# Heavy quark jets in evolving anisotropic matter

Carlos A. Salgado
IGFAE - U. Santiago de Compostela

OPORTUNIUS RESEARCH PROFESSOR

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[Work in collaboration with João Barata, Xoan Mayo and Andrey Sadofyev]

Yocto/LHC
Yoctosecond imaging of QCD collectivity using jet observables
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CENTROS gus

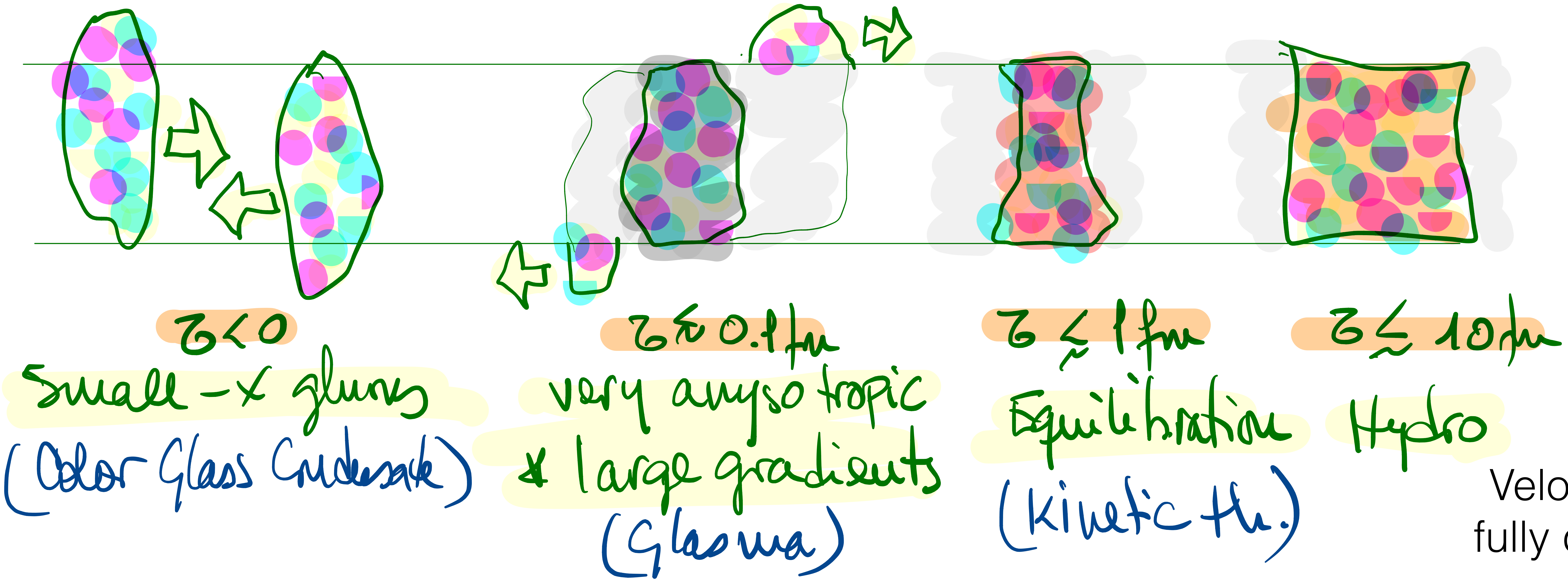
EXCELENCIA MARÍA DE MAEZTU 2024-2029

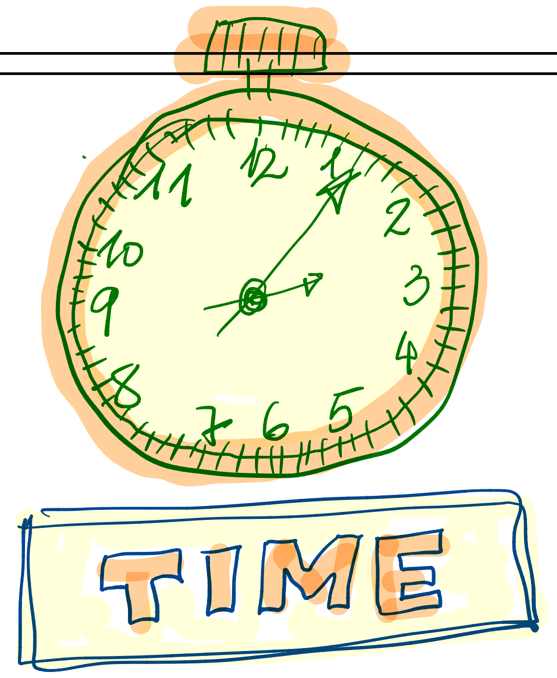
Happy Birthday HP!!!

Hard Probes 2004

International Conference on
Hard and Electromagnetic Probes
of High Energy Nuclear Collisions

# (A possible) Time evolution of a HIC



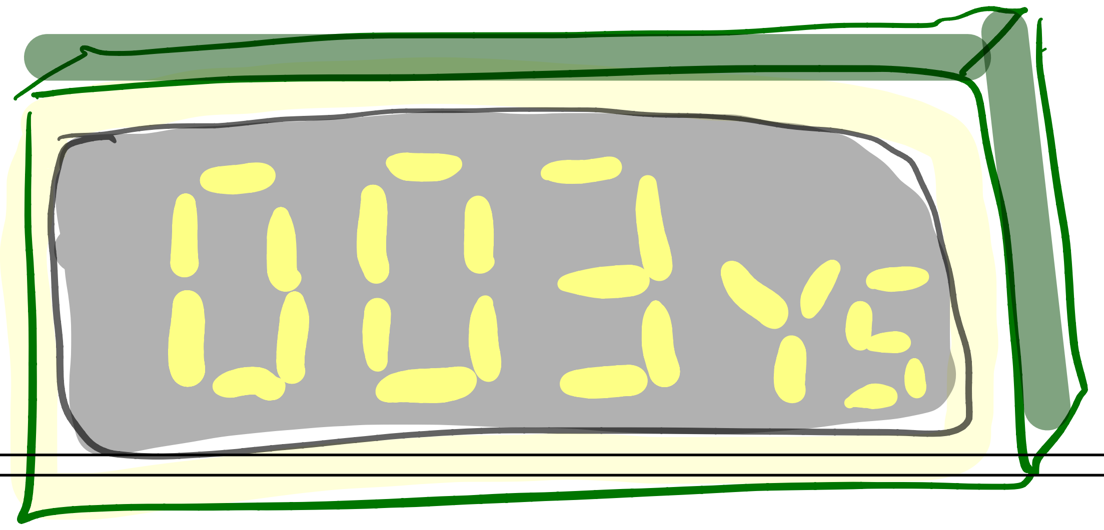




Velocity fields fully developed

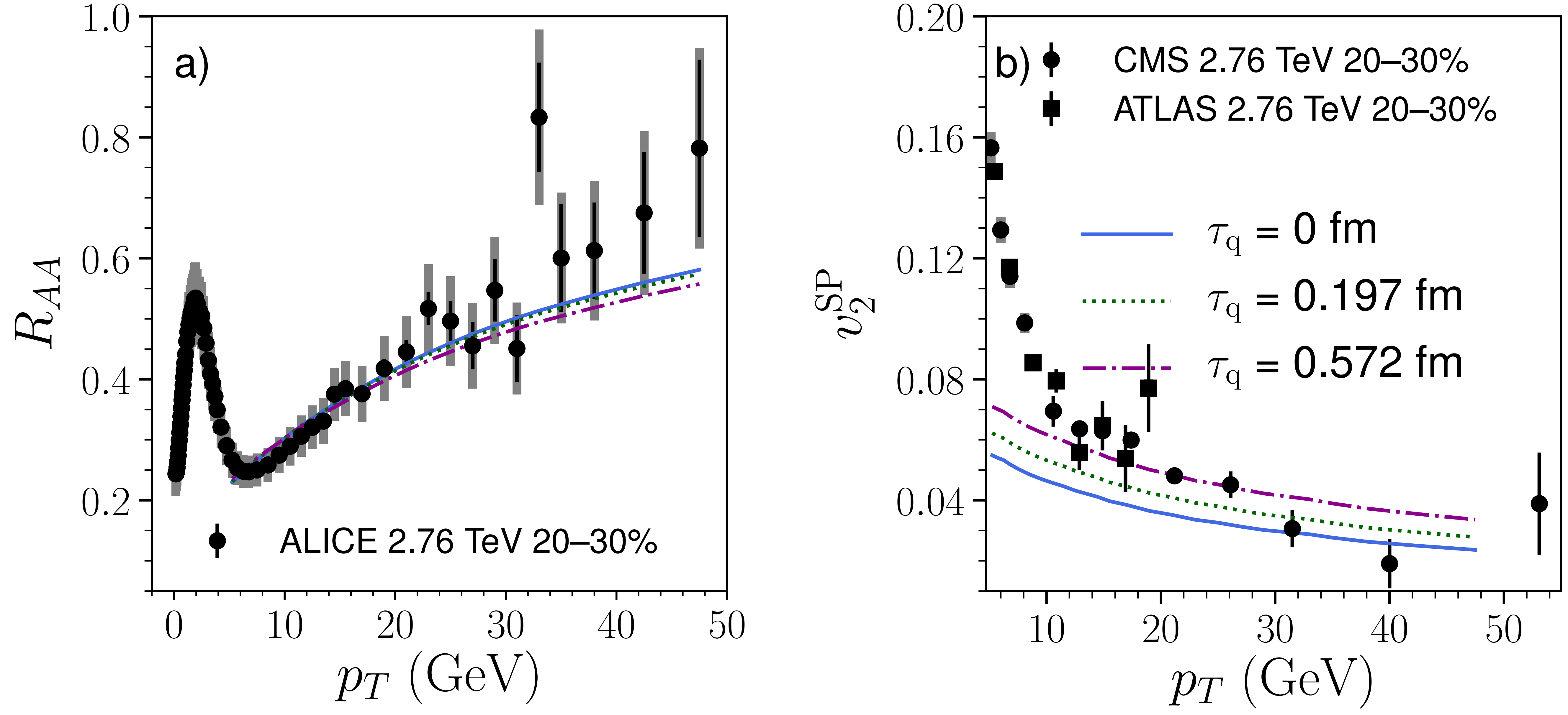
In contrast to usual HEP, **time and distance are relevant variables** in heavy-ion collisions

