Field Theory and the Electroweak Standard Model

— lecture 3 —

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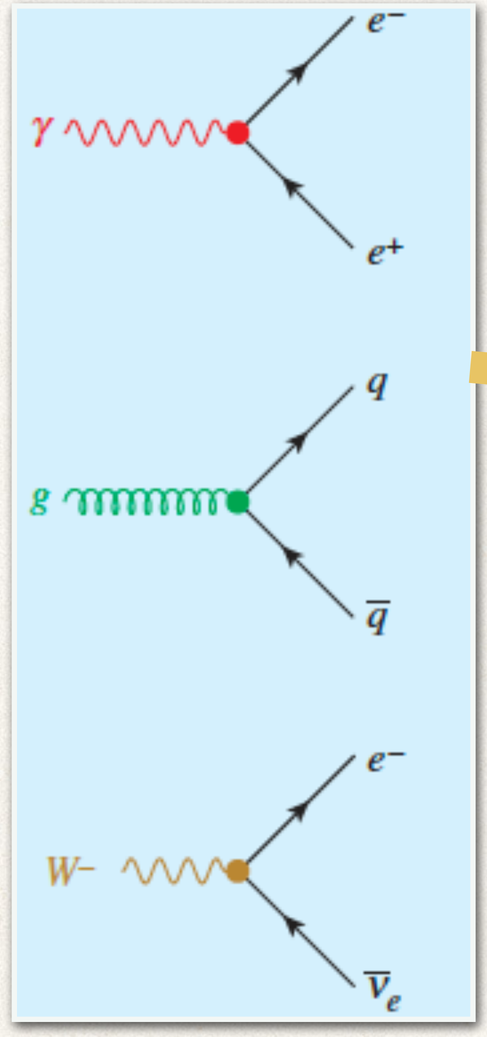
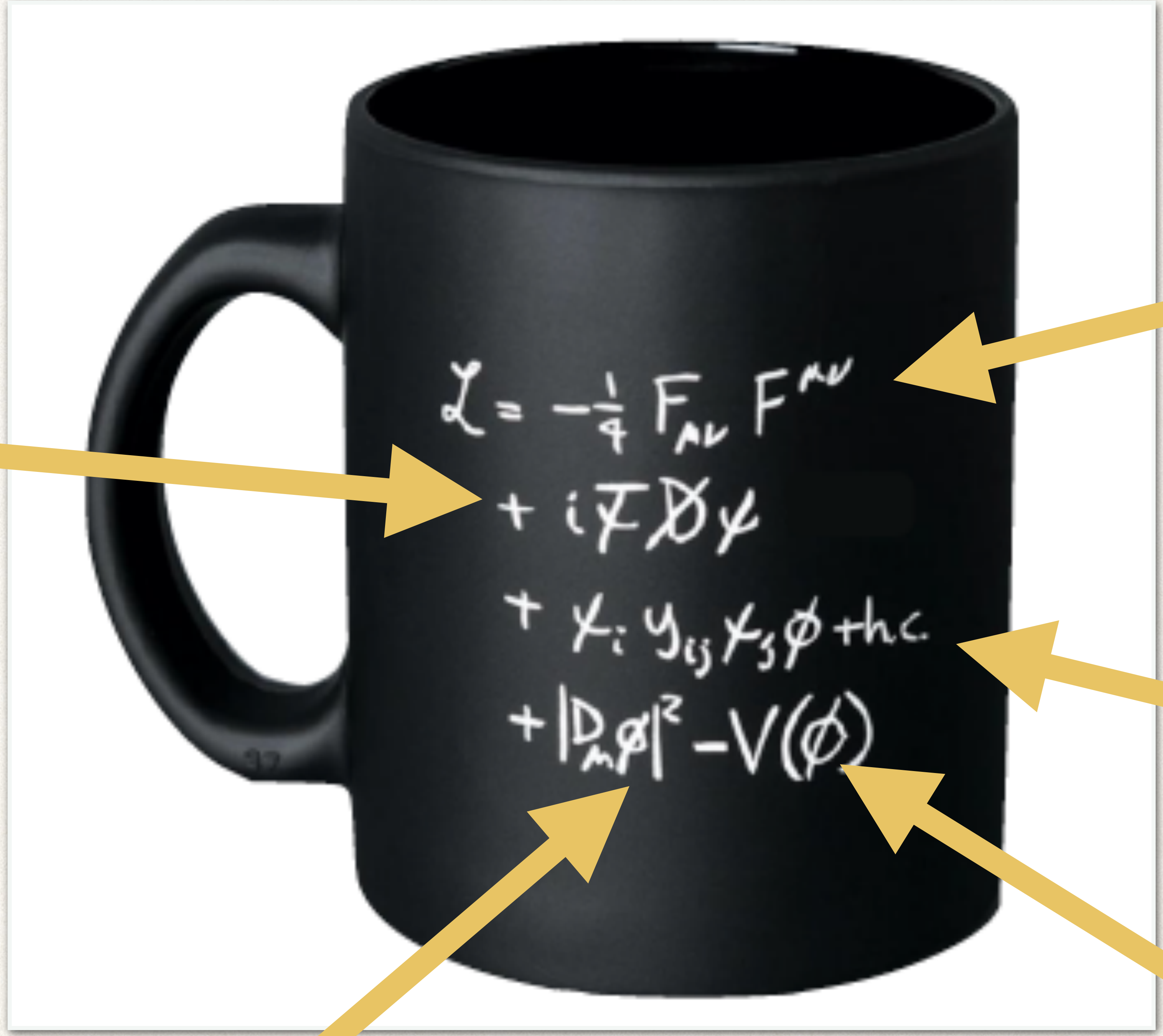
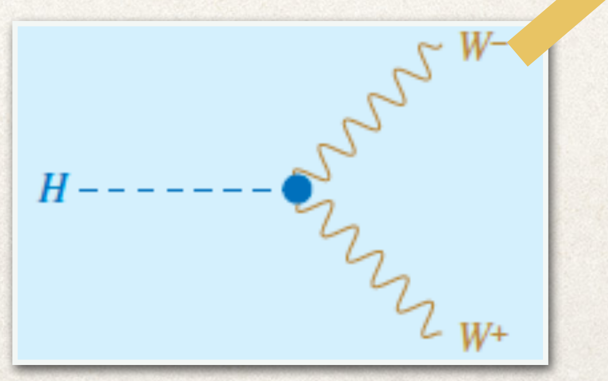
SM on a mug

