The modified dilepton production rate from charged pion-pair annihilation in the inhomogeneous chiral condensed phase

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Abstract - We investigate the dilepton production rates from annihilation processes of charged pion pairs with modified pion dispersion relations in the inhomogeneous chiral condensed phase. We assume a dual chiral density wave as an inhomogeneous chiral condensate, and obtain the dispersion relations of the Nambu-Goldstone modes in the inhomogeneous chiral condensed phase using a low energy effective Lagrangian based on the O(4) symmetry. We evaluate the electron-positron production rates by charged pion-pair annihilations as functions of an invariant mass using the obtained dispersion relations. Basically, the production rate in the inhomogeneous chiral condensed phase has a steeper overall slope with respect to an invariant mass than that in the homogeneous chiral condensed phase. Also, there may be a possibility that the production rate diverges when the invariant mass is just twice the pion mass.

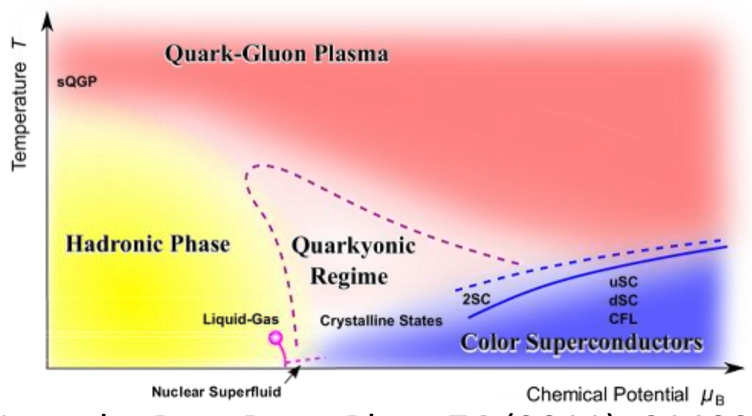
Introduction

QCD phase diagram

low temperature and
high density region

↓

Possibility of existence of
various phases



K. Fukushima and T. Hatsuda, Rep. Prog. Phys. **74** (2011), 014001

Chiral condensation

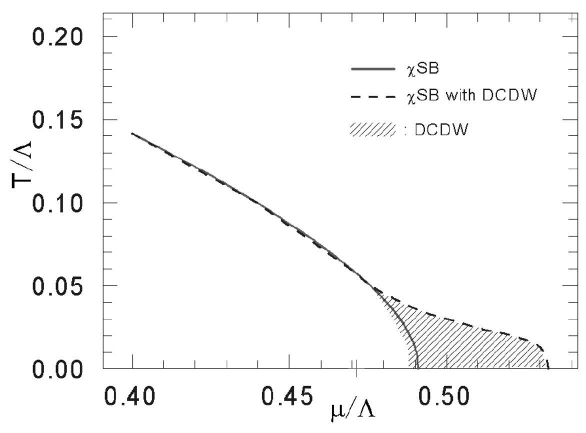
$$\phi = \Delta \sim \langle \bar{\psi}\psi \rangle$$

basically spatially homogeneous

↓

$\phi = \Delta e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$ or $\phi = \Delta\cos(\boldsymbol{q}\cdot\boldsymbol{r})$

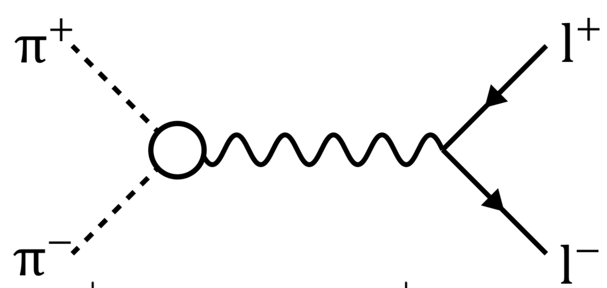
Inhomogeneous chiral condensed phase



Phase diagram in previous studies using the NJL model
E. Nakano and T. Tatsumi, Phys. Rev. D **71**, 114006 (2005).