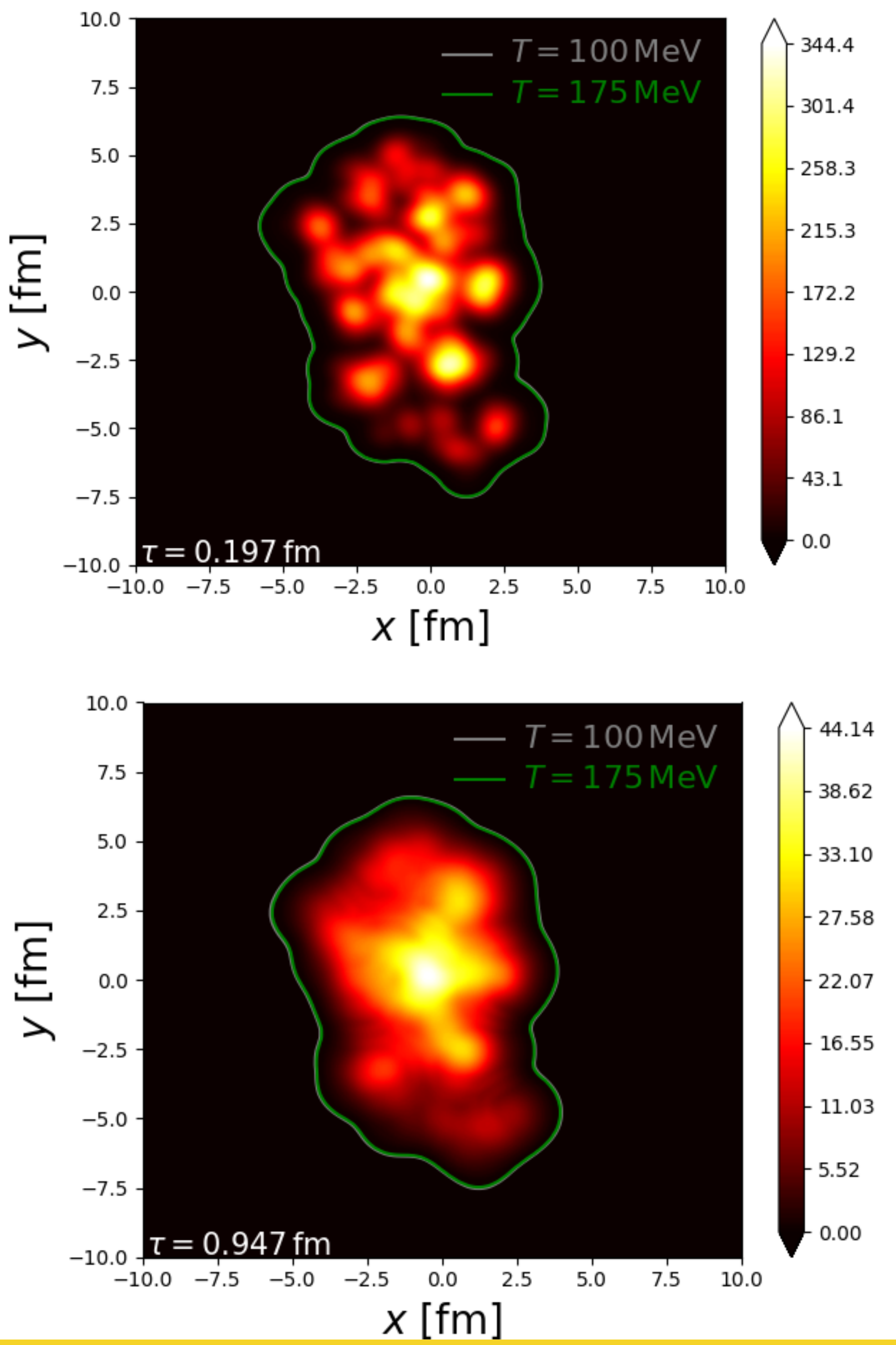
**Measure time evolution - in equilibrium and out of equilibrium**

# First ~3ys…




Can we access the initial stages with jet quenching?




[Andres, Armesto, Niemi, Paatelainen, Salgado 2019]

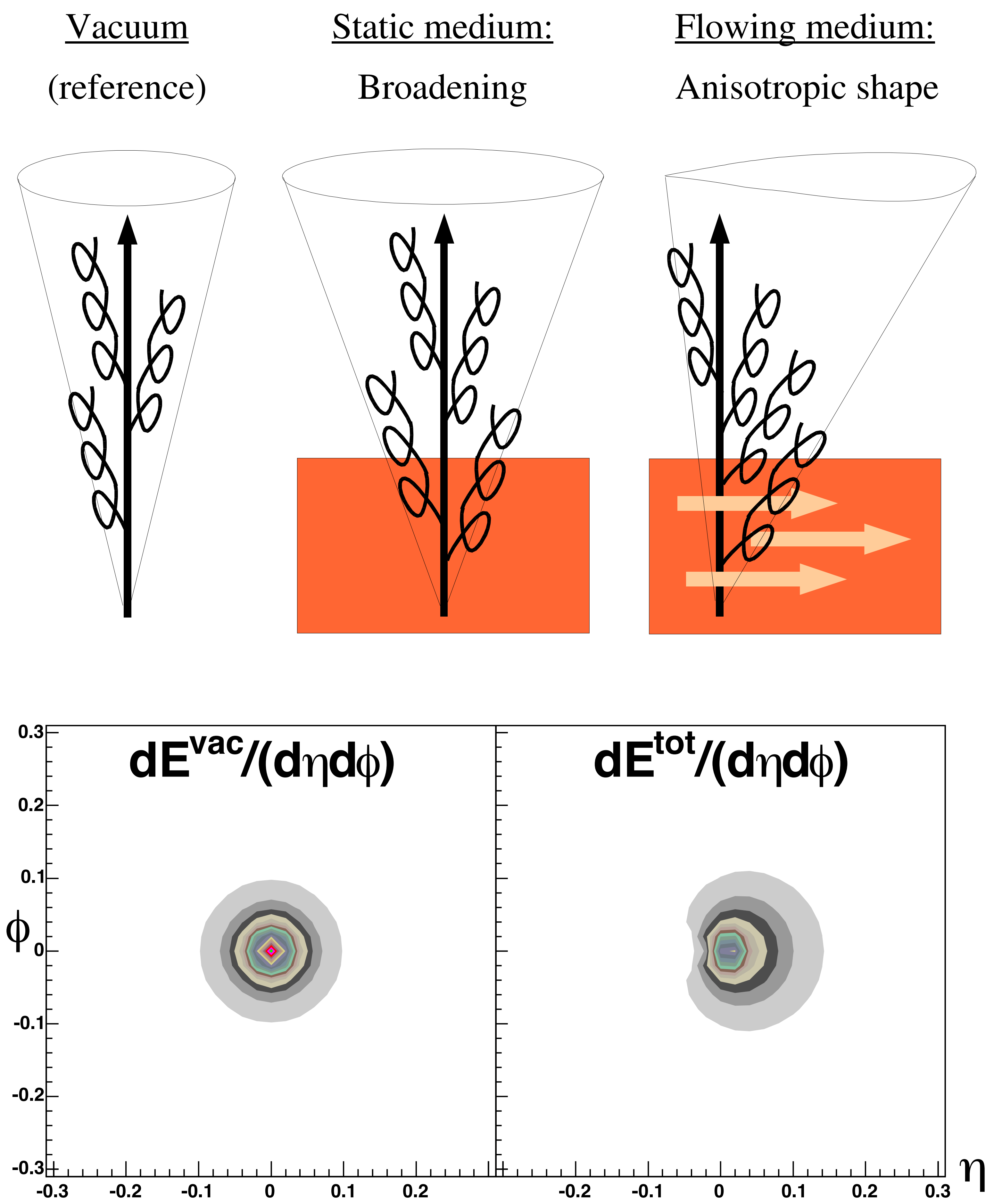
Phenomenological analyses need delay time in energy loss to fit RAA and v2 simmultaneously
**Would Flow/gratient effects at late times modify this conclusion?**