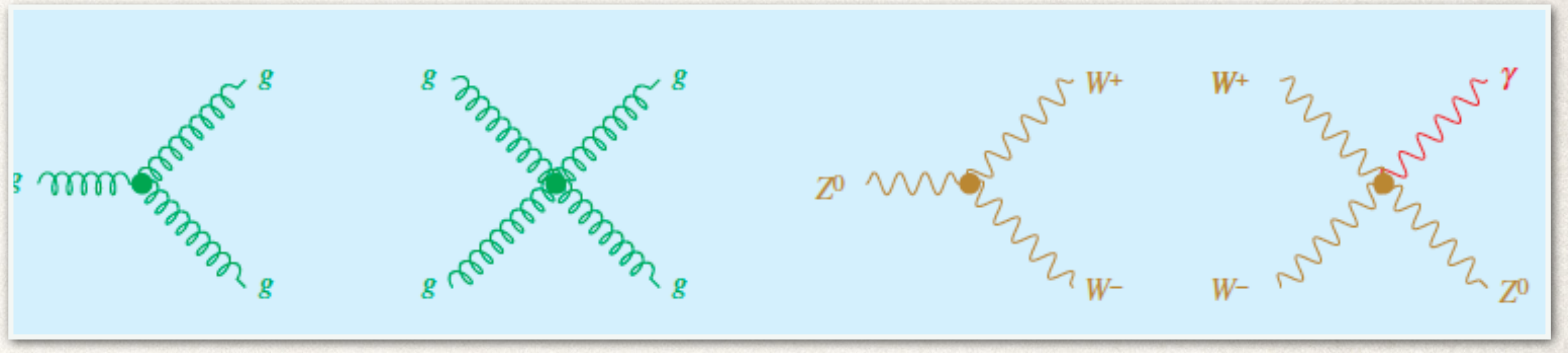
- Lagrangian before SSB

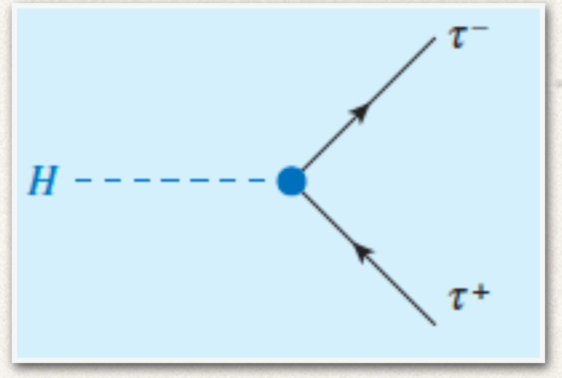


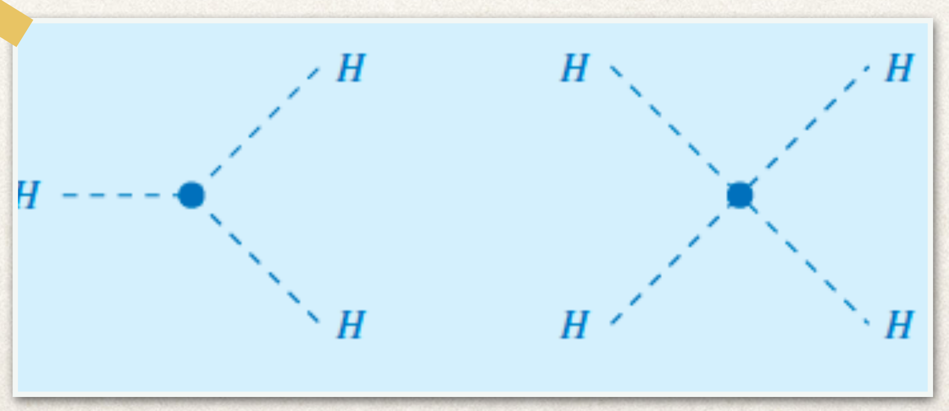




Summation over
$SU(3)$, $SU(2)_L$, $U(1)_Y$
fields implicitly assumed

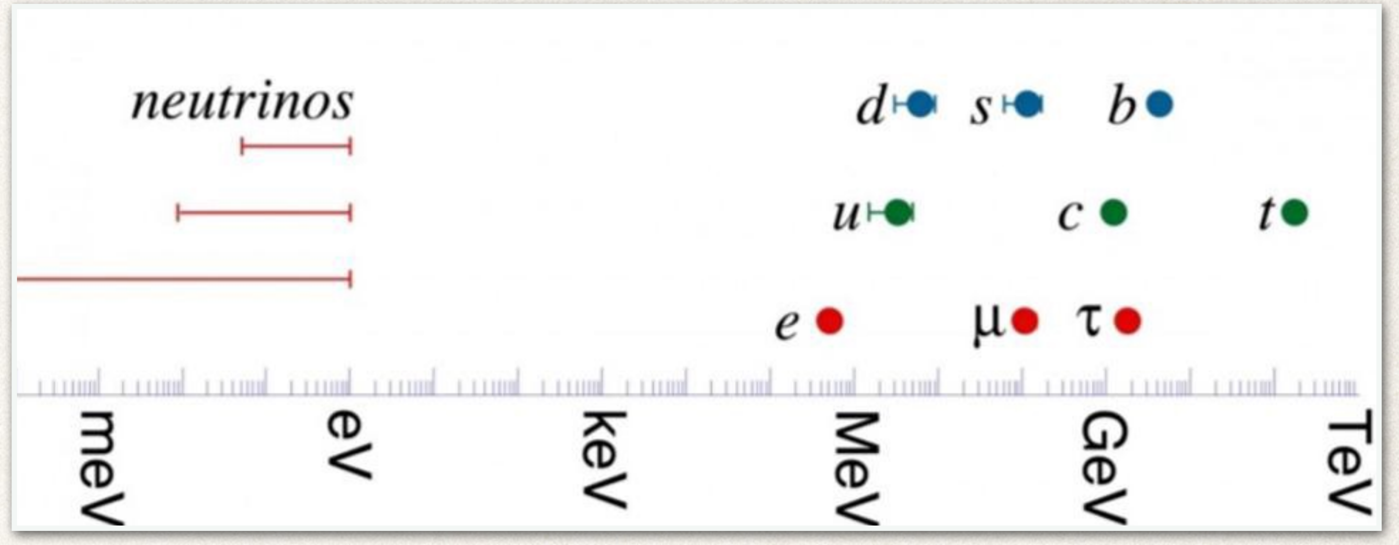






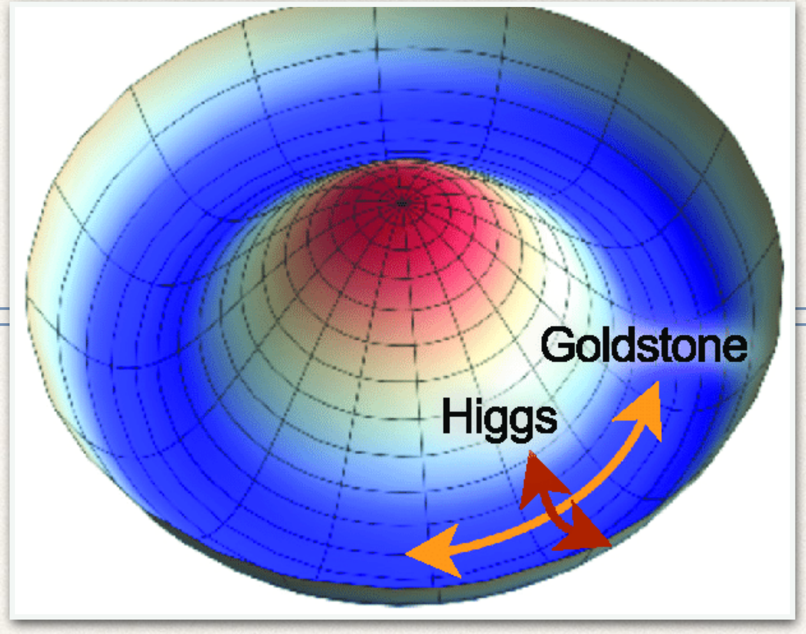
Neutrinos

➜ see lectures by S. Lavignac



- Neutrinos are known to oscillate, which implies that (at least two) have (tiny) mass
- Canonical SM formulation with massless neutrinos clearly fails here
- However, simple extensions the SM that could yield neutrino masses have been proposed
  - Neutrino as a Dirac particle (Dirac spinors)
    - SU(2) singlet RH neutrinos ν_R, mimicking the construction in the quark sector → Dirac mass term, but note $Y_{\nu_R} = 0 \rightarrow$ sterile neutrino (only gravity)
    - neutrinos get mass through EW SSB, would require neutrino Yukawa couplings of (at least) 4-5 orders of magnitude smaller than electron Yukawa coupling
  - Neutrino as a Majorana particle (Majorana spinor -> Dirac spinor for particle = antiparticle)
