Medium-induced coherent gluon radiation and heavy flavor suppression in pA collisions

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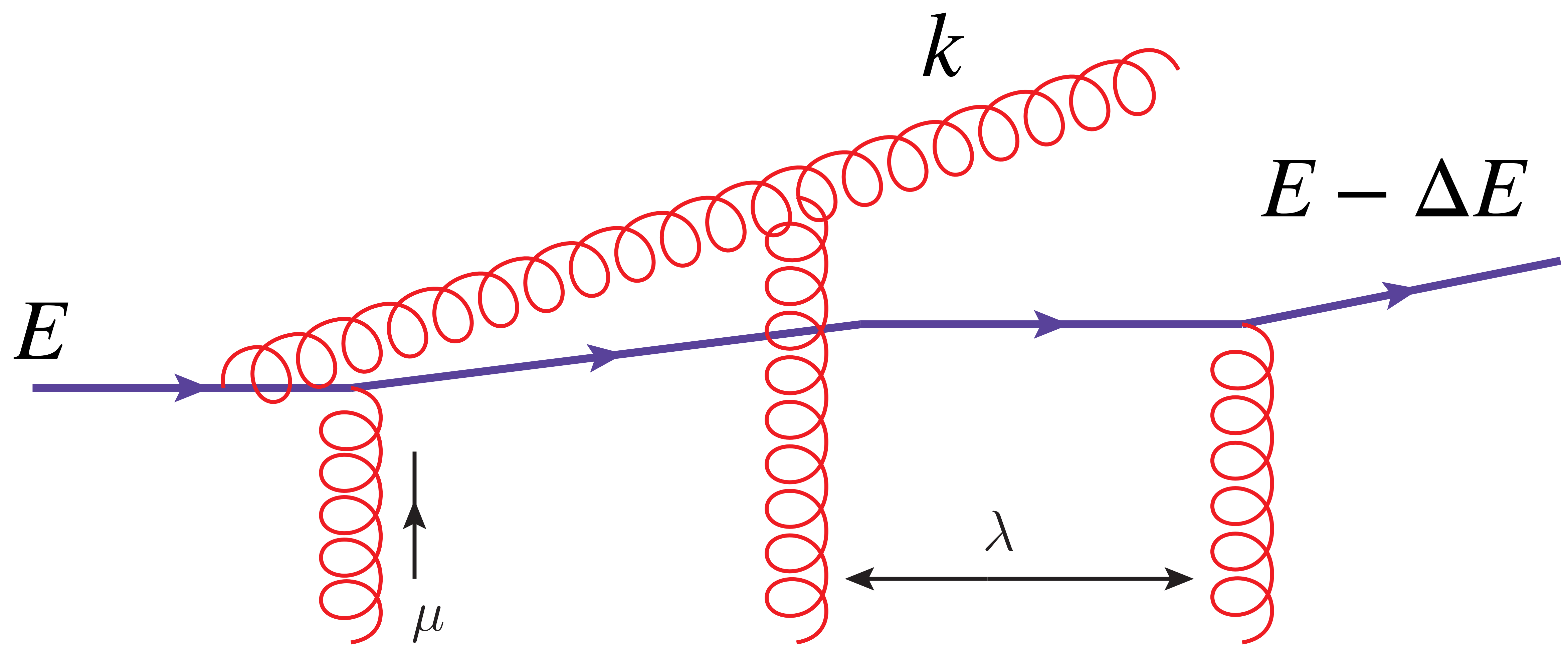
Parton energy loss in medium

J. D. Bjorken, FERMILAB-PUB-82-059-THY (1982)
M. Gyulassy and X. N. Wang, NPB420, 583-614 (1994)

E-loss happens via scattering with medium or induced gluon radiation:



E-loss is characterized by transport (diffusion) coefficient $\hat{q} = \mu^2/\lambda$:

- ✓ λ: parton's mean-free path in the medium.
- ✓ μ: typical momentum transferred from 1 soft scattering.
- ✓ $\langle k_\perp^2 \rangle \sim \hat{q}\, t_f$ with $t_f \sim k^+/k_\perp^2$: transverse momentum broadening in the medium.

E-loss in three distinct regimes

Depending on the gluon formation time t_f :

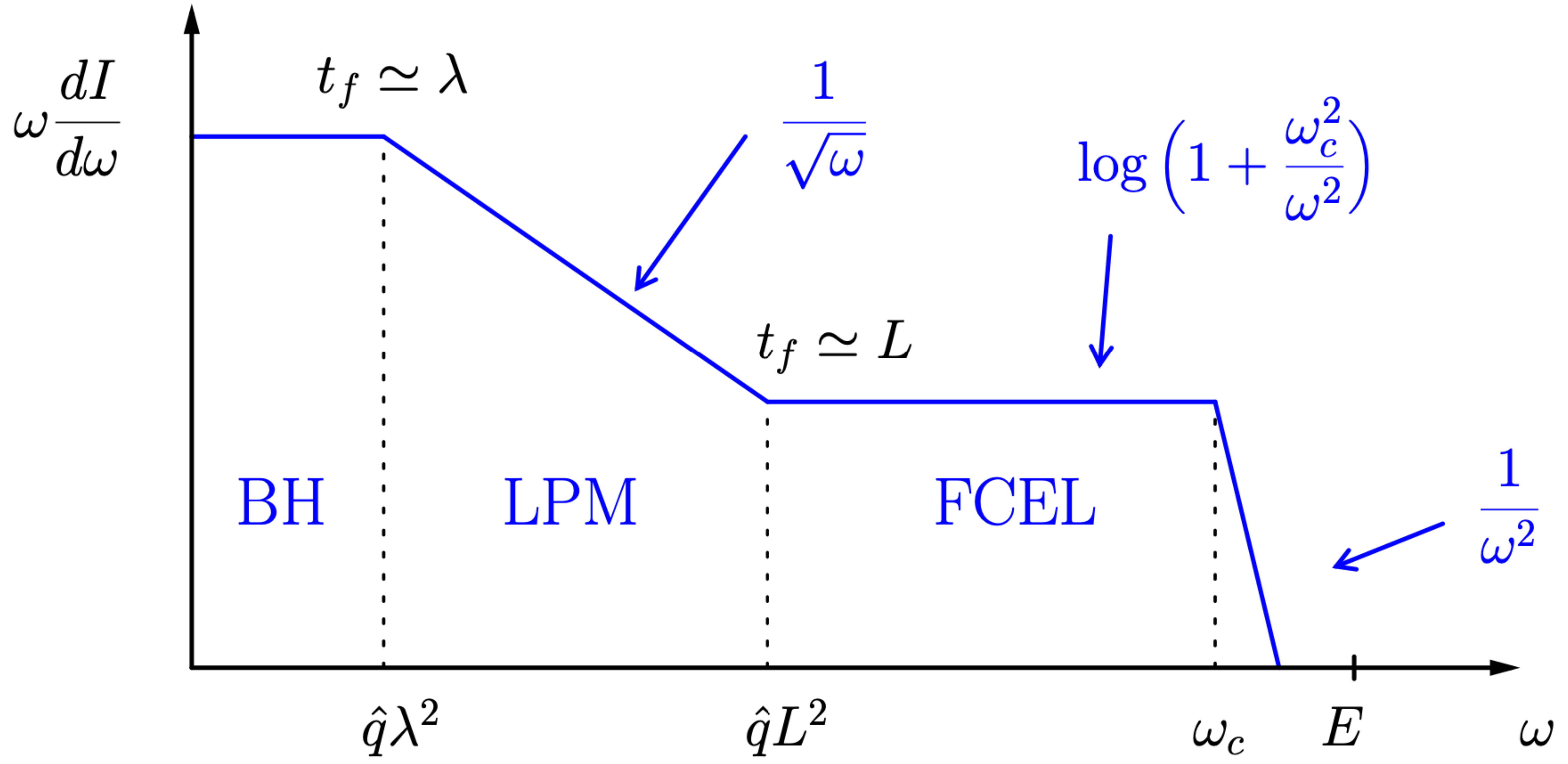
➡ **Bethe-Heitler regime ($t_f \ll \lambda$)**: each scattering center acts as an indep. source.

➡ **Landau-Pomeranchuk-Migdal regime ($\lambda \ll t_f \ll L$)**: a group of t_f/λ scattering centers acts as a single radiator.

➡ **Fully coherent (Long formation time or factorization) regime ($L \ll t_f$)**: all scattering centers in the medium act coherently as a source of radiation.

