QCD jets evolution in non equilibrium plasmas

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in collaboration with

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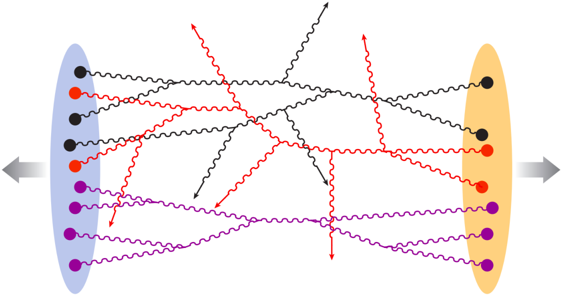
- After a heavy-ion collision, an out-of-equilibrium high-populated system of gluons is produced.

Phys. Rev. D 55 (1997). Jalilian-Marian et al.

Nucl. Phys. B 529 (1998). Kovchegov and Mueller

- In the weak coupling limit, this system of gluons evolves following the so called "bottom up thermalization".

Phys. Lett. B 502 (2001). Baier et al.



Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

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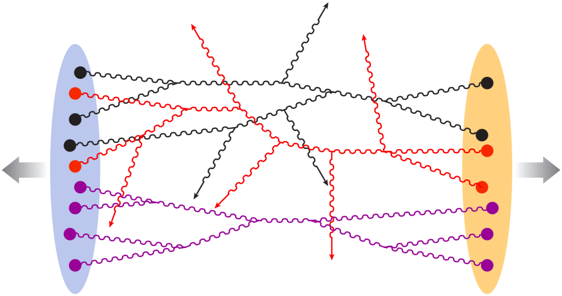
- A hard scattering between two partons of the nucleus can produce a jet.
- This highly energetic probes propagate throughout the system.
  - It is affected by the system (jet quenching).
  - It may perturbe the themalization/hydrodynamization process (for example, jet wake).



Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

- The thermalization process has been explored numerically using the Effective Kinetic Theory (EKT).

JHEP 01 (2003). Arnold, Moore, and Yaffe

Phys. Rev. Lett. 113.18 (2014). Kurkela and Lu

- Recent works propose the Boltzmann Equation in Diffussion Approximation (BEDA) as an alternative approach.

Physics Letters B 834 (2022). SBC, Salgado, and Wu

JHEP 06 (2024). SBC, Salgado, and Wu

- We will use this kinetic theory (BEDA) as an unified framework to **the evolution of the bulk and the jet during the thermalization time**.

- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f]\,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

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- We consider a spatially homogeneous system.

$$f^a(\mathbf{p}) = f^a(p)$$

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- The thermalization can be studied following the time evolution of the screening mass, the jet quenching parameter and the effective temperature and net quark chemical potential[1].

$$m_D^2 = m_D^2[f] \qquad \hat{q} = \hat{q}[f]$$

$$T_*(t) \equiv \frac{\hat{q}}{2\alpha_s N_c m_D^2 \ln \frac{\langle p_t^2 \rangle}{m_D^2}} \qquad \mu_* = \mu_*[f]$$

[1] All quarks are assumed to have identical distribution. In general each flavour would have its own μ_* associated.

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

$$C^a_{2\leftrightarrow 2} = \frac{1}{4}\hat{q}_a(t)\nabla_{\mathbf{p}} \cdot \left[\nabla_{\mathbf{p}} f^a + \frac{\mathbf{v}}{T^*(t)} f^a(1+\epsilon_a f^a)\right] + \mathcal{S}_a$$

$$\mathcal{S}_q = \frac{2\pi\alpha_s^2 C_F^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}}{p}\left[\mathcal{I}_c f(1-F) - \bar{\mathcal{I}}_c F(1+f)\right],$$

$$\mathcal{S}_{\bar{q}} = \mathcal{S}_q\big|_{F\leftrightarrow\bar{F}}, \qquad \mathcal{S}_g = -\frac{N_f}{2C_F}(\mathcal{S}_q + \mathcal{S}_{\bar{q}}),$$

