Thermalization and Hydrodynamization in Small and Large Systems

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1. Introduction

- Transport model : approximation solution to Boltzmann Transport Equation.

$$\underbrace{\frac{\partial}{\partial t}f(t,\vec{r},\vec{p})}_{\text{Time evolution}}+\underbrace{\frac{\vec{p}}{m}\nabla_{\vec{r}}f(t,\vec{r},\vec{p})}_{\text{Diffusion}}-\underbrace{\nabla_{\vec{r}}U(\vec{r})\nabla_{\vec{p}}f(t,\vec{r},\vec{p})}_{\text{External field}}=\underbrace{\left(\frac{\partial f}{\partial t}\right)_{coll}}_{\text{Interparticle collision}}$$

- Applicable to far-from-equilibrium quark-gluon plasma.
- Bottom up approach to quark-gluon plasma thermal and chemical equilibration.
- Based on Hadronic transport model SMASH[1] reconfigured for quark-gluon plasma case.

Goal : Create a realistic initial condition generator model for hydrodynamic simulation with the minimum number of free parameters.

[1] J. Weil, V. Steinberg, J. Staudenmaier, L. G. Pang, D. Oliinychenko, J. Mohs, M. Kretz, T. Kehrenberg, A. Goldschmidt, B. Bäuchle, J. Auvinen, M. Attems, and H. Petersen, Phys. Rev. C 94, 054905 (2016)

2. Stochastic Collision Model

- Inter-particle collisions are calculated using stochastic method*

$$P_{2\to N}=v_{rel}\frac{\sigma_{2\to N}}{N_{test}}\frac{\Delta t}{\Delta^3 x}\qquad P_{3\to 2}=\frac{1}{4E_1E_2E_3}\frac{(s-m_1^2-m_2^2)^2-4m_1^2m_2^2}{\Psi_3 8\pi s}\sigma_{2\to 3}\frac{\Delta t}{\Delta^3 x}$$

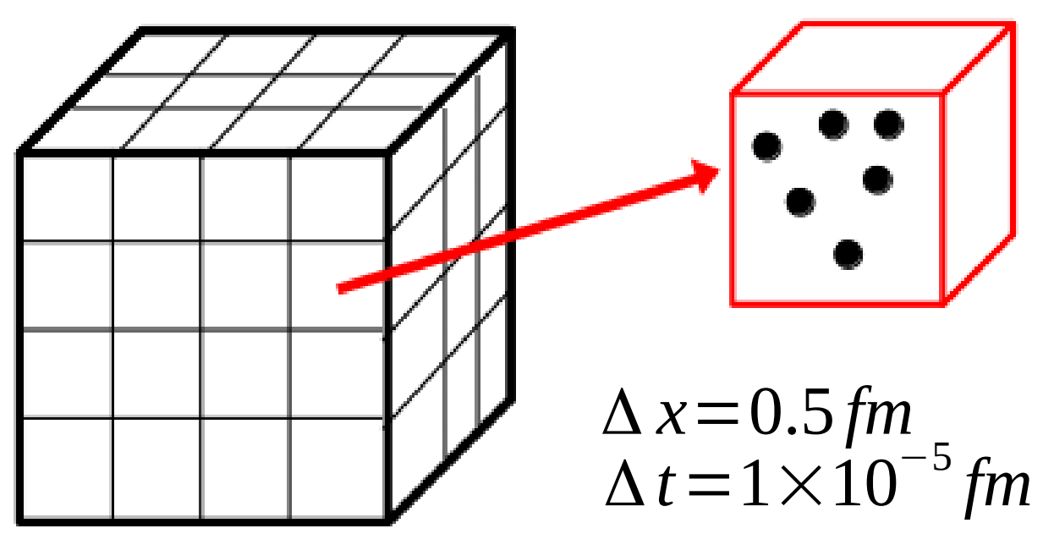
→ Only allow interaction in the same cell.
→ Assume only 1 collision for each timestep.