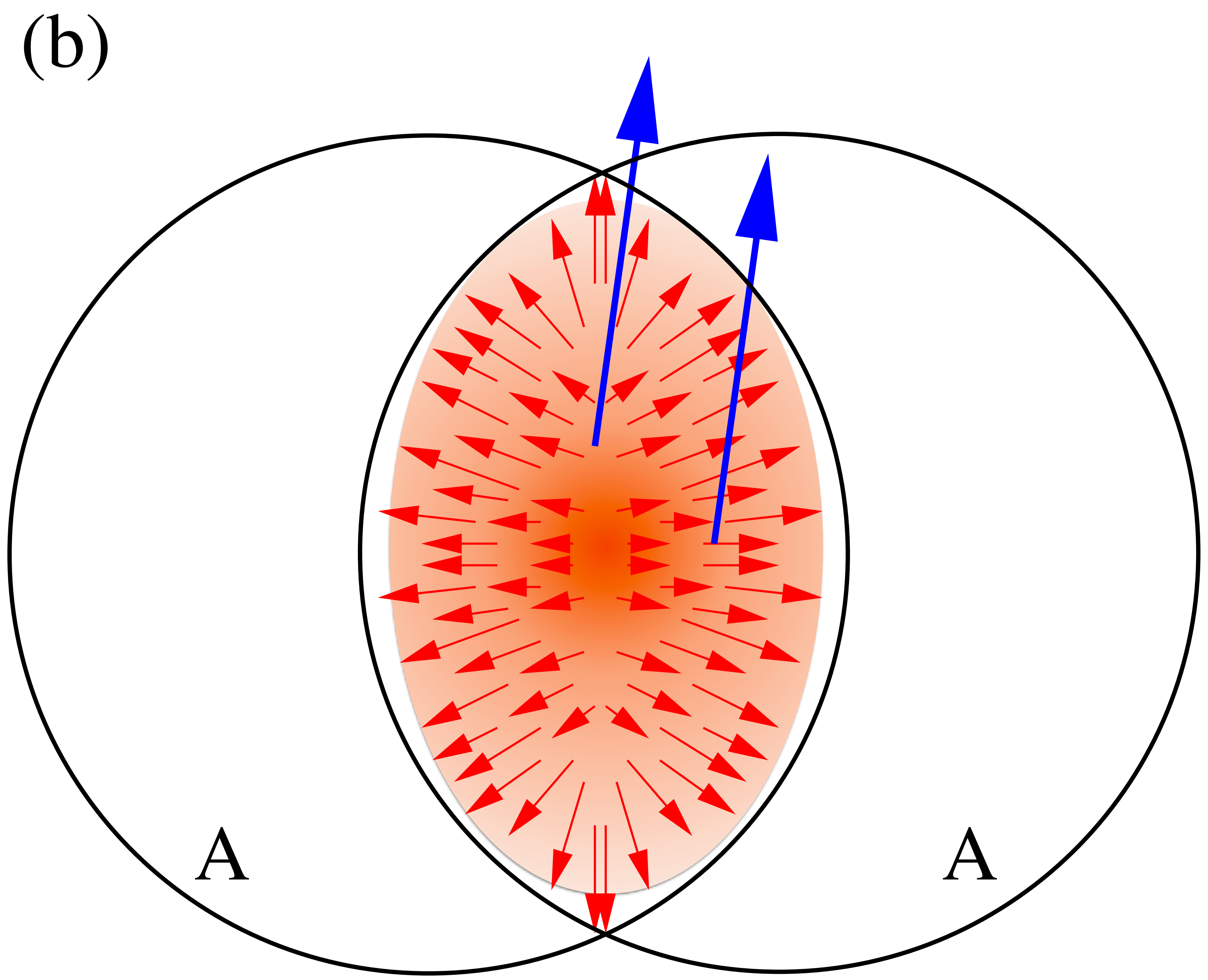
**Is there an interplay between time evolution properties (velocity fields, gradients…) and jets developping in the medium?**

[So that we can measure these properties with jet observables for different times]

# 20 yrs ago… [rather ad-hoc implementation]

[Armesto, Salgado, Wiedemann 2004]

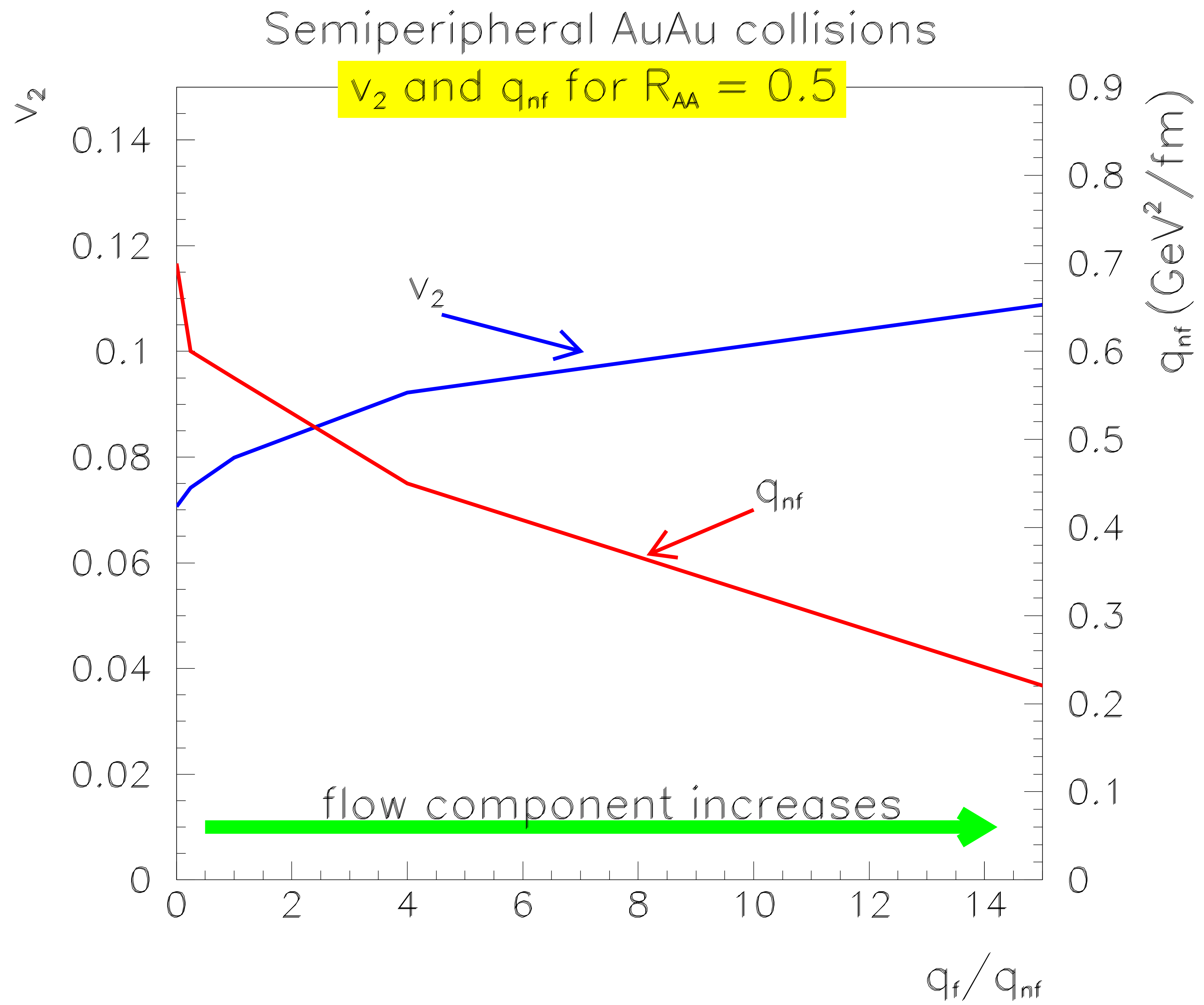



What is the effect of the velocity fields and the (density/temperature) gradients in jet quenching observables?

