Jet momentum broadening beyond the jet quenching parameter from QCD kinetic theory

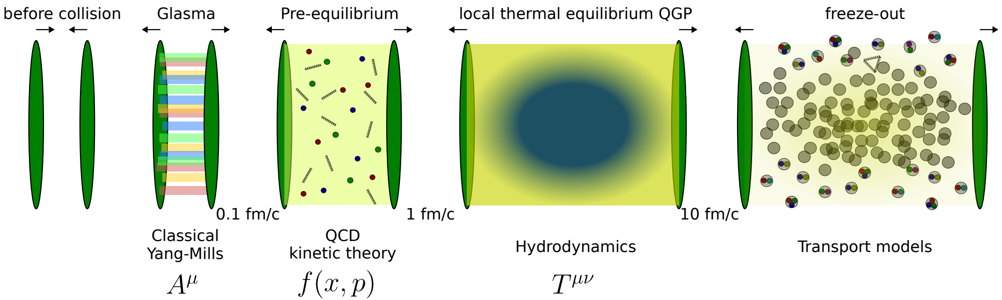
2303.12595 & 2312.00447 with K. Boguslavski, A. Kurkela, T. Lappi, J. Peuron
and based on work in preparation with Alois Altenburger & Kirill Boguslavski

Florian Lindenbauer

TU Wien

23.09.2024, Hard Probes 2024, Nagasaki, Japan

Time-evolution of the QGP in heavy-ion collisions



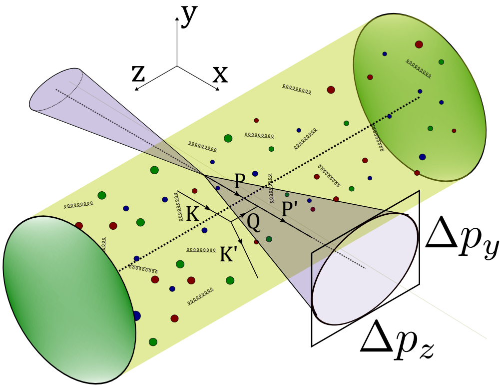
Interested in pre-equilibrium stages ("Initial stages")

$\rightarrow$ **QCD out of equilibrium**

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

How can we study the initial stages?

- To study initial stages
  $\rightarrow$ very **energetic** or **heavy** probes
- Here depicted: **jets**



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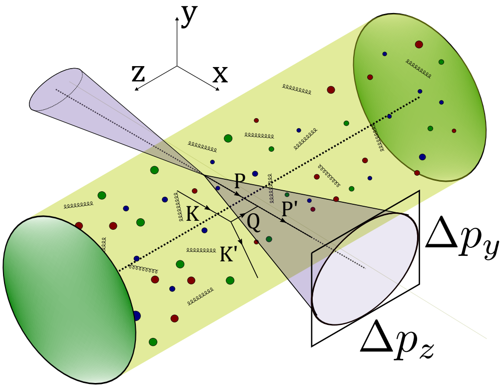
  → very **energetic** or **heavy** probes
- Here depicted: **jets**
  - **Highly energetic partons** created in initial collision
  - Splits into many particles
    → then measured in the detectors
  - Imprints of **medium interactions**



Jet energy loss through medium-induced radiation

- **Energy loss** dominated by **gluon radiation**

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- Depends on effective propagator
  Input: **dipole cross section**

$$C(\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \left(1 - e^{i\mathbf{b}\cdot\mathbf{q}_\perp}\right)$$

$$C(\mathbf{q}_\perp) = (2\pi)^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

See also JHEP 07 (2020) [Andres, Apolinário, Dominguez], JHEP 10 (2021) [Moore, Schlichting, Schlusser, Soudi], PRD 105 (2022) [Schlichting, Soudi]

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- **Harmonic approximation**: (small b limit)
  Dependence on single medium parameter $\hat{q}$

$$C(\mathbf{b}) \approx \frac{1}{4}\hat{q}\mathbf{b}^2 + \ldots$$

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- Quantifies **momentum broadening**

$$\hat{q} = \frac{d\langle p_\perp^2\rangle}{dL} = \frac{d\langle p_\perp^2\rangle}{dt} = \int d^2 q_\perp\, q_\perp^2 \frac{d\Gamma^{\mathrm{el}}}{d^2 q_\perp}$$

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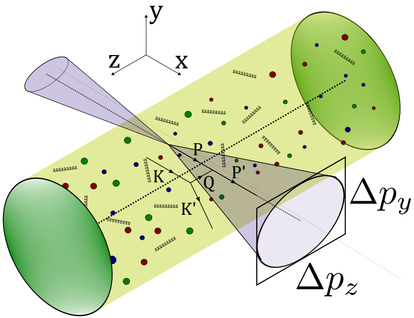

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Estimates of $\hat{q}$

ELSEVIER

Physics Letters B

Volume 803, 10 April 2020, 135318

Jet quenching as a probe of the initial stages in heavy-ion collisions ☆

Carlota Andres [a], Néstor Armesto [b], Harri Niemi [c d], Risto Paatelainen [e d], Carlos A. Salgado [b]

A B S T R A C T

Jet quenching provides a very flexible variety of observables which are sensitive to different energy- and time-scales of the strongly interacting matter created in heavy-ion collisions. Exploiting this versatility would make jet quenching an excellent chronometer of the yoctosecond structure of the evolution process. Here we show, for the first time, that a combination of jet quenching observables is sensitive to the initial stages of heavy-ion collisions, when the approach to local thermal equilibrium is expected to happen. Specifically, we find that in order to reproduce at the same time the inclusive particle production suppression, R_{AA}, and the high-p_T azimuthal asymmetries, v_2, energy loss must be strongly suppressed for the first ~ 0.6 fm. This exploratory analysis shows the potential of jet observables, possibly more sophisticated than the ones studied here, to constrain the dynamics of the initial stages of the evolution.

