Fluid-dynamic approach to heavy-quark diffusion in the quark-gluon plasma

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Based on
Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108 (2023) 11, 116011
Capellino, Beraudo, Dubla, Floerchinger, Pawlowski, Masciocchi, Selyuzhenkov, PRD 106 (2022) 3, 034021

Heavy quarks as probes of the QGP

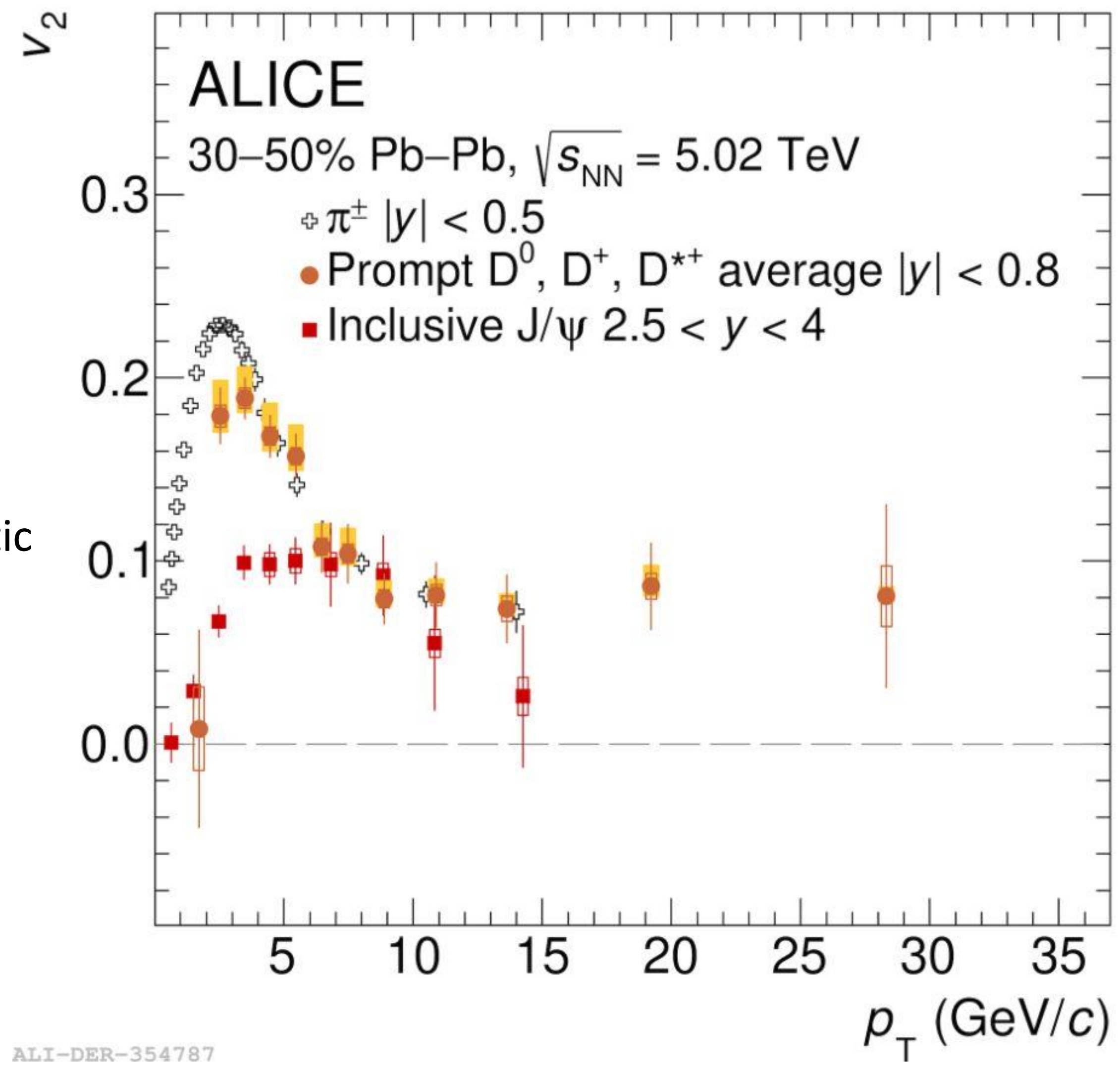
ALICE PLB 813 (2021) 136054

- Produced via hard scatterings at the beginning of the collision before the QGP is formed: **they go through all the stages of the expanding fireball**

- In the low p_T region they provide a window to study **equilibration processes**

Significant measurements of J/ψ and D mesons of positive elliptic flow

- Do **charm quarks** interact long enough with the medium to be considered part of the medium itself?



Thermalization

If particles have enough time to interact with each other they will eventually relax to (at least local) thermal equilibrium.

- **Chemical equilibrium:**

the particle multiplicity is given by a thermal distribution at a **unique (local) chemical potential** ($\mu = 0$)

HQs: initial hard production far from chemical equilibrium with the QGP! **Fugacity factor $e^{q\alpha}$** needed.

- **Kinetic equilibrium:**

the momentum distribution of the particles approaches a Maxwell-Boltzmann distribution described by a **unique (local) temperature**.

HQs: possibly get quite close to local kinetic equilibrium in the QGP within the lifetime of the fireball.

$$f(E,x) \sim \underbrace{e^{-E/T(x)} e^{q\alpha}}_{\text{in kinetic equilibrium}} + \underbrace{\delta f(E,x)}_{\text{out of kinetic equilibrium}}$$

A new approach: fluid dynamics for heavy quarks

Supported by lQCD calculations [Altenkort *et al.* PRL 130 (2023) 23, 231902], we assume heavy quarks had enough time to interact with the light thermal partons of the medium and to approach local kinetic equilibrium

→ we treat them with a **fluid-dynamic approach**!

We write down a current associated to the **conservation of QQbar pairs** in the medium: *effective* symmetry

→ **Number of QQbar pairs fixed by initial hard production!**

$$N^\mu = n\, u^\mu + \nu^\mu \qquad \nabla_\mu N^\mu = 0$$

HQ density (n)

HQ diffusion current (ν^μ)

Equation of motion (Israel-Stewart type):

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \alpha$$

gives rise to the fugacity factor (α)

Relaxation time (τ_n)

HQ diffusion coefficient (κ_n)

Fluid-dynamic transport coefficients

We computed the **relaxation time** and **diffusion coefficient** associated to heavy quarks by integrating the first moment of the Fokker-Planck equation

