# Jet entropy as a probe of jet collimation

**24th September 2024, HP24**

João Barata, BNL

Based on: 2305.10476, 24xx.xxxx with J.-P. Blaizot, Y. Mehtar-Tani

# Entropy and correlations inside jets

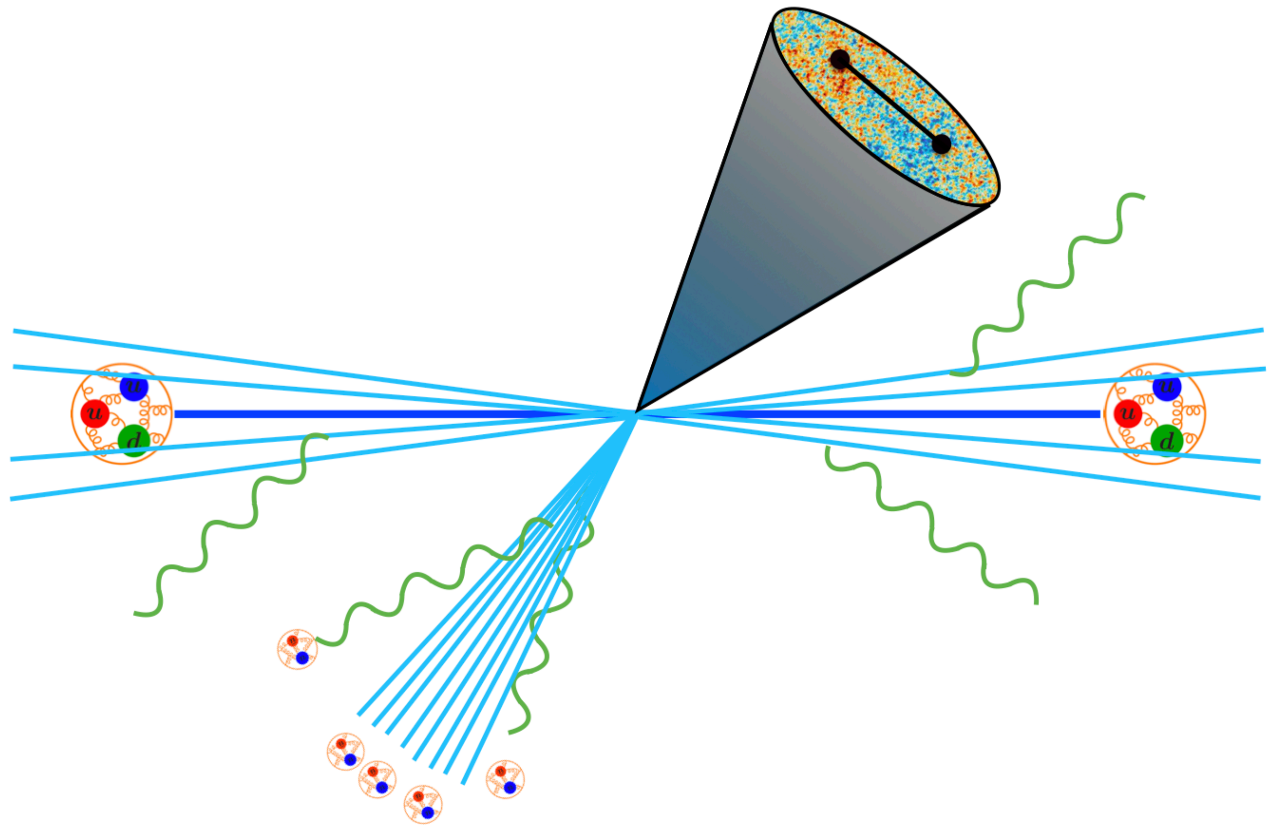
Most jet quenching observables come from projections of the final particle distribution inside jets

$$\frac{d\sigma}{\sigma do} \propto \int_\Omega \frac{dN}{d\Omega}\delta(\hat{o}-o)$$

It would be interesting to have observables which are (directly) sensitive to correlations present in the final state

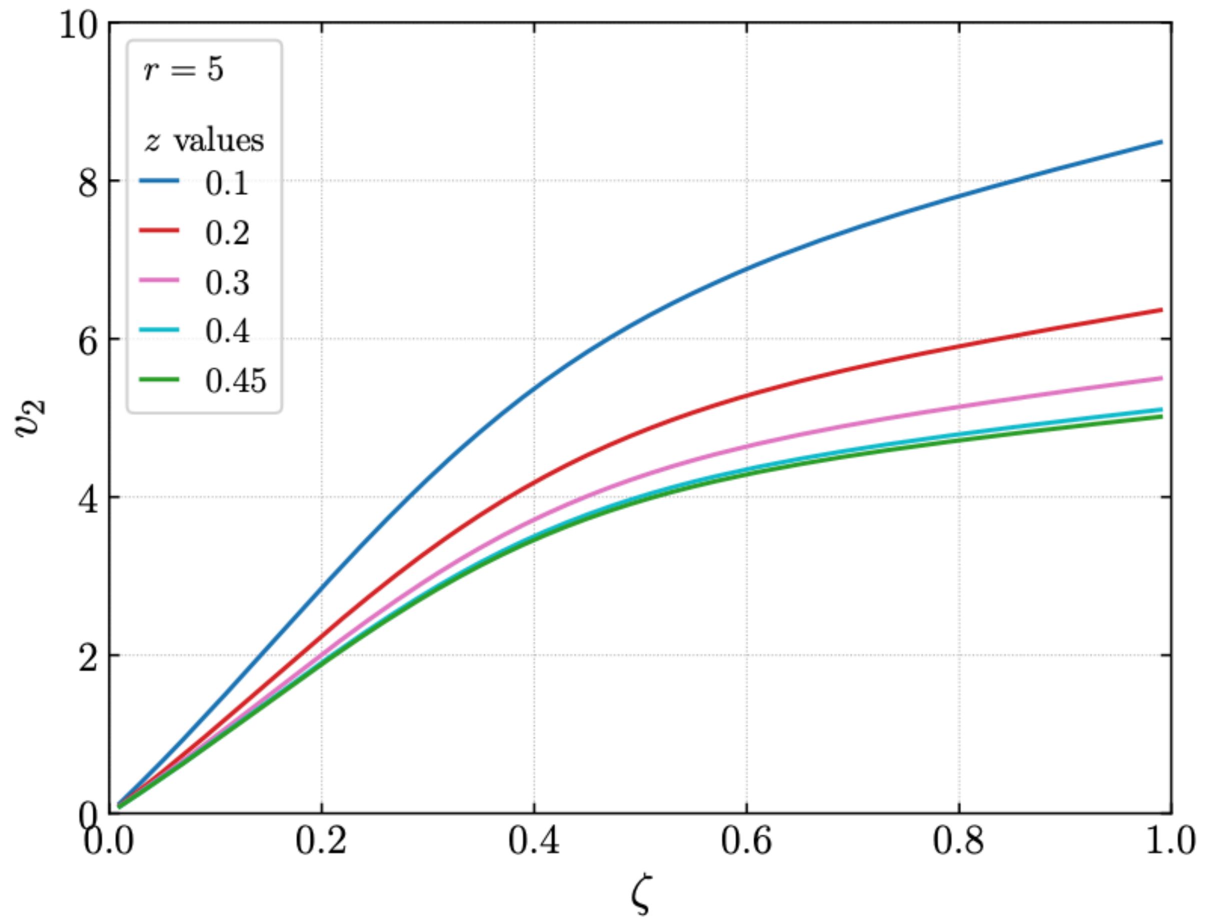
Examples:

[See Tuesday's parallel sessions]



Correlators of Ligh-Ray Operators (inclusive)

[See talk by João Silva]