- Update particles
  → Propagate particles in a straight line
  (Δt assumed to be small enough to ignore force)

  → Update momentum based on

$$\frac{dp}{dt}=\vec{E}+\vec{v}\,X\,\vec{B}$$

For the sake of simplicity, consider no external field



$\Delta x=0.5\,fm$
$\Delta t=1\times10^{-5}\,fm$

This allows simulation including multipartonic interactions

3. Model Description

- Initial condition (Glauber Model).
  Initial particles are from mini-jets produced by binary collision between nucleons
  - Particle number

$$N=\boxed{4}\,v\,\sigma_{jet}\int d^2x_T\,dz\,dt\,n_A(\vec{x}_T,z-vt)\,n_B(\vec{x}_T,z+vt)\ \text{with}\ \sigma_{jet}=\int_{P^2_{tcutoff}}^{4P^2_{CoM}}dt\frac{d\sigma}{dt}$$

Each collision produces 2 parton x 2 to include higher order perturbation

Minijet cutoff @ 2 GeV and only until leading order

and $$n_A(\mathbf{x}_{T1},z_1)=\frac{\gamma\,n_0}{1+Exp\left(\left(\sqrt{x_{T1}^2+(\gamma z_1)^2}-R_A\right)/d\right)}$$

With d = 0.54 fm
$R_A = 1.12A^{1/3} - 0.86\,A^{-1/3}$
n_0 = normalization constant so that

$$A=\int d^2x_T\,dz\,n(x_T,z)$$

  - Momentum distribution

$$\frac{d\sigma_{jet}}{dp_T^2dy_1dy_2}=K\sum_{a,b}x_1f_a(x_1,p_T^2)x_2f_b(x_2,p_T^2)\frac{d\sigma_{ab}}{d\hat{t}}$$

With f_1 and f_2 partonic distribution function (NNPDF[2])

- Sanity check
  Including all 2-to-2 channels
  $p_{cut\text{-}off}$ = 1.8 GeV (from Au + Au @ 200 GeV PYTHIA)

$$\sigma_{pp}=\int dx_1dx_2dt\sum_{all\,channels}x_1f_1(x_1)x_2f_2(x_2)\frac{d\sigma}{dt}=46.3622\,mb$$

(COMPETE[3] prediction = 51.79 mb and STAR[4] experiment = 54.67 mb)

- Coupling constant α_s fixed at 0.3

- 2-to-2 Channels
  based on perturbative QCD up to tree level.
  Includes 4 channels : gg <-> gg, gq <-> gq, qq <-> qq, gg <-> $q\bar{q}$

- 2-to-3 Channels
  based on improved Gunion-Bertsch approximation[5]

$$|M_{2\to3}|^2=12g^2|M_{2\to2}|^2(1-x)^2\left[\frac{k_\perp}{k_\perp^2+x^2M^2}+\frac{q_\perp-k_\perp}{(q_\perp-k_\perp)^2+x^2M^2}\right]$$

under assumption $k_\perp\ll\sqrt{s},\ q_\perp\ll\sqrt{s},\ xq_\perp\ll k_\perp\ with\ x=\frac{k_\perp}{\sqrt{s}}e^y\Rightarrow$ soft gluon exchange and soft gluon radiation

