# System-size dependence of $\Lambda_b/B$ in high-energy pp collisions

Based on Y. Dai, S. Zhao and MH, arXiv:2402.03692


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# Introduction: heavy quark fragmentation

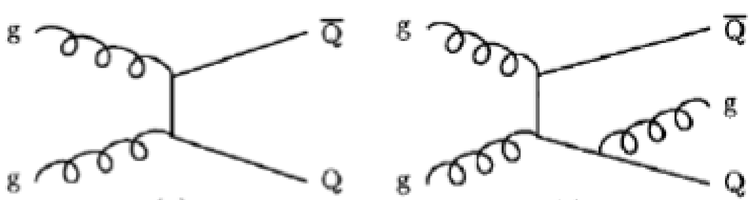
- HQ $m_Q \gg \Lambda_{QCD}$ ➔ production separated from hadronization in pp: factorization

$$\frac{d\sigma^{H_c}}{dp_T^{H_c}}(p_T;\mu_F,\mu_R) = \boxed{\mathrm{PDF}(x_1,\mu_F)\cdot\mathrm{PDF}(x_2,\mu_F)} \otimes \boxed{\frac{d\sigma^c}{dp_T^c}(x_1,x_2,\mu_R,\mu_F)} \otimes \boxed{D_{c\to H_c}(z=p_{H_c}/p_c,\mu_F)}$$

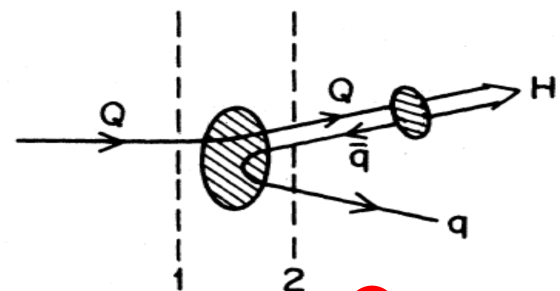
Parton distribution functions (PDFs)

Hard scattering cross section (pQCD)

Fragmentation function (hadronization)




(a) (c)




$$D_Q^H(z) = \frac{N}{z[1-(1/z)-\epsilon_Q/(1-z)]^2}$$

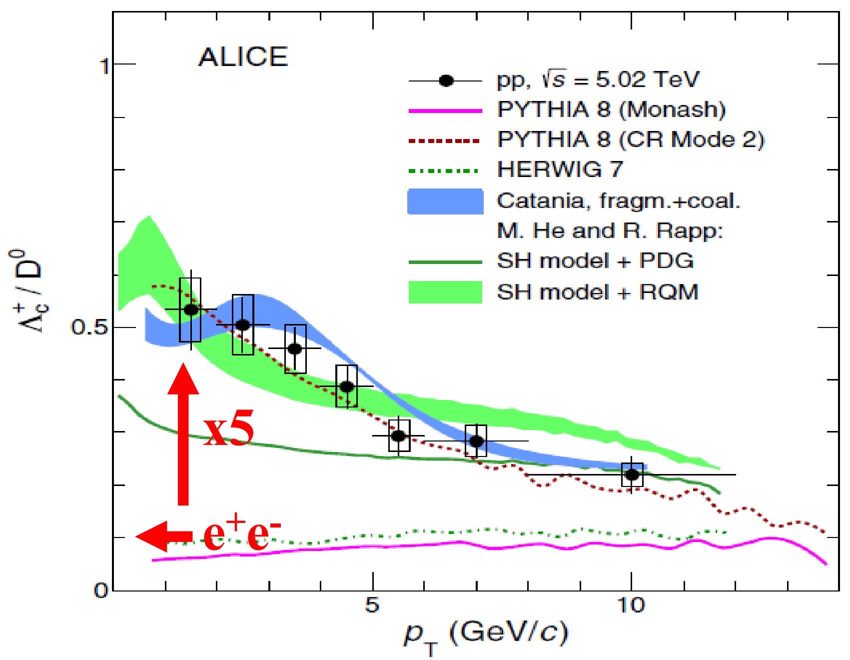
Peterson, PRD27, 105 (1983)

- Hadronization: fragmentation function, including fragmentation weights constrained by $e^+e^-$ data, then applied to pp ➔ universal?

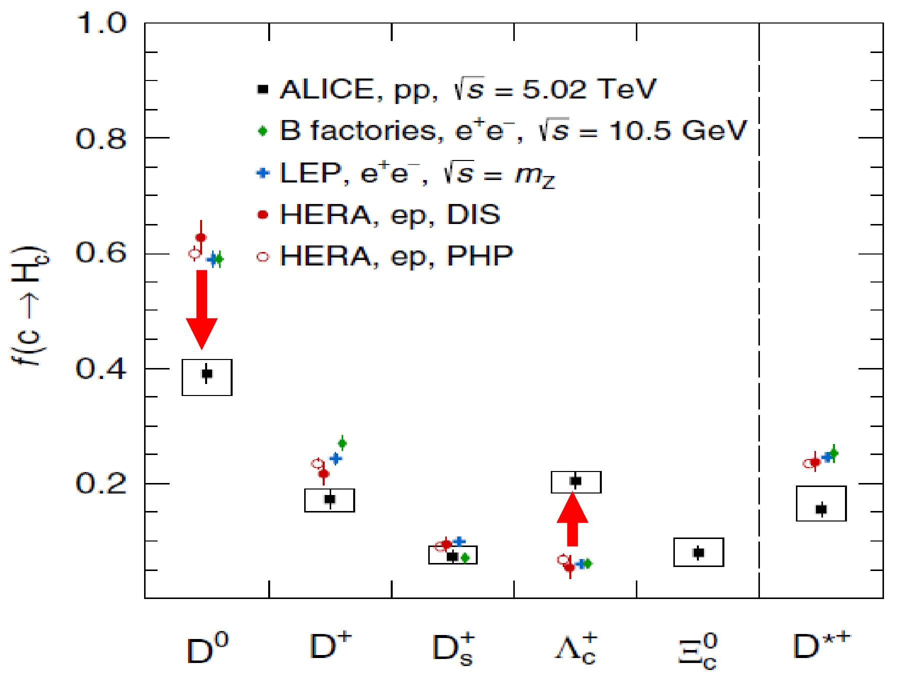
# Charm quark fragmentation: $\Lambda_c/D$

- Charm fragmentation fractions non-universal $e^+e^-$ → mini.bias pp
- $f(c \rightarrow \Lambda_c)$ ↑ vs $f(c \rightarrow D^0)$ ↓ → $\Lambda_c/D$ much enhanced!