The gluon radiation spectrum:

$$dI = \frac{d\sigma_{\rm rad}}{d\sigma_{\rm el}} = \frac{\sum |M_{\rm rad}|^2}{\sum |M_{\rm el}|^2} \frac{dk^+ dk_\perp^2}{2k^+(2\pi)^3}$$

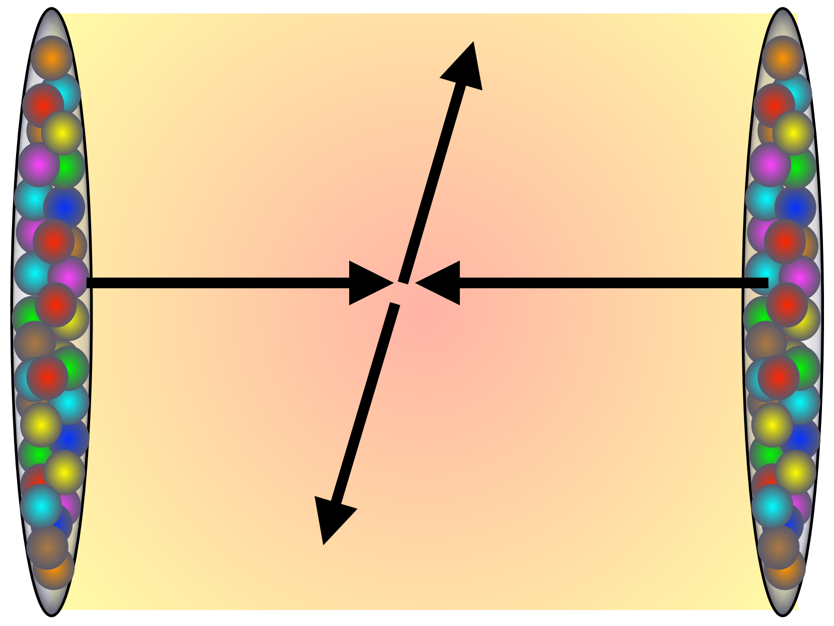


Parametric dependence of LPM and FCEL

Baier, Dokshitzer, Mueller, Peigne, Schiff, NPB484, 265 (1997)
Zakharov, JETP Lett.63, 952 (1996)
Wang and Guo, NPA696, 788-832 (2001)
Gyulassy, Levai and Vitev, NPB 571, 197 (2000)
…

❖ **LPM E-loss (initial state or final state):**

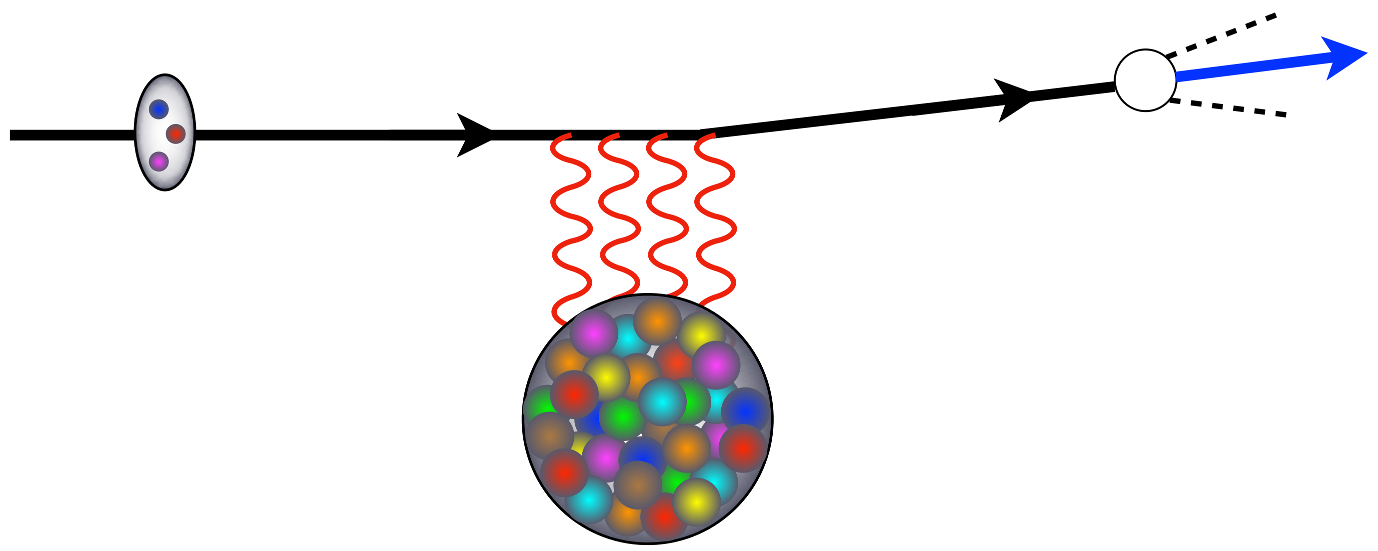
$$\Delta E_{\rm LPM}^{\rm BDMPS} = \langle \epsilon \rangle \sim \alpha_s \hat{q} L^2$$



✓ Important for hadron production in nuclear DIS, and jet in QGP.
✓ The fractional E-loss: $\Delta E/E \to 0$ as $E \to \infty$.

❖ **Fully Coherent E-loss (initial state & final state):**

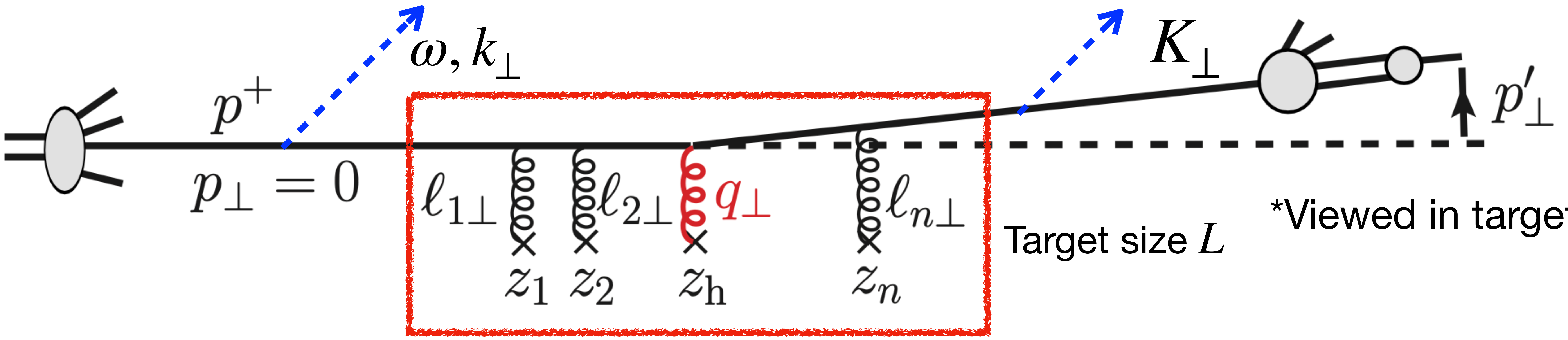
$$\Delta E_{\rm FCEL} \sim \alpha_s \frac{\sqrt{\hat{q}L}}{Q_{\rm hard}} E$$



✓ Important for hadron production in pA collisions.
✓ $\Delta E/E$ cannot vanish as $E \to \infty$: important at all energies.

Setup of FCEL

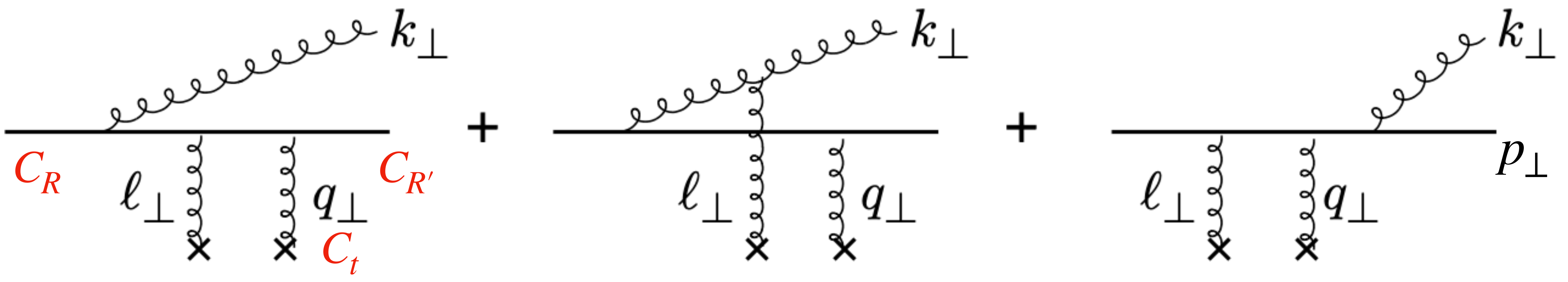
Forward scattering of fast asymptotic parton with $E\,(\to\infty)$ crossing a nuclear medium



- Parent parton from the projectile undergoes:
  - ✓ single hard scattering with $q_\perp$ an exchanged momentum.
  - ✓ multiple soft scatterings: $l_\perp^2 = \left(\sum l_{i\perp}\right)^2 \sim \hat{q}L \ll q_\perp^2, K_\perp^2$.
- Radiated gluons: soft ($x = k^+/p^+ \ll 1$) and small angle ($k_\perp \ll k^+$) radiation
- Hadron of $p'_\perp = zK_\perp$ is tagged.
- Recoiled parton assumed to be soft; kinematics remains the same.

