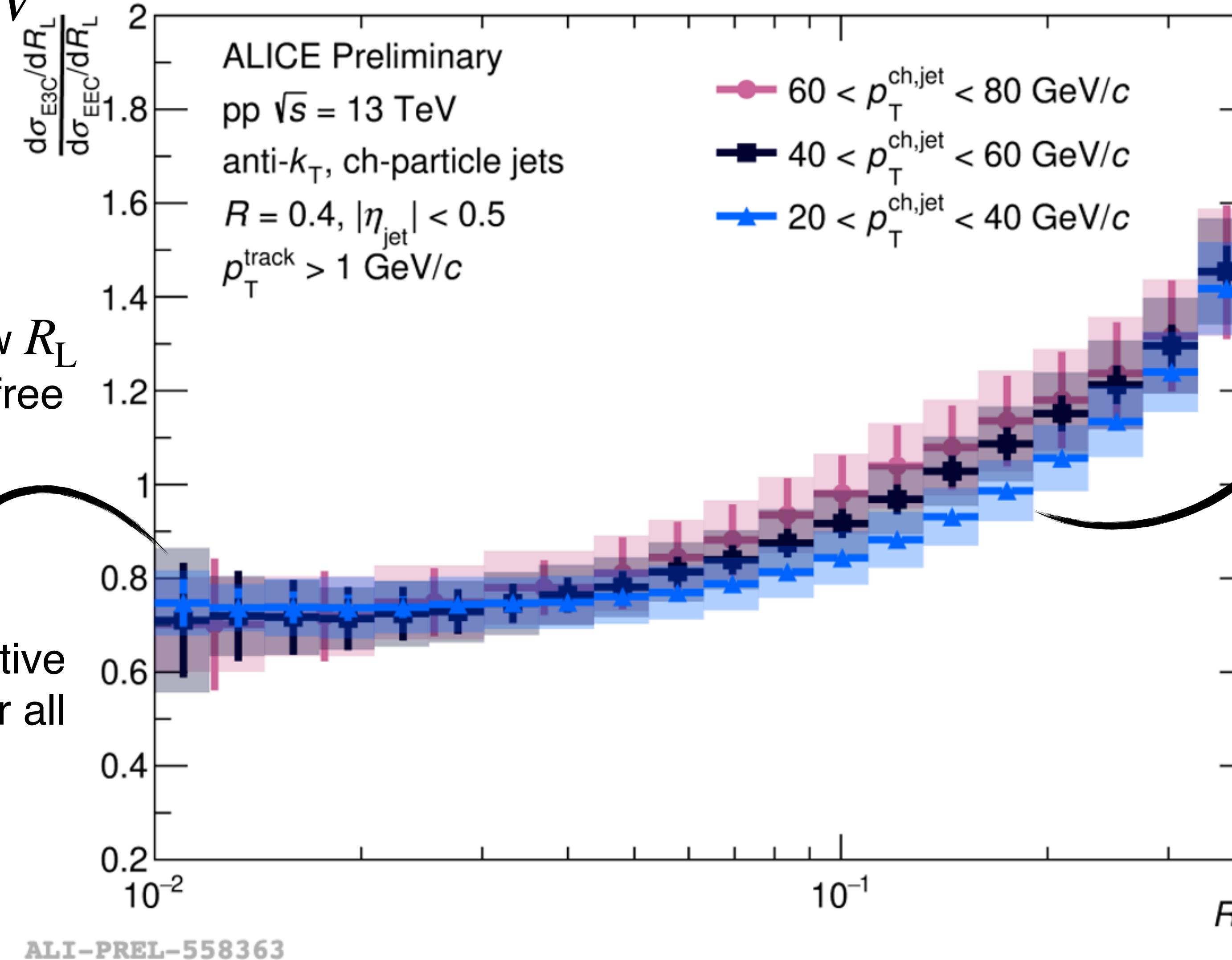
- * Similar behavior as the EEC



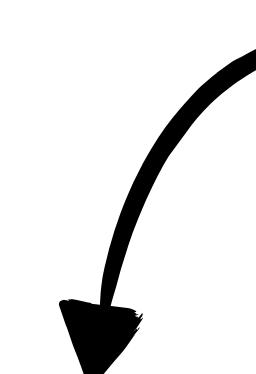
- * γ_3 is the anomalous dimension of E3C operator!

Results: E3C/EEC

pp $\sqrt{s} = 13 \text{ TeV}$



- * Trivial correlation low R_L regime comes from free streaming hadrons



- Slope in non-perturbative regime is the same for all jet p_T bins.

Both E3C & EEC are normalized by the area (in the measured range) and bin width.

$$\begin{array}{c} R_L \\ \diagdown \\ R_S \end{array} \quad \begin{array}{c} R_M \\ \diagup \\ R_L \end{array} \quad \propto \alpha_s \ln(R_L)$$

Slope in perturbative regime $\sim R_L^{\gamma_3 - \gamma_2}$

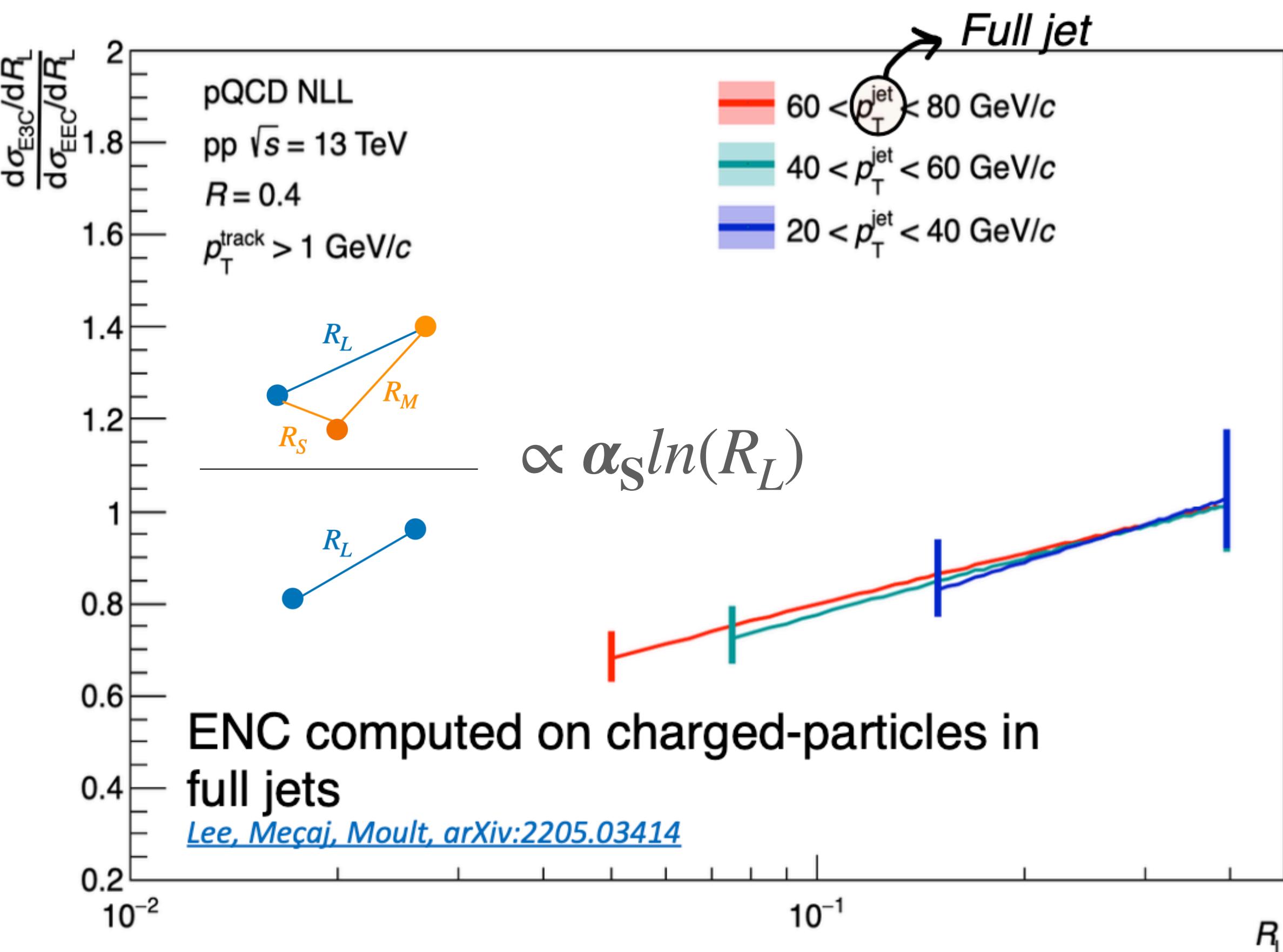
$$* \gamma_3 - \gamma_2 \propto \alpha_s \implies \text{Slope} \propto \alpha_s$$

* Change in slope with jet p_T — **indicative of running of coupling**

* pQCD prediction:
 $\gamma_{N+1} > \gamma_N$
Reproduced in data!

α_S extraction at ALICE : E3C/EEC Ratio

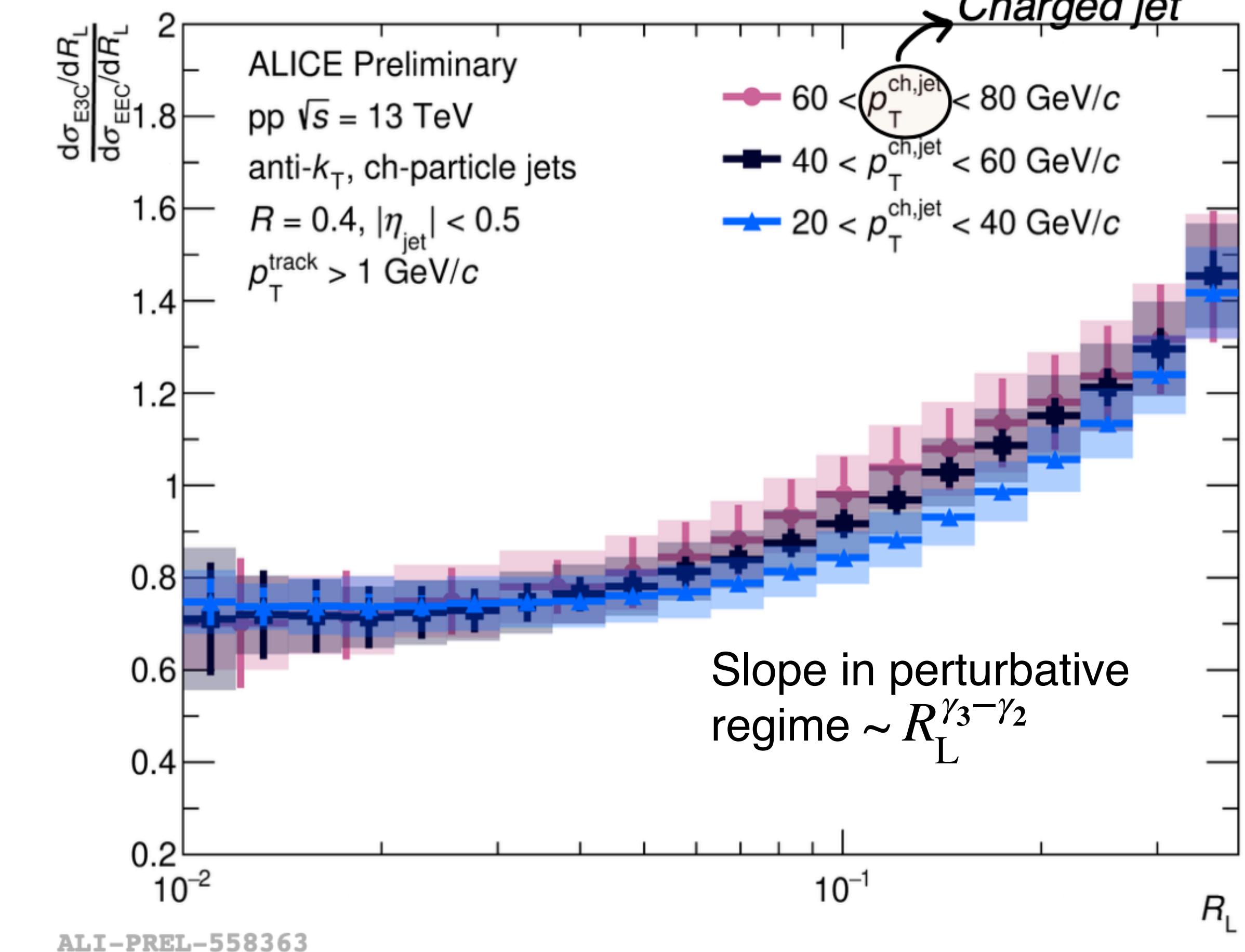
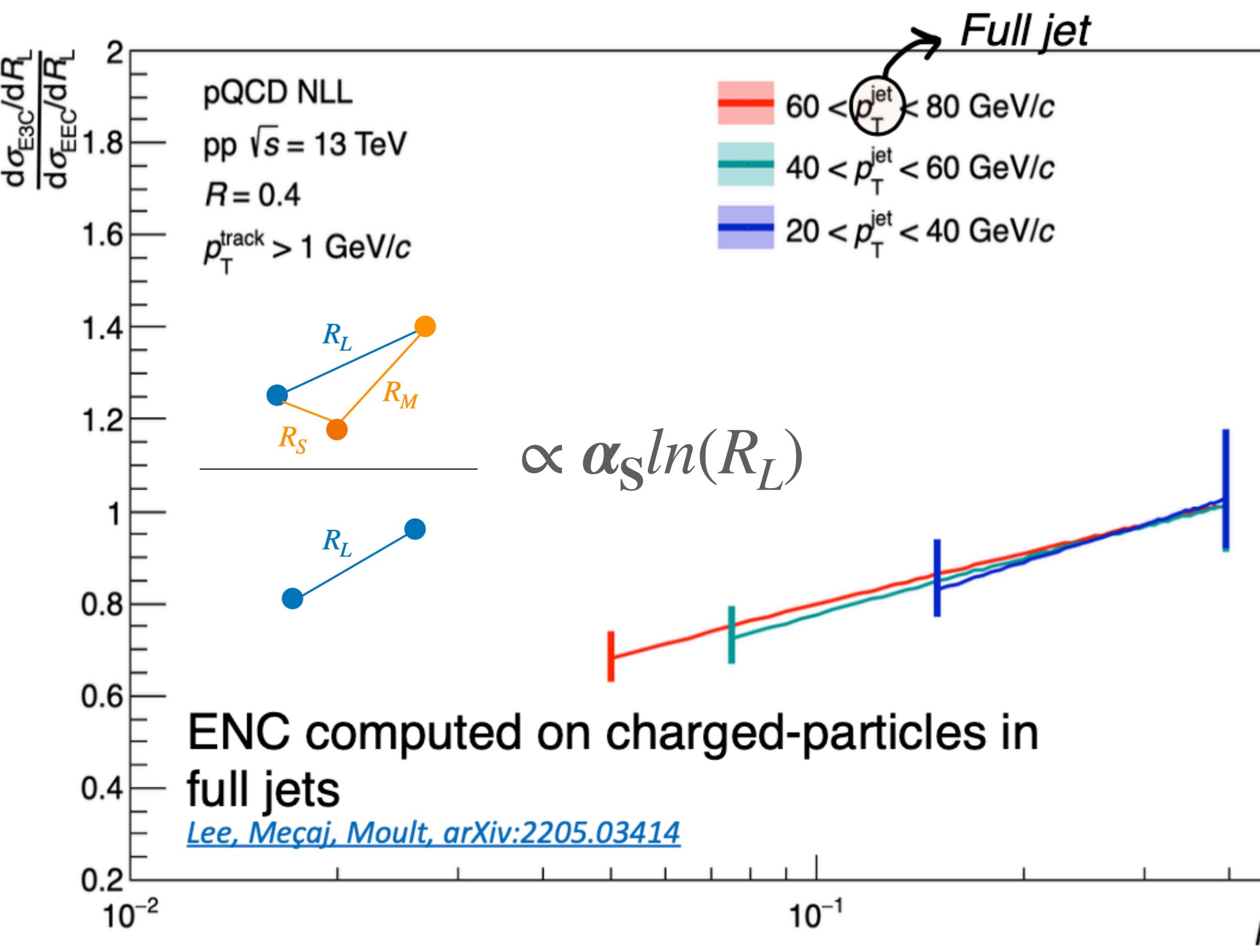
pp $\sqrt{s} = 13 \text{ TeV}$



α_S extraction at ALICE : E3C/EEC Ratio

pp $\sqrt{s} = 13 \text{ TeV}$

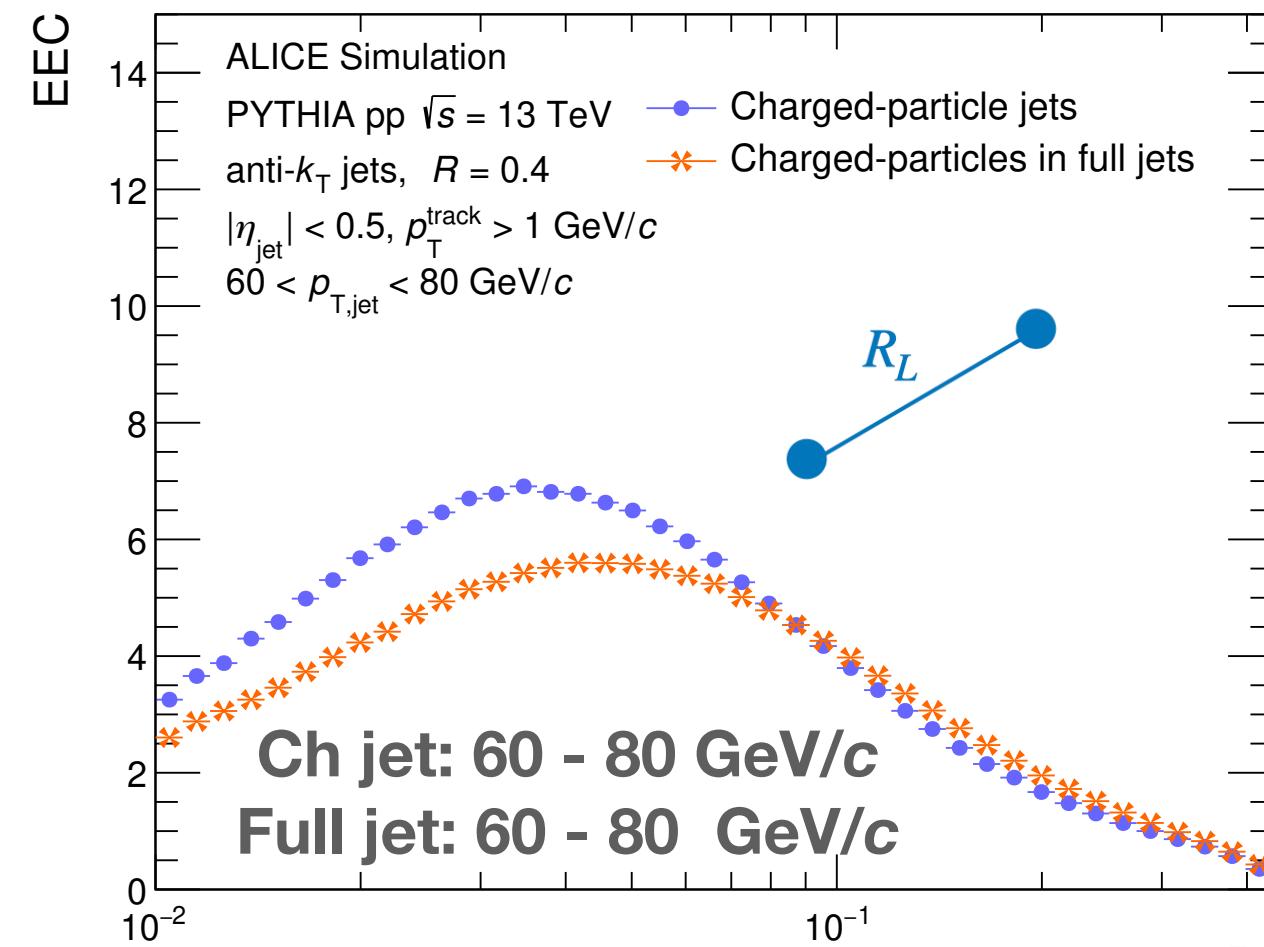
Cause uncertainty in mapping



- * Trends between theory and data agree
- * Current work: extract γ and map to α_S

Can we build a Charged to Full Jet mapping for the ENC?

Do's and dont's

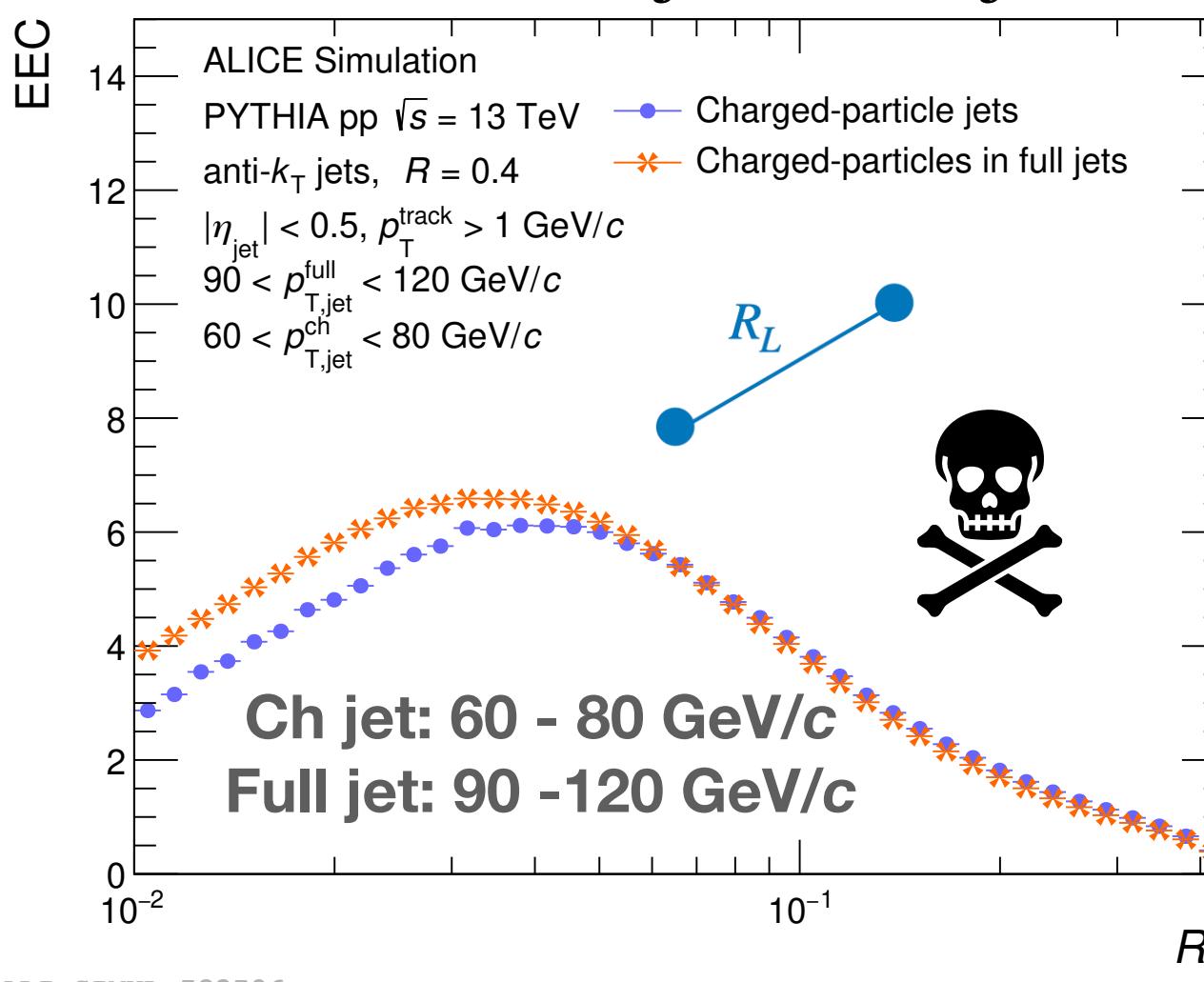


ALI-SIMUL-582588

$$2/3 * p_{T,jet}^{\text{full}} = p_{T,jet}^{\text{ch}}$$

R_L

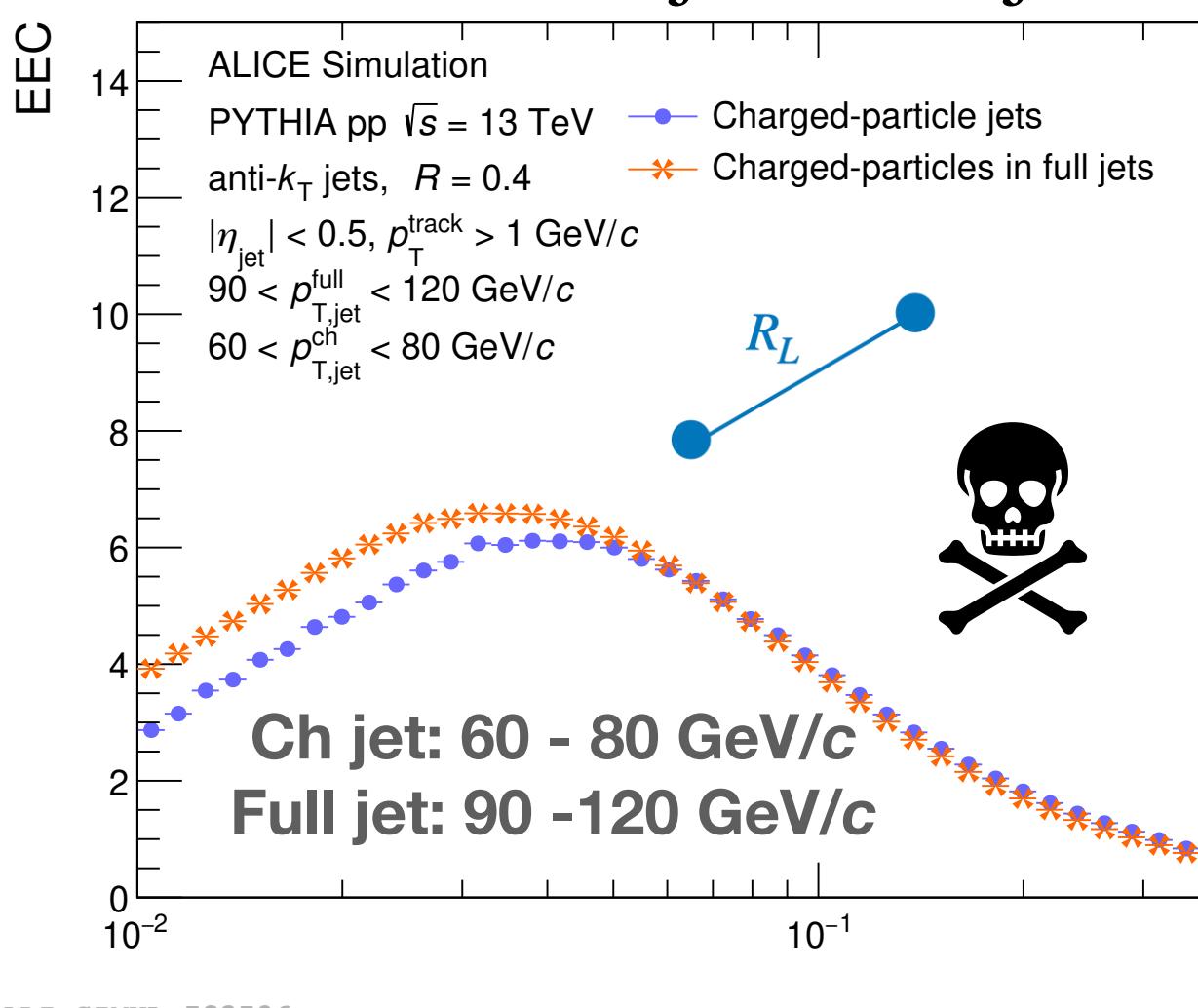
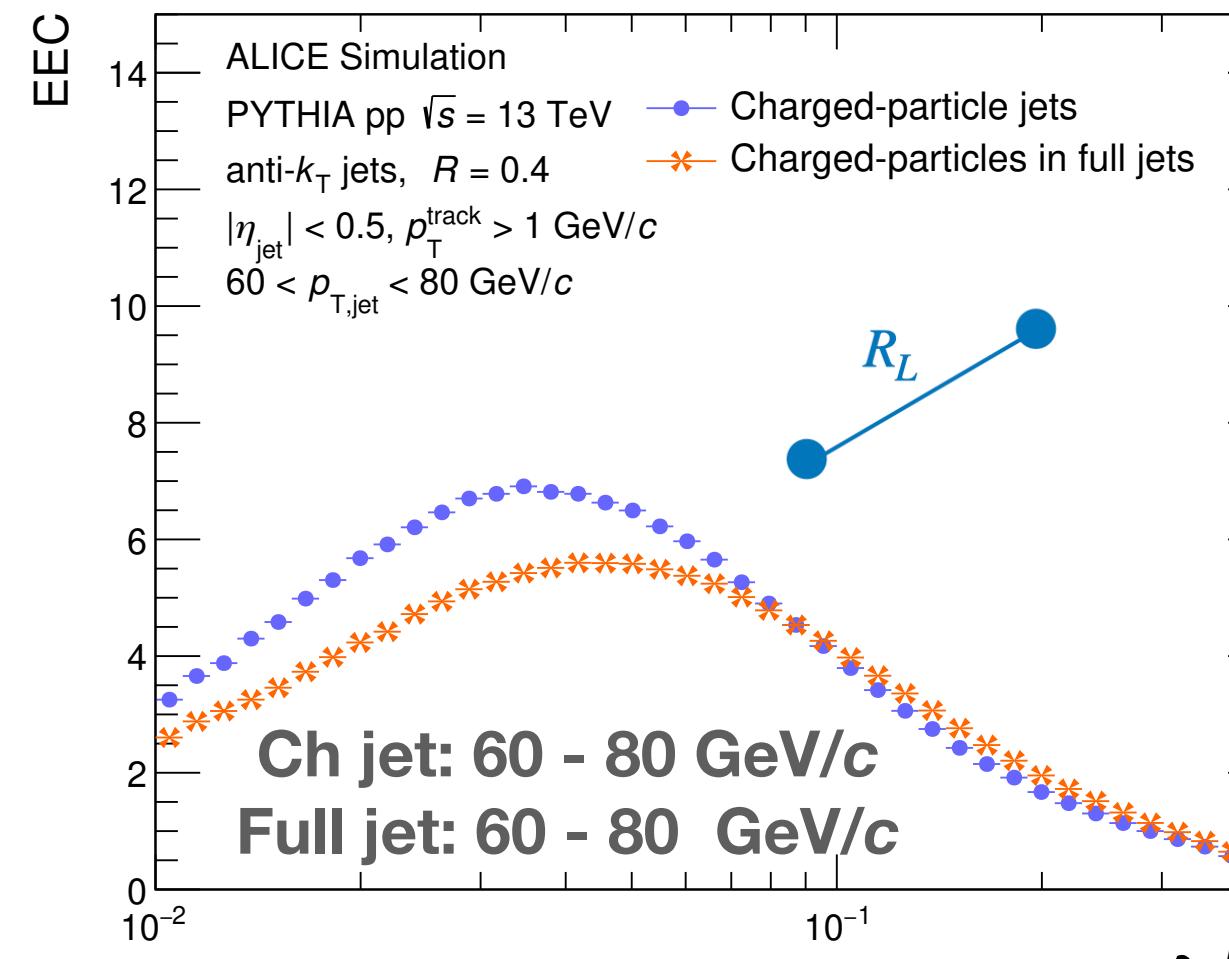
Detailed description: A mathematical equation is overlaid on the plot, showing the relationship between the full jet transverse momentum and the charged-particle transverse momentum. An arrow points from the text down towards the data point at $R_L \approx 10^{-1}$.