    - Observed ν_L : LH component of a light Majorana neutrino with small mass generated by a seesaw mechanism?

Quantization of a spontaneously broken gauge theory (1)



- Abelian Higgs model:

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right) e^{i\xi(x)/v} = \frac{1}{\sqrt{2}}\left(v + \rho(x) + i\xi(x) + \dots\right)$$

Higgs Goldstone

Unitary gauge

$$\phi(x) \to \phi'(x) = e^{(-i\xi(x)/v)}\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right)$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{gv}\partial_\mu \xi(x)$$

Lagrangian in this gauge:

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}g^2 v^2 A_\mu A^\mu - \mu^2\rho^2 + \frac{1}{2}g^2 A\mu A^\mu \rho^2 + g^2 v A_\mu A^\mu \rho - \lambda v \mu \rho^3 - \frac{\lambda}{4}\rho^4 + \frac{1}{4}\mu^2 v^2$$

Advantage: physical fields!

Propagator of the A^μ field

$$\frac{i\left(-g_{\mu\nu} + k_\mu k_\nu / m_A^2\right)}{k^2 - m_A^2 + i\epsilon}$$

behaves as $\frac{1}{m_A^2}$ as $k \to \infty$; compare with $\frac{1}{k^2}$ for massless photon

What about other gauges?

Quantization of a spontaneously broken gauge theory (2)

