

# $N-c\bar{c}$ interaction from lattice QCD

Yan Lyu (吕岩)  
iTHEMS, RIKEN  
Sep. 24, 2024

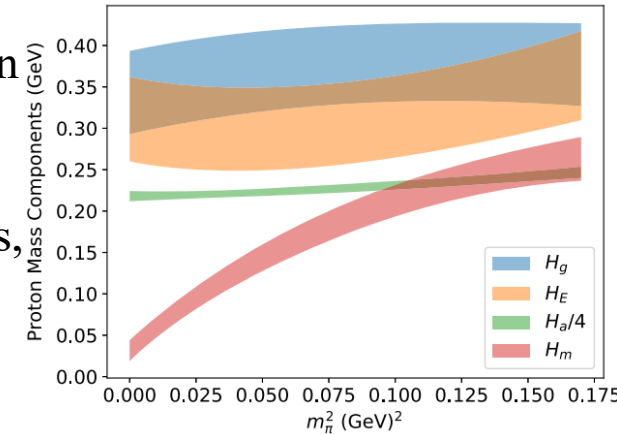
In Collaboration with T. Doi, T. Hatsuda, and T. Sugiura

# $N$ - $c\bar{c}$ interaction and hadron masses

## ➤ The proton mass

M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Phys. Lett. B 78 (1978)  
X.-D. Ji, Phys. Rev. Lett. 74 (1995)

- $c\bar{c}$  is an ideal probe to detect gluonic structure of proton
- The  $N$ - $J/\psi$  forward scattering amplitude gives an access to the trace anomaly contribution of proton mass, as containing  $\langle N | G_{\mu\nu} G^{\mu\nu} | N \rangle$



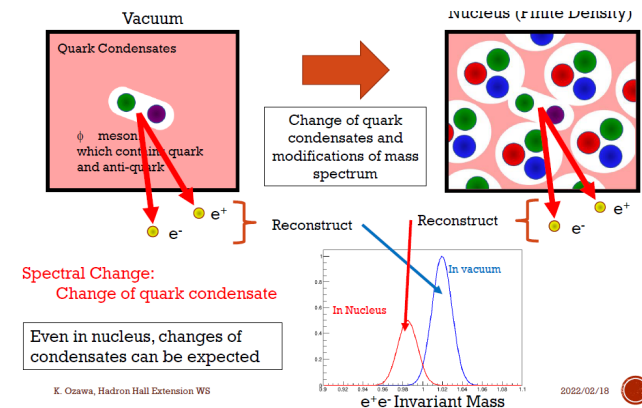
Y.-B. Yang *et. al.*, Phys. Rev. Lett. 121 212001 (2018)

## ➤ The $J/\psi$ mass modification in nuclear medium

A. Hayashigaki, Prog. Theor. Phys. 101 (1999), T. Hatsuda, and S. H. Lee, Phys. Rev. C 46 R34 (1992)

- To be measured via  $e^+e^-$  decay at J-PARC
- Closely related to the charm quark condensate
- Proportional to  $N$ - $J/\psi$  scattering length

$$\delta m_{J/\psi} \simeq - \frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\text{nm}}$$