Induced gluon spectrum in LLA

For massive particle, $p_\perp \to m_\perp$



(1) **Initial state interaction** (2) (3) **Final state interaction**

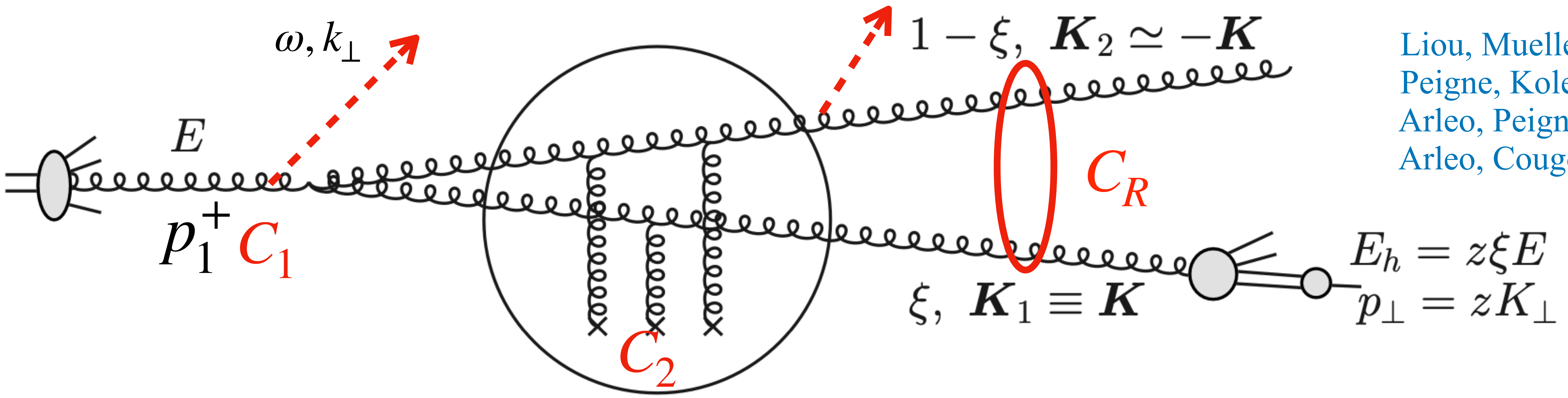
- ❖ The "induced" $k_\perp$-integrated gluon spectrum $dI/d\omega$, given by interference terms $\mathrm{Re}[(1+2)(3)^*]$ in leading-log approximation (**LLA**).
- ❖ $|(1+2)|^2$ and $|(3)|^2$ cancel out (power suppressed) in $dI/d\omega$.

$$\omega \frac{dI}{d\omega}\bigg|_{2\to 1} \approx F_c \frac{\alpha_s}{\pi}\left[\ln\left(1+\frac{l_{A\perp}^2 E^2}{\omega^2 p_\perp^2}\right) - \mathrm{pp}\ (l_{A\perp} \to l_{p\perp})\right]$$

Arleo, Peigne, Sami, PRD83, 114036 (2011)
Peigne, Arleo, Kolevatov, PRD93, 014006 (2016)
Munier, Peigne, Petreska, PRD95, 014014 (2017)
Armesto, Ma, Martinez, Mehtar-Tani and Salgado,PLB717, 280 (2012)

$F_c = C_R + C_{R'} - C_t$ with $R(R')$, t being a color rep. of incoming (outgoing) and t-channel particle. **Color charge = Casimir**.

Induced gluon spectrum for $2 \to 2$ in LLA (1/2)

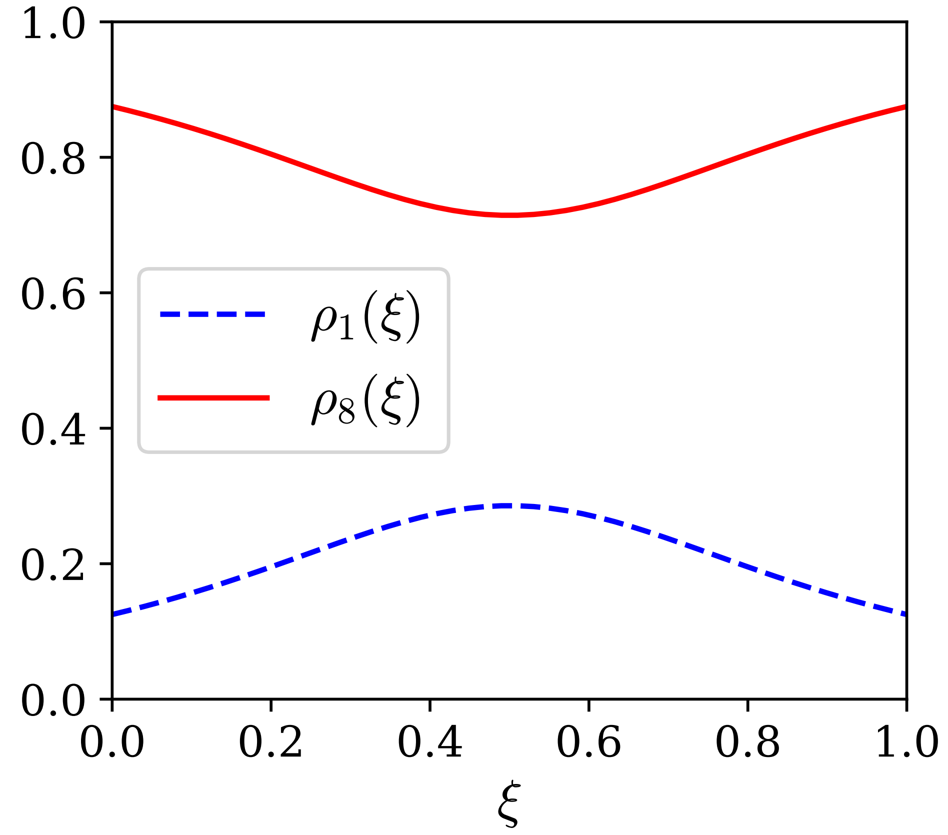


Liou, Mueller, PRD89, no.7, 074026 (2014)
Peigne, Kolevatov, JHEP01, 141 (2015)
Arleo, Peigne, PRL125, no.3, 032301 (2020)
Arleo, Cougoulic, Peigne, JHEP09, 190 (2020)

❖ **Simplification**: The induced soft gluon cannot probe the dijet constituents but see their global color state R in LLA (**Point-like Dijet Approximation**) with $\xi \sim 1/2$:

$$\omega \frac{dI}{d\omega}\bigg|_{2\to2} = \sum_R \rho_R F_R \frac{\alpha_s}{\pi} \left[\ln\left(1 + \frac{l_{A\perp}^2 E^2}{\omega^2 K_\xi^2}\right) - \text{pp} \right]$$