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

Drag coefficient

Momentum-diffusion coefficient

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

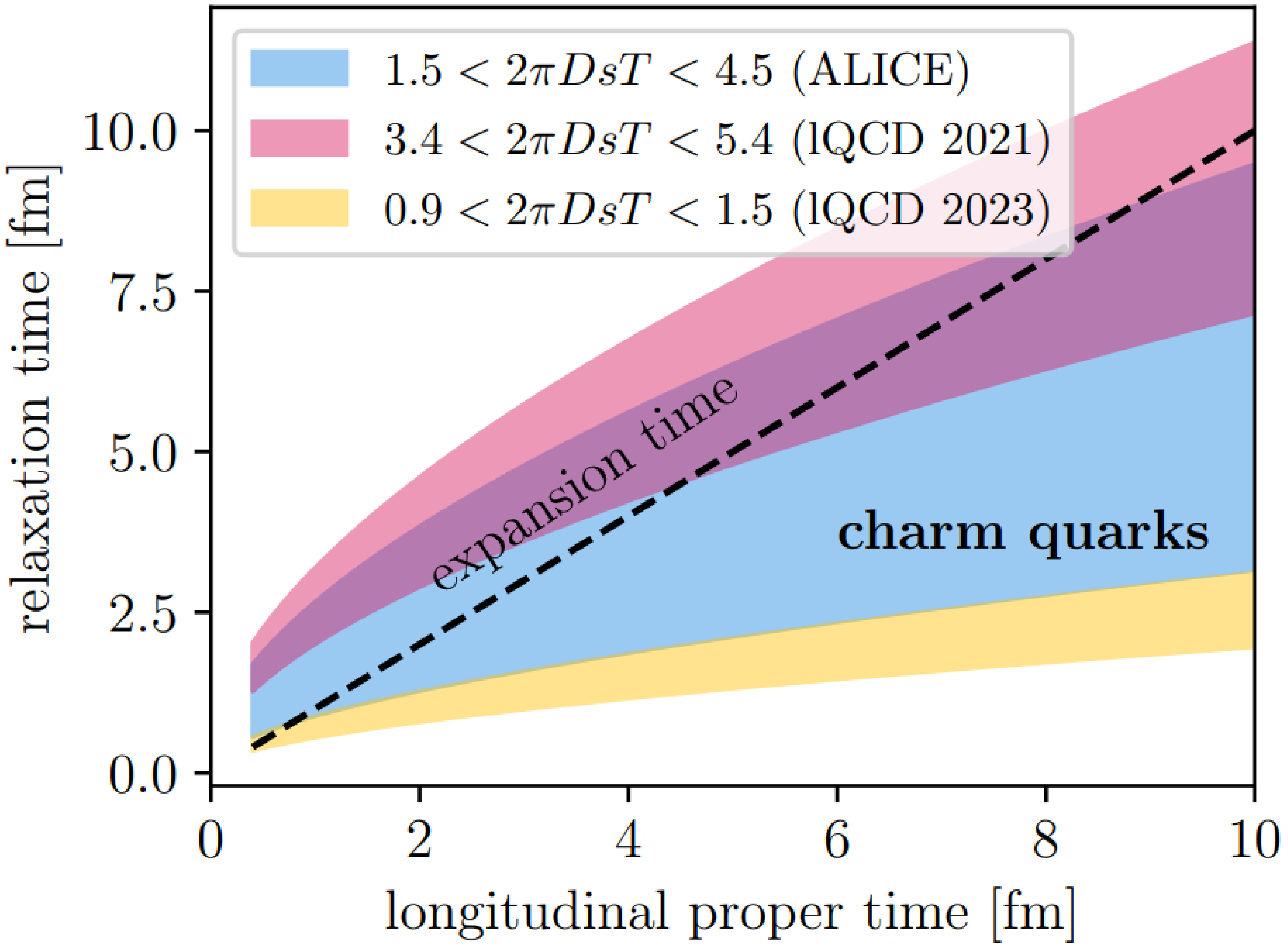
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Where the **spatial diffusion coefficient** is defined as

$$D_s = \lim_{k\to 0} \frac{T}{M A(k)}$$

Capellino *et al.* PRD 106 (2022) 034021

Charm quark relaxation time



Capellino *et al.* PRD 106 (2022) 034021 (updated with new lQCD results)

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Bjorken flow: $v_x = v_y = 0 \quad v_z = z/t$

lQCD 2021: Altenkort *et al.* PRD 103 (2021) 014511
lQCD 2023: Altenkort *et al.* PRL 130 (2023) 23, 231902
ALICE fits to data: ALICE JHEP 01 (2022) 174

Relaxation time is much shorter than typical expansion time of the QGP in Bjorken flow
→ **Fluid-dynamic description of charm looks meaningful!**

Fluid dynamics with a conserved charge

We implement the conservation of the energy-momentum tensor and charm current

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu N^\mu = 0$$

together with a lQCD-inspired Equation of State and Israel-Stewart equations of motion for the dissipative currents.

Equation of State: **Hadron Resonance Gas of charm states** $n(T,\alpha) = \frac{T}{2\pi^2} \sum_{i \in \mathrm{HRGc}} q_i M_i^2 e^{q_i \alpha} K_2(M_i/T)$

We assume that the **charm fugacity** and **diffusion current** are a **perturbation** with respect to $T, u^\mu, \pi^{\mu\nu}, \Pi$

→ they don't contribute significantly to the thermodynamics of the bulk evolution of the system!

E.g. in the non-dissipative case

$$P = P(T) \qquad n = n(T,\alpha)$$
$$\epsilon = \epsilon(T)$$

→ The evolution of T and u^μ is not influenced by α

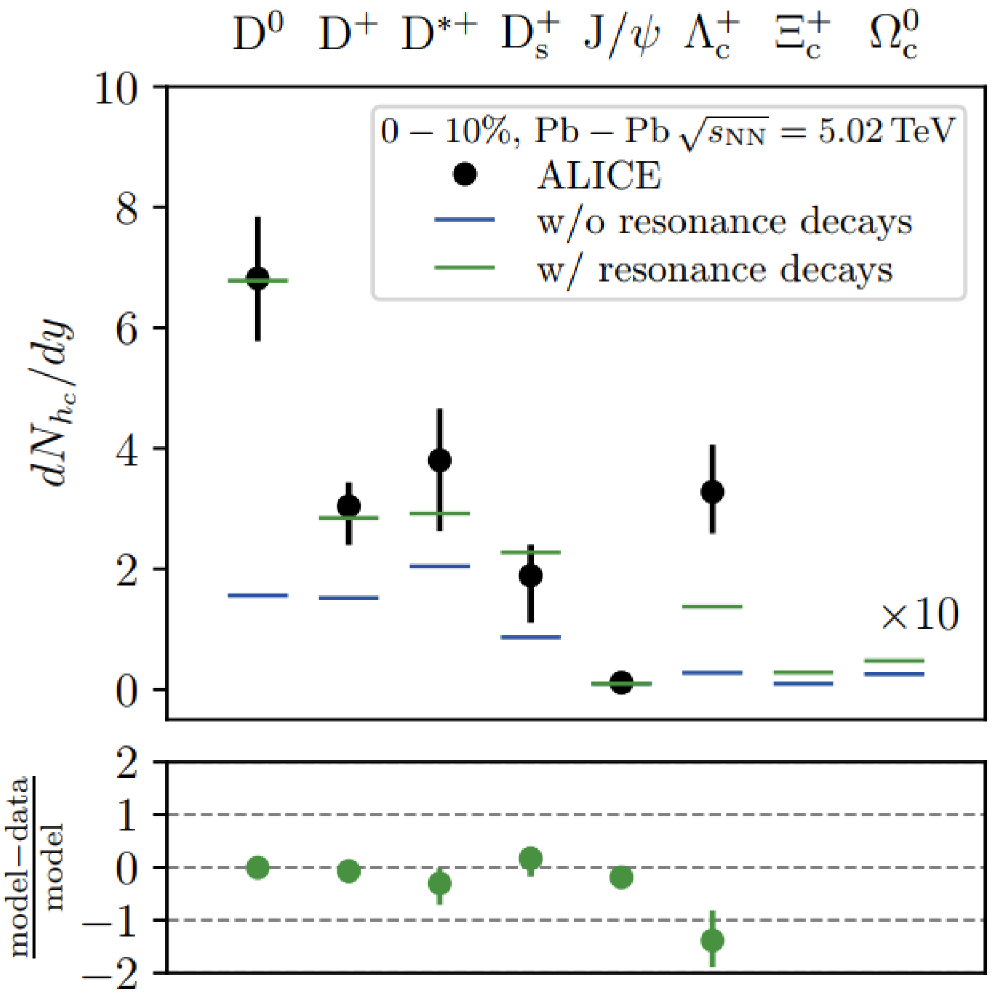
Charm-hadron integrated yields *dN/dy*

- ❑ Resonance decays are taken into account
Mazeliauskas *et al.* Eur. Phys. J. C (2019) 79: 284

- ❑ **Mesons are compatible with the experimental data within uncertainties**

- ❑ Deviation of 2.4σ for Λ_c^+:
  - missing higher resonance states?
    He, Rapp, PLB 795, 117 (2019)
    Andronic *et al.* JHEP 07, 035 (2021)
  - coalescence mechanisms?
    Plumari *et al*. Eur. Phys. J. C 78, 348 (2018)
    Beraudo *et al*., Eur. Phys. J. C 82, 607 (2022)

- ❑ Prediction for not-yet-measured open-charm states Ξ_c^+, Ω_c^0



Capellino *et al.* PRD 108 (2023) 11, 116011
ALICE JHEP 01 (2022) 174, ALICE PLB 849 (2024) 138451, ALICE PLB 839 137796 (2023), ALICE PLB 827 136986 (2022)

Momentum distributions

We compute spectra with a **Cooper-Frye prescription at T_{fo} = 156.5 MeV**

Notice: dissipative corrections on the freeze-out surface are missing!