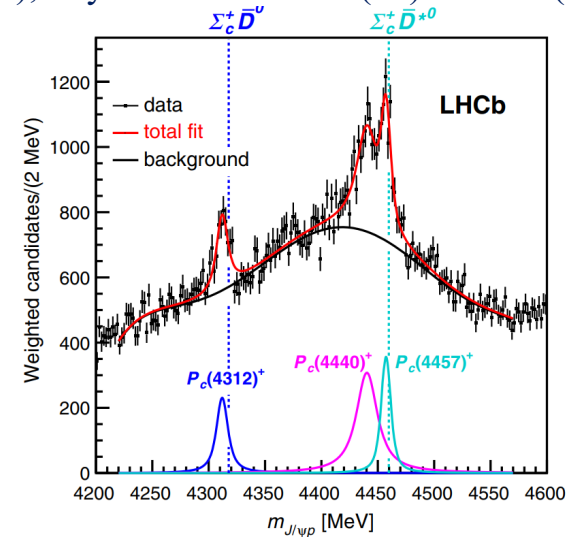


# $N-c\bar{c}$ interaction and multi-quark/hadron state

## ➤ The pentaquark states $P_c$

R. Aaij, *et al.* [LHCb Coll.], Phys. Rev. Lett. 115 072001 (2015); Phys. Rev. Lett. 122 (22) 222001 (2019)

- Appear as peaks in the  $pJ/\psi$  spectrum
- Lots of discussions on their quantum number, production/decay properties...
- A full coupled-channel study needs  $N-c\bar{c}$  interaction



## ➤ Predicted charmonium-nucleus bound states ( $A-c\bar{c}$ )

- The existence of  $A-c\bar{c}$  is in agreement among theories
- Predicted binding energies are disparate
- A reliable prediction calls for accurate  $N-c\bar{c}$  interaction

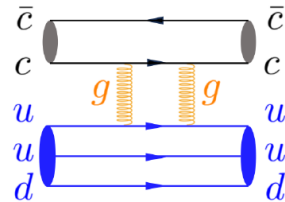
S. Brodsky, I. Schmidt, and G. Teramond, Phys. Rev. Lett. 64 (1990)

G. Krein, A. W. Thomas, and K. Tsushima, Prog. Part. Nucl. Phys. 100 (2018)

# $N$ - $c\bar{c}$ interaction

➤ Theoretically, a characteristic long-range behavior is expected

- Multiple-gluon exchange is believed to manifest as two-pion exchange



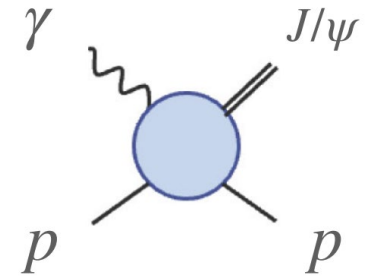
J. Castella and G. Krein, Phys. Rev. D **98**, 014029 (2018)  
H. Fujii and D. Kharzeev, Phys. Rev. D **60**, 114039 (1999)  
G. Bhanot and M. E. Peskin, Nucl. Phys. B **156**, 391(1979)

➤ Experimentally, large uncertainties remain

- Vector-meson dominance:  $a_{NJ/\psi} \sim O(1 - 10) \times 10^{-3}$  fm
- Low-energy unitary:  $a_{NJ/\psi} \sim O(1)$  fm