ALICE, PRL127(2021)202301; PRC104(2021)054905




ALICE, PRD105(2022)L011103

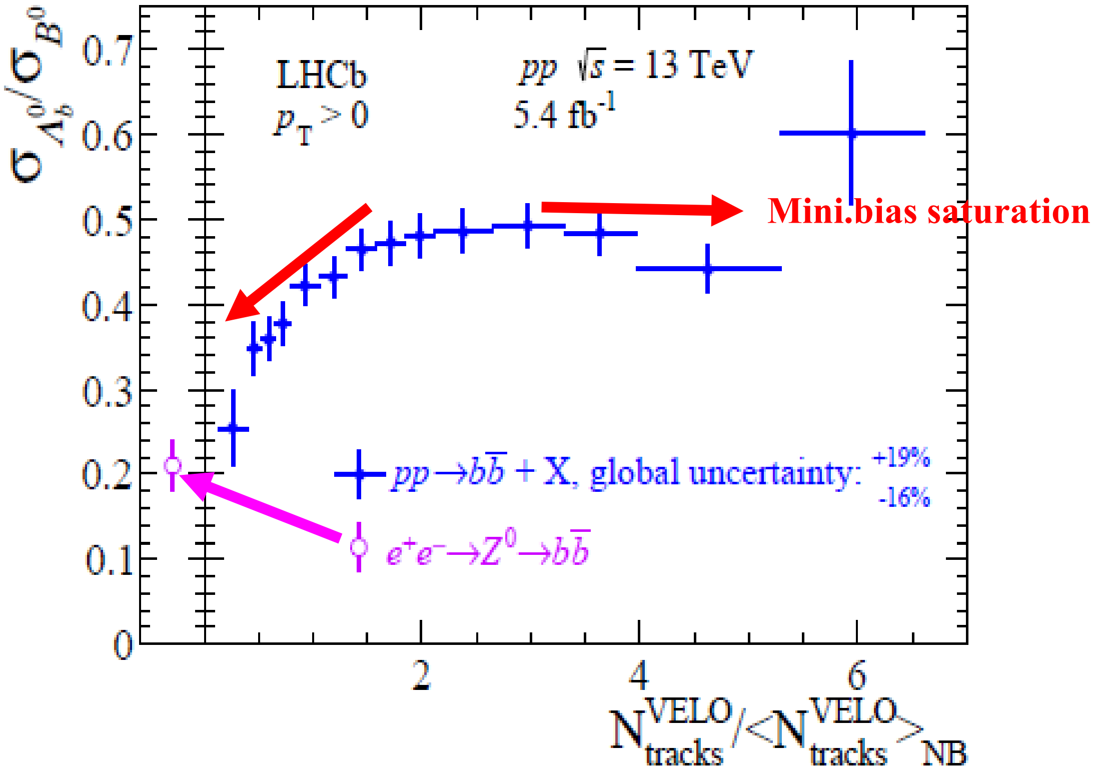



$$f_u + f_d + f_s + f_{baryon} = 1$$

# Bottom quark fragmentation: $\Lambda_b$/B

- $\Lambda_b$/B vs $dN_{ch}/d\eta$: system-size scan of pp collisions

LHCb Collab., PRL132 (2024) 081901




- decreasing from saturation value in mini.bias toward smaller size
- tending to $e^+e^-$ value at very low $dN_{ch}/d\eta$

# Bottom-hadrons: Statistical Hadronizaton Model

- QCD hadronic population from partons: born into equilibrium = maximum entropy state

Khazeev & Satz, EPJC 52,187 (2007)

- Statistical Hadronization Model (SHM) for bottom-hadron production in pp

→ bottom quarks produced in early hard processes, bottom-hadron yields not in absolute equilibrium (unlike light hadrons)

→ *relative* chemical equilibrium achieved between different bottom-hadron species primary production yields $N_i \propto$ statistical thermal densities

- Grand-canonical vs canonical ensemble

→ for large enough system size/high enough $dN_{ch}/d\eta$, relative fluctuation of quantum charges small → grand-canonical ensemble SHM, e.g., mini.bias pp

→ for small system-size with low multiplicity $dN_{ch}/d\eta$, exact conservation of quantum charges important → canonical ensemble SHM

# Grand-canonical SHM for b-hadrons

- grand-canonical thermal density for primary b-hadrons (Boltzmann)

$$n_i^{\text{primary}} = \frac{d_i}{2\pi^2} \gamma_s^{N_s^i} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right)$$

- $\gamma_s$=0.6 -- strangeness suppression factor
- $T_H$=170 MeV -- hadronization temperature

- PDG: 5 B, 4 $B_s$, 5 $\Lambda_b$, 2 $\Sigma_b$, 4 $\Xi_b$, 1 $\Omega_b$

$\Lambda_b^0$ $\quad I(J^P) = 0(\frac{1}{2}^+)$

$I(J^P)$ not yet measured; $0(\frac{1}{2}^+)$ is the quark model prediction.

Mass $m = 5619.60 \pm 0.17$ MeV
$m_{\Lambda_b^0} - m_{B^0} = 339.2 \pm 1.4$ MeV
$m_{\Lambda_b^0} - m_{B^+} = 339.72 \pm 0.28$ MeV
Mean life $\tau = (1.471 \pm 0.009) \times 10^{-12}$ s
$c\tau = 441.0\ \mu$m

$\Lambda_b(5912)^0$ $\quad J^P = \frac{1}{2}^-$

Mass $m = 5912.20 \pm 0.21$ MeV
Full width $\Gamma$ < 0.66 MeV, CL = 90%

$\Lambda_b(5920)^0$ $\quad J^P = \frac{3}{2}^-$

Mass $m = 5919.92 \pm 0.19$ MeV (S = 1.1)
Full width $\Gamma$ < 0.63 MeV, CL = 90%

$\Lambda_b(6146)^0$ $\quad J^P = \frac{3}{2}^+$

Mass $m = 6146.2 \pm 0.4$ MeV
Full width $\Gamma = 2.9 \pm 1.3$ MeV
Full width $\Gamma = 526.55 \pm 0.34$ MeV

$\Lambda_b(6152)^0$ $\quad J^P = \frac{5}{2}^+$

Mass $m = 6152.5 \pm 0.4$ MeV
Full width $\Gamma = 2.1 \pm 0.9$ MeV
Full width $\Gamma = 532.89 \pm 0.28$ MeV
Full width $\Gamma = 6.34 \pm 0.32$ MeV

- RQM: 25 B, 20 $B_s$, 30 $\Lambda_b$, 46 $\Sigma_b$, 75 $\Xi_b$, 42 $\Omega_b$ Ebert et al., PRD 84 (2011) 014025

TABLE II. Masses of the $\Lambda_Q$ ($Q = c,\ b$) heavy baryons (in MeV).

| | | $Q = c$ | | $Q = b$ | |
|---|---|---|---|---|---|
| $I(J^P)$ | $Qd$ state | $M$ | $M^{exp}$ [1] | $M$ | $M^{exp}$ [1] |
| $0(\frac{1}{2}^+)$ | 1S | 2286 | 2286.46(14) | 5620 | 5620.2(1.6) |
| $0(\frac{1}{2}^+)$ | 2S | 2769 | 2766.6(2.4)? | 6089 | |
| $0(\frac{1}{2}^+)$ | 3S | 3130 | | 6455 | |
| $0(\frac{1}{2}^+)$ | 4S | 3437 | | 6756 | |
| $0(\frac{1}{2}^+)$ | 5S | 3715 | | 7015 | |
| $0(\frac{1}{2}^+)$ | 6S | 3973 | | 7256 | |
| $0(\frac{1}{2}^-)$ | 1P | 2598 | 2595.4(6) | 5930 | |
| $0(\frac{1}{2}^-)$ | 2P | 2983 | $2939.3(^{1.4}_{1.5})$? | 6326 | |
| $0(\frac{1}{2}^-)$ | 3P | 3303 | | 6645 | |
| $0(\frac{1}{2}^-)$ | 4P | 3588 | | 6917 | |
| $0(\frac{1}{2}^-)$ | 5P | 3852 | | 7157 | |
| $0(\frac{3}{2}^-)$ | 1P | 2627 | 2628.1(6) | 5942 | |
| $0(\frac{3}{2}^-)$ | 2P | 3005 | | 6333 | |
| $0(\frac{3}{2}^-)$ | 3P | 3322 | | 6651 | |
| $0(\frac{3}{2}^-)$ | 4P | 3606 | | 6922 | |
| $0(\frac{3}{2}^-)$ | 5P | 3869 | | 7171 | |
| $0(\frac{3}{2}^+)$ | 1D | 2874 | | 6190 | |
| $0(\frac{3}{2}^+)$ | 2D | 3189 | | 6526 | |
| $0(\frac{3}{2}^+)$ | 3D | 3480 | | 6811 | |
| $0(\frac{3}{2}^+)$ | 4D | 3747 | | 7060 | |
| $0(\frac{5}{2}^+)$ | 1D | 2880 | 2881.53(35) | 6196 | |
| $0(\frac{5}{2}^+)$ | 2D | 3209 | | 6531 | |
| $0(\frac{5}{2}^+)$ | 3D | 3500 | | 6814 | |

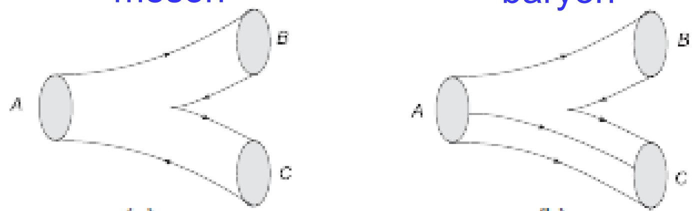
TABLE III. Masses of the $\Sigma_Q$ ($Q = c,\ b$) heavy baryons (in MeV).