# Collective flow induces quenching (?)

[Armesto, Salgado, Wiedemann 2004]




Proposal was to define $\hat{q}$ with the boosted component of $T^{\mu\nu}$

$$\hat{q} = c\,\epsilon^{3/4}(p) \qquad \longrightarrow \qquad \hat{q} = c\,\epsilon^{3/4}(T^{n_\perp n_\perp})\,.$$

$$T^{n_\perp n_\perp} = p(\epsilon) + [\epsilon + p(\epsilon)]\,\frac{\vec{\beta}_\perp^2}{1-\beta^2}\,.$$

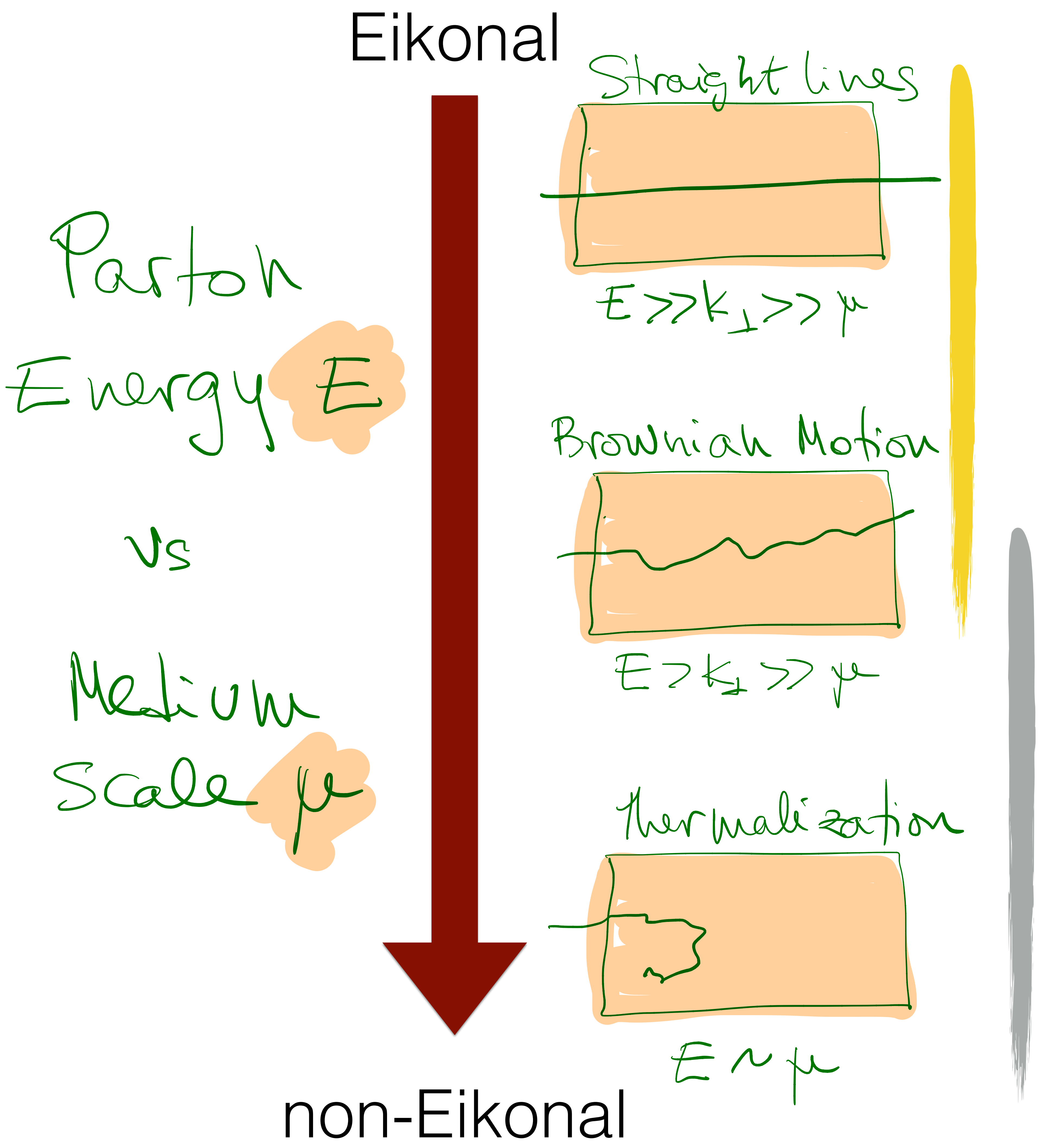
**Nestor Armesto talk Hard Probes 2004**

- Effect on $v_2$ is not large, (Wang, '03; Drees, Feng, Jia, '03); flow effects may mimic a higher density.

[Majumder, Muller, Bass 2007 propose $\hat{q}^{\mu\nu}$]

# FORMALISM

# In-medium parton propagation




**Medium is a background field: color rotation**

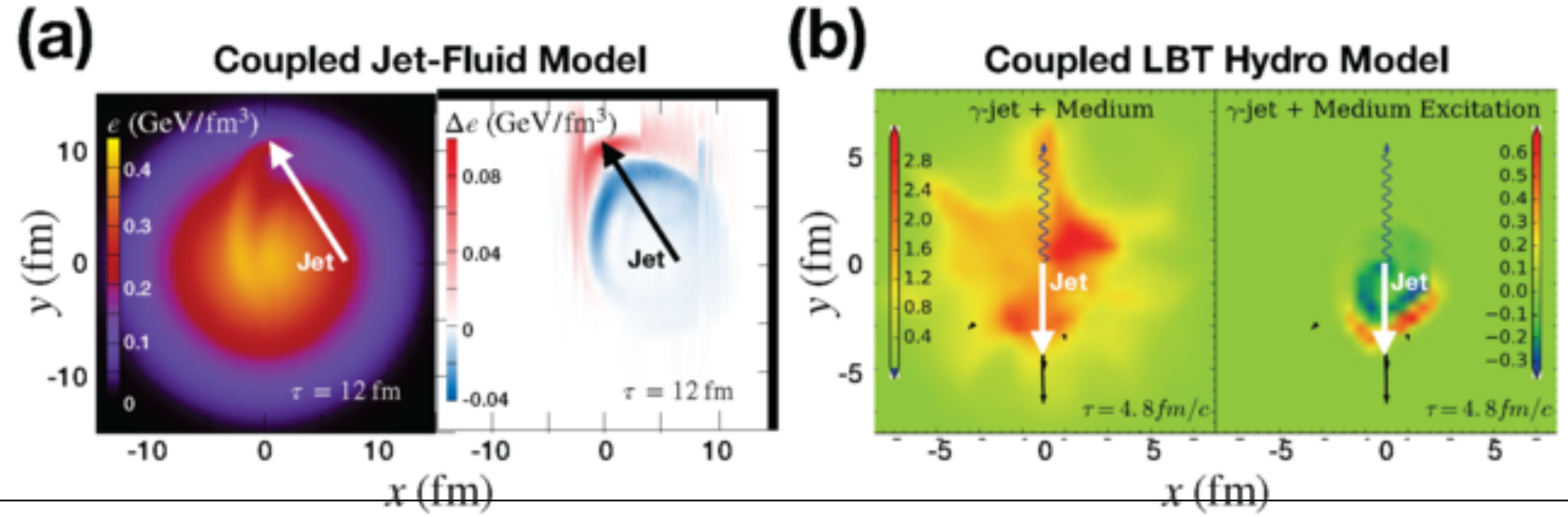
[Energy of the parton unmodified]

$$W(x_\perp) = \mathcal{P} \exp\left\{ig\int d\xi\, n\cdot A(\xi, x_\perp)\right\}$$

$$G(x_\perp; y_\perp) = \mathcal{P}\int \mathcal{D}\mathbf{r}\, \exp\left\{i\frac{E}{2}\int d\xi \left[\frac{d\mathbf{r}}{d\xi}\right]^2 + ig\int d\xi\, n\cdot A(\xi, \mathbf{r})\right\}$$

Medium is **dynamical**

[Energy exchanged with the medium]




[Tachibana 2019]