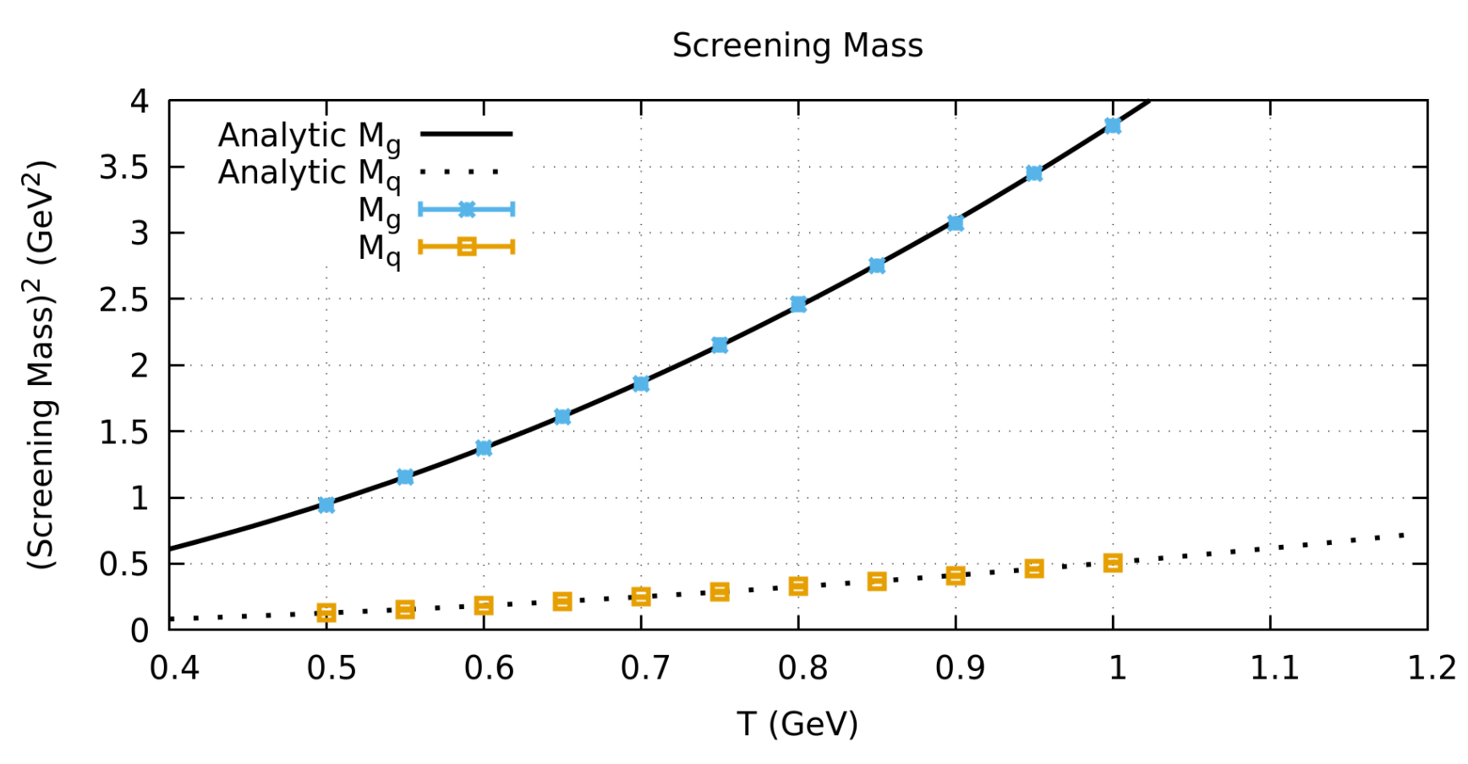
Includes gg <-> ggg, qg <-> qgg, and qq <-> qqg extra channels

- Infrared divergence regularization
  based on dynamic colour-screening mass in the one-loop approximation[6] in SU(3) and up to α_s order

$$m_q^2=4\alpha_s\pi\frac{N_c^2-1}{2N_c}\left(\frac{1}{2(N_c^2-1)}\frac{1}{V}\sum_i^{gluon}\frac{1}{p_i}+\frac{1}{g_{spin}g_{(anti)quark}N_cn_f}\frac{1}{V}\sum_i^{quark}\frac{1}{p_i}\right)$$

$$m_g^2=16\alpha_s\pi\left(\frac{N_c}{2(N_c^2-1)}\frac{1}{V}\sum_i^{gluon}\frac{1}{p_i}+\frac{n_f}{g_{spin}g_{(anti)quark}N_cn_f}\frac{1}{V}\sum_i^{quark}\frac{1}{p_i}\right)$$

Comparison to analytic calculation using Boltzmann distribution.