e.g. $gg \to Q\bar{Q} : \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$



probability for dijet to be produced in color state R

$F_R = C_1 + C_R - C_2$ with C_R global Casimir charge in R

Dijet inv. mass

Phenomenology in LLA

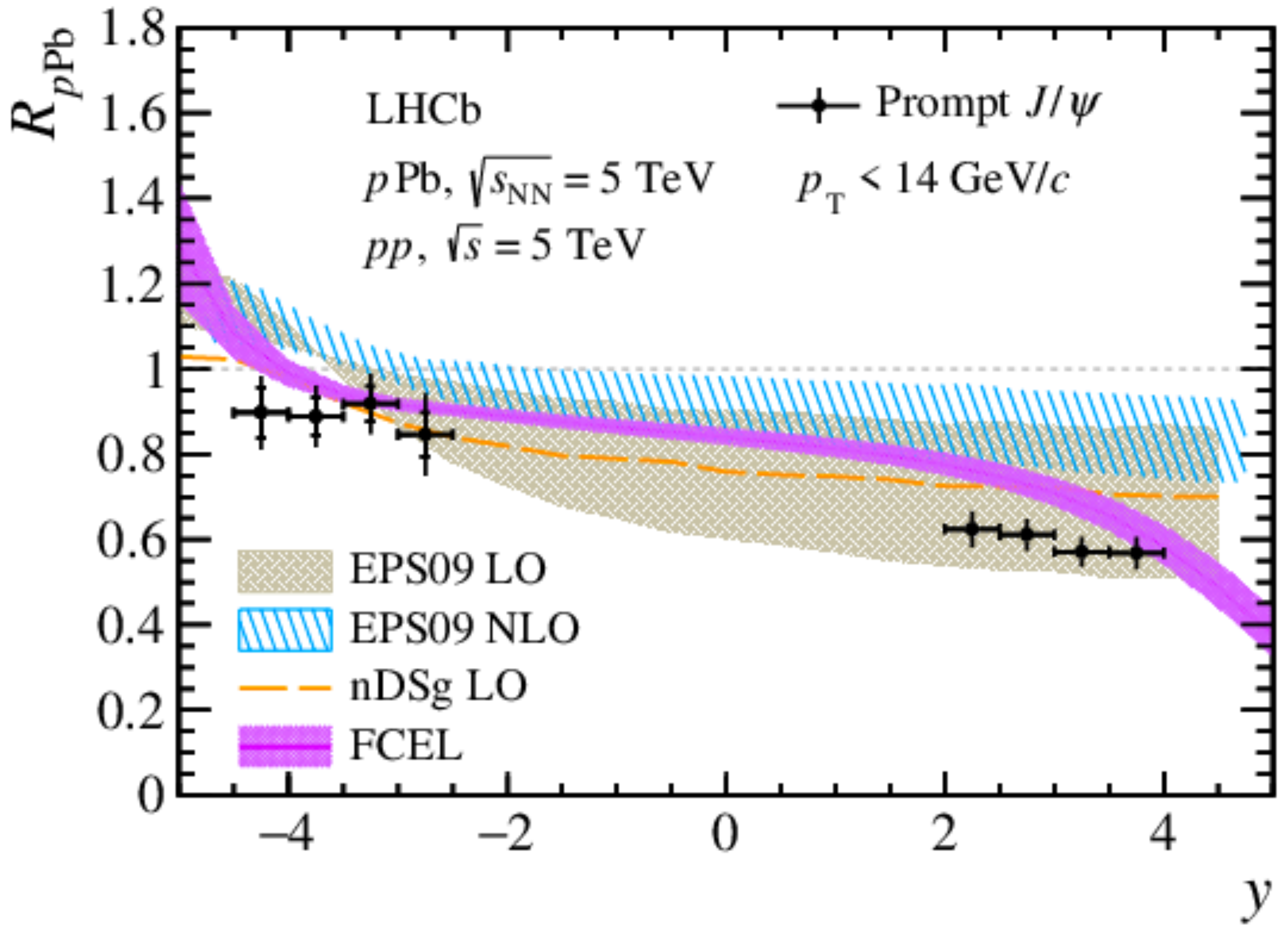
$$E\frac{d\sigma_{pA\to h+X}}{d^3p} = A\int_0^{\epsilon_{\max}} d\epsilon\, \mathscr{P}(\epsilon)\, E\frac{d\sigma_{pp\to h+X}}{d^3p}\Big|_{E\to E+\epsilon}$$

Energy (rapidity) shift

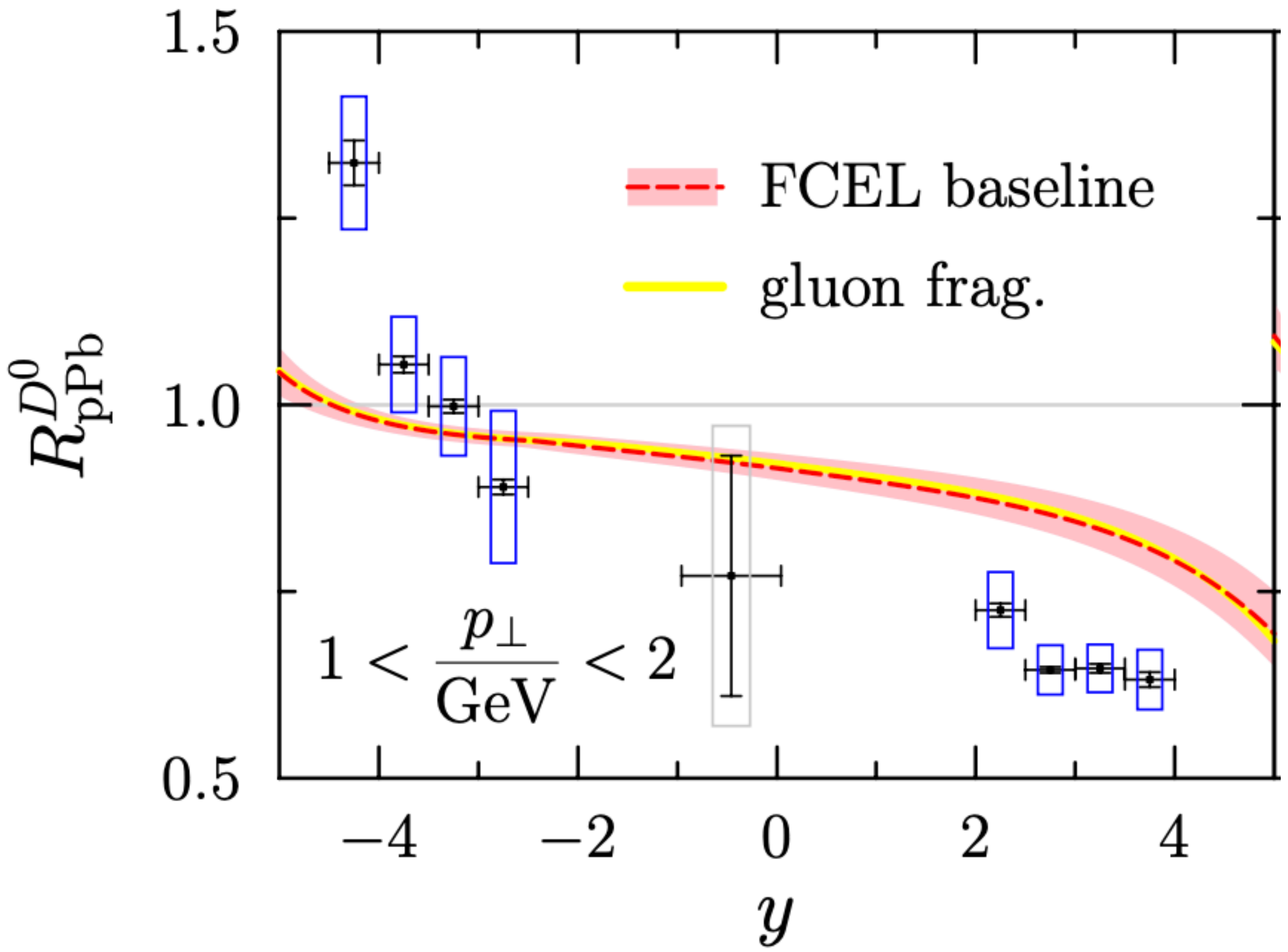
Probability distribution (**quenching weight**) in double-log approx. (DLA): Baier, Dokshitzer, Mueller, Schiff, JHEP09, 033 (2001)

$$\mathscr{P}(\epsilon) \simeq \frac{dI}{d\epsilon}\exp\left\{-\int_\epsilon^\infty d\omega \frac{dI}{d\omega}\right\}$$

Arleo and Peigne, PRL109, 122301 (2012), JHEP03, 122 (2013)



Arleo, Jackson, Peigne, JHEP01, 164 (2022)



Beyond LLA (1/2)

Jackson, Peigne, **KW**, JHEP05, 207 (2024)

$$\frac{dI}{dx} = \Phi_{\alpha\beta}\, S(x)_{\beta\alpha} = \mathrm{Tr}\left[\Phi \cdot S(x)\right] \quad \text{with} \quad x = \omega/E$$

- Φ : color density matrix, quantifying the entanglement between color components of $2 \to 2$ amplitudes.

