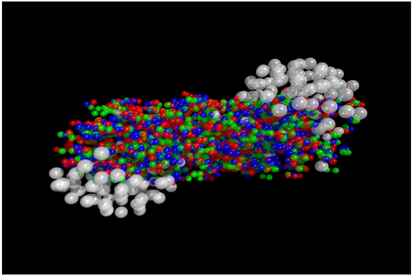
Quantifying the degree of hydrodynamic behaviour in heavy-ion collisions

Clemens Werthmann
Ghent University

in collaboration with: Victor Ambruș
Sören Schlichting

based on: PRD 107, 094013
PRL 130, 152301
and WiP



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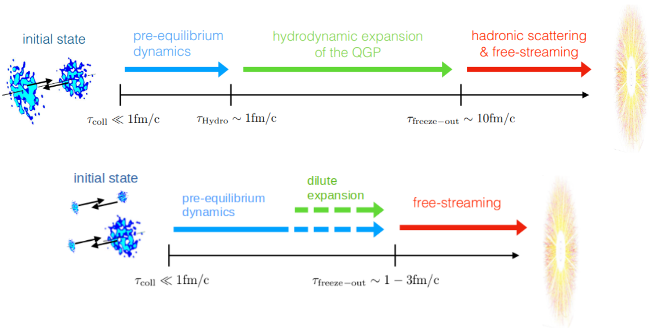
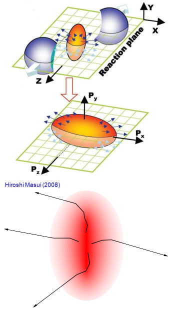
Dynamical modelling in small vs. large systems



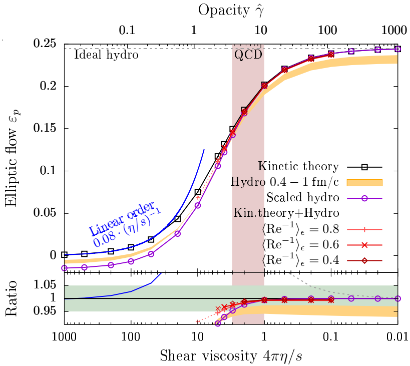


- large systems: dominated by hydrodynamic QGP, leaves imprints of thermalization and collectivity in final state observables:
  $\mathbf{v_n}$, $\langle p_T \rangle$, particle yields, ...
- small systems: might not fully equilibrate $\Rightarrow$ applicability of hydro unclear
- kinetic theory can describe off-equilibrium systems, applicable to free-streaming <u>and</u> hydrodynamic systems
  $\Rightarrow$ in comparison to hydrodynamics, can discern where it is accurate

Applicability of hydrodynamics in terms of opacity

previous study: in kinetic theory + hydro simulations, results for flow observables accurate on 5% level if opacity $\hat{\gamma} \gtrsim 3$ $\hat{=}$ central O+O

Ambruș, Schlichting, Werthmann PRD 107 (2023) 094013 and PRL 130 (2023) 152301



problem: Flow results from dynamical response to initial state geometry, which is poorly constrained in small systems

$$v_n = \kappa_{n,n} \cdot \epsilon_n$$

New Aim

- find observables that untangle effects of response and geometry on flow
- look for model-independent quantification of hydrodynamicity
- verify these in event-by-event simulations

Model and Setup: Kinetic Theory

- microscopic description in terms of averaged on-shell phase-space distribution of massless bosons:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\text{eff}}} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y)$$

  - boost invariance
  - initialized with vanishing longitudinal pressure and no transverse momentum anisotropies

- time evolution: Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\text{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\text{eq}}) \;, \quad \tau_R = 5\frac{\eta}{s}T^{-1}$$

  results will depend only on initial state and opacity

- dimensionless parameter: opacity ∼"total interaction rate"

  Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{1}{a\pi} R \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

  - encodes dependencies on viscosity, transverse size and energy scale

Initial conditions

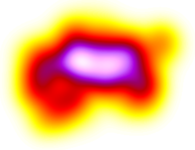
- initial conditions with event-by-event fluctuations (TRENTO model)

  Moreland, Bernhard, Bass PRC 92 (2015) 011901(R)

- pre-generated nucleon positions to account for correlations like α-clustering
- reasons for O+O:
  - intermediate system size ($\hat{\gamma} \sim 3$)
  - same collision system ran at RHIC and LHC for the first time!