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ABSTRACT

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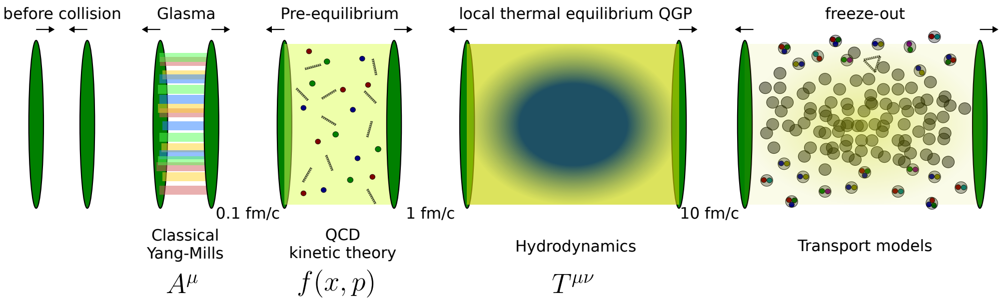
$\hat{q} = 0$ during initial stages [Phys.Lett.B 803 (2020) [Andres, Armesto, Niemi, Paatelainen, Salgado]]

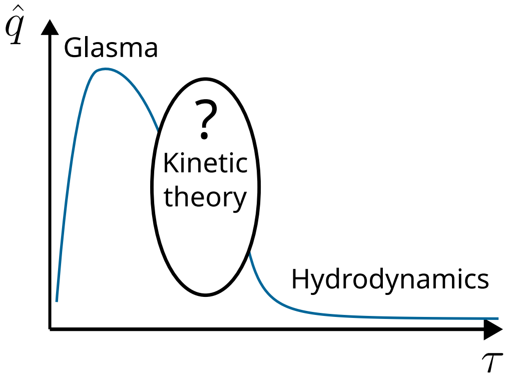
Estimates of $\hat{q}$



Motivation

- Studies find initial stages $\hat{q}(\tau < 0.6\,\mathrm{fm}) \equiv 0$ needed for description of jet v_2 and R_{AA}[1]
- However, studies in the initial Glasma stage find $\hat{q}$ very large[2]
- **Goal: $\hat{q}$ during hydrodynamization**
  $\rightarrow$ between Glasma and hydro
- **Question:**
  Supports large Glasma values?

Schematic overview of $\hat{q}$ evolution



[1] [Phys.Lett.B 803 (2020) [Andres, Armesto, Niemi, Paatelainen, Salgado]]

[2] [Phys.Lett.B 810 (2020) [Ipp, Müller, Schuh], Phys.Rev.C 105 (2022) [Carrington, Czajka, Mrowczynski], Phys.Rev.D 107 (2023) [Avramescu, Baran, Greco, Ipp, Müller, Ruggieri]]

Effective kinetic theory description of the QGP

- Plasma without quarks
- Gluons with **distribution function** $f(t, \mathbf{p})$

- Azimuthal symmetry around beam axis $\hat{z}$,
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- Plasma without quarks
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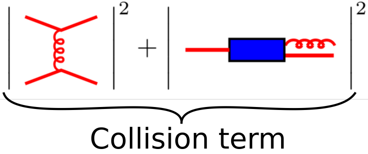
- Time evolution described by **Boltzmann equation** at leading-order[3]

$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) f =$$



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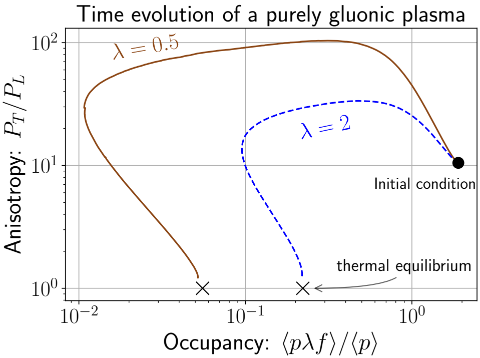
Bottom-up thermalization in heavy-ion collisions

- Initial condition[4], with $\lambda = g^2 N_C$

$$f(p_\perp, p_z) = \frac{2A}{\lambda} \frac{\langle p_T \rangle}{\sqrt{p_\perp^2 + \xi_0{}^2 p_z^2}} \times \exp\left(\frac{-2}{3\langle p_T \rangle^2} \left(p_\perp^2 + \xi_0{}^2 p_z^2 \right) \right)$$

$\xi_0 \sim$ anisotropy, $\langle p_T \rangle = 1.8 Q_s$,
$Q_s \sim$ saturation scale



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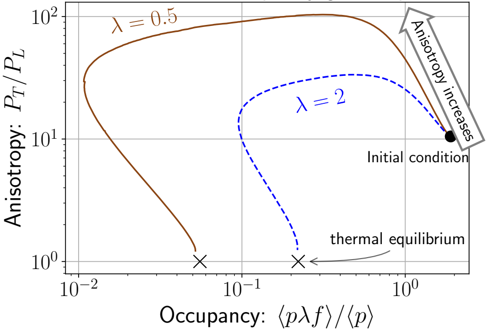
  - **Phase 1**: Anisotropy increases

Time evolution of a purely gluonic plasma



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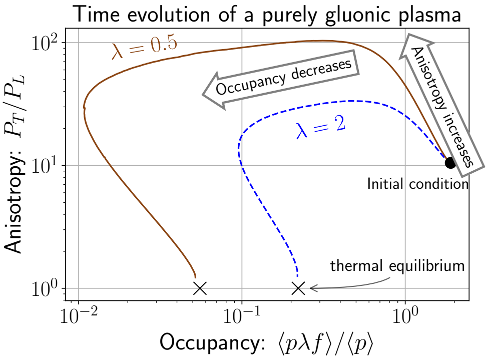

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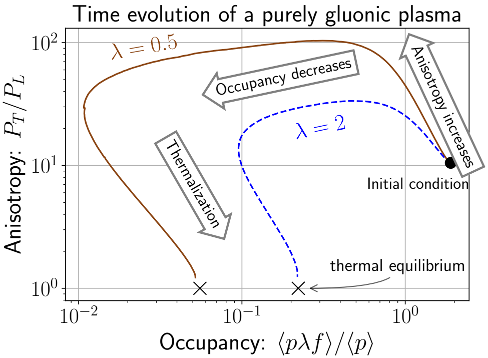