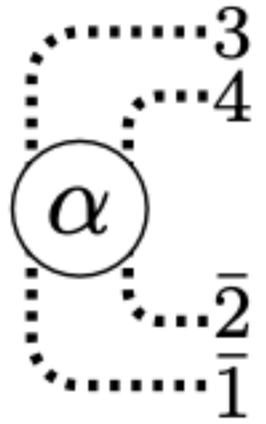
$$\Phi_{\alpha\beta} = \frac{\mathrm{Tr}_{\mathrm{Dirac}}(v_\alpha v_\beta^*)}{\mathrm{Tr}_{\mathrm{color}}\mathrm{Tr}_{\mathrm{Dirac}}|M|^2}$$

Dirac structure — Color structure

$$M_{12\to34} = \sum_\alpha v_\alpha \langle\alpha|$$

s-channel

$$\langle\alpha| = \frac{1}{\sqrt{K_\alpha}}$$



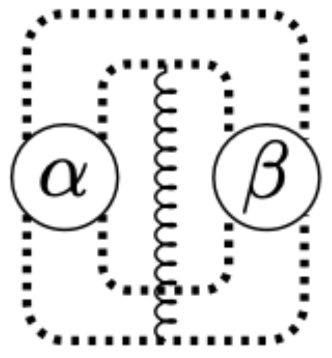
K_α : dimesions of irreps. α

- S : soft color matrix

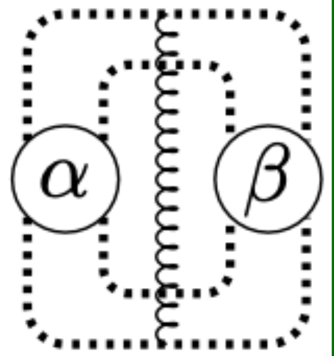
$$S(x)_{\alpha\beta} \equiv \frac{\alpha_s}{\pi x}\left(\mathscr{L}_\xi B_{\alpha\beta} + \mathscr{L}_{\bar{\xi}} \bar{B}_{\alpha\beta}\right)$$

Soft color connections

$$B_{\alpha\beta} = \frac{2}{\sqrt{K_\alpha K_\beta}}$$



$$\bar{B}_{\alpha\beta} = \frac{2}{\sqrt{K_\alpha K_\beta}}$$



Soft factors

$$\mathscr{L}_\xi \approx \log\left(1 + \frac{\xi^2 l_{A\perp}^2}{x^2 m_\perp^2}\right) - \mathrm{pp}$$

Beyond LLA (2/2)

Jackson, Peigne, **KW**, JHEP05, 207 (2024)

$$\frac{dI}{dx} = \Phi_{\alpha\beta}\, S(x)_{\beta\alpha} = \mathrm{Tr}\left[\Phi \cdot S(x)\right] \quad \text{with} \quad x = \omega/E$$

The soft color matrix S can be cast into:

$$\mathcal{L}_\xi B + \mathcal{L}_{\bar\xi} \overline{B} = \frac{\mathcal{L}_\xi + \mathcal{L}_{\bar\xi}}{2} B_+ + \frac{\mathcal{L}_\xi - \mathcal{L}_{\bar\xi}}{2} B_-$$

Diagonal color matrix

Off-diagonal color matrix

$$(B_+)_{\alpha\beta} = \langle \alpha \,|\, 2T_1(T_4 + T_3) \,|\, \beta \rangle = \langle \alpha \,|\, T_1^2 + (T_4 + T_3)^2 - T_2^2 \,|\, \beta \rangle = (C_1 + C_\alpha - C_2)\delta_{\alpha\beta}$$

❖ In LLA (Point-like Dijet Approximation) with $\xi \sim 1/2$, $\mathcal{L}_\xi \sim \mathcal{L}_{\bar\xi} \implies B_- \sim 0$:

➡ **No color transition**

❖ Beyond LLA or $\xi \neq 1/2$, the induced gluon can change the color state of a pair:

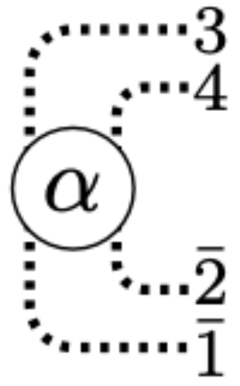
➡ **Color transitions** ($R \to R'$)

Matching with LLA results

dI/dx is independent of the color basis, but S can be diagonalized in some basis.

(1) Matching for $\xi = 1/2$: Diagonal in s-channel basis

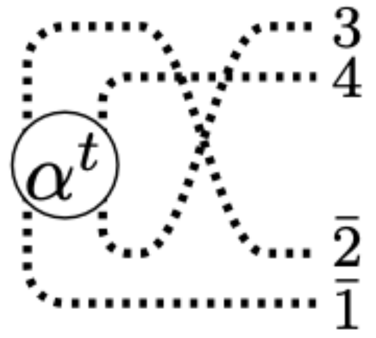
$$\left.\frac{dI}{dx}\right|_{\xi=1/2} = \frac{\alpha_s}{\pi x}\sum_{\alpha} \Phi_{\alpha\alpha}\,(C_1 + C_\alpha - C_2)\,\mathscr{L}_{\xi=1/2}$$

$\langle\alpha| = \frac{1}{\sqrt{K_\alpha}}$ 

ρ_α : probability of the s-channel irreps. α

(2) Matching in $\xi = 0$ limit: Diagonal in t-channel basis

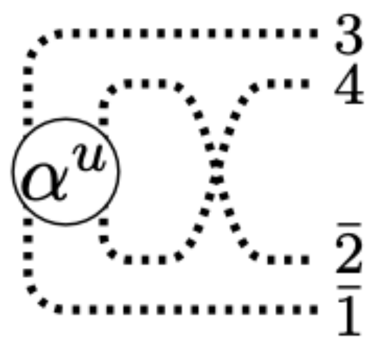
$$\left.\frac{dI}{dx}\right|_{\xi=0} = \frac{\alpha_s}{\pi x}\sum_{\alpha^t} \Phi^t_{\alpha_t\alpha_t}\,(C_1 + C_3 - C_{\alpha^t})\,\mathscr{L}_{\bar{\xi}=1}$$

$\langle\alpha^t| \equiv \frac{1}{\sqrt{K_{\alpha^t}}}$ 

$\rho^t_{\alpha^t}$: probability of the t-channel irreps. α_t

(3) Matching in $\xi = 1$ limit: Diagonal in u-channel basis

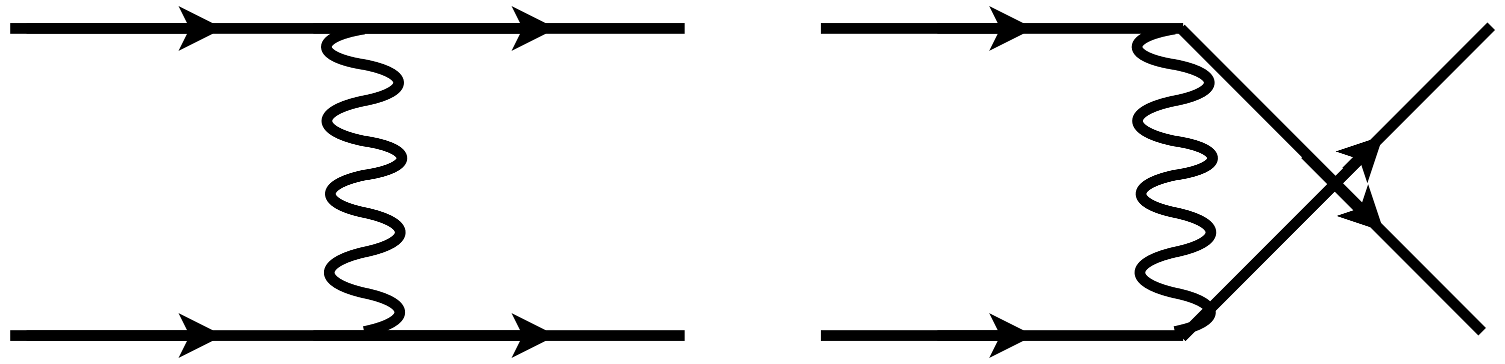
$$\left.\frac{dI}{dx}\right|_{\xi=1} = \frac{\alpha_s}{\pi x}\sum_{\alpha^u} \Phi^u_{\alpha^u\alpha^u}\,(C_1 + C_4 - C_{\alpha^u})\,\mathscr{L}_{\xi=1}$$

$\langle\alpha^u| \equiv \frac{1}{\sqrt{K_{\alpha^u}}}$ 

$\rho^u_{\alpha^u}$: probability of the u-channel irreps. α_u

Demonstration: Fully Coherent Energy Loss vs. Gain

Quark-quark scattering processes



$\alpha = (\bar{\mathbf{3}}, \mathbf{6})$ in s-channel

$\alpha = (\mathbf{1}, \mathbf{8})$ in t,u-channel

$$\left.\frac{dI}{dx}\right|_{\xi=1/2} = \frac{\alpha_s}{\pi x}\left[\rho_{\bar{3}}\, C_{\bar{3}} + \rho_6\, C_6\right]\, \mathscr{L}_{\xi=1/2}$$