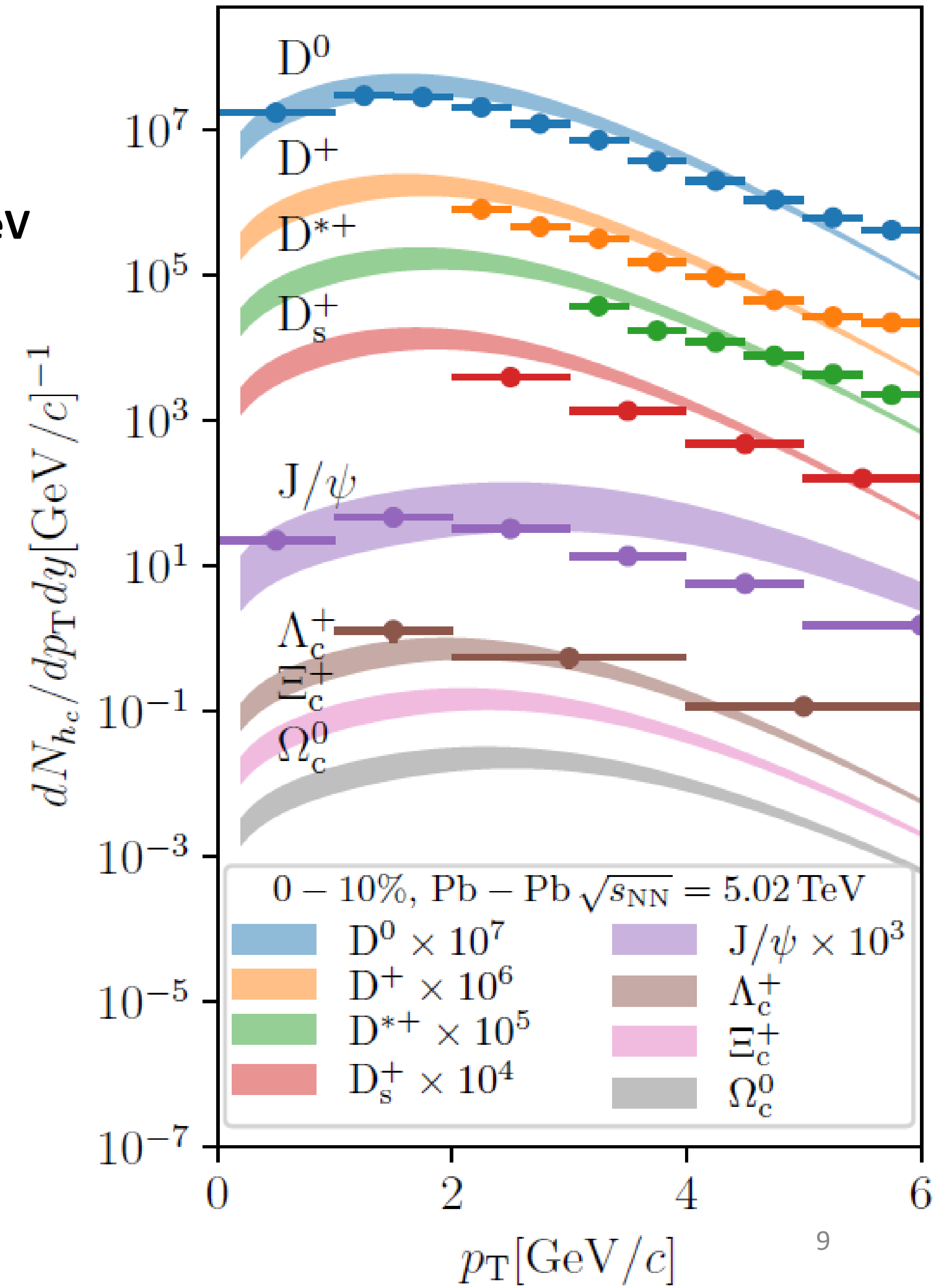
→ Expanding in terms of

$$f^{(hc)} = f^{(hc)}_{\mathrm{eq}} + \delta f^{(hc)}_{\mathrm{bulk}} + \delta f^{(hc)}_{\mathrm{shear}} + \delta f^{(hc)}_{\mathrm{diff}}$$

multi-fluid setup? Maximum entropy method?

- **The fluid-dynamic description of charm captures the physics of D mesons up to 4-5 GeV and J/ψ up to 3 GeV!**
- Expected deviation for Λ_c^+ as seen in integrated yields
- deviation for J/ψ : primordial J/ψ

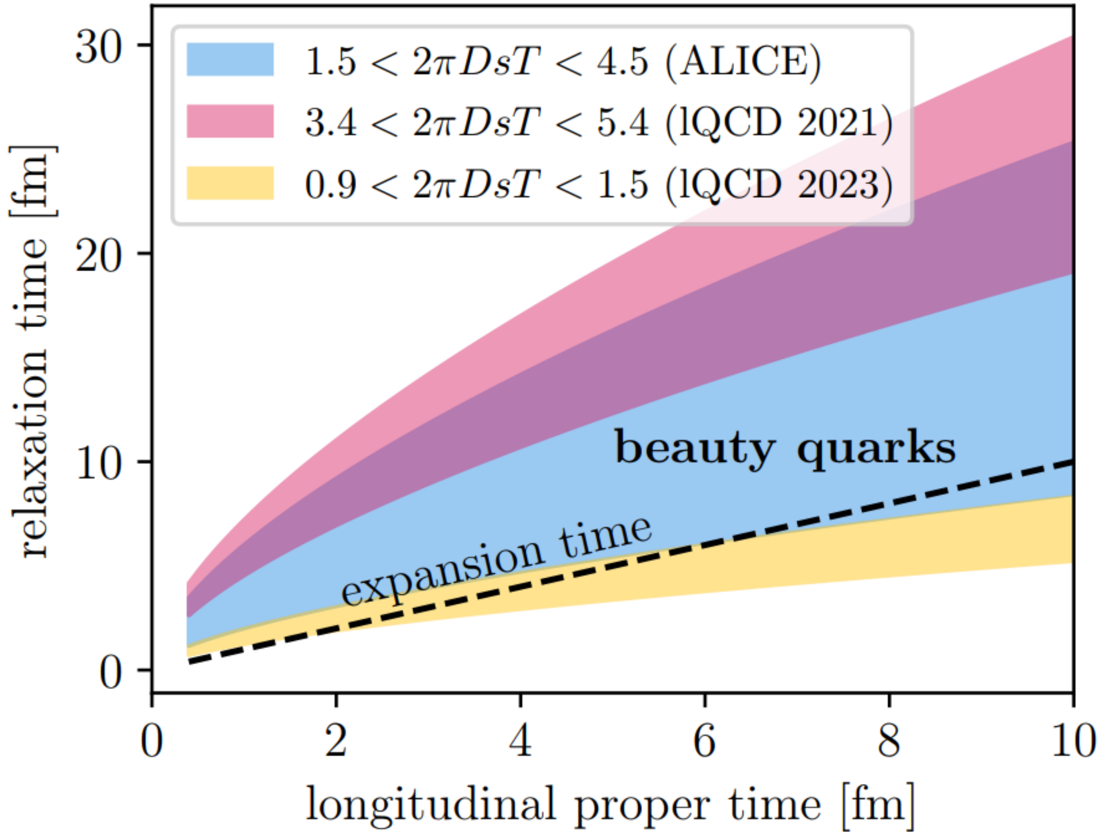
Capellino *et al.* PRD 108 (2023) 11, 116011
ALICE JHEP 01 (2022) 174
ALICE (2023), arXiv:2303.13361 [nucl-ex]
ALICE PLB 839, 137796 (2023)
ALICE PLB 827, 136986 (2022)



What about beauty quarks?

Recent LQCD result <u>don't exclude</u> hydrodynamization in late stages of the fireball.

Fluid dynamics of beauty quarks?



