Perturbative & non-perturbative aspects of the multiplicity distribution in a jet

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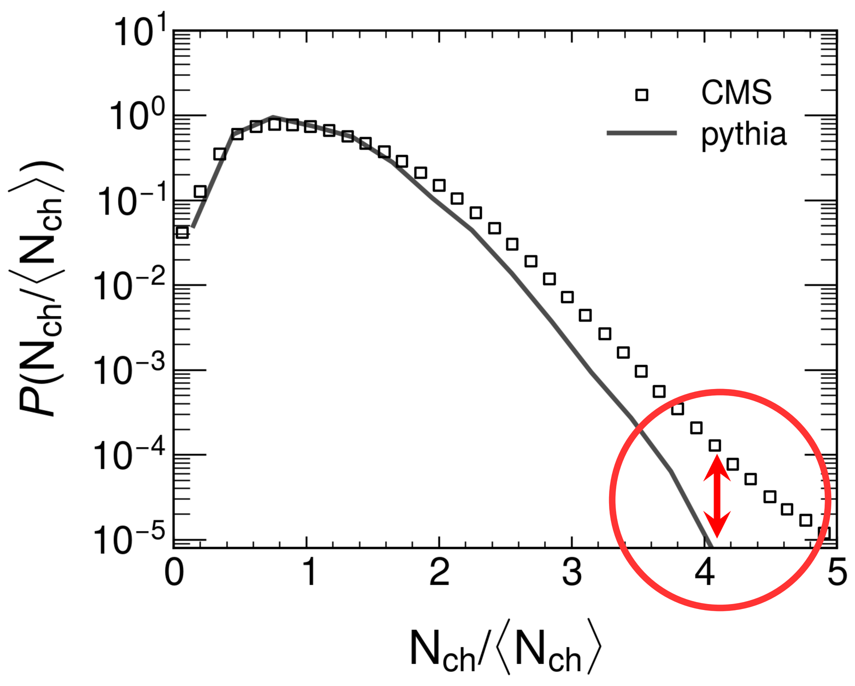
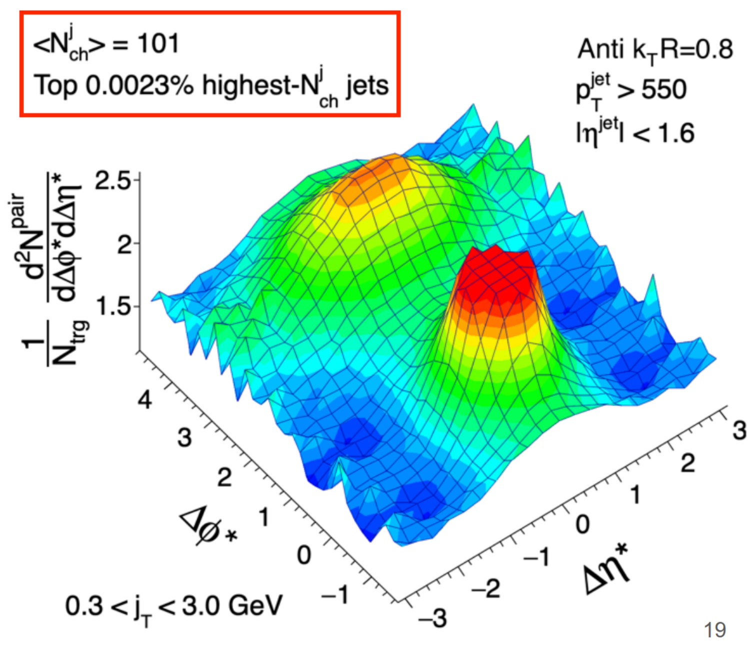
1. To understand jets with very high multiplicities





- Jets with extreme multiplicities ($N_{\rm ch} > 100$) have been measured and display non-trivial particle correlations [1]. **A new system to study QCD in the "many" limit and attracts theoretical interest [2].**
- However, Pythia8 (MPI + color reconnection) underestimates $N_{\rm ch}$ distribution in **the interested region** by order of magnitude! Also, it is hard to trigger on jets with $N_{\rm ch} > 100$ with event generator.
- **This poster**: looking for semi-analytic understanding of such extreme $N_{\rm ch}$ fluctuations using a simplified system of pure gluons.