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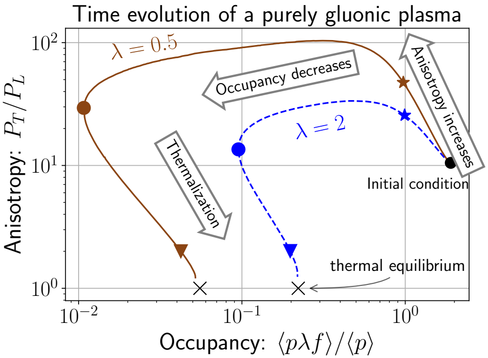
Markers represent **different stages**



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Jet momentum broadening in kinetic theory

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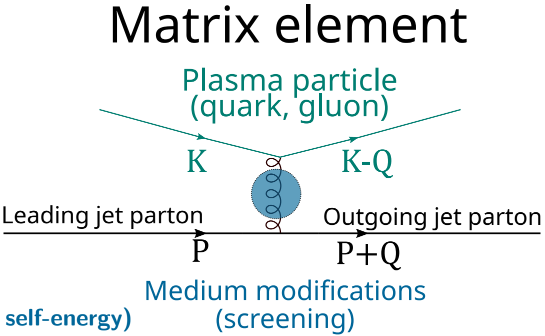
Outgoing plasma particle

$$\hat{q}^{ij} = \int_{\substack{q_\perp < \Lambda_\perp \\ p \to \infty}} \mathrm{d}\Gamma_{\mathrm{PS}}\, q^i q^j \, |\mathcal{M}|^2 f(k)\,(1 + f(k'))$$

Incoming plasma particles with momentum k

Matrix element
with medium corrections (HTL self-energy)

appropriate phase-space measure



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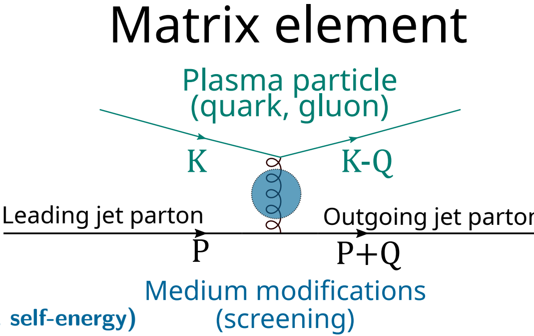
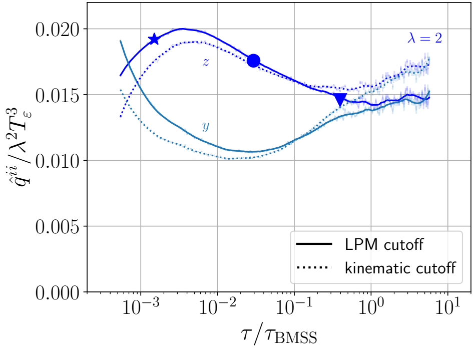


Results for $\hat{q}$

- Cutoff models for dependence on jet energy and effective temperature
  - $\Lambda_\perp^{\mathrm{LPM}}(E, T_\varepsilon) = \zeta^{\mathrm{LPM}} g \times (E T_\varepsilon^3)^{1/4}$
  - $\Lambda_\perp^{\mathrm{kin}}(E, T_\varepsilon) = \zeta^{\mathrm{kin}} g \times (E T_\varepsilon)^{1/2}$



[Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

Plot for $E = 100\,\mathrm{GeV}$

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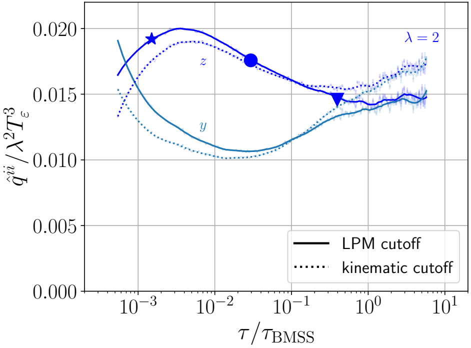
- Logarithmic cutoff $\Lambda_\perp$ dependence[6]

  $$\hat{q}^{xx}(\Lambda_\perp \gg T_\varepsilon) \simeq a_x \ln \frac{\Lambda_\perp}{Q_s} + b_x$$

  (and similar for $\hat{q}^{yy}$)



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6 [Values available at https://zenodo.org/records/10419537]

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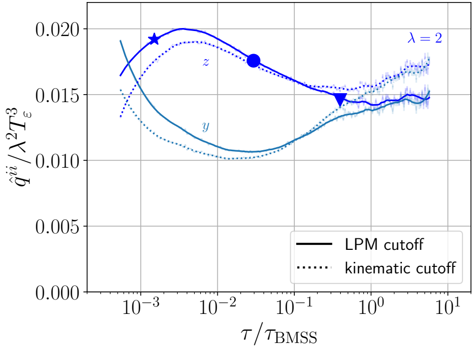
- Mostly $\hat{q}^{zz} > \hat{q}^{yy}$
  $\rightarrow$ **Enhanced broadening along beam axis**
- Similar results for both cutoffs



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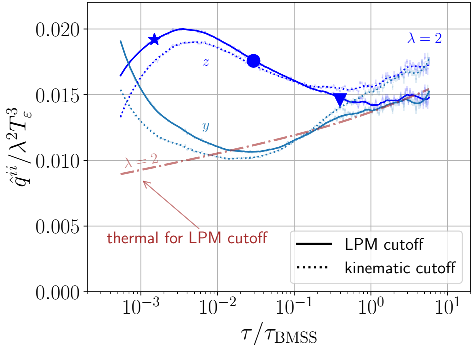