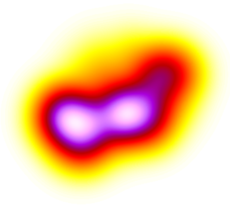
Pb+Pb 2.76 TeV

Alvioli, Drescher, Strikman PLB 680 (2009) 225



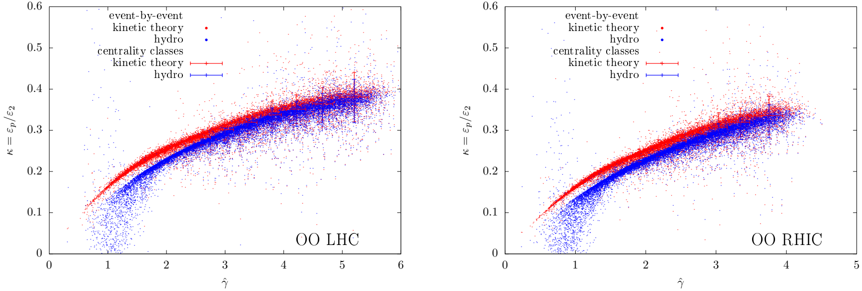
O+O 7 TeV

Loizides, Nagle, Steinberg SoftwareX 1-2 (2015) 13



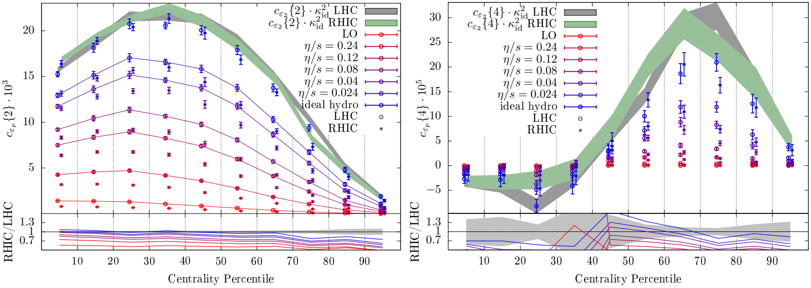
(example profiles from 20-30% centrality class)

Event-by-event flow responses



- main difference between RHIC and LHC is energy scale
- variation in geometry introduces spread of flow response
- still mostly depends on $\hat{\gamma}$ with $\varepsilon_p^{\text{hydro}} \nearrow \varepsilon_p^{\text{RTA}}$ as before

Flow cumulants in O+O



- larger opacity: larger magnitude of flow response and better agreement between RHIC and LHC
- centrality dependence of $\kappa(\hat{\gamma})$ introduces modulation
- flow fluct. dominated by avg. response to geometry fluct.

$$\langle(\epsilon_p)^n\rangle = \langle(\kappa\epsilon_2)^n\rangle = \bar{\kappa}^n\langle(\epsilon_2)^n\rangle + \ldots$$

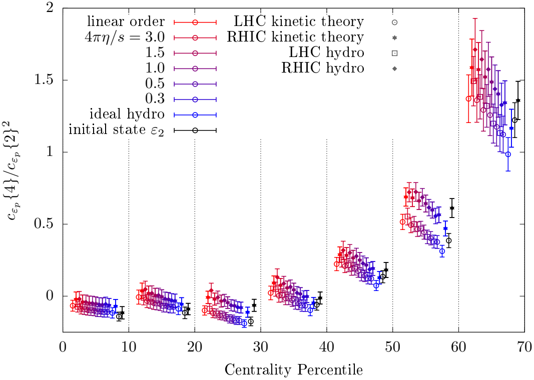
Cumulant ratios probe geometry

If $\langle(\epsilon_p)^n\rangle \approx \bar{\kappa}^n \langle(\epsilon_2)^n\rangle$, then $\bar{\kappa}$ cancels in ratios:

$$\frac{c_{\epsilon_p}\{4\}}{c_{\epsilon_p}\{2\}^2} = \frac{\langle(\epsilon_p)^4\rangle - 2\langle(\epsilon_p)^2\rangle^2}{\langle(\epsilon_p)^2\rangle^2} \approx \frac{\langle(\epsilon_2)^4\rangle - 2\langle(\epsilon_2)^2\rangle^2}{\langle(\epsilon_2)^2\rangle^2} = \frac{c_{\epsilon_2}\{4\}}{c_{\epsilon_2}\{2\}^2}$$