| | | $Q = c$ | | $Q = b$ | |
|---|---|---|---|---|---|
| $I(J^P)$ | $Qd$ state | $M$ | $M^{exp}$ [1] | $M$ | $M^{exp}$ [1] |
| $1(\frac{1}{2}^+)$ | 1S | 2443 | 2453.76(18) | 5808 | 5807.8(2.7) |
| $1(\frac{1}{2}^+)$ | 2S | 2901 | | 6213 | |
| $1(\frac{1}{2}^+)$ | 3S | 3271 | | 6575 | |
| $1(\frac{1}{2}^+)$ | 4S | 3581 | | 6869 | |
| $1(\frac{1}{2}^+)$ | 5S | 3861 | | 7124 | |
| $1(\frac{3}{2}^+)$ | 1S | 2519 | 2518.0(5) | 5834 | 5829.0(3.4) |
| $1(\frac{3}{2}^+)$ | 2S | 2936 | $2939.3(^{1.4}_{1.5})$? | 6226 | |
| $1(\frac{3}{2}^+)$ | 3S | 3293 | | 6583 | |
| $1(\frac{3}{2}^+)$ | 4S | 3598 | | 6876 | |
| $1(\frac{3}{2}^+)$ | 5S | 3873 | | 7129 | |
| $1(\frac{1}{2}^-)$ | 1P | 2799 | $2802(^{4}_{7})$ | 6101 | |
| $1(\frac{1}{2}^-)$ | 2P | 3172 | | 6440 | |
| $1(\frac{1}{2}^-)$ | 3P | 3488 | | 6756 | |
| $1(\frac{1}{2}^-)$ | 4P | 3770 | | 7024 | |
| $1(\frac{1}{2}^-)$ | 1P | 2713 | | 6095 | |
| $1(\frac{1}{2}^-)$ | 2P | 3125 | | 6430 | |
| $1(\frac{1}{2}^-)$ | 3P | 3455 | | 6742 | |
| $1(\frac{1}{2}^-)$ | 4P | 3743 | | 7008 | |
| $1(\frac{3}{2}^-)$ | 1P | 2798 | $2802(^{4}_{7})$ | 6096 | |
| $1(\frac{3}{2}^-)$ | 2P | 3172 | | 6430 | |
| $1(\frac{3}{2}^-)$ | 3P | 3486 | | 6742 | |
| $1(\frac{3}{2}^-)$ | 4P | 3768 | | 7009 | |
| $1(\frac{3}{2}^-)$ | 1P | 2773 | 2766.6(2.4)? | 6087 | |
| $1(\frac{3}{2}^-)$ | 2P | 3151 | | 6423 | |
| $1(\frac{3}{2}^-)$ | 3P | 3469 | | 6736 | |
| $1(\frac{3}{2}^-)$ | 4P | 3753 | | 7003 | |
| $1(\frac{5}{2}^-)$ | 1P | 2789 | | 6084 | |

# Strong decay systematics of excited states

- $^3P_0$ model: A → B + C via creating a q-qbar pair of $J^{PC}=0^{++}$

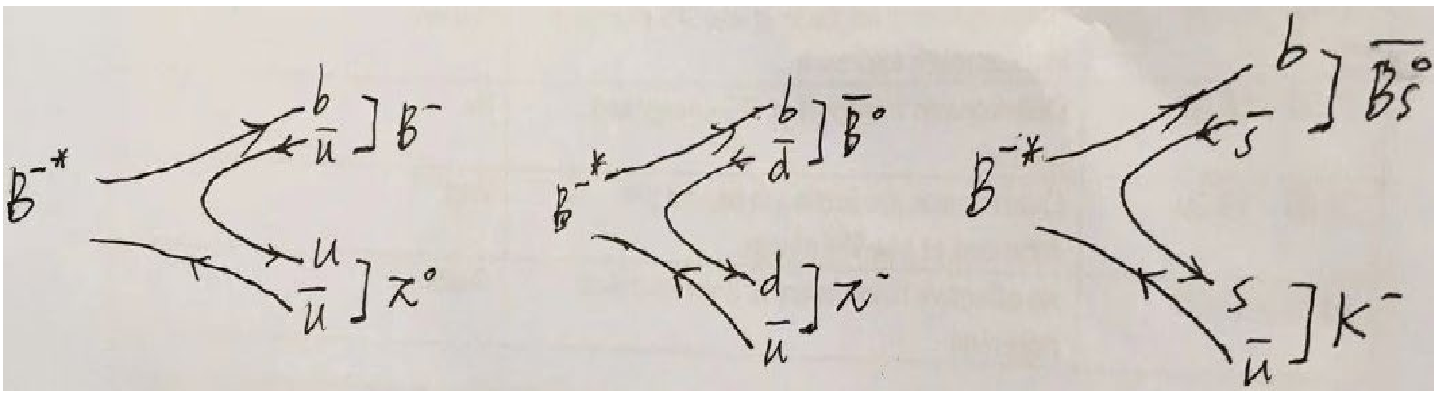
meson baryon




(a) (b)

$$\Gamma_{A\to BC} = \Phi_{A\to BC}(q_0)\sum_{\ell}|\langle BCq_0\ell J|T^{\dagger}|A\rangle|^2$$

- Branching Ratio ∝ # of possible diagrams once a decay channel opens up (mesons)

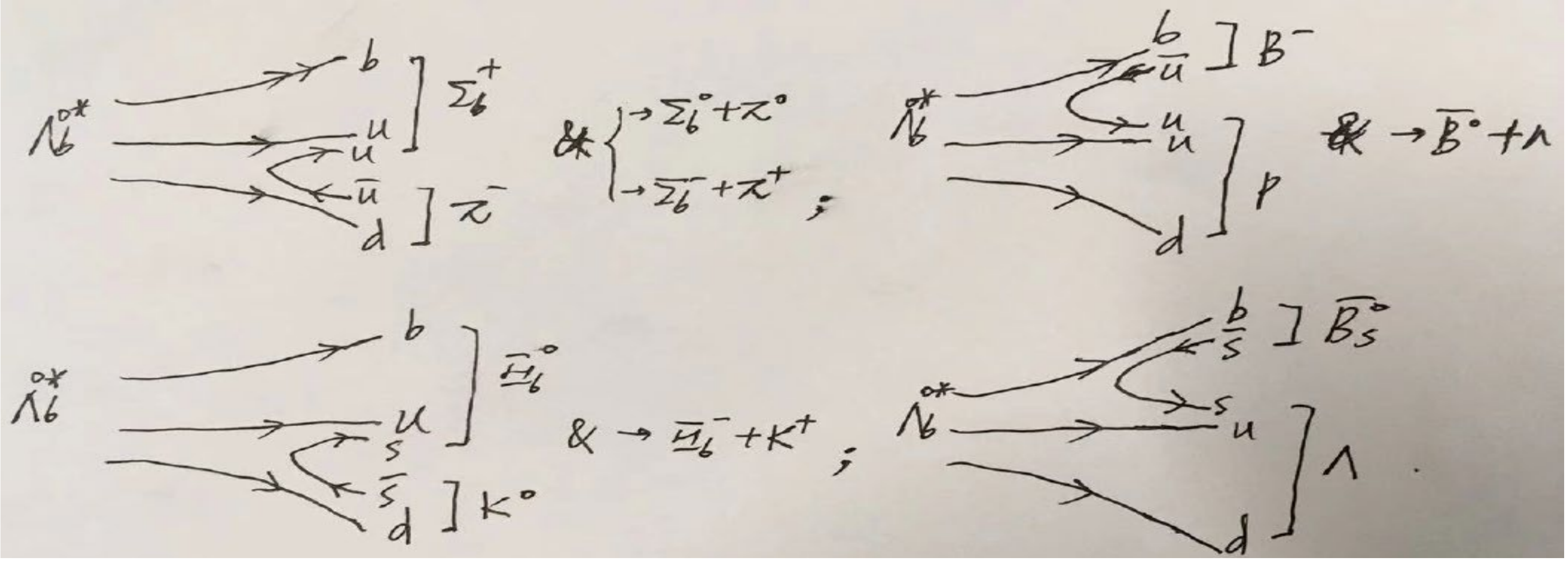



- probability of producing a q-qbar pair ∝ $\exp(-2m/T_H)$
  ➔ $\exp(-2m_q/T_H) : \exp(-2m_s/T_H) = 1 : 1/3$ [$m_q$~8, $m_s$~100 MeV]
  ➔ diagrams involving s-sbar counted as 1/3

- E.g. BR($B^{-*}$→$B^-$+$\pi^0$)= BR($B^{-*}$→$B^0$bar+$\pi^-$)=1/(1+1+1/3)=43%;
  BR($B^{-*}$→$B^0_s$bar+$\pi^0$)=1/3/(1+1+1/3)=14%; BR($B_s^*$→$B^-$+K)=1/(1+1+1/3)=43%

# Strong decay systematics: BR's estimation

- Branching Ratio ∝ # of possible diagrams once a decay channel opens up (baryons)



- E.g. BR($\Lambda_b^{0*} \rightarrow \Sigma_b + \pi^- \rightarrow \Lambda_b^0 + 2\pi$)=3/(3+2+2*1/3+1/3)=54%
  BR($\Lambda_b^{0*} \rightarrow B^- + p$) =1/(3+2+1/3+2*1/3)=16%
  BR($\Lambda_b^{0*} \rightarrow \Xi_b + K$) =2/3/(3+2+1/3+2*1/3)=11%
  BR($\Lambda_b^{0*} \rightarrow B^0_s$bar + $\Lambda$) =1/3/(3+2+1/3+2*1/3)=6%

- done for all RQM excited mesons/baryons ($\Sigma_b^*$, $\Xi_b^*$, $\Omega_b^*$)
  numbers comparable to (limited) results in PDG & computed in $^3P_0$ with full wave func.