- Abelian Higgs model:

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right) e^{i\pi(x)/v} = \frac{1}{\sqrt{2}}\left(v + \rho(x) + i\pi(x) + \dots\right)$$

$$(D_\mu\phi)^*(D^\mu\phi) = \frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}(\partial_\mu\pi)^2 + \frac{1}{2}g^2v^2A_\mu A^\mu + gvA_\mu\partial^\mu\pi + \text{interaction terms}$$

Eliminate the kinetic mixing term $gv\,A_\mu\partial^\mu\pi$ by gauge fixing ($M_A = gv$)

$$\mathscr{L}_{GF} = -\frac{1}{2\zeta}(\partial^\mu A^a_\mu - \xi M_A \pi)^2$$

R_ξ gauges
(manifestly renormalizable)

Propagator of the A^μ field

$$\frac{i\left(-g_{\mu\nu} + (1-\xi)k_\mu k_\nu/(k^2 - \xi m_A^2)\right)}{k^2 - m_A^2 + i\epsilon}$$

behaves as $\frac{1}{k^2}$ as $k \to \infty$

Propagator of the π field

$$\frac{i}{k^2 - \xi m_A^2 + i\epsilon}$$

Quantization of a spontaneously broken gauge theory (2)

	A^μ	π
$\xi \to \infty$	$\dfrac{i\left(-g_{\mu\nu} + k_\mu k_\nu / m_A^2\right)}{k^2 - m_A^2 + i\epsilon}$	decoupled
$\xi = 1$	$\dfrac{-ig_{\mu\nu}}{k^2 - m_A^2 + i\epsilon}$	$\dfrac{-i}{k^2 - m_A^2 + i\epsilon}$