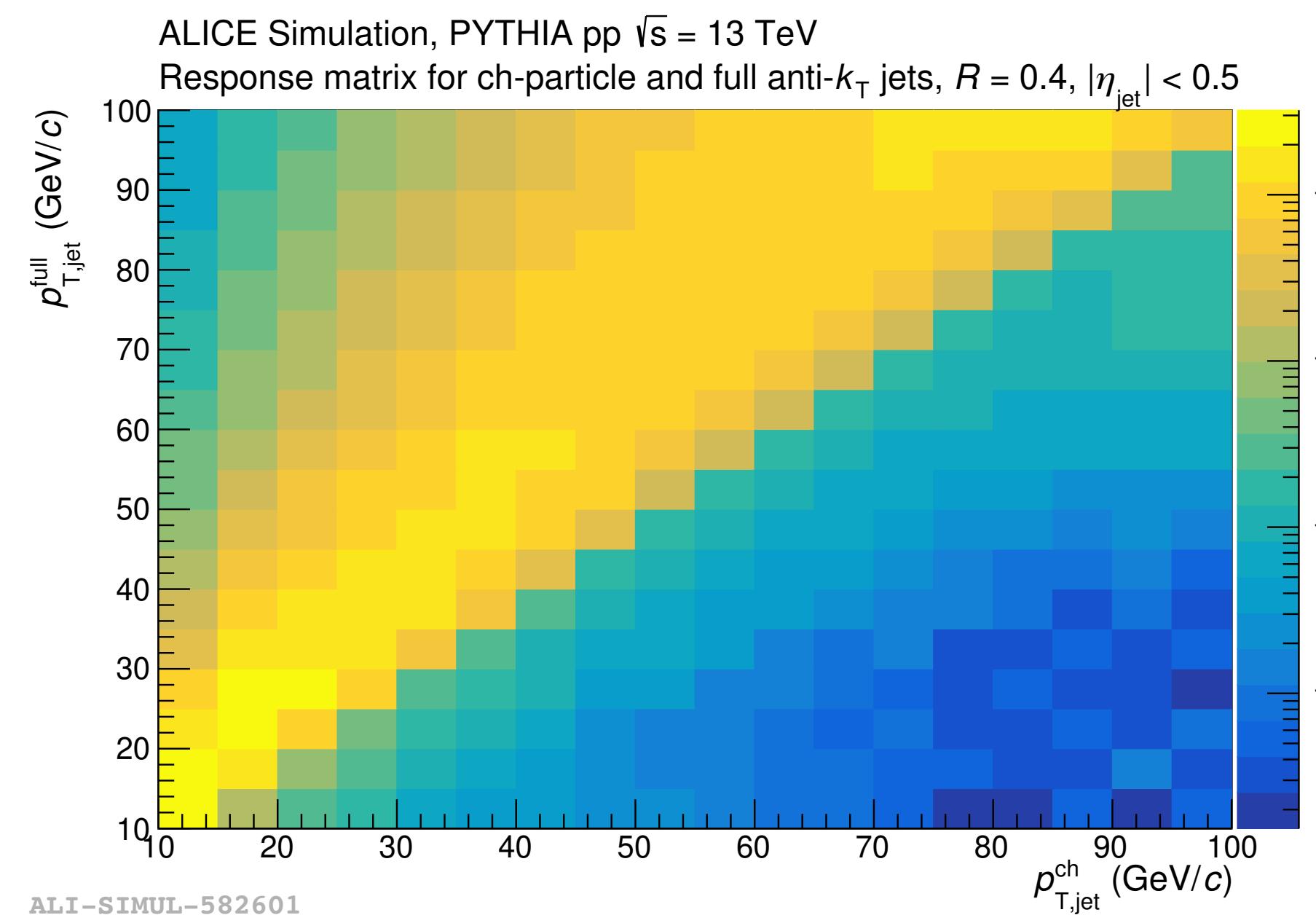
ALI-SIMUL-582596

Can we build a Charged to Full Jet mapping for the ENC?

Do's and dont's



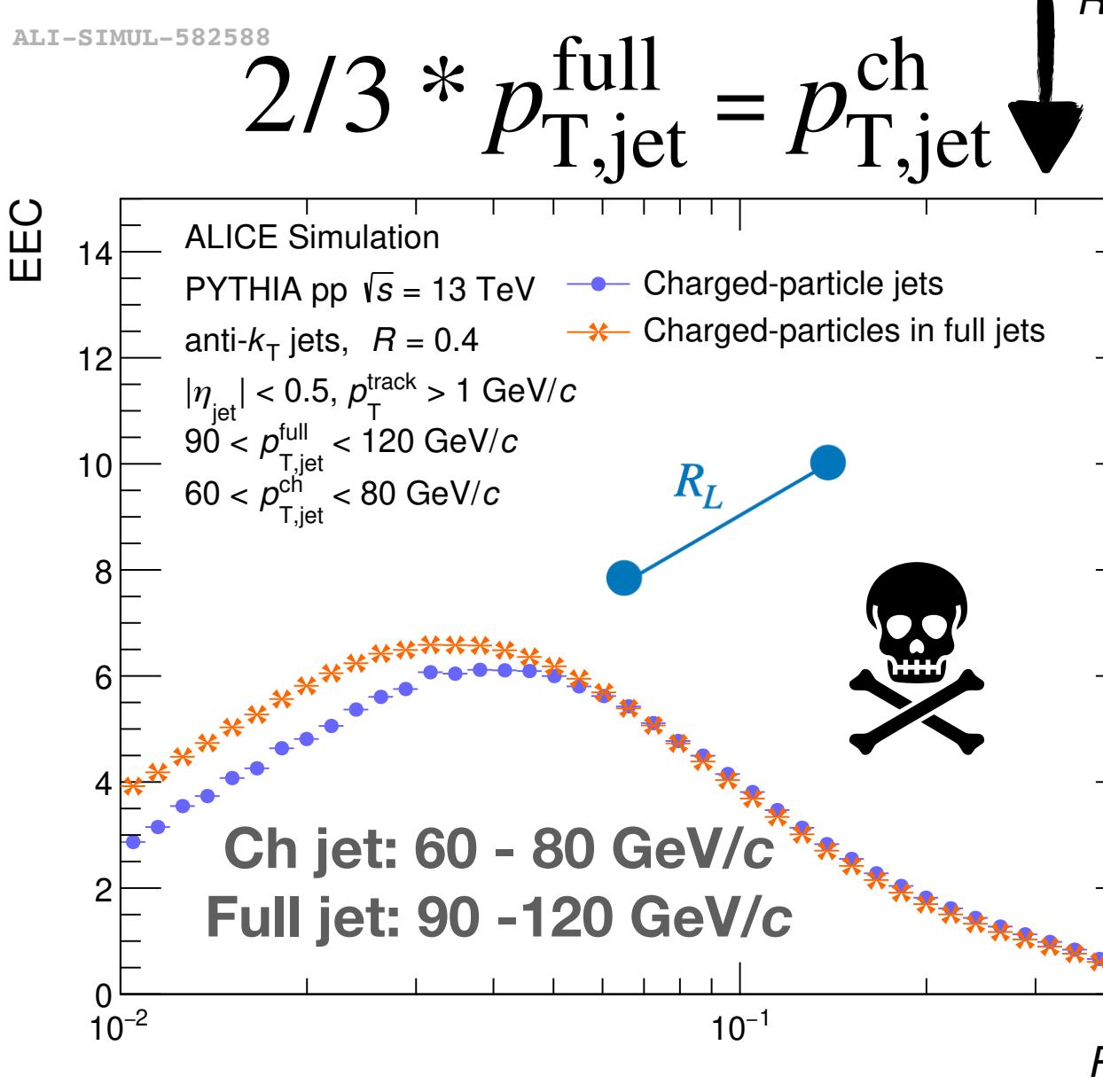
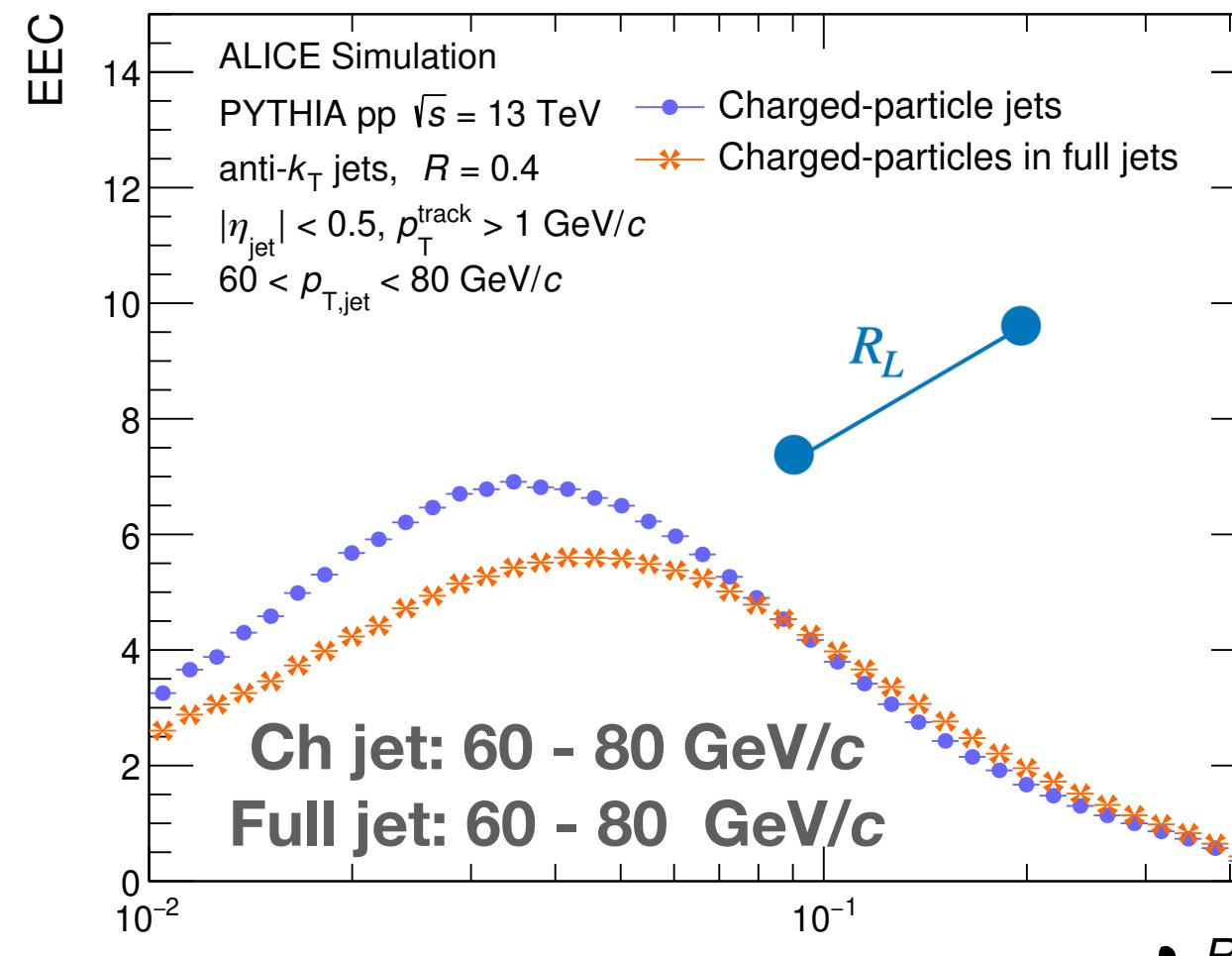
- * Trivial scaling does not work
 - as expected



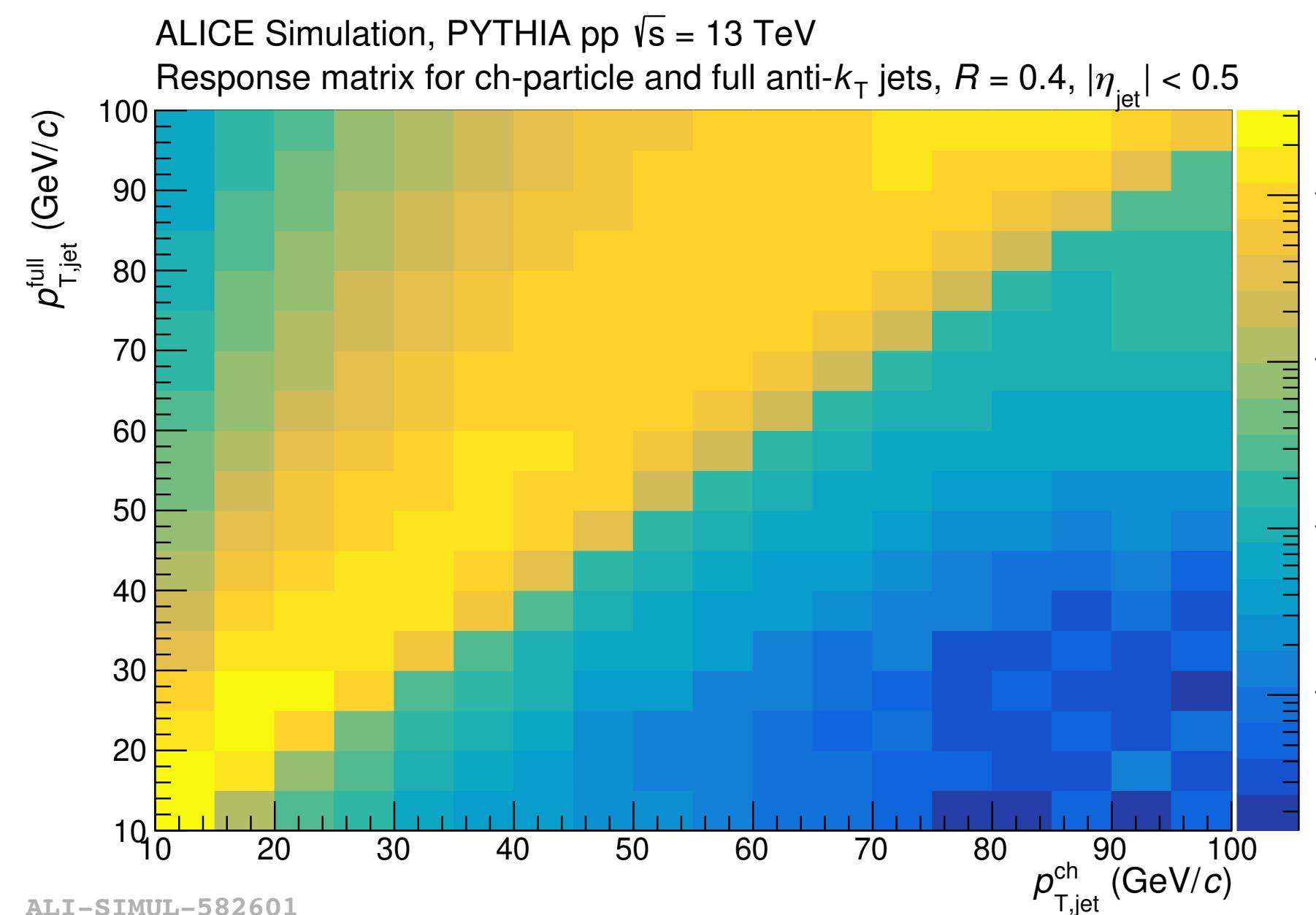
- * To extract α_S precisely using E3C/EEC, we need to account for jet energy scale differences

Can we build a Charged to Full Jet mapping for the EEC?

Do's and don'ts

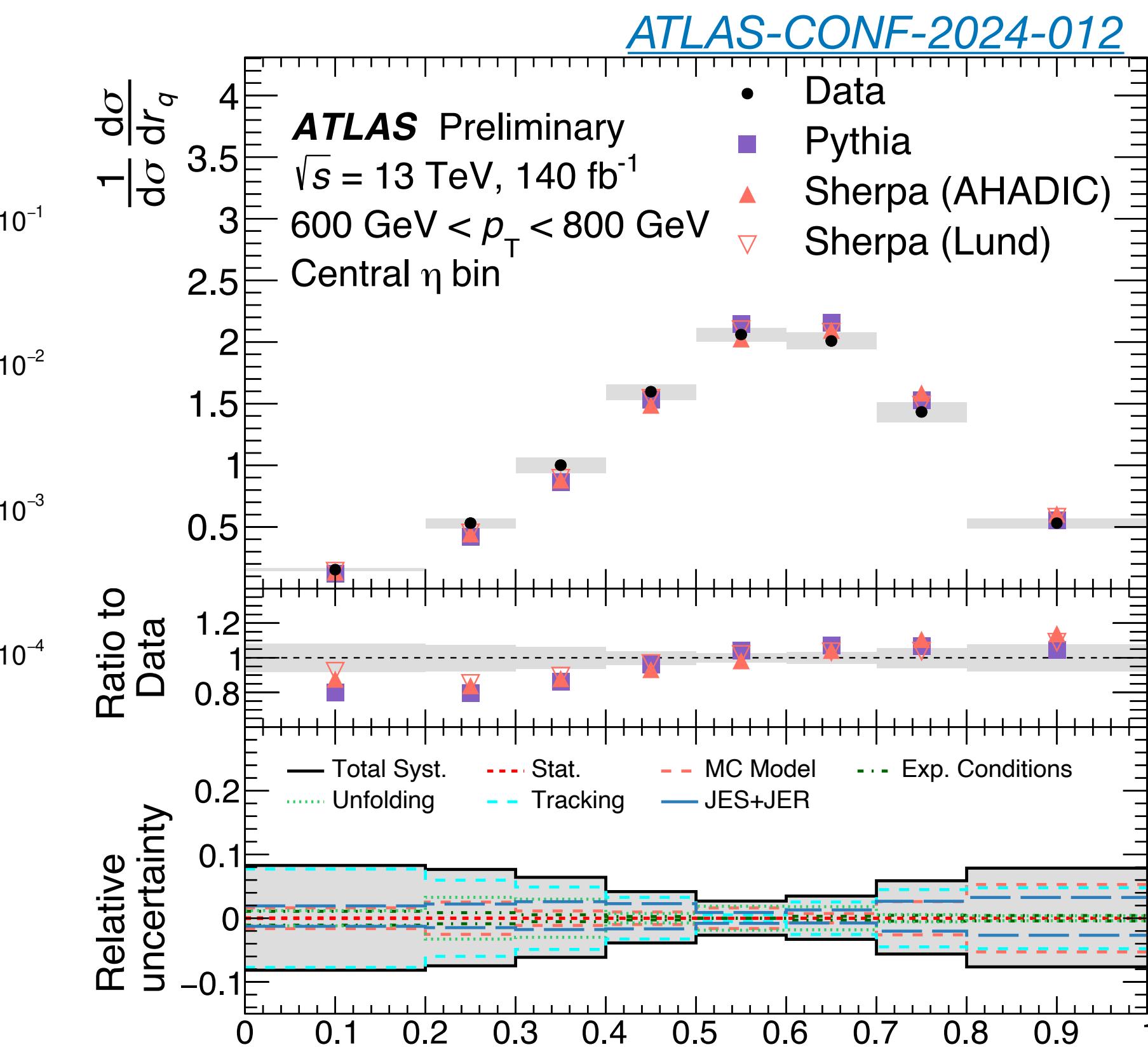


- * Trivial scaling does not work – as expected



- * To extract α_S precisely using E3C/EEC, we need to account for jet energy scale differences

- * Track functions – one avenue for creating a mapping!



$$r_q = \sum(\vec{p}_{T,trk}) / p_{T,jet}$$

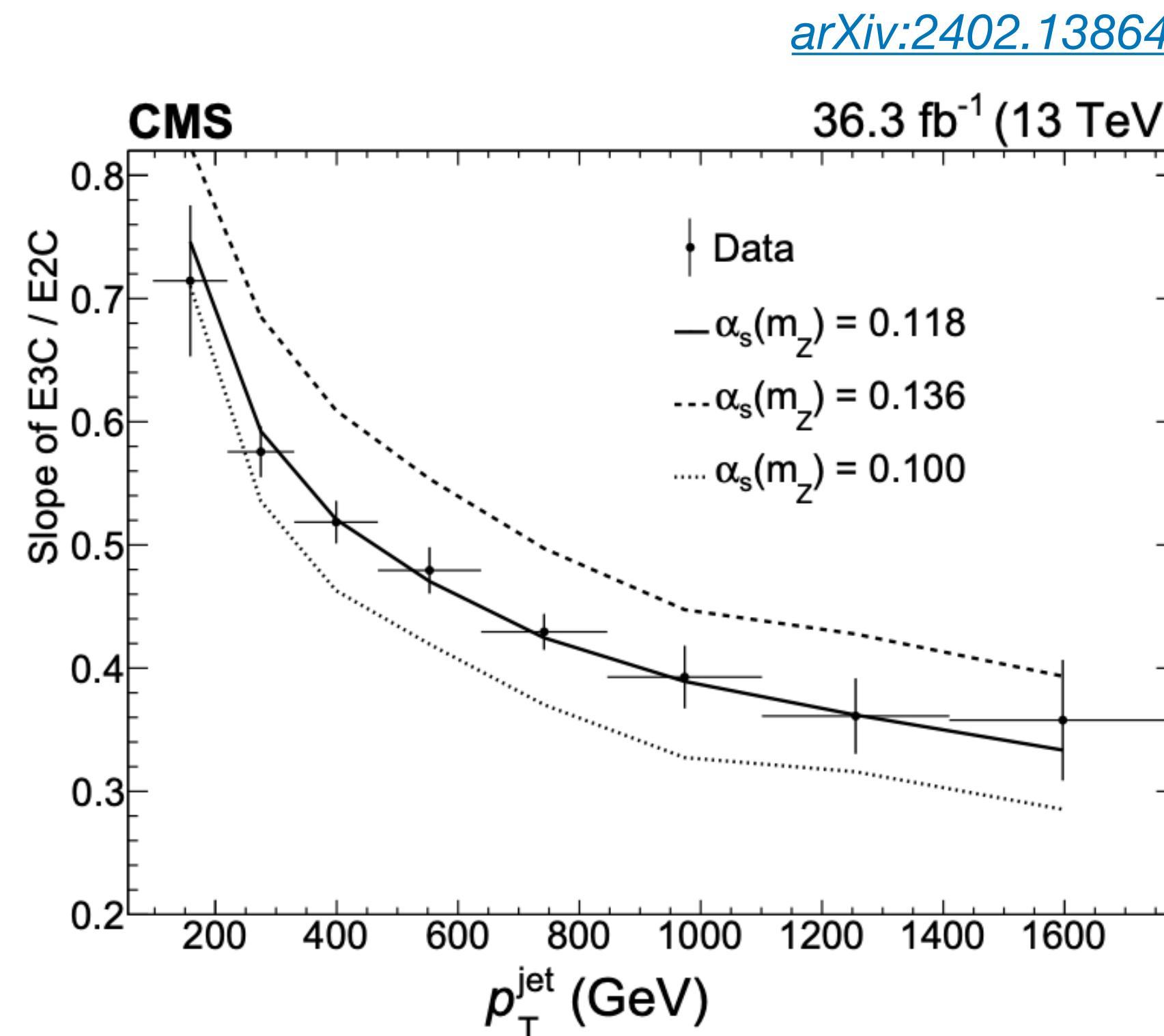
α_S extraction at ALICE : E3C/EEC Ratio

Motivation

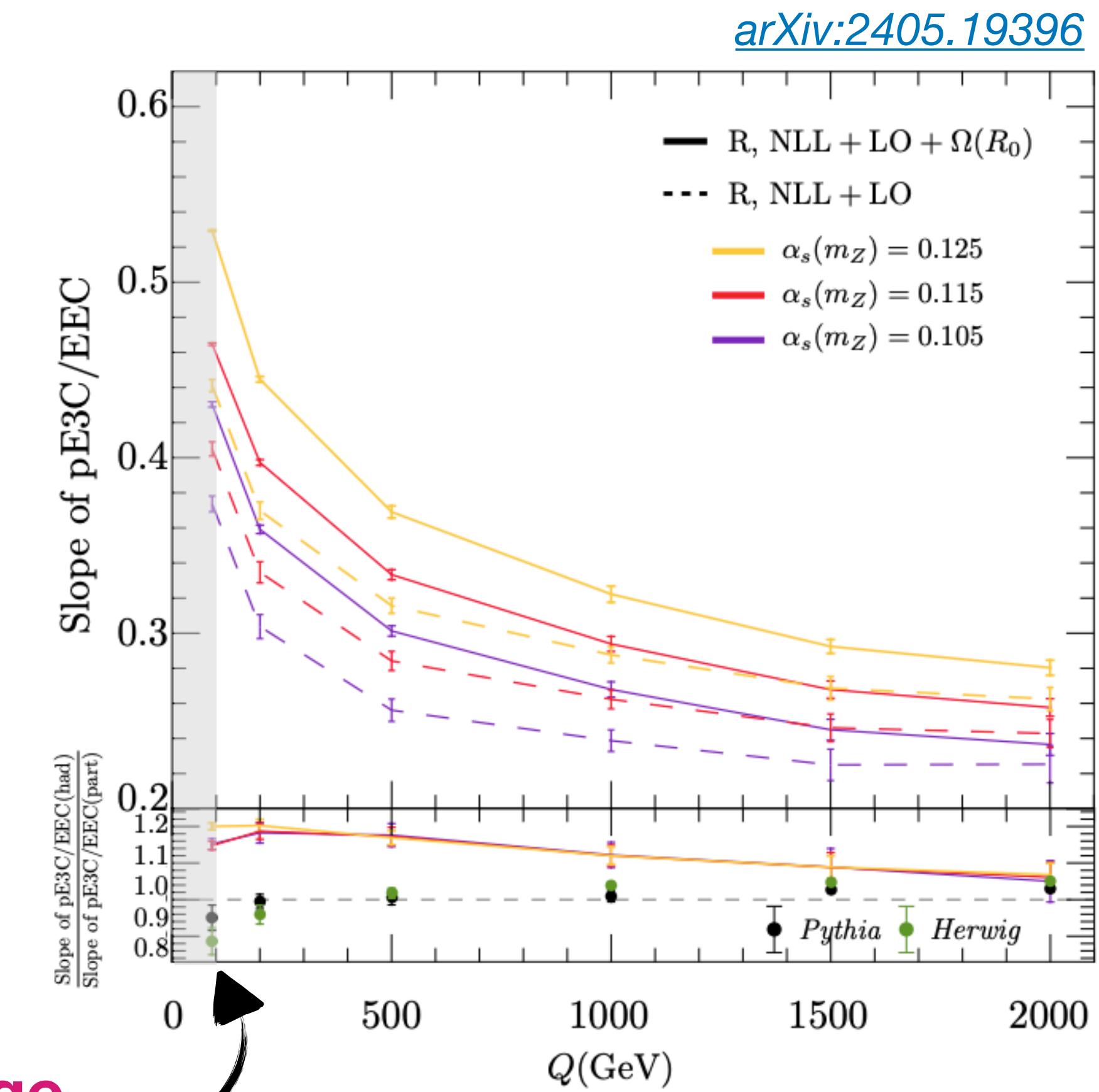
$$\propto \alpha_S \ln(R_L)$$

$$\alpha_S(m_Z) = 0.1229^{+0.0014}_{-0.0012} (\text{stat})^{+0.0030}_{-0.0033} (\text{theo})^{+0.0023}_{-0.0036} (\text{exp})$$

4% precision!



ALICE
jet energy range



* Map to α_S – access non-perturbative corrections at our energy scales!

Ongoing work

pp $\sqrt{s} = 13 \text{ TeV}$

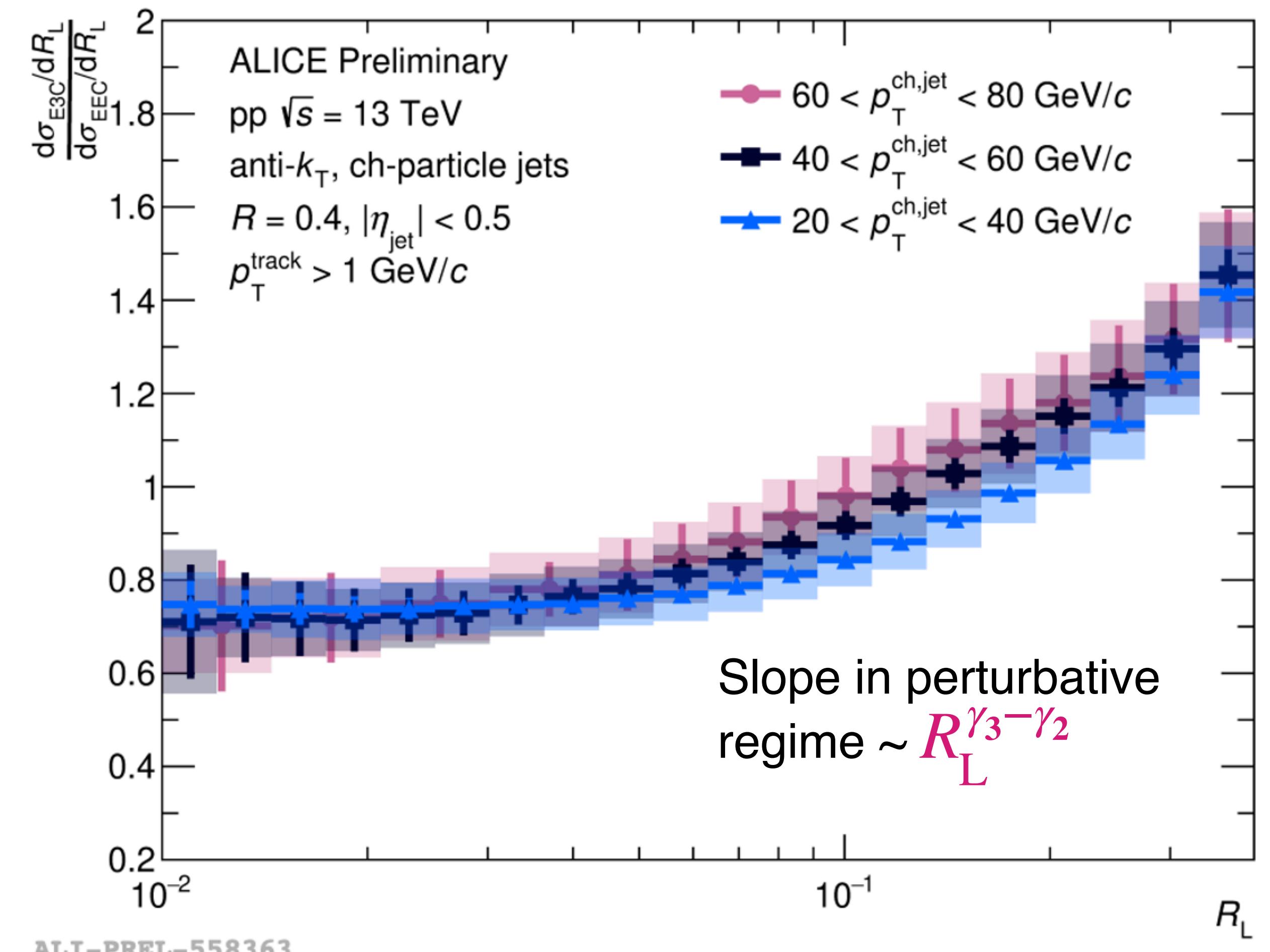
* Preliminary results used ***bin-by-bin*** correction method (possible due to excellent R_L resolution at ALICE)

$$f_{corr}(R_L^{\det}, p_{T,\text{jet}}^{\det}) = ENC_{\det}/ENC_{\text{true}}$$

$$ENC_{\text{true}}(p_{T,\text{jet}}^{\text{true}}) = (1/f_{corr})ENC_{\det}(p_{T,\text{jet}}^{\det})$$

* Map charged to full jets to set the jet energy scale (via track functions or via a model dependent mapping)

