Evolution of structure functions at NLO without PDFs

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Center of Excellence in Quark Matter

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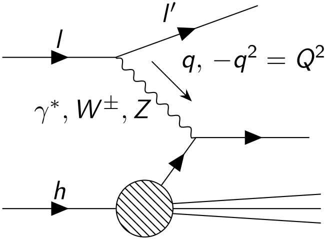
UNIVERSITY OF JYVÄSKYLÄ

Centre of Excellence in Quark Matter

Background

- Deep Inelastic Scattering (DIS):



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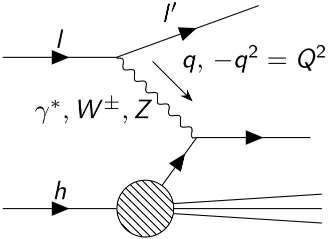

- Measured cross sections expressed in terms of structure functions

- Structure functions expressed in terms of parton distribution functions (PDFs)
  $F_i(x, Q^2) = \sum_j C_{ij}(Q^2, \mu^2) \otimes f_j(\mu^2) \quad j = q, \bar{q}, g \qquad \mu =$ factorization scale

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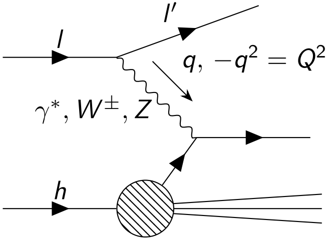

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- The conventional procedure:
  - PDFs are fitted to DIS data (to structure functions)
  - Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution: PDFs to higher Q^2

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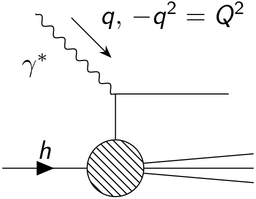
- The novelty of our work:
  - Momentum space
  - Full three-flavor basis at NLO
- Continuation for LO physical basis 2304.06998

Straightforward example with only two observables



$$F_i(x, Q^2) = \sum_j C_{F_i f_j}(Q^2, \mu^2) \otimes f_j(\mu^2),$$

where $F_i = F_2, F_{\rm L}/\frac{\alpha_{\rm s}}{2\pi}$, and $f_j = \Sigma, g$

Quark singlet:
$\Sigma(x, \mu^2) = \sum_q^{n_{\rm f}} \left[q(x, \mu^2) + \overline{q}(x, \mu^2)\right]$, $n_{\rm f} = 3$
Gluon PDF: $g(x, \mu^2)$

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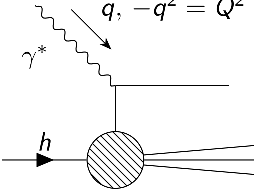

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First step: invert the linear mapping (difficult because $f \otimes g = \int_x^1 \frac{{\rm d}z}{z} f(z) g\left(\frac{x}{z}\right)$)

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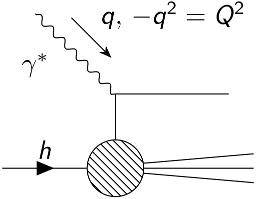

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DGLAP evolution in physical basis

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$$= \sum_j \frac{\mathrm{d}C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d}\log(Q^2)} \otimes \sum_k C^{-1}_{F_k f_j}(Q^2, \mu^2) \otimes F_k(Q^2) + \mathcal{O}(\alpha_\mathrm{s}^3)$$

Scheme and scale dependence at NLO

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$\mathcal{P}_{ij}$'s determined by:

- Splitting functions
- Coefficient functions
  $\longrightarrow$ The scheme and scale dependence exactly cancels between these two

Inverting the gluon PDF at NLO

Simple example without quarks

Invert $g(x)$ from $\widetilde{F}_{\mathrm{L}} = C^{(1)}_{F_{\mathrm{L}}g} \otimes g + \frac{\alpha_{\mathrm{s}}}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g$ $\widetilde{F}_{\mathrm{L}}(x, Q^2) \equiv \frac{2\pi}{\alpha_{\mathrm{s}}} \frac{F_{\mathrm{L}}(x, Q^2)}{x}$

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$$g(x) = \hat{P}(x)\left[\widetilde{F}_{\rm L}(x) - \frac{\alpha_{\rm s}}{2\pi} C^{(2)}_{F_{\rm L}g} \otimes g\right]$$