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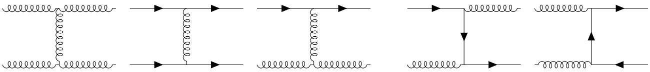
Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

Fokker-Planck term | Source term



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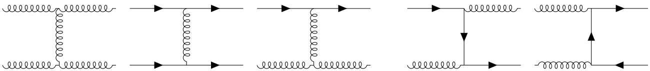


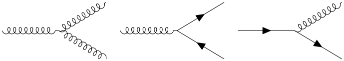
- The $1 \leftrightarrow 2$ kernel can be computed in the deep LPM regime

Nucl. Phys. B 483 (1997). Baier et al.

Phys. Rev. D 78 (2008). Arnold and Dogan

$$C^a_{1\leftrightarrow 2} = \int_0^1 \frac{dx}{x^3} \sum_{b,c} \left[\frac{\nu_c}{\nu_a} C^c_{ab}(\mathbf{p}/x; \mathbf{p}, \mathbf{p}(1-x)/x) - \frac{1}{2} C^a_{bc}(\mathbf{p}; x\mathbf{p}, (1-x)\mathbf{p}) \right]$$

- The $C^a_{bc}(\mathbf{p}; x\mathbf{p}, (1-x)\mathbf{p})$ describes the collinear splitting $a \leftrightarrow bc$. The three possible processes involved are the three QCD interaction vertices.



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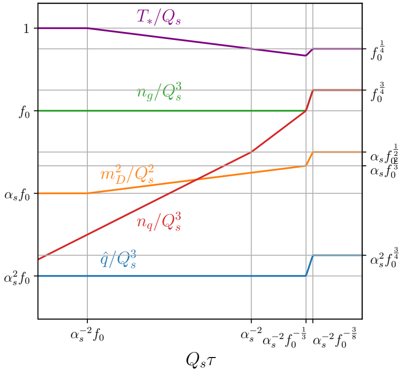
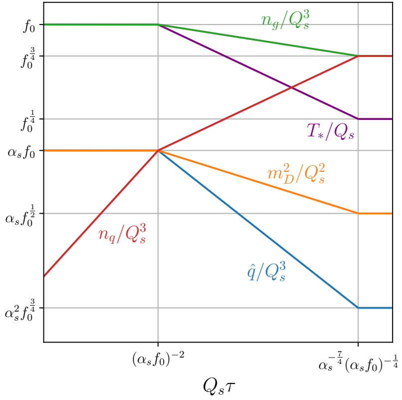
- The parametric evolution has been well understood in both under and overpopulated limits

Physics Letters B 834 (2022). SBC, Salgado, and Wu

JHEP 06 (2024). SBC, Salgado, and Wu



Parametric estimation for $f_0 \ll 1$



Parametric estimation for $f_0 \gg 1$

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- We can explore the jet as a perturbation to an spatially homogeneous medium.

$$f(\mathbf{p},\mathbf{x},t) = f_{back}(\mathbf{p},t) + \delta f(\mathbf{p},\mathbf{x},t)$$

- After linearizing the BEDA, the background will evolve by itself

$$\partial_t f_{back} = C^a_{2\leftrightarrow 2}[f_{back}] + C^a_{1\leftrightarrow 2}[f_{back}],$$

meanwhile the perturbation

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})\delta f^a(\mathbf{p},\mathbf{x},t) = \delta C^a_{2\leftrightarrow 2}[f,\delta f] + \delta C^a_{1\leftrightarrow 2}[f,\delta f].$$

$$C^a_i[f] = C^a_i[f_{back}] + \underbrace{\tilde{C}^a_i[f_{back}]\delta f(\mathbf{p},\mathbf{x},t)}_{\equiv \delta C^a_i[f,\delta f]} + \dots$$

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- This allows us to integrate out the spatial dependence