Fourier harmonics of intra-jet particle distribution

# Entropy and correlations inside jets

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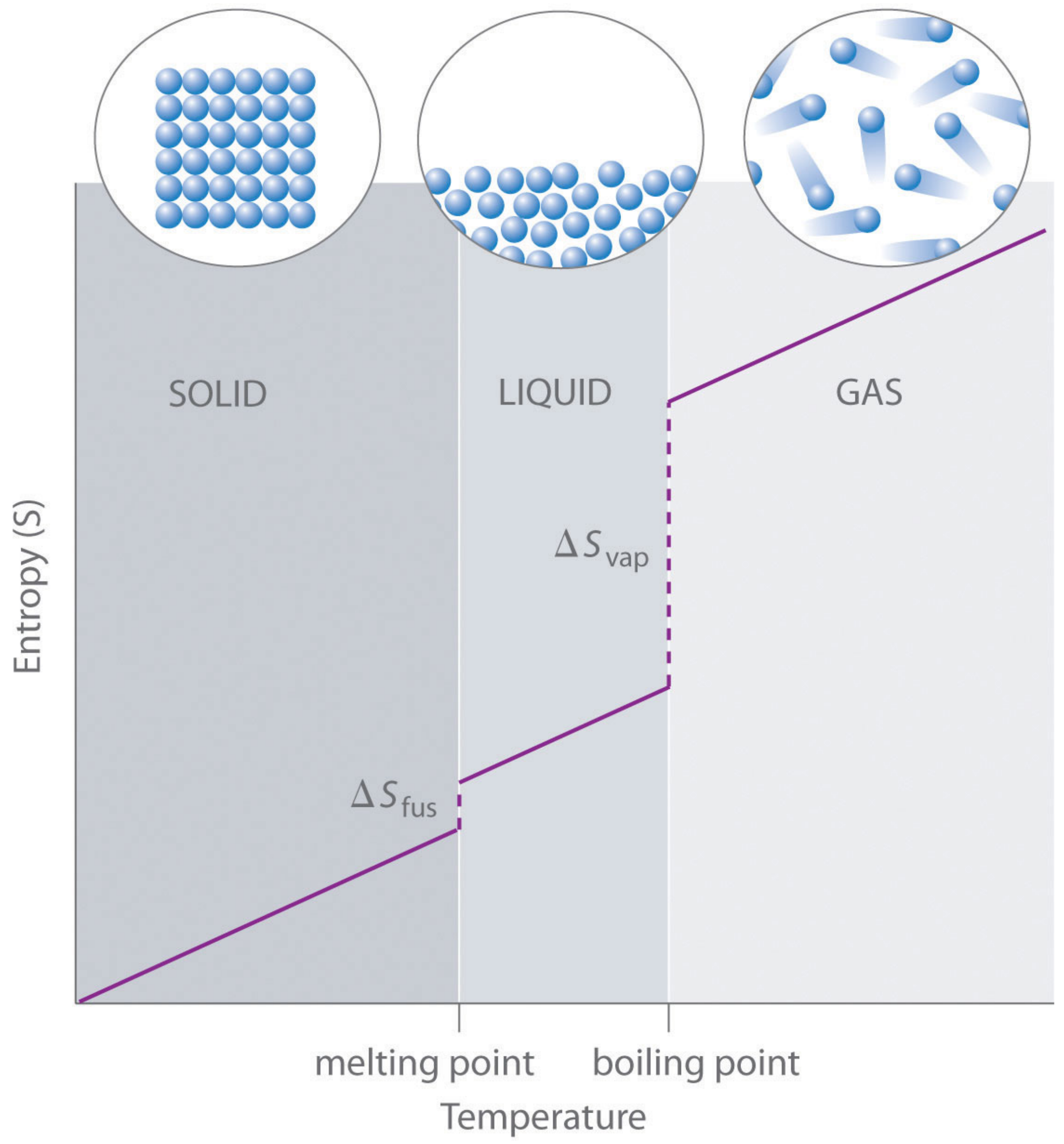
The entropy associated to the particle distribution inside jets is another example of such an observable




- The entropy encapsulates information about multipoint correlators (exclusive distributions)
- Classically, it "measures" the number of available dofs
- For mixed states, as in jets, other non-equivalent measures can be introduced to probe the structure of the state
- Entropy is the canonical entanglement measure

Entanglement

Confinement ⟷ Entanglement ⟷ Proton Structure

[Klebanov, Kutasov, Murugan, 0709.2140]

[Kharzeev, Levin, 1702.03489]

What can we learn from the entropy of a jet?

# Jet entropy in vacuum

# Jet entropy

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One can define the jet entropy from the associated jet density matrix $\rho_{\text{pure}} = |\psi\rangle\langle\psi|$

Creates n particles inside the jet

$$\rho_n\left(\{p_i\}_{i=1}^n, \{p_i'\}_{j=1}^m\right) = \sum_{\{a_i,\lambda_i,f_i\}_{i=1}^n} \sum_{\{a_j',\lambda_j',f_j'\}_{j=1}^m} C^\dagger\left(p_1^{a_1\lambda_1 f_1}, \dots, p_n^{a_n\lambda_n f_n}\right) \times I\left(p_1^{a_1\lambda_1 f_1}, \dots, p_n^{a_n\lambda_n f_n}; p_1'^{\,a_1'\lambda_1' f_1'}, \dots, p_m'^{\,a_m'\lambda_m' f_m'}\right) \times C\left(p_1'^{\,a_1'\lambda_1' f_1'}, \dots, p_m'^{\,a_m'\lambda_m' f_m'}\right) + \dots$$

Feynman-Vernon functional incorporating the interactions between the jet constituents and everything else

Not IRC safe quantity

However, it is natural to consider the entropy associated to the hardest (collinear) partons inside the jet

$$\rho_n\left(\{p_i\}_{i=1}^n, \{p_i'\}_{j=1}^m\right) = \sum_{\{a_i,\lambda_i,f_i\}_{i=1}^n} \sum_{\{a_j',\lambda_j',f_j'\}_{j=1}^m} C_H^\dagger\left(p_1^{a_1\lambda_1 f_1}, \dots, p_n^{a_n\lambda_n f_n}\right) \times I\left(p_1^{a_1\lambda_1 f_1}, \dots, p_n^{a_n\lambda_n f_n}; p_1'^{\,a_1'\lambda_1' f_1'}, \dots, p_m'^{\,a_m'\lambda_m' f_m'}\right) \times C_H\left(p_1'^{\,a_1'\lambda_1' f_1'}, \dots, p_m'^{\,a_m'\lambda_m' f_m'}\right) + \dots$$

$$\rho = \sum_{n=1}^{\infty} \rho_n\,, \quad \rho_n = \int dP_n |p_1 \cdots p_n\rangle\langle p_1 \cdots p_n|$$

$$I\left(p_1^{a_1\lambda_1 f_1}, \dots, p_n^{a_n\lambda_n f_n}; p_1'^{\,a_1'\lambda_1' f_1'}, \dots, p_m'^{\,a_m'\lambda_m' f_m'}\right) = 0$$

unless $n = m, p_i = p_i'$ and $a_i = a_i'$ for all $i$,

[Breuer, Petruccione; Nagy, Soper; Neill, Waalewijn]

# Jet entropy

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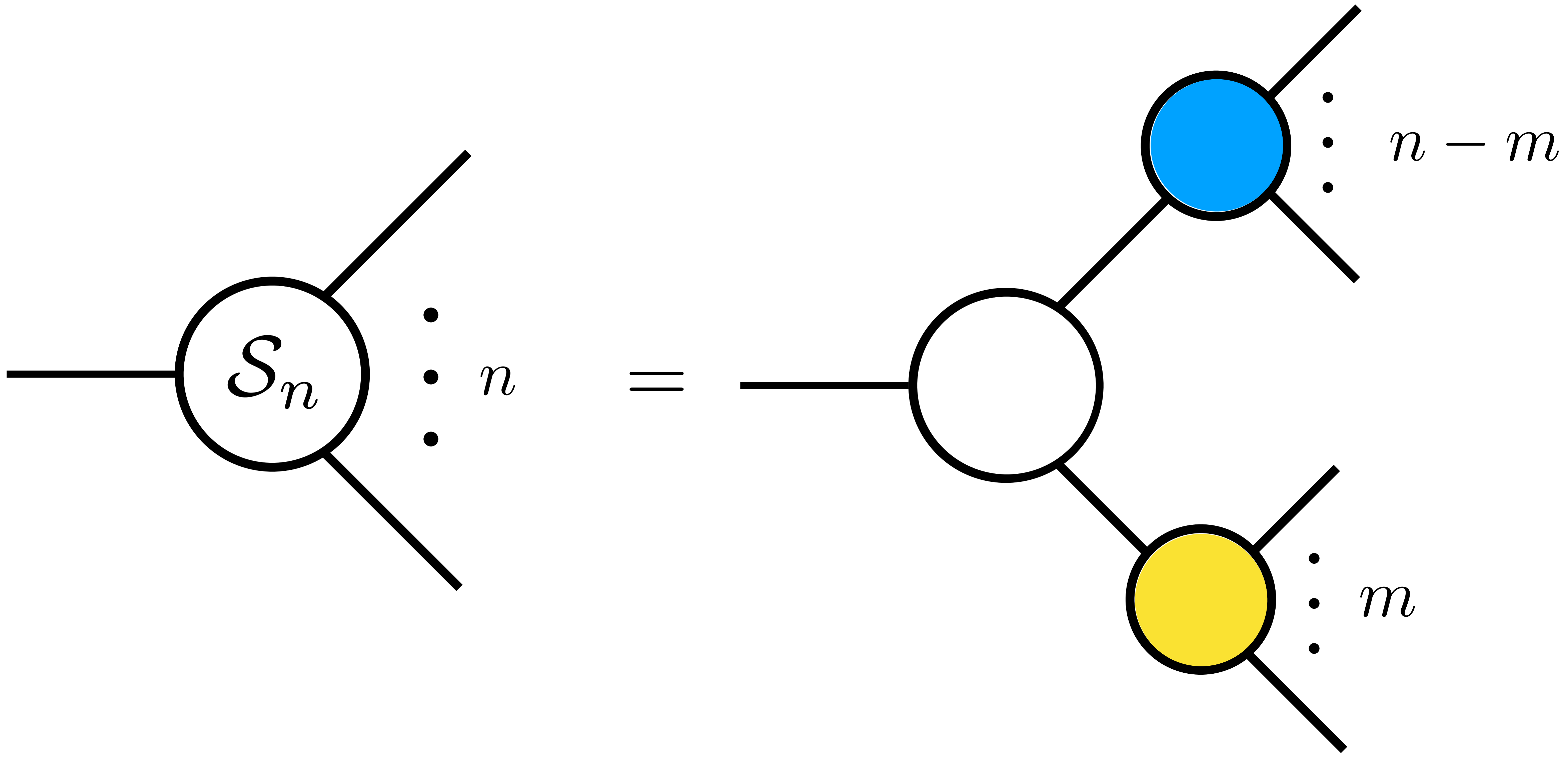
One way to define the hardest constituents is to work only with subjets which have energy fractions and angles larger than cutoffs $z_c, R_c$ , then one can write

$$\mathcal{S} = -\mathrm{Tr}\rho \log \rho$$

$$\mathcal{S} = -\sum_n \int d\Pi_n \frac{dP}{d\Pi_n} \log \frac{dP}{d\Pi_n} = \sum_n \mathcal{S}_n$$