Plug in $g(x) = \hat{P}(x)\widetilde{F}_{\rm L}(x) + \mathcal{O}(\alpha_{\rm s})$ to the right hand side

$$g(x) = \hat{P}(x)\widetilde{F}_{\rm L}(x) - \frac{\alpha_{\rm s}(Q^2)}{2\pi}\hat{P}(x)\left[C^{(2)}_{F_{\rm L}g} \otimes \hat{P}\widetilde{F}_{\rm L}\right] + \mathcal{O}\left(\alpha_{\rm s}^2\right)$$

Six observable basis (work in preparation)

- Full three-flavor basis: $u, \bar{u}, d, \bar{d}, s = \bar{s}$, and g
  $\longrightarrow$ Need six linearly independent DIS structure functions

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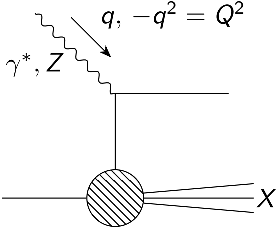
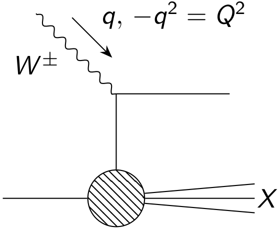
- We choose the NLO structure functions:





Neutral current γ^*, Z

- γ^* exhange $\to F_2$ and F_{L}
- Z boson exhange $\to F_3$

Charged current $W^\pm$

- W^- exhange $\to F_3^{\mathrm{W}^-}$ and $F_{2\mathrm{c}}^{\mathrm{W}^-}$
- $\Delta F_2^W = F_2^{\mathrm{W}^-} - F_2^{W^+}$

Comparison with conventional DGLAP evolution

Physical basis evolution

- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Perturbative truncation
  $\longrightarrow$ sum rule not exact
- Parametization of observable quantities

Evolution with PDFs

- Factorization scheme and scale
- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Easy to enforce an exact sum rule
- Parametization of non-observable quantities

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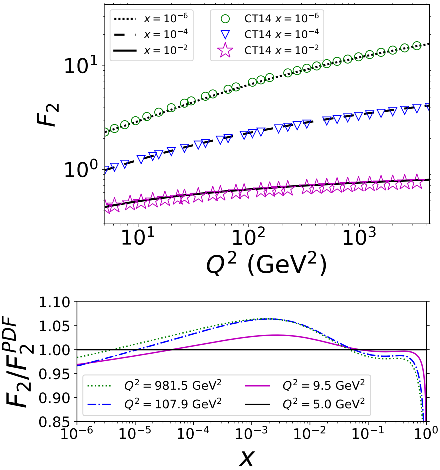
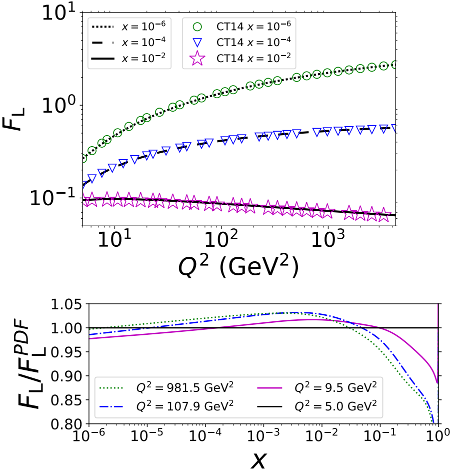


F_2/F_2^{CT14}

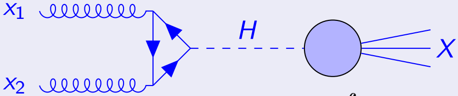


$F_{\mathrm{L}}/F_{\mathrm{L}}^{\mathrm{CT14}}$

- Similar Q^2 evolution
- Differences in values from:
  - uncertainty in PDFs from scheme and scale (error band not shown)
  - perturbative truncation

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion



$$\sigma(p+p \longrightarrow \mathrm{H}+X) = \int \mathrm{d}x_1 \mathrm{d}x_2 g(x_1,\mu) g(x_2,\mu) \hat{\sigma}_{gg\to H+X}(x_1, x_2, \frac{m_H^2}{\mu^2}),$$

where m_H is the Higgs mass, $g(x_1,\mu)$ and $g(x_2,\mu)$ are the gluon PDFs

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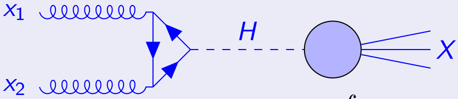

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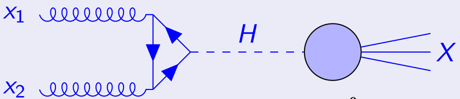