2. Multiplicity distribution and generating function

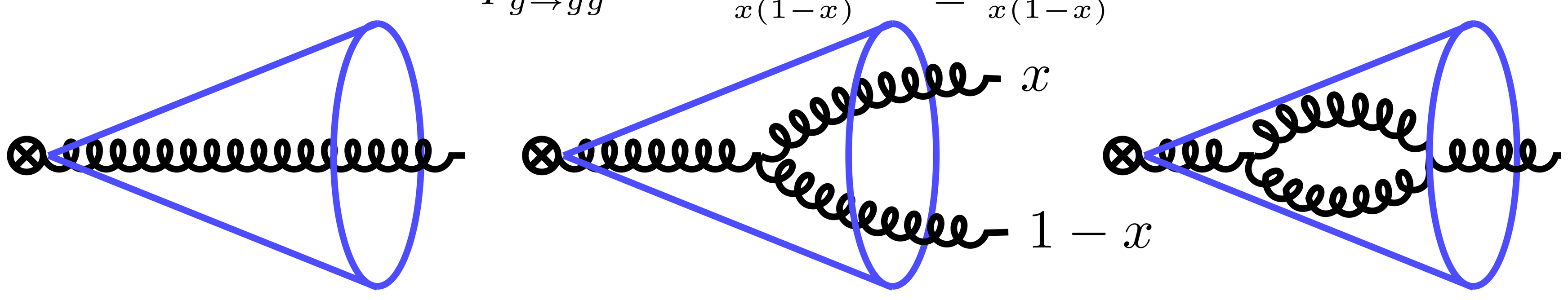
Let $P(n;\omega_J,R)$ be the probability that a parton with energy ω_J produces an exclusive jet of radius R and contain n particles in the cone. We define the multiplicity generating function $Z(s)$ of exclusive jet as

$$Z(s;\omega_J,R) = \sum_{n=0}^{\infty} e^{-ns}P(n;\omega_J,R)\,,\quad P(n) = \int_{s_0-i\pi}^{s_0+i\pi} Z(s)e^{ns}\frac{ds}{2\pi i}\,,$$

$$P_a(n) = \sum_{m=0}^{n} P_b(m)P_c(n-m) \Longleftrightarrow Z_a(s) = Z_b(s)Z_c(s)\,.$$

Generating function is useful in convolving the multiplicity distributions of two partons in a jet. For example, for exclusive jet ▽

$$P_{g\to gg} = \frac{1+x^4+(1-x)^4}{x(1-x)} \equiv \frac{g(x)}{x(1-x)}$$



LO $Z^{(0)}(s;\omega_J)$ | A splitting in the cone $Z^{(0)}(s,x\omega_J)Z^{(0)}(s,(1-x)\omega_J)$ | Loop correction is scaleless in DR

3. Non-perturbative modelling

Consider a parton with invariant mass (virtuality) Q_0 and energy ω.

- In the rest frame where $\omega = Q_0$, it will isotropically fragment into hadrons that carry energies $\Lambda_{\rm QCD} \lesssim E < Q_0$.
- If the parton is boosted with energy $\omega \gg Q_0$, then almost all the produced hadrons will be squeezed into a cone of size Q_0/ω.
- From the considerations above, an ansatz for the average number of particles found in a cone of size R is

$$\langle n\rangle_{\omega,Q_0} = n_0(Q_0)\left(1+\frac{cQ_0^2}{\omega^2R^2}\right)^{-1}.$$

  At $Q_0 = 4$ GeV, with $n_0(Q_0) = 7.7$ and $c = 0.6$, it gives a reasonable description of the Pythia8 simulations ($gg \to$hadrons).
- If number of particles in the cone follows a Poisson distribution, then a simple model for $P(n)$ and $Z(n)$ at scale Q_0 could be

$$P^{(0)}(n;\omega,R) = \frac{1}{n!}\left(\langle n\rangle_{\omega,Q_0}\right)^n e^{-\langle n\rangle_{\omega,Q_0}}\,,$$

$$Z^{(0)}(s;\omega,R) = \exp\{\langle n\rangle_{\omega,Q_0}\left(e^{-s}-1\right)\}\,.$$