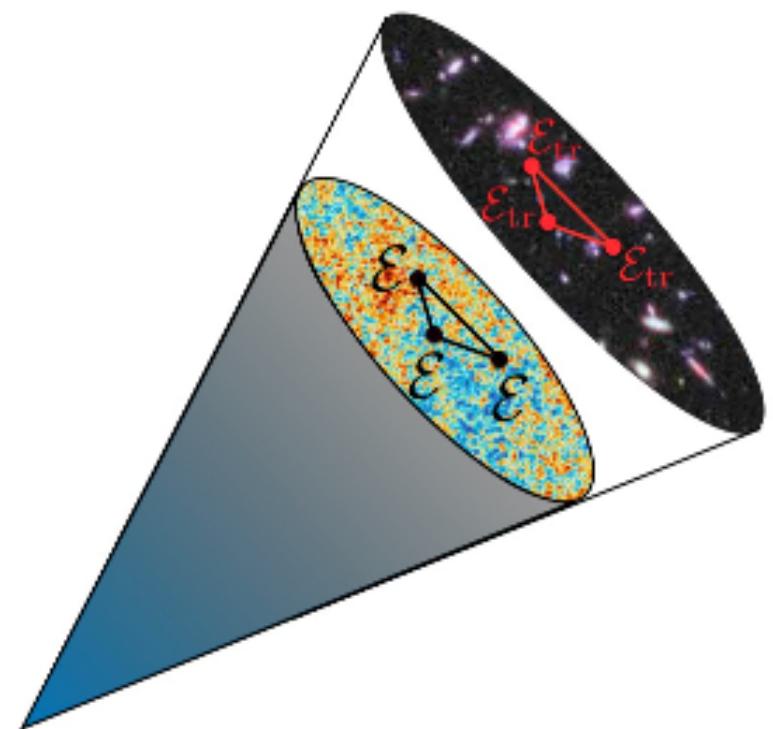
* Perform 3D unfolding to extract anomalous dimensions and relate to α_S



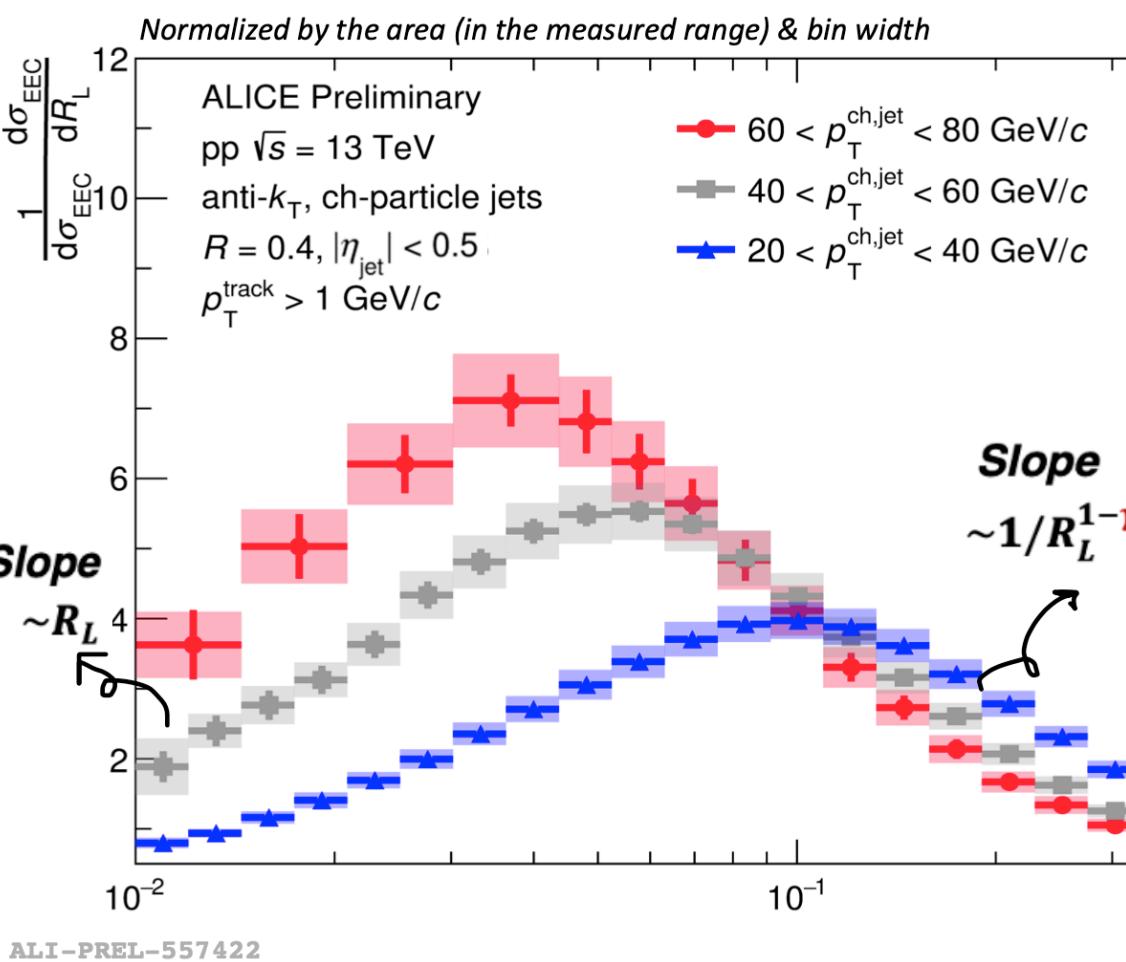
Both E3C & EEC are normalized by the area (in the measured range) and bin width.

Talk outline

Why study jet substructure
with energy correlators?



Outlook and summary



Energy Correlator
measurements at ALICE

Extending Energy Correlator
measurements to
Heavy Ions

- See jet EEC session earlier today!
- See C. Andres' talk in this session!
- See A. Rai's poster!



Energy Correlators in Heavy Ion Collisions

Active area of research!

Resolving the Scales of the Quark-Gluon Plasma with Energy Correlators

Carlota Andres,¹ Fabio Dominguez,² Raghav Kunnawalkam

Marquet,¹ and Ian Moult⁶

¹*F-91128 Palaiseau, France*

²*Enerxías (IGFAE),*

Avda de Compostela 15782, Spain

³*New Haven, CT*

⁴*Upton NY*

⁵*Yale University, Nashville, TN*

New Haven, CT 06511

Advancing the understanding of energy-energy correlators in heavy-ion collisions

João Barata,^a Paul Cauca,^b Alba Soto-Ontoso,^c and Robert Szafron^a

^a*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

^b*SUBATECH UMR 6457 (IMT Atlantique, Université de Nantes, IN2P3/CNRS), 4 rue Alfred Kastler, 44307 Nantes, France*

^c*CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*

Probing the Short-Distance Structure of the Quark-Gluon Plasma with Energy Correlators

Zhong Yang,¹ Yayun He^{2,3}, Ian Moult,⁴ and Xin-Nian Wang⁵

Flavor Hierarchy of Jet Energy Correlators inside the Quark-Gluon Plasma

Wen-Jing Xing,¹ Shanshan Cao,^{1,*} Guang-You Qin,^{2,†} and Xin-Nian Wang^{2,‡}

¹*Institute of Frontier and Interdisciplinary Science,*

Shandong University, Qingdao, Shandong 266237, China

²*Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE),
Central China Normal University, Wuhan, 430079, China*

(Dated: September 20, 2024)

A Coherent View of the Quark-Gluon Plasma from Energy Correlators

Carlota Andres,^a Fabio Dominguez,^b Jack Holguin,^a Cyrille Marquet,^a Ian Moult^c

^a*CPHT, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France*

^b*Instituto Galego de Física de Altas Enerxías (IGFAE), Universidade de Santiago de Compostela 15782, Spain*

Seeing Beauty in the Quark-Gluon Plasma with Energy Correlators

Carlota Andres,¹ Fabio Dominguez,² Jack Holguin,¹ Cyrille Marquet,¹ and Ian Moult³

¹*CPHT, CNRS, École polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France*

²*Instituto Galego de Física de Altas Enerxías (IGFAE),*

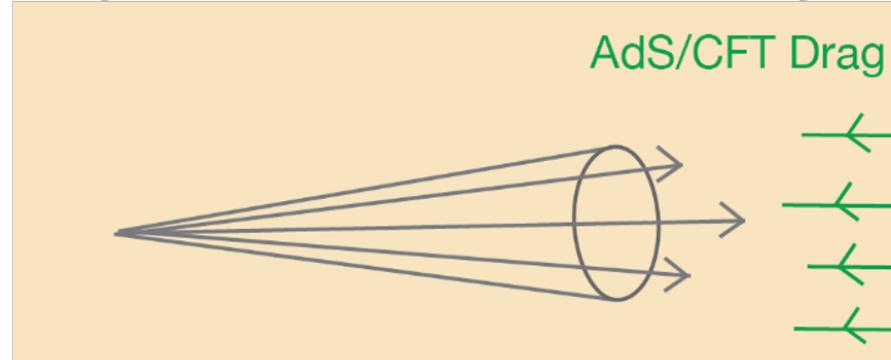
Universidade de Santiago de Compostela, Santiago de Compostela 15782, Spain

³*Department of Physics, Yale University, New Haven, CT 06511*

E3C/EEC in Heavy Ion Collisions

A study in the Hybrid Model

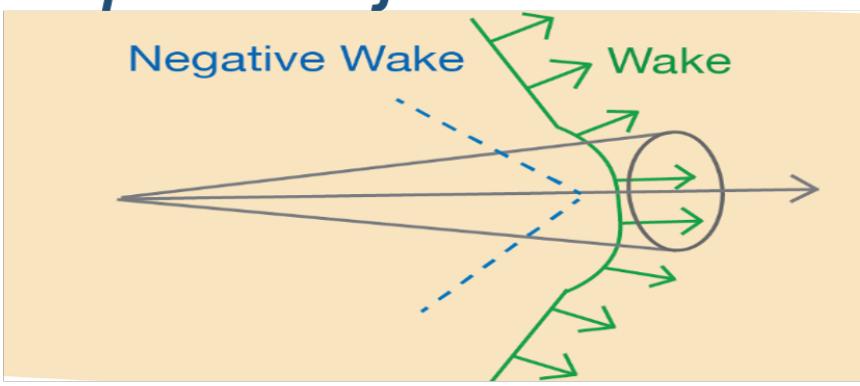
Impact of medium on jet



“Jet Energy Loss”

Drag Force

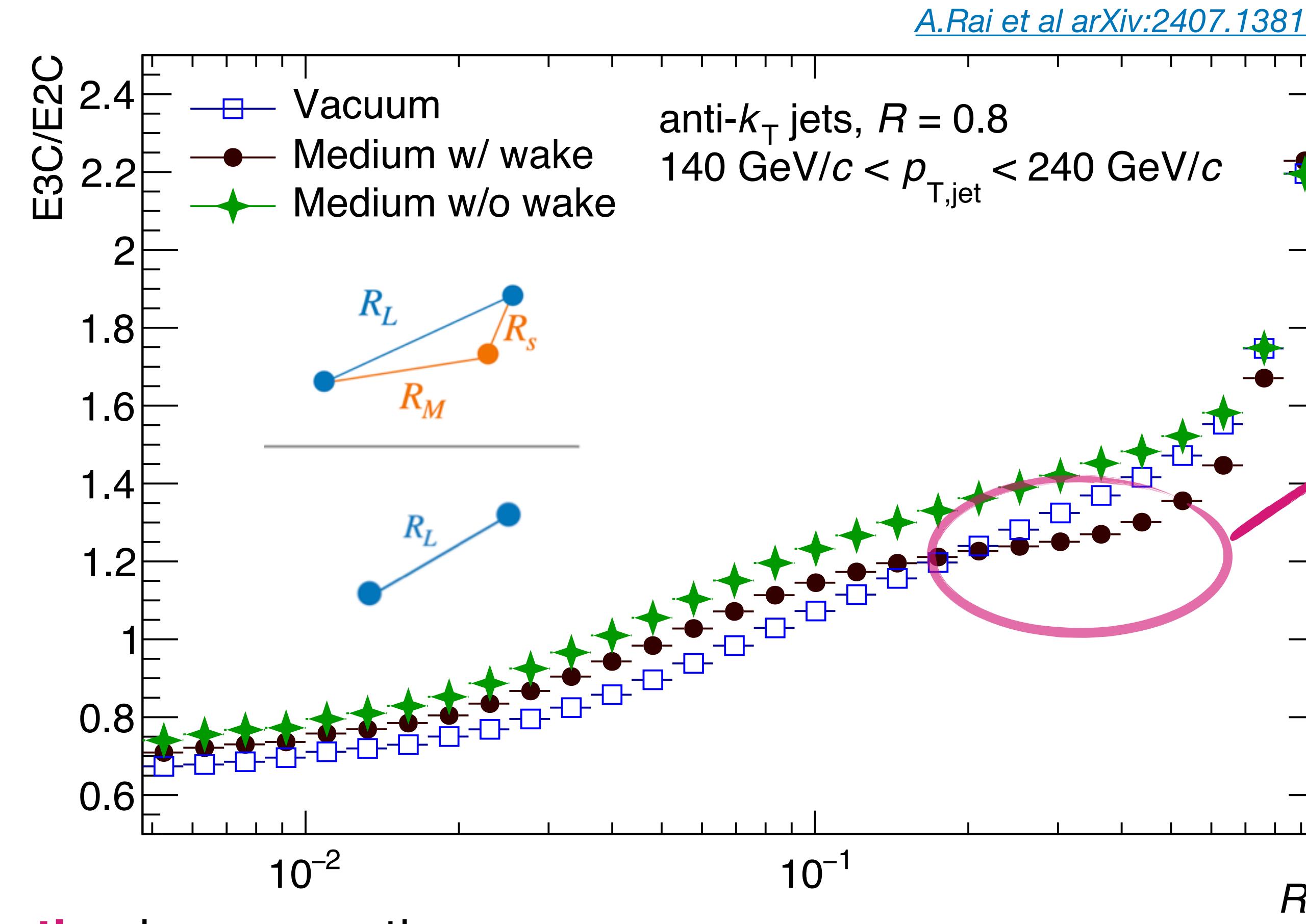
Impact of jet on medium



“Medium Response”

Hydrodynamic Wake

[A.Rai et al arXiv:2407.13818](#)



- * **“Flat”** region corresponds to the presence of the **wake**
- * In pp, we access the anomalous dimensions of vacuum QCD. Interpretation for HIC?

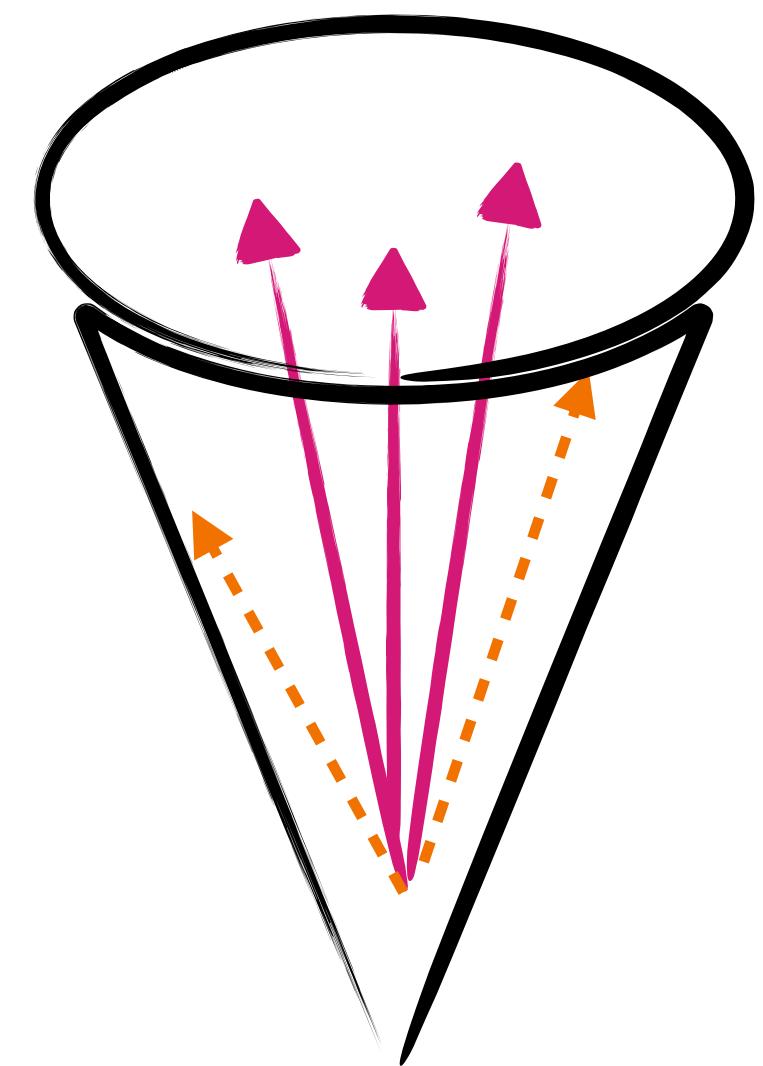
* **Experimentally interesting** because ratios are more robust to detector effects!

* Ratios will also cancel some uncorrelated background effects

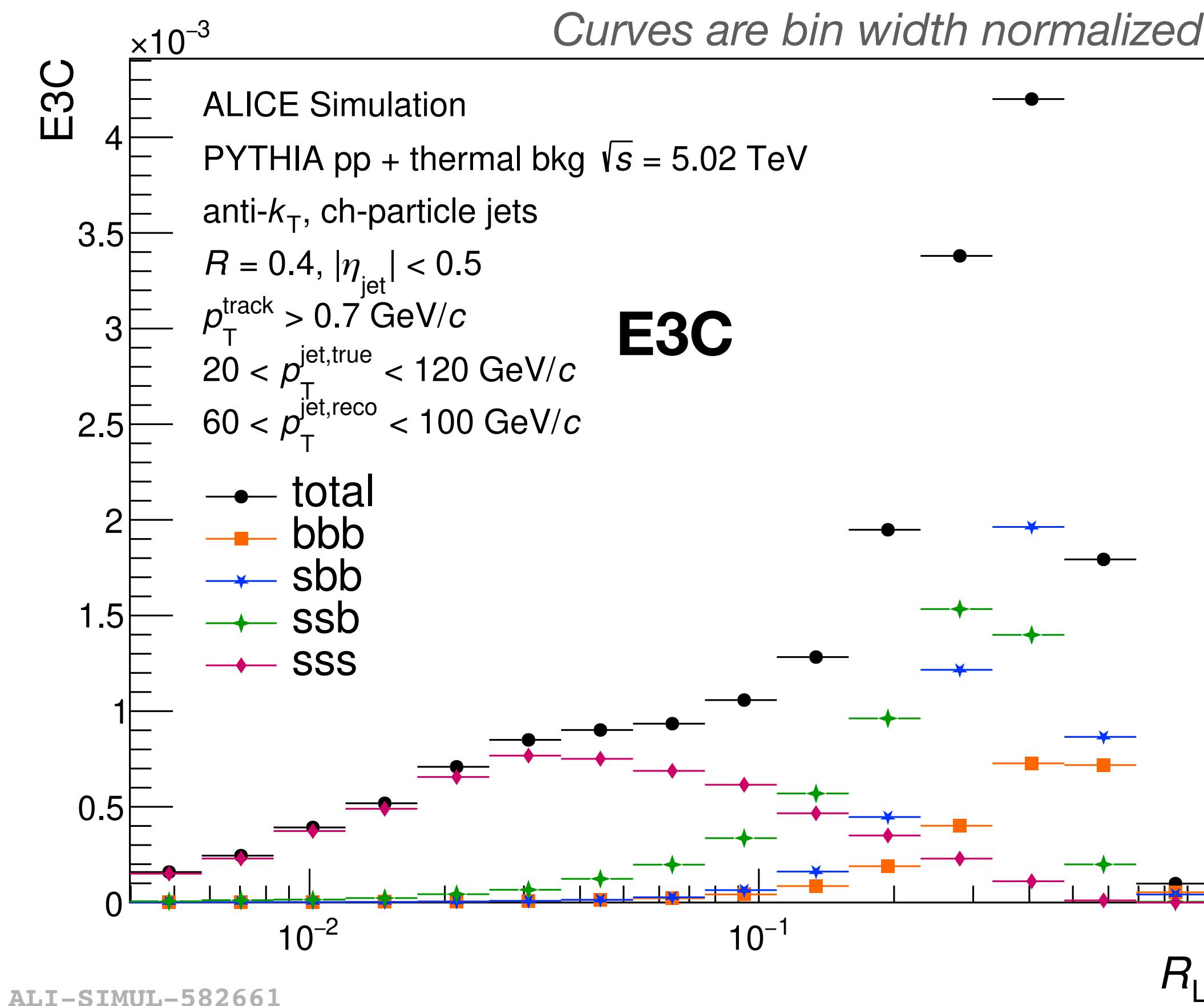
Background Subtraction in Heavy Ion Collisions

Pythia study: Energy Correlators in a Thermal Toy

- * Embedding **Pythia jets** in a **thermal toy** – now we have **signal (s)** and **background (b)** particles in our jets (ρA subtraction already performed)



Adding uncorrelated
background to Pythia jet,
New Jet = **Signal + Bkg**



* Goal: Correct the **black curve (total)** so it matches the **pink curve (sss)**

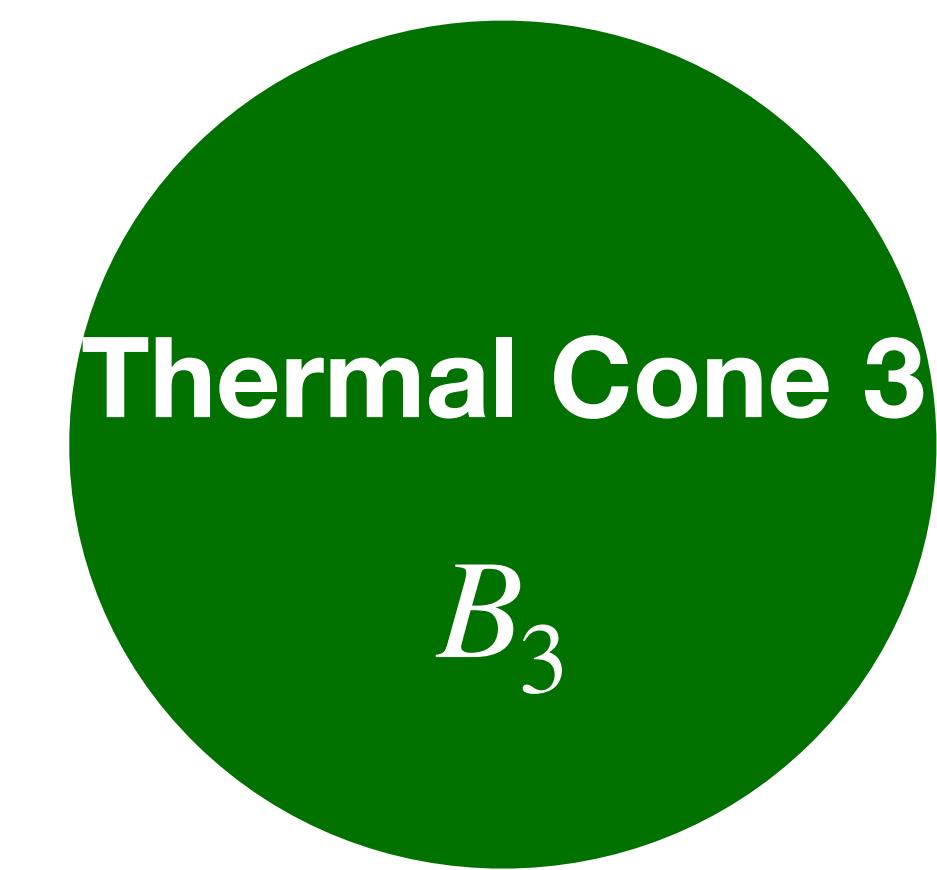
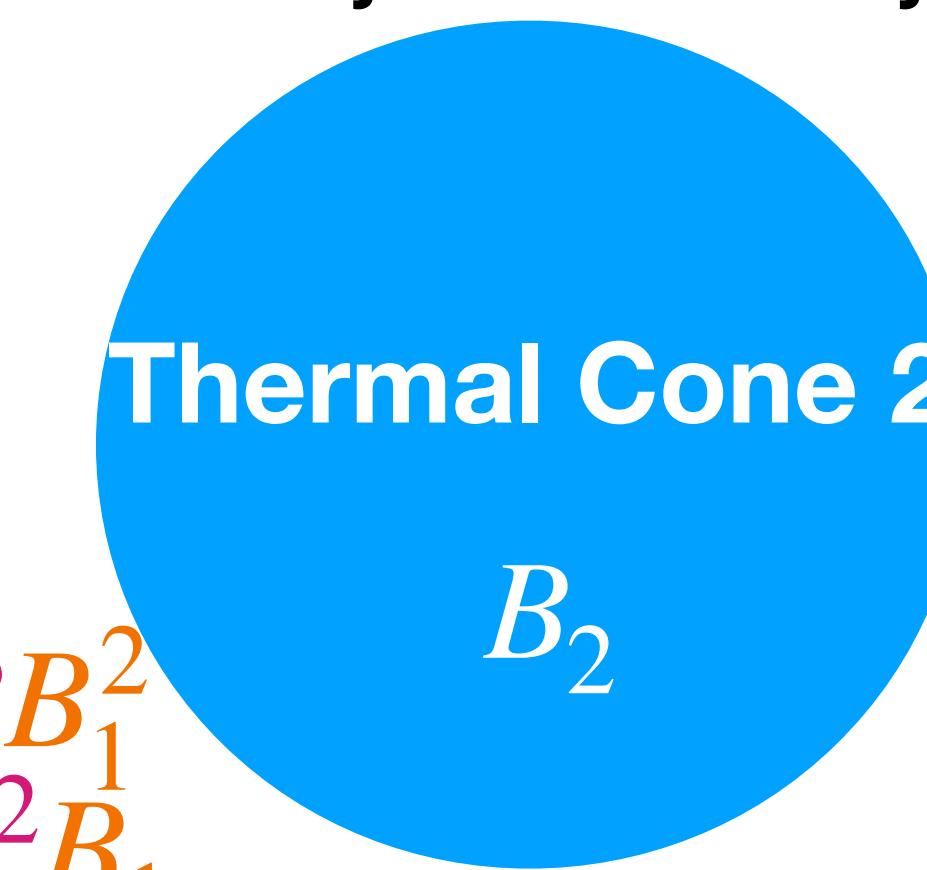
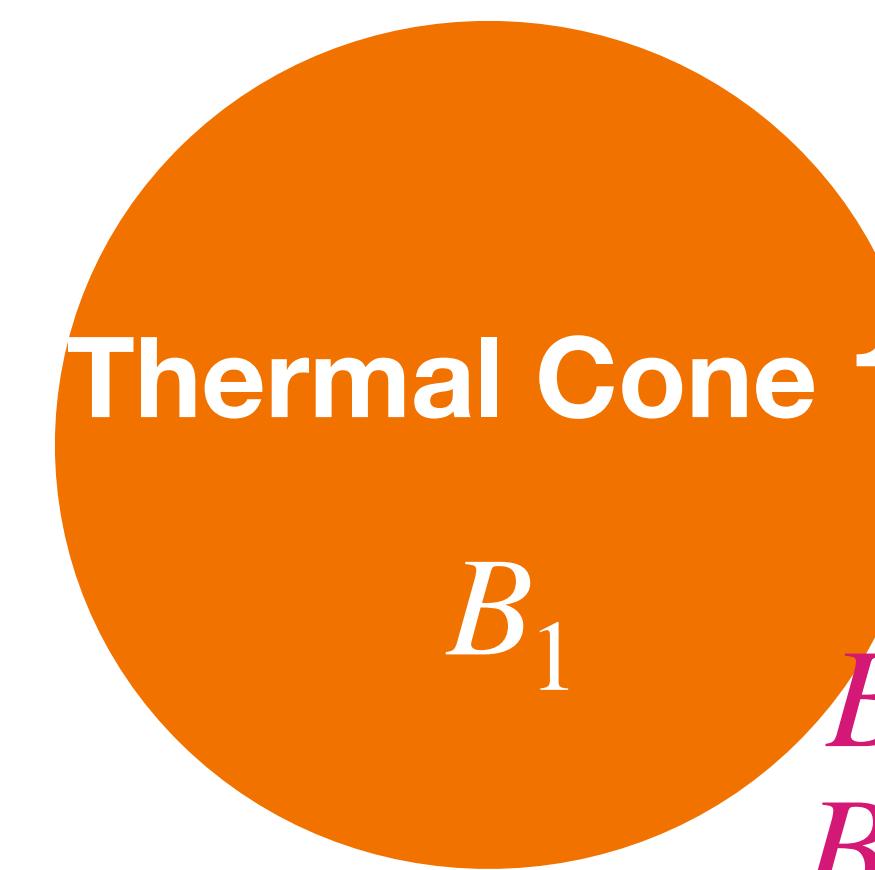
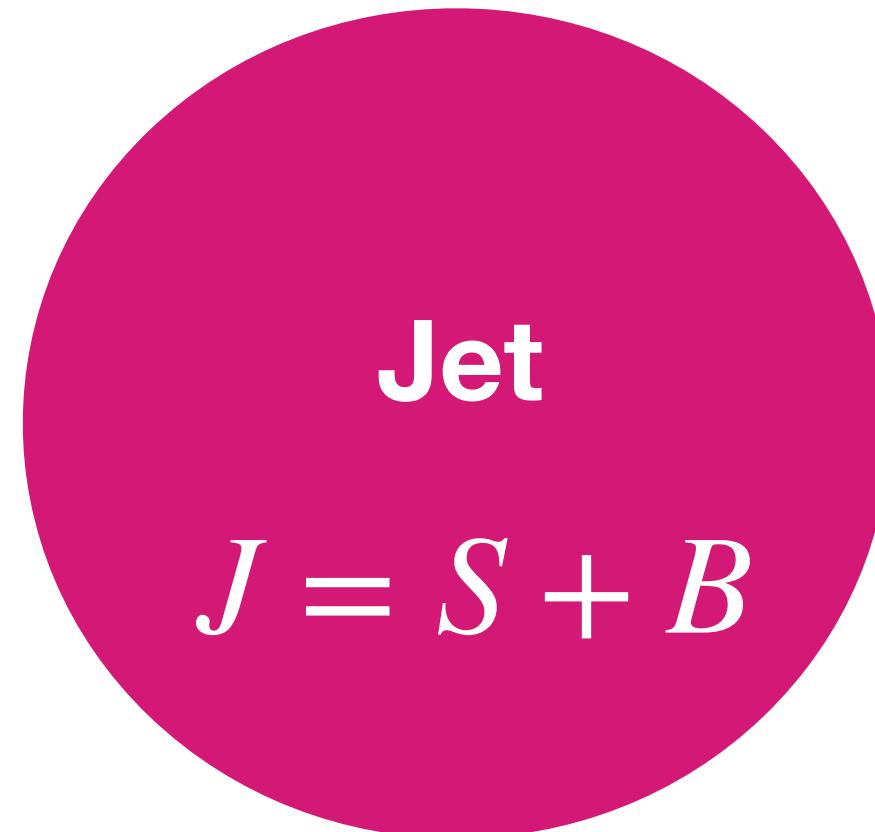
* How to achieve this?
- Correct uncorrelated bkg using min bias/mixed events
- Correct for jet-selection bias

**Similar study for EEC in backup

Background Subtraction for E3C

A multi-cone story

Cones centered on jet axis of jet radius R



$$J = S + B$$

$$(B_1)^3 = B_1 B_1 B_1$$

$$(B_1)^2 B_2 = B_1 B_1 B_2$$

$$B_1 B_2 B_3$$

$$J^3 = S^3 + \boxed{B^3} + \boxed{3S^2B} + \boxed{3SB^2}$$

$$\boxed{J^2 B_1 = (S + B)^2 B_1}$$

$$\boxed{JB_1 B_2 = \underline{S B_1 B_2} + \underline{B B_1 B_2}}$$

$$\boxed{J(B_1)^2 = (S + B)(B_1)^2}$$

$$\boxed{B_1(B_2)^2 = \underline{B_1 B_2 B_2}}$$

We need a 3rd cone because of this term!