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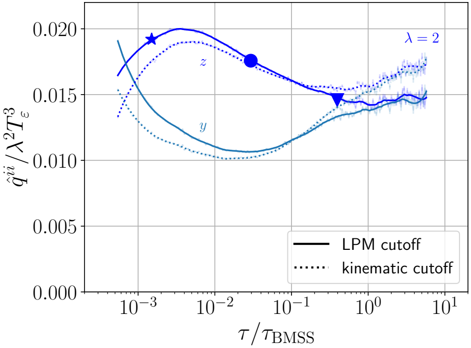

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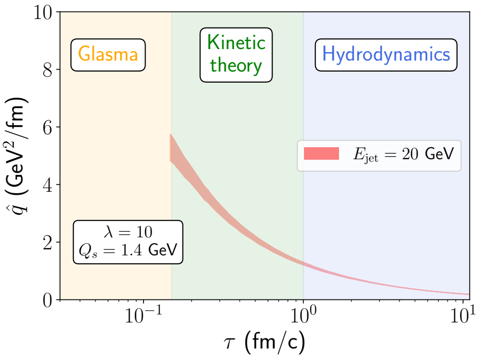
Time evolution of jet quenching parameter

- Model cutoff variation for fixed jet energy
- Dependence on initial conditions and cutoff (bands)



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Time evolution of jet quenching parameter

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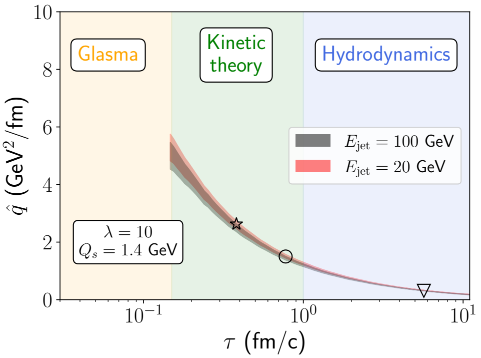

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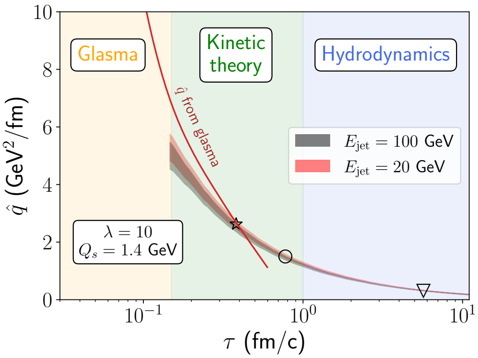
- Little jet energy dependence
- Supports **large values** from **Glasma**[6] and lower values in hydrodynamic stage



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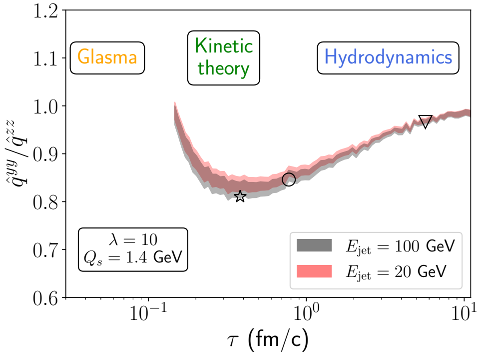
Time evolution of jet quenching parameter

- Model cutoff variation for fixed jet energy
- Dependence on initial conditions and cutoff (bands)
- Little jet energy dependence
- Broadening **anisotropy** up to 15 %
- Possible impact on polarization[6], azimuthal and spin observables[7]



[Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

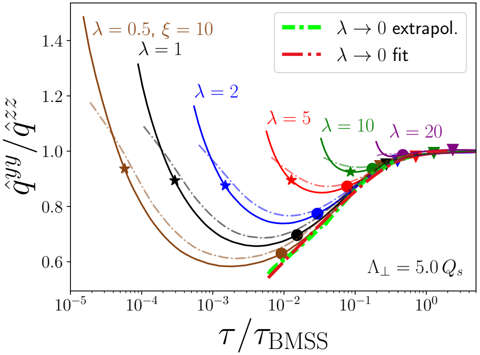
[6] [JHEP 08 (2023) [Hauksson, Iancu]]

[7] [arXiv:2407.04774 [Barata, Salgado, Silva]]

Limiting attractors in $\hat{q}$ anisotropy

Phys.Lett.B 852 (2024)

(Boguslavski, Kurkela, Lappi, FL, Peuron)



- Different time scales associated with

  bottom-up thermalization $\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$
- Weak coupling (bottom-up) attractor important already at moderate λ

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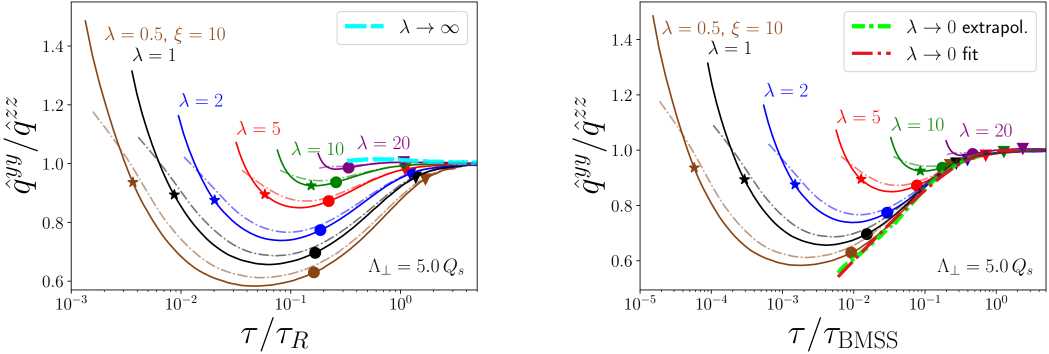


- Different time scales associated with hydrodynamical attractor $\tau_R = \frac{4\pi\eta/s}{T}$ and bottom-up thermalization $\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$
- Weak coupling (bottom-up) attractor important already at moderate λ