X. Liu et al. '07, Ferretti et al., '18, Z. Wang et al., '23

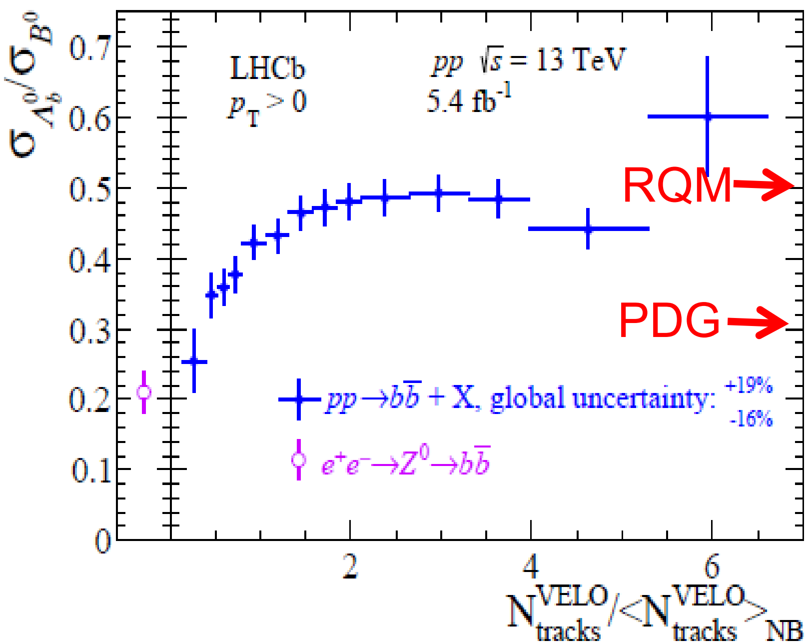
# Ground-state b-hadron densities/ratios

- total density of weakly-decaying b-hadrons @ $T_H$=170 MeV

$$n_\alpha = n_\alpha^{\text{primary}} + \sum_i n_i^{\text{primary}} \cdot BR(i \to \alpha)$$

| $n_\alpha\ (\cdot 10^{-12}\ \text{fm}^{-3})$ | $B^-$ | $\bar{B}^0$ | $\bar{B}_s^0$ | $\Lambda_b^0$ | $\Xi_b^{0,-}$ | $\Omega_b^-$ |
|---|---|---|---|---|---|---|
| PDG | 1.0094 | 1.0089 | 0.29308 | 0.31591 | 0.10097 | 0.002341 |
| RQM | 1.2045 | 1.2041 | 0.32513 | 0.61702 | 0.19548 | 0.0063204 |

LHCb Collab., PRL132 (2024) 081901




- fragmentation ratios at large $dN_{ch}/d\eta$:
  RQM favored!

| $r_\alpha$ | $\bar{B}^0/B^-$ | $\bar{B}_s^0/B^-$ | $\Lambda_b^0/B^-$ | $\Xi_b^{0,-}/B^-$ |
|---|---|---|---|---|
| PDG | 0.9995 | 0.2904 | 0.3129 | 0.1000 |
| RQM | 0.9994 | 0.2699 | 0.5122 | 0.1623 |

# Canonical ensemble (CE) SHM

- Canonical ensemble partition function: strict conservation of quantum charges (electric charge, baryon-number, strangeness, charm-, bottom-number)

$$Z(\vec{Q}) = \int_0^{2\pi} \frac{d^5\phi}{(2\pi)^5} e^{i\vec{Q}\cdot\vec{\phi}} \exp[\sum_j \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} \gamma_b^{N_{bj}} e^{-i\vec{q}_j\cdot\vec{\phi}} z_j]$$

$$z_j = (2J_j + 1)\frac{V T_H}{2\pi^2} m_j^2 K_2(\frac{m_j}{T_H}) \qquad \vec{Q} = (Q, N, S, C, B)$$

correlation volume ~ system size

- Primary hadron yield: CE vs GCE

$$\langle N_j \rangle^{CE} = \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} \gamma_b^{N_{bj}} z_j \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})}$$

$$= \langle N_j \rangle^{GCE} \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})}$$

chemical factor <1: canonical suppression for charged hadron with $\vec{q}_j \neq 0$

- E.g. exact baryon-number conservation requires: simultaneous creation of a pair of baryon and antibaryon ➔ energy-expensive $\exp(-2m_N/T_H)$ ➔ canonical suppression for baryon production

# Canonical suppression: chemical factors

| $CF$ | $V_C$=5 fm$^3$ | 10 | 20 | 30 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|---|
| $\bar{B}^0$ | 0.0097194 | 0.023927 | 0.058660 | 0.094845 | 0.16493 | 0.32591 | 0.56988 |
| $B^-$ | 0.0078259 | 0.021863 | 0.056893 | 0.093168 | 0.16331 | 0.32438 | 0.56858 |
| $\bar{B}_s^0$ | 0.0039920 | 0.013624 | 0.045935 | 0.082725 | 0.15364 | 0.31546 | 0.56101 |
| $\Lambda_b^0$ | 0.0049325 | 0.014844 | 0.047305 | 0.084415 | 0.15574 | 0.31768 | 0.56300 |
| $\Xi_b^{0-}$ | 0.0021863 | 0.0089128 | 0.037336 | 0.073498 | 0.14477 | 0.30720 | 0.55402 |
| $\Omega_b^-$ | 0.0004649 | 0.0030092 | 0.019475 | 0.047296 | 0.11221 | 0.27231 | 0.52265 |
| $\bar{B}_s^0/\bar{B}^0$ | 0.41072 | 0.56939 | 0.78307 | 0.87221 | 0.93155 | 0.96793 | 0.98443 |
| $\Lambda_b^0/\bar{B}^0$ | 0.50749 | 0.62039 | 0.80643 | 0.89003 | 0.94427 | 0.97474 | 0.98793 |
| $\Xi_b^{0-}/\bar{B}^0$ | 0.22494 | 0.37250 | 0.63648 | 0.77493 | 0.87776 | 0.94259 | 0.97217 |

At a small volume/system size,

- CF of **$B_s$ & $\Lambda_b$ < B**, canonical strangeness & baryon suppression
- CF of **$\Omega_b < \Xi_b < \Lambda_b$**, increasing strangeness content despite common baryon

# Canonical suppression: chemical factors

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|---|---|---|---|---|---|---|---|
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As volume/system size increases,