At leading logarithmic accuracy and in a physical gauge, we can compute the entropy recursively




$$dP_n(p_t, R) = \frac{\alpha_s C_A}{2\pi} \frac{P(z)}{\theta} e^{-\Delta(R,\theta)} dP_{n-m}(zp_t, \theta) dP_m((1-z)p_t, \theta)$$

# Jet entropy

[Neill, Waalewijn, 1811.01021]

Using this approximation, the entropy can be directly computed LL accuracy and is satisfies the implicit equation

$$\mathcal{S} = -\log\left(e^{-\Delta(R,R_c)}\right) e^{-\Delta(R,R_c)} \longrightarrow \text{Entropy associated to not branching}$$

"Linear" evolution due to cascading individually on each leg (multiplicity)

$$+\int_{z,\theta} \bar{\alpha} e^{-\Delta(R,\theta)} \{\mathcal{S}(zE) + \mathcal{S}((1-z)E)\}$$

"Non-linear" piece coming from the logarithm

$$+\int_{z,\theta} \bar{\alpha} e^{-\Delta(R,\theta)} \left(\Delta(R,\theta) - \log\frac{4\pi^2}{z^2(1-z)^2\theta^2E^2}\bar{\alpha}\Lambda^2\right)$$

At DL accuracy and for YMs theory, a closed form solution can be found:

$$\mathcal{S}(x) = (I_0(x) - 1) + 2\log\frac{ER}{E_cR_c}\left(\frac{2}{x}I_1(x) - 1\right) \qquad x = 2\sqrt{\frac{2\alpha_s N_c}{\pi}\log\frac{E}{E_c}\log\frac{R}{R_c}}$$

Intra-jet multiplicity

# Entropy evolution in the medium

# Single parton evolution in the medium

2305.10476, JB, Blaizot, Mehtar-Tani

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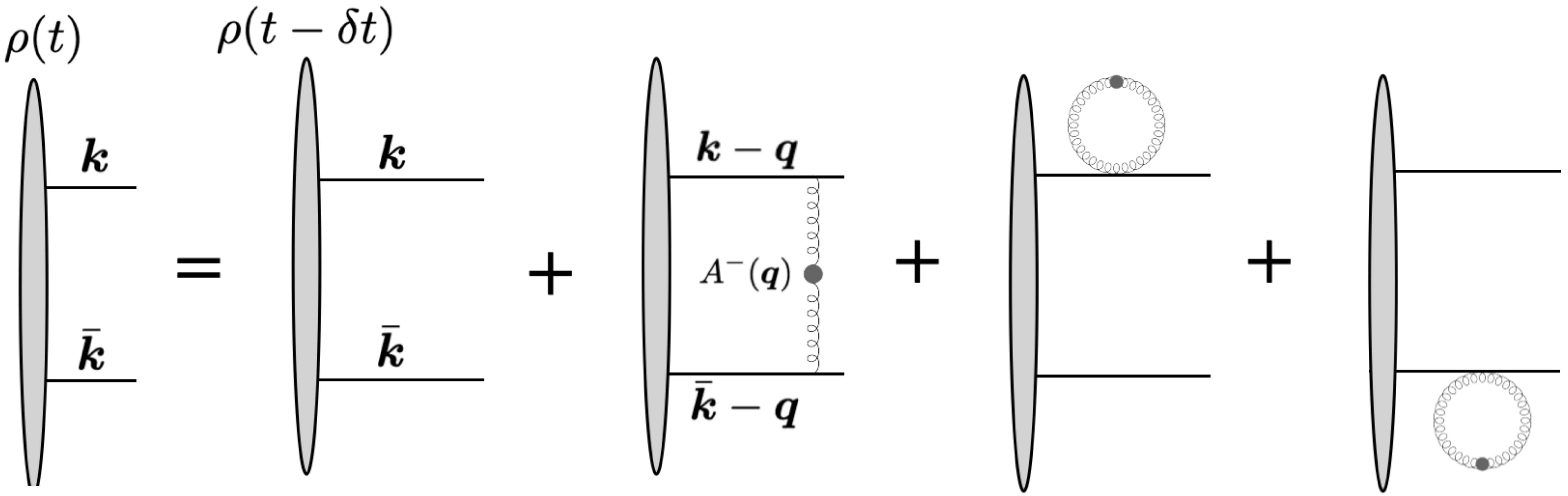
For a single parton, the density matrix satisfies a simple evolution equation

$$\rho \equiv \mathrm{tr}_A(\rho[A]) = \Big\langle |\psi_A(t)\rangle\langle\psi_A(t)| \Big\rangle_A$$

$$\rho = |\boldsymbol{k}\rangle\langle\bar{\boldsymbol{k}}|$$

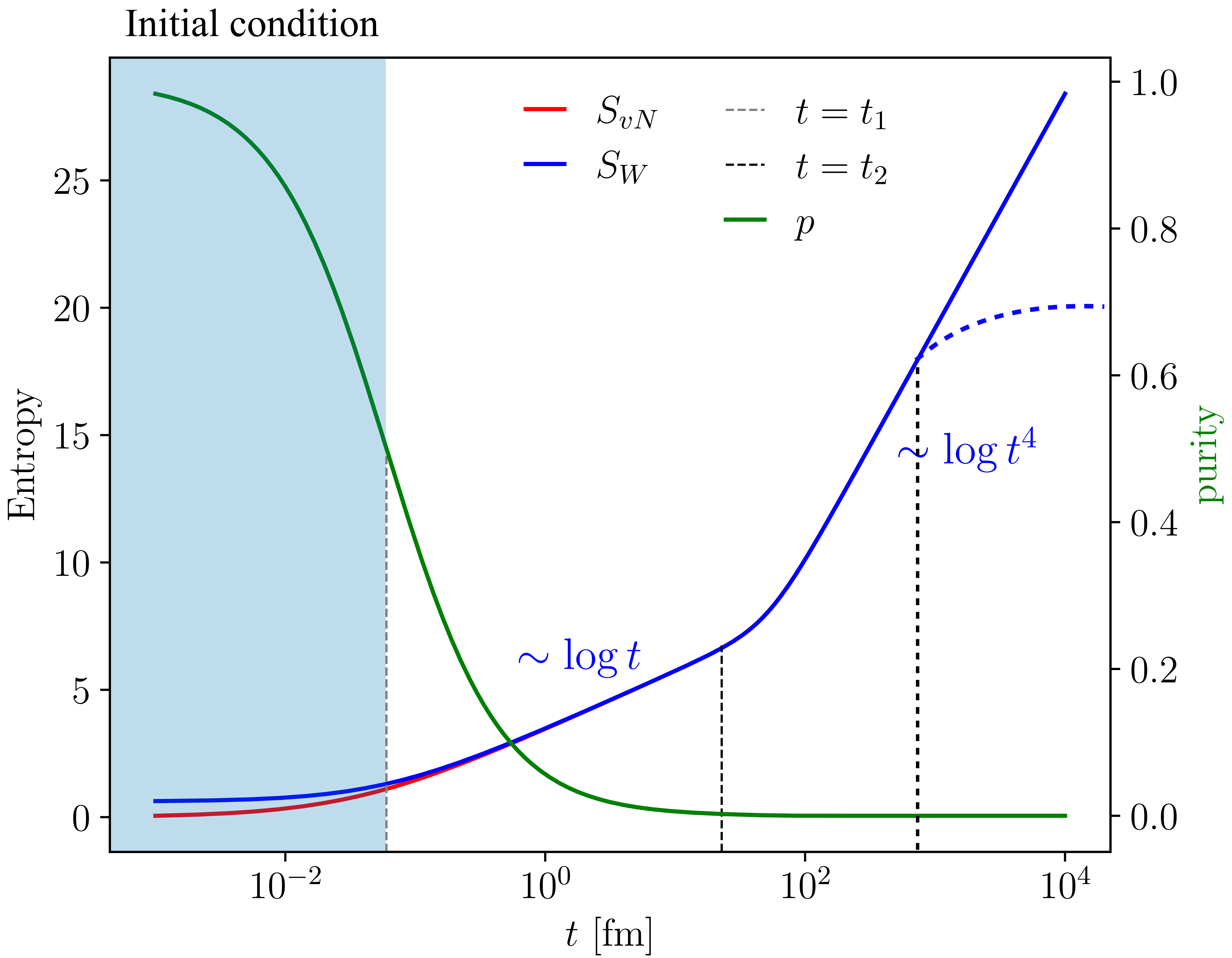



$$\langle \boldsymbol{k}|\rho_{\mathrm{s}}(t)|\bar{\boldsymbol{k}}\rangle = C_F \int_{\boldsymbol{q}} \int_0^t dt'\, e^{i\frac{(\boldsymbol{k}^2-\bar{\boldsymbol{k}}^2)}{2E}(t-t')} \times \gamma(\boldsymbol{q}) \left[\langle \boldsymbol{k}-\boldsymbol{q}|\rho_{\mathrm{s}}(t')|\bar{\boldsymbol{k}}-\boldsymbol{q}\rangle - \langle \boldsymbol{k}|\rho_{\mathrm{s}}(t')|\bar{\boldsymbol{k}}\rangle\right]$$

$$\boldsymbol{K} = \frac{\boldsymbol{k}+\bar{\boldsymbol{k}}}{2}, \quad \boldsymbol{\ell} = \boldsymbol{k}-\bar{\boldsymbol{k}} \qquad\qquad \boldsymbol{b} \equiv \frac{\boldsymbol{r}+\bar{\boldsymbol{r}}}{2}, \quad \boldsymbol{x} \equiv \boldsymbol{r}-\bar{\boldsymbol{r}}$$

