A Unified Adiabatic Description of Hydrodynamization in Kinetic Theory

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based on arXiv:2405.17545

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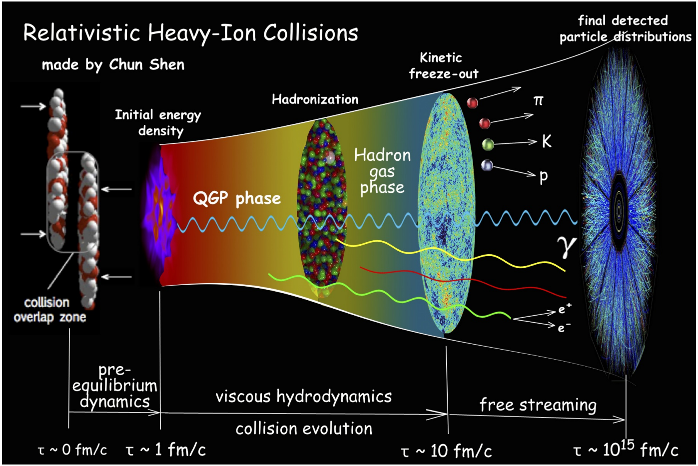
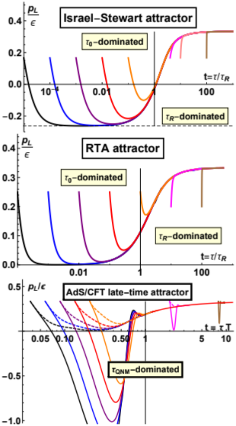
Hydrodynamization in Heavy Ion Collisions



Pre-Hydrodynamic Attractors

- How can we describe early out-of-equilibrium pre-hydro?
  - QCD kinetic theory
  - Holography
  - Classical field simulations
- "Attractor" behavior shown for kinetic theory, AdS/CFT, and Israel-Stewart theory



Kurkela, van der Schee, Wiedemann, Wu, arXiv:1907.08101

Scaling in Kinetic Theory

- Distribution functions quickly take "scaling" form for most of pre-equilibrium evolution:

$$f(\boldsymbol{p}, \tau) = \tau^{\alpha} w(\tau^{\beta} p_{\perp}, \tau^{\gamma} p_z)$$

Berges, Boguslavski, Schlichting, Venugopalan arXiv:1303.5650

- Follows distinct stages of bottom-up thermalization

  Baier, Mueller, Schiff, Son

  arXiv:hep-ph/0009237

- w is time-independent during scaling and acts as an attractor

stage	α	β	γ
free-streaming	0	0	1
"BMSS"	$-\frac{2}{3}$	0	$\frac{1}{3}$
"dilute"	-1	0	0
hydrodynamization	0	$\frac{1}{3}$	$\frac{1}{3}$

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Adiabatic Hydrodynamization Framework

- Is there a generic way of understanding why and in what form attractors arise?
- Idea: view attractors as the time-dependent ground state of an effective "Hamiltonian"

Brewer, Yan, Yin, arXiv:1910.00021

Early-Time Dynamics

$$\tau^{\alpha(\tau)} w(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z, \tau)$$

$$\downarrow$$

Prescaling "Ground State"

$$\tau^{\alpha_S(\tau)} w(\tau^{\beta_S(\tau)} p_\perp, \tau^{\gamma_S(\tau)} p_z)$$

Mazeliauskas, Berges, arXiv:1810.10554;

Mikheev, Mazeliauskas, Berges, arXiv:2203.02299

Adiabatic Hydrodynamization Framework

- To do this, cast the Boltzmann equation

$$\frac{\partial f}{\partial \tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} = -C[f],$$

in the form of a Schrödinger-like equation:

$$Hw \equiv -\partial_y w, \quad \text{where } y \equiv \log\left(\frac{\tau}{\tau_0}\right)$$

- If H is "sufficiently adiabatic", excited state parts of w with eigenvalue ϵ will decay as $\sim e^{-\epsilon y}$.
- This eventually leaves w in the time dependent ground state of H.