$\Rightarrow$ ratio sensitive mostly to geometry

Giacalone, Yan, Noronha-Hostler, Ollitrault PRC 95 (2017) 1, 014913



Hydrodynamization observable: definition

- cancel geometry: comparing systems with same geometry (and same η/s):

$$\frac{c_2^{\mathrm{RHIC}}\{2k\}}{c_2^{\mathrm{LHC}}\{2k\}} \approx \frac{\bar{\kappa}^{2k}_{\mathrm{RHIC}}}{\bar{\kappa}^{2k}_{\mathrm{LHC}}} \qquad \frac{\hat{\gamma}_{\mathrm{RHIC}}}{\hat{\gamma}_{\mathrm{LHC}}} = \left(\frac{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{RHIC}}}{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{LHC}}} \right)^{1/4}$$

- use logarithm to turn ratios into differences:

hydrodynamization observable

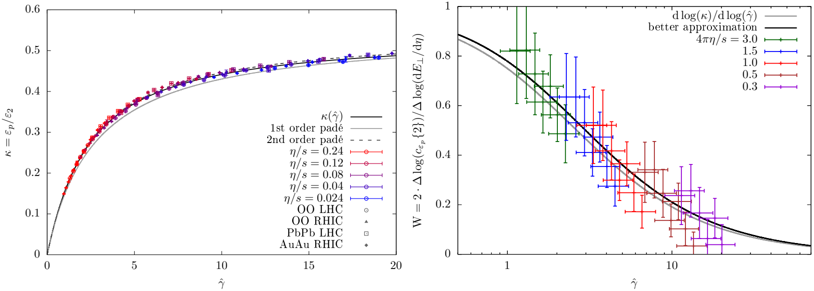
$$\mathrm{W} = \frac{2}{k} \frac{\Delta \log(c_2\{2k\})}{\Delta \log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$$

- small $\hat{\gamma}$: linear buildup, $\frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}} \lesssim 1$
  large $\hat{\gamma}$: saturation, $\frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}} \to 0$

Hydrodynamization observable: Proof of principle

crosscheck of W-observable:

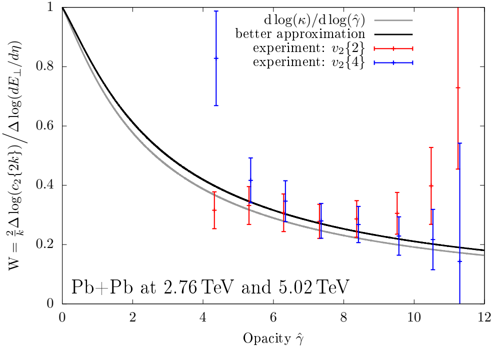
1. extract $\kappa(\hat{\gamma})$ from fit to simulation results
2. compute $\frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$: smooth monotonous transition from 1 to 0
3. compare with simulation data for W-observable: agreement!



previous hydrodynamization criterion

$\hat{\gamma} \sim 3$ corresponds to $\mathrm{W} \sim 0.5$

- first test with LHC data: results agree with theory ($\hat{\gamma}$ from Trento initial conditions, η/s chosen s.t. flow matches)



- centrality dependence off for $v_2\{2\}$ (nonflow?), but accurate for $v_2\{4\}$

Summary

- applicability of hydrodynamics can be assessed by comparing to kinetic theory, but uncertainties in initial state obscure results
- effects of initial state and dynamical response on flow can be untangled using appropriate observables:
  - cumulant ratios for initial state geometry
  - W-observable for hydrodynamization via slope of flow response curve

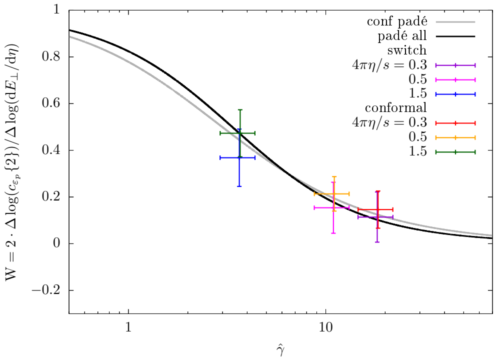
$$\mathrm{W} = \frac{2}{k}\frac{\Delta \log(c_2\{2k\})}{\Delta \log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$$

- verified discriminative power in event-by-event simulations
- criterion for hydrodynamic behaviour in experiment: $\mathrm{W} \lesssim 0.5$

Backup

Non-conformal effects

- probing non-conformal effects in hydro simulations with chiral eos
  - losing theoretical control over setup
- might need to adjust calibration curve; at the very least still applicable for mid-central collisions at $\hat{\gamma} \gtrsim 4$



Hydrodynamics in real collision systems

Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to "real-life" collisions?