Need of precise measurements for spectra and flow coefficients to study thermalization of bottom quarks in the QGP

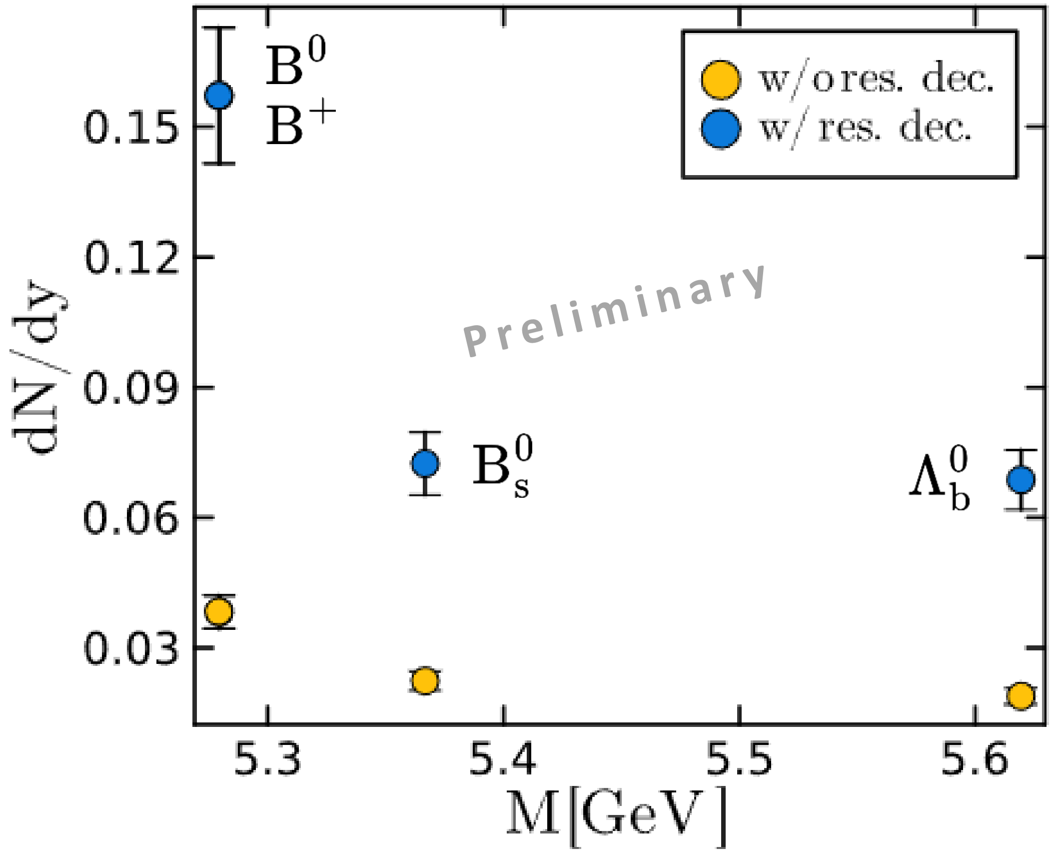
→ **Run3 at the LHC: new data for pp and Pb-Pb collisions**

Capellino *et al.* PRD 106 (2022) 034021

Open beauty hadron integrated yields

Fluid dynamics of beauty quarks in PbPb collisions at 5.02 TeV

Cooper-Frye prescription at T_{fo} = 156.5 MeV



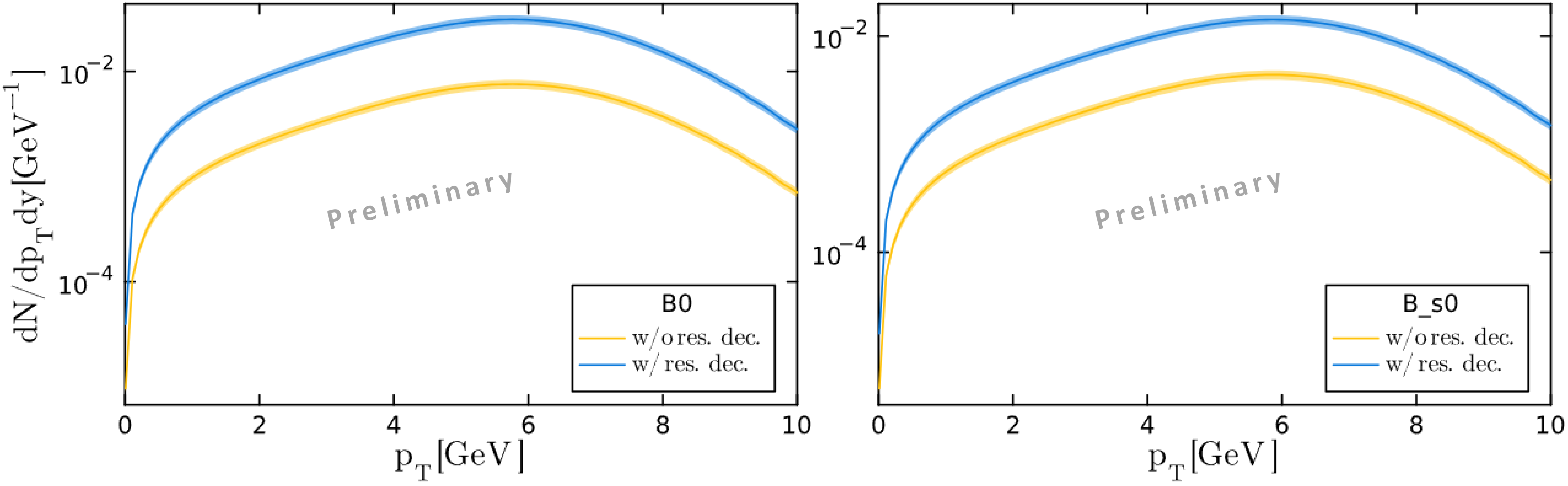
- Uncertainty from $b\bar{b}$ production cross section
  fugacity = $0.86 \cdot 10^9 - 1.05 \cdot 10^9$
- Resonance decays are taken into account
  Mazeliauskas et al. Eur. Phys. J. C (2019) 79: 284
- Results compatible with SHMb
  Andronic et al. QM 2022 arXiv:2209.14562
- Presence of **currently unknown open bottom** states would lead to a change in hadrons' relative abundance

Open beauty mesons

Fluid dynamics of beauty quarks in PbPb collisions at 5.02 TeV

Cooper-Frye prescription at T_{fo} = 156.5 + resonance decays

- Caveat: no spatial diffusion included yet!