JHEP 07 (2021). Schlichting and Soudi

$$\partial_t \bar{\delta f}^a(\mathbf{p},t) = \delta C^a_{2\leftrightarrow 2}[f,\bar{\delta f}] + \delta C^a_{1\leftrightarrow 2}[f,\bar{\delta f}],$$

$$\bar{\delta f}^a(\mathbf{p},t) \equiv \int d^3\mathbf{x}\,\delta f^a(\mathbf{p},\mathbf{x},t)$$

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- We have derived the linearized version of the BEDA.
- The Focker-Planck term writes

$$\delta C^a_{2\leftrightarrow 2,FP} = \frac{\delta\hat{q}}{4}\nabla_{\mathbf{p}}\cdot\left[\nabla_{\mathbf{p}} f^a_0 + \frac{\mathbf{v}}{T^0_*} f^a_0 (1+\epsilon^a f^a_0)\right] +$$
$$\frac{\hat{q}_0}{4}\nabla_{\mathbf{p}}\cdot\left[\nabla_{\mathbf{p}}\delta f^a + \frac{\mathbf{v}}{T^0_*}\left(\delta f^a(1+\epsilon^a 2 f^a_0) - \frac{\delta T_*}{T^0_*} f^a_0(1+\epsilon^a f^a_0)\right)\right]$$

- The source term

$$\delta S_{q^i} = \frac{2\pi\alpha_s^2 C_F^2 \mathcal{L}}{p}\left[\left(\delta\mathcal{I}^i_c f_0(1-F^i_0) - \delta\bar{\mathcal{I}}^i_c F^i_0(1+f_0)\right) + \right.$$
$$\left.\delta f\left(\mathcal{I}^i_{c,0}(1-F^i_0) - \bar{\mathcal{I}}^i_{c,0}F^i_0\right) - \delta F^i\left(\mathcal{I}^i_{c,0} f_0 + \bar{\mathcal{I}}^i_{c,0}(1+f_0)\right)\right]$$

$$\delta S_{\bar{q}^i} = \frac{2\pi\alpha_s^2 C_F^2 \mathcal{L}}{p}\left[\left(\delta\bar{\mathcal{I}}^i_c f_0(1-\bar{F}^i_0) - \delta\mathcal{I}^i_c \bar{F}^i_0(1+f_0)\right) + \right.$$
$$\left.\delta f\left(\bar{\mathcal{I}}^i_{c,0}(1-\bar{F}^i_0) - \mathcal{I}^i_{c,0}\bar{F}^i_0\right) - \delta\bar{F}^i\left(\bar{\mathcal{I}}^i_{c,0} f_0 + \mathcal{I}^i_{c,0}(1+f_0)\right)\right]$$

$$\delta S_g = -\frac{1}{2C_F}\sum_{i=1}^{N_f}\left(S_{q^i} + S_{\bar{q}^i}\right)$$

- The $1 \leftrightarrow 2$ collision kernel is also straightforward:

$$\delta C^a_{bc}(\mathbf{p};\mathbf{k},\mathbf{l}) \equiv \Gamma^a_{bc}(x)\sqrt{\frac{\hat{q}_0}{p}}\left[\frac{1}{2}\frac{\delta\hat{q}}{\hat{q}_0}\mathcal{F}^a_{bc,0}(\mathbf{p};\mathbf{k},\mathbf{l}) + \delta\mathcal{F}^a_{bc}(\mathbf{p};\mathbf{k},\mathbf{l})\right].$$

$$\begin{aligned}\delta\mathcal{F}^a_{bc}(\mathbf{p};\mathbf{k},\mathbf{l}) \equiv & \delta f^a\left[(1+\epsilon^b f^b_0)(1+\epsilon^c f^c_0) - \epsilon^a f^b_0 f^c_0\right] + \\ & \delta f^b\left[\epsilon^b f^a_0(1+\epsilon^c f^c_0) - f^c_0(1+\epsilon^a f^a_0)\right] + \\ & \delta f^c\left[\epsilon^c f^a_0(1+\epsilon^b f^b_0) - f^b_0(1+\epsilon^a f^a_0)\right].\end{aligned}$$