# Scattering amplitudes

**Color dipole** - The simplest configuration

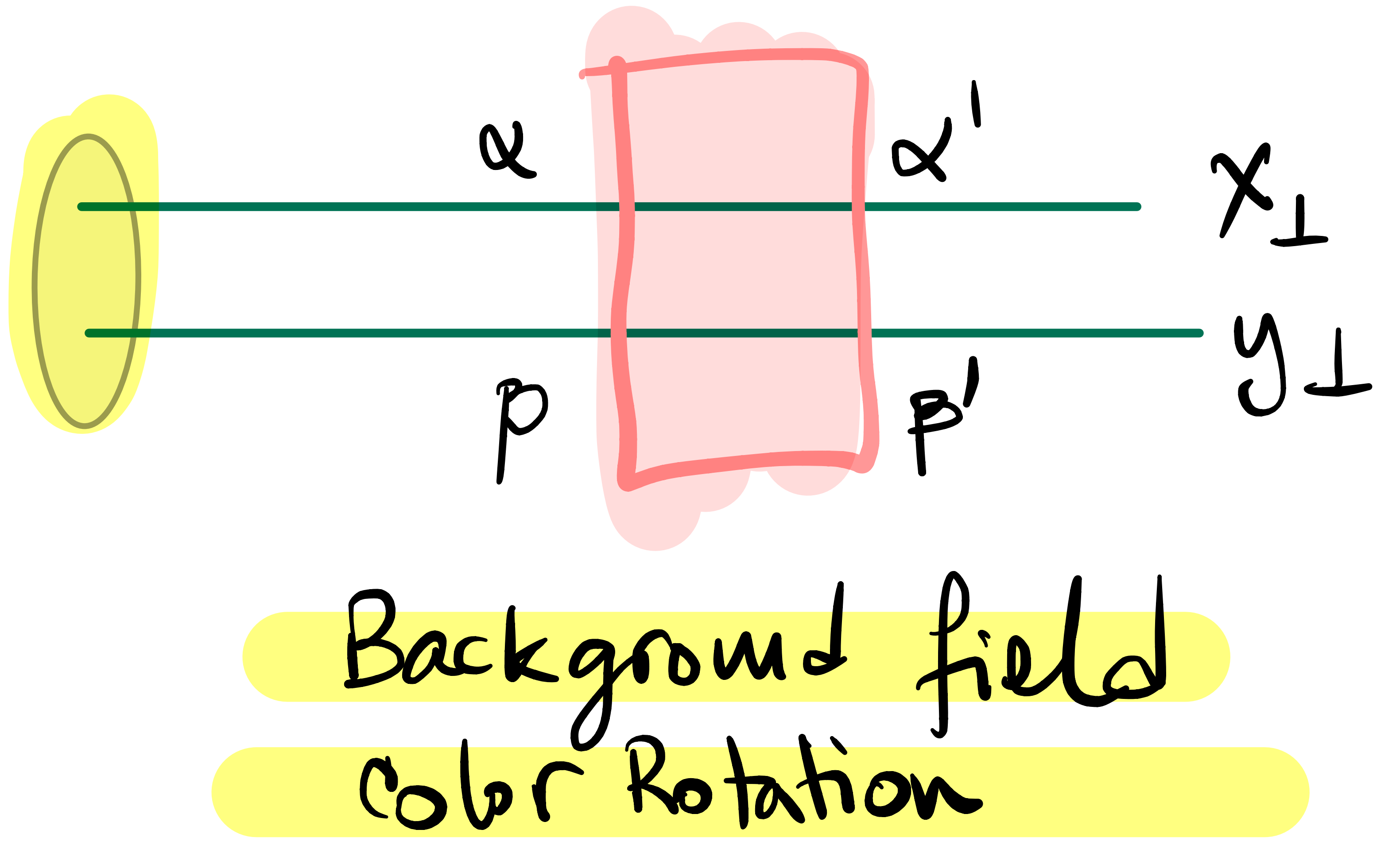
S-Matrix

$$|\alpha'\beta'\rangle = S_{\alpha'\beta'\alpha\beta}|\alpha\beta\rangle = W_{\alpha'\alpha}(x_\perp)W^\dagger_{\beta'\beta}(y_\perp)|\alpha\beta\rangle$$

Survival Probability —

$$S(x_\perp, y_\perp) = \frac{1}{N_c}\,\mathrm{tr}\left[W(x_\perp)W^\dagger(y_\perp)\right]$$

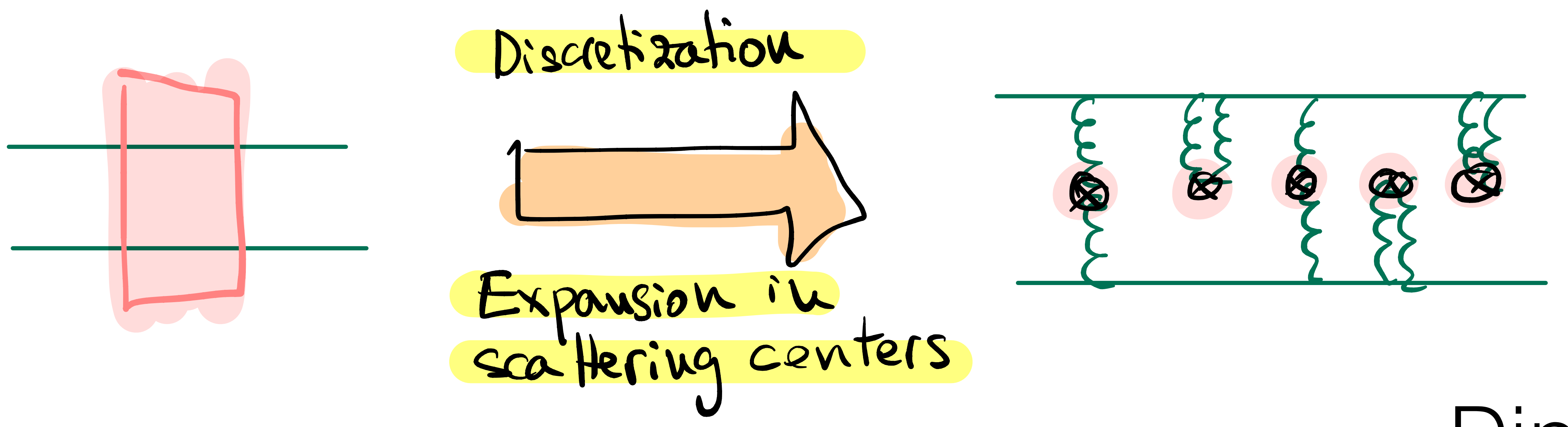
Average over Configurations $\quad \frac{1}{N_c}\left\langle \mathrm{tr}\left(W(x_\perp)W^\dagger(y_\perp)\right)\right\rangle_{med}$




**Medium averages needed - model of the medium**

# Medium averages

**A recoil-less medium $\sim$ a collection of static scattering centers**




$$\frac{1}{N^2-1}\mathrm{Tr}\left\langle W_A(\mathbf{x})W_A(\mathbf{y})\right\rangle = \exp\left\{-\frac{1}{2}\int_{t_0}^{t} ds\, n(s)\,\sigma(\mathbf{x}-\mathbf{y})\right\}$$

Dipole cross section $\sigma(\mathbf{r}) = \int_{\mathbf{q}} |v(\mathbf{q})|^2\left(1-e^{i\mathbf{q}\mathbf{r}}\right)$

In the harmonic approximation $S(\mathbf{x}_\perp,\mathbf{y}_\perp) \simeq \exp\left\{-\frac{1}{4}\hat{q}L(\mathbf{x}_\perp-\mathbf{y}_\perp)\right\}$

Where the second moment of the distribution defines the jet quenching parameter

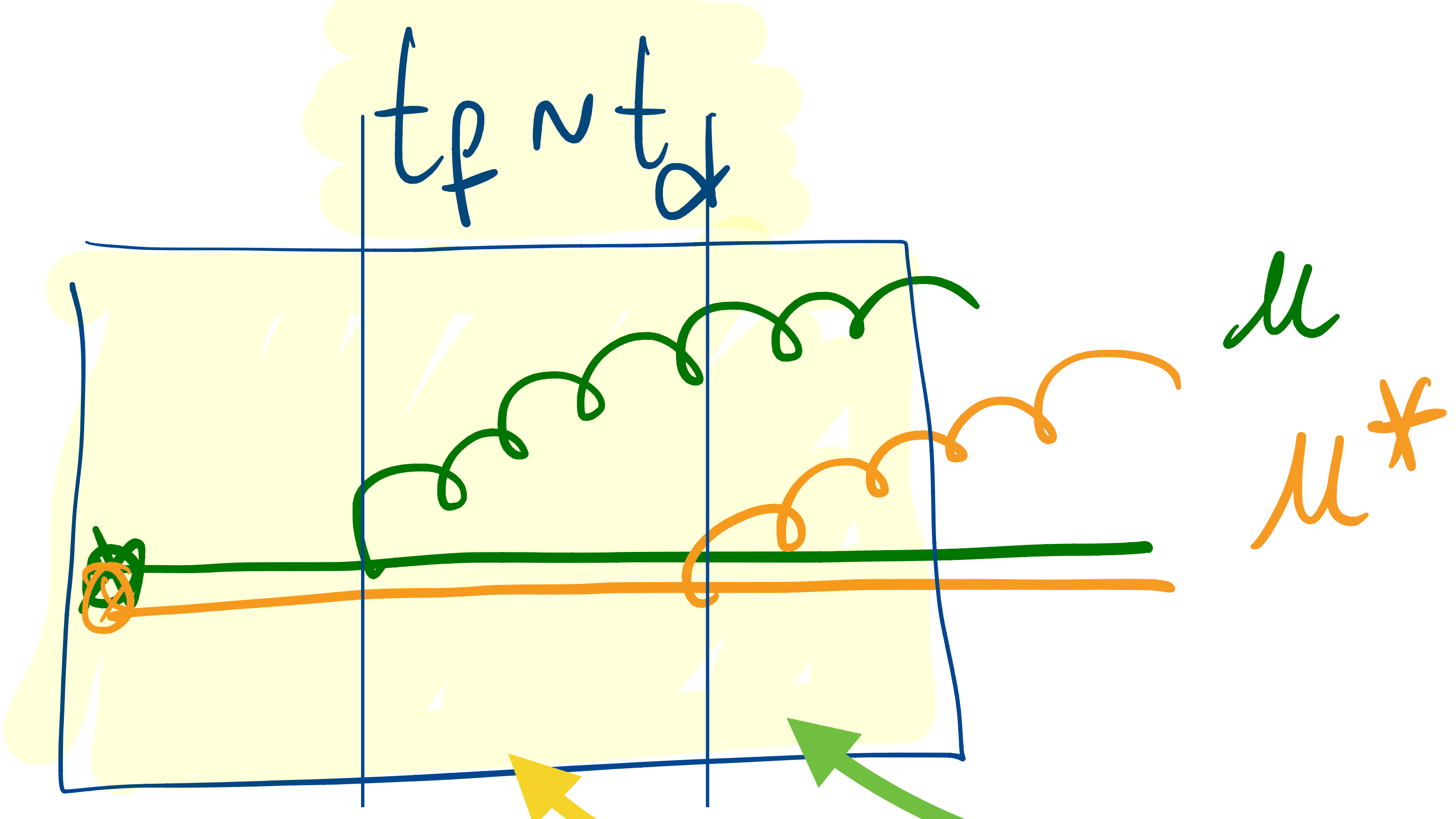
$$\hat{q} = \frac{\partial}{\partial L}\int_{\mathbf{q}} \mathbf{q}^2 S(\mathbf{q})$$