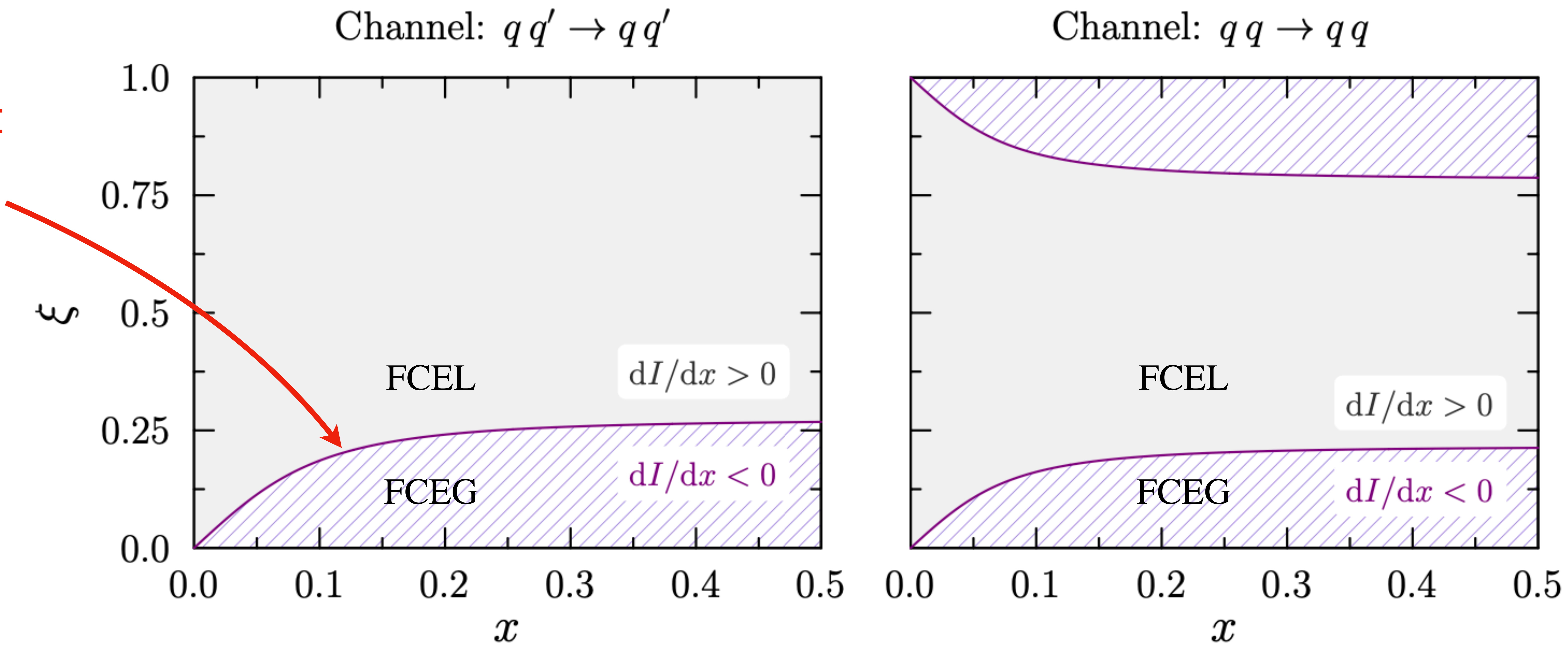
$$\left.\frac{dI}{dx}\right|_{\xi=0} = \frac{\alpha_s}{\pi x}\left[\rho_1^t\,(2C_F) + \rho_8^t\,(2C_F - N_c)\right]\, \mathscr{L}_{\bar{\xi}=1}$$

Negative

$$\left.\frac{dI}{dx}\right|_{\xi=1} = \frac{\alpha_s}{\pi x}\left[\rho_1^u\,(2C_F) + \rho_8^u\,(2C_F - N_c)\right]\, \mathscr{L}_{\xi=1}$$

Negative

No energy loss at the boundary



Summary

- FCEL(G) is significant for all hadron production, including heavy flavors, in pA collisions at all energies:

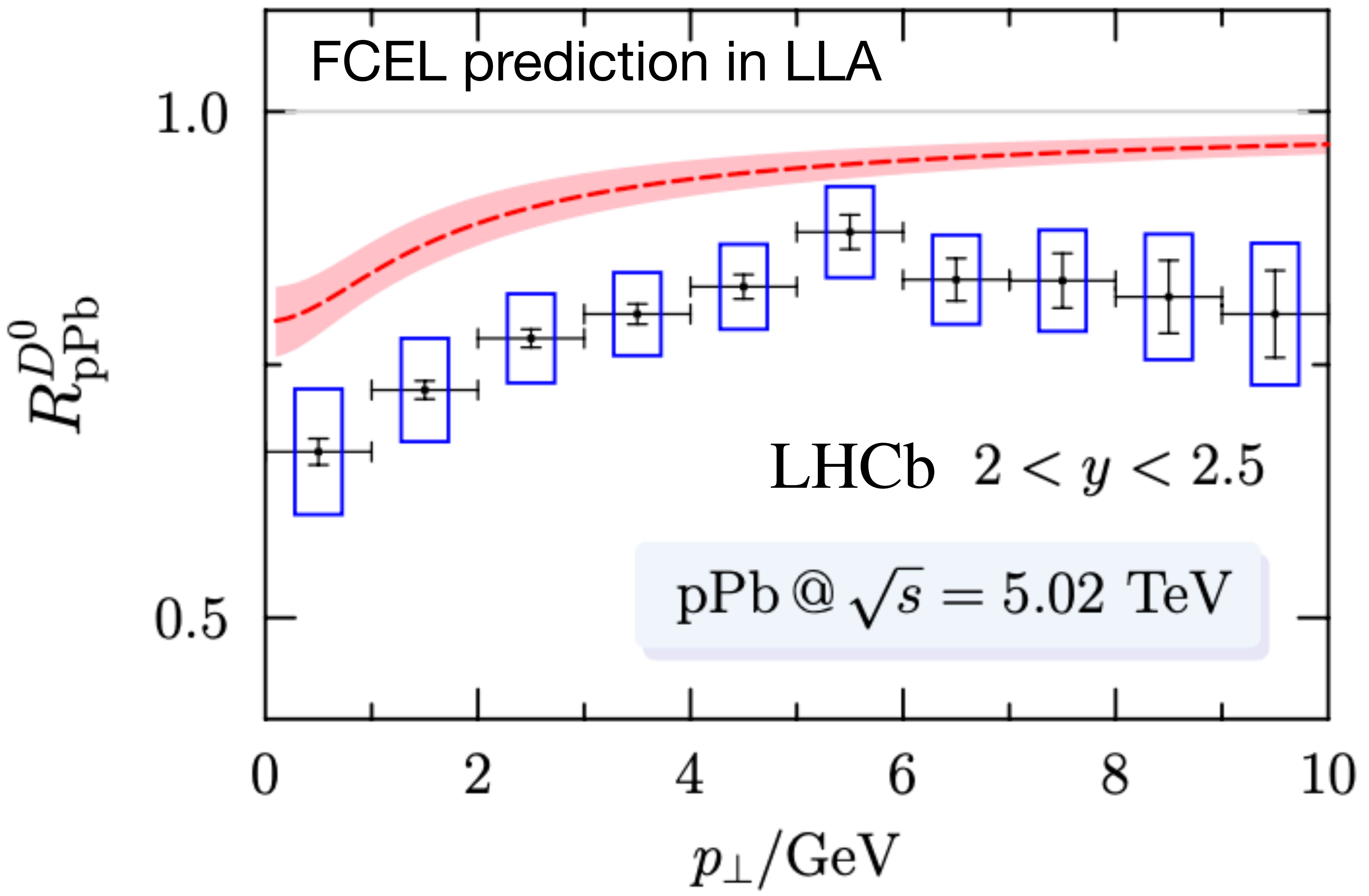
$$\Delta E_{\rm FCEL} \sim \alpha_s \frac{\sqrt{\hat{q}L}}{Q_{\rm hard}} E \gg \Delta E_{\rm LPM} \sim \alpha_s \hat{q} L^2$$

- Pushing forward precise calculations of the induced gluon radiation spectrum beyond LLA enables us to look into the color transition of a parton pair in the nuclear medium.

- Building a quenching weight from the induced gluon spectrum beyond LLA is underway.

- Stay tuned for improving our predictions beyond LLA!

Arleo, Jackson, Peigne, JHEP01, 164 (2022)



Thank you!