J : jet

S : signal particles in jet

B : bkg particles in jet

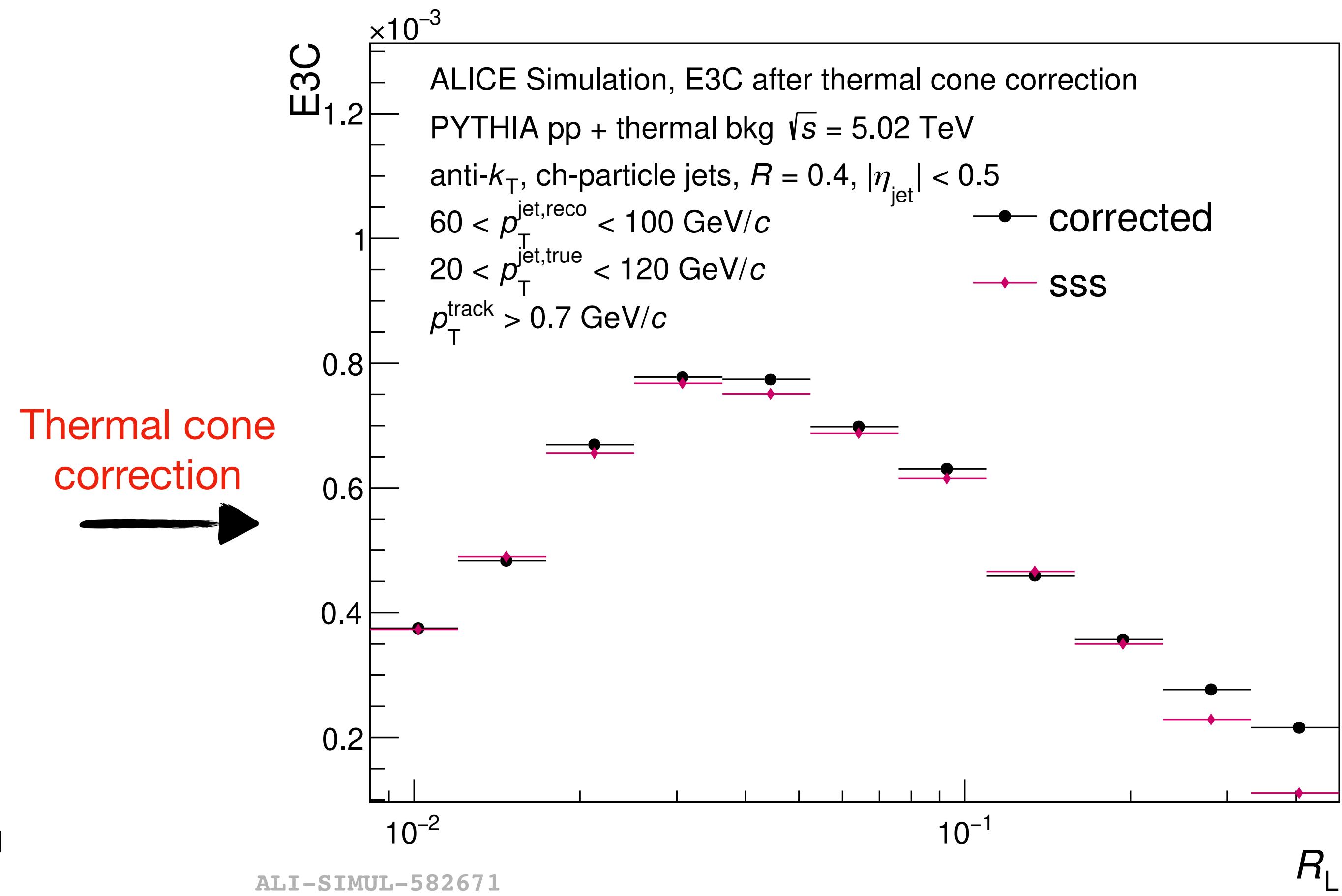
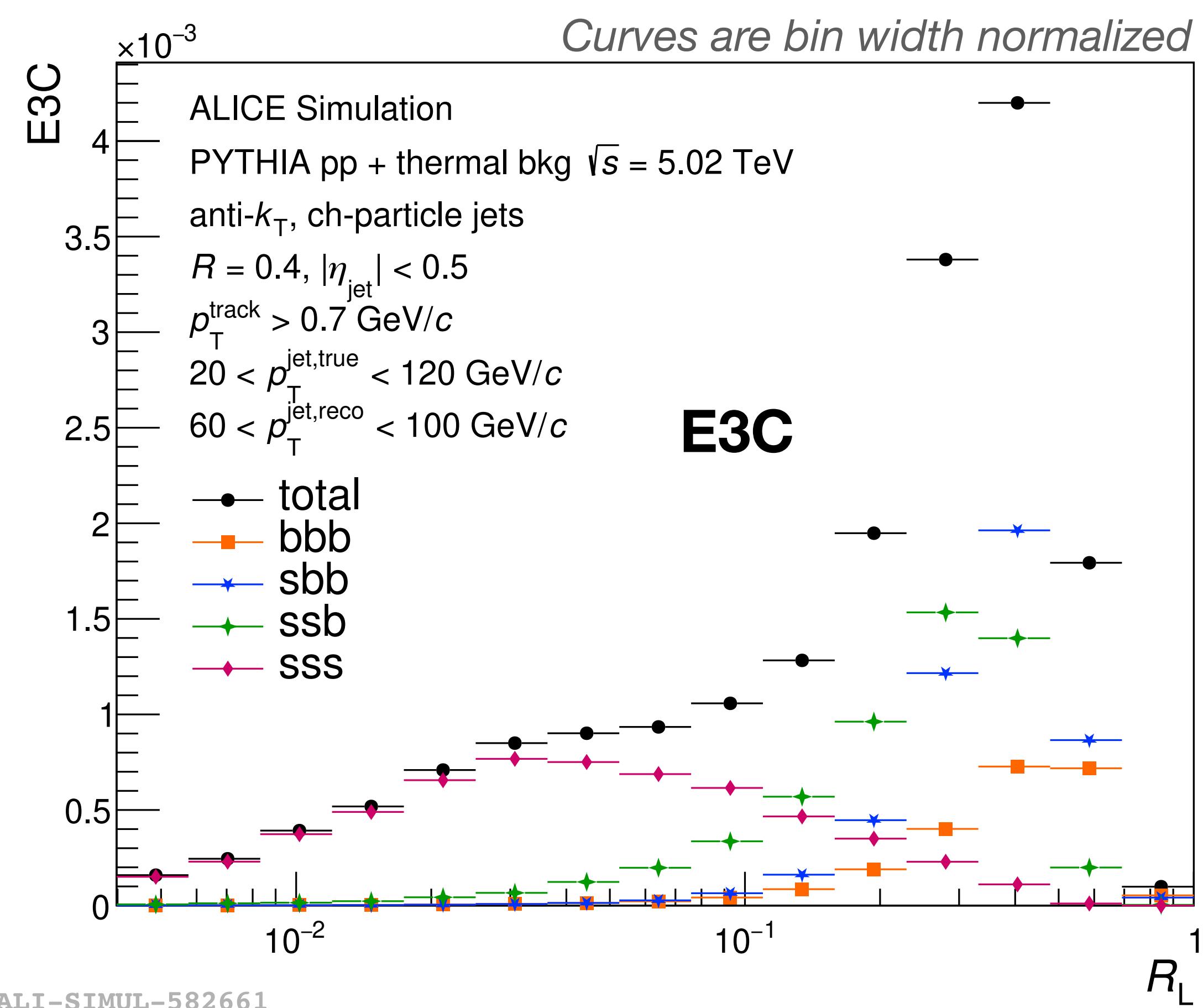
B_1 B_2 B_3 : bkg particles in thermal cones

Takeaway: We need a third cone to correct for E3Cs!

**Similar method for EEC but only 2 cones needed

Background Subtraction in Heavy Ion Collisions

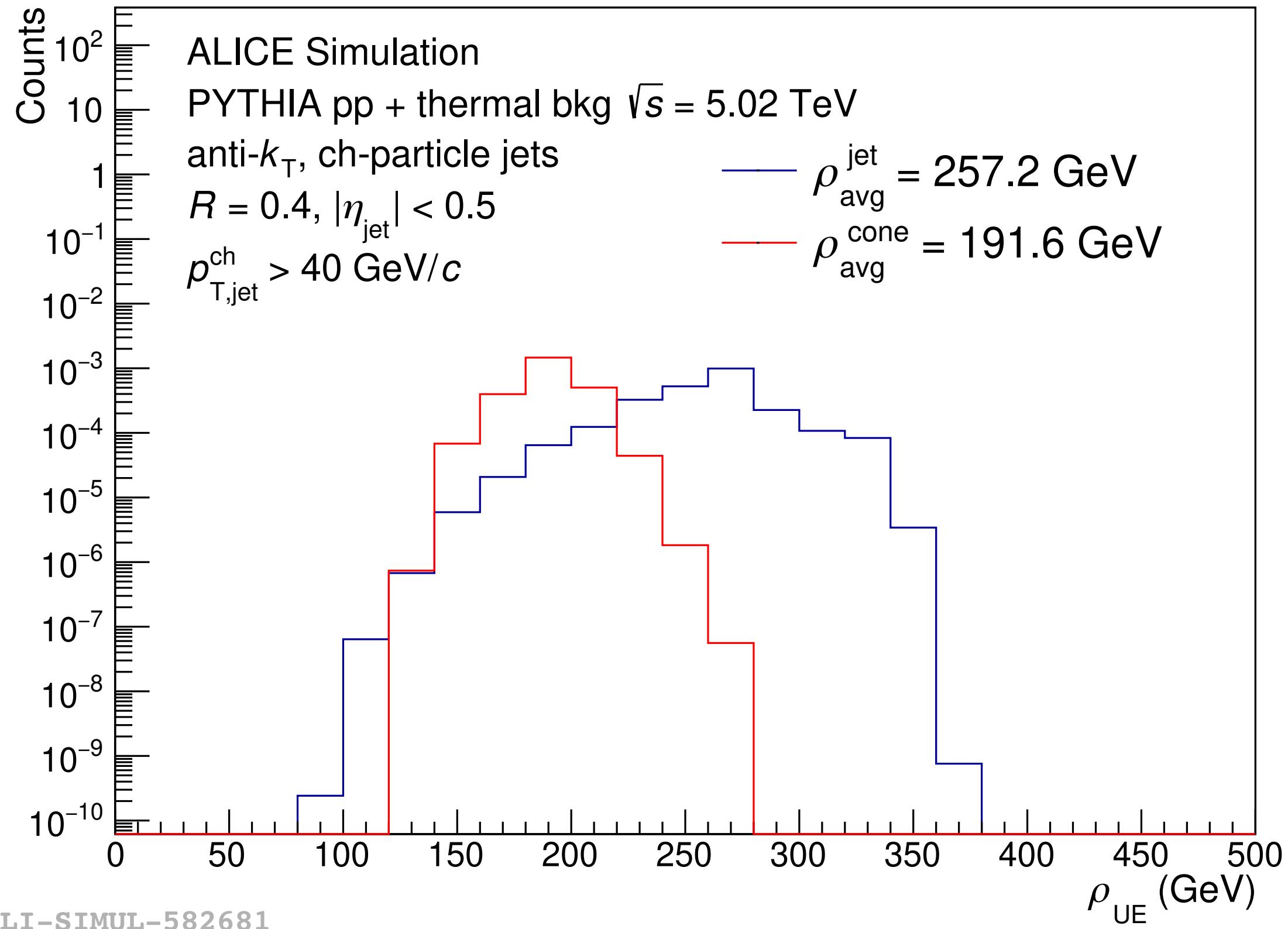
Pythia study: Energy Correlators in a Thermal Toy



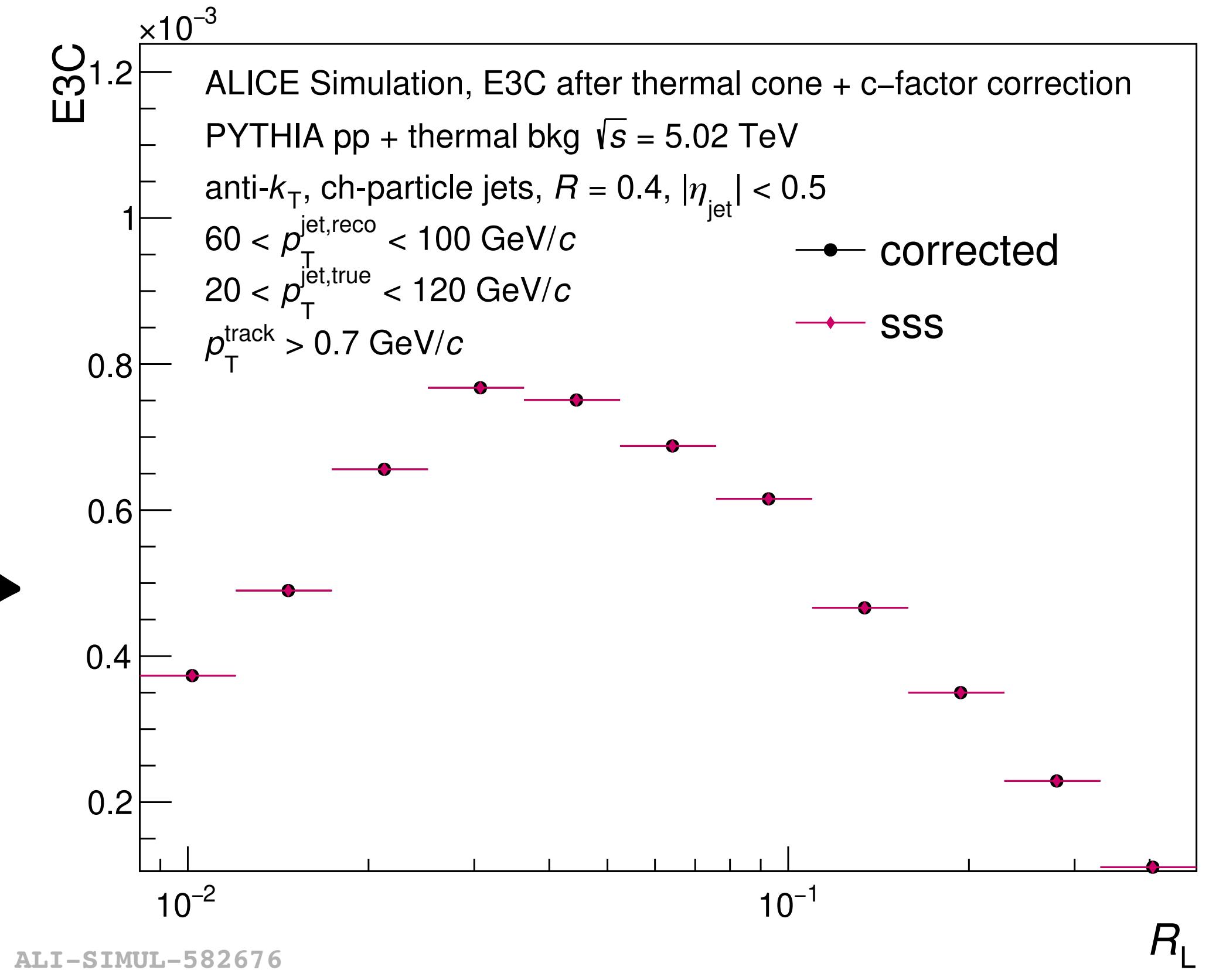
* Thermal cone correction works but doesn't give complete closure — culprit is the **jet selection bias from upward underlying event fluctuations**

Background Subtraction for E3C

Pythia study: Jet selection bias and c-factor correction



Thermal
cone
+
c-factor
correction

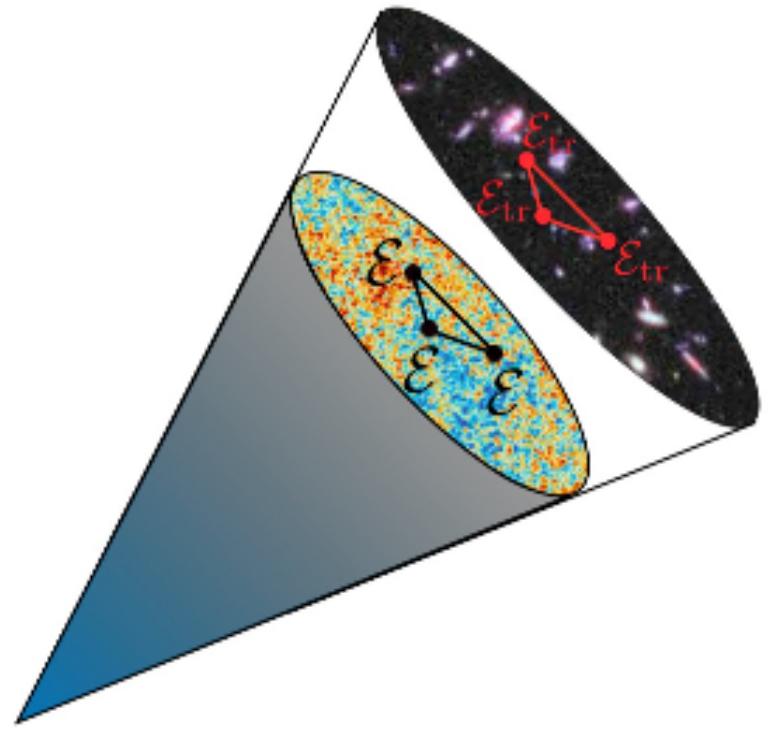


- * Underestimation of ρ_{UE}^{jet} related to **jet selection bias** – more likely to select jets that sit on an “upward fluctuation of the background” – more background particles in jet

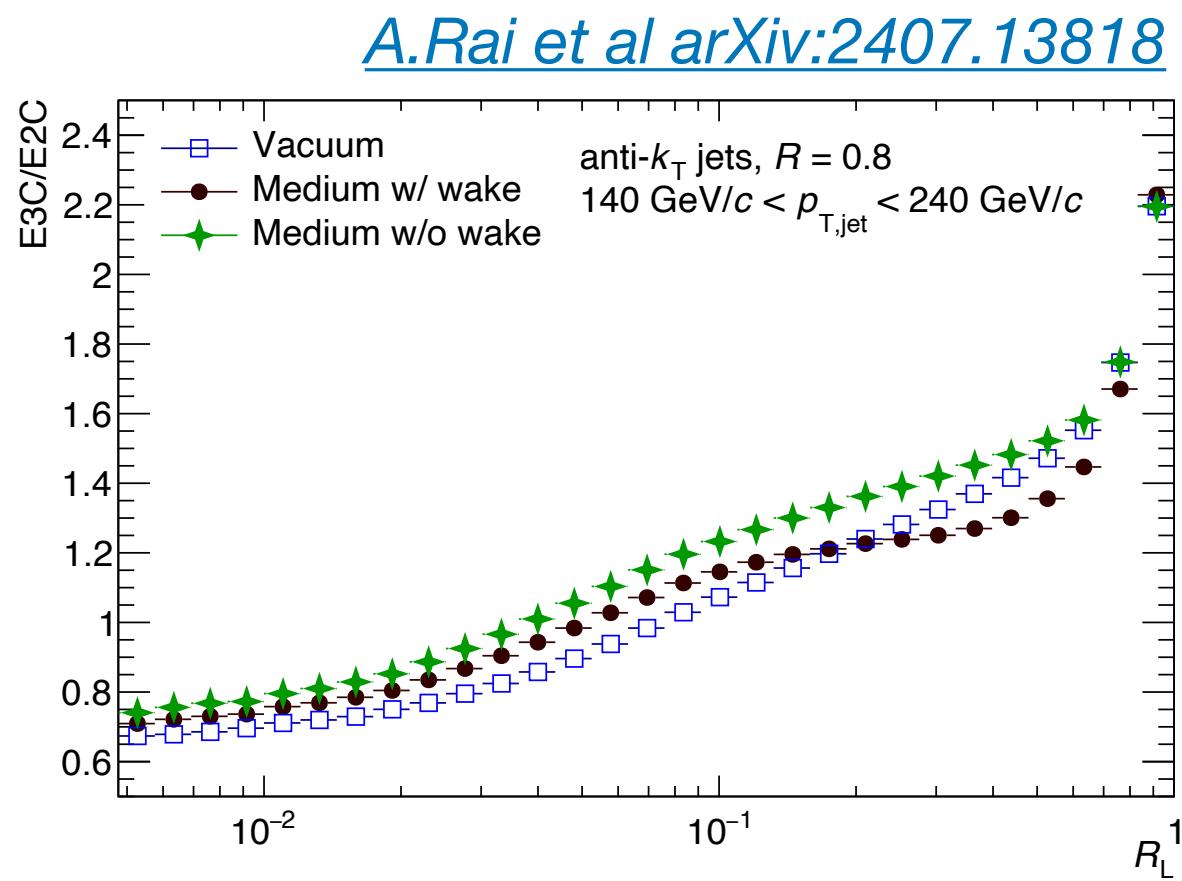
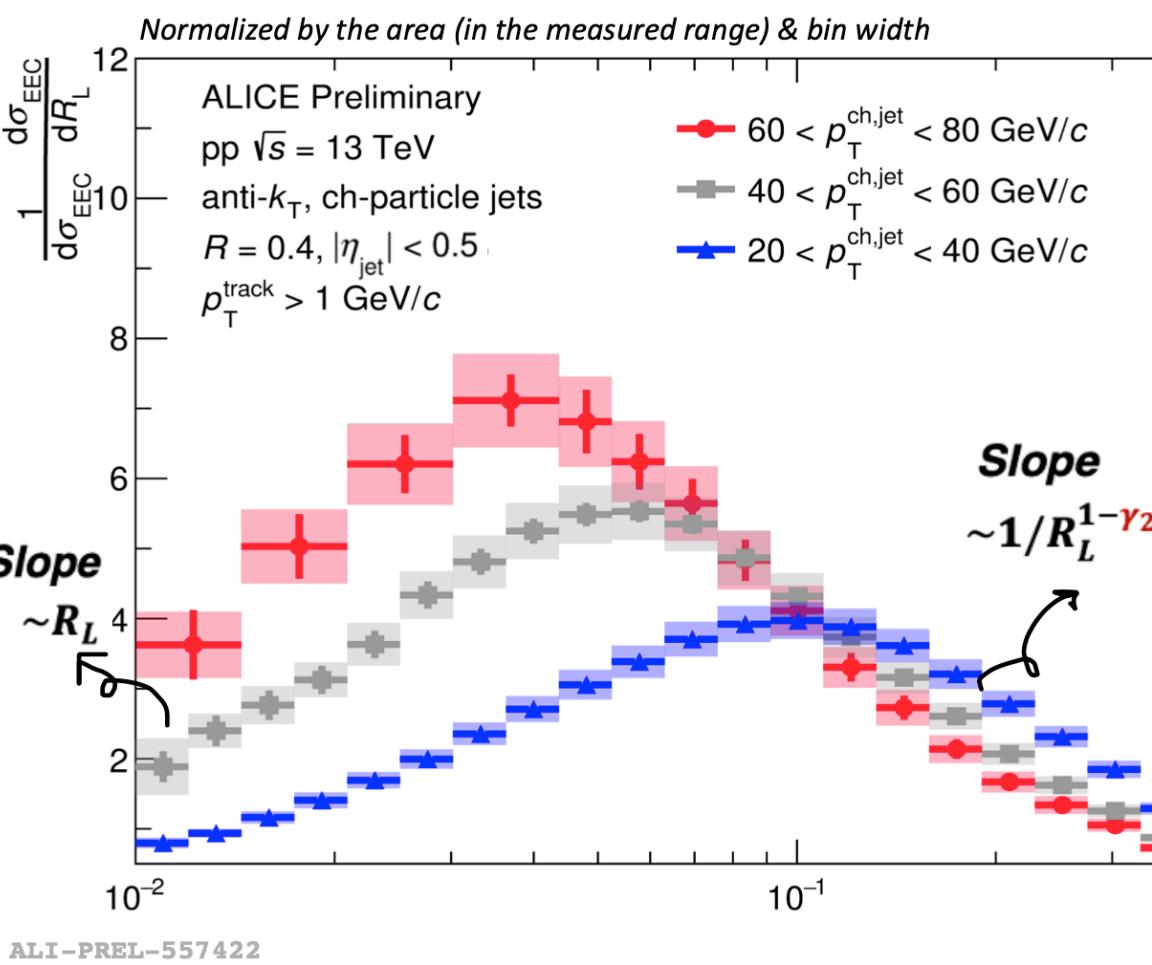
- * c-factor = $E3C(R_L)_{bkg}^{\text{cone}} / E3C(R_L)_{bkg}^{\text{jet}}$
- * Accounting for background UE density differences gives closure!

Talk outline

Why study jet substructure
with Energy Correlators?



Energy Correlator
measurements at ALICE



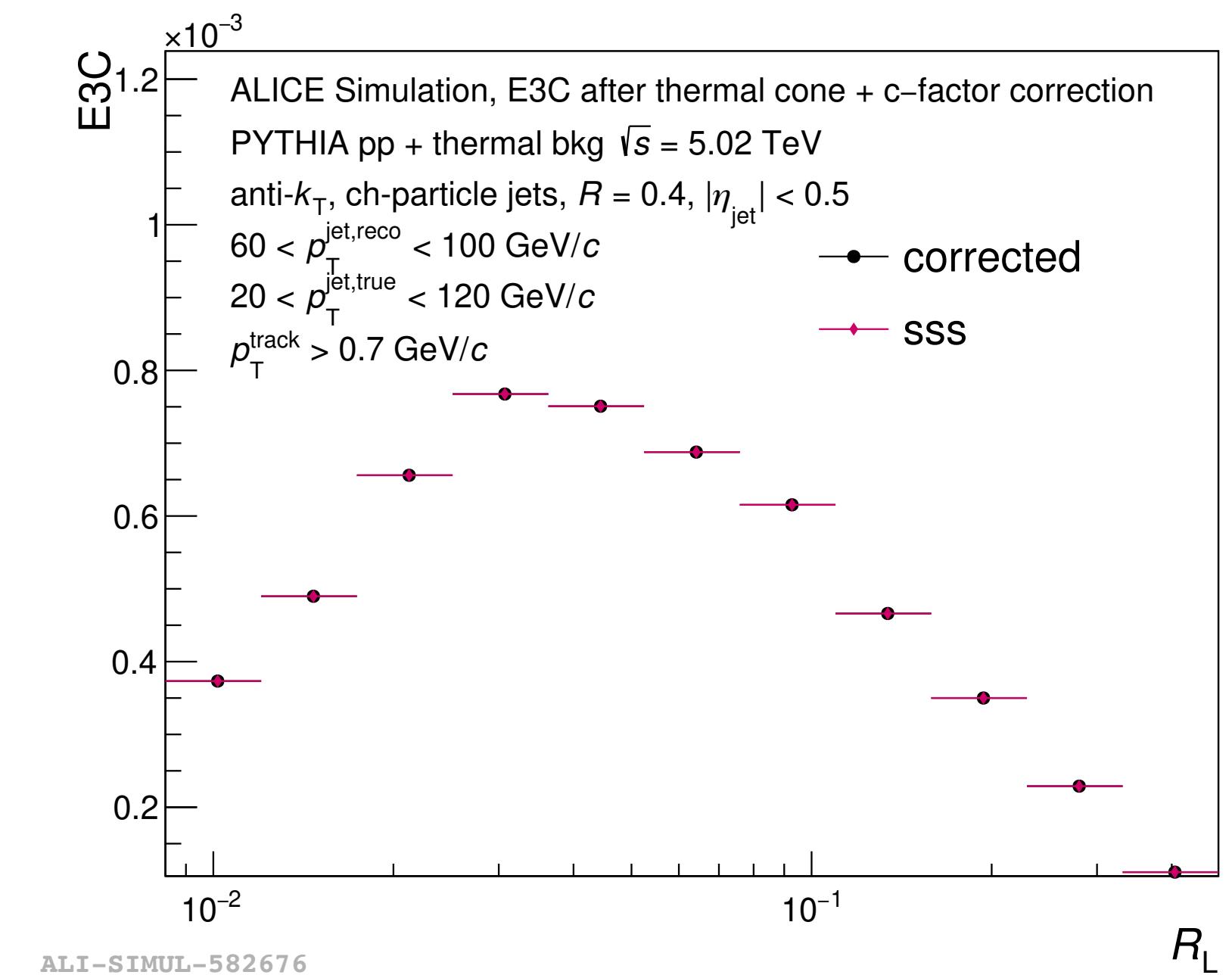
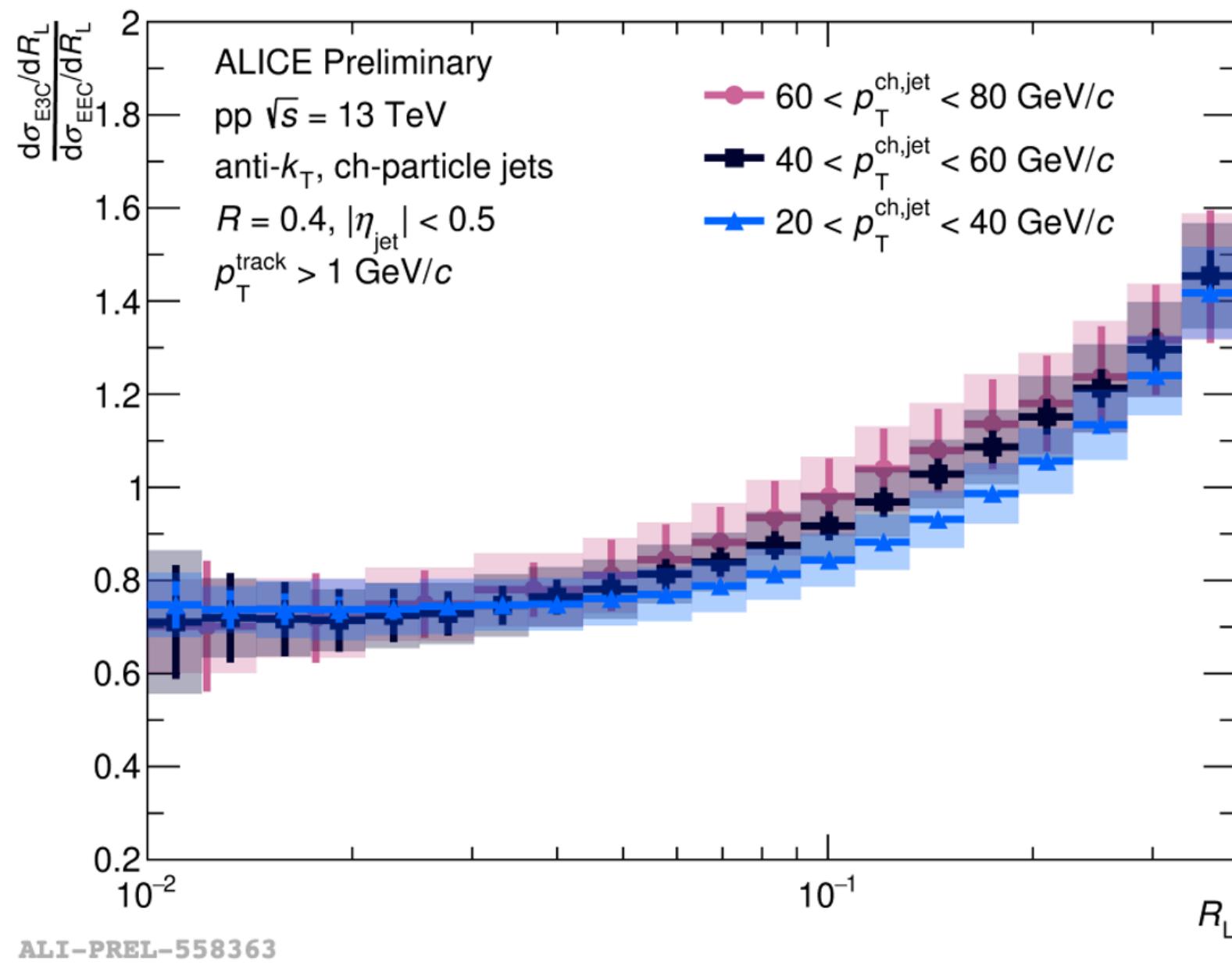
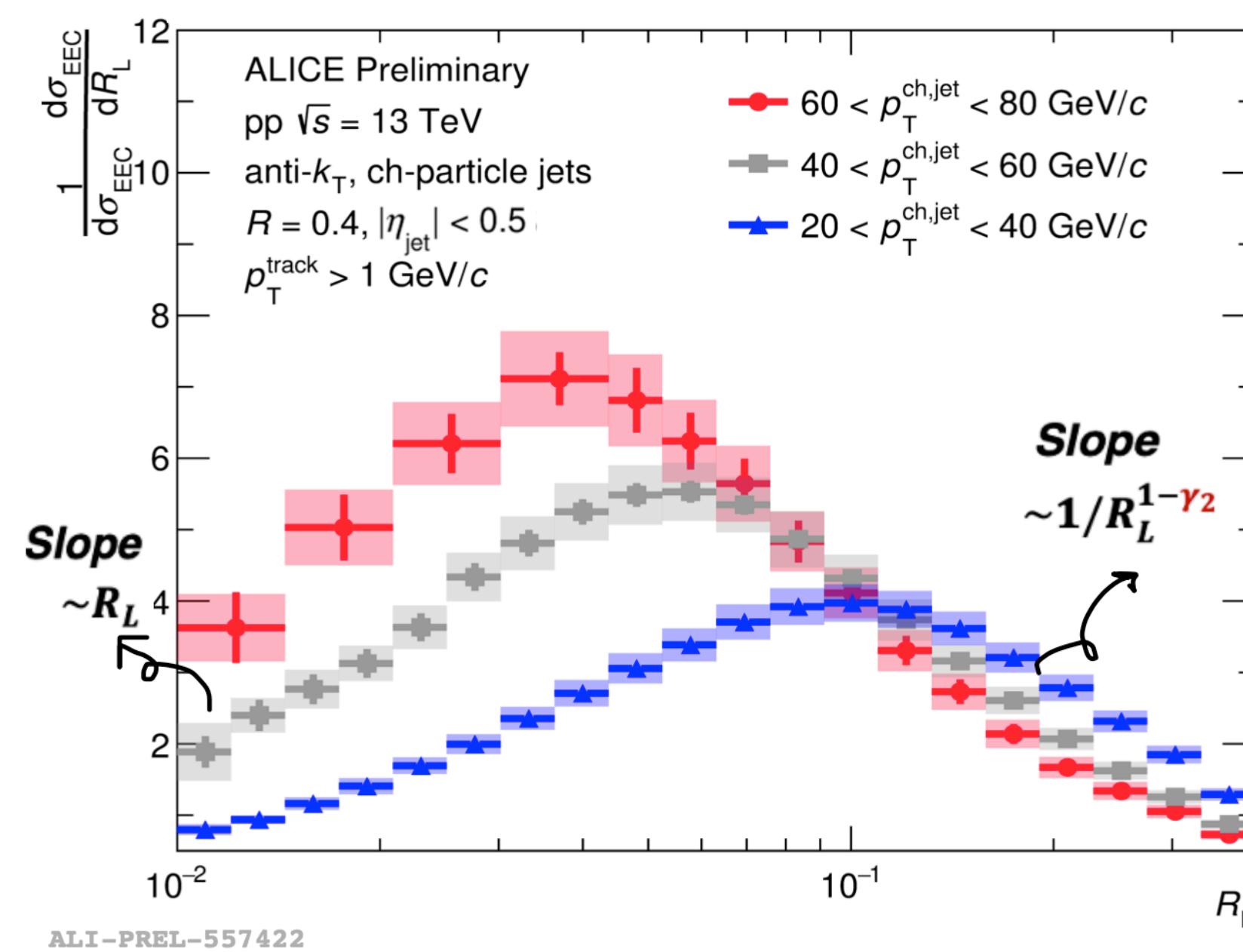
Extending Energy Correlator
measurements to
Heavy Ions



Outlook and summary

Summary and Outlook

Energy Correlators in vacuum QCD and beyond



Energy Correlators show clear
separation of energy scales
inside jets

Ratios of projected correlators are
sensitive to running of the strong coupling, α_S . We can extract the slopes of E3C/EEC and relate them to α_S by mapping to the correct jet energy scale

We understand how to perform background subtraction in the QGP for the correlators.