Momentum broadening kernel

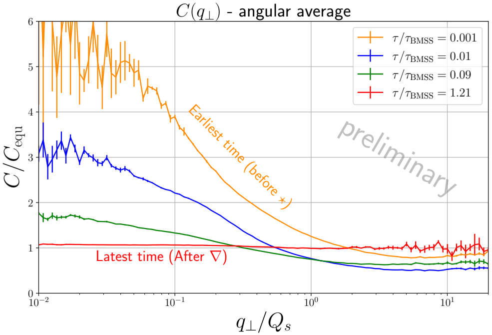
$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\ |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

Normalize using (Landau-matched) thermal kernel, with small $q_\perp$ form[8]

$$C_{\mathrm{equ}}(q_\perp \ll T) = \frac{C_R g^2 T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$

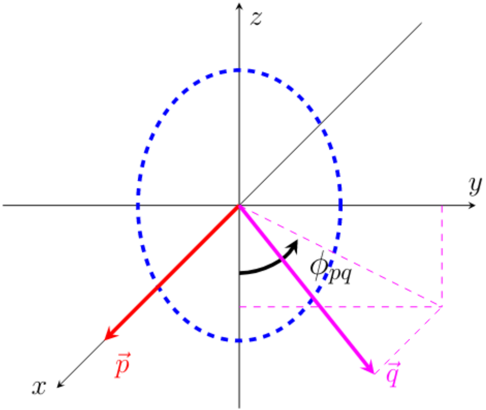
- **Momentum transfer of soft momenta enhanced**
- Late times (red curve):Thermal

[8] [JHEP 05 (2002) [Aurenche, Gelis, Zaraket]]



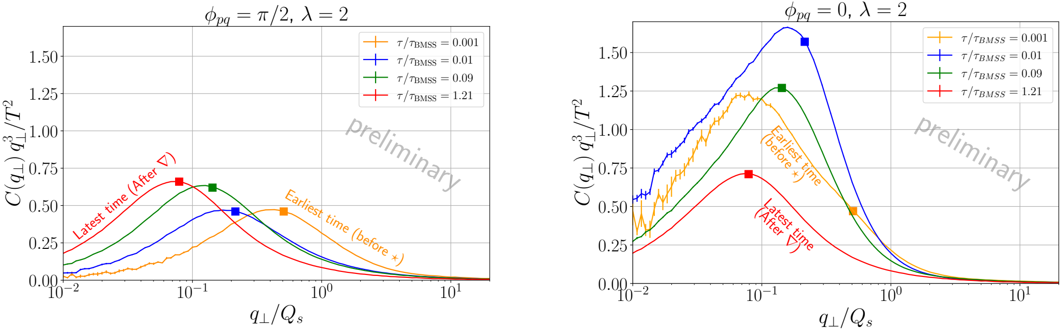
Here, $\lambda = 2$.

Angular dependence and contribution to $\hat{q}$



- Consider **Angular dependence**
- $\phi_{pq} = 0 \rightarrow$ along beam axis

Angular dependence and contribution to $\hat{q}$



- Contribution to $\hat{q} = \int \mathrm{d}^2\mathbf{q}_\perp\, q_\perp^2\, C(\mathbf{q}_\perp)/(2\pi)$
- **Peaked at Debye mass** ■ for later times
- Along beam ($\phi_{pq} = 0$): Much larger and different form at early times

Conclusions and outlook

- Studied **momentum broadening of jets**
  $\rightarrow \hat{q}$ and $C(\mathbf{q}_\perp)$ during initial stages in heavy-ion collisions
- Values of $\hat{q}$ within $\sim 20\%$ of thermal estimate
- $\hat{q}$ from kinetic theory **supports large Glasma values**
- More momentum broadening along the beam axis ($\hat{q}^{zz} > \hat{q}^{yy}$)
- $C(\mathbf{q}_\perp)$ at small $\mathbf{q}_\perp$ is enhanced compared to thermal (especially along beam)

Outlook

- Obtain gluon emission spectrum from pre-equilibrium $\hat{q}$ (with Barata, Sadofyev)
- Inclusion of quarks in plasma background (with Mazeliauskas, Takacs, Zhou)

[Code and data: https://zenodo.org/records/10419537, https://zenodo.org/records/10409474]

Thank you very much for your attention!

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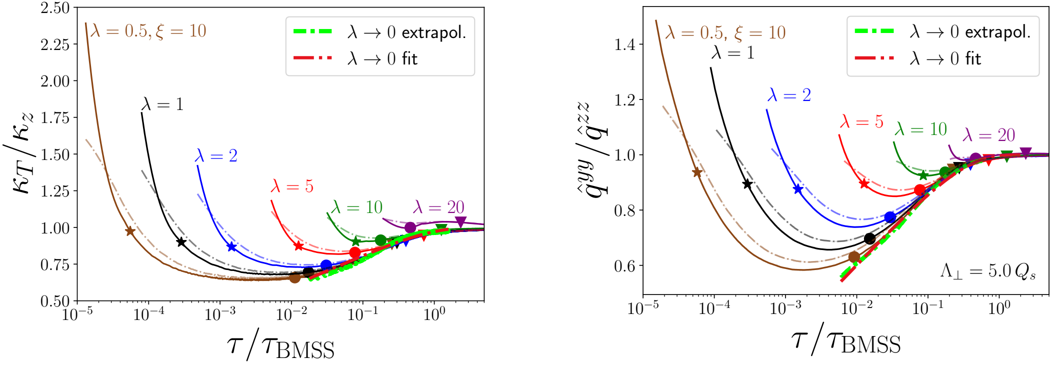
Anisotropies fall on universal curve . . .

[Phys.Lett.B 852 (2024)]

[Boguslavski, Kurkela, Lappi, FL, Peuron]



- Approach to universal curve when scaled with $\tau_{\mathrm{BMSS}} = \alpha^{-13/5} / Q_s$
- Many quantities plotted as function of different time . . .