Linearized BEDA II

- The $1 \leftrightarrow 2$ collision kernel is also straightforward:

$$\delta C^a_{bc}(\mathbf{p};\mathbf{k},\mathbf{l}) \equiv \Gamma^a_{bc}(x)\sqrt{\frac{\hat{q}_0}{p}}\left[\frac{1}{2}\frac{\delta\hat{q}}{\hat{q}_0}\mathcal{F}^a_{bc,0}(\mathbf{p};\mathbf{k},\mathbf{l}) + \delta\mathcal{F}^a_{bc}(\mathbf{p};\mathbf{k},\mathbf{l})\right].$$

$$\begin{aligned}\delta\mathcal{F}^a_{bc}(\mathbf{p};\mathbf{k},\mathbf{l}) \equiv & \delta f^a\left[(1+\epsilon^b f_0^b)(1+\epsilon^c f_0^c) - \epsilon^a f_0^b f_0^c\right] + \\ & \delta f^b\left[\epsilon^b f_0^a(1+\epsilon^c f_0^c) - f_0^c(1+\epsilon^a f_0^a)\right] + \\ & \delta f^c\left[\epsilon^c f_0^a(1+\epsilon^b f_0^b) - f_0^b(1+\epsilon^a f_0^a)\right].\end{aligned}$$

- This set of equations completes the equation used for study thermalization in the soft sector of jets

JHEP 10 (2015). Iancu and Wu

- Previous works have studied the behaviour of this jets in EKT for (non-evolving) thermal backgrounds.

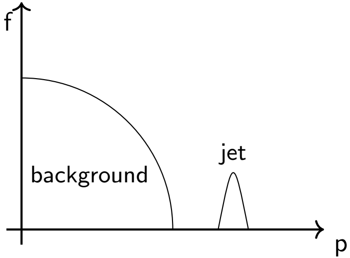
JHEP 07 (2021). Schlichting and Soudi

JHEP 05 (2023). Mehtar-Tani, Schlichting, and Soudi

- Similar methods have been used to study minijets in both thermal and non thermal backgrounds.

JHEP 06 (2024). Zhou, Brewer, and Mazeliauskas

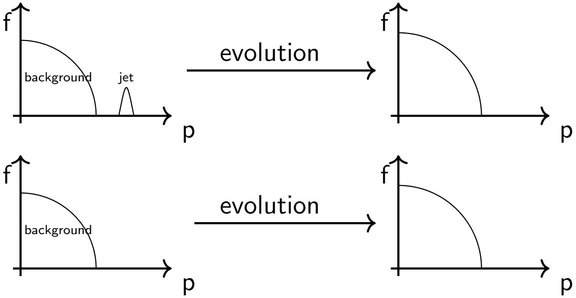
- We will explore the evolution of jets in both types of mediums.



- In the numerical results, the perturbation will contain 10% of the total energy of the system.

Studying the jet as a perturbation III

- For thermal background, the linearized BEDA is solved.
- For non thermal background we compute the perturbation as sketched below.



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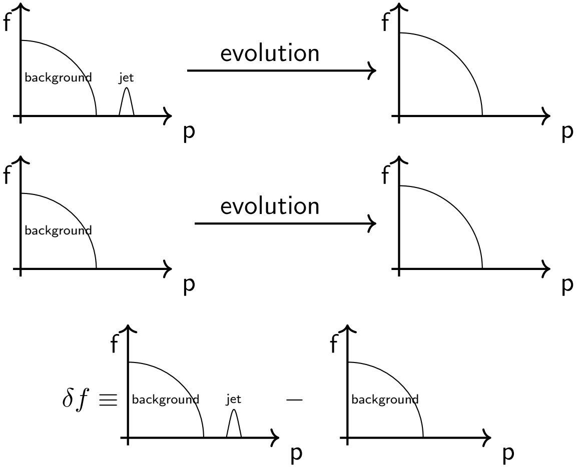
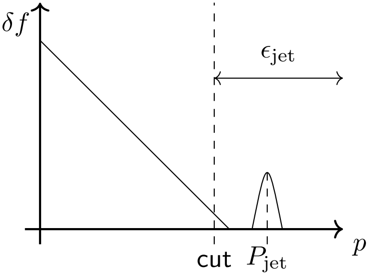