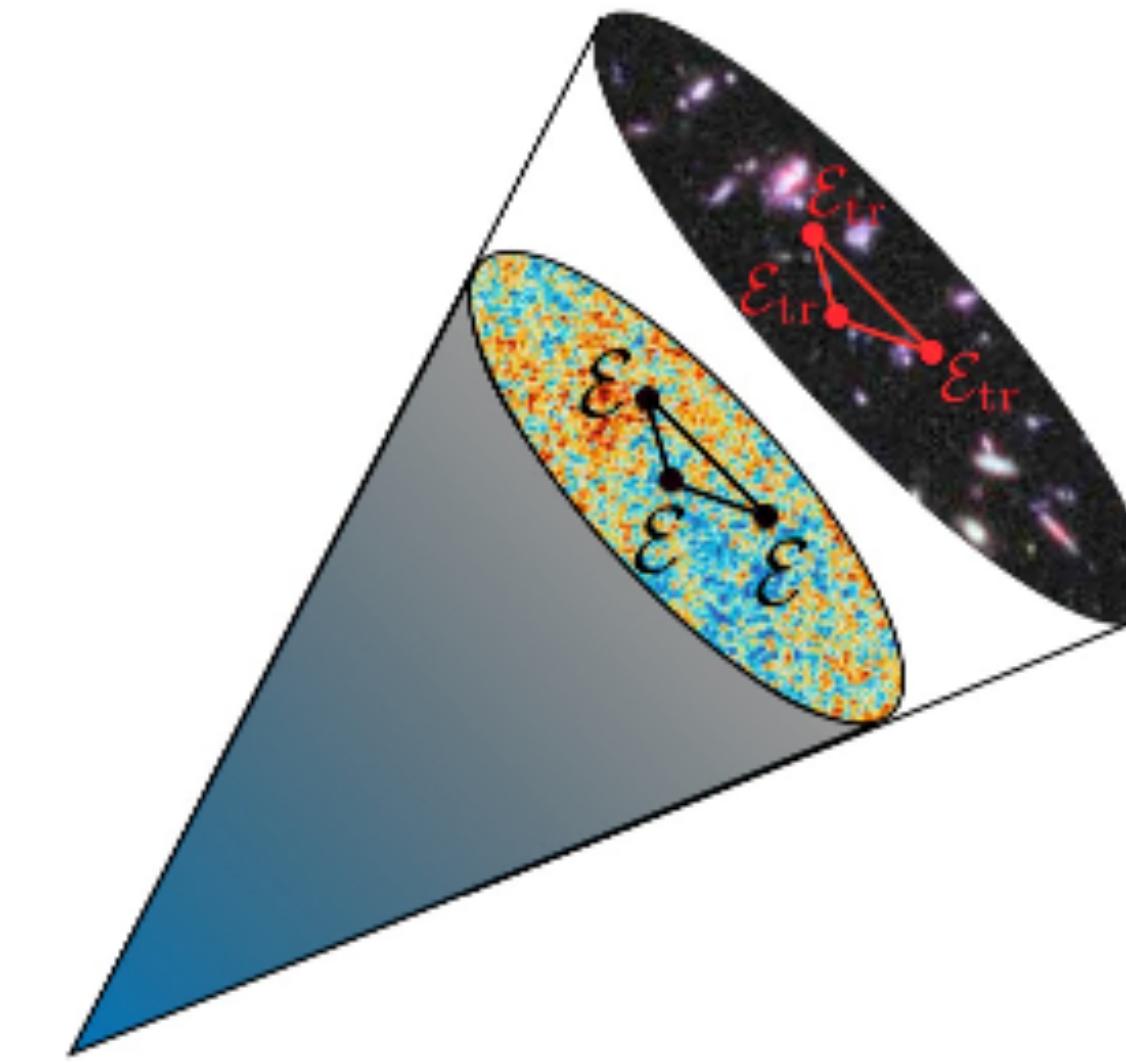
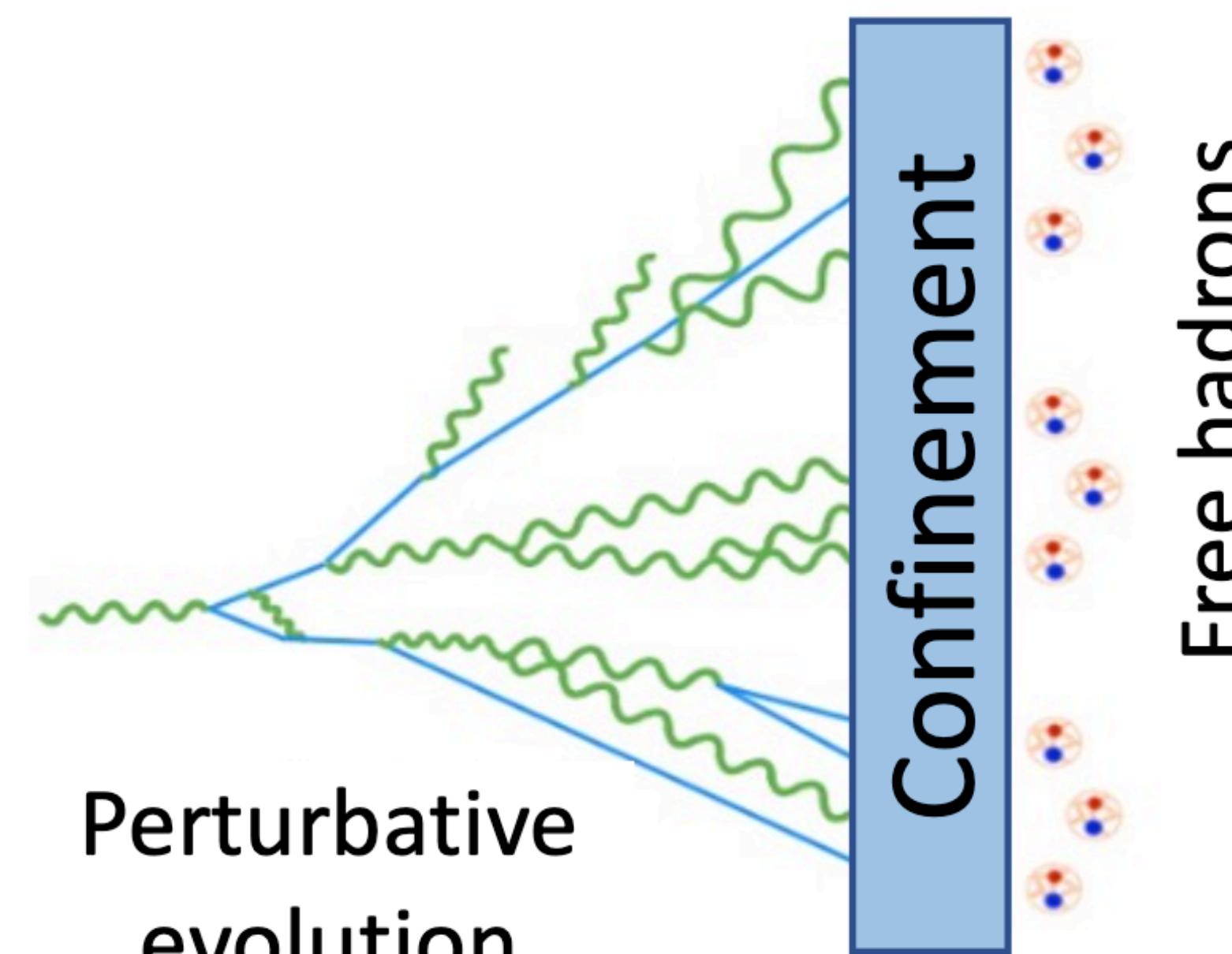
Time to go looking for jet modification in the QGP!

Backup

Jets and Jet Substructure

Looking inside jets

- * Modeled by iterative splittings of hard scattered partons during the initial collision



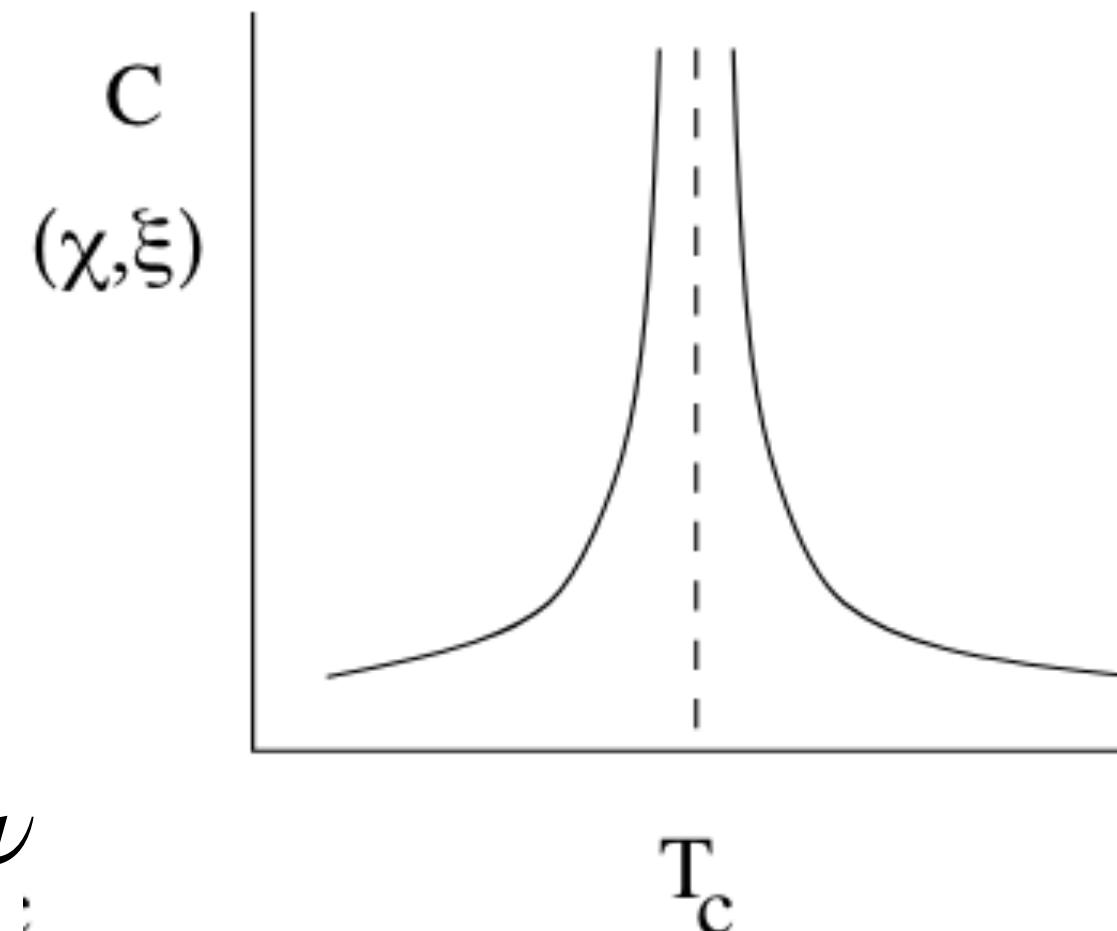
- * **Multi-scale objects:** QCD evolution imprinted on jets as they go from perturbative to non-perturbative scales

Correlation Functions

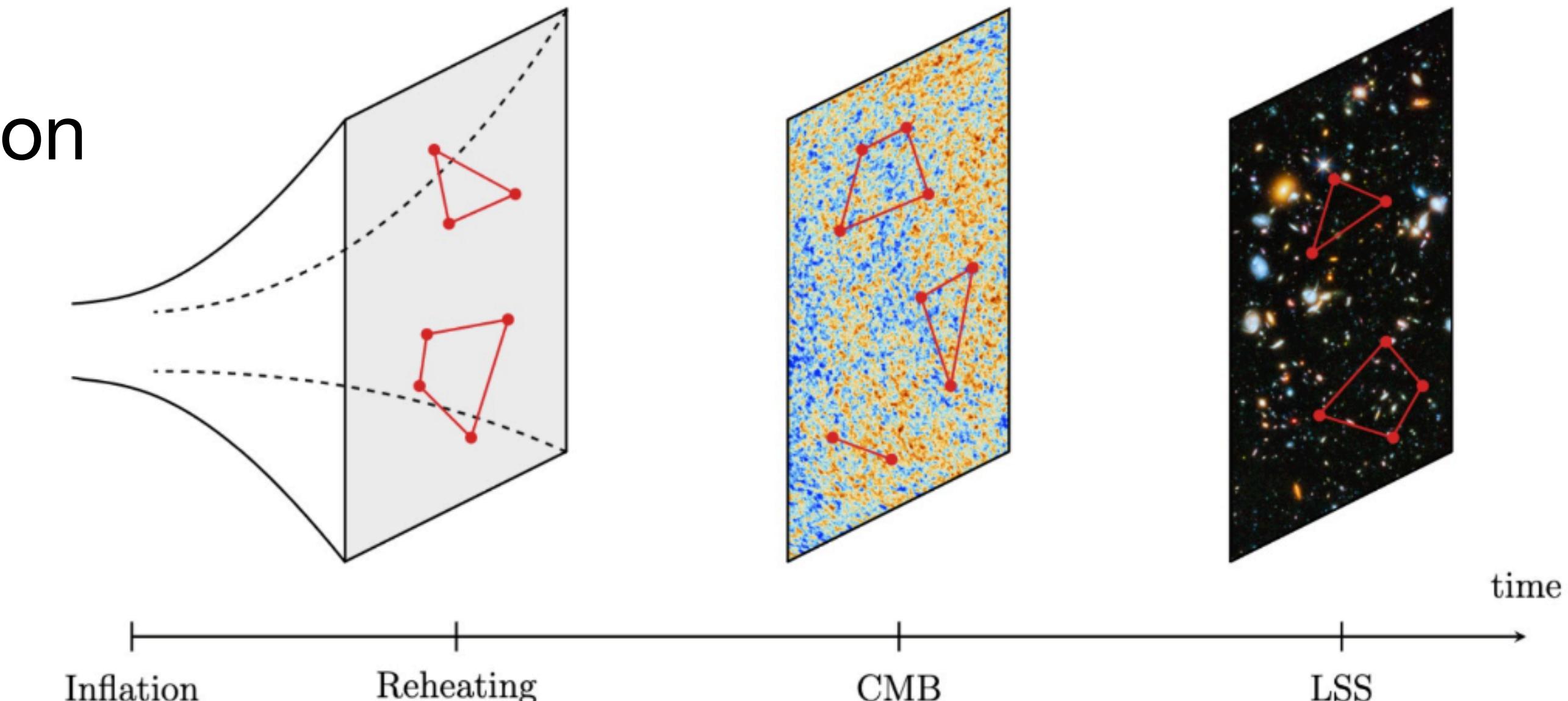
Some intuition from other systems

* Correlation functions indicate **phase transitions**

Eg 1: Ferromagnetic transition, correlation length $\rightarrow \infty$



$$\xi \sim 1/(T - T_C)^\nu:$$

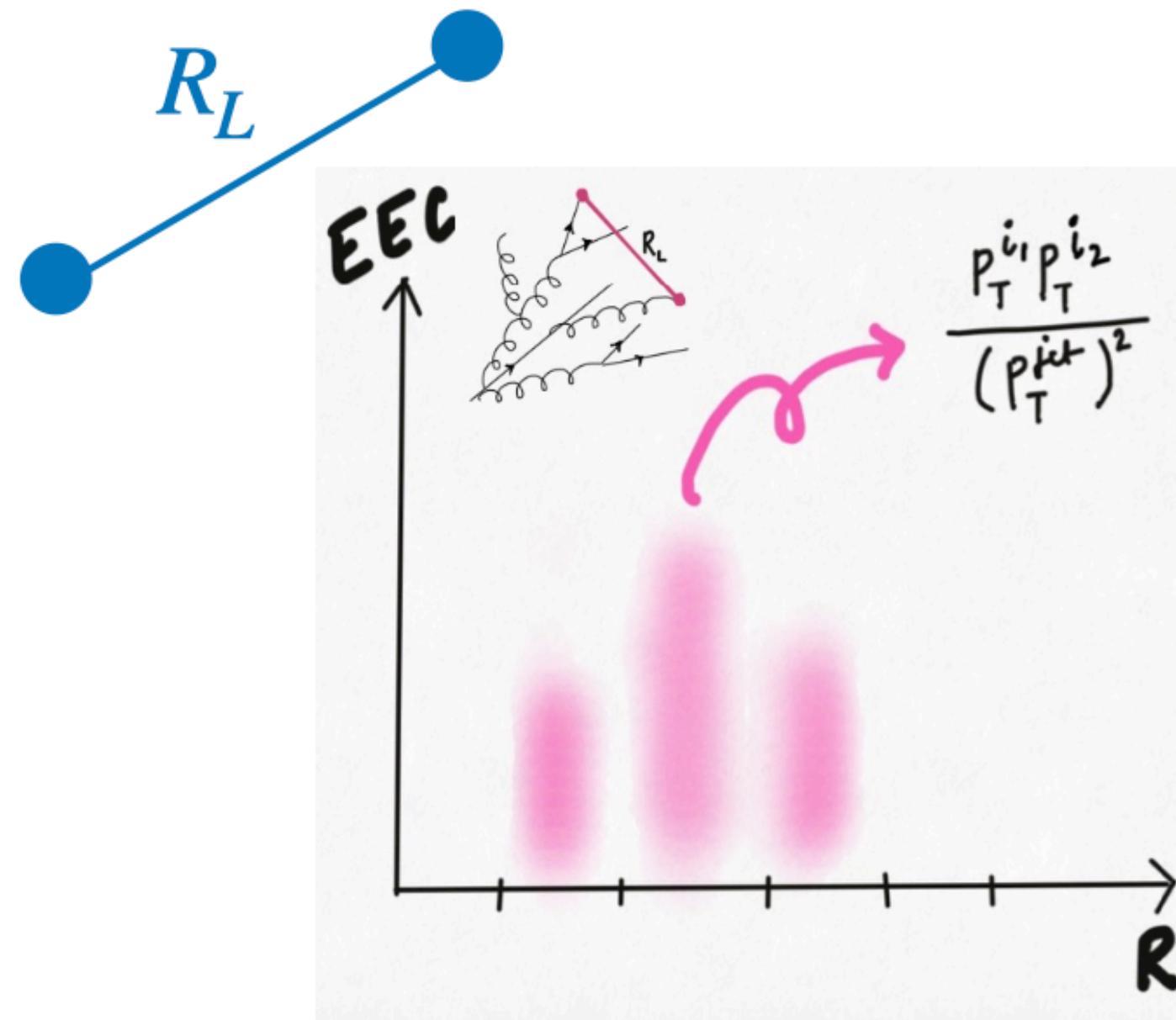


Eg 2: Use correlations to trace back to possible inflation scenarios

Energy Correlators in Jets

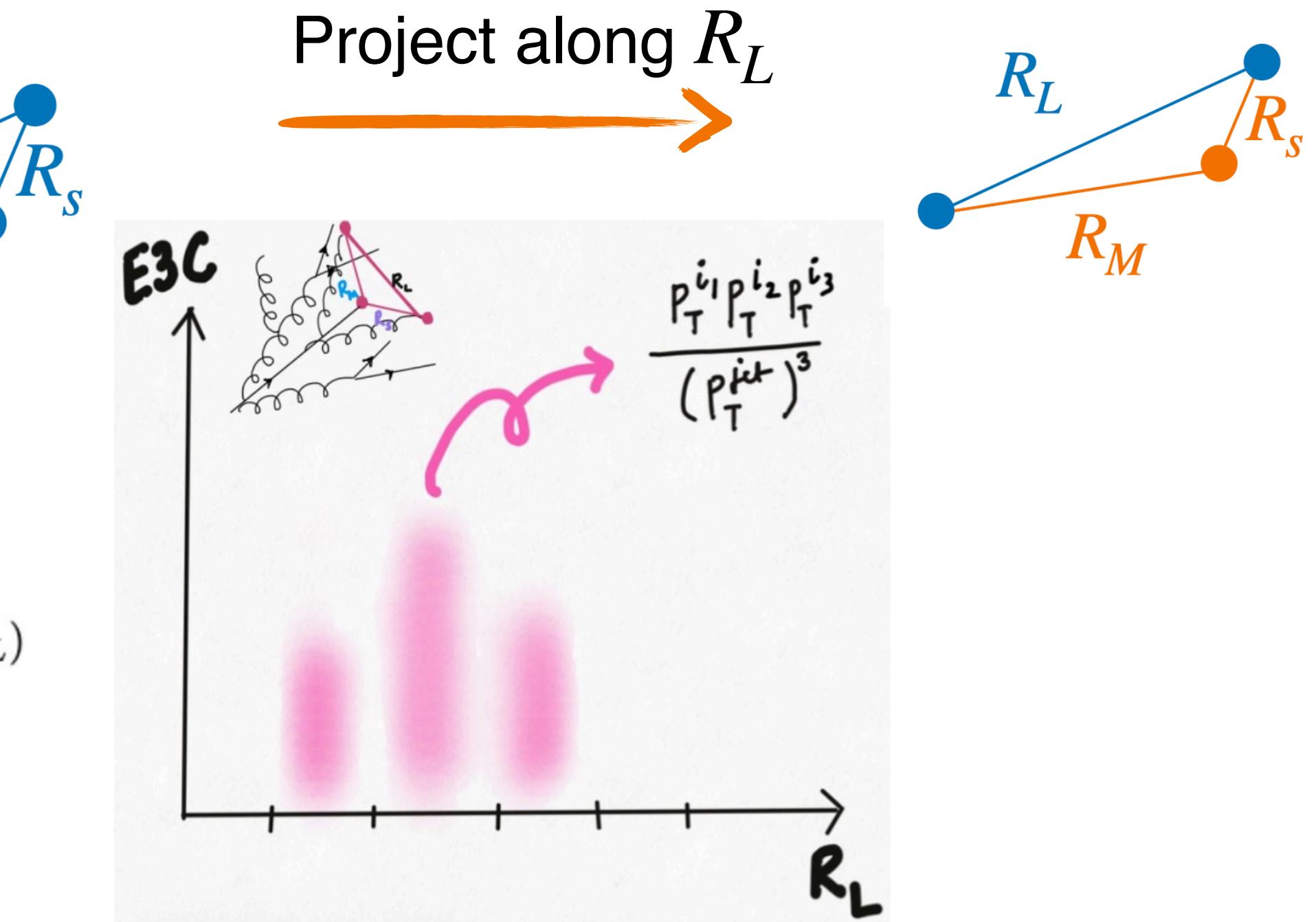
Constructing the correlators

EEC (or E2C): 2-point correlator



$$ENC(R_L) = \sum_{i_1, i_2, \dots, i_k}^{i_N} \int dR_L \frac{p_T^{i_1} p_T^{i_2} \dots p_T^{i_N}}{p_{T,jet}^N} \delta(R_L - \Delta \hat{R}_L)$$

E3C: Projected 3-point correlator

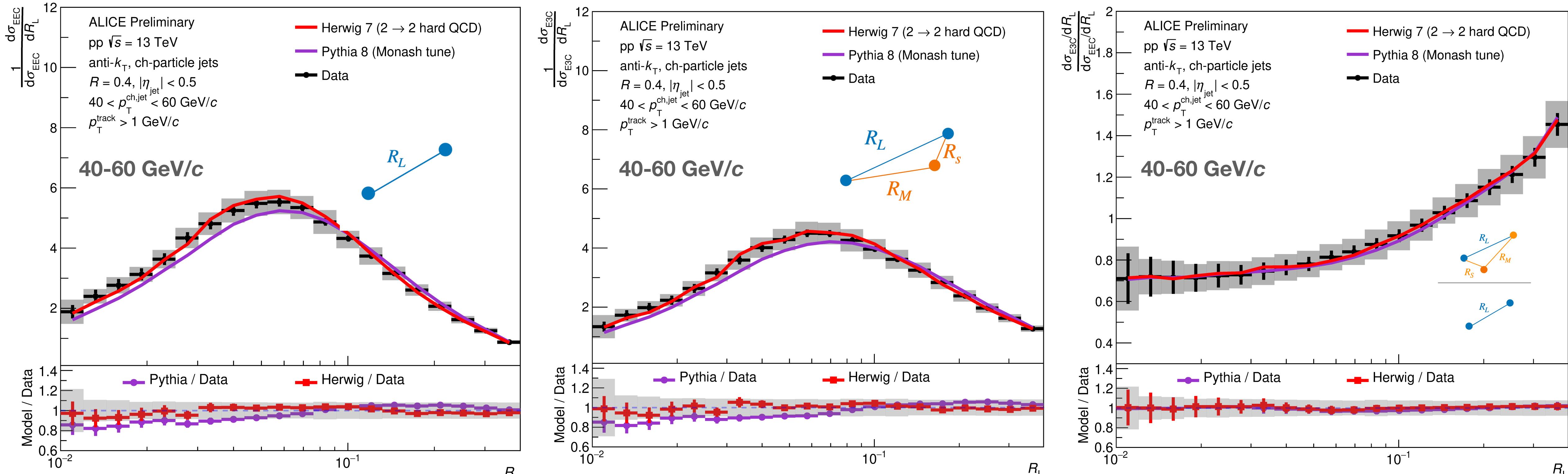


1. Find a jet
2. Create pairs from jet constituents
3. Fill an energy-weighted histogram

1. Find a jet
2. Create triplets from jet constituents
3. Find the largest distance between any pair in the triplets
4. Fill an energy-weighted histogram