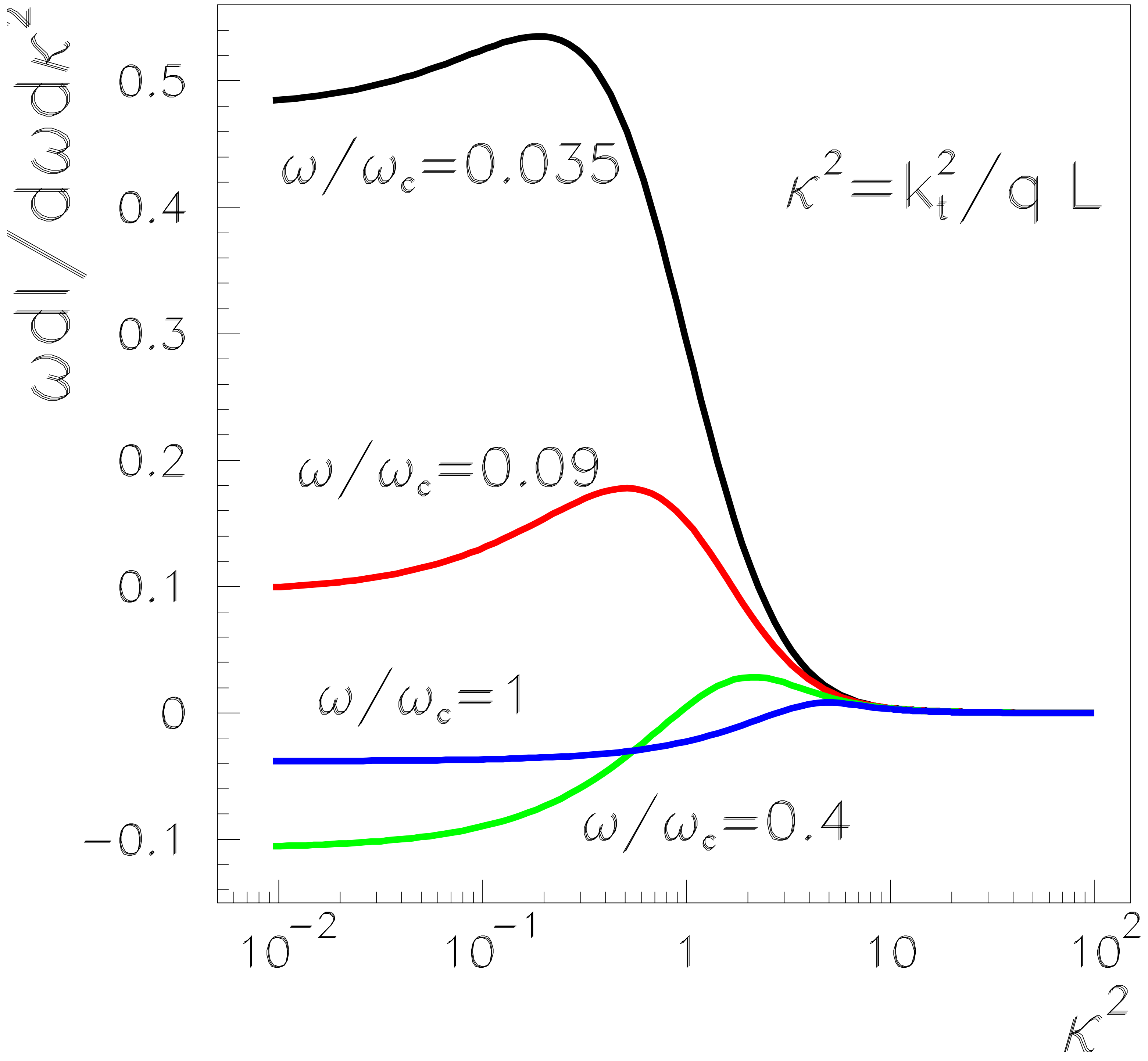
# Medium-induced radiation

[Zakharov, Baier, Dokshitzer, Mueller, Peigne, Schiff, Wiedemann, Gyulassy, Levai, Vitev, and many others… starting in the mid-90's]

For fluctuation with $t_f \sim t_d$ the gluon is resolved: **medium-induced radiation**

$$t_{\rm form} \sim \frac{1}{\omega\theta^2}, \quad t_{\rm d} \sim \sqrt{\frac{\hat{q}}{\omega}}, \quad \theta_{\rm d} \sim \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$



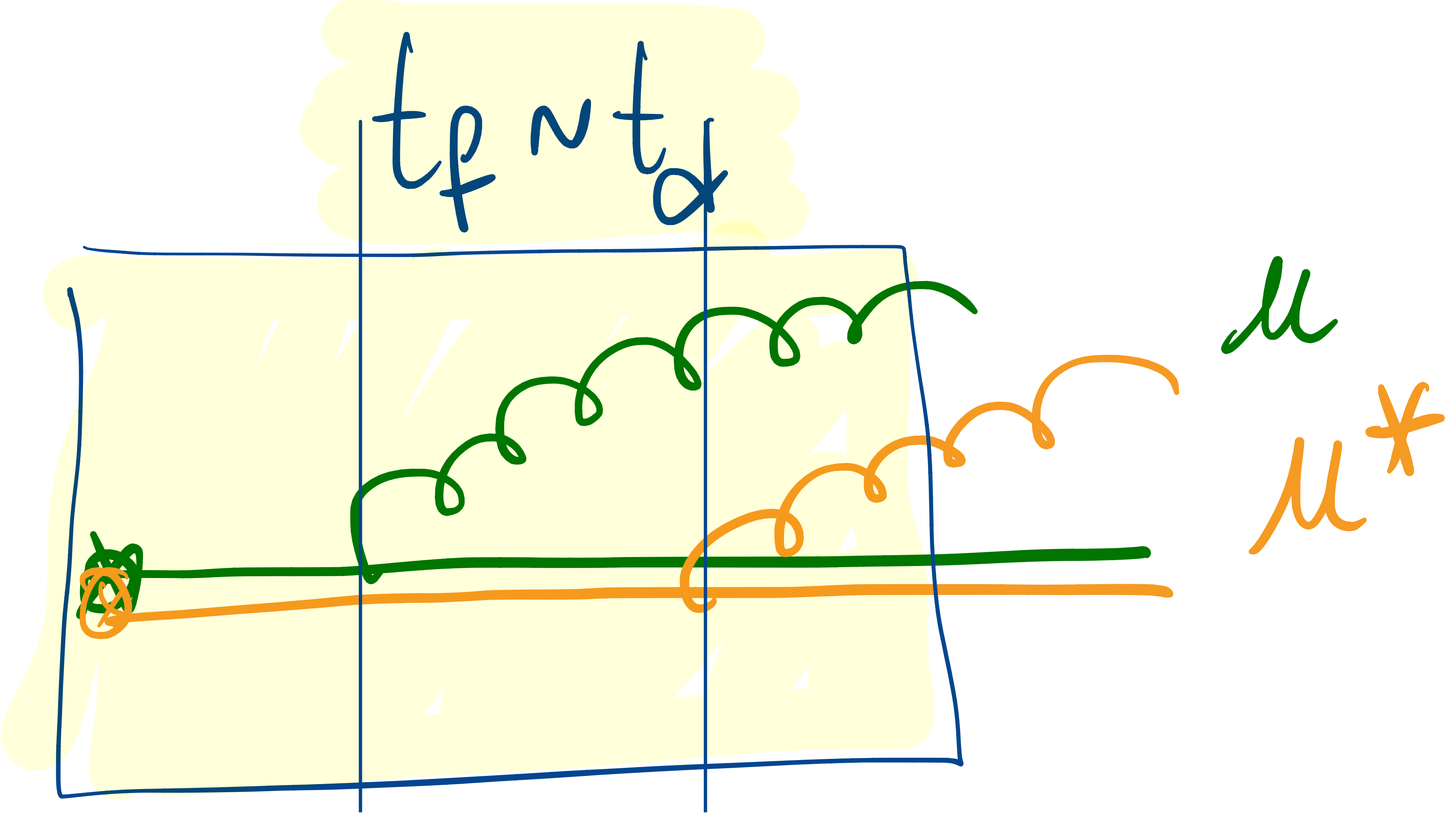

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} \sim \frac{\alpha_s C_R}{\omega^2} \mathrm{Re} \int_{t',t} \int_{\mathbf{p},\mathbf{q}} \mathbf{p}\cdot\mathbf{q}\,\tilde{\mathcal{K}}(t',\mathbf{q};t,\mathbf{p})\,\mathcal{P}(L,\mathbf{k};t',\mathbf{q})$$

$$\mathcal{K}(t',\mathbf{z};t,\mathbf{y}) = \int \mathcal{D}\mathbf{r}\,\exp\left[\int_t^{t'} ds\left(\frac{i\omega}{2}\dot{\mathbf{r}}^2 - \frac{1}{2}n(s)\sigma(\mathbf{r})\right)\right]$$