Dilepton production from charged pion-pair annihilation

High density matter in heavy ion collisions
→ lepton emission
→ dilepton production from charged pion-pair annihilation



$$\pi^+ + \pi^- \to l^+ + l^-$$

↓ Dependence on charged pion's dispersion

Pion's dispersion in inhomogeneous chiral condensed phase
→Changes of dilepton production rates

Dispersion Relations

Chiral SU(2) symmetry
→ isospin O(4) symmetry

$$\phi = \begin{pmatrix} \sigma \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

↓

Low energy effective Lagrangian based on O(4) symmetry

$$\mathcal{L} = |\partial_t \phi|^2 - V(\phi)$$

$$V(\phi) = a|\phi|^2 + b|\phi|^4 + c|\nabla\phi|^2 + d|\nabla^2\phi|^2 + e|\nabla\phi|^2 \cdot |\phi|^2 + f|\phi|^6 + g(\phi\cdot\nabla\phi)^2$$

Inhomogeneous chiral condensate (dual chiral density wave)

$$\phi_0 = \Delta \begin{pmatrix} \cos(\boldsymbol{q}\cdot\boldsymbol{r}) \\ 0 \\ 0 \\ \sin(\boldsymbol{q}\cdot\boldsymbol{r}) \end{pmatrix} = \Delta e^{-i\boldsymbol{q}\cdot\boldsymbol{r}L_{A3}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

↓ Add [amplitude fluctuation δ, phase fluctuations $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$

$$\phi = (\Delta + \delta) e^{-i\alpha_1 L_{V1}} e^{-i\alpha_2 L_{V2}} e^{-i\beta_1 L_{A1}} e^{-i\beta_2 L_{A2}} e^{-i\beta_3 L_{A3}} \begin{pmatrix} \cos(\boldsymbol{q}\cdot\boldsymbol{r}) \\ 0 \\ 0 \\ \sin(\boldsymbol{q}\cdot\boldsymbol{r}) \end{pmatrix}$$

↓ Euler-Lagrange equation

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} + \nabla^2 \frac{\partial\mathcal{L}}{\partial(\nabla^2\phi)} = 0$$

- dispersion relations for δ, β_3 mixing

$$m_0^2 \equiv 4(b + eq^2 + 3f\Delta^2)\Delta^2$$

$$A = 4e\Delta^2 q^2 \left(\frac{4d}{M^2} - \frac{eg\Delta^4}{M^4}\right), \quad B = \frac{16e^4\Delta^8 q^4}{M^6}$$

$$\omega^2 = \begin{cases} m_0^2 + 4dq^2\left(1 + \frac{e^2\Delta^4}{dm_0^2}\right)k^2\cos^2\theta + g\Delta^2 k^2 + Ak^4\cos^2\theta - Bk^4\cos^4\theta + dk^4 \\ 4dq^2\left(1 - \frac{e^2\Delta^2}{4ds}\right)k^2\cos^2\theta - Ak^4\cos^2\theta + Bk^4\cos^4\theta + dk^4 \end{cases}$$

- dispersion relations for β_1, α_2 mixing (the same for β_2, α_1)

$$\omega^2 = 4dq^2k^2\cos^2\theta + dk^4 \pm 4dqk^3\cos\theta$$

↓ spatial inversion symmetry
Add pion mass($m_\pi \approx$ 139 MeV) ➡ Assuming as follows

Charged Pion: $\omega^2 = 4dq^2k^2\cos^2\theta + 4d'qk^3|\cos\theta| + dk^4 + m_\pi^2$
$d' = +d$ or $-d$