Comparison to MC generators

pp $\sqrt{s} = 13 \text{ TeV}$



ALI-PREL-557442

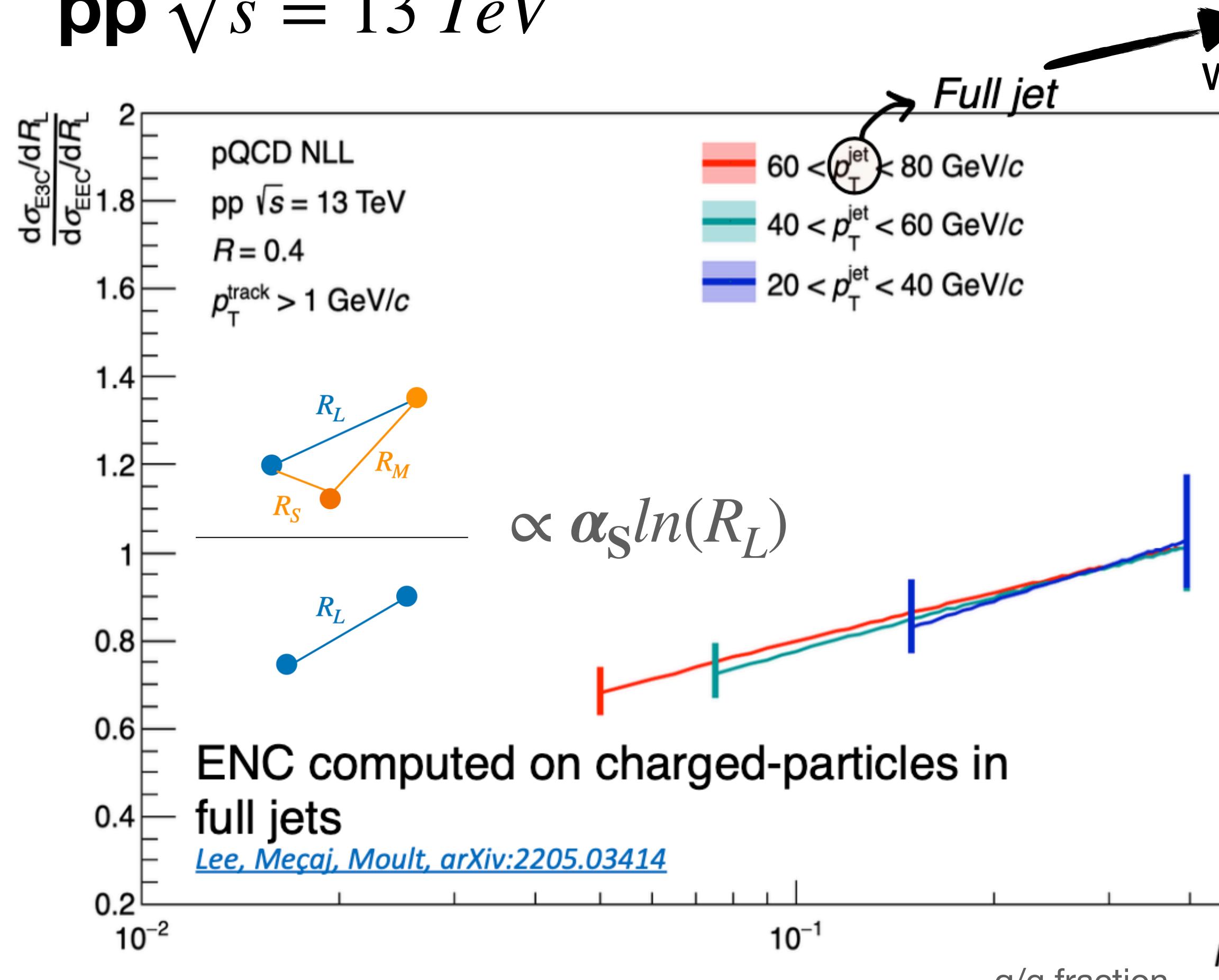
ALI-PREL-557457

ALI-PREL-557472

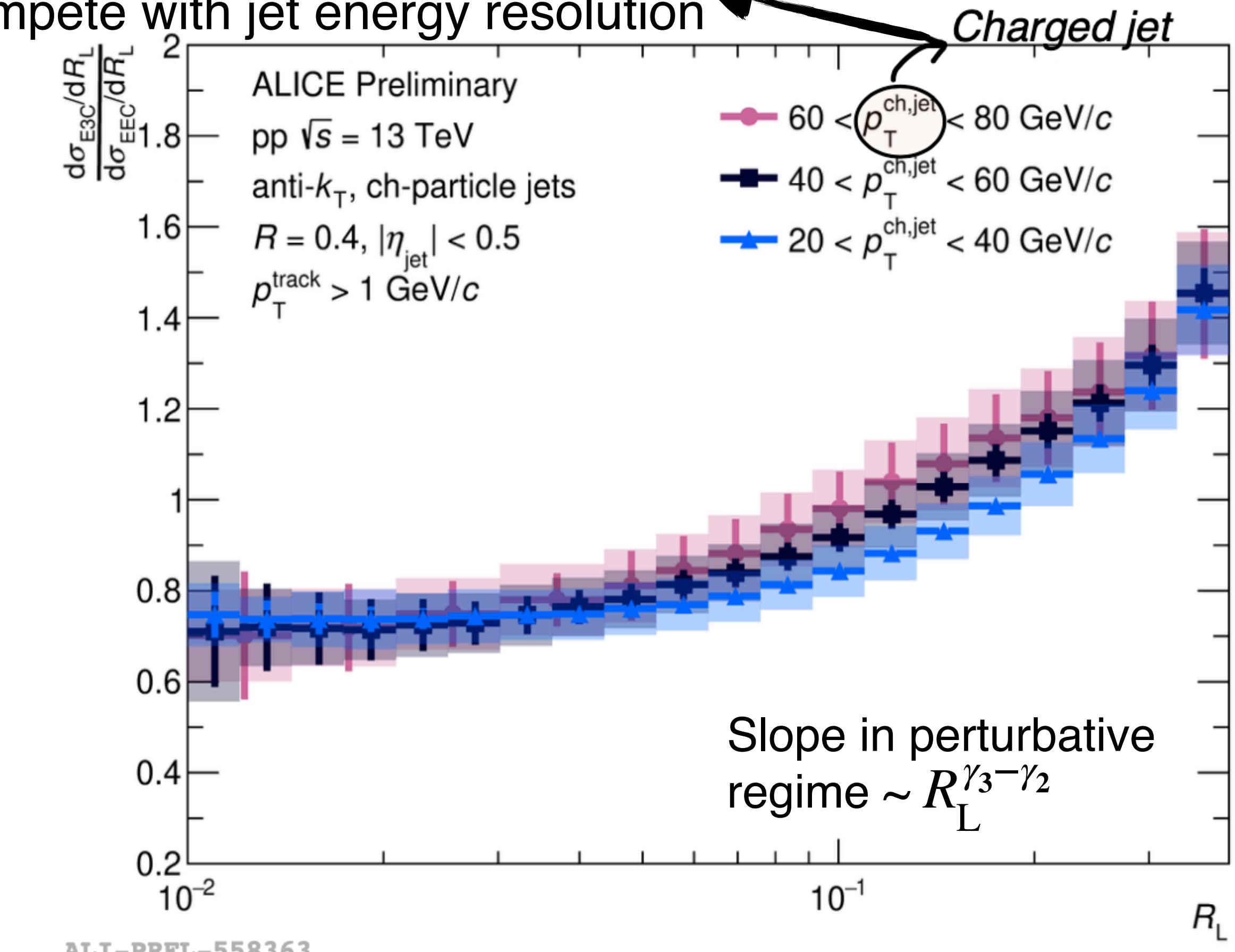
- * **Herwig** shows better agreement for EEC. Differences more pronounced in the hadronization region (non-perturbative corrections matter!)
- * The ratio to MC sits at unity for E3C/EEC – non-perturbative power corrections as modeled in MC largely cancel. E3C/EEC isolates perturbative physics

α_S extraction at ALICE : E3C/EEC Ratio

pp $\sqrt{s} = 13 \text{ TeV}$



Cause uncertainty in mapping,
will compete with jet energy resolution



At LL collinear in pQCD

$$\text{ENC}(R_L) = -\frac{d}{dR_L} \left[(1,1) \exp \left(-\frac{\gamma^{(0)}(N+1)}{\beta_0} \ln \left(\frac{\alpha_s(R_L\mu)}{\alpha_s(\mu)} \right) \right) (x_q, x_g) H_J(\mu) \right]$$

q/g fraction

Jet production crossx

Jet p_T scale

QCD beta function (1 loop)

[PhysRevLett.130.051901](https://arxiv.org/abs/130.051901)

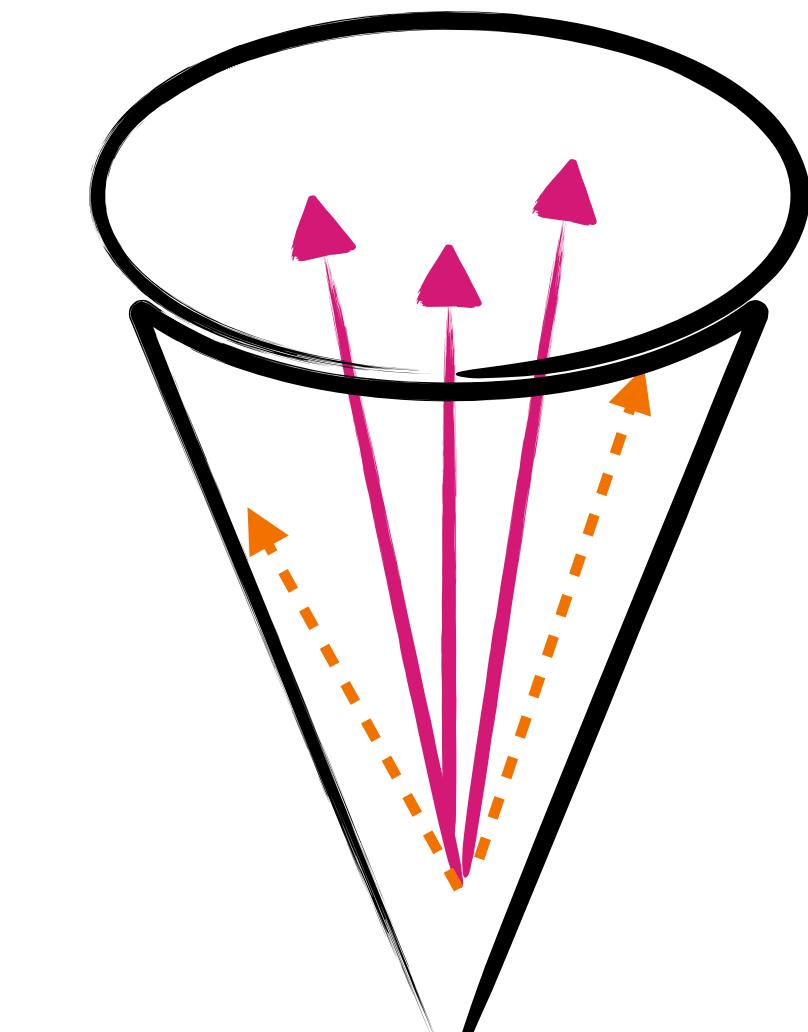
- * Trends between theory and data agree
- * Current work: extract γ and map to α_S

Background Subtraction in Heavy Ion Collisions

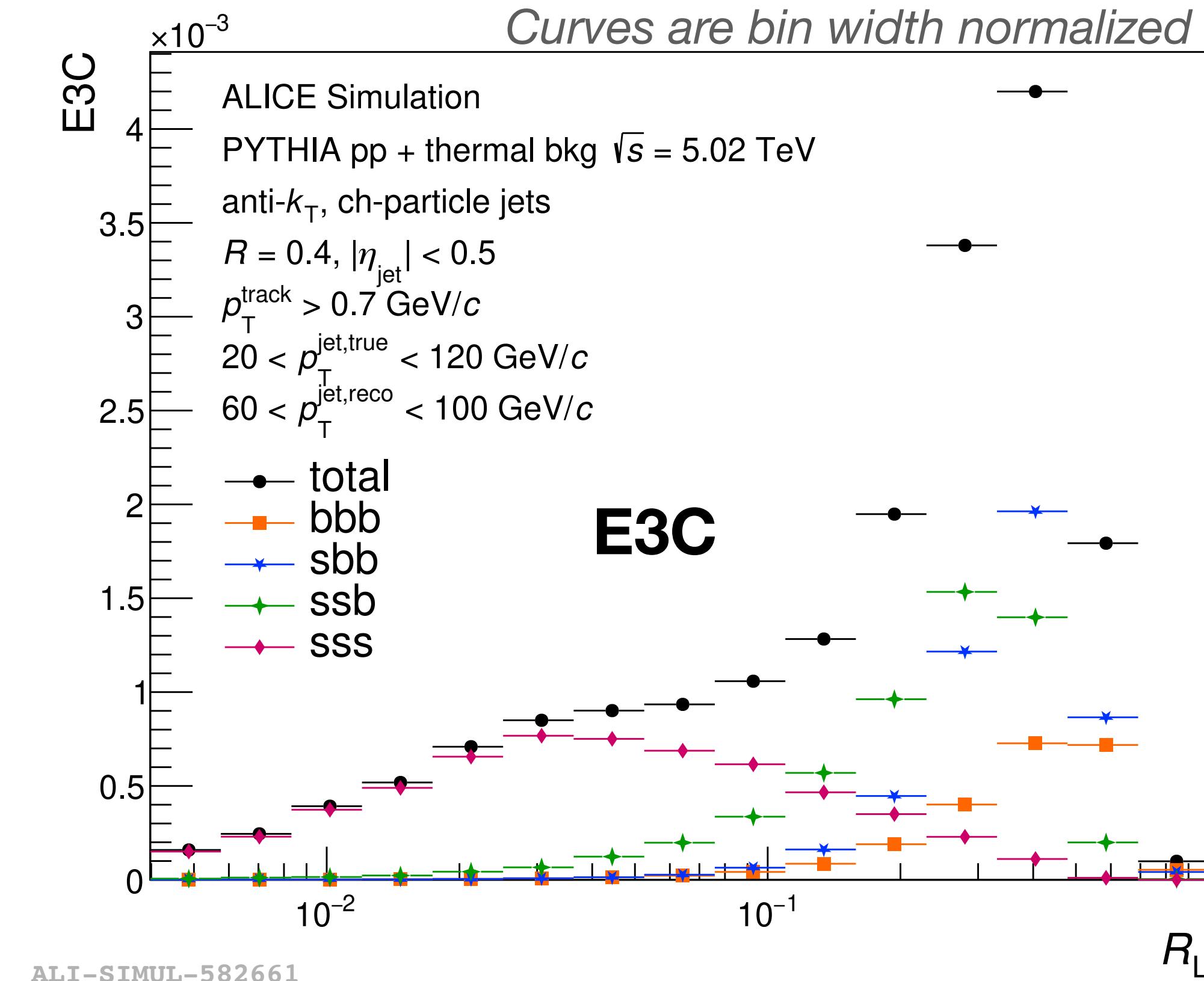
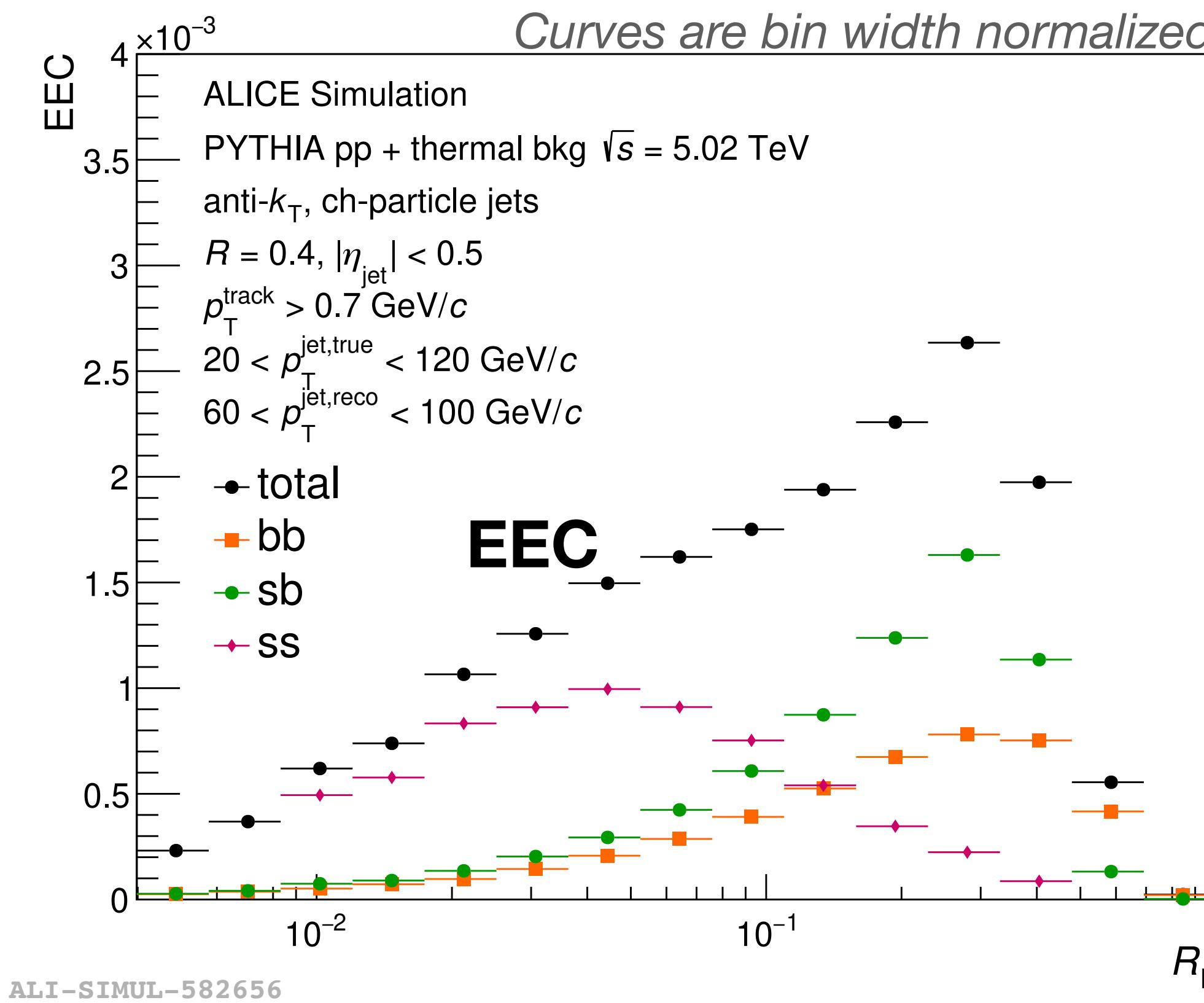
Energy Correlators in a Thermal Toy

See talk by J. Viinikainen

See talk by A. Tamis



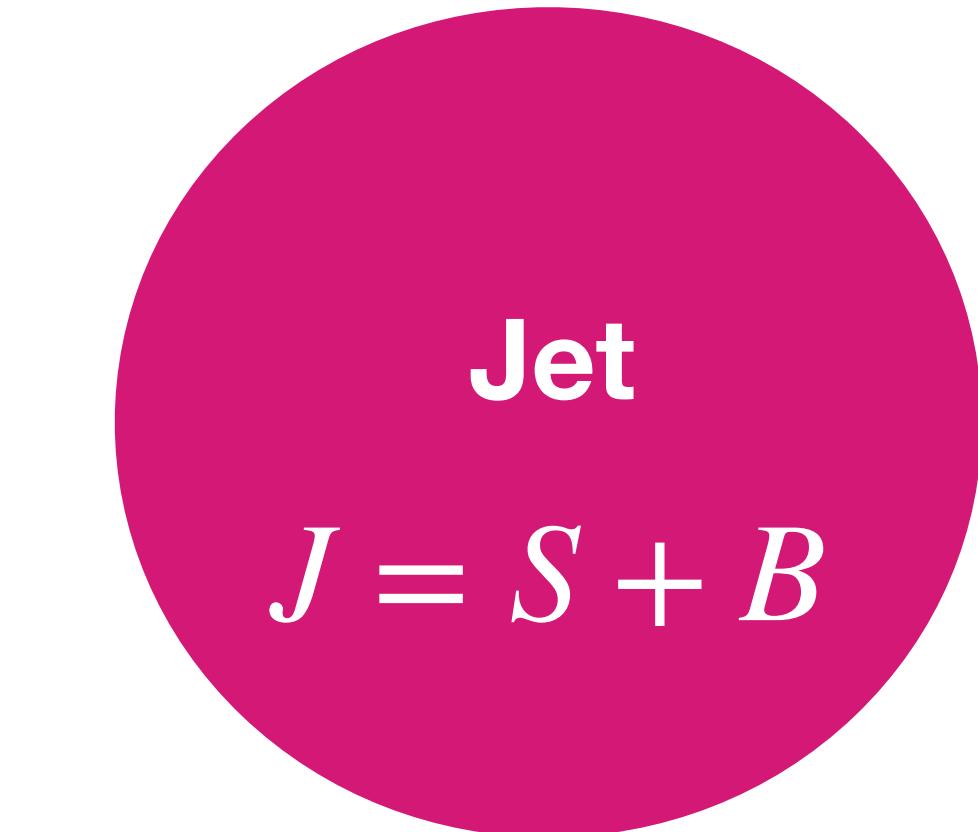
Adding uncorrelated
background to Pythia jet,
New Jet = **Signal + Bkg**



- * Embedding Pythia jets in a thermal toy – now we have **signal (s)** and **background (b)** particles
- * Want to reduce this problem to a pp jet p_T scale unfolding problem

Background Subtraction: EEC

A multi-cone story



$$J = S + B$$
$$J^2 = S^2 + B^2 + 2SB$$

Want to correct
this background

$$S^2 = 2S_1S_2 + 2S_1S_3 + \dots$$

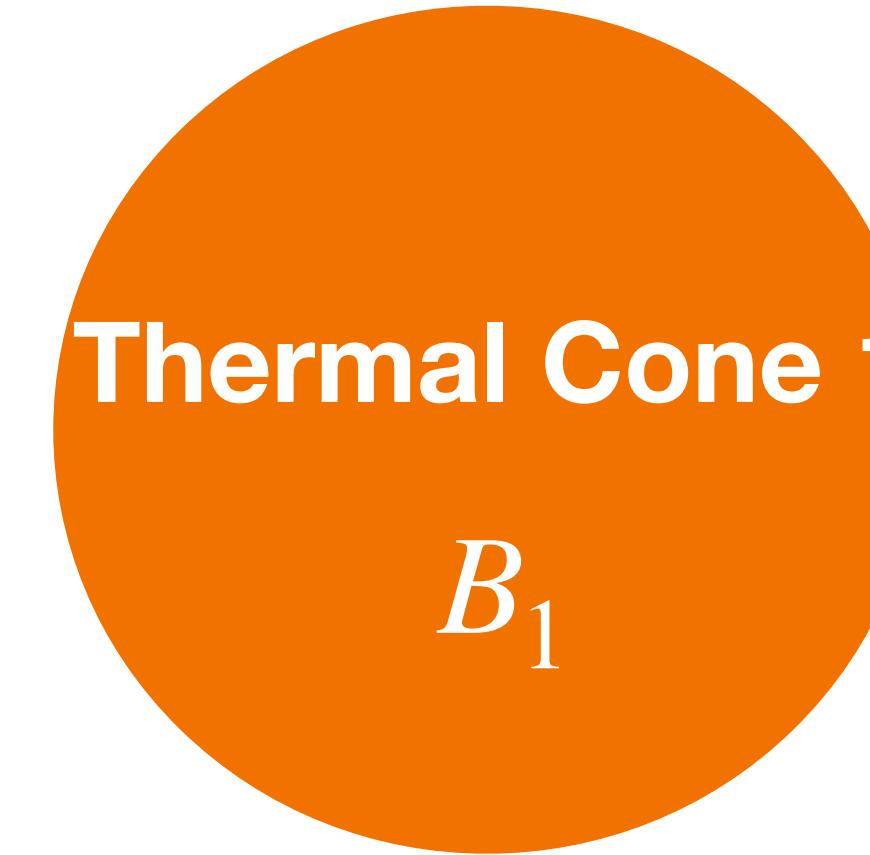
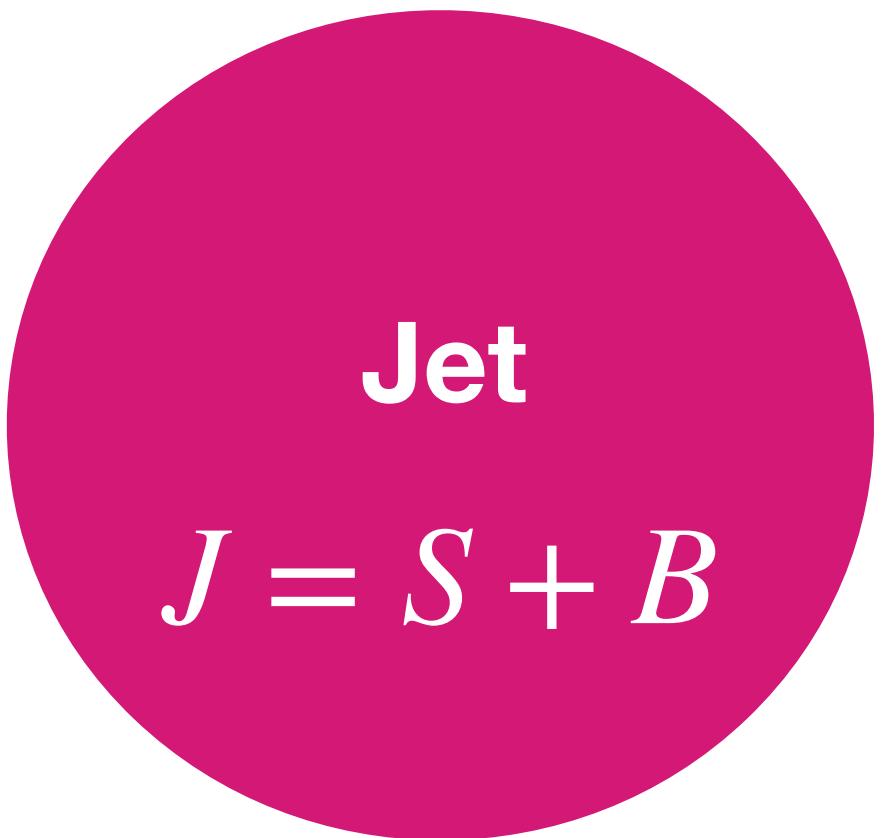
$$B^2 = 2B_1B_2 + 2B_1B_3 + \dots$$

$$2SB = 2S_1B_1 + 2S_1B_2 + \dots$$

Note: S^2 includes terms such as $S_1S_1 + S_2S_2 + \dots$ but $R_L = 0$ for these
 B^2 includes terms such as $B_1B_1 + B_2B_2 + \dots$ but $R_L = 0$ for these

Background Subtraction for EEC

A multi-cone story



$$J = S + B$$

$$J^2 = S^2 + \boxed{B^2} + \boxed{2SB}$$

$$(B_1)^2 = B_1 B_1$$

$$JB_1 = (S + B)B_1$$

$$S^2 = 2S_1S_2 + 2S_1S_3 + \dots$$

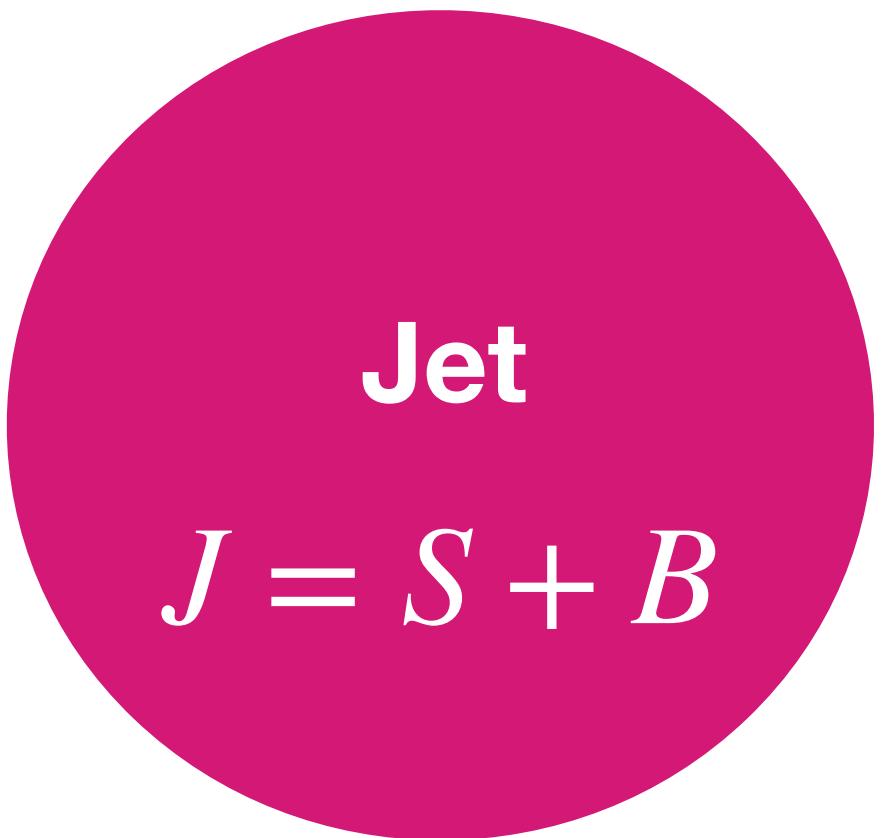
$$B^2 = 2B_1B_2 + 2B_1B_3 + \dots$$

$$2JB_1 = 2(S + B)B_1$$

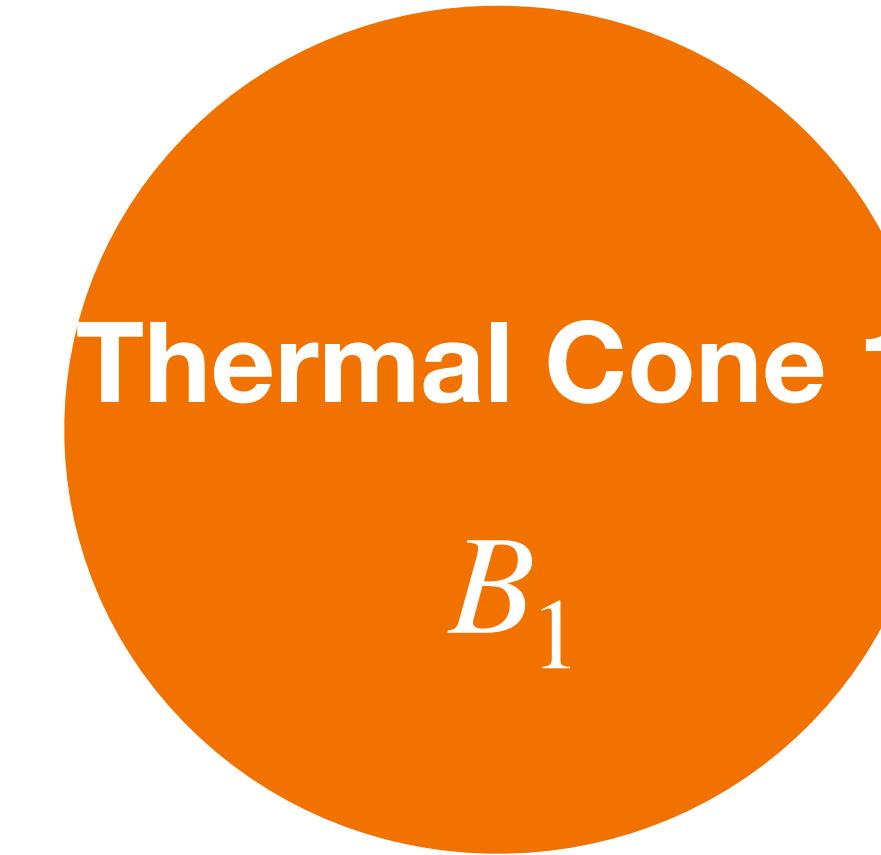
$$(B_1)^2 = 2(B_1B_2)_1 + 2(B_1B_3)_1 + \dots$$

Background Subtraction for EEC

A multi-cone story



$$J = S + B$$
$$J^2 = S^2 + \boxed{B^2} + \boxed{2SB}$$



$$(B_1)^2 = B_1 B_1$$
$$JB_1 = (S + B)B_1$$

However, $\textcolor{magenta}{BB}_1 \neq \textcolor{orange}{BB}$

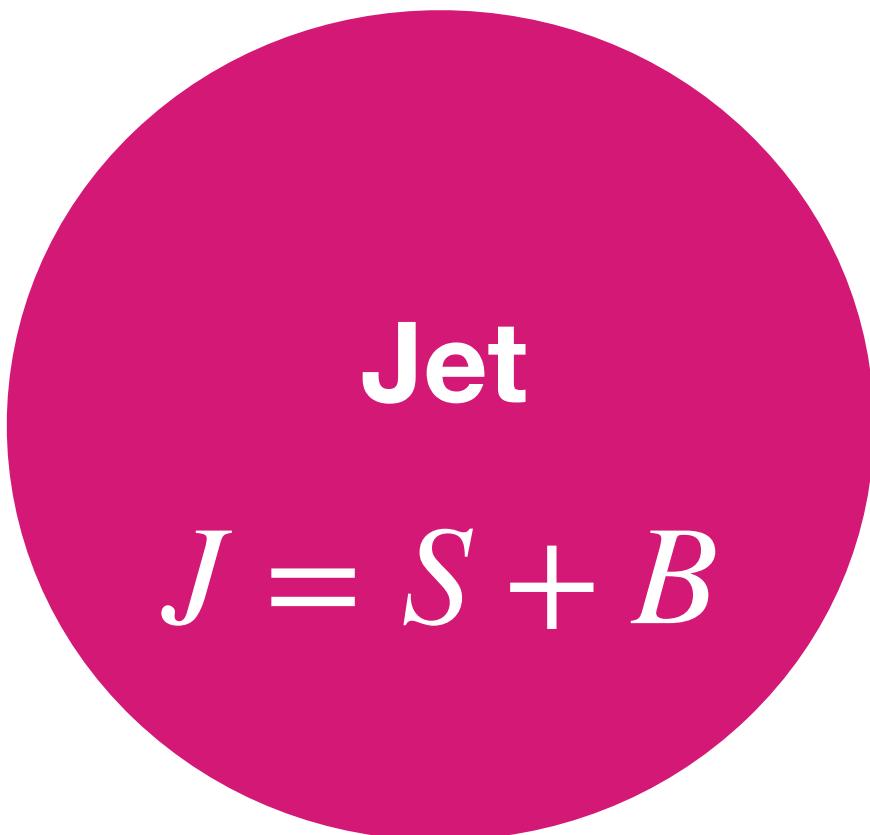
$$S^2 = 2S_1S_2 + 2S_1S_3 + \dots$$

$$B^2 = 2B_1B_2 + 2B_1B_3 + \dots$$

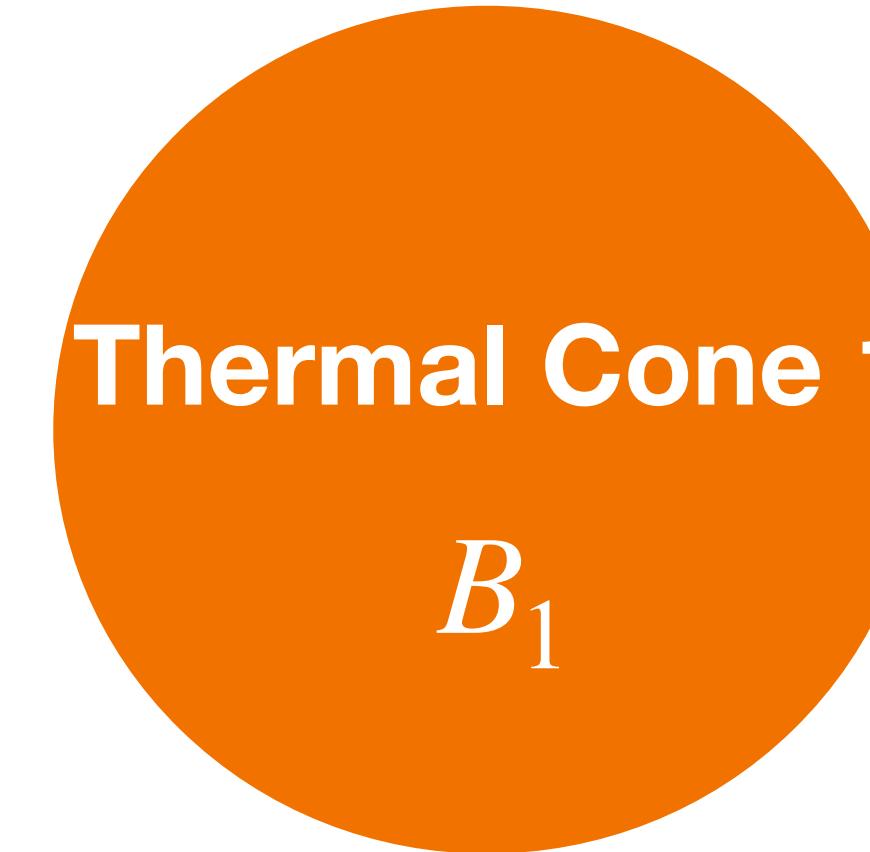
$$2JB_1 = 2(S + B)B_1$$
$$(B_1)^2 = 2(B_1B_2)_1 + 2(B_1B_3)_1 + \dots$$