- Coupling constant α_s fixed at 0.3

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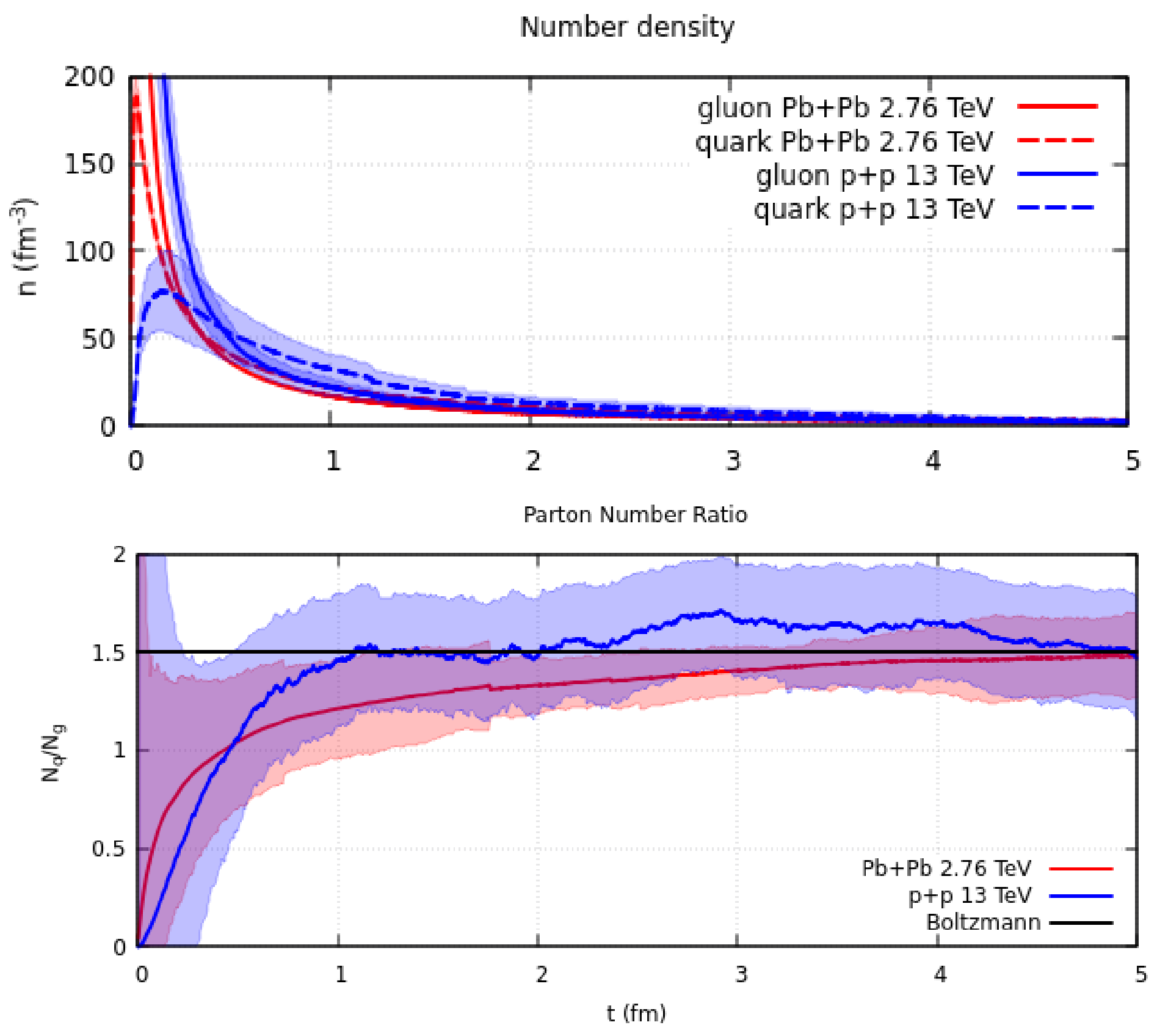
[2] Richard D. Ball, et al arXiv:1207.1303 [hep-ph]
[3] J.R. Cudell, et al., COMPETE Collaboration, Phys. Rev. Lett. 89 (2002) 201801.
[4] STAR Collaboration Physics Letters B 808 (2020) 135663