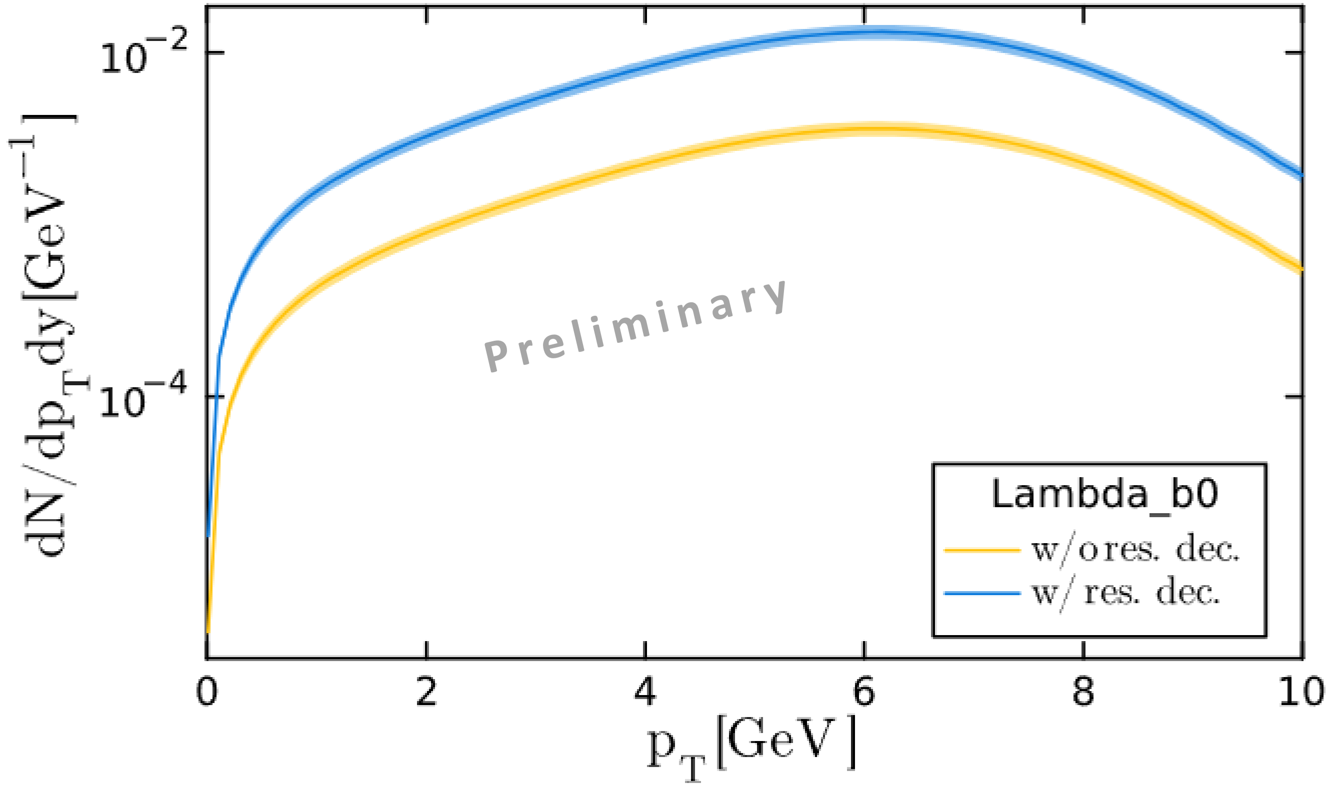


Beauty baryons

Fluid dynamics of beauty quarks in PbPb collisions at 5.02 TeV

Cooper-Frye prescription at T_{fo} = 156.5 + resonance decays

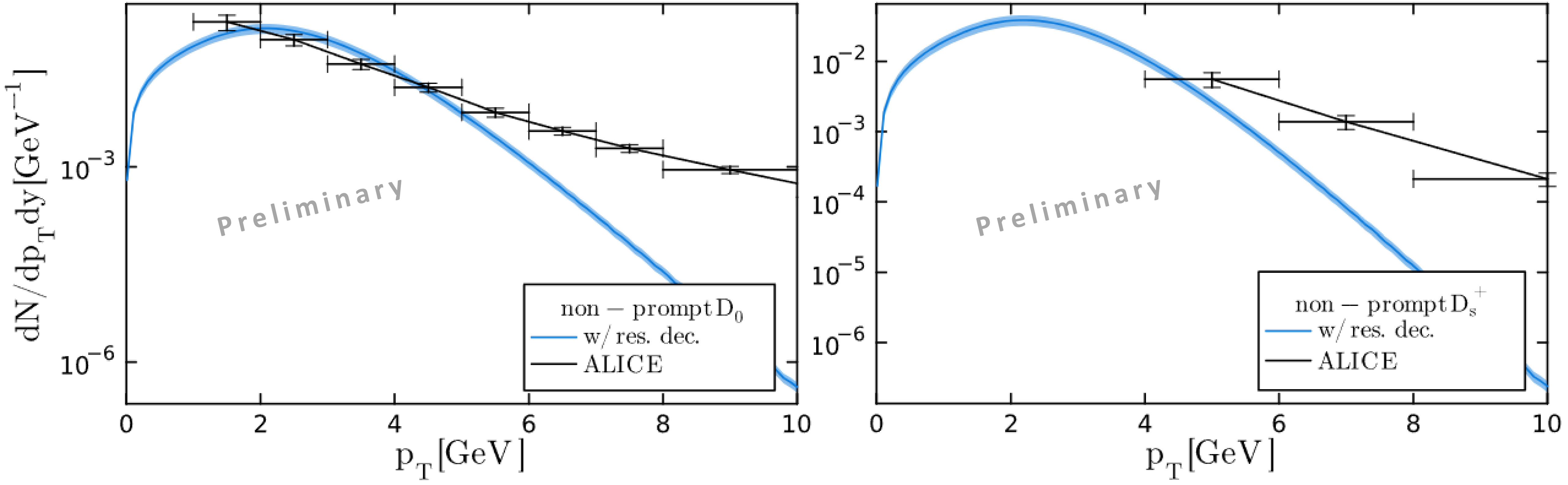
- Caveat: no spatial diffusion included yet!



Presence of yet **unknown resonances** would enhance the integrated yield (as in the charm sector)

Non-prompt charm hadrons

- Pythia8 decay list to implement weak beauty decays
- Momentum distribution compatible with experimental data within uncertainties up to 5 GeV (uncertainty on branching ratio not displayed here)



ALICE JHEP 12 (2022) 126, 2022
ALICE PLB 846 (2023) 137561, 2023

Summary and outlook

Summary

- ✓ We presented an approach based on **fluid dynamics** to describe **charm and beauty** quark dynamics in the QGP
- ✓ Theoretical studies and experimental data point towards **full thermalization of charm quarks** within the lifetime of the QGP at LHC energies
- ✓ Integrated yields as well as the momentum distributions of charm hadrons are **consistent with experimental data up to** p_T **of 4-5 GeV** → **tension in the charm baryon sector**
- ✓ **Beauty**: fluid dynamics not excluded, preliminary results **consistent with non-prompt charm mesons experimental data**

Outlook

- ❑ Full treatment with **viscous corrections** at the freeze-out is necessary
- ❑ Extension to **flow coefficients + Bayesian analysis**

Thank you for your attention!

Back up

Equation of motion for the HQ diffusion current

We derive hydrodynamic equations of motion from kinetic theory (Boltzmann) in a **Fokker-Planck approximation**

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

By integrating the first moment of the equation

$$\int dP p^\nu p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \int dP p^\nu \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

We obtain a relaxation-type equation for the diffusion current

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \left(\frac{\mu}{T}\right)$$

Relaxation time (τ_n)

HQ diffusion coefficient (κ_n)

$$\tau_n = \frac{D_s I_{31}}{T P_o}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Fluid-dynamic transport coefficients

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

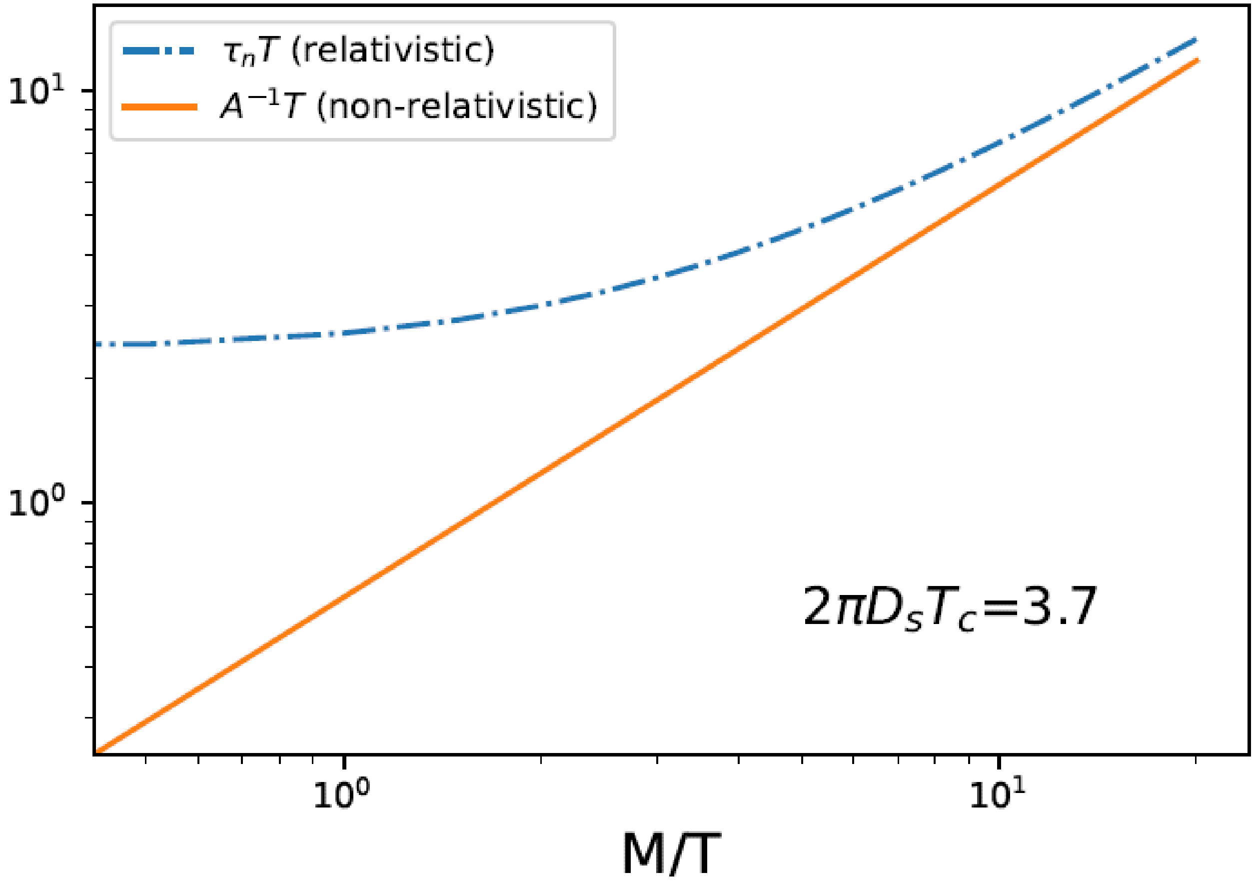
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