L. Pentchev, and I. I. Strakovsky, Eur. Phys. J. A **57** (2) 56 (2021)

D. Winney, *et al.* (Joint Physics Analysis Center), Phys. Rev. D **108** (5) 054018 (2023)



➤ Previous lattice calculations have been limited to heavy pion masses

T. Sugiura, Y. Ikeda, N. Ishii, PoS LATTICE2018, 093 (2019)

U. Skerbis, S. Prelovsek, Phys. Rev. D **99**, 094505 (2019)

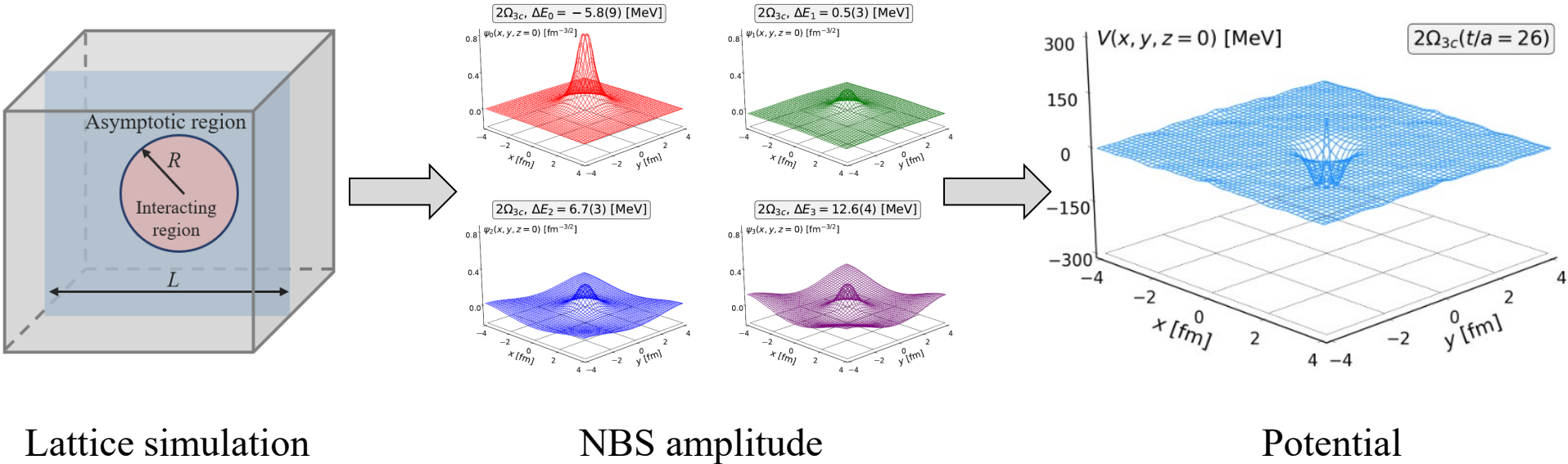
➤ This talk is going to present a first realistic lattice study on the low-energy  $N$ - $c\bar{c}$  interaction

- Examine the expected behavior of the long-range potential