# Heavy quark radiation

Leading mass correction to Wilson line $W_M(\mathbf{x}_\perp, E) = W(\mathbf{x}_\perp) \times \exp\left\{ i \frac{M^2}{2E}(s-t) \right\}$

$$\omega \frac{dN}{d\omega d^2\mathbf{k}} \sim \frac{\alpha_s C_R}{\omega^2} \mathrm{Re} \int_{t',t} \int_{\mathbf{p},\mathbf{q}} \mathbf{p}\cdot\mathbf{q}\, \tilde{\mathcal{K}}(t',\mathbf{q};t,\mathbf{p}) \mathcal{P}(L,\mathbf{k};t',\mathbf{q}) \times \exp\left\{ i \frac{M^2}{2E} x(t'-t) \right\}$$



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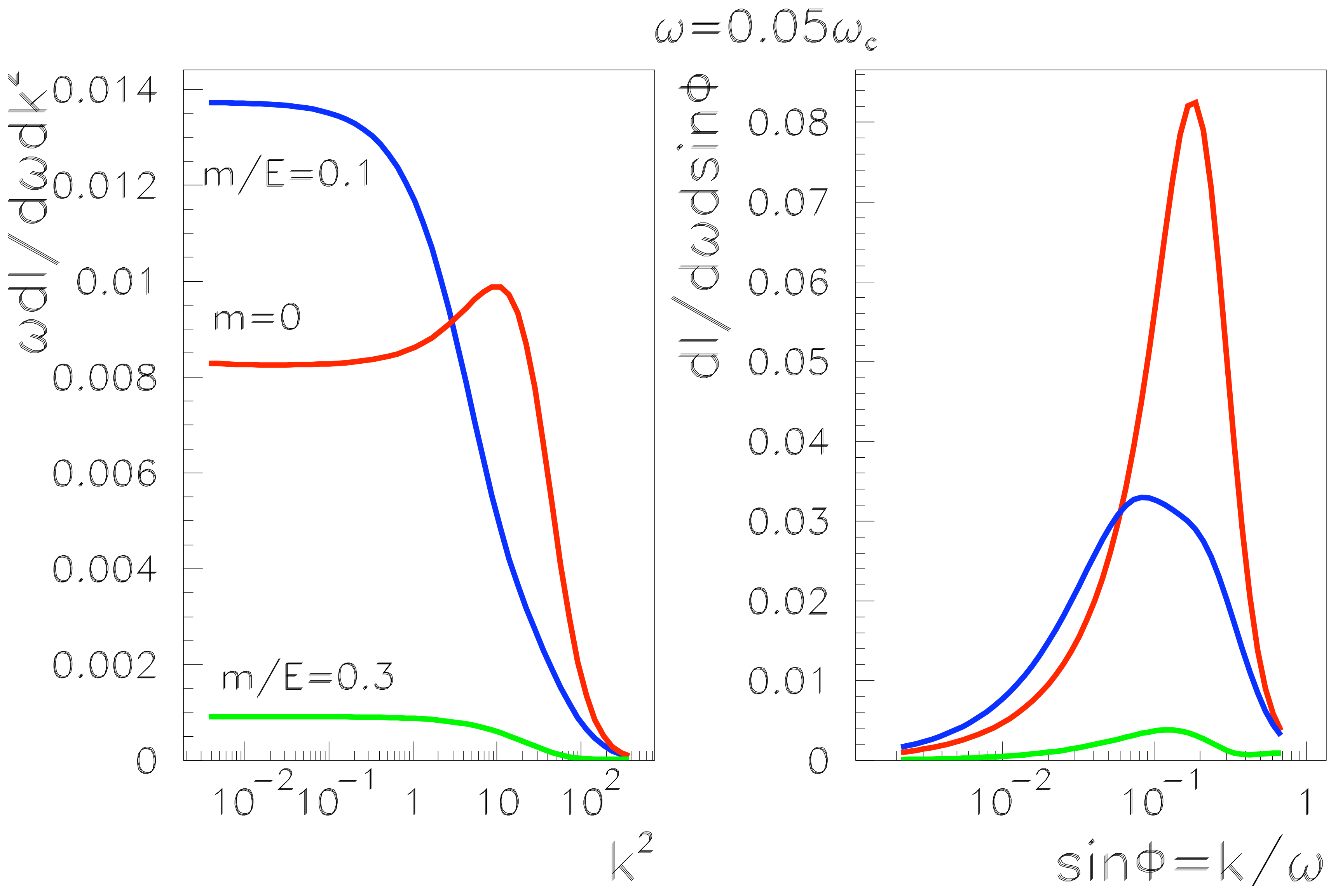
Taking $t_{\mathrm{form}} \sim \frac{1}{\theta^2 x E}$

$$\exp\left\{ i\frac{M^2}{2E}t_{\mathrm{form}} \right\} \simeq \exp\left\{ i\left(\frac{\theta_{\mathrm{DC}}}{\theta}\right)^2 \right\}$$

Radiation suppressed for $\theta < \theta_{\mathrm{DC}}$




[Armesto, Salgado, Wiedemann, 2003]

**For the medium, interplay with LPM radiation angle - competing effects**

# NOW WITH VELOCITY FIELDS

With gradients and velocity fields

$$gA^{a\mu}(q) = \sum_i u_i^\mu e^{-iq\cdot x_i} t_i^a v_i(q) (2\pi)\delta(q_0 - \boldsymbol{q}\cdot\boldsymbol{u} - q_z u_z)$$

- controls the jet-medium interaction ($v_i(q)$)
- controls de inhomogeneity ($\sum_i$)
- velocity of the sources ($u_i^\mu$)

[Stolen from Xoan Mayo - previous talk]

# “Directional broadening”

PRELIMINARY

The broadening is given by the average of two path integrals

$$W_L(\boldsymbol{p}, \boldsymbol{p}_{in}; \bar{\boldsymbol{p}}, \bar{\boldsymbol{p}}_{in}) = \left\langle \mathcal{G}(\boldsymbol{p}, L; \boldsymbol{p}_{in}, 0)\, \mathcal{G}^{\dagger}(\bar{\boldsymbol{p}}, L; \bar{\boldsymbol{p}}_{in}, 0) \right\rangle$$

The resummation of multiple scatterings can be done with

$$\frac{\partial}{\partial L} W_L(\boldsymbol{p}; \bar{\boldsymbol{p}}) = i \left( \frac{E}{\sqrt{E^2 - m^2}} \boldsymbol{u} \cdot (\boldsymbol{p} - \bar{\boldsymbol{p}}) - \frac{\boldsymbol{p}^2 - \bar{\boldsymbol{p}}^2}{2E} \right) W_L(\boldsymbol{p}; \bar{\boldsymbol{p}}) - \int_{\boldsymbol{l}\bar{\boldsymbol{l}}} \mathcal{K}(\boldsymbol{l}, \bar{\boldsymbol{l}}) W_L(\boldsymbol{l}; \bar{\boldsymbol{l}})$$

… and the kernels …

$$\begin{aligned}
\mathcal{K}(\boldsymbol{l}, \bar{\boldsymbol{l}}) = -\frac{\mathcal{C}\, E^2}{E^2 - m^2} \int_{\boldsymbol{q}\bar{\boldsymbol{q}}\boldsymbol{x}} \Big[ & v(\boldsymbol{q}, \boldsymbol{x}, L) v(\bar{\boldsymbol{q}}, \boldsymbol{x}, L) \rho(\boldsymbol{x}, L) e^{-i\boldsymbol{x}\cdot(\boldsymbol{q}-\bar{\boldsymbol{q}})} \delta^{(2)}(\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{l}) \delta^{(2)}(\bar{\boldsymbol{p}} - \bar{\boldsymbol{q}} - \bar{\boldsymbol{l}}) \\
& - \frac{1}{2} v(\boldsymbol{q}, \boldsymbol{x}, L) v(\bar{\boldsymbol{q}}, \boldsymbol{x}, L) \rho(\boldsymbol{x}, L) e^{-i\boldsymbol{x}\cdot(\boldsymbol{q}+\bar{\boldsymbol{q}})} \delta^{(2)}(\boldsymbol{p} - \boldsymbol{l}) \delta^{(2)}(\bar{\boldsymbol{p}} - \boldsymbol{q} - \bar{\boldsymbol{q}} - \bar{\boldsymbol{l}}) \\
& - \frac{1}{2} v(\boldsymbol{q}, \boldsymbol{x}, L) v(\bar{\boldsymbol{q}}, \boldsymbol{x}, L) \rho(\boldsymbol{x}, L) e^{i\boldsymbol{x}\cdot(\boldsymbol{q}+\bar{\boldsymbol{q}})} \delta^{(2)}(\boldsymbol{p} - \boldsymbol{q} - \bar{\boldsymbol{q}} - \boldsymbol{l}) \delta^{(2)}(\bar{\boldsymbol{p}} - \bar{\boldsymbol{l}}) \Big] . \qquad (12)
\end{aligned}$$