Extracting the energy loss

- We can get an estimation of the energy loss of the jet computing the energy up to some momentum cut.



$$\epsilon_{\text{loss}} \equiv 1 - \frac{\epsilon_{\text{jet}}}{\epsilon_{\text{jet},0}}$$

- The balance equation attempts to reproduce the behaviour of the IR cascade with a simpler description than the full Boltzmann equation.

Phys. Rev. Lett. 111 (2013). Blaizot, Iancu, and Mehtar-Tani

- This equation can be solved exactly if one simplifies the BDMPS-Z splitting rate.

$$x\frac{dN}{dx} \approx D_0(x,\tau) \propto \frac{\tau}{\sqrt{x}(1-x)^{\frac{3}{2}}} e^{-\frac{\pi\tau^2}{1-x}}, \quad \tau \equiv \alpha\sqrt{\omega_c}2E$$

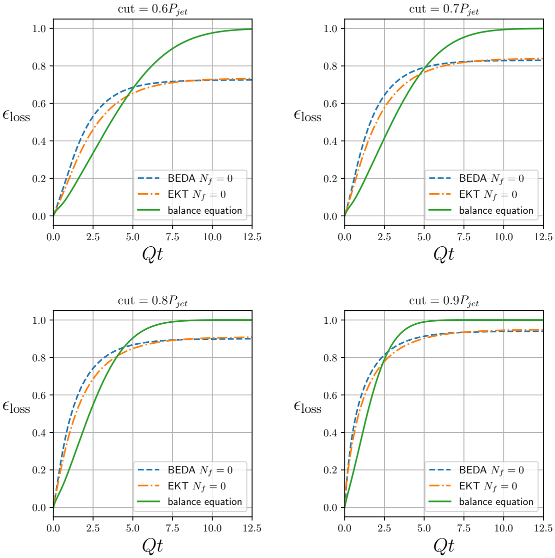
- The result of this can be interpreted as those from the Quenching Weights for the solf gluon emision if one takes the limit $x \to 1$.

JHEP 09 (2001). Baier et al.

Phys. Rev. D 68 (2003). Salgado and Wiedemann

$$\epsilon D(\epsilon) \approx \alpha\sqrt{\frac{\omega_c}{\epsilon}} \exp\{-\frac{\pi\alpha^2\omega_c}{2\epsilon}\}$$

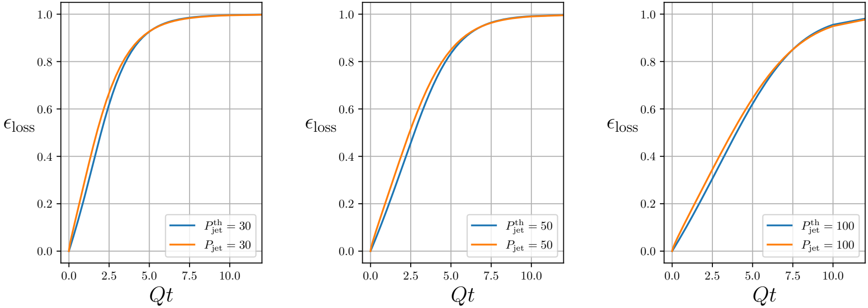
- We will compare the results from both EKT and BEDA with those from the balance equation in order to see how accurate it is.



$$\epsilon_{\text{loss}} \equiv 1 - \frac{\epsilon_{\text{jet}}}{\epsilon_{\text{jet},0}}$$

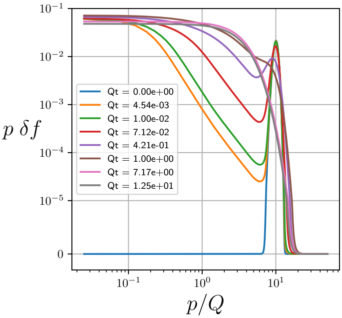
- **BEDA and EKT agrees better at larger times / larger mediums**, when the LPM effect is more important.