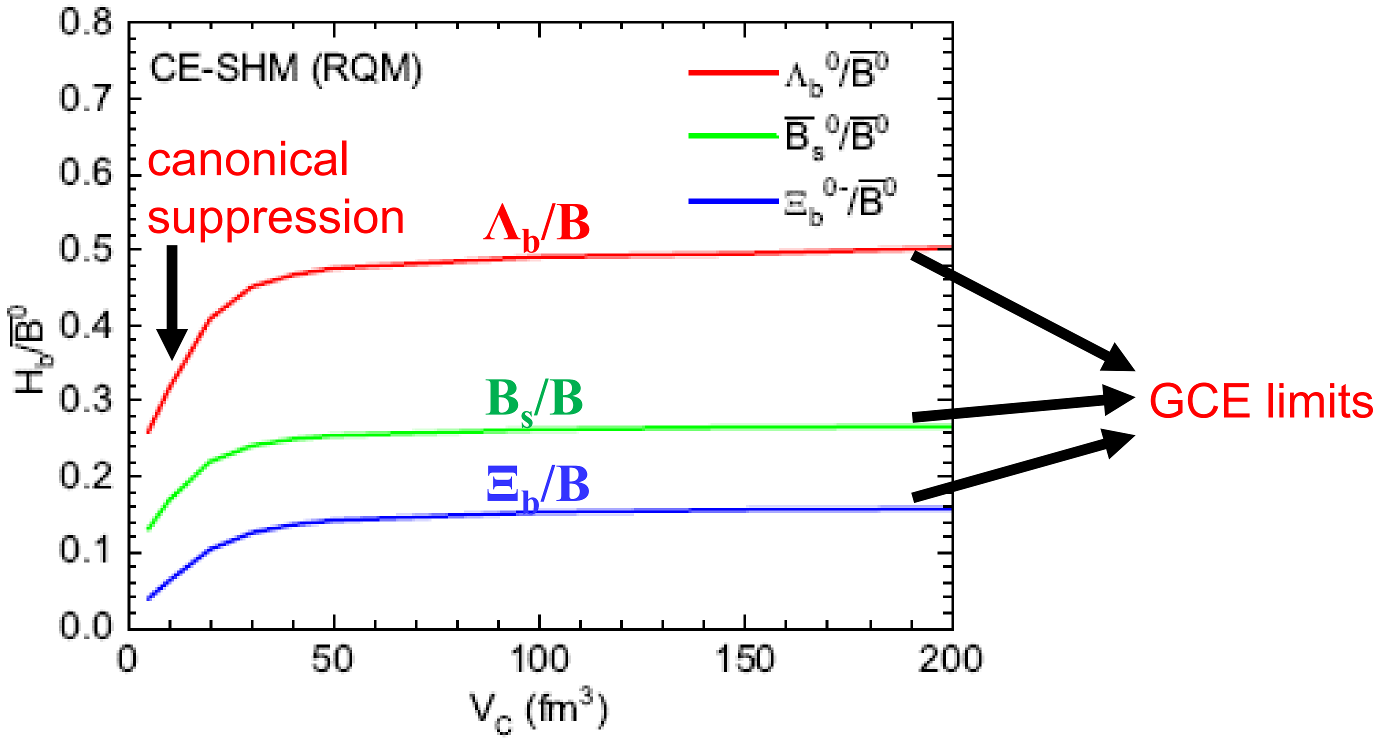
- canonical strangeness & baryon suppression attenuates → CF of **$B_s$ , $\Lambda_b$, $\Xi_b$ , $\Omega_b$ increase**
- same residual CF at large V: common canonical bottom number suppression

# CE-SHM densities with feeddowns

| $n_\alpha(\cdot 10^{-5}\ \mathrm{fm}^{-3})$ | $V_C$=5 fm$^3$ | 10 | 20 | 30 | 50 | 100 | 200 | GCE |
|---|---|---|---|---|---|---|---|---|
| $\bar{B}^0$ | 1.1220 | 2.7920 | 6.9508 | 11.313 | 19.759 | 39.148 | 68.534 | 120.41 |
| $B^-$ | 0.96934 | 2.6261 | 6.8105 | 11.181 | 19.635 | 39.038 | 68.452 | 120.45 |
| $\bar{B}_s^0$ | 0.14641 | 0.47267 | 1.5299 | 2.7242 | 5.0273 | 10.285 | 18.263 | 32.513 |
| $\Lambda_b^0$ | 0.29886 | 0.90201 | 2.8845 | 5.1551 | 9.5210 | 19.435 | 34.453 | 61.702 |
| $\Xi_b^{0-}$ | 0.043883 | 0.17479 | 0.72393 | 1.4247 | 2.8132 | 5.9882 | 10.818 | 19.548 |
| $\Omega_b^-$ | 0.00028060 | 0.0018164 | 0.011755 | 0.028549 | 0.067730 | 0.16437 | 0.31548 | 0.63204 |
| $\bar{B}_s^0/\bar{B}^0$ | 0.13049 | 0.16929 | 0.22010 | 0.24080 | 0.25443 | 0.26273 | 0.26648 | 0.27002 |
| $\Lambda_b^0/\bar{B}^0$ | 0.26635 | 0.32307 | 0.41499 | 0.45568 | 0.48186 | 0.49644 | 0.50271 | 0.51243 |
| $\Xi_b^{0-}/\bar{B}^0$ | 0.039110 | 0.062602 | 0.10415 | 0.12594 | 0.14238 | 0.15296 | 0.15785 | 0.16235 |

- As volume/system size reduces, **$B_s$/B, $\Lambda_b$/B** suppressed by a factor 2; **$\Xi_b$/B** suppression stronger, two-fold role of baryon + strangeness
- All ratios tend to the corresponding GCE-SHM values at large system size

# Ground-state b-hadron ratios vs Volume

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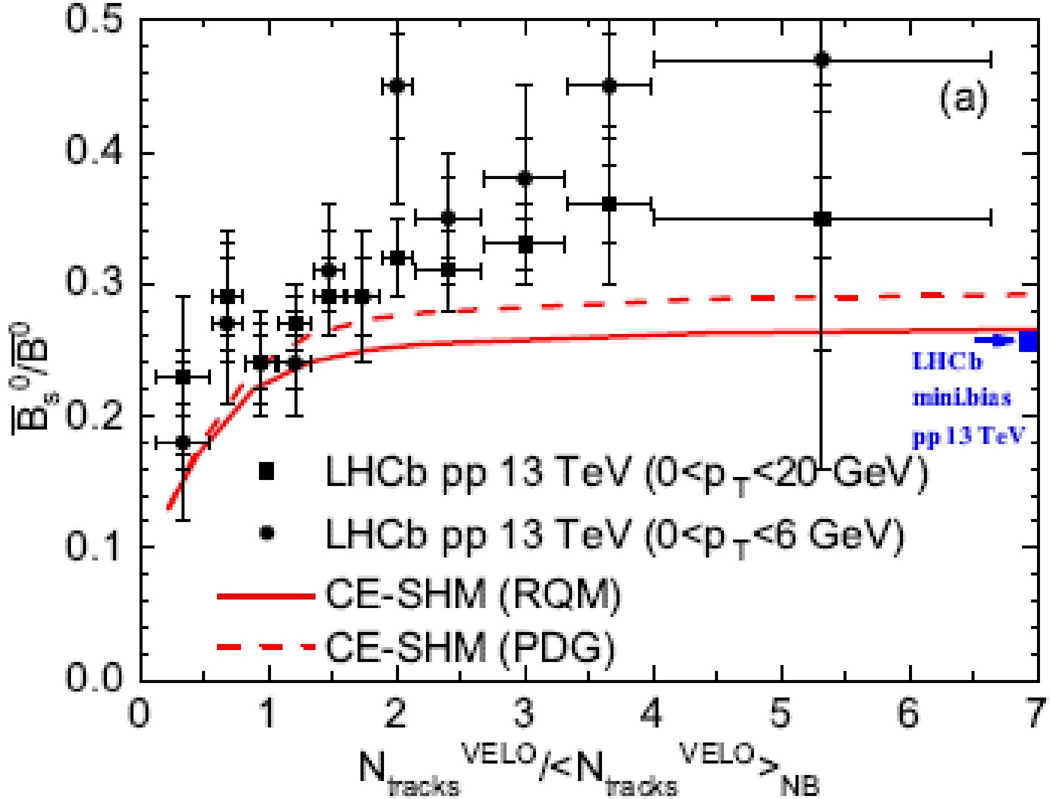
# $B^0_s/B$ vs $dN_{ch}/d\eta$