Boltzmann Equation

For the kinetic theory of gluons in this work, we will assume:

- **Longitudinal expansion only**: boost-invariance and no transverse expansion
- **Elastic scatterings only**: neglect the inelastic 1-to-2 part of the collision kernel
- **Small-angle scatterings only**

That is,

$$\frac{\partial f}{\partial \tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} = \lambda_0 l_{Cb}[f]\left(I_a \nabla_p^2 f + I_b \nabla_p \cdot \left(\hat{p}(f + \cancel{f^2})\right)\right)$$

where $I_a = \int_p f(1+f)$, $I_b = \int_p \frac{2f}{p}$, and $\lambda_0 = \frac{g_s^4 N_c^2}{4\pi}$.

Adiabatic Hydrodynamization: An Exact Limit

Brewer, Scheihing-Hitschfeld, and Yin (BSY) showed that for a simplified version of this theory,

$$\frac{\partial f}{\partial \tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} = \lambda_0 l_{Cb}[f]\left(I_a \nabla_p^2 f + \cancel{I_b \nabla_p \cdot (\hat{p} f)}\right)$$

it is possible to write f as

$$f(p_\perp, p_z, \tau) = A(\tau) w\left(\frac{p_\perp}{B(\tau)}, \frac{p_z}{C(\tau)}, \tau\right) = A(\tau) w(\zeta, \xi, \tau)$$

such that the resulting H is time-independent with ground state

$$w = \frac{1}{\sqrt{2\pi}} e^{-(\zeta^2+\xi^2)/2}$$

Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

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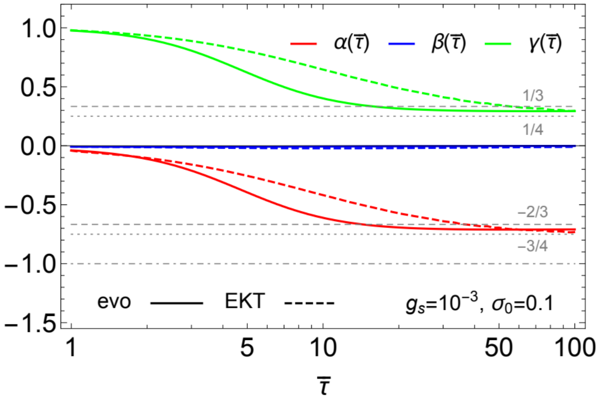
AH Scaling Exponents Match Full QCD EKT

The time-dependent rescalings which make H time-independent [1] reproduce scaling exponents calculated from full QCD EKT [2]



[1] Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

[2] Mazeliauskas, Berges, arXiv:1810.10554

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Adiabatic Hydrodynamization: Generalizing

If we reintroduce I_b, we no longer know an analytic solution to the eigenvalue problem for H,

$$H = \alpha + \beta\zeta\partial_\zeta + (\gamma - 1)\xi\partial_\xi - q\left[\frac{1}{B^2}\left(\frac{1}{\zeta}\partial_\zeta + \partial_\zeta^2\right) + \frac{1}{C^2}\partial_\xi^2\right] - \frac{\lambda}{p}\left(2 + \zeta\partial_\zeta + \xi\partial_\xi\right)$$

However, if we expand f and H on a basis

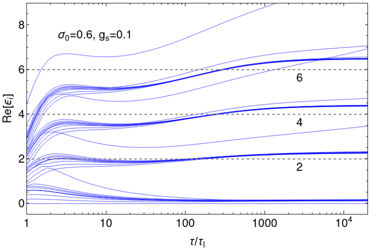
$$\psi_i = P_i(\zeta, \xi)e^{-\left(\xi^2/2+\zeta\right)}$$

and take $p \approx p_\perp$, we can solve the system numerically (but this will prevent us from solving until hydrodynamization).