- Determine near-threshold scattering properties

# HAL QCD method

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)

N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)



$$\left( \frac{1}{8\mu} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

$$R(\mathbf{r}, t) = \sum_n a_n \psi_n(\mathbf{r}) e^{-\Delta E_n t}$$

# Lattice setup

- (2+1)-flavor configurations
  - Iwasaki gauge action
  - $O(a)$ -improved Wilson quark action for  $uds$  quark
  - Relativistic heavy quark action for  $c$  quark



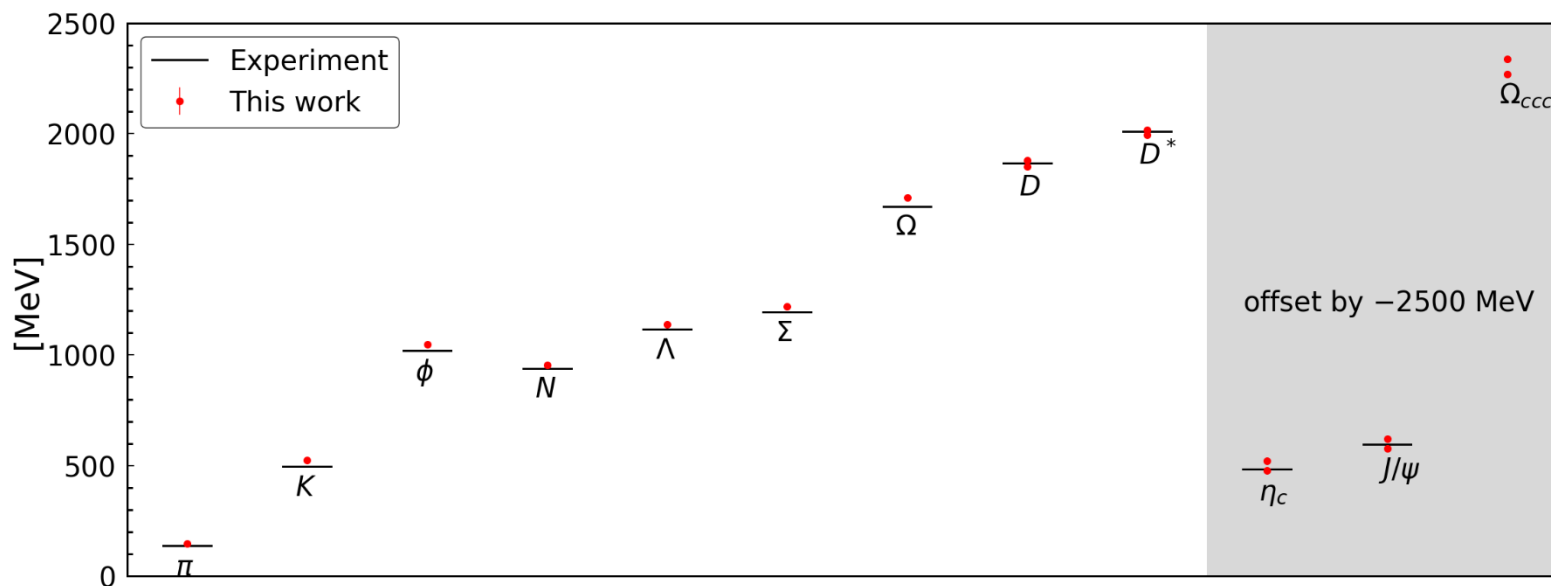
Fugaku supercomputer (440 PFlops)