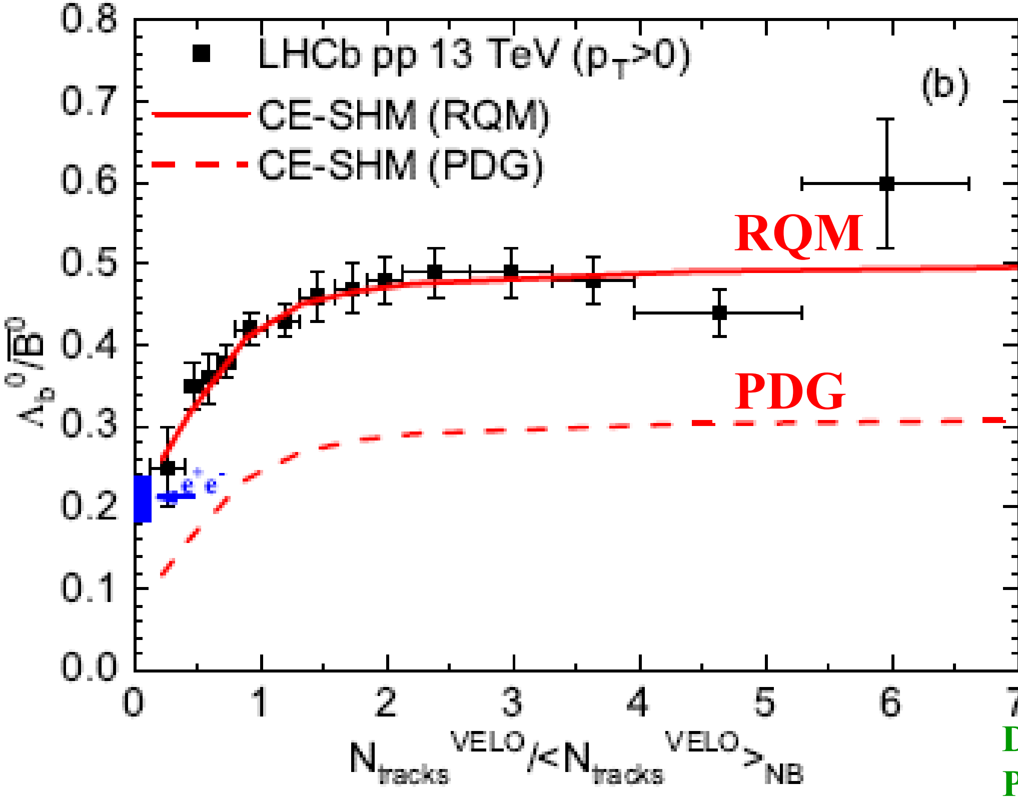



Data taken from LHCb Collab., PRL131 (2023) 061901

$$V_C/\langle V_C\rangle = N^{\text{VELO}}_{\text{tracks}}/\langle N^{\text{VELO}}_{\text{tracks}}\rangle_{\text{NB}} \qquad \langle V_C\rangle = 22.6\ \text{fm}^3$$

- **$B^0_s/B$ vs $dN_{ch}/d\eta$** increasing from small multiplicity to saturation at large size
- RQM a bit smaller than PDG

# $\Lambda_b^0/B$ vs $dN_{ch}/d\eta$




Data taken from LHCb Collab., PRL132 (2024) 081901

$$V_C/\langle V_C\rangle = N_{\rm tracks}^{\rm VELO}/\langle N_{\rm tracks}^{\rm VELO}\rangle_{\rm NB} \qquad \langle V_C\rangle = 22.6\ {\rm fm}^3$$

- $\Lambda_b^0/B$ vs $dN_{ch}/d\eta$ increasing from $e^+e^-$ value with small multiplicity to saturation/GCE limit at large size
- RQM strongly favored by data

# Summary

- Bottom-hadron production in high-energy pp collisions: *relative* chemical equilibrium via statistical hadronization

- System-size dependence of $\Lambda_b/B$: canonical suppression of $\Lambda_b$ arising from exact conservation of baryon number toward smaller size

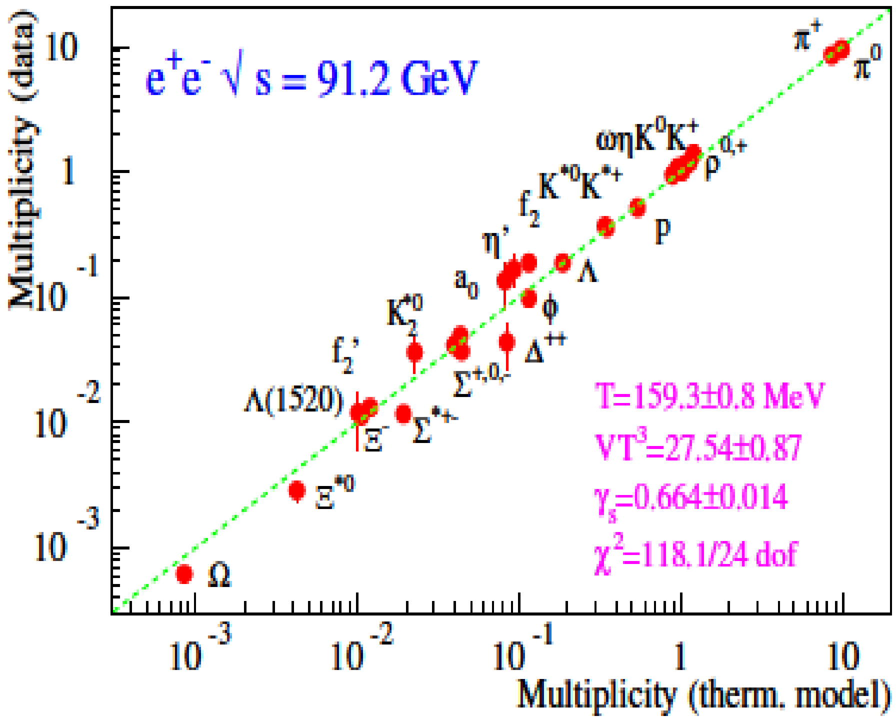
- role of many "missing" heavy-baryons highlighted
  → awaiting discovery!

# Back-up: SHM for light hadrons in $e^+e^-$

- Canonical ensemble SHM for light hadrons in $e^+e^-$

$$Z(\mathbf{Q}) = \frac{1}{(2\pi)^N} \int_{-\pi}^{+\pi} d^N\phi \; \mathrm{e}^{i\mathbf{Q}\cdot\boldsymbol{\phi}}$$
$$\times \exp\left[\frac{V}{(2\pi)^3} \sum_j (2S_j+1) \int d^3p \; \log\left(1 \pm \gamma_s^{N_{sj}} \mathrm{e}^{-\sqrt{p^2+m_j^2}/T_i - i\mathbf{q}_j\cdot\boldsymbol{\phi}}\right)^{\pm 1}\right]$$