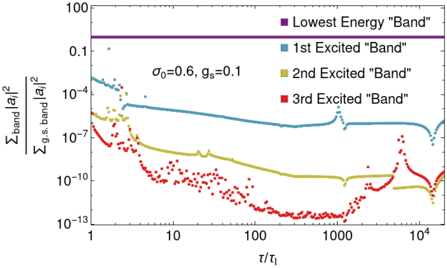
Adiabatic Hydrodynamization: Generalizing

With $A(y), B(y), C(y)$ as in the analytic solution, H is now time-dependent, but quasi-adiabatic (dominated by a set of low-energy modes):

Energies:



Weights:



Rajagopal, Scheihing-Hitschfeld, RS, arXiv:2405.17545

An adiabatic interpretation for the reduction of degrees of freedom still holds!

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Extending to Hydrodynamization

- We want a basis expanded near the time-dependent ground state of H at all times until hydrodynamization.
- At early times (as in BSY), we can find $C(y)$ such that the ground state is near

$$w \propto e^{-p_z^2/2C^2(y)}$$

- At late times, we know the system will equilibrate to

$$w \propto e^{-p/T}$$

- Writing $w \propto e^{-p/D(y)} e^{-u^2 r(y)^2/2}$ with $u \equiv p_z/p$ will allow us to smoothly interpolate between early and late time with a prudent choice of $r(y)$ and $D(y)$.

Choosing a Basis Near the Ground State

Therefore we write

$$f(p, p_z, y) = A(y)w\left(\frac{p}{D(y)}, \frac{p_z}{p}, r(y), y\right) = A(y)w(\chi, u, r, y)$$

and expand f and H on the basis

$$\psi_{n,l} = P_{n,l}(\chi, u, r(y))e^{-\chi}e^{-u^2 r(y)^2/2}$$

and make "reasonable" choices for $A(y), D(y), r(y)$ both such that

$$\psi_{0,1} = P_{0,1}(r)e^{-\chi}e^{-u^2 r^2/2}$$

is somewhat near the ground state, and such that the system evolves approximately adiabatically.

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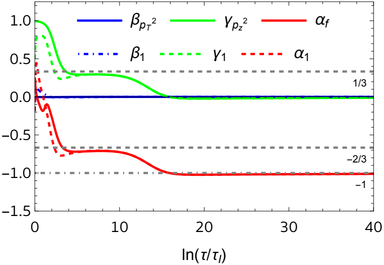
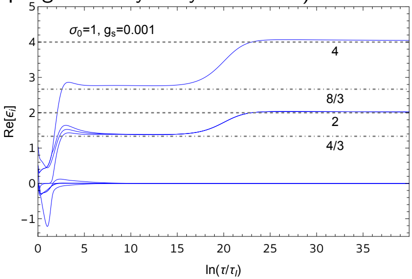
Scaling Exponents: Weak Coupling ($g_s = 10^{-3}$)

At very weak couplings, we see “BMSS” and “dilute” fixed points, characteristic of the first two stages of bottom-up.

(System evolution is too slow at this weak coupling to see hydrodynamization)





Rajagopal, Scheihing-Hitschfeld, RS, arXiv:2405.17545

Energy levels match analytic prediction from BSY:

$$\epsilon_{n,m} = 2n(1-\gamma) - 2m\beta$$

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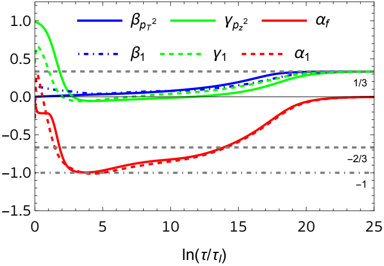
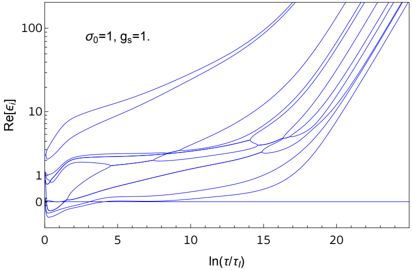
Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

Scaling Exponents: Stronger Coupling ($g_s = 1$)

At somewhat stronger couplings, we can evolve the system until hydrodynamization, and at late times a unique ground state emerges!