[5] Jan Uphoff, Oliver Fochler, Zhe Xu, Carsten Greiner arXiv:1302.5250 [hep-ph]
[6] S. M. H. Wong, Phys. Rev. C 54, 2588 (1996)

4. Chemical Equilibration

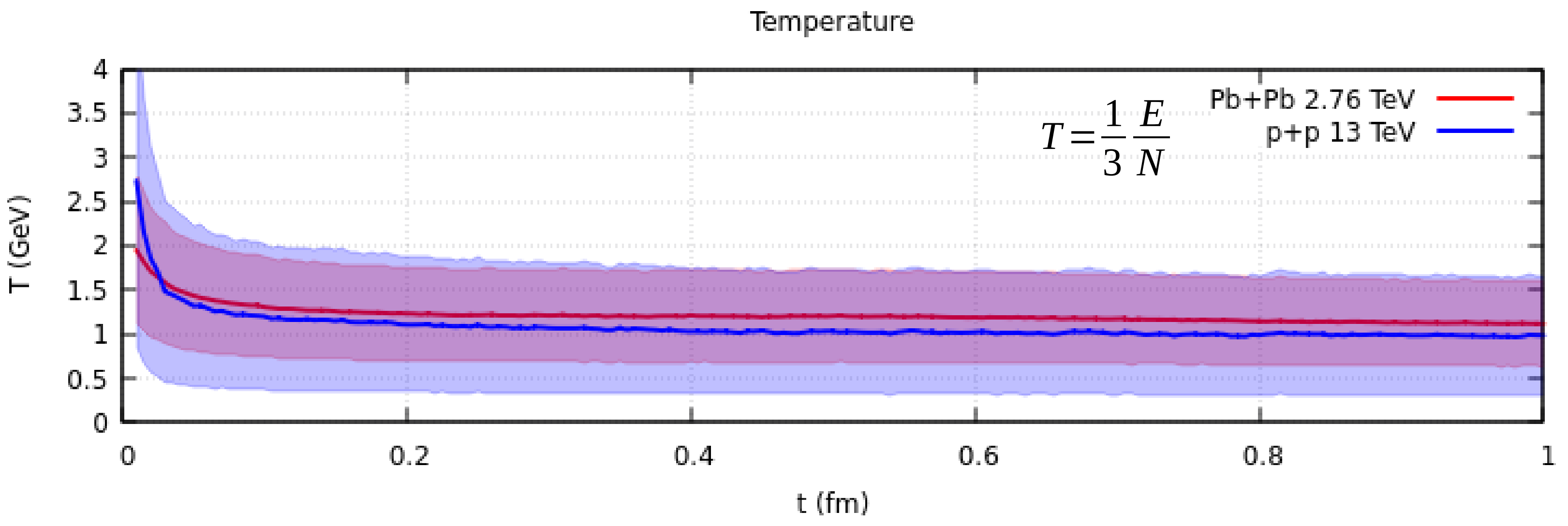
We consider 2 collision systems
1) Pb-Pb @ 2.76 TeV
2) p-p @ 13 TeV

- Central collision for both systems
- Focusing only for mid-rapidity region -0.5 < η < 0.5
- Number density for both quark and gluons are understandably constantly decreasing for an expanding medium
- Ratio between number of quarks and gluons saturates toward Boltzmann limit between 1-2 fm