Bottom-up vs. hydrodynamic attractor

- Often universal behavior in τ/τ_R,

$$\tau_R = \frac{4\pi\eta/s}{T}.$$

- Conformal (first order) relativistic hydrodynamics [9]:

$$\frac{P_L}{P_T} = 1 - 8\underbrace{\frac{\eta/s}{\tau T}}_{\sim\tau_R/\tau}$$

Two different pictures emerge:

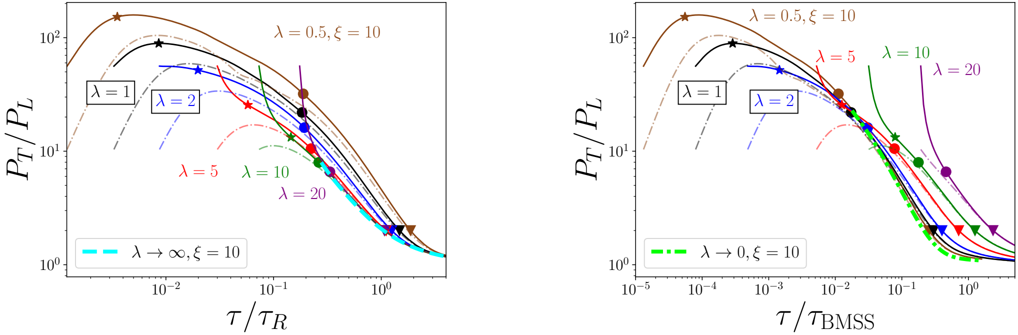
- **Bottom-up** expects thermalization around $\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$
- **Hydrodynamics** expects thermalization around $\tau_R = \frac{4\pi\eta/s}{T}$

[9] [[Romatschke, Romatschke] (2019)]

Pressure ratio

$$\tau_R = \frac{4\pi\eta/s}{T},\ \tau_{\rm BMSS} = \alpha_s^{-13/5}/Q_s$$

- **Kinetic theory simulations** for **different couplings** $0.5 \leq \lambda \leq 20$ and **initial conditions**.

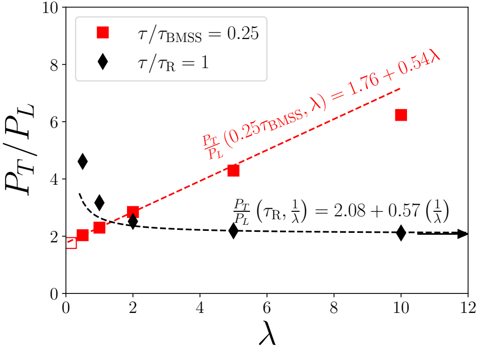


- Attractor for each λ (insensitive to IC)
- Curves **approach limiting attractors** after •

Extrapolation to limiting attractors

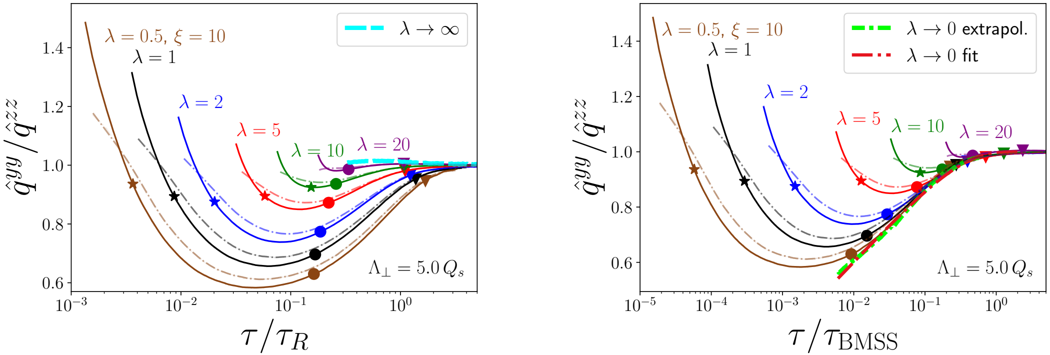
$$\tau_R = \frac{4\pi\eta/s}{T}$$
$$\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$$

- Obtain limiting attractors by extrapolating at fixed τ/τ_R or $\tau/\tau_{\mathrm{BMSS}}$
- **Bottom-up attractor**: Linear extrapolation to $\lambda \to 0$
- **Hydro attractor**: Linear extrapolation to $1/\lambda \to 0$



$\hat{q}$ and the limiting attractors

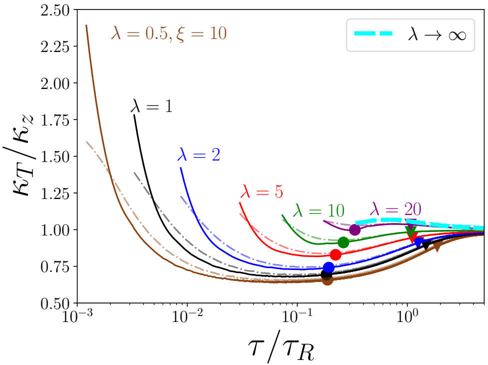
[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

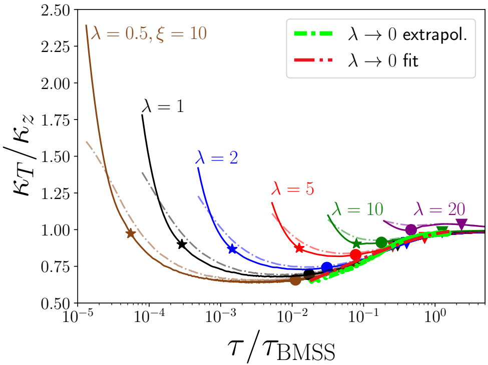


- Approach to weak coupling attractor even at moderate couplings

κ and the limiting attractors

[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]