- strict conservation of quantum charges $\mathbf{Q} = (Q, N, S, C, B)$

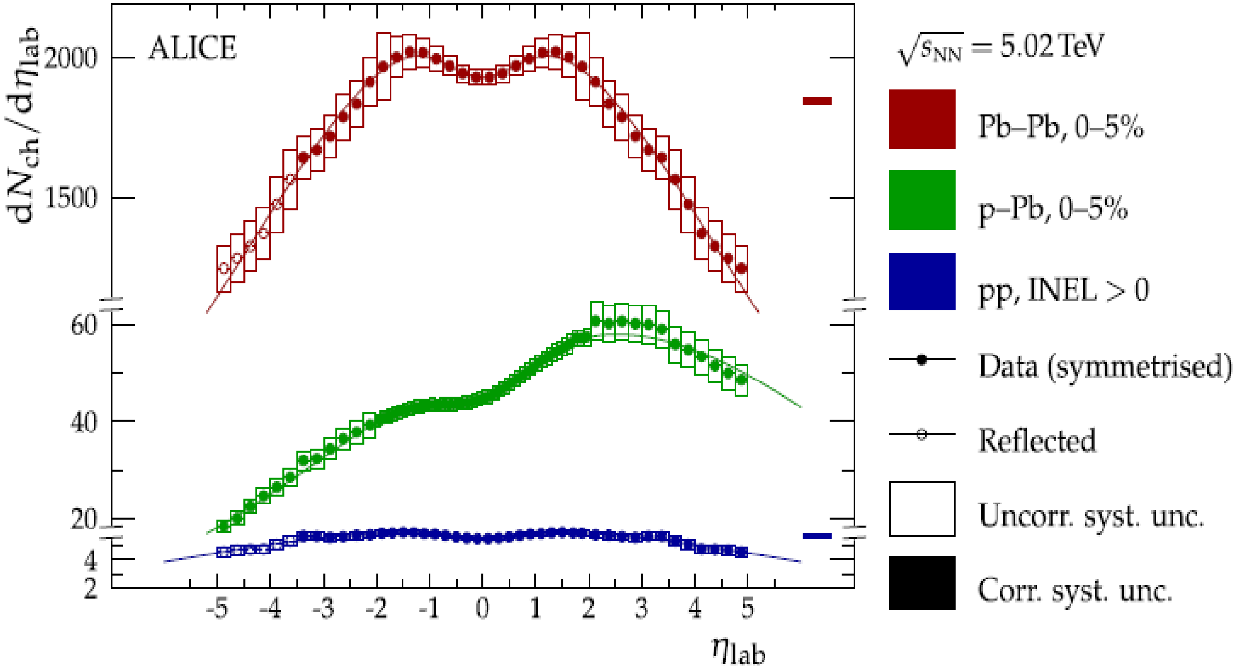



- hadron yield

$$< N_i >^{CE} = < N_i >^{GCE} \frac{Z(\vec{Q} - \vec{q}_i)}{Z(\vec{Q})}$$

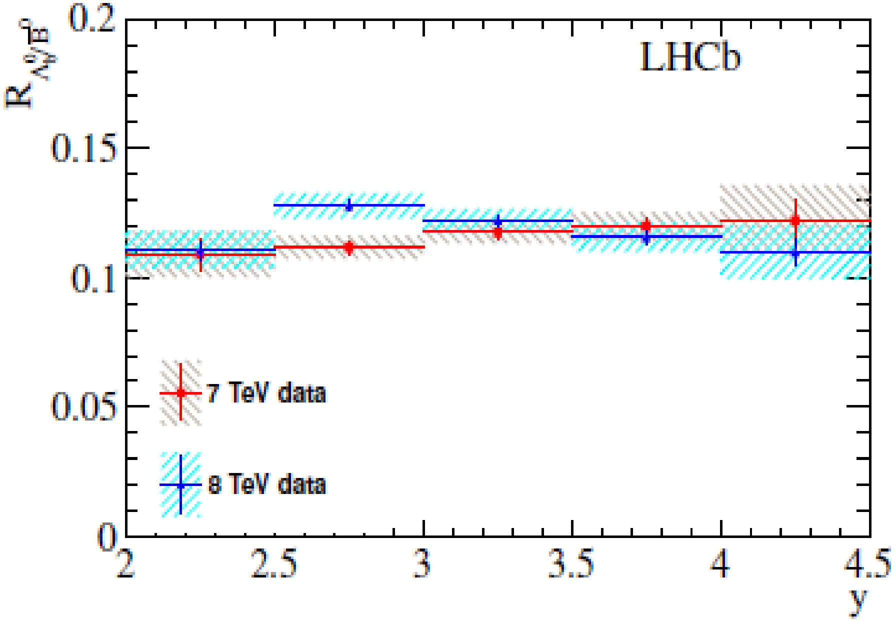
Becattini, Nucl.Phys. A702(2002)336

# Back-up: Boost-invariance in pp collisions




First, the ALICE measurement of charged-particle multiplicity dNch/dη in 5.02 TeV minimum bias pp collisions [PLB845(2023)137730] indeed shows a rather flat behavior in a rather wide rapidity window, from η=0 up to η≈±4; see the blue curve in the following figure copied from Fig.1 of ALICE PLB845(2023)137730 (the dip at pseudo-rapidity η=0 disappears when translated into rapidity y using the appropriate Jacobian, and the narrower rapidity distribution in PbPb collisions is due to the QGP interaction/thermalization that reduces the kinematic spread).

# Back-up: Boost-invariance of Lb/B in pp




Second, LHCb has directly measured the $\Lambda_b$/B (multiplied by the pertinent branching fractions) at varying rapidity slices for y=[2, 4.5] in 7 and 8 TeV minimum bias pp collisions, as shown in the following figure (copied from Fig.6 of LHCb Chin. Phys. C40, No.1(2016) 011001), where

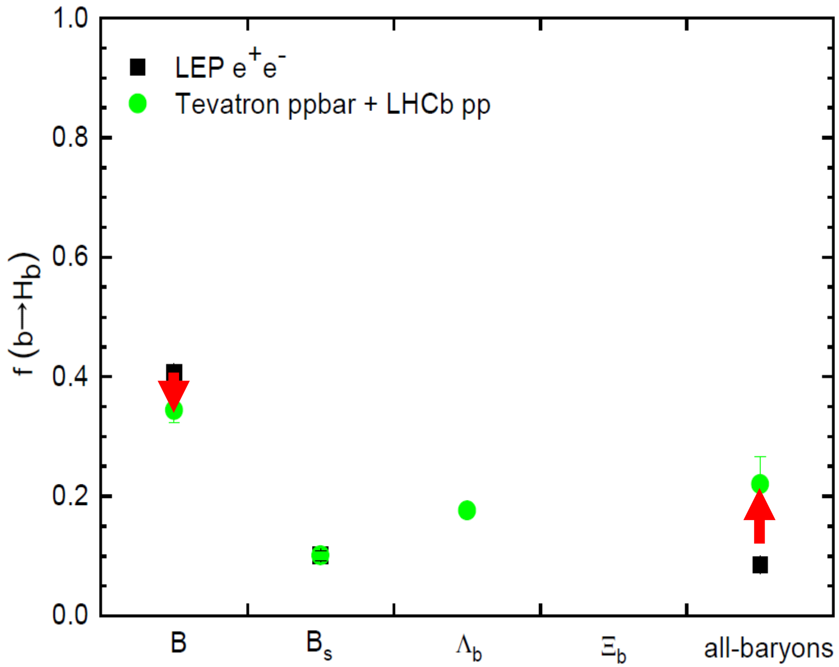
$$R_{\Lambda_b^0/\overline{B}^0} \equiv \frac{\sigma(\Lambda_b^0)\mathcal{B}(\Lambda_b^0 \to J/\psi\, pK^-)}{\sigma(\overline{B}^0)\mathcal{B}(\overline{B}^0 \to J/\psi\, \overline{K}^{*0})},$$

which translates to $\Lambda_b$/B ~ 0.5 (i.e. the grand-canonical saturation value shown in Fig.2(b) in our manuscript) when plugging in the pertinent branching fractions. This ratio is indeed roughly constant in the whole range of rapidity covered by the measurement.

# Bottom quark fragmentation

- Bottom fragmentation fractions non-universal $e^+e^- \rightarrow pp$

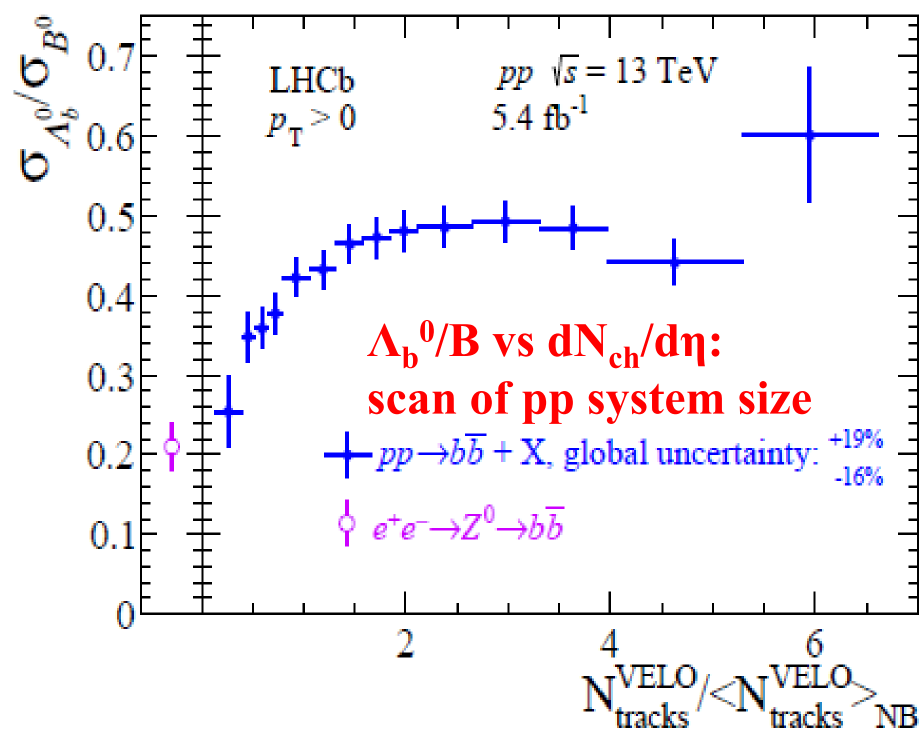
Heavy Flavor Averaging Group, EPJC 81, 226 (2021)




$$f_u + f_d + f_s + f_{baryon} = 1$$

- $e^+e^- \rightarrow$ mini.bias pp:
  b→ baryons enhanced vs
  b→B reduced

LHCb Collab., PRL132 (2024) 081901




- increasing with $dN_{ch}/d\eta$ toward saturation at mini.bias value,
- tending to $e^+e^-$ value at very low $dN_{ch}/d\eta$