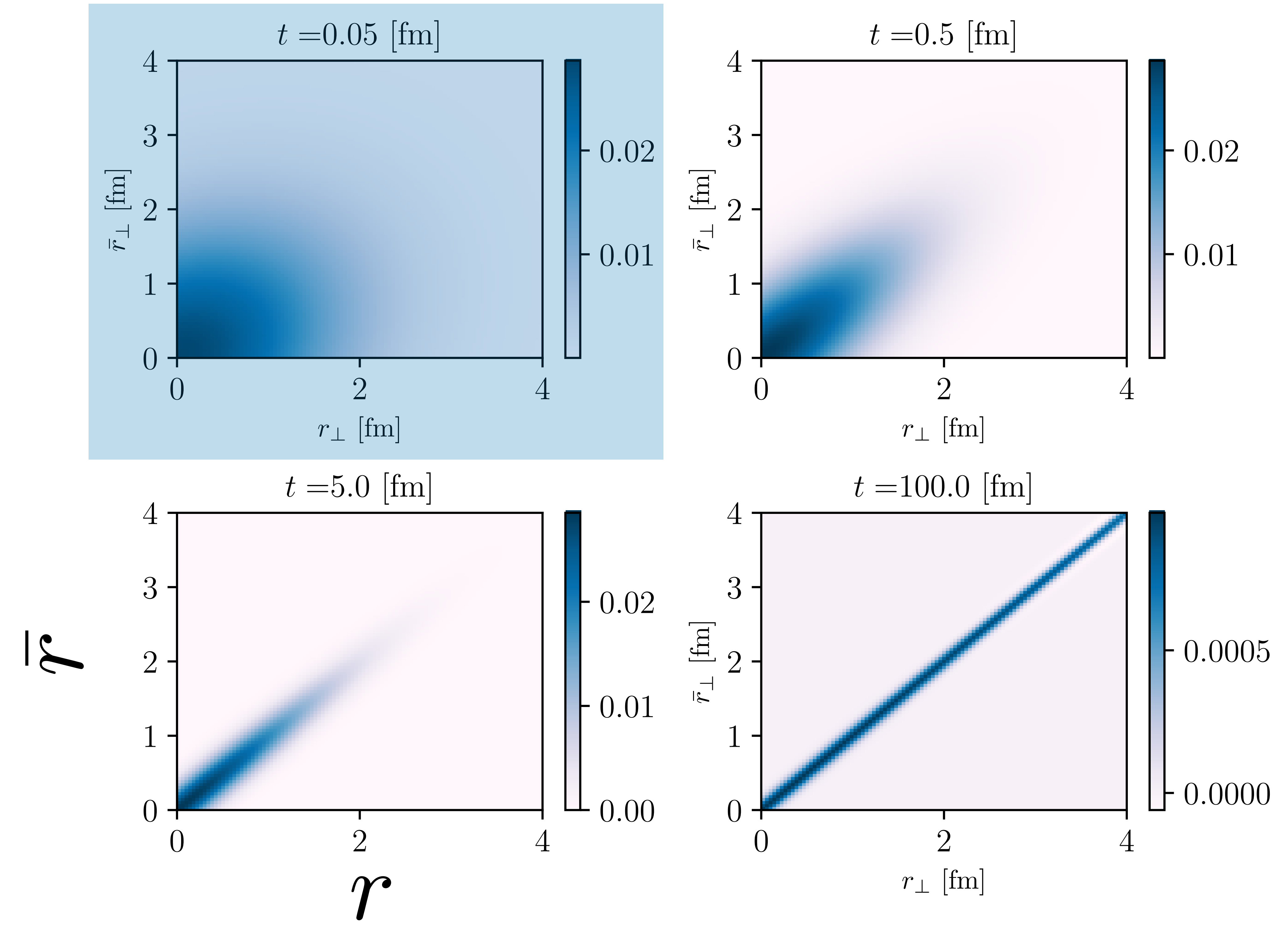
# Single parton evolution in the medium

2305.10476, JB, Blaizot, Mehtar-Tani




Asymptotically, one has that

$$S \sim \log\langle \boldsymbol{k}^2\rangle\langle \boldsymbol{r}^2\rangle \sim \log \text{Phase Space}$$



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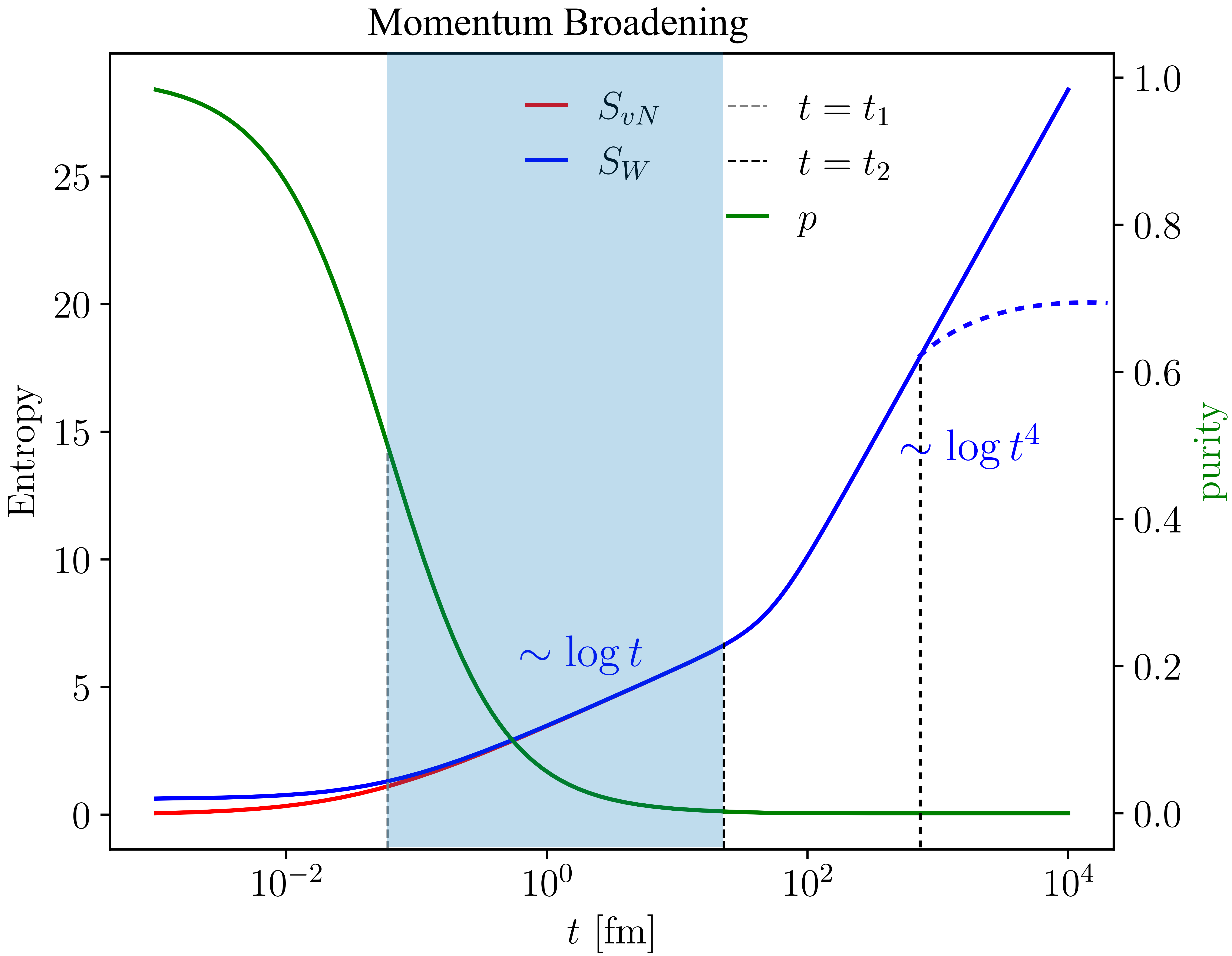