$$I_{31} = \frac{1}{3}\int dP p^0 p^2 f_0(p)$$

$$p^0 \sim M \qquad I_{31} \sim M P_0$$

$$\tau_n \sim \frac{D_s M P_0}{T P_0} = D_s \frac{M}{T}$$

$$\tau_n = (2\pi D_s T)\frac{1}{2\pi}\frac{M}{T} = A^{-1} T$$



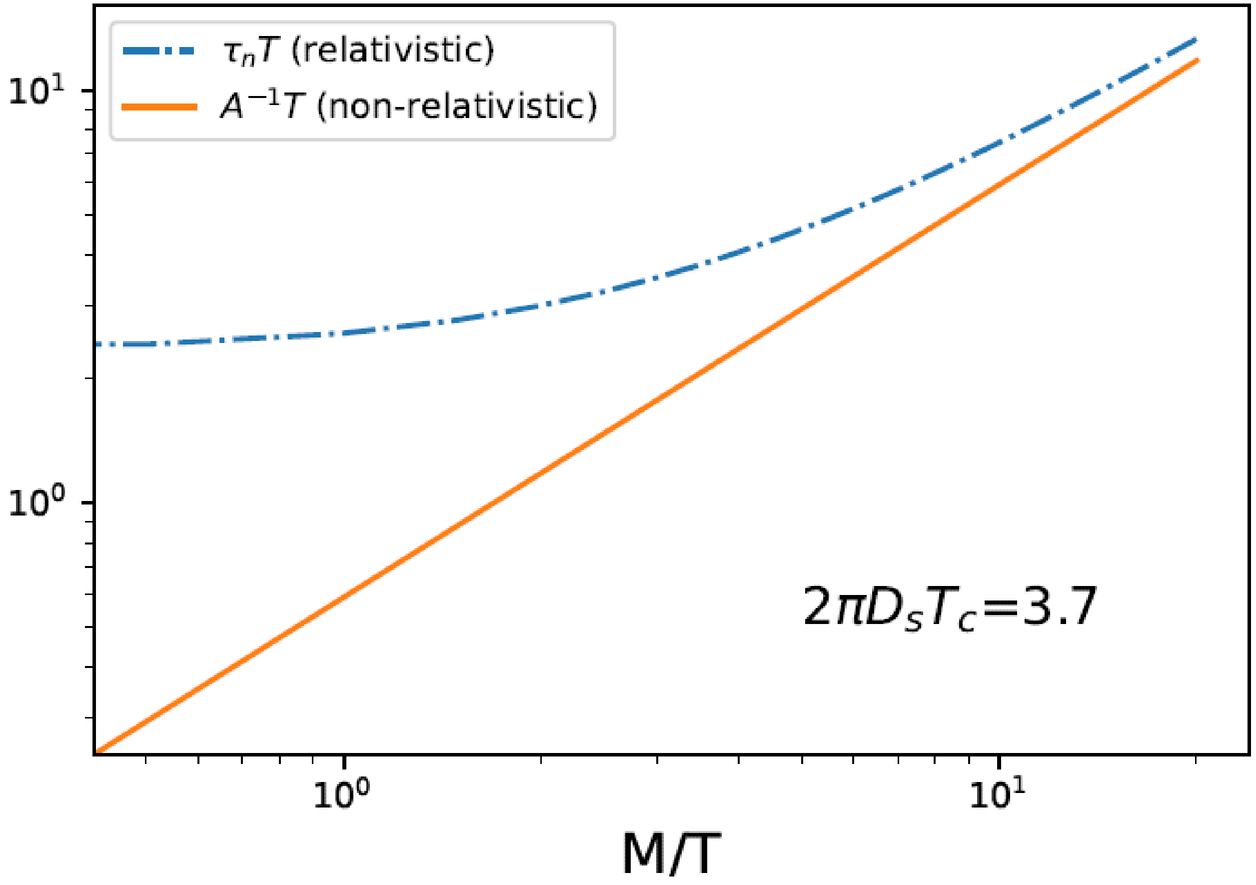
Important check: the hydrodynamic **relaxation time** is consistent with the relaxation time found within the Fokker-Planck approach in the non-relativistic limit.

Fluid-dynamic transport coefficients

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

The relation between **spatial diffusion coefficient** and **momentum-diffusion coefficient** - usually found in a non-relativistic setup - arises naturally also in a relativistic framework.



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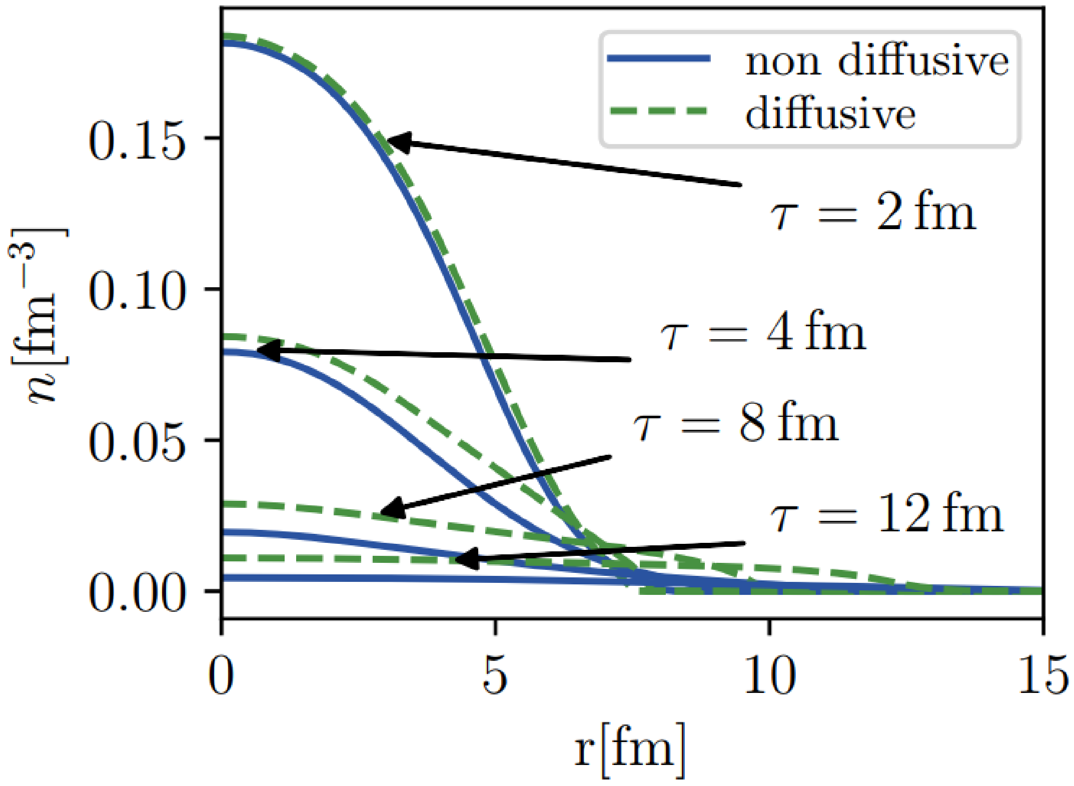
Evolution of charm fields