Backup

Resumming all orders in opacity

$$dI_{\text{induced}} = dI_{pA} - dI_{pp} = \sum_{n=1} dI^{(n)}$$

At any order in n (=opacity: # of scatterings)

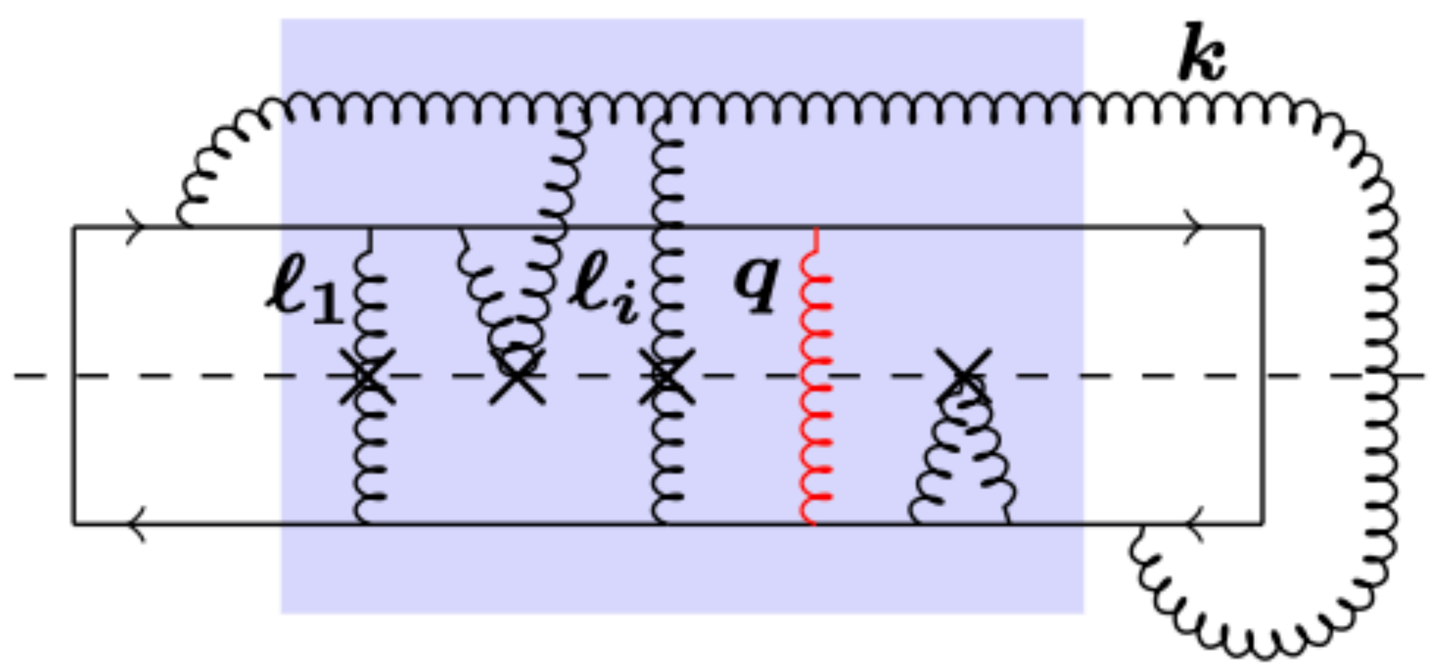
$$x\frac{\mathrm{d}I^{(\tilde{n})}}{\mathrm{d}x} = \frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{k}}{\pi}\left[\prod_{i=1}^{\tilde{n}}\int\frac{\mathrm{d}z_i}{N\lambda_g}\int\mathrm{d}^2\boldsymbol{\ell}_i\,V(\boldsymbol{\ell}_i)\right]C_{\tilde{n}}(\boldsymbol{k},\boldsymbol{q})$$

Re(initial final*): **fully coherent!**

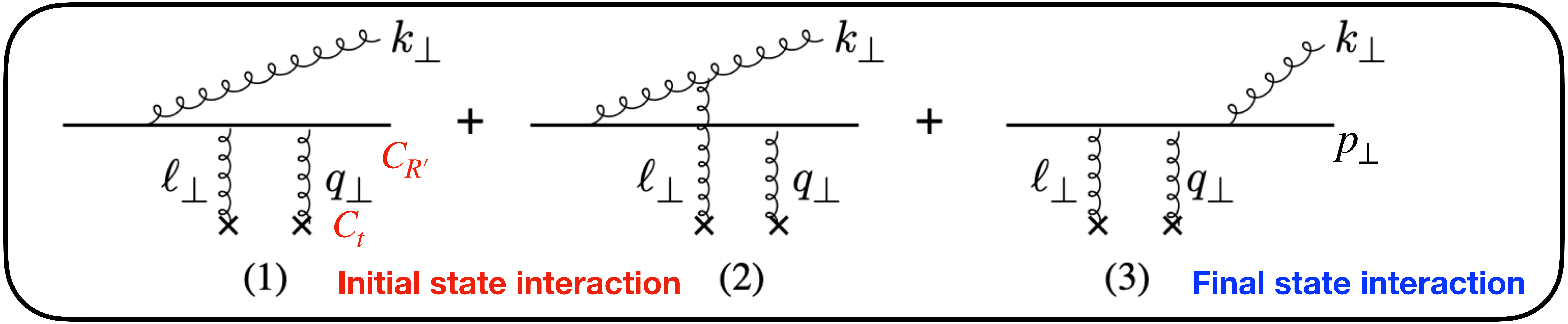
Static Coulomb potential: V

Elastic mean free path: λ_g

$$C_{\tilde{n}}(\boldsymbol{k},\boldsymbol{q}) = \frac{2}{\left|\mathcal{M}^{\text{vac}}_{qg\to q}\right|^2} \cdots$$



The origin of large log



$$\omega \frac{dI}{d\omega}\bigg|_{2\to 1} \approx F_c \frac{\alpha_s}{\pi}\left[\ln\left(1+\frac{l_{A\perp}^2 E^2}{\omega^2 p_\perp^2}\right) - \mathrm{pp}\right]$$

The leading logs arise from an integral over $k_\perp$:

$$x q_\perp \ll k_\perp \ll \sqrt{\hat{q}L}$$

The soft gluon does not probe the relative displacement of the core charge.