K.-I. Ishikawa, *et al.* (PACS Collaboration), *Proc. Sci.*, LATTICE2015 075 (2016)

Y. Namekawa *et al.* (PACS Collaboration), *Proc. Sci.*, LATTICE2016 125 (2017)

$L^3 \times T$	$a$ [fm]	$La$ [fm]	$m_\pi$ [MeV]	$m_K$ [MeV]
$96^3 \times 96$	0.0846	8.1	146	525

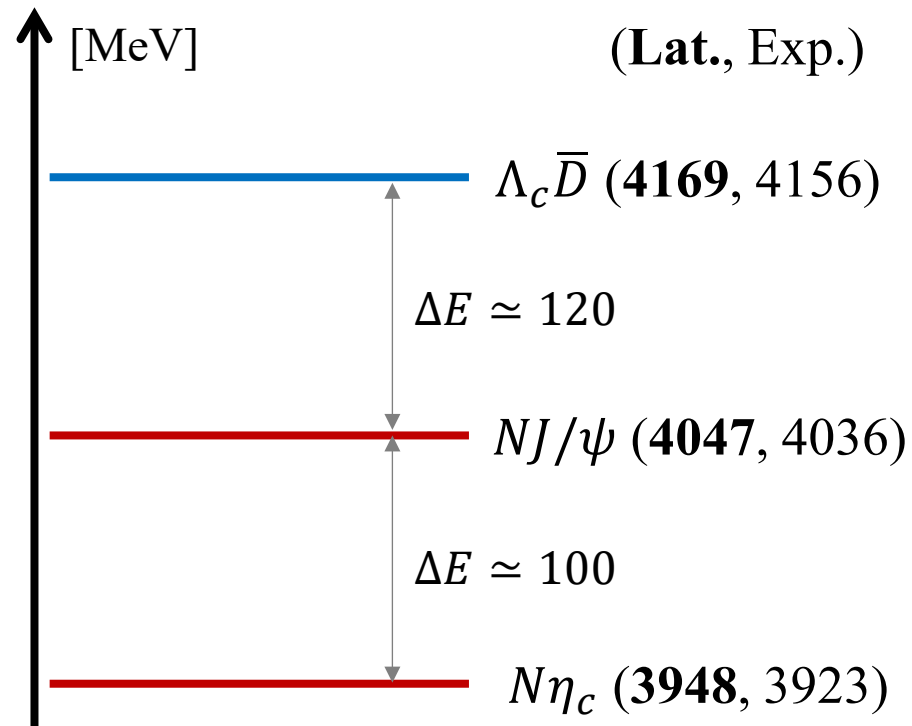
## ➤ Hadron mass



# Energy levels

## ➤ Relevant energy levels

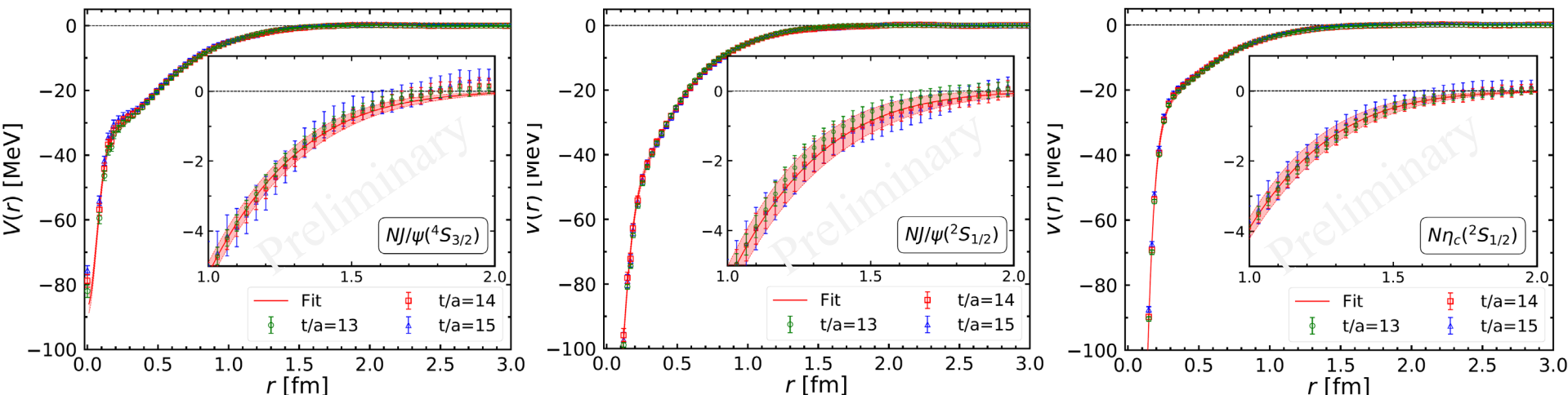
- The inelastic  $\Lambda_c \bar{D}$  is around 120 MeV above, and is suppressed at large  $t$
- The transition between  $NJ/\psi$  to  $N\eta_c$  is suppressed by heavy quark spin symmetry, and is further suppressed kinematically for the highest spin state
- Decay through  $c\bar{c}$  annihilation is OZI suppressed



- ## ➤ As we focus on near-threshold scatterings, single-channel calculations are expected to be sufficient

# Potential

➤  $S$ -wave  $N$ - $c\bar{c}$  potential at  $t/a = 14 \pm 1$  ( $t \simeq 1.2$  fm)



- The attractive potentials weakly depend on  $t$  → small systematic errors
- Attractive core, partially because of no Pauli exclusion, similar as  $N\phi$ ,  $N\Omega$
- The long-range potentials exhibit similar decreasing behavior → a common mechanism operates at long distances
- For later use, a pure phenomenological multiple Gaussian can describe the lattice data with  $\chi^2/\text{dof} = 0.5-1.5$  (red bands)