Rajagopal, Scheihing-Hitschfeld, RS, arXiv:2405.17545

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Conclusions

- Casting a Boltzmann equation in the form of a Schrödinger-like equation can provide an intuitive explanation for attractors (adiabatic hydrodynamization).
- In a simplified QCD kinetic theory, we can interpolate between various known "fixed point" scaling regimes using a single basis.
- Between each stage of hydrodynamization, there is a loss of memory as the set of low-energy states becomes smaller.
- Next Steps:
  - Generalize Boltzmann equation: add 1-to-2 scatterings and transverse expansion. (In progress: KR, BS, RS)
  - Use in Bayesian analysis of heavy ion collisons. (In progress: G. Nijs, BS, RS)

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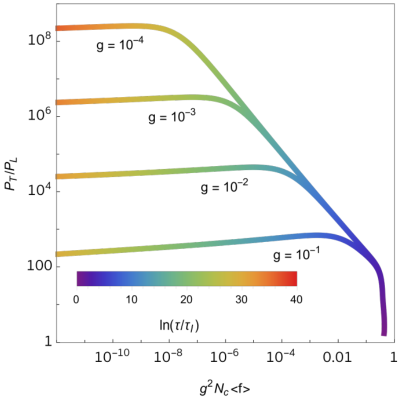
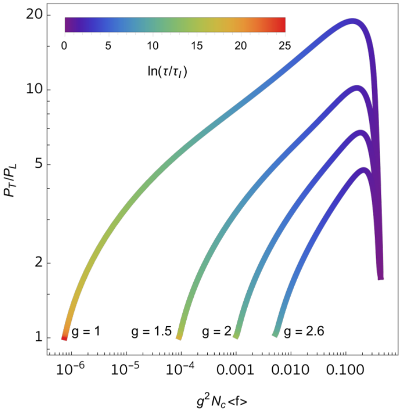
Supplemental: Anisotropy/Occupancy





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Supplemental: Choosing an Adiabatic “Frame”

- $A(y)$ can be chosen to satisfy number conservation: $\frac{\partial_y A}{A} = 3\frac{\partial_y D}{D} - 1$
- We choose $D(y)$ to decay towards $\langle \frac{2}{p} \rangle$, which at late times is the effective temperature.
- We would like $\psi_{1,0}$ to approximately describe f as much as possible.
- Therefore, we choose $r(y)$ such that if $f \propto \psi_{1,0}$, the evolution equation of the $\langle u^2 \rangle$ moment will be exactly satisfied:

$$\int_{\boldsymbol{p}} u^2 \partial_y f = \int_{\boldsymbol{p}} (p_z \partial_{p_z} - C[f]) f$$

Supplemental: "Translating" Scaling Exponents

Two notions of $\beta(y), \gamma(y)$.

- Physical scaling from moments:

$$\beta_{\langle p_T^2 \rangle} = -\frac{1}{2}\partial_y \ln\langle p_\perp^2 \rangle\,,$$
$$\gamma_{\langle p_z^2 \rangle} = -\frac{1}{2}\partial_y \ln\langle p_z^2 \rangle\,,$$
$$\alpha_{\langle f \rangle} = \partial_y \ln\langle f \rangle$$

- Scaling from choice of $A(y), D(y), R(y)$:

$$\beta_1 \equiv -\frac{1}{2}\partial_y \ln\langle p_\perp^2 \rangle_1 = -\frac{1}{2}\partial_y \ln\left(D^2 - \frac{D^2 J_2(r)}{J_0(r)}\right)\,,$$
$$\gamma_1 \equiv -\frac{1}{2}\partial_y \ln\langle p_z^2 \rangle_1 = -\frac{1}{2}\partial_y \ln\frac{D^2 J_2(r)}{J_0(r)}\,,$$
$$\alpha_1 \equiv \partial_y \ln\langle f_1 \rangle_1 = -1 - \partial_y \ln\frac{D^3 J_0(r)^2}{J_0(\sqrt{2}r)}$$

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