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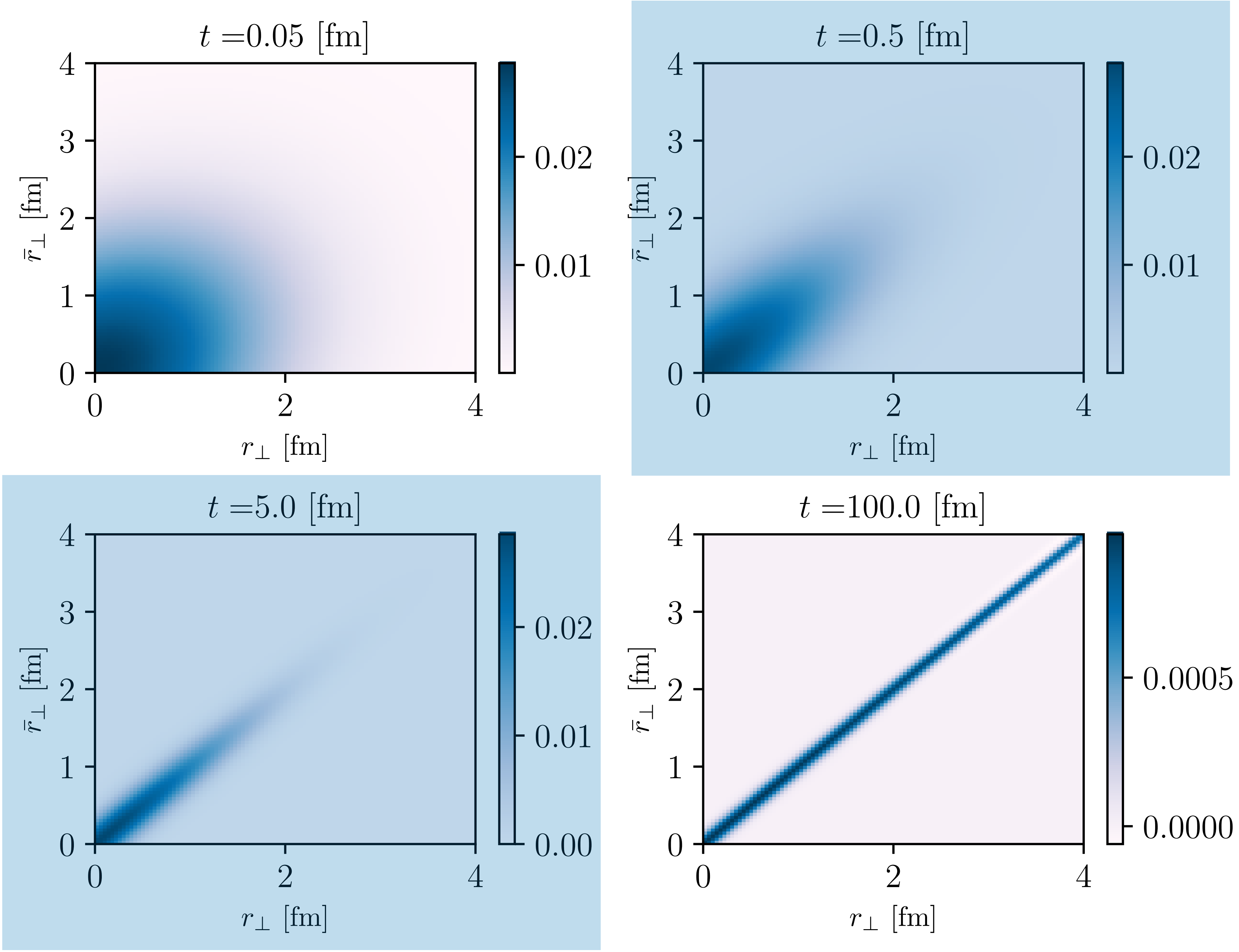

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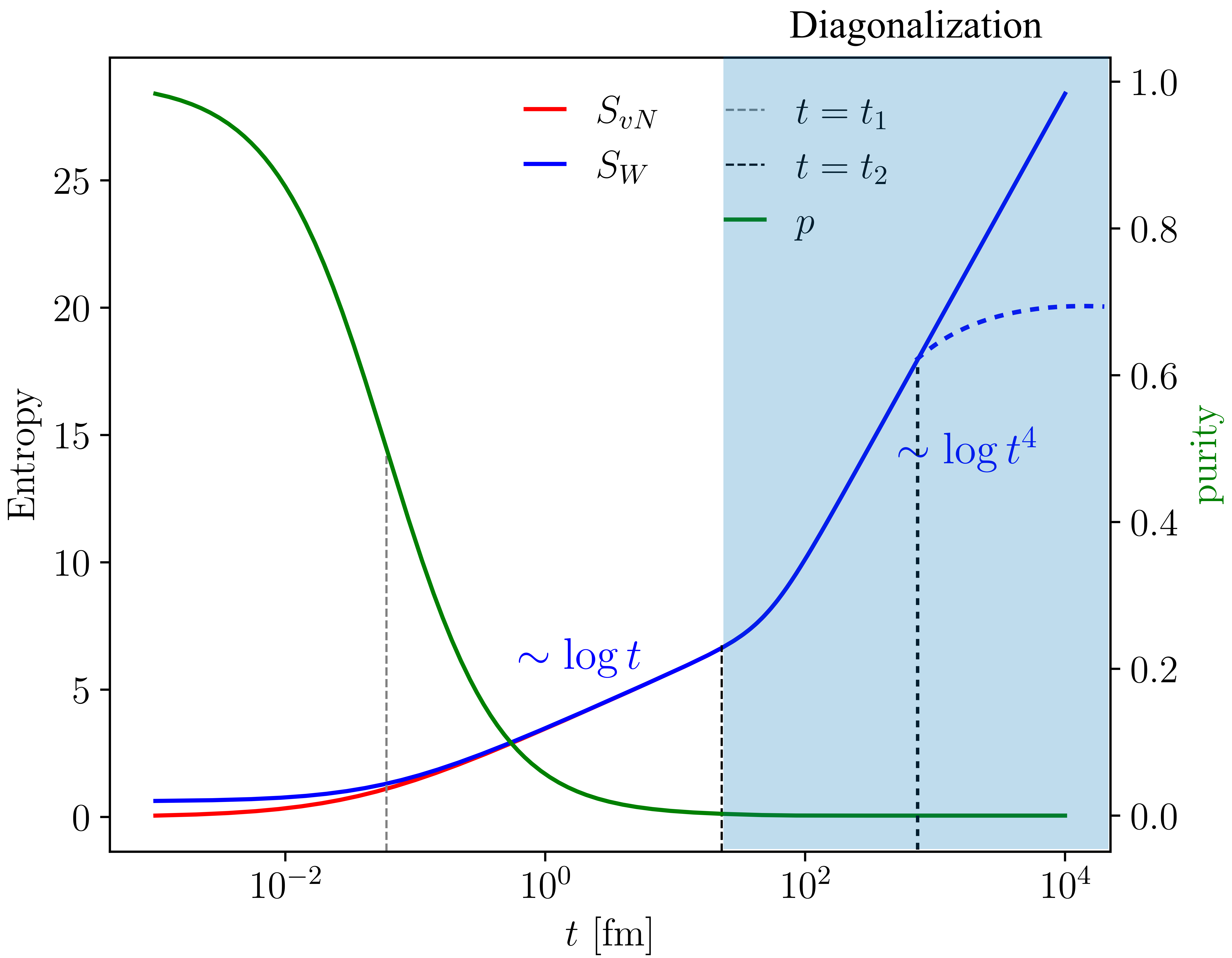
2305.10476, JB, Blaizot, Mehtar-Tani

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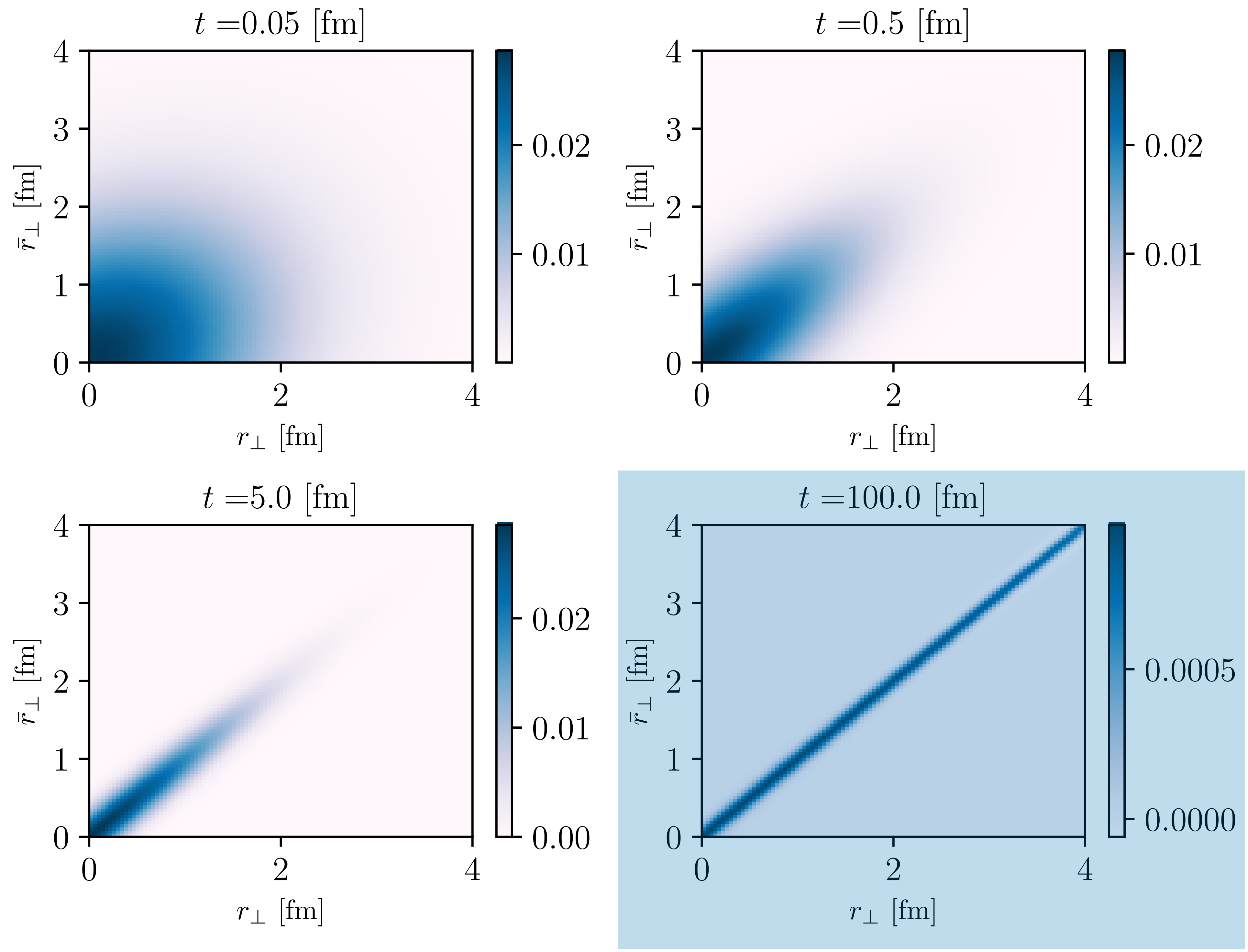