$$\mathrm{Pb+Pb}: \overset{30-40\%}{\hat{\gamma} \sim 5.7} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{2.78\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_{\perp}^{(0)}/\mathrm{d}\eta}{1280\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{2.7} - \overset{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

$$\mathrm{O+O}: \overset{30-40\%}{\hat{\gamma} \sim 2.2} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{1.13\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_{\perp}^{(0)}/\mathrm{d}\eta}{55\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{1.4} - \overset{0-5\%}{3.1}$$

probes transition region to hydrodynamic behaviour

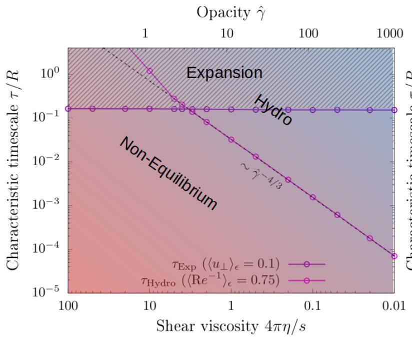
$$\mathrm{p+Pb}: \overset{\mathrm{min.bias}}{\hat{\gamma} \sim 1.5} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.81\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_{\perp}^{(0)}/\mathrm{d}\eta}{24\,\mathrm{GeV}}\right)^{1/4} \overset{\mathrm{high\ mult.}}{\lesssim 2.7}$$

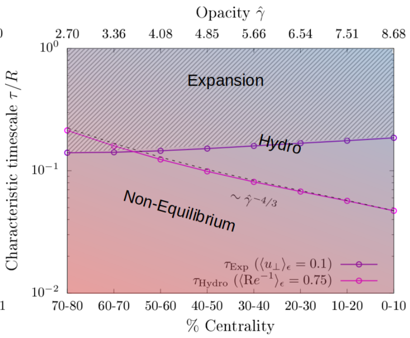
very high multiplicity events approach regime of applicability, but do not reach it

$$\mathrm{p+p}: \overset{\mathrm{min.bias}}{\hat{\gamma} \sim 0.7} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.12\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_{\perp}^{(0)}/\mathrm{d}\eta}{7.1\,\mathrm{GeV}}\right)^{1/4}$$

far from hydrodynamic behaviour

Hydrodynamization in viscosity and centrality dependence





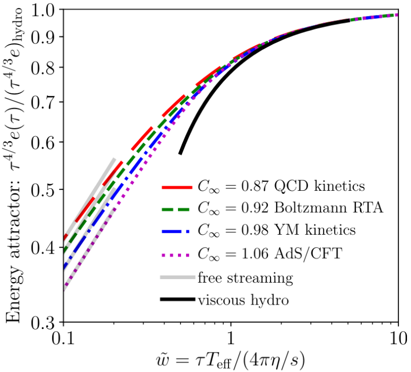
- transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- Hydro appicable when $\mathrm{Re}^{-1} < \mathrm{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\mathrm{Hydro}}/R \approx 1.53\, \hat{\gamma}^{-4/3} \left[(\mathrm{Re}_c^{-1})^{-3/2} - 1.21 (\mathrm{Re}_c^{-1})^{0.7} \right]$$

- hydrodynamization before transv. Expansion for $\hat{\gamma} \gtrsim 3$

What might happen when going beyond RTA?

- more complex kernels will introduce further parameter dependence, but opacity dependence might still be "leading order approximation"
- in Bjorken flow, equilibration happens in very similar ways across different model descriptions:



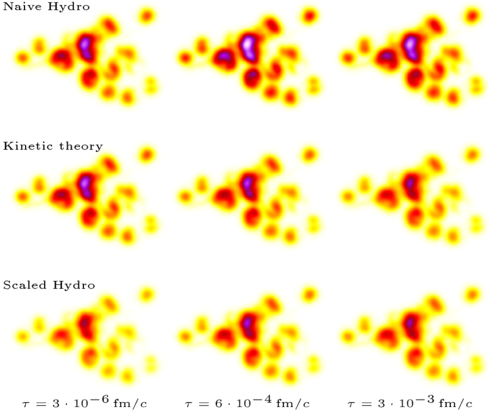
Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

Model and Setup: Hydrodynamics

- 2nd order Müller-Israel-Steward type hydrodynamics (vHLLE) with RTA transport coefficients

  Karpenko, Huovinen, Bleicher Comput. Phys. Commun. 185, 3016 (2014)

- How to define initial state? Hydro deviates at early times!



- solution: hydro initial condition <u>scaled</u> according to attractor curve prediction of early time behaviour

  Ambruș, Schlichting, Werthmann PRD 107 (2023) 094013

Initializing on the attractor

- accuracy depends on timescale separation of pre-equilibrium and transv. expansion