Only radiation softer than $l_\perp$ can be shaken off by in-medium scatterings

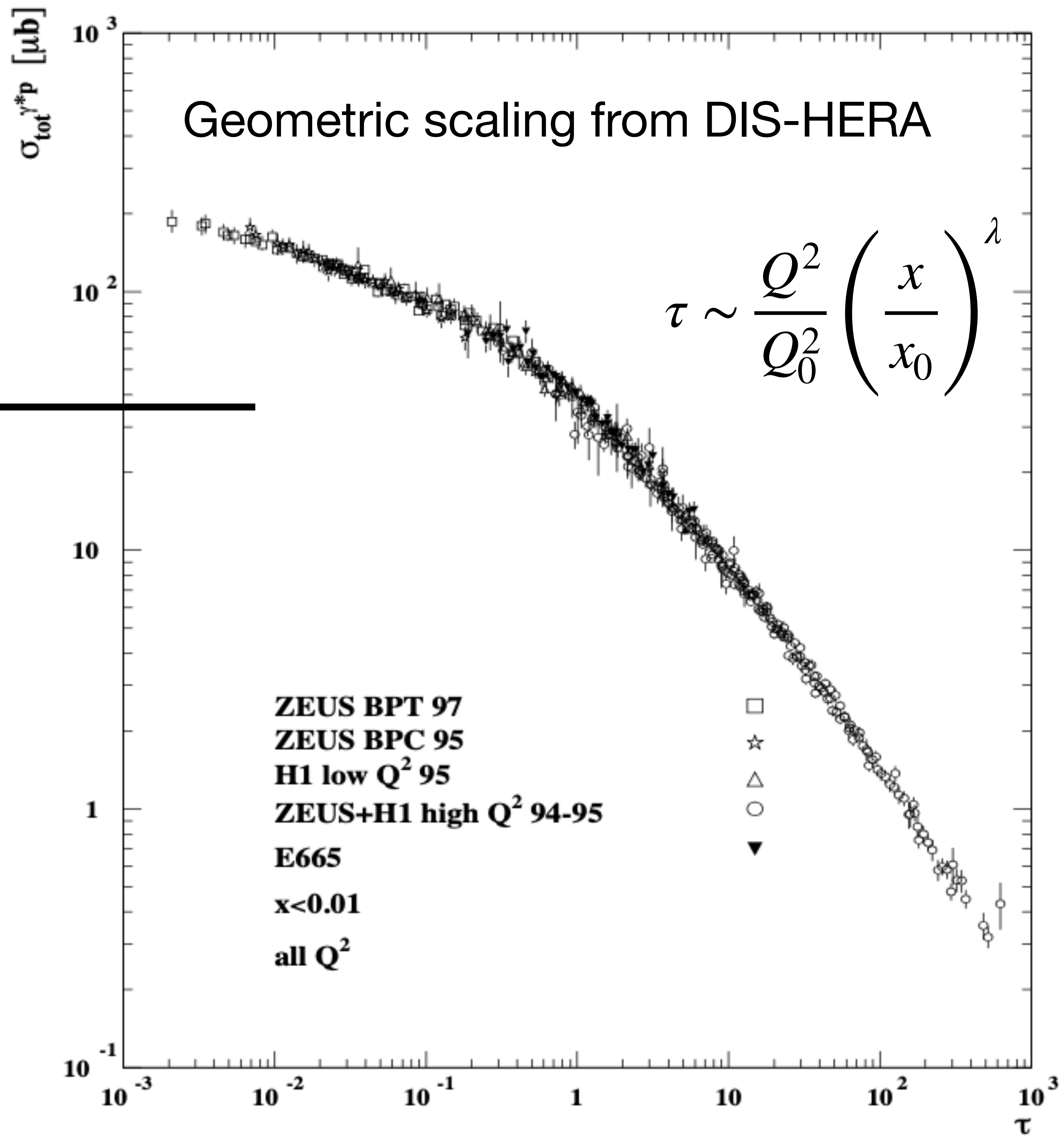
Transport coefficient

- $l_\perp^2 \simeq \hat{q}L$ is the only free parameter in the model.
- Parametrization of the transport coefficient

$$\hat{q} \sim \hat{q}_0 \left(\frac{10^{-2}}{x}\right)^{0.3}$$

  - $\hat{q}_0 = 0.07 - 0.09\,\mathrm{GeV}^2/\mathrm{fm}$: fixed by fitting data
  - QCD evolution is not considered for simplicity
  - L: determined by Glauber theory

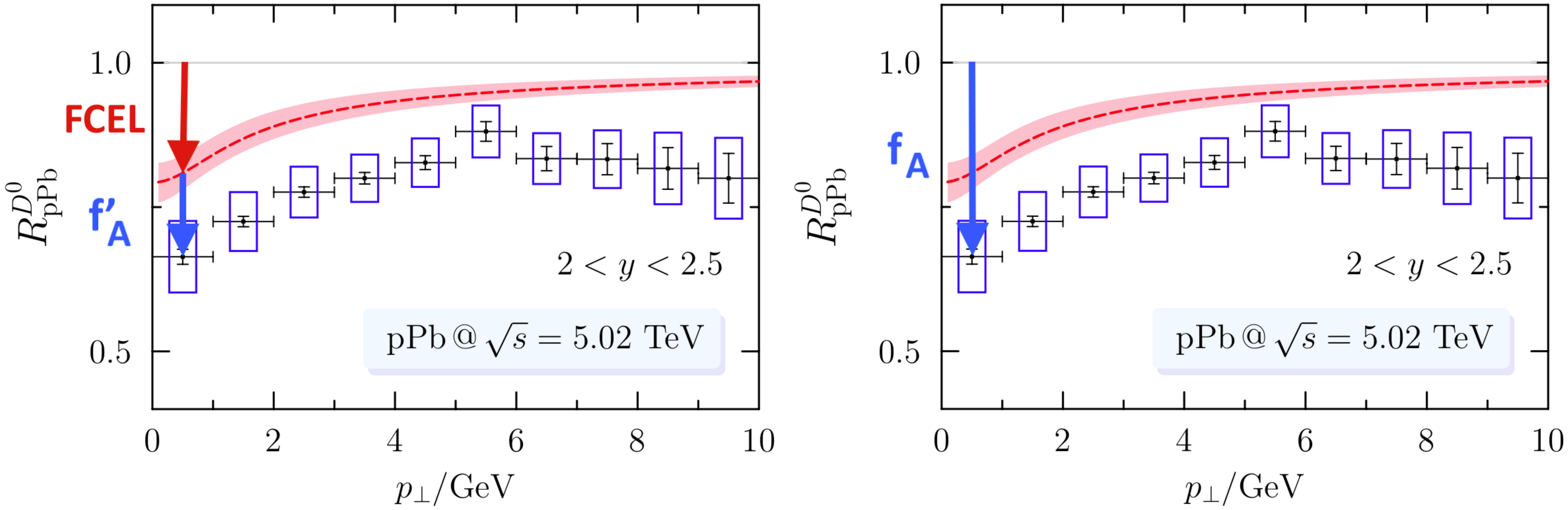
- In the small-x limit, we could read $\hat{q}L \sim Q_s^2$, but cannot derive it analytically. Baier, Dokshitzer, Mueller, Peigne and Schiff, NPB484, 265 (1997)



Golec-Biernat and Wusthoff, PRD59, 014017 (1998), PRD60, 114023 (1999)
Stasto, Golec-Biernat, Kwiecinski, PRL86, 596 (2001)

Perspectives

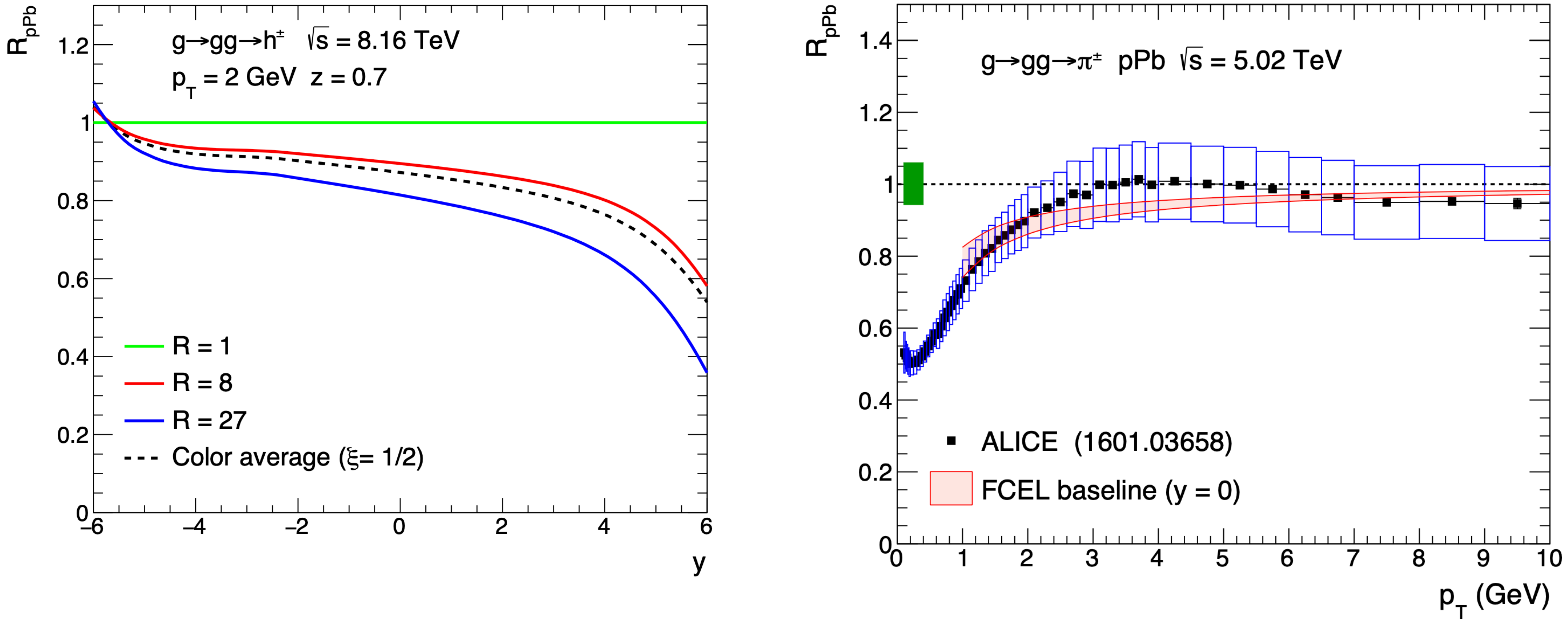
Arleo, Jackson, Peigne, **KW**, in progress



- $\chi^2(f'_A \,|\, \text{FCEL} \cap \text{LHCb})$ vs. $\chi^2(f_A \,|\, \text{no FCEL} \cap \text{LHCb})$
- nPDFs can be reweighed by implementing both FCEL and nPDFs.
- Remark: all hadron production in pA collisions can be affected by nPDFs and FCEL.

Sample: Light hadron production

Arleo, Peigne, PRL125, no.3, 032301 (2020)
Arleo, Cougoulic, Peigne, JHEP09, 190 (2020)



❖ Assume that $gg \to gg$ is a dominant channel.

❖ The suppression patterns depend on R of a produced parton pair.