# Entropy beyond the single parton limit

2307.01792, JB, Du, Li, Qian, Salgado

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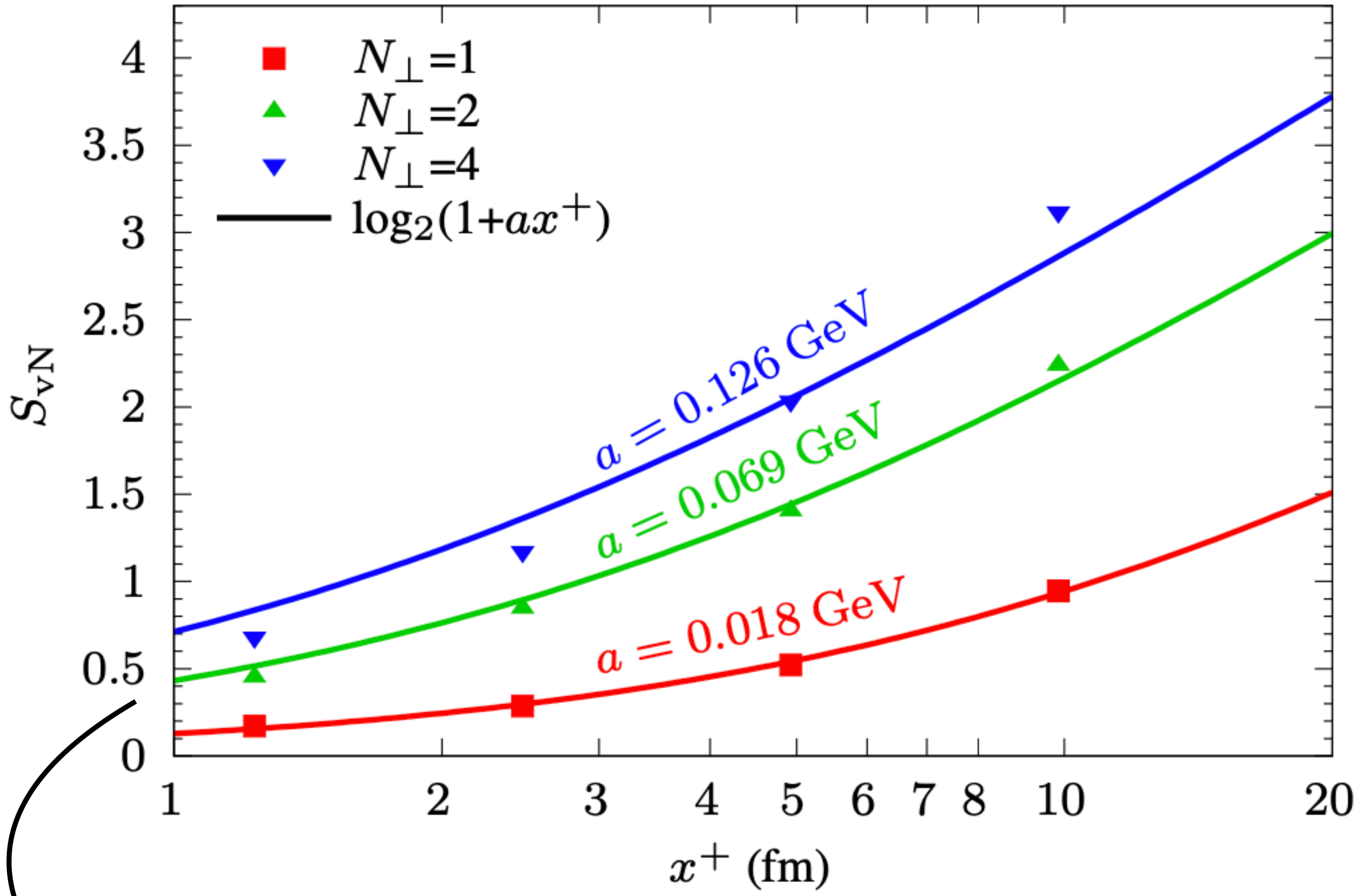
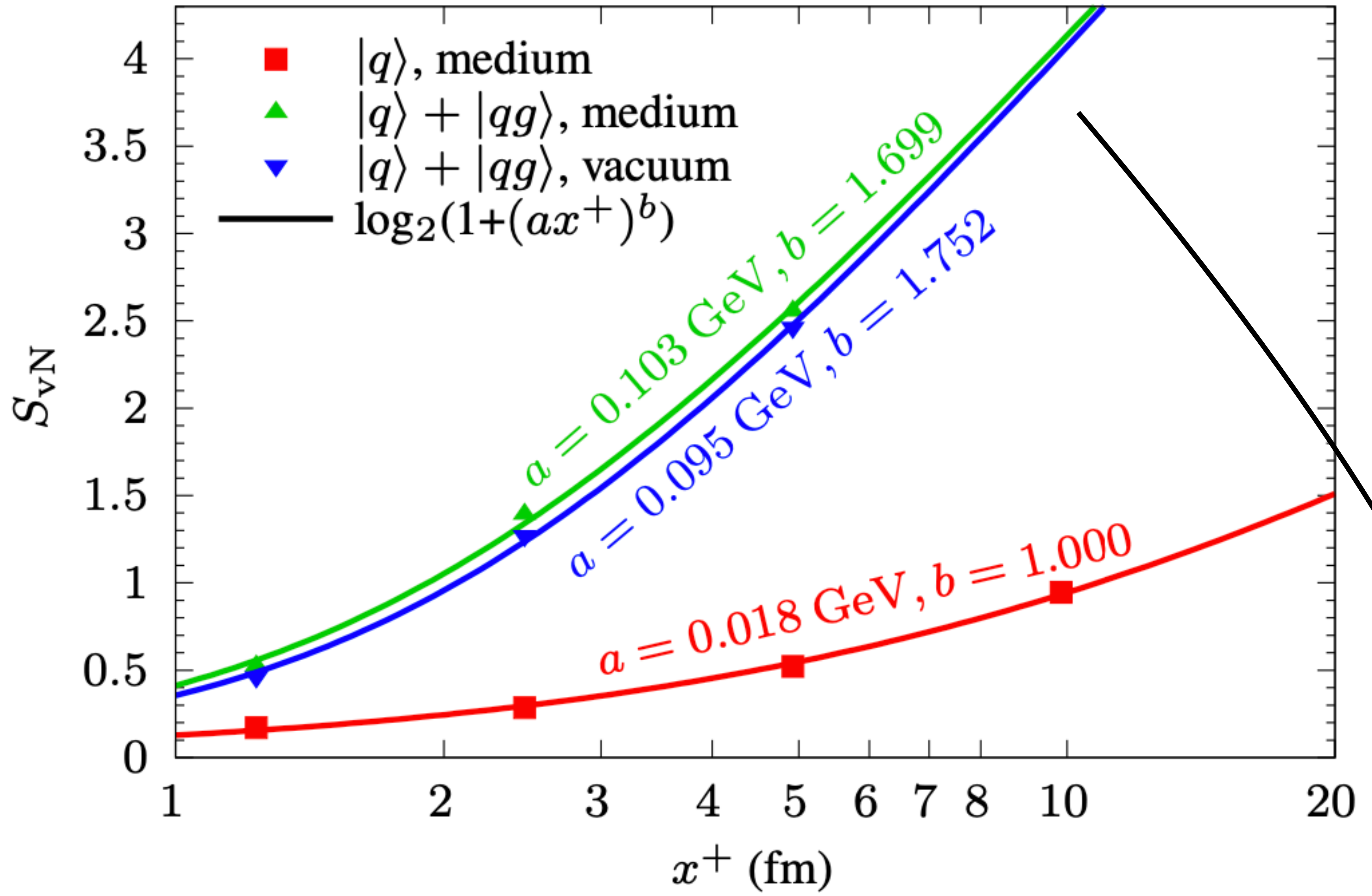
Only one Quark




Quark and Gluon



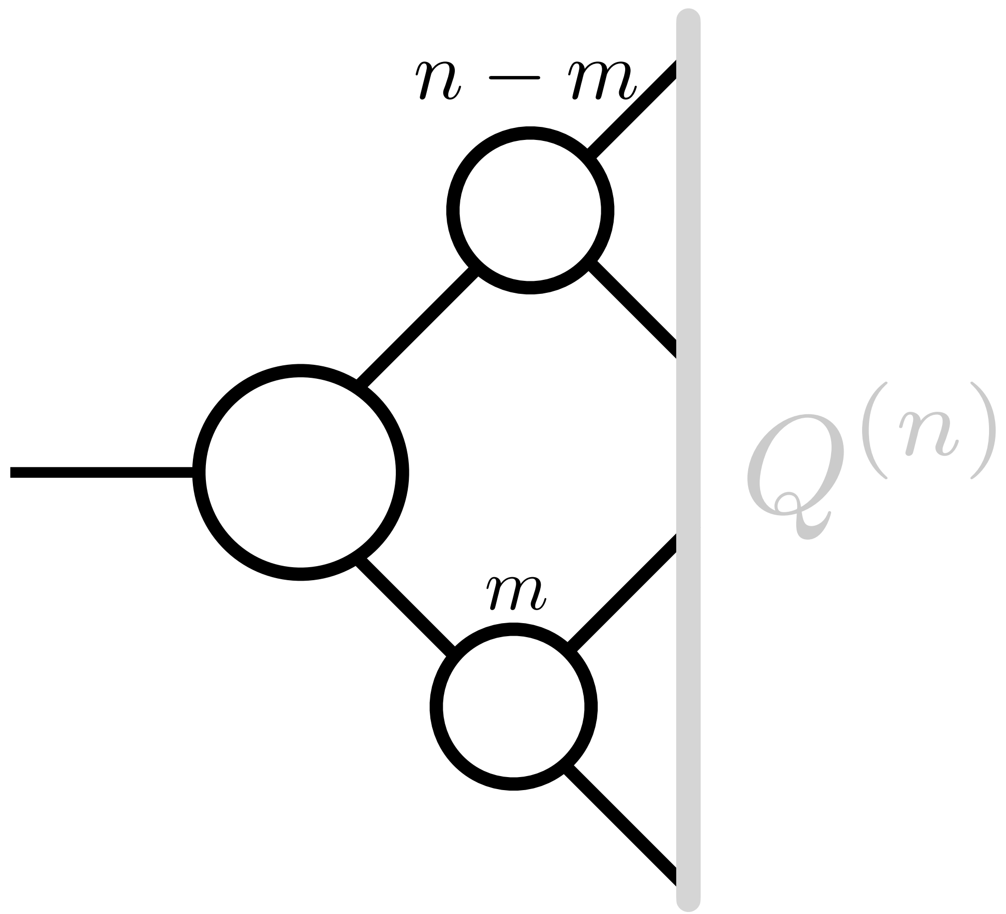

[For details see talks by Wenyang Qian (plenary) and Carlos Lamas]