# Statistical Hadronization Model (SHM)

- QCD hadronic population from partons: born into equilibrium = maximum entropy state
- Hadron yields governed by partition function of a free hadron resonance gas (HRG)
- Grand-canonical ensemble SHM for light hadrons in Pb-Pb

$$Z^{GC}(T,V,\mu_Q) = Tr[e^{-\beta(H-\sum_i \mu_{Q_i} Q_i)}]$$

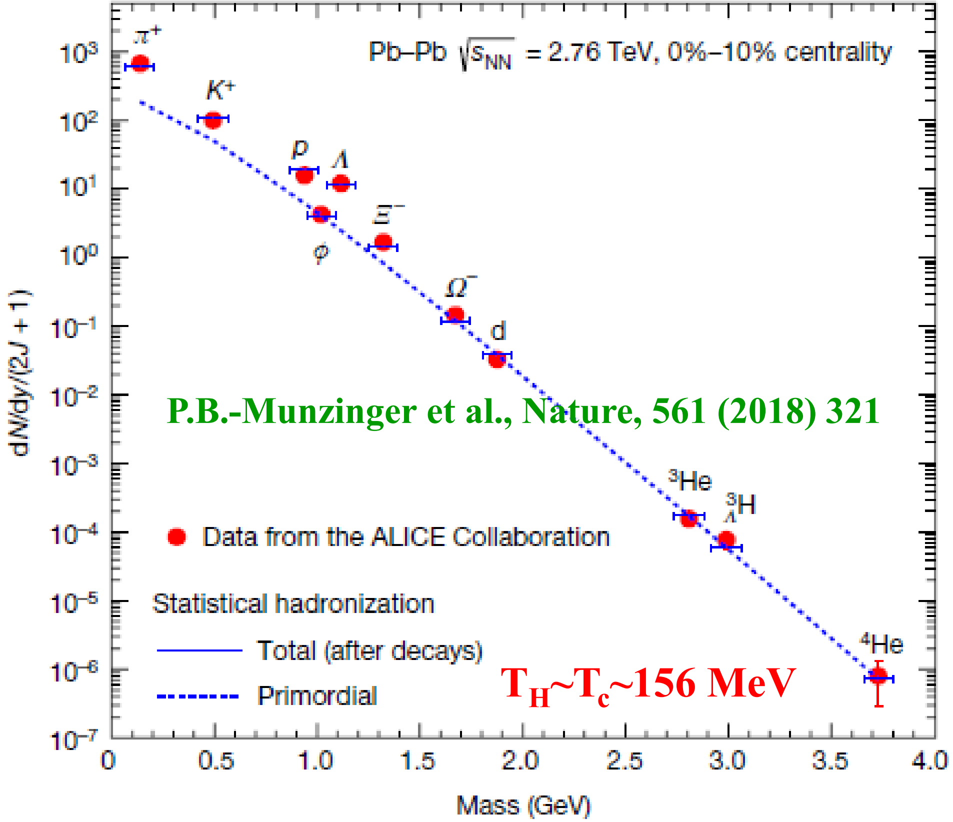
$$logZ^{GC}(T,V,\mu_Q) = \sum_i logZ_i(T,V,\mu_{Q_i})$$

- one-particle partition function

$$logZ_i(T,V,\mu_Q) = \frac{Vg_i}{2\pi^2}\int_0^{\infty} \pm p^2 dp log[1 \pm \lambda_i e^{-\beta E_i}]$$
$$= \frac{Vg_i}{2\pi^2}\sum_{n=1}^{\infty}\frac{(\pm 1)^n}{n}\lambda_i^n m_i^2 K_2(\frac{nm_i}{T})$$

- hadron yield

$$< N_i > = \frac{g_i VT}{2\pi^2}\sum_n^{\infty}\frac{(\pm 1)^{n+1}}{n}\lambda_i^n m_i^2 K_2(\frac{nm_i}{T})$$



# SHM for Bottom-hadrons in pp

- QCD hadronic population from partons: born into equilibrium = maximum entropy state

  ➔ hadron thermodynamic state not reached by dynamical collisions among hadrons/partons, but rather a generic fingerprint of hadronization

  Khazeev & Satz, EPJC 52,187 (2007)

  ➔ SHM applicable to pp collisions

- Applying SHM to bottom-hadron production in pp

  ➔ bottom quarks produced in early hard processes, bottom-hadron yields not in absolute equilibrium (unlike light hadrons)

  ➔ *relative* chemical equilibrium achieved between different bottom-hadron species production yields $N_i \propto$ statistical thermal densities

- Grand-canonical vs canonical ensemble

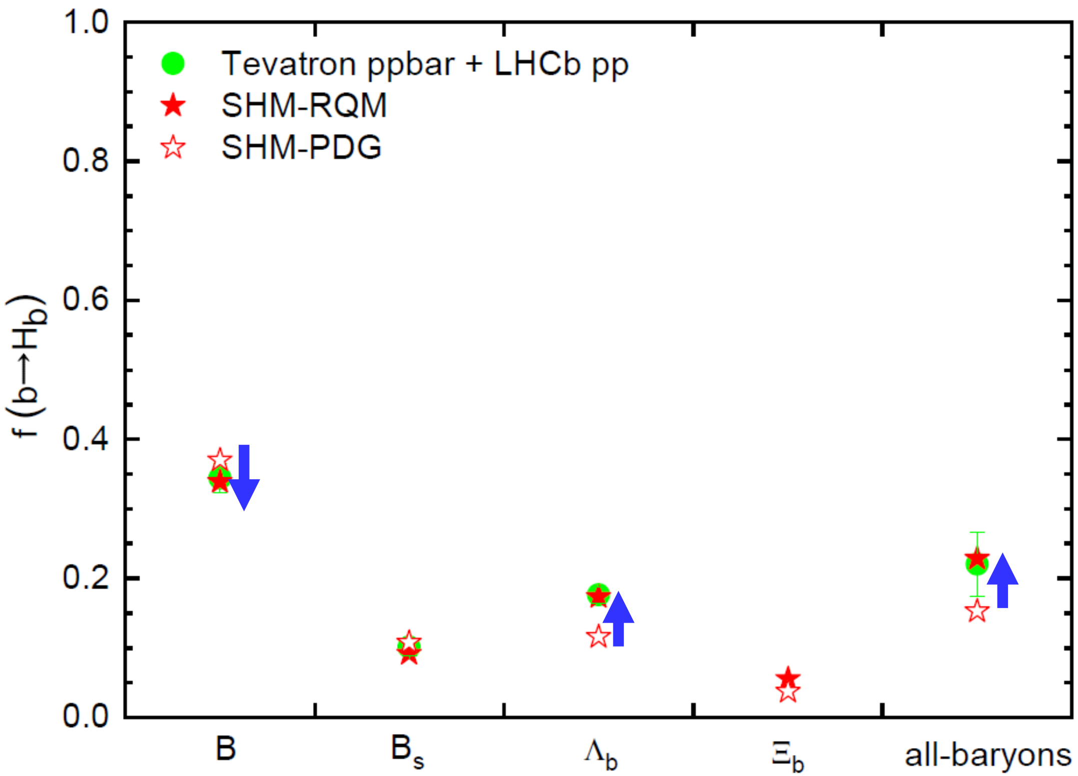
  ➔ For large enough system size/high enough $dN_{ch}/d\eta$, relative fluctuation of quantum charges small → grand-canonical ensemble SHM

  ➔ for small system-size with low multiplicity $dN_{ch}/d\eta$, exact conservation of quantum charges important → canonical ensemble SHM

# Ground-state b-hadron fractions

- fragmentation fractions $f_u + f_d + f_s + f_{\Lambda_b^0} + f_{\Xi_b^{0,-}} + f_{\Omega_b^-} = 1$

| $f_\alpha$ | $B^-$ | $\bar{B}^0$ | $\bar{B}_s^0$ | $\Lambda_b^0$ | $\Xi_b^{0,-}$ |
|---|---|---|---|---|---|
| PDG | 0.3697 | 0.3695 | 0.1073 | 0.1157 | 0.03698 |
| RQM | 0.3391 | 0.3389 | 0.09152 | 0.1737 | 0.05503 |




- PDG → RQM: $\Lambda_b^0$ & $\Xi_b^{0,-}$ enhanced by ~50%
- RQM: good agreement with mini.bias p-pbar & pp data