# $\hat{q}$ is a tensor

PRELIMINARY

The information from these expressions can be encoded computing the (generalized) jet quenching parameter

$$\hat{q}_{ij} = \frac{\partial}{\partial L}\langle \boldsymbol{p}_i \boldsymbol{p}_j \rangle = -\frac{1}{\mathcal{N}} \int_{\boldsymbol{p}} \boldsymbol{p}_i \boldsymbol{p}_j \int_{\boldsymbol{l}\bar{\boldsymbol{l}}} \mathcal{K}(\boldsymbol{l}, \bar{\boldsymbol{l}})\Big|_{\bar{\boldsymbol{p}}=\boldsymbol{p}} W_L(\boldsymbol{l}; \bar{\boldsymbol{l}})$$

$$\simeq \left(1 - \frac{EL}{\sqrt{E^2 - m^2}} \boldsymbol{u} \cdot \boldsymbol{\nabla} g \frac{\delta}{\delta g}\right) \frac{\mathcal{C}\rho(L)\, E^2}{E^2 - m^2} \int_{\boldsymbol{q}} \boldsymbol{q}_i \boldsymbol{q}_j [v(\boldsymbol{q}, L)]^2$$

$$\simeq \frac{1}{2}\left(1 - \frac{EL}{\sqrt{E^2 - m^2}} \boldsymbol{u} \cdot \boldsymbol{\nabla} g \frac{\delta}{\delta g}\right) \hat{q}_0 \left[\left(1 - \frac{m^2 \boldsymbol{u}^2}{2E^2}\right)\delta_{ij} - \boldsymbol{u}_i \boldsymbol{u}_j \frac{m^2}{E^2}\right]$$

[Also considered in Hauksson, Iancu (2023) and Barata, Salgado, Silva (2024) - see next talk!]

# Directional radiation

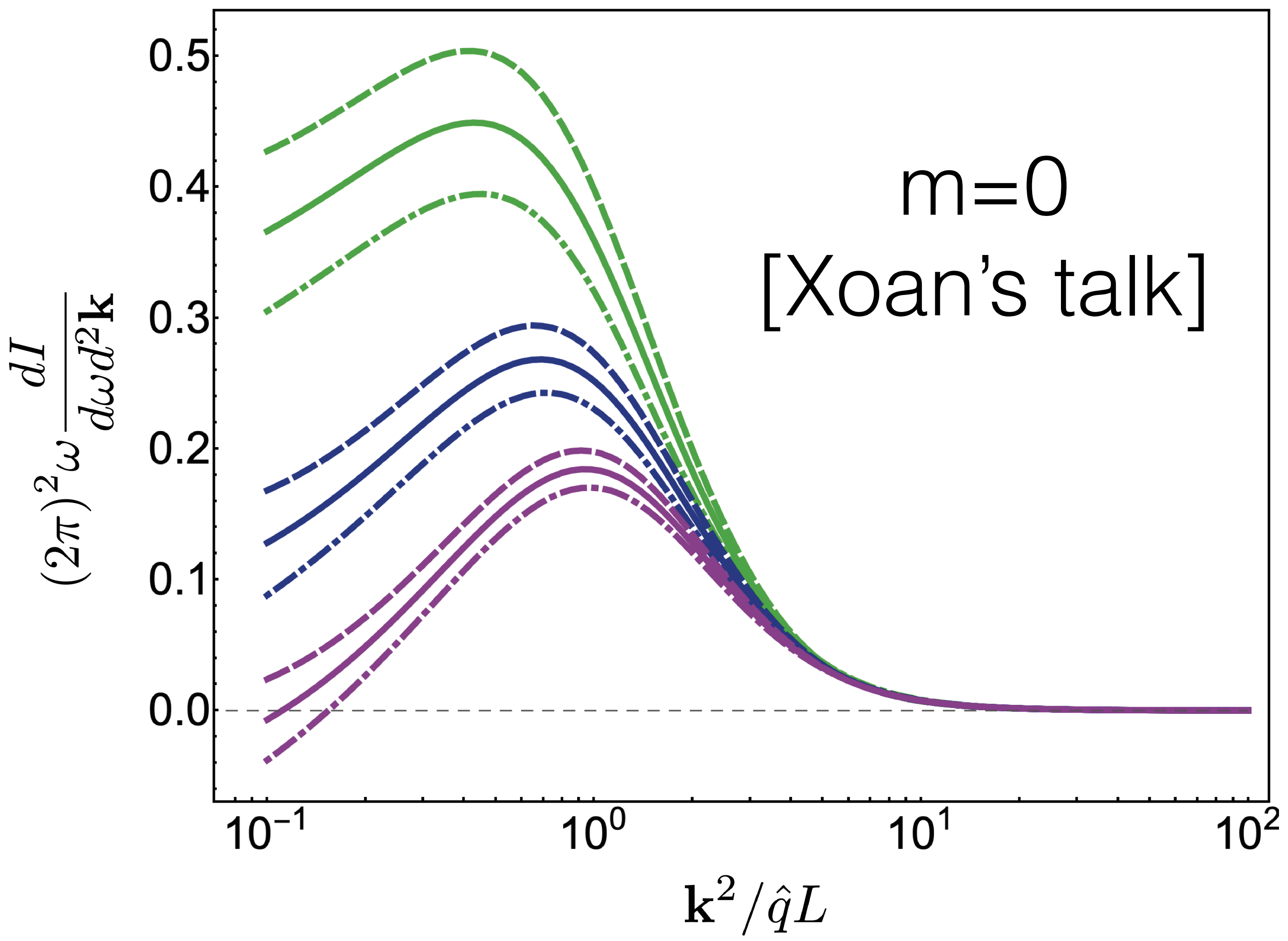
PRELIMINARY

$$2(2\pi)^3\omega E\frac{d\mathcal{N}}{d\omega dE d^2\boldsymbol{k}} \simeq \frac{2g^2C_F}{4\pi\omega^2}\mathrm{Re}\left(\frac{E}{\sqrt{E^2-m^2}}\right)^2\int_0^\infty d\bar{z}_s\int_0^{z_s}dz_s\int_{\boldsymbol{y}} e^{i\frac{m^2x^2}{2\omega}(z_s-\bar{z}_s)}$$
$$\times\left(\boldsymbol{\nabla}_{\boldsymbol{x}}\cdot\boldsymbol{\nabla}_{\bar{\boldsymbol{x}}}\right)\bar{S}_2\left(\boldsymbol{k},\boldsymbol{k},z_f;\bar{\boldsymbol{x}},\boldsymbol{y},\bar{z}_s\right)\bar{\mathcal{K}}\left(\boldsymbol{y},0,\bar{z}_s;\boldsymbol{x},0,z_s\right)\Big|_{\boldsymbol{x}=\bar{\boldsymbol{x}}=0}.$$

We can now also define a tensorial jet quenching parameter

$$\mathcal{V}_{gq}(\boldsymbol{x}) \simeq \frac{1}{4}\hat{q}_\parallel x_1^2 + \frac{1}{4}\hat{q}_\perp x_2^2$$

$$\hat{q}_\parallel = \hat{q}\left[1+\frac{1}{2}\frac{m^2}{E^2}\left(1-\frac{1}{2}\boldsymbol{u}^2-\left(1-\frac{\hat{q}_0}{2\hat{q}}\right)\boldsymbol{u}^2\right)\right]$$
$$\hat{q}_\perp = \hat{q}\left[1+\frac{1}{2}\frac{m^2}{E^2}\left(1-\frac{1}{2}\boldsymbol{u}^2\right)\right]$$



# Conclusions

**Gradients (T, density, etc…) and flow velocities modify jet properties —broadening and medium-induced radiation**

- Softer particles are bent in the gradient / velocity direction - effect is subleading in energy
- Additional source of energy loss that could be very important phenomenologically ($R_{AA}$ vs $v_2$)

**For the massive case**

- Jet quenching parameter becomes a **tensor** both for broadening and radiation
- Some observable consequences in the next talk by João Silva
- **Dead cone effect is then also directional** - mass effect in radiation depends on the relative direction of the propagation of the quark and the fluid velocity

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# Acknowledgements