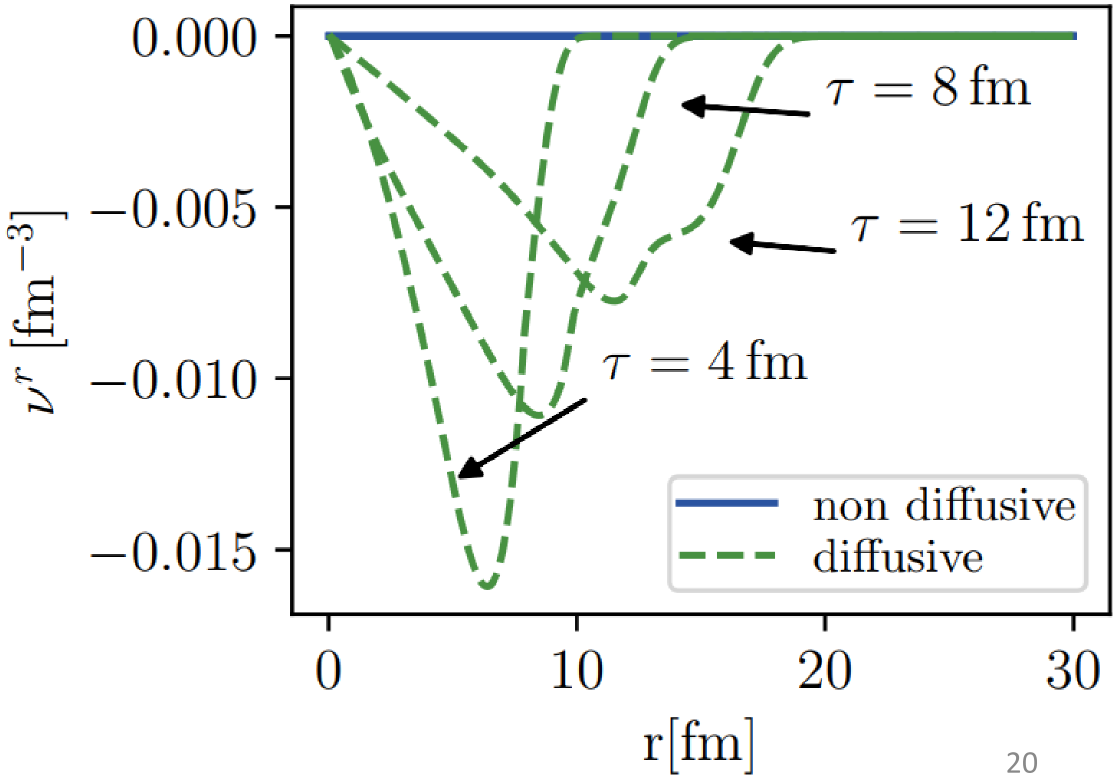
The evolution of the charm density and diffusion current depend on the **spatial diffusion coefficient** D_s .

The ratio $|\nu^r|/n$ is not always <<1 ⟶ The larger D_s , the larger the out-of-equilibrium corrections coming from ν^r are throughout the QGP evolution.