4. Perturbative correction to generating function

The LO+NLO contribution to the multiplicity generating function of an exclusive jet (LO expression is given by the non-perturbative model):

$$Z^{(0)}(s;\omega_J) + Z^{(1)}(s;\omega_J)\ \text{(dimension regularization (DR) in } d=4-2\epsilon)$$

$$= Z^{(0)}(s;\omega_J)\left[1+\frac{\alpha_s(\mu)C_A}{2\pi}g(1)Z^{(0)}(s;0)\frac{1}{2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\mathcal{L}+\frac{1}{2}\mathcal{L}^2\right)\right]$$

$$-\left(\frac{1}{\epsilon}+\mathcal{L}\right)\frac{\alpha_s(\mu)C_A}{2\pi}\int_0^1 dx\frac{g(x)Z^{(0)}(s;x\omega_J)Z^{(0)}(s;(1-x)\omega_J)}{(1-x)_+}$$

$$+\frac{\alpha_s(\mu)C_A}{2\pi}\int_0^1 dxg(x)Z^{(0)}(s;x\omega_J)Z^{(0)}(s;(1-x)\omega_J)$$

$$\times 2\left(\frac{\ln(x)}{1-x}+\left[\frac{\ln(1-x)}{1-x}\right]_+\right)$$

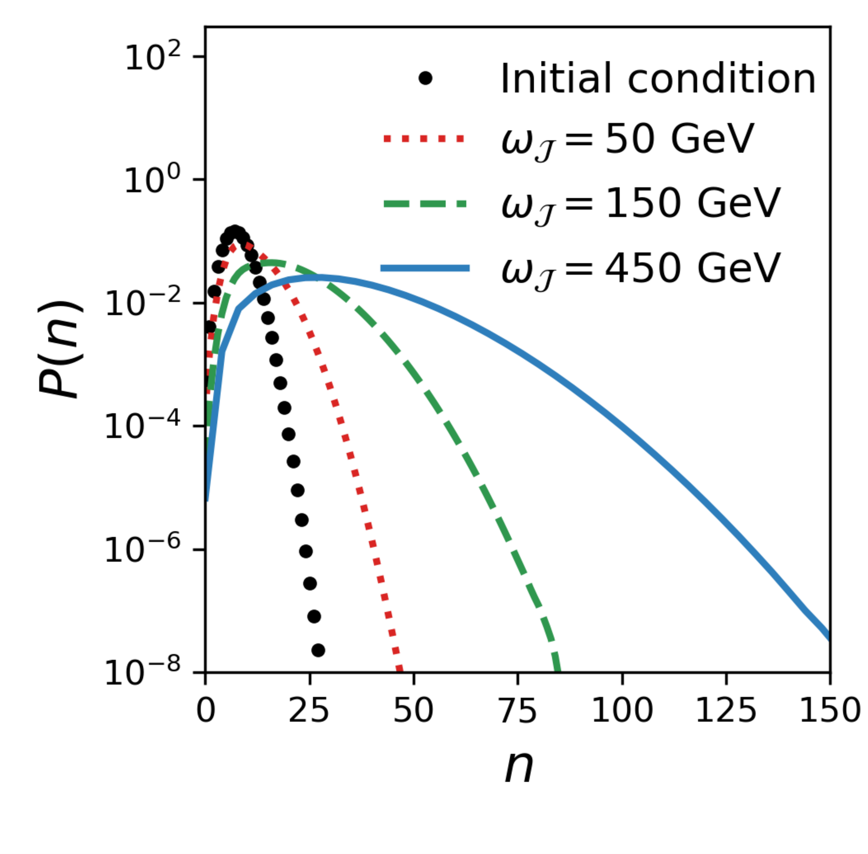
$\mathcal{L} = \ln\left[\mu^2e^{\gamma_E}/(\omega_J^2R^2)\right]$ with μ the renormalization scale