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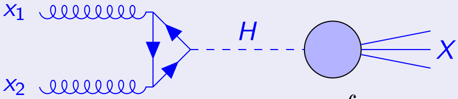

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Harland-Lang and Thorne 1811.08434:

explicit μ dependence vanishes and terms $\log\left(Q^2/m_H^2\right)$ are left behind

$\longrightarrow$ no need to choose relation between μ and Q or m_H

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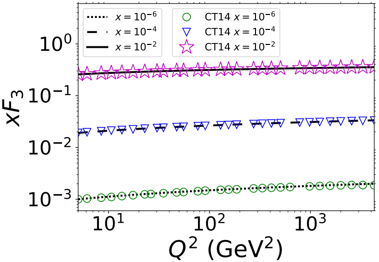
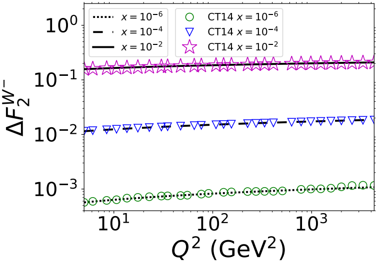
- Scheme dependence of PDFs play a role at NLO in α_s
  $\longrightarrow$ Scheme and scale dependence avoided in the physical basis
- What next:
  - Express LHC cross sections, e.g. Drell-Yan, in physical basis
  - Include heavy quarks

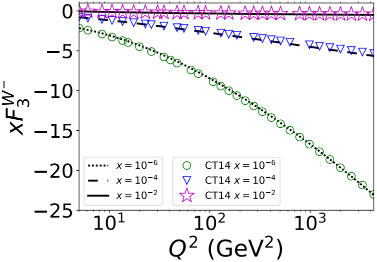
Backup: NLO evolution for F_3, ΔF_2^{W}, $F_3^{\mathrm{W}^-}$, and $F_{2c}^{\mathrm{W}^-}$



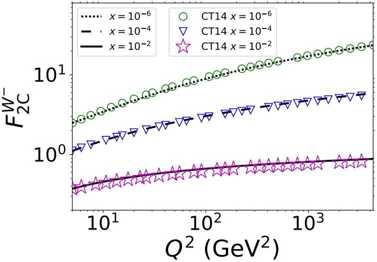
F_3 CT14 NLO



ΔF_2^{W} CT14 NLO

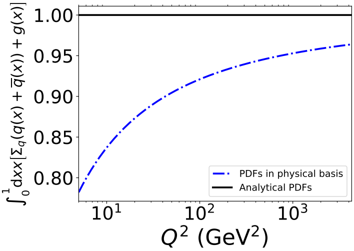


$F_3^{\mathrm{W}^-}$ CT14 NLO

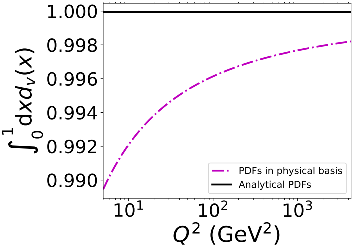


$F_{2c}^{\mathrm{W}^-}$ CT14 NLO

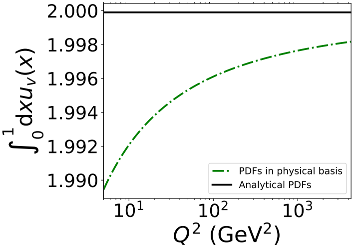
Backup: Sum rule



Momentum sum rule

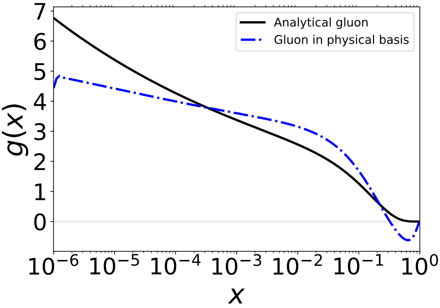


$\int_0^1 \mathrm{d}x d_v(x)$

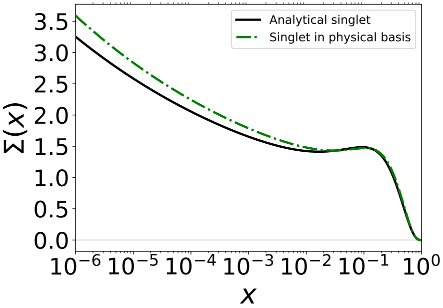


$\int_0^1 \mathrm{d}x u_v(x)$

Backup: Gluon PDF and quark singlet in physical basis



Gluon PDF



Quark singlet