- Similar to $\hat{q}$: Approach to weak coupling attractor even at moderate λ

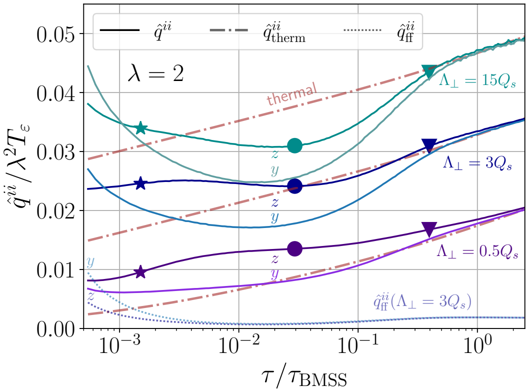
Bose-enhanced terms

- $\hat{q}$ for fixed coupling $\lambda = 2$ and varying cutoffs $\Lambda_\perp$
- 2D distribution

$$f(\mathbf{k}) \sim \delta(k_z)$$

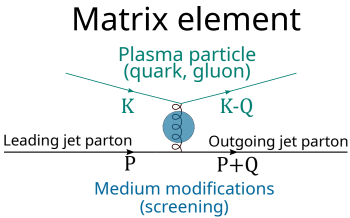
Leads to $\hat{q}^{zz}_{\rm ff} = 0$

- Reason for different ordering: Bose-enhanced part $\hat{q}_{\rm ff}$ = term quadratic in $f(\mathbf{k})$



Screening in the matrix element of $\hat{q}$

- Scattering matrix element includes **in-medium propagator**
- Receives **self-energy corrections**
- Anisotropic hard thermal loop (HTL) self-energy $\rightarrow$ unstable modes[10]
- **Approximation: Use isotropic HTL matrix element**
  Similar approximation also in EKT implementations[11]



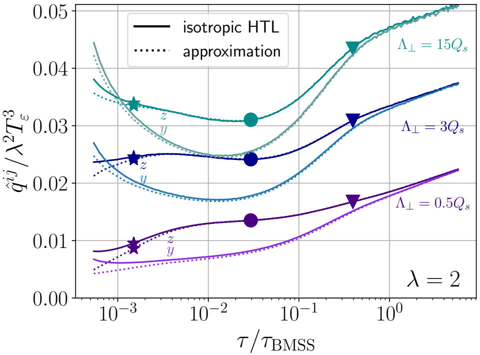
[10] [Phys.Rev.D 68 (2003) [Romatschke, Strickland]]
[11] [Phys.Rev.Lett. 115 (2015) [Kurkela, Zhu]; Phys.Rev.Lett. 122 (2019) [Kurkela, Mazeliauskas]; Phys.Rev.D 104 (2021) [Du, Schlichting]]

Screening approximation to the matrix element

- Compare with simple screening approximation

$$\frac{(s-u)^2}{t^2} \rightarrow \frac{(s-u)^2}{t^2}\frac{q^4}{(q^2+\xi_T^2 m_D^2)^2}$$

- Longitudinal[12] $\xi_L = e^{5/6}/\sqrt{8}$
- Transverse broadening: $\xi_T = e^{1/3}/2$
- **Good agreement**

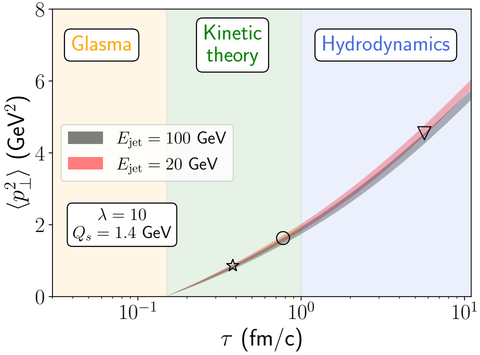


s, u, t: Mandelstam variables

[12] [Phys.Rev.D 89 (2014) [York, Kurkela, Lu, Moore]]

What about total momentum broadening?

- Per definition, $\hat{q} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}\tau}$
- Naïvely $\Delta p_\perp^2 = \int \mathrm{d}\tau\, \hat{q}(\tau)$ over lifetime of jet
- Think of $\hat{q}$ as medium parameter.



Toy model for underoccupation

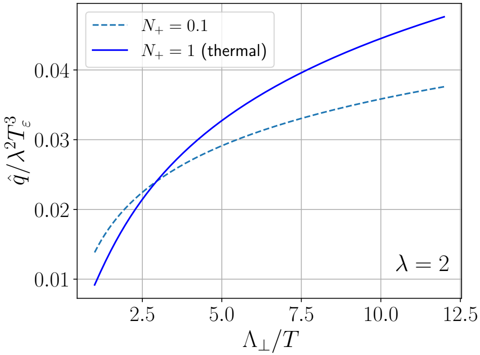
Scaled thermal distribution

- Scaled thermal distribution

$$f(k;T) = \frac{N_+}{\exp(k/T) - 1}$$

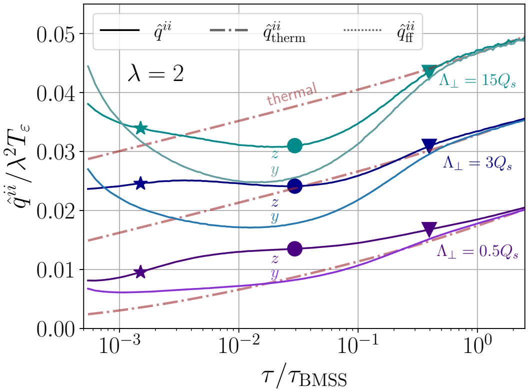
Explains ordering $\hat{q}_{\mathrm{therm}} \lessgtr \hat{q}$ for underoccupancy

[arXiv:2312.00447 [Boguslavski, Kurkela, Lappi, FL, Peuron]]



Cutoff dependence and comparison with equilibrium

- $\hat{q}$ for fixed coupling $\lambda = 2$ and varying cutoffs $\Lambda_\perp$
- **Ordering $\hat{q}^{yy} \lessgtr \hat{q}^{zz}$ depends on cutoff**
- Energy-matched **equilibrium over- or underestimates $\hat{q}$, depending on cutoff**



Making sense of the cutoff

- Cutoff $\Lambda_\perp$ restricts transverse momentum transfer $q_\perp < \Lambda_\perp$ (needed in eikonal limit $p \to \infty$)

$$\hat{q} \sim \int \mathrm{d}^2 q_\perp \, q_\perp^2 \underbrace{\frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}}_{1/q_\perp^4 \text{ for large } q_\perp} \sim \int \frac{\mathrm{d} q_\perp}{q_\perp}$$