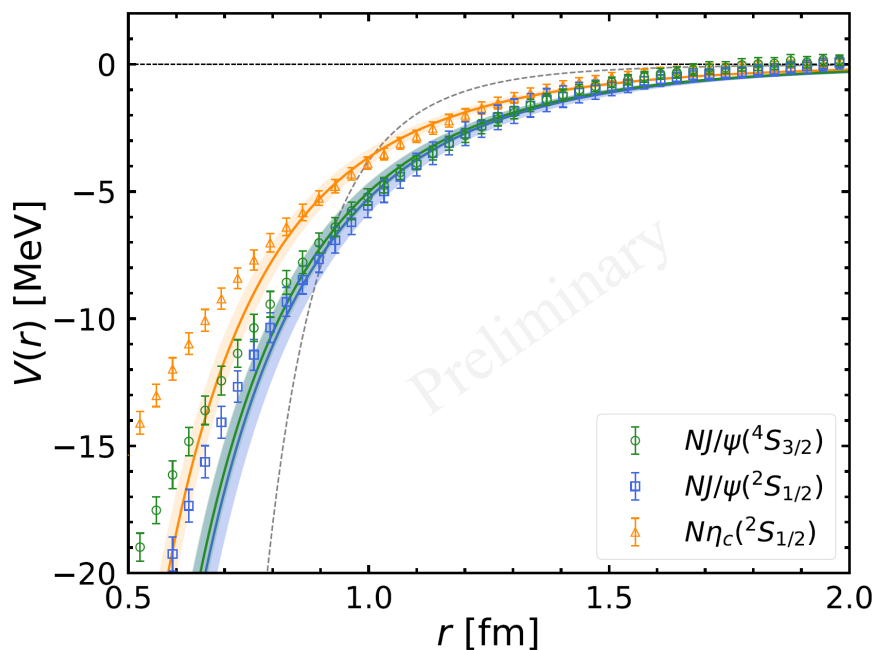


# Long-range potential

➤ The  $N-Q\bar{Q}$  potential is believed to be induced by multiple gluon exchange

- A QCD analogy of the van der Waals interaction between atoms/molecules
- Photon can propagate over long distances, leading to  $V(r) \sim -\frac{\alpha}{r^7}$  (the dashed line)
- Gluon can NOT propagate over long distances, where color-neutral hadrons involves especially the lightest two-pion state, leading to  $V(r) \sim -\alpha \frac{e^{-2m_\pi r}}{r^2}$  (bands)

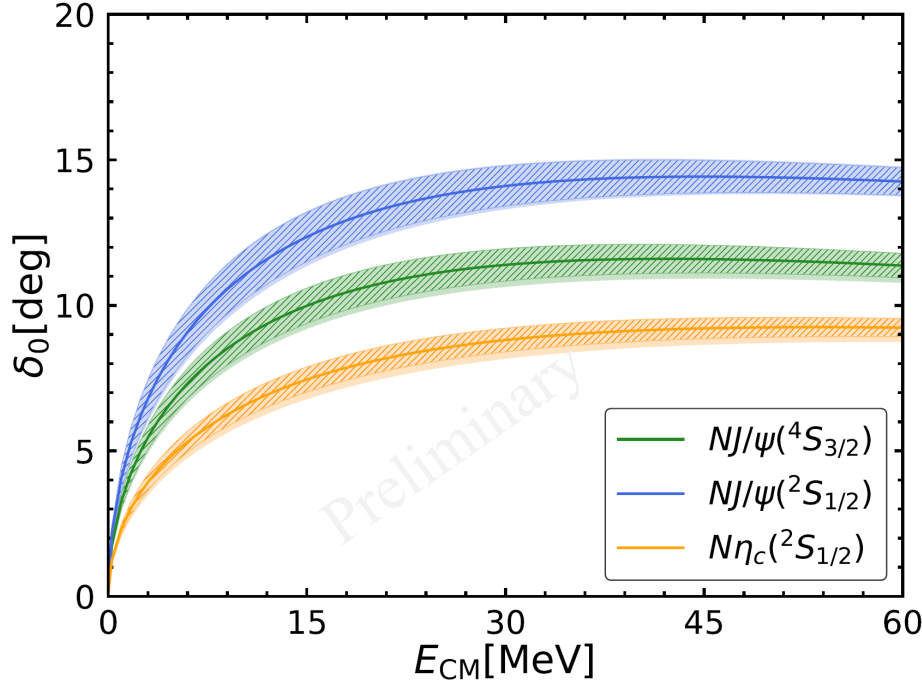
J. Castella and G. Krein, Phys. Rev. D **98**, 014029 (2018); H. Fujii and D. Kharzeev, Phys. Rev. D **60**, 114039 (1999)  
G. Bhanot and M. E. Peskin, Nucl. Phys. B **156**, 391(1979)



- The long-range  $N-c\bar{c}$  potential is consistent with the two-pion exchange interaction