charm density

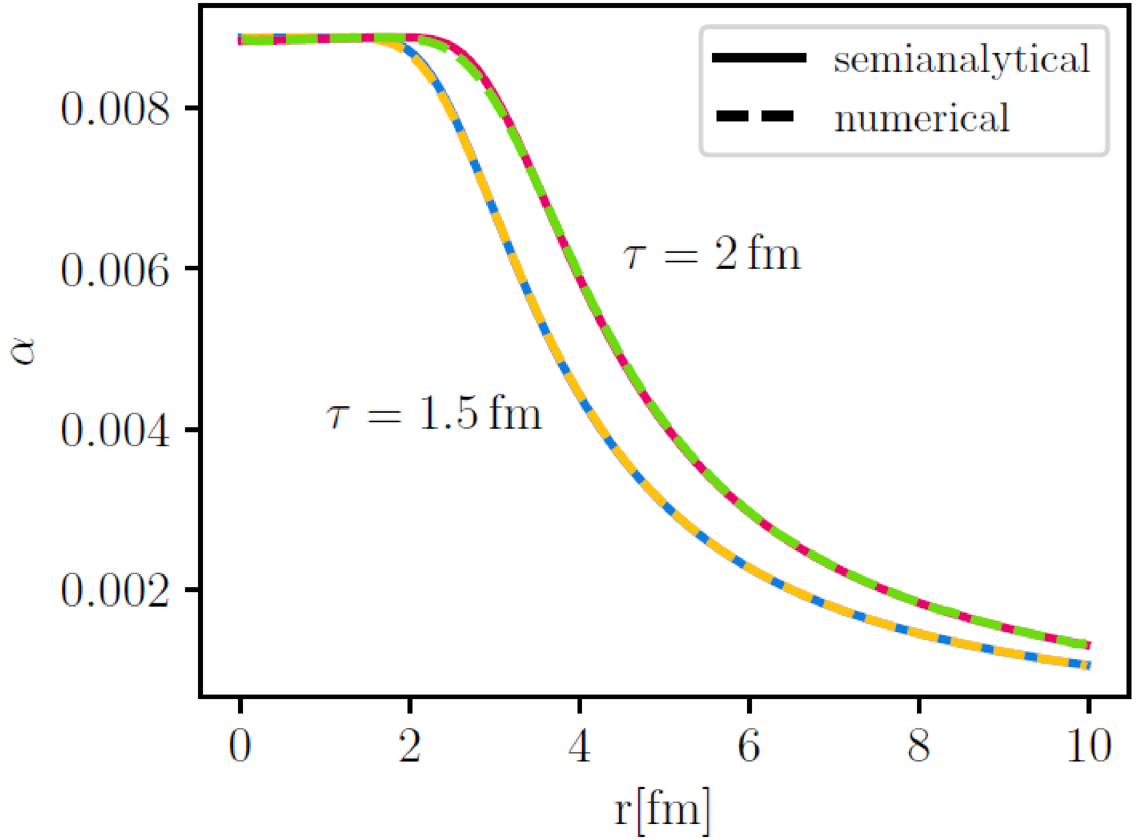


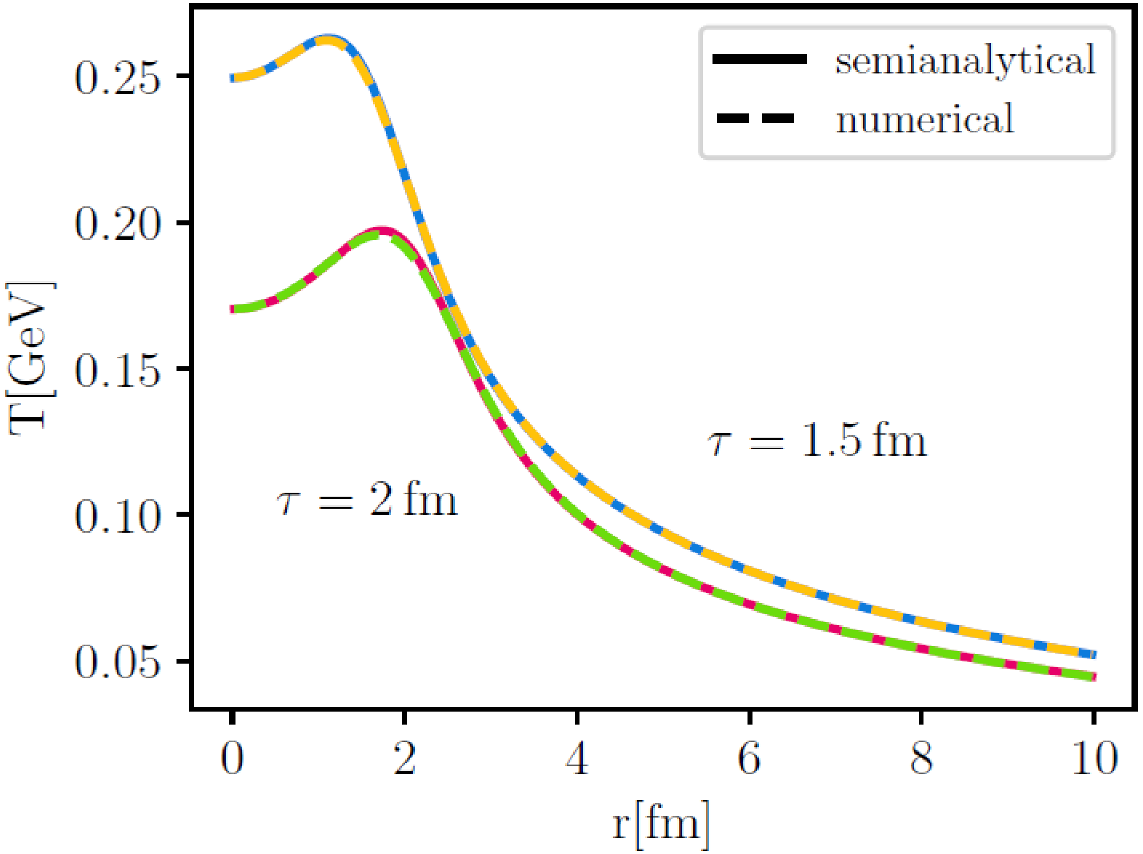
charm diffusion current



Code validation

We validated our numerical framework against Gubser flow.

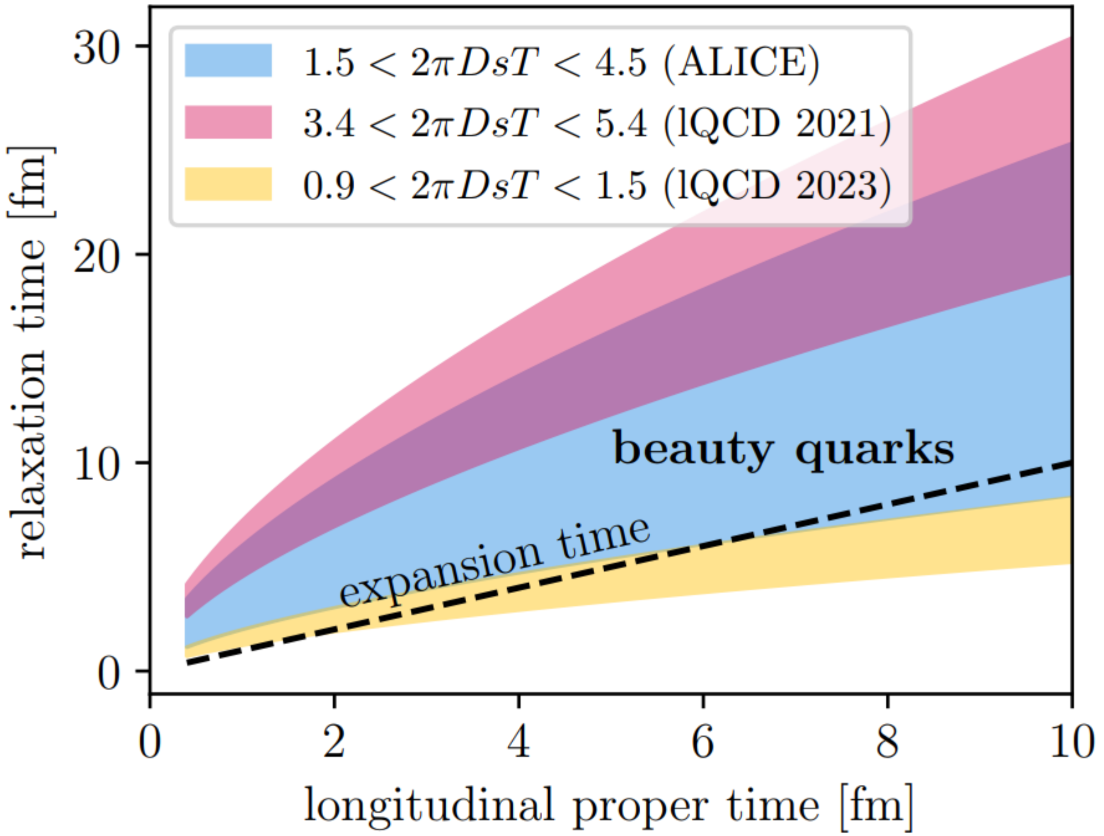


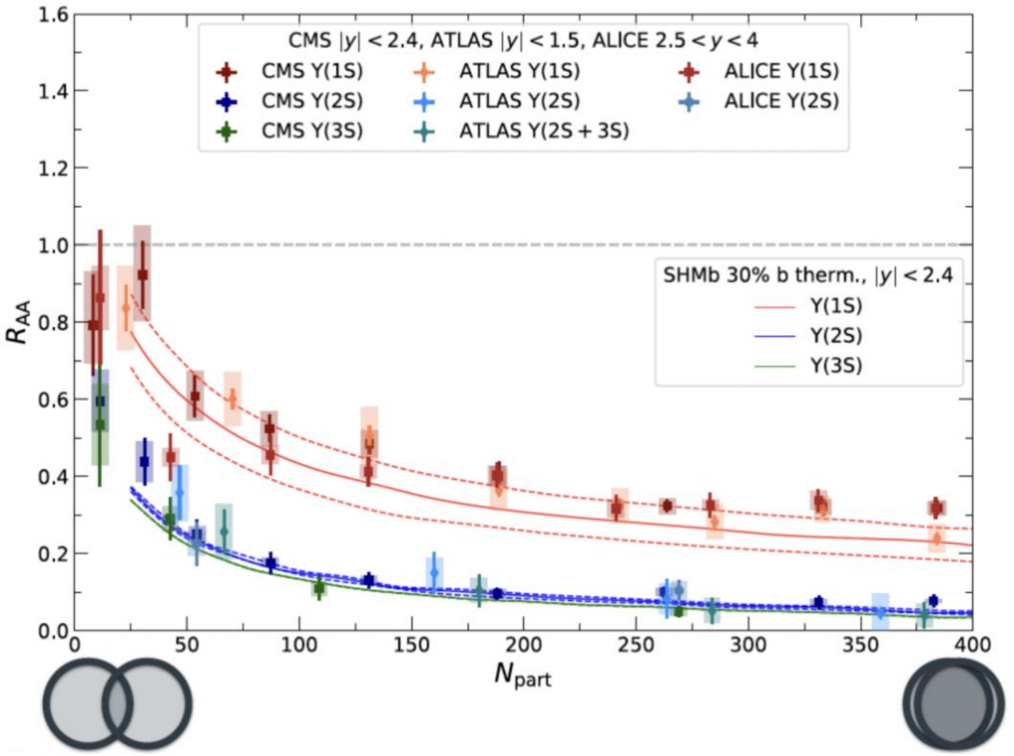


The solid lines correspond to the semianalytic Gubser solution, while the dashed lines are the numerical result with N = 200 discretization points. We have here chosen the maximal radius to be 10 fm.

What about bottom quarks?

- Need of precise measurements for spectra and flow coefficients to study thermalization of bottom quarks in the QGP → Run3 at the LHC: new data for pp and Pb-Pb collisions
- ϒ described by SHM if 30% of bottom quarks assumed to thermalize → **partial thermalization?**
- Presence of currently unknown open bottom states will lead to a reduction of the bottomonia yields





Capellino *et al.* PRD 106 (2022) 034021
Andronic *et al.* QM 2022

Fluid dynamics of beauty quarks
WORK IN PROGRESS