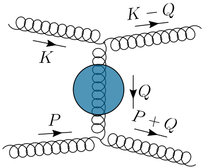
Making sense of the cutoff

- Cutoff $\Lambda_\perp$ restricts transverse momentum transfer $q_\perp < \Lambda_\perp$ (needed in eikonal limit $p \to \infty$)
- Cutoff should grow with jet energy
- **kinematic cutoff** $\Lambda_\perp^{\mathrm{kin}}(E, T) = \zeta^{\mathrm{kin}} g(ET)^{1/2}$
  obtained from comparing leading log behavior for large p and $\Lambda_\perp$
- **LPM cutoff** $\Lambda_\perp^{\mathrm{LPM}}(E, T) = \zeta^{\mathrm{LPM}} g(ET^3)^{1/4}$
  Estimate for momentum broadening during LPM 'formation time':
  $Q_\perp^2 \sim \hat{q} t^{\mathrm{form}}$, $t^{\mathrm{form}} \sim \sqrt{E/\hat{q}}$, approximately $\hat{q} \sim g^4 T^3$

[arXiv:2312.00447 [Boguslavski, Kurkela, Lappi, FL, Peuron]]

Elastic collision term

$$\mathcal{C}^{2\leftrightarrow 2}[f(\mathbf{p})] = \int_{\mathbf{k}\mathbf{k}'} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}'\mathbf{k}')\right|^2 \left\{ f_{\mathbf{p}} f_{\mathbf{k}} \left[1 + f_{\mathbf{p}'}\right] \left[1 + f_{\mathbf{k}'}\right] - f_{\mathbf{p}'} f_{\mathbf{k}'} \left[1 + f_{\mathbf{p}}\right] \left[1 \pm f_{\mathbf{k}}\right] \right\},$$



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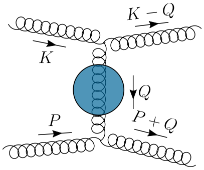
with medium effects included in the **vaccum matrix element**

$$\frac{|\mathcal{M}|^2}{4\lambda^2 d_A} = 9 + \frac{(s-t)^2}{\underline{u^2}} + \frac{(u-s)^2}{\underline{t^2}} + \frac{(t-u)^2}{s^2}.$$



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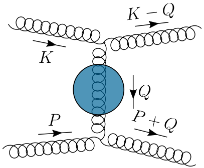

via the replacement (“**Debye-like screening**”)

$$\frac{(u-s)^2}{\underline{t^2}} \to \frac{(u-s)^2}{t^2} \frac{q^4}{(q^2 + \xi^2 m_D^2)^2},$$

$\xi = e^{5/6}/\sqrt{8}$

m_D . . . Debye mass

s, u, t . . . Mandelstam variables

as simple **isotropic approximation** for **hard thermal loop propagator**.

[Phys.Rev.D 89 (2014) [Abraao York, Kurkela, Lu, Moore]]

Screening prescription revisited

$$\partial_\tau f = -\mathcal{C}^{2\leftrightarrow 2} - \mathcal{C}^{1\leftrightarrow 2}$$

$$\mathcal{C}^{2\leftrightarrow 2}[f(\mathbf{p})] = \int_{\mathbf{k}\mathbf{k}'} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}'\mathbf{k}')\right|^2 \left\{ f_{\mathbf{p}} f_{\mathbf{k}} \left[1 + f_{\mathbf{p}'}\right] \left[1 + f_{\mathbf{k}'}\right] - f_{\mathbf{p}'} f_{\mathbf{k}'} \left[1 + f_{\mathbf{p}}\right] \left[1 \pm f_{\mathbf{k}}\right] \right\},$$

with medium effects included in the vaccum matrix element

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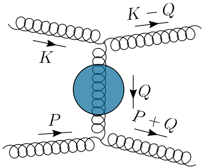

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with medium effects included in the vaccum matrix element

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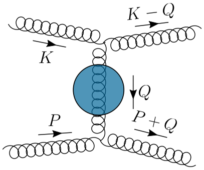


via the replacement (“**isoHTL screening**”)

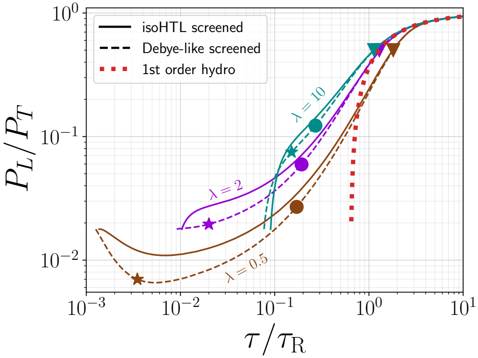
$$\frac{(u-s)^2}{\underline{t^2}} \to \left| G^{\mathrm{HTL,ret}}_{\mu\nu}(P-P')\,(P+P')^\mu (K+K')^\nu \right|^2$$

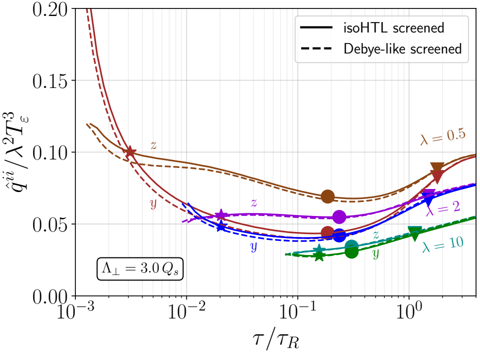
with G the **isotropic hard thermal loop propagator**.

[JHEP 01 (2003) [Arnold, Moore, Yaffe], arXiv:2407.09605 [Boguslavski, FL]]

Effects of isoHTL screening

[arXiv:2407.09605 [Boguslavski, FL]]





- Left: Screening affects pressure ratio at early times
- Right: Minimal effect on $\hat{q}$