# Physical observables

## ➤ Scattering phase shifts



$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2$$

channel	$a_0$ [fm]	$r_{\text{eff}}$ [fm]
$NJ/\psi(^4S_{3/2})$	$0.30(2) \begin{pmatrix} +0 \\ -2 \end{pmatrix}$	$3.25(12) \begin{pmatrix} +6 \\ -9 \end{pmatrix}$
$NJ/\psi(^2S_{1/2})$	$0.38(4) \begin{pmatrix} +0 \\ -3 \end{pmatrix}$	$2.66(21) \begin{pmatrix} +0 \\ -10 \end{pmatrix}$
$N\eta_c(^2S_{1/2})$	$0.21(2) \begin{pmatrix} +0 \\ -1 \end{pmatrix}$	$3.65(20) \begin{pmatrix} +0 \\ -6 \end{pmatrix}$

## ➤ A direct phenomenological application

- The  $J/\psi$  mass modification in nuclear medium is related to the spin-averaged scattering length of  $N$ - $J/\psi$  scattering

A. Hayashigaki, Prog. Theor. Phys. 101 (1999)

$$\delta m_{J/\psi} \simeq -\frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\text{nm}} = -19(3) \text{ MeV}$$

# Comparison

## ➤ The spin averaged $N\text{-}J/\psi$ scattering length

$a$ [fm]	Year	Author	Method
0.046(5)	2016	Gryniuk-Gryniuk	Photoproduction (VMD)
$3\sim 25 \cdot 10^{-3}$	2021	Pentchev-Strakovsky	Photoproduction (VMD)
$O(1)$	2023	JPAC	Photoproduction (unitarity)
0.05	1992	Kaidalov-Volkovitsky	QCD multipole expansion
0.24	1997	Brodsky-Miller	QCD multipole expansion
$\geq 0.37$	2005	Sibirtsev-Voloshin	QCD multipole expansion
0.10(2)	1999	Hayashigaki	QCD sum rule
$0.2\sim 3 \cdot 10^{-3}$	2020	Du-Baru-Guo et. al.	Coupled channel
0.71(48)	2006	Yokokawa-Sasaki etal	LQCD (quenched)
0.1(7)	2008	Liu-Lin-Organos	LQCD (full, extrapolate to phys. Pt.)
0.33(5)	2010	Kawanai-Sasaki	LQCD (quenched)
0.47(1)	2019	Sugiura-Ikeda-Ishii	LQCD ( $m_\pi = 700$ )
$\simeq 0$	2019	Skerbis-Prelovsek	LQCD ( $m_\pi = 266$ )
<b>0.33(4)</b>	<b>2024</b>	<b>The present work</b>	<b>LQCD (<math>m_\pi = 146</math>)</b>

# Summary & Outlook

---

- **Summary:** we present a first realistic study on low-energy  $N-c\bar{c}$  interaction based on (2+1)-flavor LQCD w/ nearly physical light quark masses and physical charm quark mass
  - The potential is found to be attractive at all distances
  - Long-range potential → consistent w/ TPE
  - Detailed analysis for near-threshold scattering properties
  
- **Outlook:**
  - The physical point simulations
  - The long-range potential for other hadron pairs

*Thanks for your attention*

# Backup

---

# Estimations for other systematic errors

---

- With a single set of configurations, systematic errors associated with  $(L, a, m_\pi)$  can only be estimated qualitatively
- Finite volume
  - From the potential,  $V(r > 2 \text{ fm}) = 0$ , while our lattice size  $L = 8.1 \text{ fm}$
  - Another estimation from two-pion exchange,  $\exp(-2m_\pi L/2) \simeq 0.3\%$
- Finite cutoff
  - Non-perturbative  $O(a)$ -improvement for light quarks,  $O\left((a\Lambda_{\text{QCD}})^2\right) \sim O(1)\%$
  - The remaining cutoff error for charm quark,  $O(\alpha_s^2 a\Lambda_{\text{QCD}}) \sim O(1)\%$
  - Even completely cutoff  $V(r)$  at  $r < 0.1 \text{ fm} \rightarrow 2\%$
- Slightly unphysical  $m_\pi$ 
  - Estimated to be  $O(1)\%$  by comparing lattice results obtain at  $m_\pi = 700 \text{ MeV}$

T. Sugiura, Y. Ikeda, and N. Ishii, *PoS LATTICE2018* (2019) 093

# FV analysis $N\eta_c$ 1/2