- Entropy growth determined by momentum broadening, with evolution parameter agreeing with analytic estimate
- Entropy evolution is dominated by gluon radiation compared to medium effects or momentum broadening

# Jet entropy in the QGP from pQCD

In matter, the jet entropy will be modified by different types of effects

**Energy loss**




**Phase space modifications**

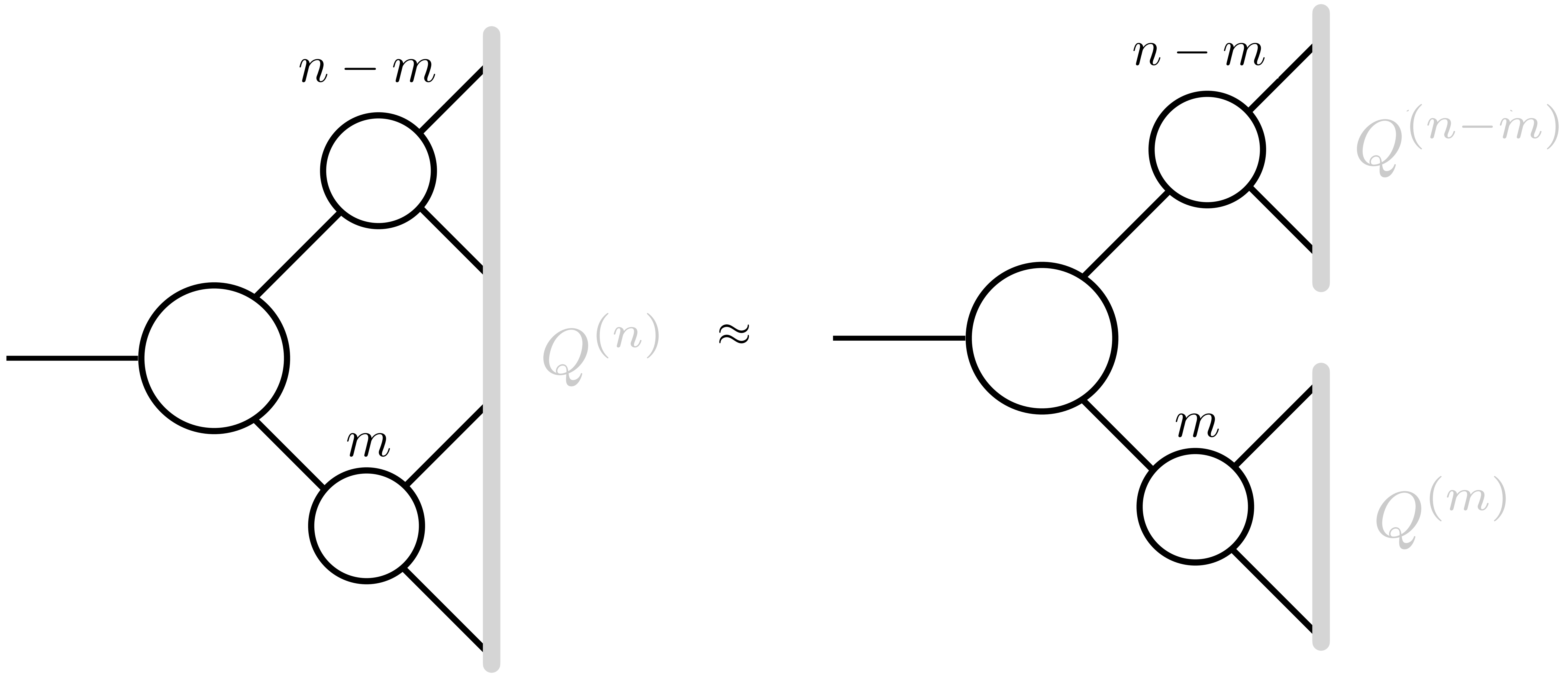
Can be incorporated directly;
current description insufficient for modeling

**Induced radiation**

Can not be resumed using jet calculus rules;
Power corrections to vacuum result (sub-leading)

# Jet entropy in the QGP from pQCD

**Energy loss:** we consider a simple picture based on the quenching weights approximation, where energy is lost incoherently



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# Jet entropy in the QGP from pQCD

It follows that the entropy takes now the form

$$\mathcal{S}_Q = \Delta(R, R_c)e^{-\Delta(R,R_c)}$$ → Entropy associated to not branching [no change]

"Linear" on energy of the opposing branch

$$+ \int_{z,\theta} \bar{\alpha} e^{-\Delta(R,\theta)} \left( \langle Q(zE) \rangle \mathcal{S}_Q(zE) + \langle Q((1-z)E) \rangle \mathcal{S}_Q((1-z)E) \right)$$

"Non-linear" dependence on energy loss of both legs

$$+ \int_{z,\theta} \bar{\alpha} e^{-\Delta(R,\theta)} \langle Q(zE) \rangle \langle Q((1-z)E) \rangle \left[ \Delta(R,\theta) - \log \frac{4\pi^2 \bar{\alpha}}{z^2 \theta^2 E^2} \Lambda^2 \right]$$

where we introduced the average quenching weight factor $\langle Q(E) \rangle = \sum_n \int_{\Pi_n} Q^{(n)} dP_n$

- In the opposite limit of fully coherent energy loss (i.e. jet = 1 charge), one directly finds $S = S_Q$

# Jet entropy in the QGP from pQCD

24xx.xxxxx, JB, Blaizot, Mehtar-Tani

How does the jet entropy evolve in the medium and why?

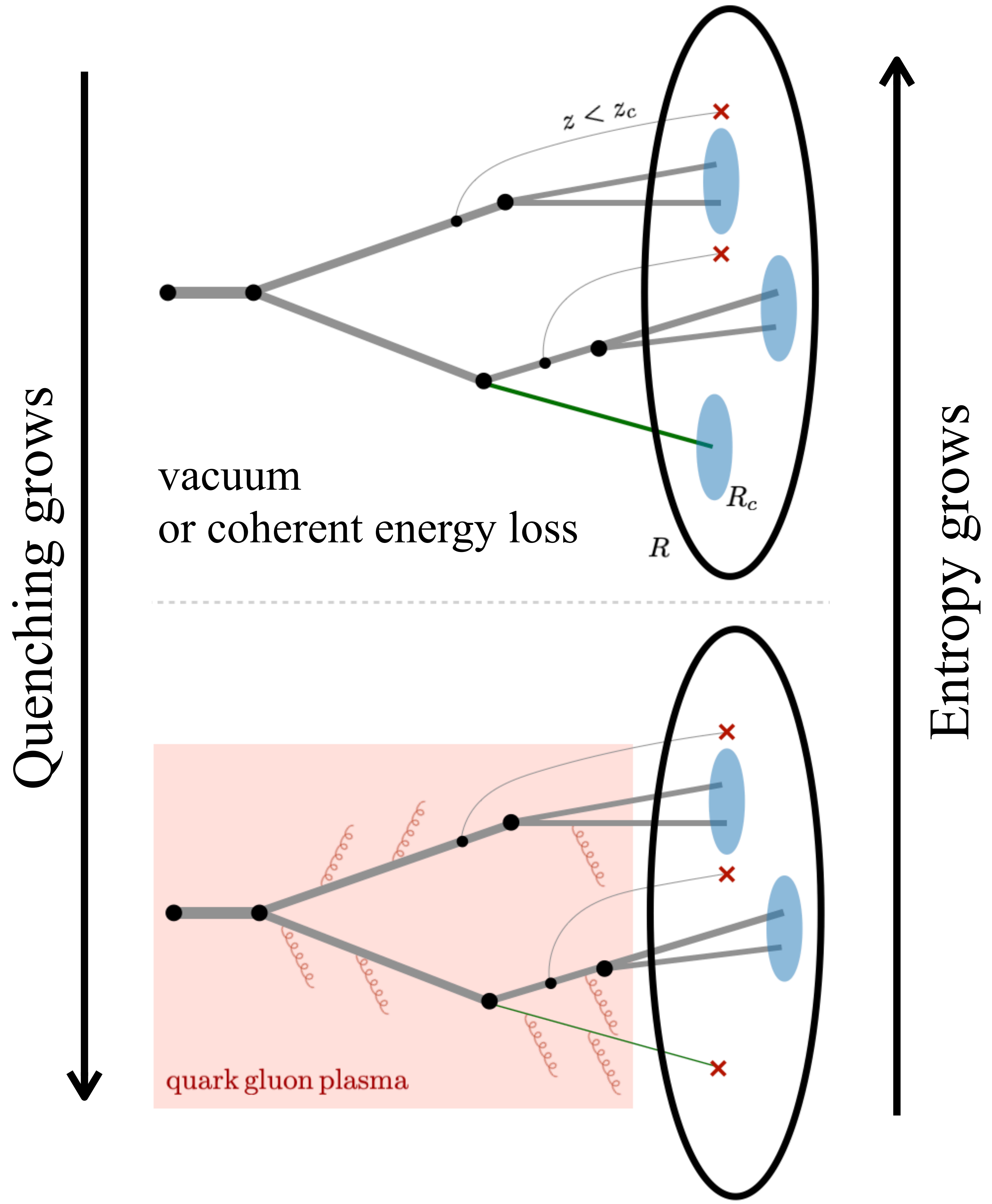
Natural competition effect between two “mechanisms”:

- More branchings lead to entropy increase due to larger phase space as in vacuum

- More branchings lead to larger number of sources which increase the quenching and reduce the entropy

Asymptotically in the evolution variable, the running can be absorbed in a redefinition of the coupling

$$S(E) = I_0\left(2\sqrt{\bar{\alpha}\log\frac{R}{R_c}\int_{z_c}^{1}\frac{\mathrm{d}z}{z}Q(z)}\right)$$




**This is a manifestation of the filtering effect of the plasma, which results in jet collimation**

# Jet entropy in the QGP from pQCD

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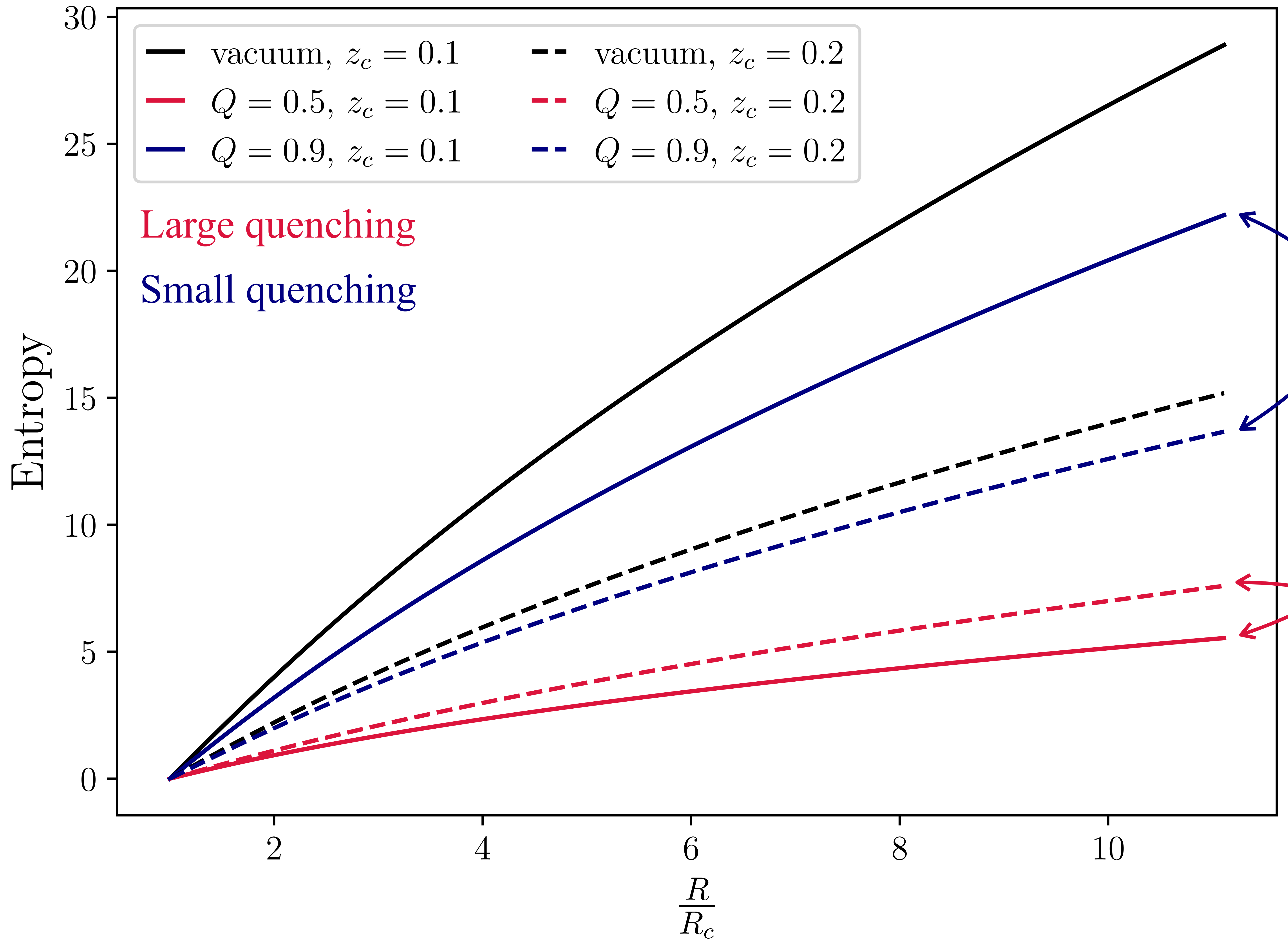
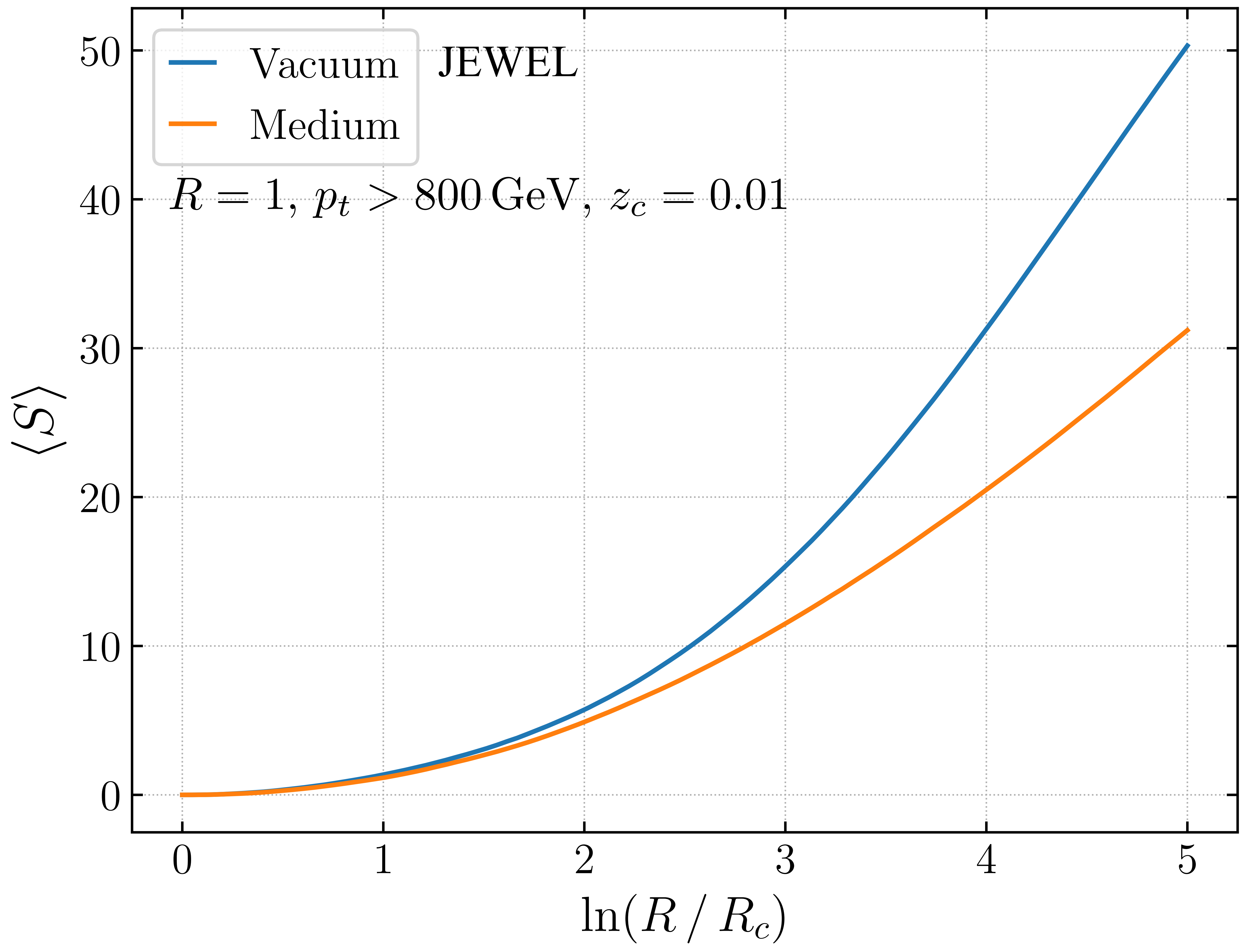



At small quenching
more sources = larger entropy

At large quenching
more sources = smaller entropy

# Jet entropy in the QGP from pQCD

[prelim., JB, J.G. Milhano, M. Ploskon, **João M. Silva**]



# Conclusion

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- Jet entropy evolution in the medium allows to explore the number of effective hard sources in the jet and the interplay with energy loss.

- Since jets are naturally associated with mixed states, one can generalized this discussion to other entanglement measures: mutual information, negativity, …

- This jet entropy corresponds to the entanglement entropy between the hard modes and the soft and medium modes. Other relative entropies can be computed within just the hard sector !