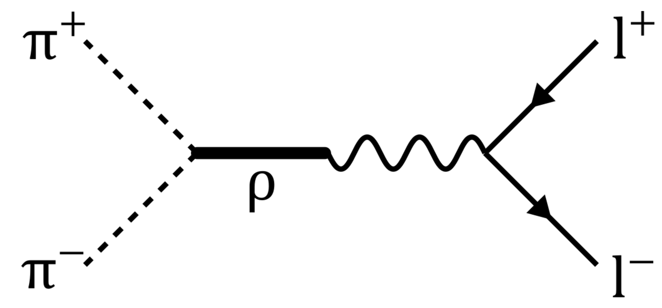
Dilepton Production Rate

Vector meson dominance

$$\pi^+ + \pi^- \to l^+ + l^-$$

↓ Assuming

$$\pi^+ + \pi^- \to \rho \to \gamma^* \to l^+ + l^-$$



We calculated $\left.\frac{d^4R}{d^3QdM}\right|_{Q=0}$ in the above diagram

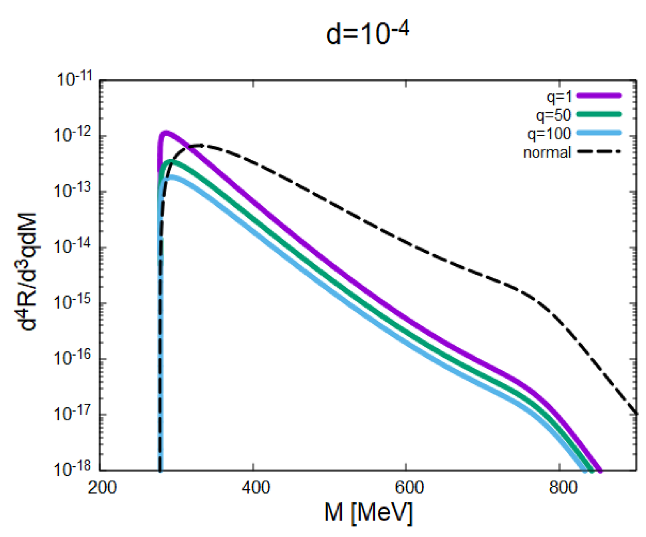
Dilepton production rate in the center of mass frame ($\boldsymbol{Q} = 0$)
for an invariant mass M ($\gamma^*: Q^\mu = (M, \boldsymbol{Q})$)

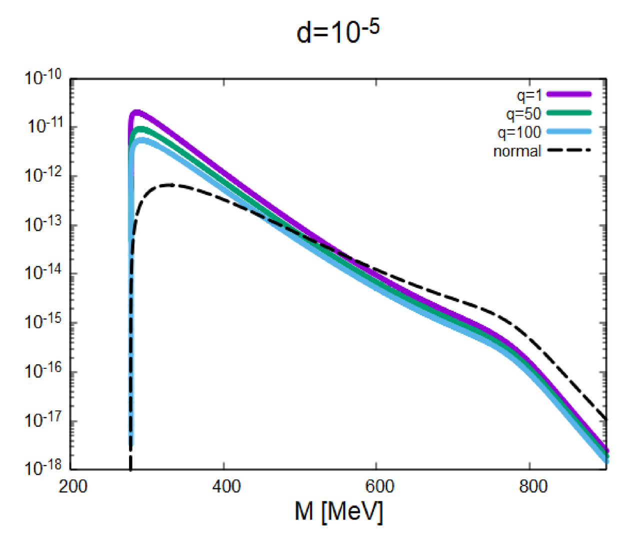
Numerical Results

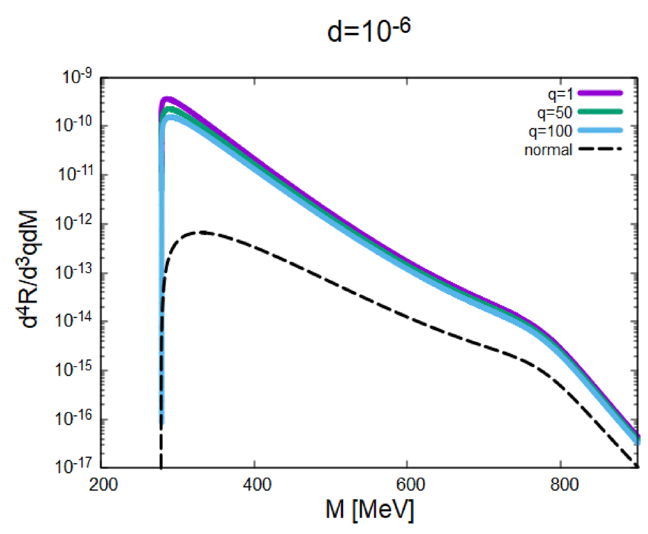
Electron-positron pair production rates (T=40 MeV)