- Jets with higher momentum need longer time to loose all of its energy.
- The jet energy loss is clearly different at early times, due to the high occupancy of the bath.
- The thermalization time of this jets is undistinguishable.

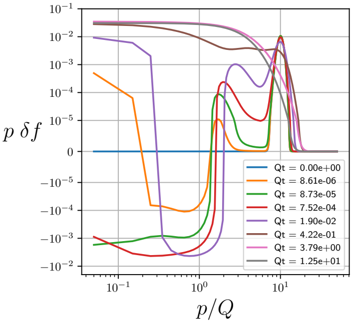


Energy loss for both thermal and overoccupied background (cut$= 0.7 P_{\rm jet}$)

- Some **negativity appears in the perturbation for non equilibrium plasma** which might be related with the jet wake in the initial stages.
- This feature only shows up for **overpopulated backgrounds**.



Gluon perturbation for thermal background



Gluon perturbation for overpopulated out of equilibrium background

- The framework of kinetic theories is a tool for explore the thermalization/hydrodynamization of the QGP.
- It can also be applied to the sudy of the evolution of jets during the initial stages of a heavy ion collision.
- Both EKT and BEDA seems to agree qualitatively in features such as the energy loss of the jet.
- The energy loss of the jet only distinguish the medium at early times.
- The medium being initially overoccupied or not might set some imprints in the wake of the jet.
- Future work in this direction should study the behaviour of more realistic jets using a more general geometry for the BEDA.

Thanks for your attention!

Back-up

Explicit forms

- Jet quenching parameter

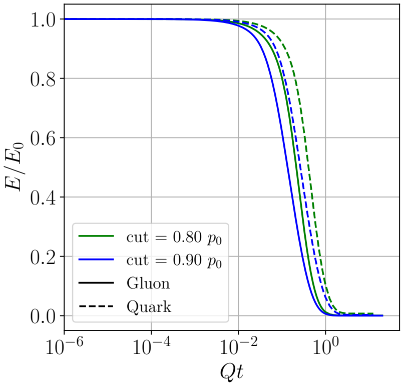
$$\hat{q}_a = 8\pi\alpha_s^2 C_a \ln\frac{\langle p_t^2\rangle}{m_D^2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\Big[N_c f\,(1+f) + \frac{N_f}{2}F(1-F) + \frac{N_f}{2}\bar{F}(1-\bar{F})\Big]$$

- Screening mass

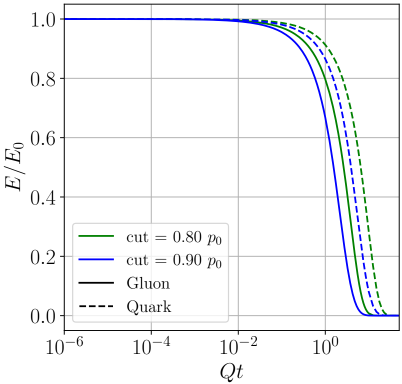
$$m_D^2 = 8\pi\alpha_s\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{|\mathbf{p}|}\left(N_c f + \frac{N_f}{2}F + \frac{N_f}{2}\bar{F}\right)$$

- Integrals $\mathcal{I}_c$

$$\mathcal{I}_c = \int\frac{d^3p}{(2\pi)^3}\frac{1}{p}[f + F + f(F-\bar{F})], \qquad \bar{\mathcal{I}}_c = \mathcal{I}_c\big|_{F\leftrightarrow\bar{F}}$$



Energy loss for thermal background



Energy loss for non thermal background