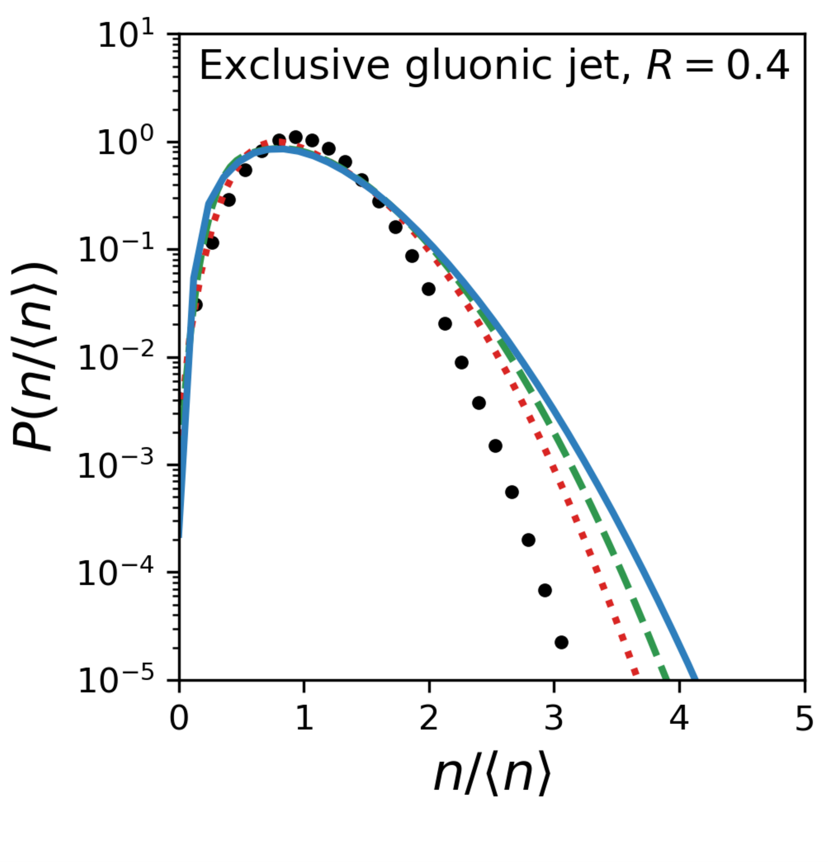
- The red block contains the first two terms of the expanded Sudakov factor, i.e., the probability of no radiation outside the cone.
- The blue block contains a non-linear convolution of $Z^{(0)}$ with a collinear logarithmic enhancement $\mathcal{L}$. It can modify the s dependence of $Z(s)$, thus changing the shape of $P(n)$.
- The orange block is finite correction. Natural scale $\mu_J \sim \omega_J R$.

5. Preliminary results

If only interested in $P(n)$ (shape), evolve Z from $\mu = Q_0$ to $\omega_J R$ using

$$\frac{\partial Z(s,\omega_J;\mu)}{\partial\ln\mu^2} = \frac{\alpha_s(\mu)C_A}{2\pi}\int_0^1 dx\frac{g(x)Z(s,x\omega_J;\mu)Z(s,(1-x)\omega_J;\mu)}{(1-x)_+}\,.$$





6. Discussion and Prospects

- We consider high multiplicities are generated by many perturbative $1 \to 2$ parton splittings in the cone and fragment independently $\Longrightarrow$ a baseline for more interesting effects.
- The collinear evolution describes a rapid increase of multiplicity in jet towards large scale $\mu_J = \omega_J R$. Qualitatively reasonable.
- Will include quarks and consider inclusive jet in the future. Need deeper investigation of the equation (e.g. $n \gg \langle n\rangle$ asymptotics). Are higher-order nonlinear effects important (e.g. $P_{g\to ggg}$), etc?
- It could be an alternative tool to study high-multiplicity jets and be complementary to Monte Carlo event generator studies.

References

[1] CMS Collaboration, 2312.17103.
[2] Wenbin Zhao, Zi-Wei Lin, Xin-Nian Wang, 2401.13137.