5. Thermalization

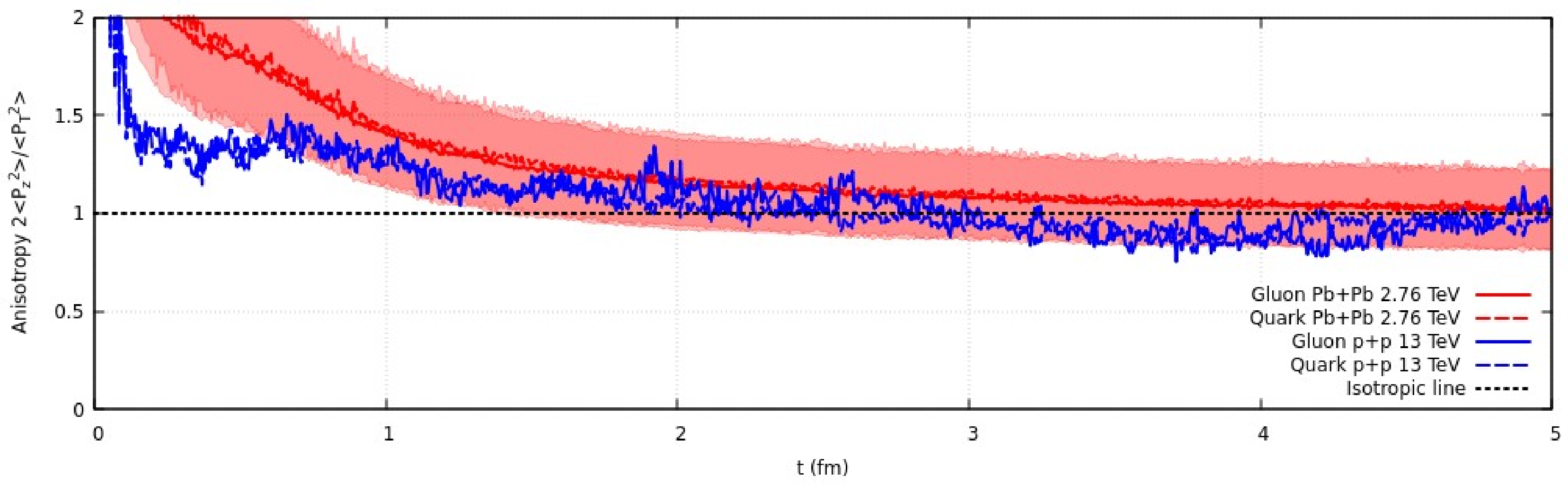
- We define effective temperature as ratio between energy and particle number to indicate thermalization



- Temperature is constantly decreasing due to the expanding medium
- Saturation behaviour can be seen well before 0.2 fm for both collision systems

6. Momentum Isotropization

Momentum isotropization is often used as the indicator for hydrodynamization



Momentum isotropization is reached ~2 fm regardless of the system size

* Error bar for p-p collision is not included since it's too large

Summary and Future Prospect

- We constructed a partonic transport model using stochastic collision model with 2-to-2 and 2-to-3 interactions up until leading order.

- The model has two parameters
  → Mini jet momentum cut-off => affect initial condition parton number and total energy
  → Coupling constant α_s => affect interaction rate

→ Similar time evolution for both small and large system
This is probably due to the higher collision energy for the small system resulting in a high enough multiplicity to give a similar evolution compared to the large system case.

- Future application to heavy quark energy loss in quark gluon plasma.
  Compare to theoretical predictions of heavy quark energy loss from both elastic* and radiative** processes.

*Peigne and Peshier (arXiv:0802.4364
**R. Abir, U. Jamil, M. G. Mustafa, D. K. Srivastava, Phys. Lett. B 715, 183 (2012).