Background Subtraction for EEC

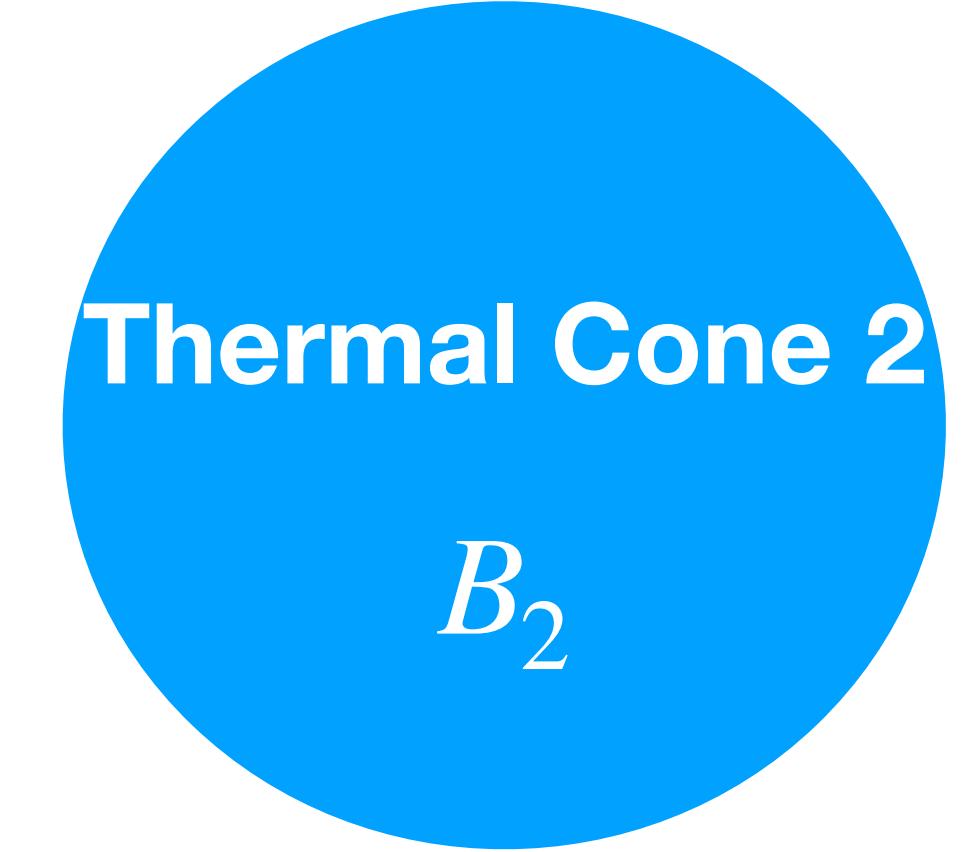
A multi-cone story



$$J = S + B$$
$$J^2 = S^2 + \boxed{B^2} + \boxed{2SB}$$



$$(B_1)^2 = B_1 B_1$$
$$JB_1 = (S + B)B_1$$



$$\boxed{B_1 \ B_2}$$

However, $\cancel{BB}_1 \neq \cancel{BB}$

Therefore, we need to a third cone that can account for this: $\cancel{BB}_1 = \cancel{B}_1 \cancel{B}_2$

$$S^2 = 2S_1S_2 + 2S_1S_3 + \dots$$

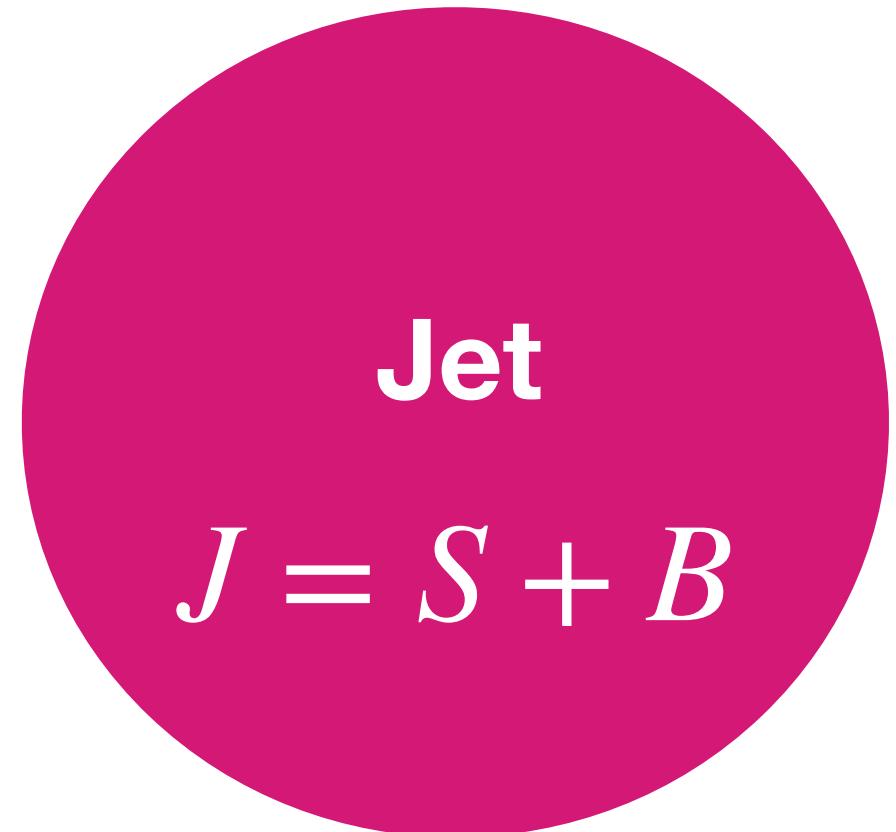
$$B^2 = 2B_1B_2 + 2B_1B_3 + \dots$$

$$2JB_1 = 2(S + B)B_1$$

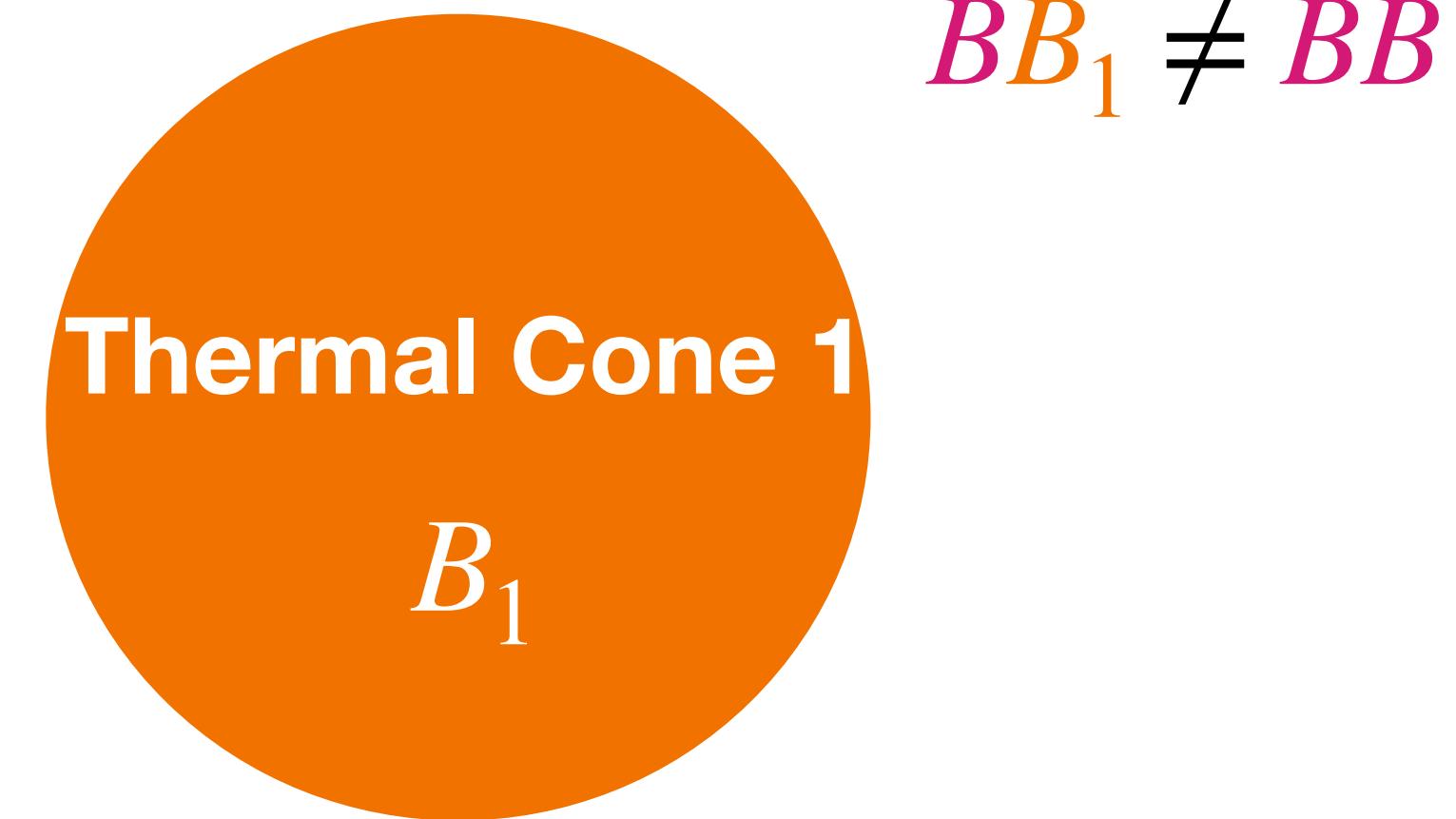
$$(B_1)^2 = 2(B_1B_2)_1 + 2(B_1B_3)_1 + \dots$$

Background Subtraction for EEC

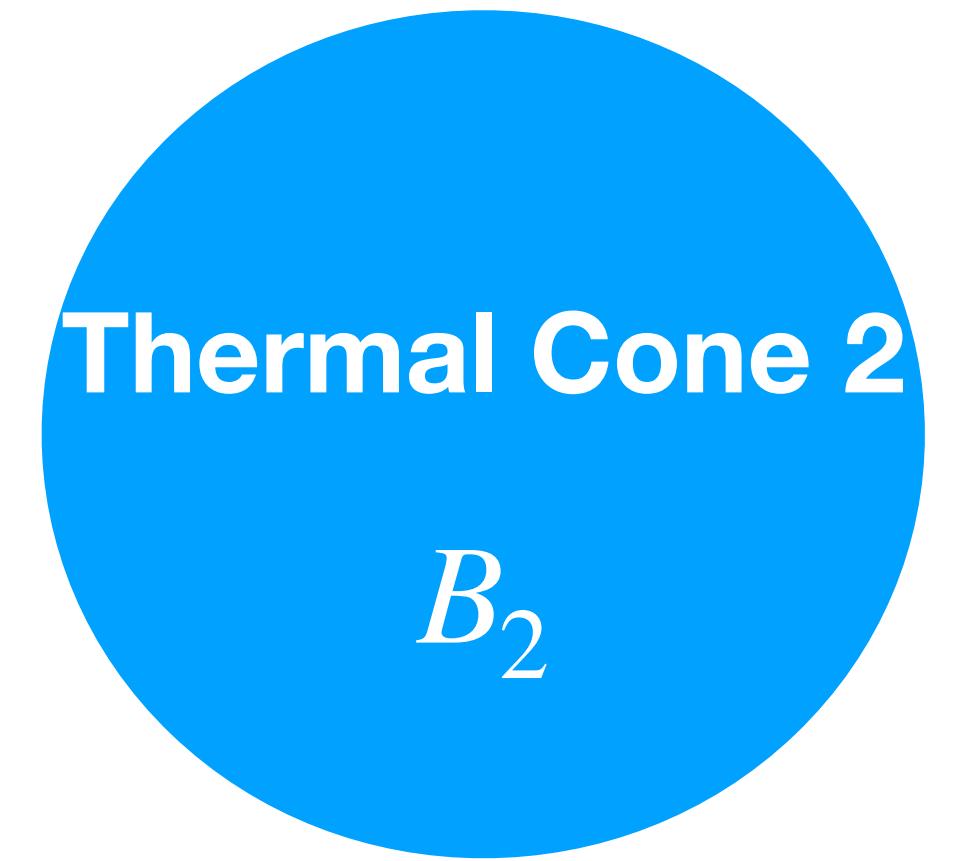
A multi-cone story



$$J = S + B$$
$$J^2 = S^2 + \boxed{B^2} + \boxed{2SB}$$



$$(B_1)^2 = B_1 B_1$$
$$JB_1 = (S + B)B_1$$

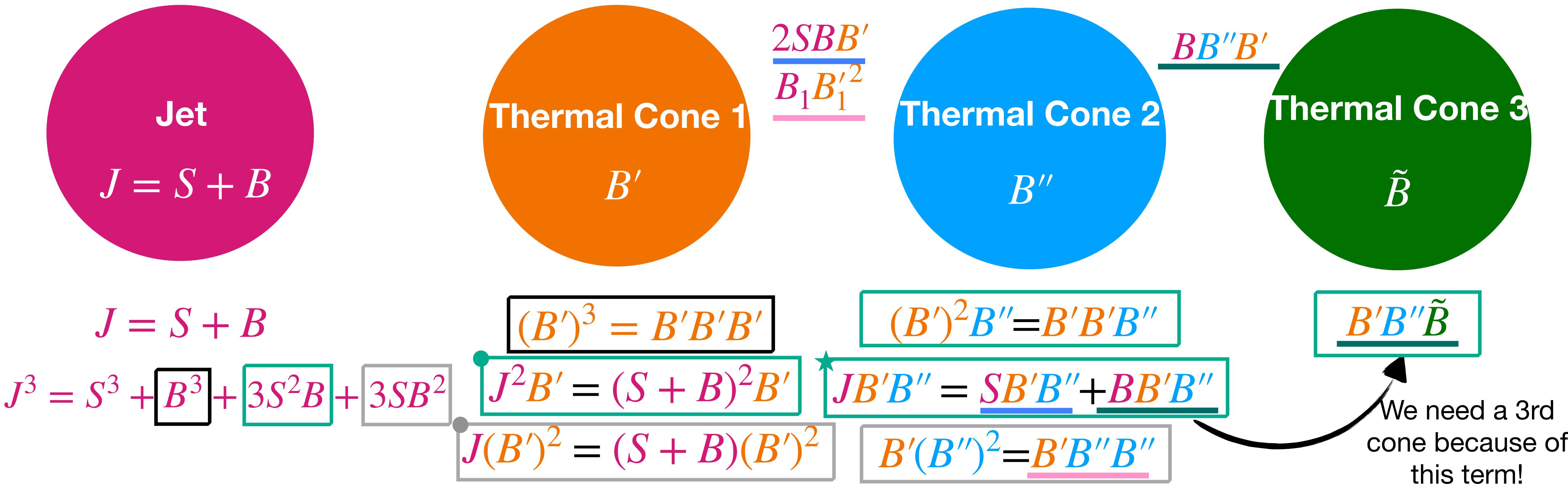


$$\boxed{B_1 B_2}$$

$$S^2 = J^2 - 2JB_1 + 2B_1 B_2$$

Background subtraction for E3C

A multi-cone story

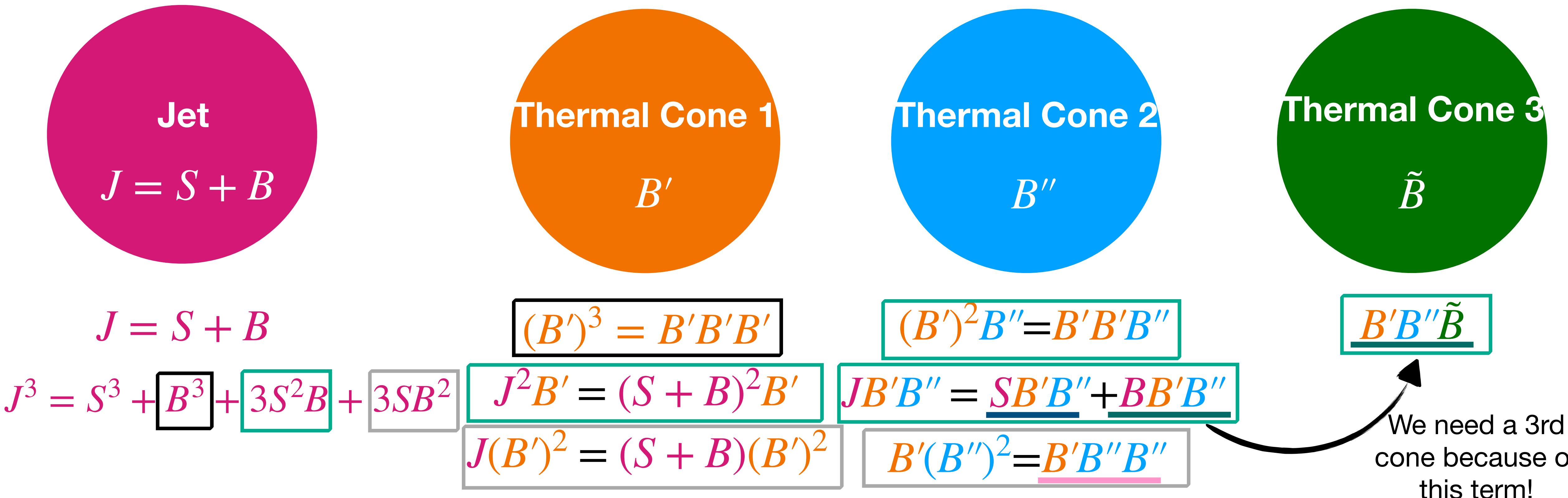


$$\begin{aligned} S^3 &= 3(S_1^2 S_2 + S_1^2 S_3 + \dots) + 6S_1 S_2 S_3 + \dots \\ B^3 &= 3(B_1^2 B_2 + B_1^2 B_3 + \dots) + 6B_1 B_2 B_3 + \dots \\ S^2 B &= (S_1^2 B_1 + S_1^2 B_2 + \dots) + 2S_1 S_2 B_1 + \dots \\ B^2 S &= (B_1^2 S_1 + B_1^2 S_2 + \dots) + 2B_1 B_2 S_1 + \dots \end{aligned}$$

- $J^2 B' = (S^2 + B^2 + 2SB)B' = S^2 B' + B^2 B' + \underline{2SBB'}$
- $J(B')^2 = \underline{B_1 B_1'^2} + B_1 B_2'^2 + \dots + S_1 B_1'^2 + S_1 B_2'^2 + \dots$
- ★ $JB'B'' = \underline{S_1 B_1' B_1''} + \dots + S_5 B_3' B_2'' + \dots + \underline{B_1 B_1' B_1'' + B_2 B_1' B_1''} + \dots$

Background subtraction for E3C

A multi-cone story

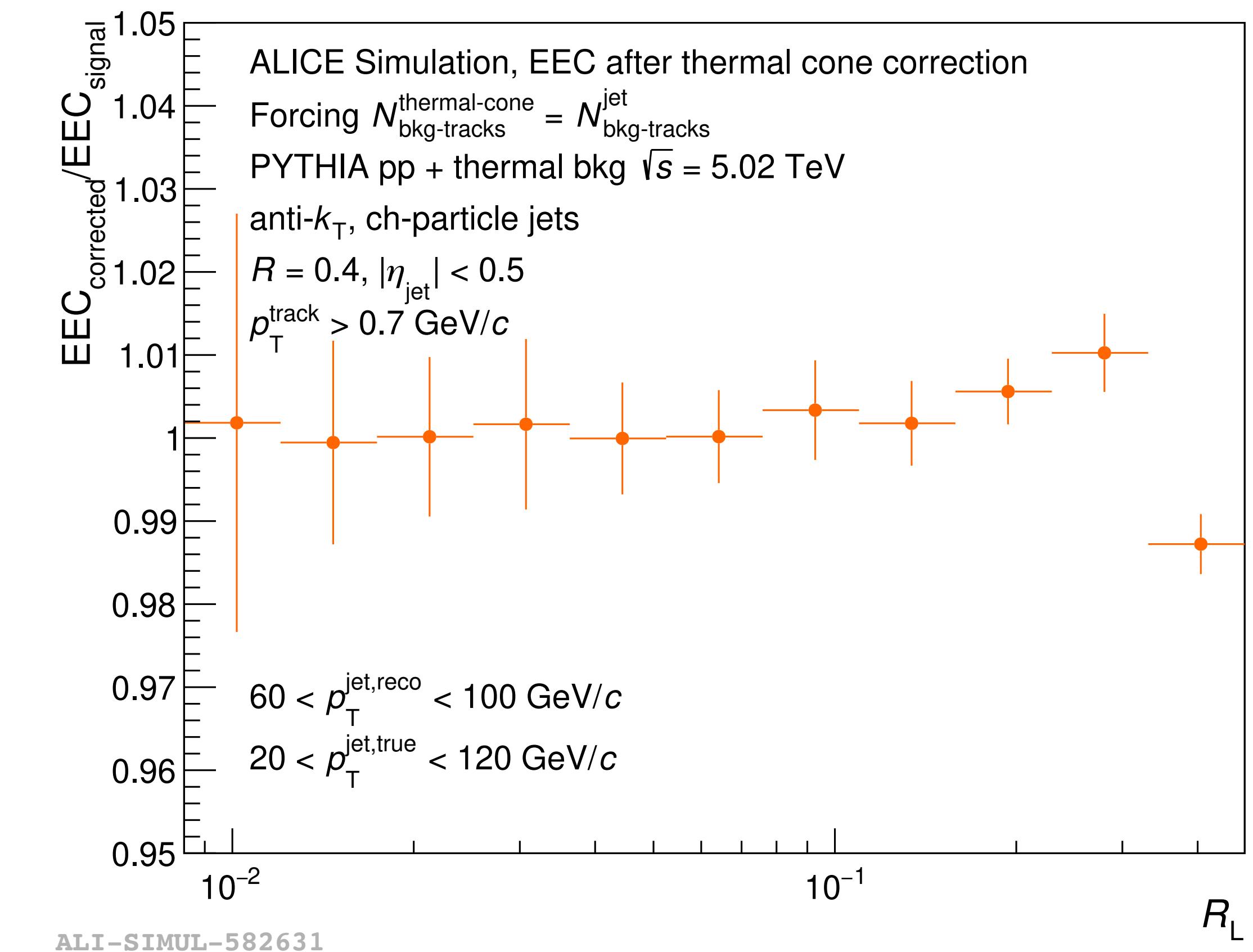
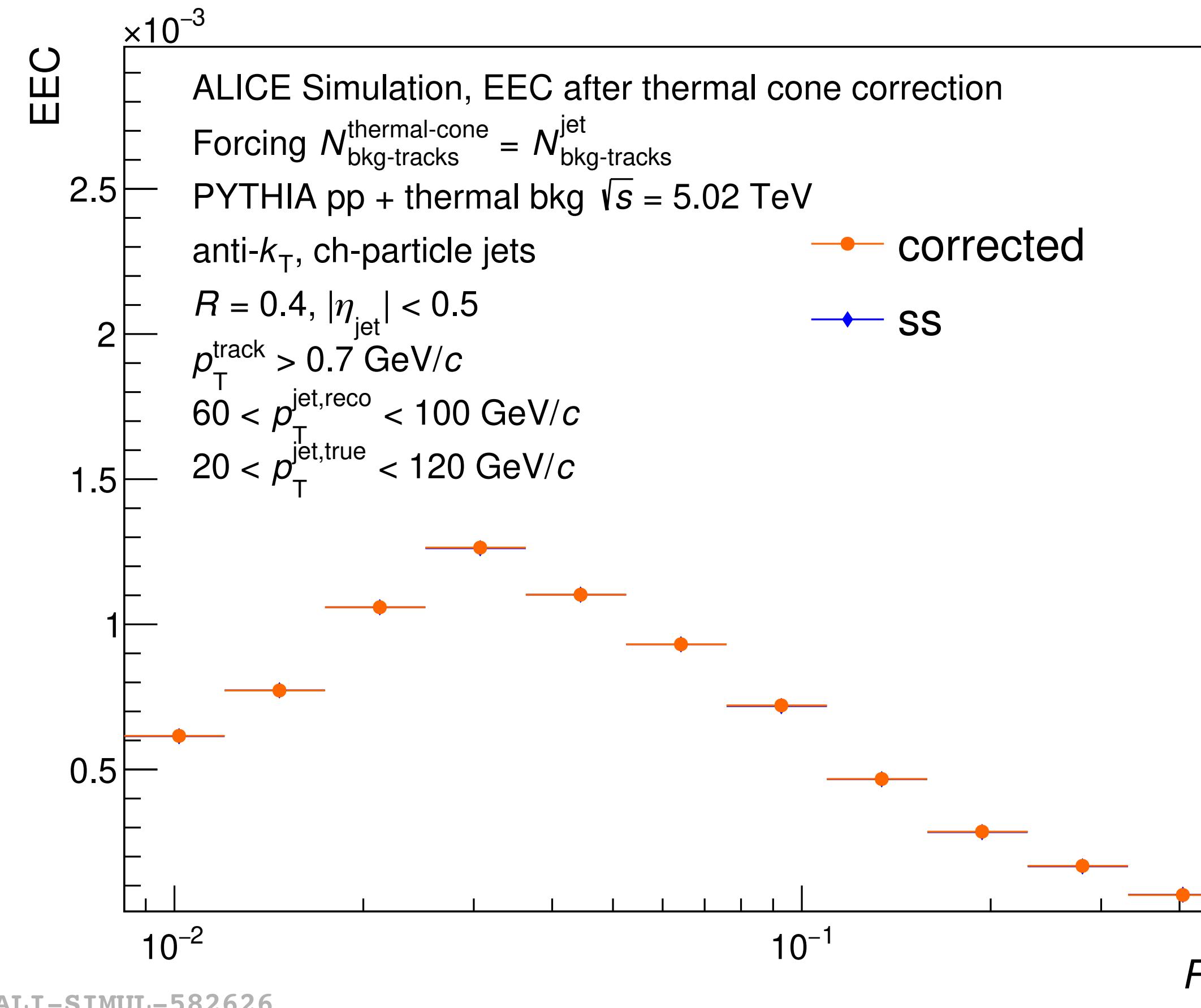


$$S^3 = J^3 - (B')^3 - 3J^2B' - 3J(B')^2 + 3(B')^2B'' + 3B'(B'')^2 + 6JB'B'' - 6B'B''\tilde{B}$$

Background Subtraction in Heavy Ion Collisions

Correcting for jet selection bias

Simple study: Add thermal tracks to your Pythia jet to artificially add background particles to the jet.
Forcing the thermal background to have the same number of tracks

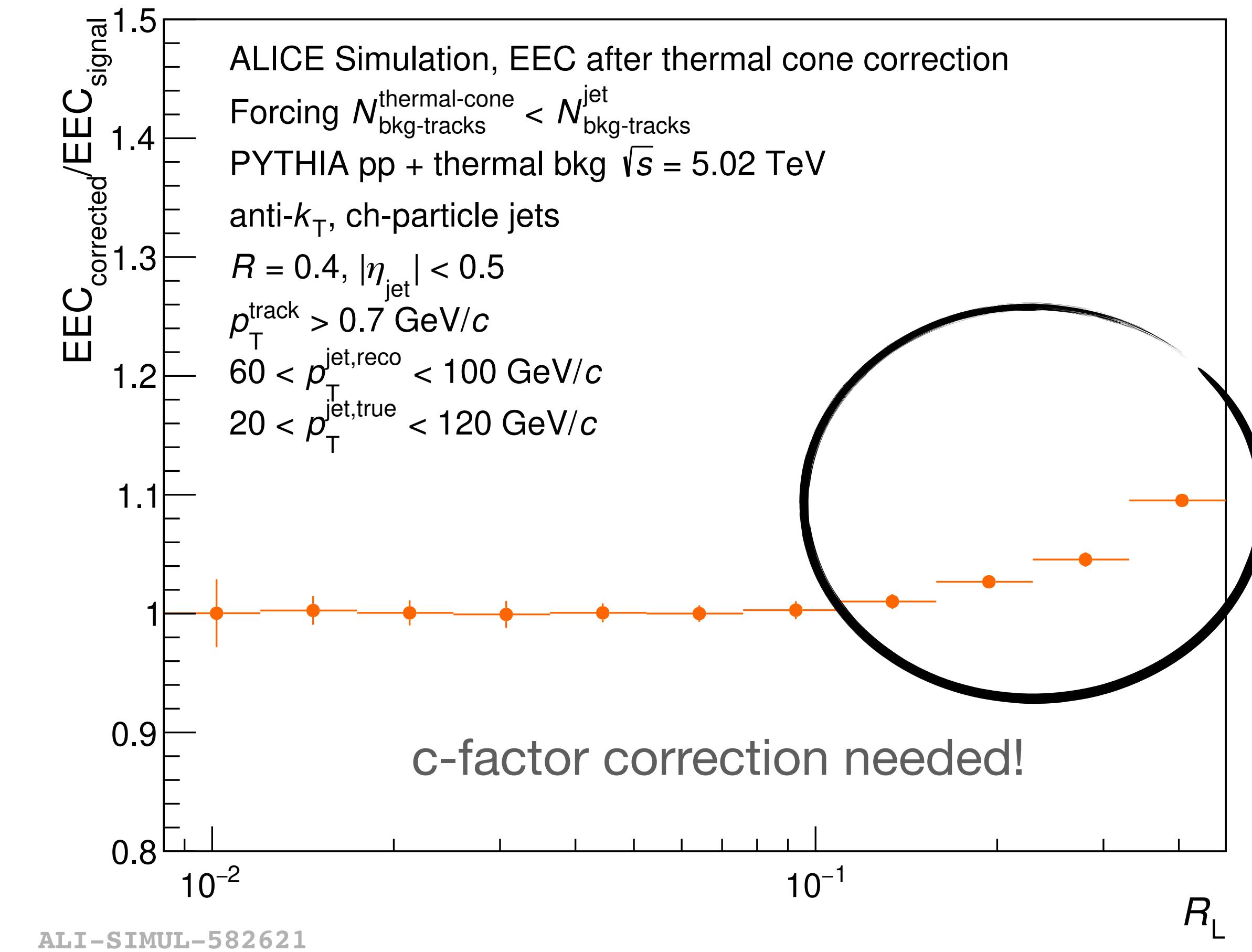
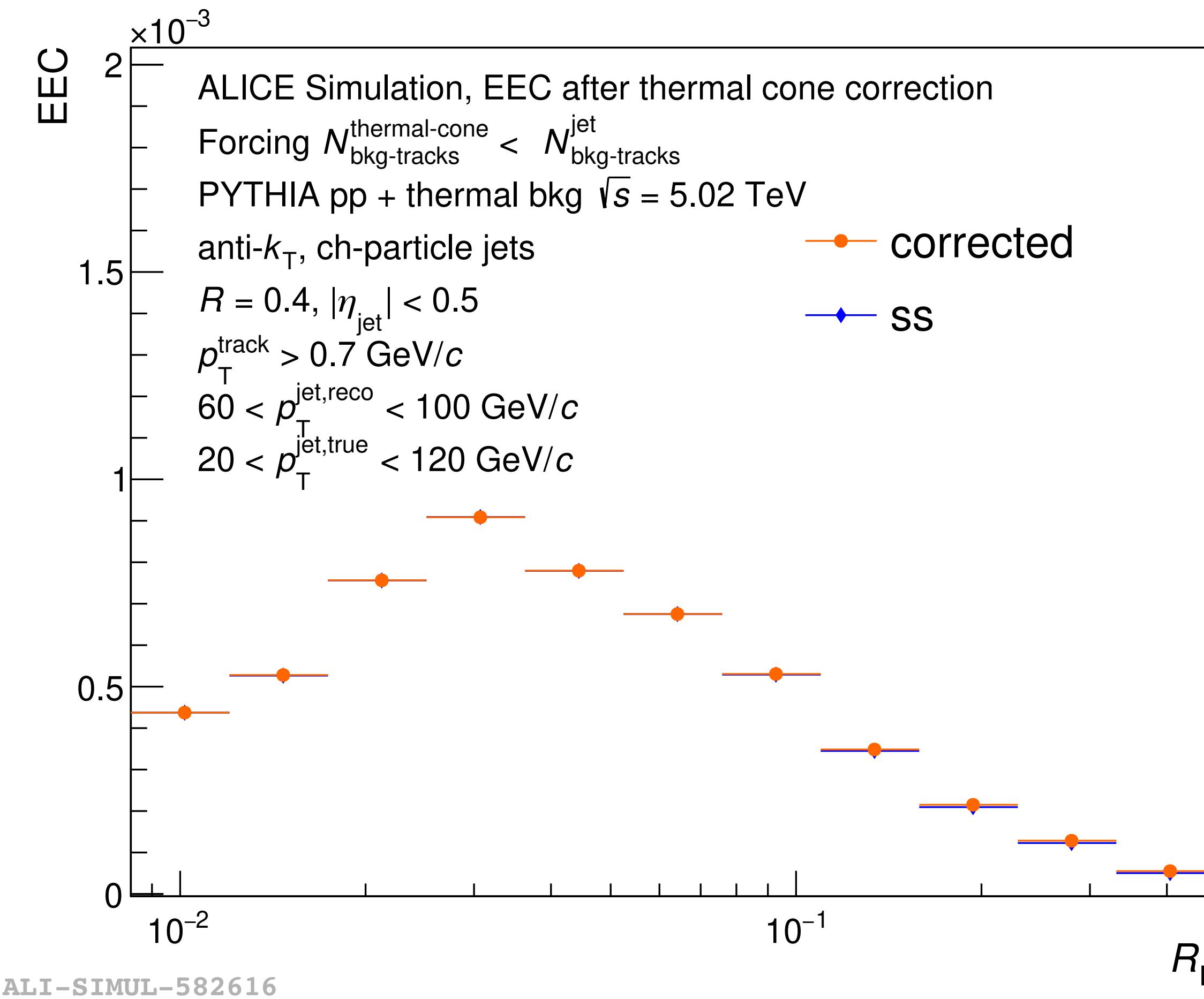


No c-factor correction needed!