- The case of $\boldsymbol{d' = -d}$

$$\omega^2 = 4dq^2k^2\cos^2\theta + 4dqk^3|\cos\theta| + dk^4 + m_\pi^2$$





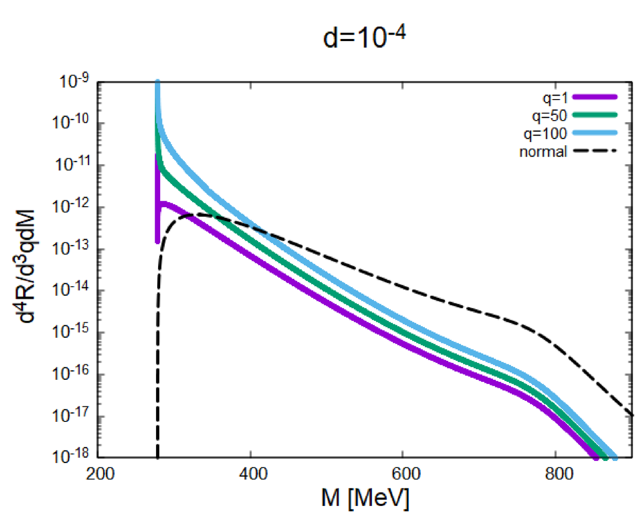


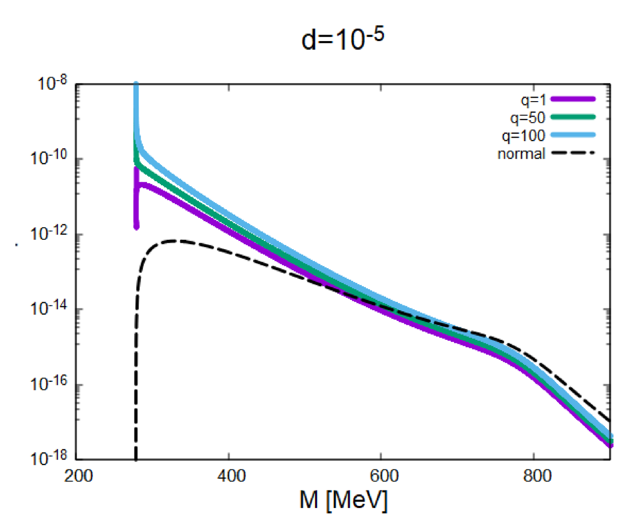
------ : by $\omega^2 = k^2 + m_\pi^2$
(homogeneous condensed phase)

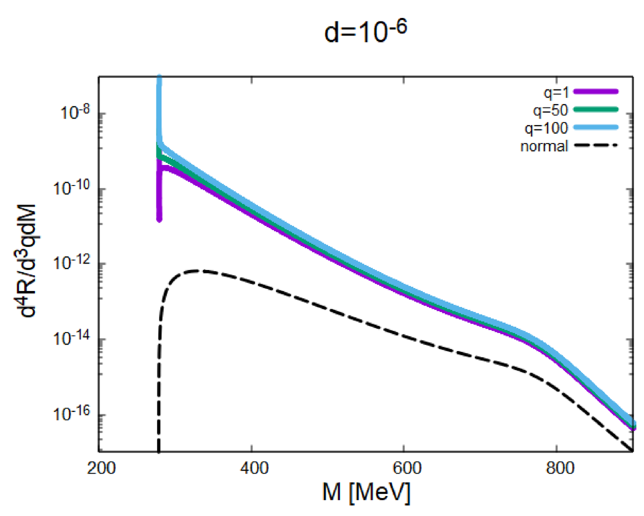
its overall slope is steeper than that of homogeneous condensed phase

- The case of $\boldsymbol{d' = -d}$

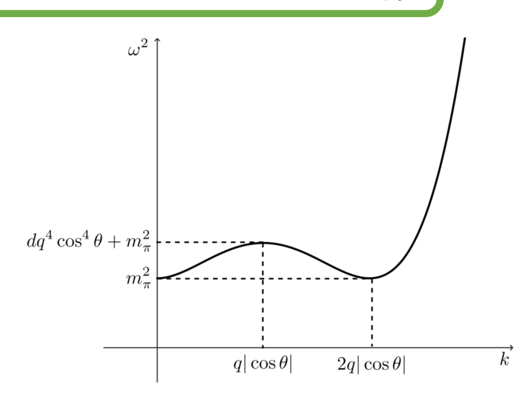
$$\omega^2 = 4dq^2k^2\cos^2\theta - 4dqk^3|\cos\theta| + dk^4 + m_\pi^2$$







divergence at $M = 2m_\pi$



Summary and Future works

We estimated the modified dilepton production rates from the charged-pion annihilation by the pion's dispersion in the inhomogeneous chiral condensed phase as one of possible experimental signatures.

From the calculation of the electron-positron pair production rate for the invariant mass,
[its overall slope is steeper than that of homogeneous condensation
the possibility of divergence at $M = 2m_\pi$

We think these can be remarkable signatures of the inhomogeneous chiral condensed phase.

Improving production rate calculation (Future work)
・πN Coupling ・change of pion mass etc.