Background Subtraction in Heavy Ion Collisions

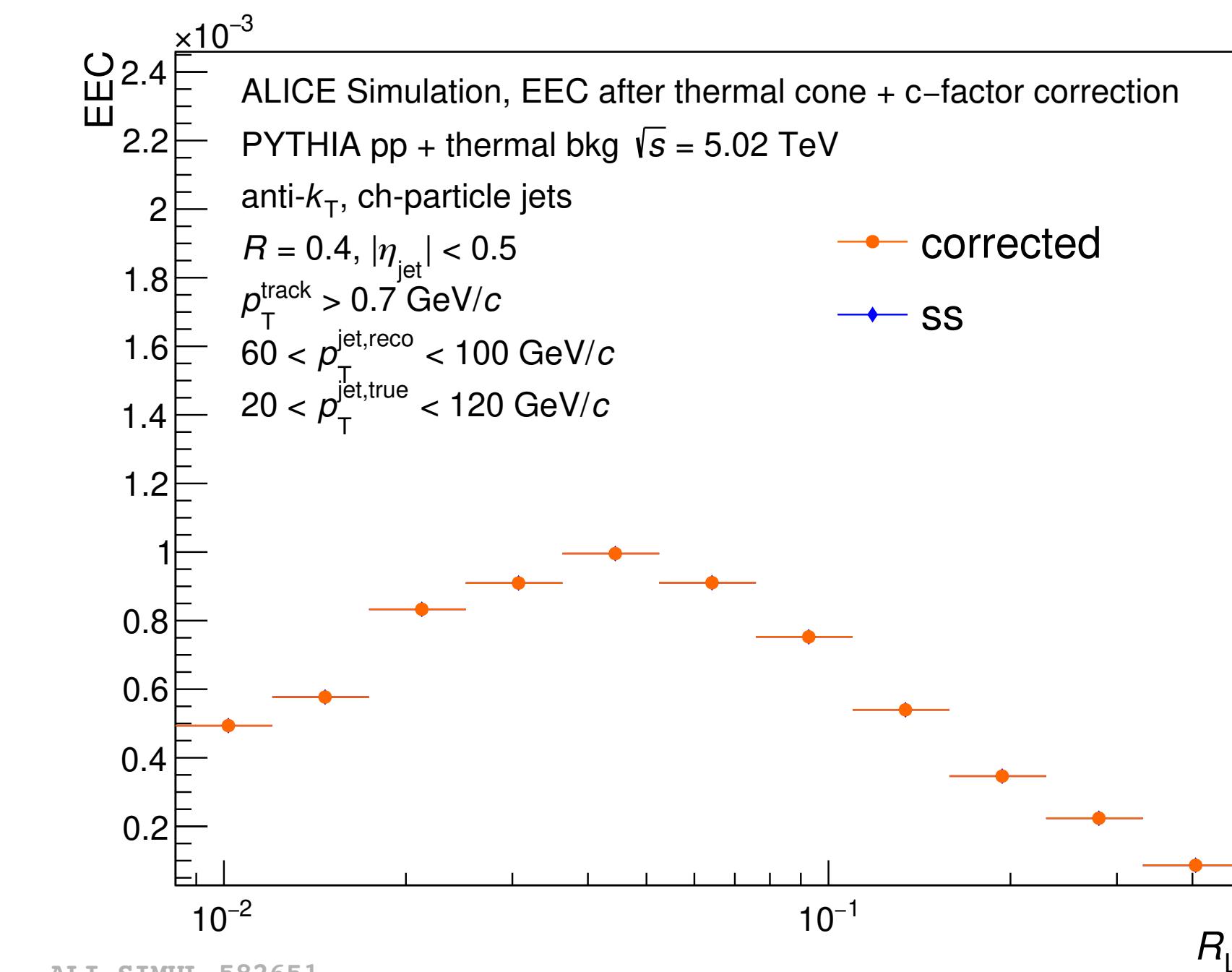
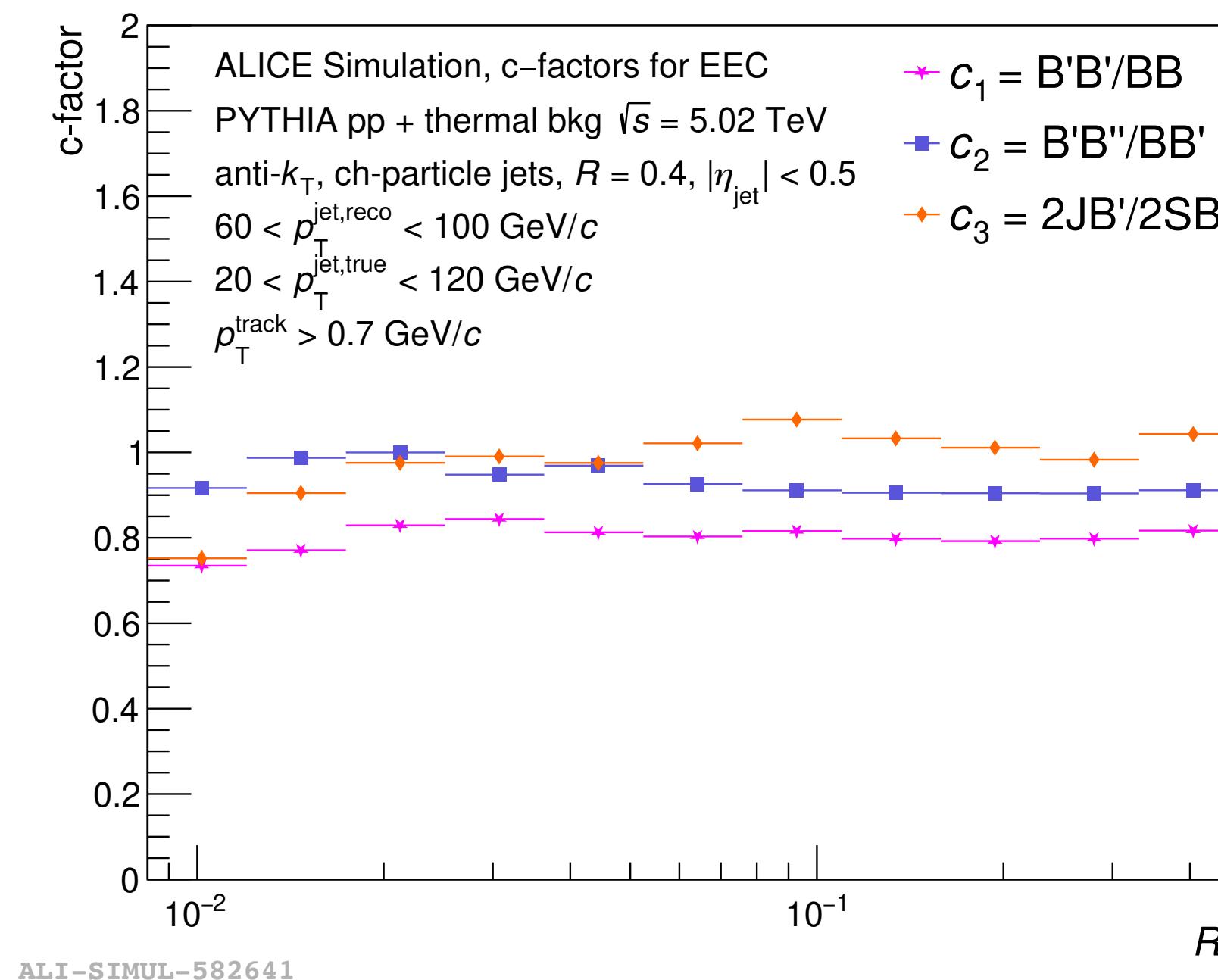
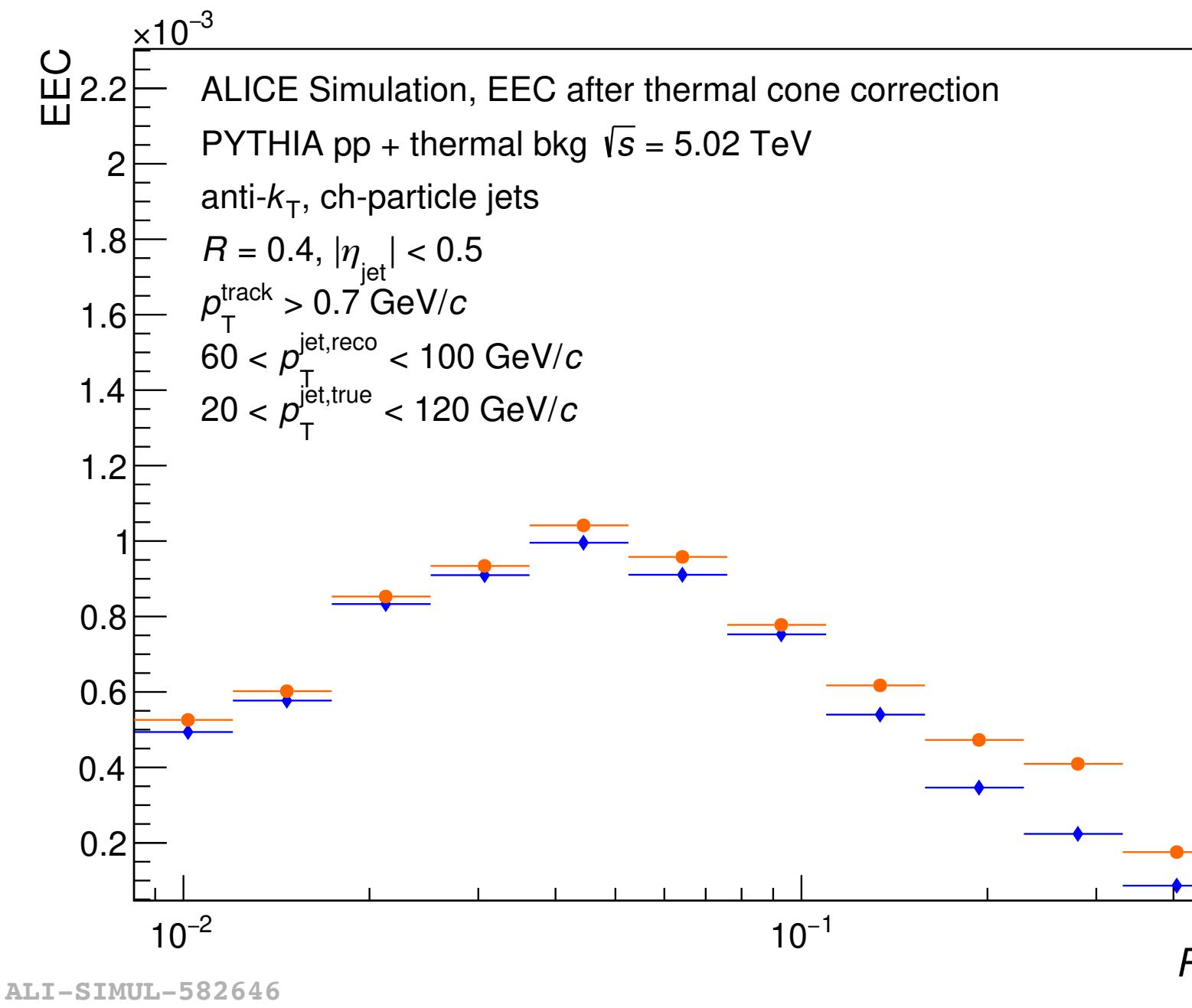
Correcting for jet selection bias

Now, add extra tracks under the jet and force the thermal background to have less tracks



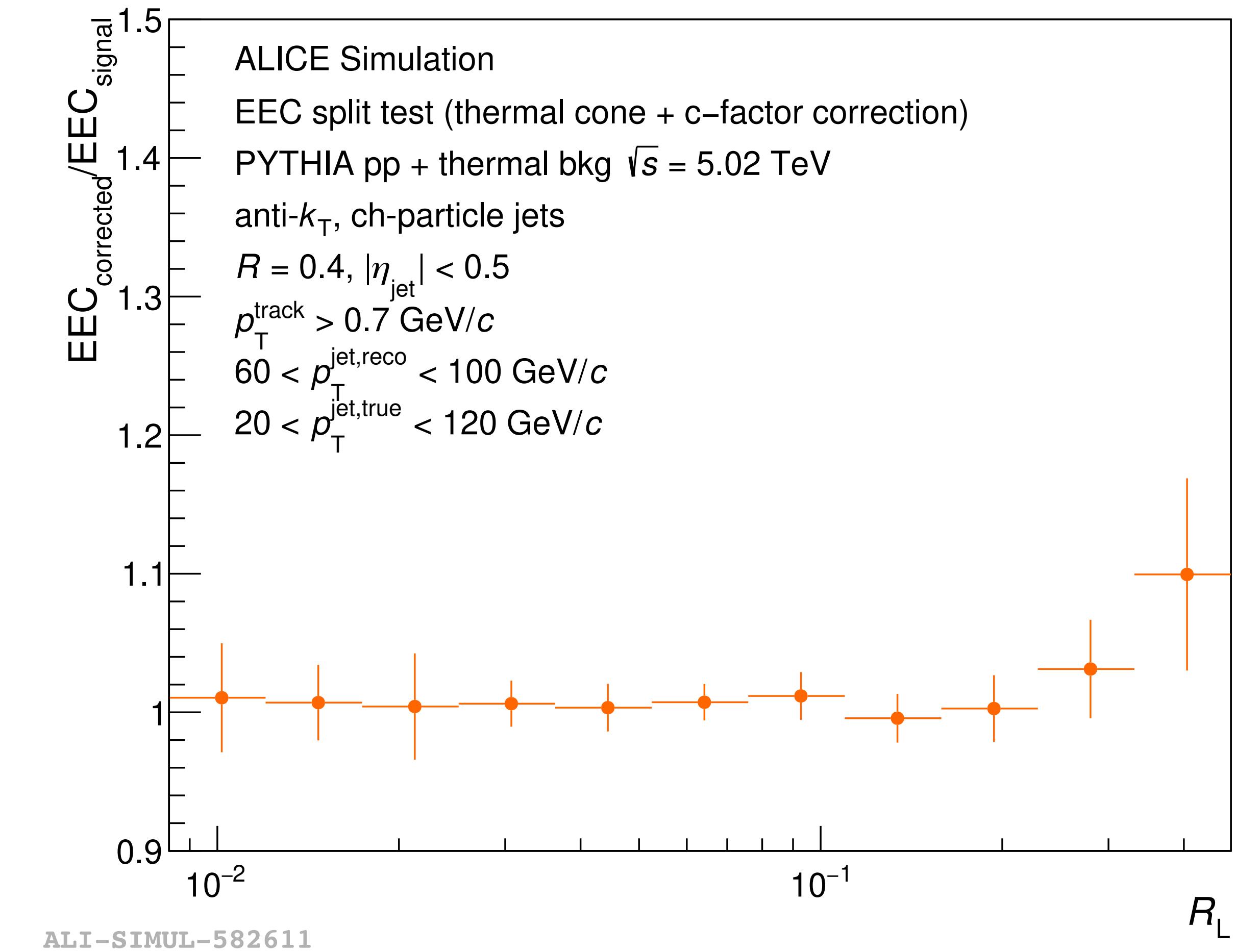
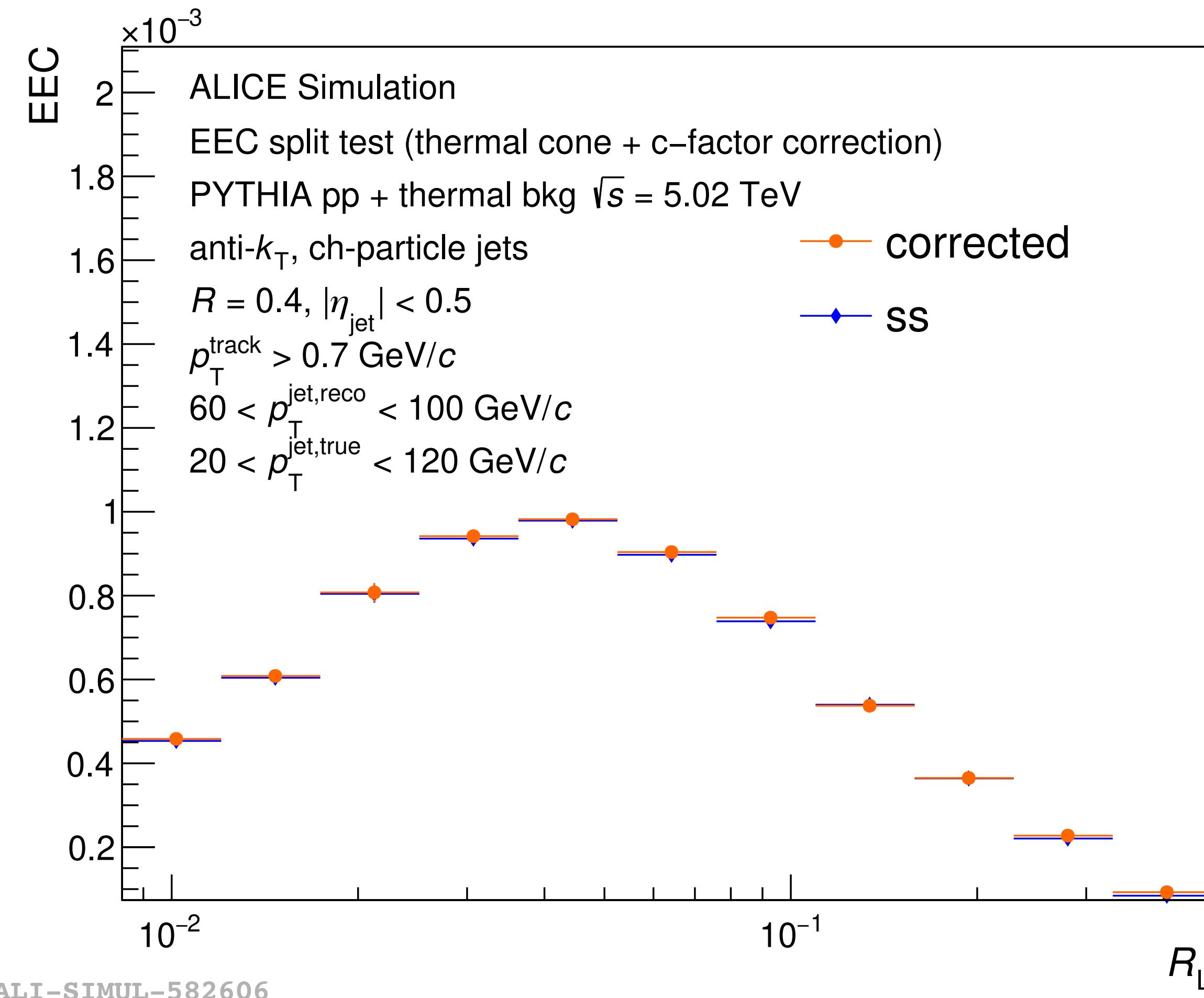
Background Subtraction in Heavy Ion Collisions

Before and after c-factor correction: EEC



Background Subtraction in Heavy Ion Collisions

Before and after c-factor correction: EEC Split Test



Background Subtraction in Heavy Ion Collisions

Jet selection bias

$$B'B''\tilde{B}/BB'B'' = c_1$$

$$(B')^2 B''/B^2 B' = c_2$$

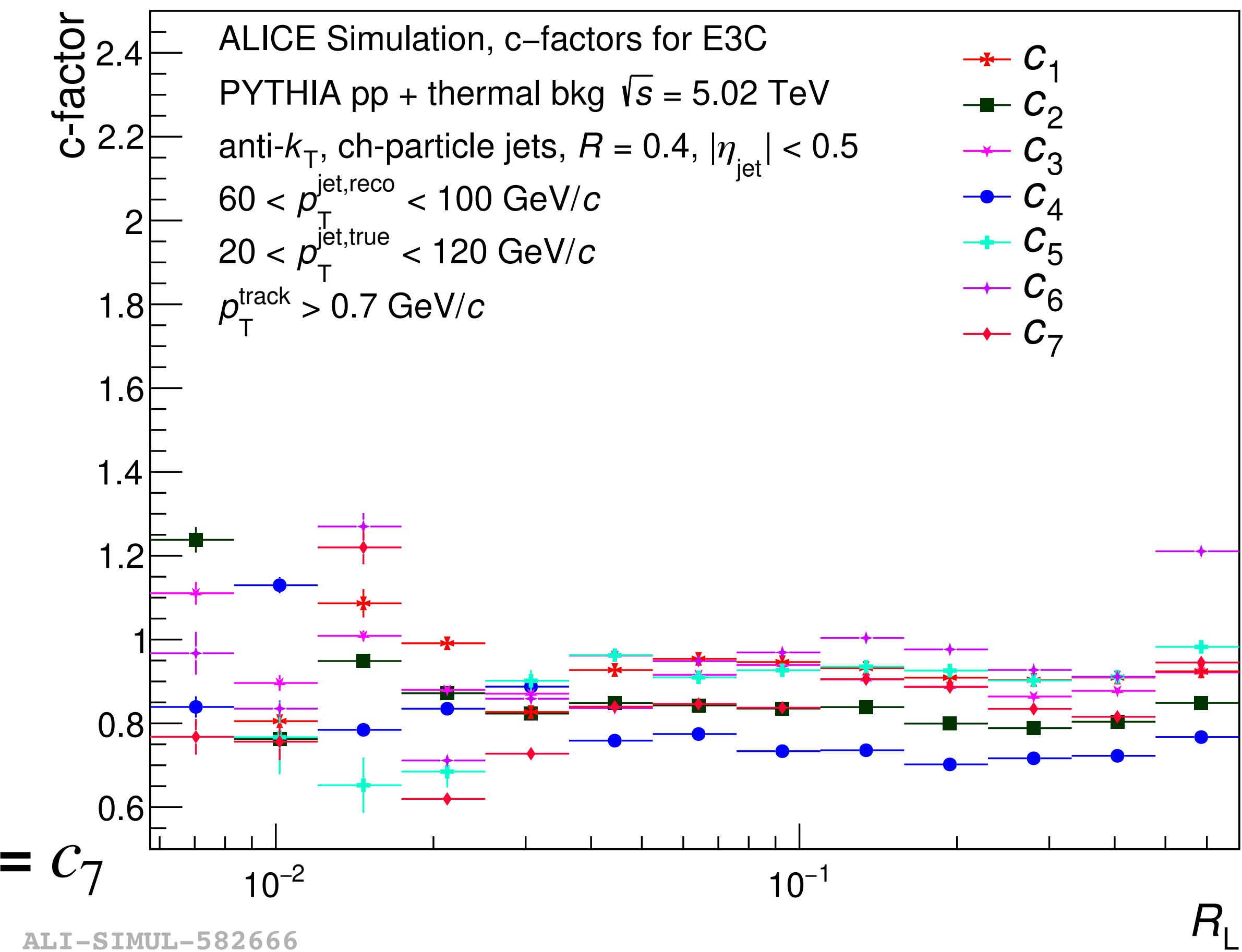
$$B'(B'')^2/B(B')^2 = c_3$$

$$(B')^3/B^3 = c_4$$

$$(6JB'B'' - 6f_1B'B''\tilde{B})/6SBB' = c_5$$

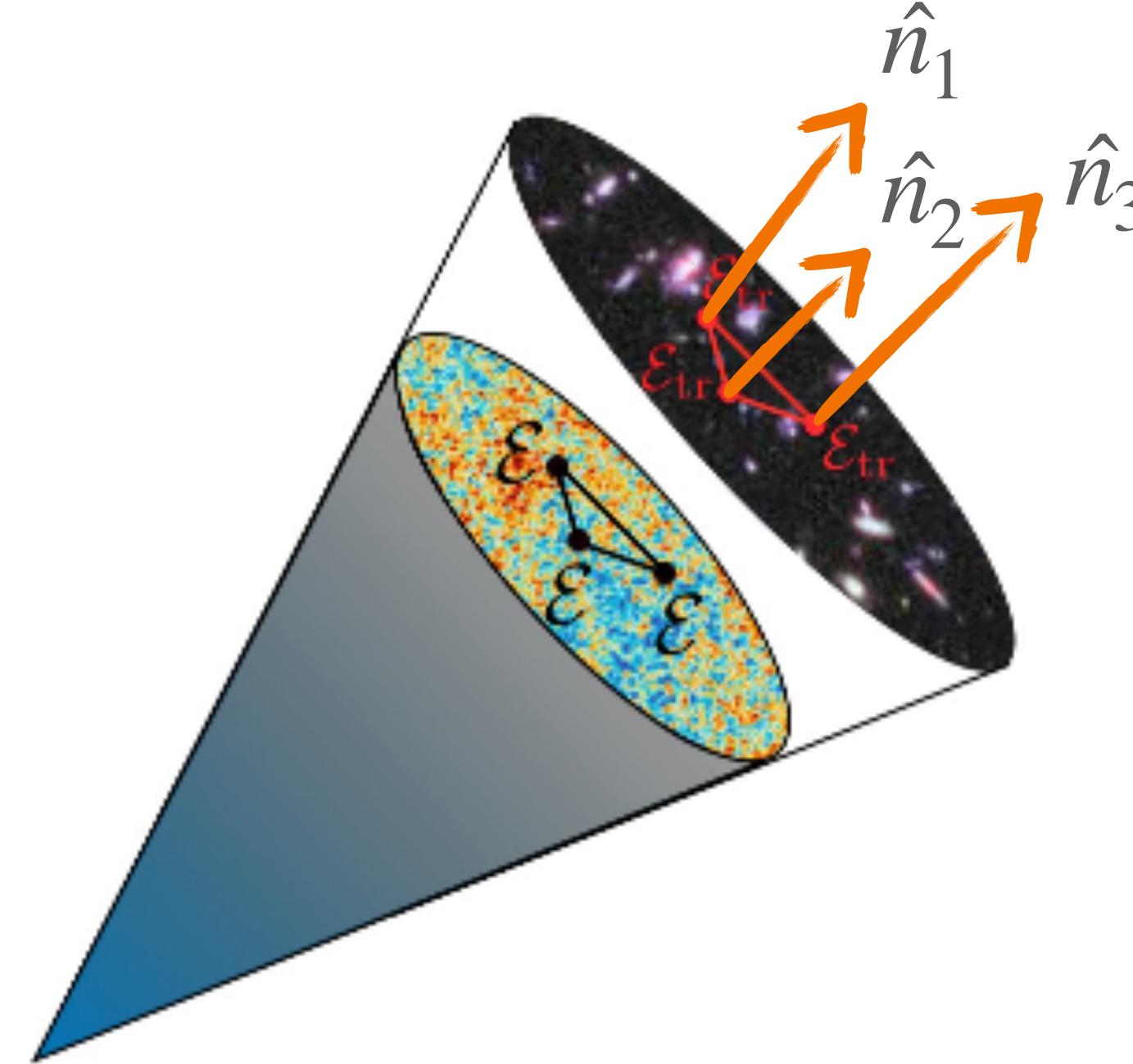
$$(3J(B')^2 - 3f_3B'(B'')^2)/3SB^2 = c_6$$

$$(3J^2B' - 3f_2(B')^2B'' - 6f_5JB'B'' + 6f_1B'B''\tilde{B})/3S^2B = c_7$$



Energy Correlators in Jets

Definition



Number of particles in the correlation

$$\text{ENC}(R_L) = \left(\prod_{k=1}^N \int d\Omega_{\vec{n}_k} \right) \delta(R_L - \Delta \hat{R}_L) \cdot \frac{1}{(E_{\text{jet}})^{(N)}} \langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \dots \mathcal{E}(\vec{n}_N) \rangle$$

Largest distance between N particles

Jet selection bias due to UE fluctuations

Upwards fluctuations pass the cut