$\frac{}{\xi}(\partial^\mu A_\mu^a - \xi M_A \pi)^2$

R_ξ gauges
(manifestly renormalizable)

Propagator of the A^μ field  $$\frac{i\left(-g_{\mu\nu} + (1-\xi)k_\mu k_\nu/(k^2 - \xi m_A^2)\right)}{k^2 - m_A^2 + i\epsilon}$$ behaves as $\frac{1}{k^2}$ as $k \to \infty$

Propagator of the π field $$\frac{i}{k^2 - \xi m_A^2 + i\epsilon}$$

Quantization of a spontaneously broken gauge theory (2)

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$\xi \to \infty$	$\dfrac{i\left(-g_{\mu\nu} + k_\mu k_\nu / m_A^2\right)}{k^2 - m_A^2 + i\epsilon}$	decoupled	**Unitary gauge**	→	**tree level**
$\xi = 1$	$\dfrac{-ig_{\mu\nu}}{k^2 - m_A^2 + i\epsilon}$	$\dfrac{-i}{k^2 - m_A^2 + i\epsilon}$	**Feynman gauge (or any other R_ξ)**	→	**loops**

$\frac{\ }{\xi}(\partial^\mu A^a_\mu - \xi M_A \pi)^2$

R_ξ gauges
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Propagator of the A^μ field  $\dfrac{i\left(-g_{\mu\nu} + (1-\xi)k_\mu k_\nu/(k^2 - \xi m_A^2)\right)}{k^2 - m_A^2 + i\epsilon}$ behaves as $\dfrac{1}{k^2}$ as $k \to \infty$

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$$\frac{1}{\xi}(\partial^\mu A_\mu^a - \xi M_A \pi)^2$$

R_ξ gauges
(manifestly renormalizable)

$$\frac{i\left(-g_{\mu\nu} + (1-\xi) k_\mu k_\nu / (k^2 - \xi m_A^2)\right)}{k^2 - m_A^2 + i\epsilon}$$

behaves as $\frac{1}{k^2}$ as $k \to \infty$

$$\frac{i}{k^2 - \xi m_A^2 + i\epsilon}$$

Analogous treatment for non-abelian gauge theories, but apart from unphysical Goldstone bosons also unphysical ghosts introduced

Counting parameters

- The Standard Model has 18 free parameters*
  - 3 couplings g, g', g_s
  - 2 parameters of the Higgs potential μ, λ
  - 9 (6 quark + 3 lepton) Yukawa couplings y_f
  - 4 parameters of the CKM matrix V_{CKM}

*though recollections may vary (one can additionally include non-canonical parameters, e.g. the Θ_{QCD} CP-violating angle or parameters of the neutrino sector bringing it up to 26 free parameters)

Counting parameters

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- Using relations between the parameters a more practical list can be obtained, e.g.:
  - 2 coupling constants $\alpha = \dfrac{e^2}{4\pi}$ and $\alpha_s = \dfrac{g_s^2}{4\pi}$
  - Fermi constant $G_F = \dfrac{g^2}{4\sqrt{2}M_W^2}$
  - 2 masses M_Z and M_H
  - 9 fermion masses m_f
  - 4 parameters of the CKM matrix V_{CKM}

SM input parameters

https://pdg.lbl.gov

- All parameters of the SM have been experimentally measured (last unmeasured was M_H)
- Consult Particle Data Group for the most up-to-date values
  - $\alpha^{-1} = 137.035999084(21)$
    $\alpha_s(M_Z) = 0.1179(9)$
    $G_F = 1.1663788(6) \times 10^{-5}$ GeV^{-2}
  - $M_Z = 91.1876 \pm 0.0021$ GeV
    $M_H = 125.25 \pm 0.17$ GeV
  - ….
- With input parameters known, one can make theoretical predictions for any SM observable!

The trouble with theoretical predictions

- The running of couplings → depending on the strength of the coupling different methods applicable
  - expansion in the coupling constant(s) in the perturbative regime
  - lattice gauge theory (e.g.) in the non-perturbative regime
- Feasibility of performing calculations in the sense of obtaining finite results (renormalisability)→ depends on the theory
  - Problem (in perturbation theory): calculations of quantum loop corrections involve integration over unconstrained momenta of the virtual particle(s) in the loop(s)

e.g. $e^+e^- \to \mu^+\mu^-$ @ 1 loop

The integration can yield
→ UV singularities when $k \to \infty$ treated by the renormalisation program
IR singularities when $k \to 0$ cancel against IR singularities from real emission diagrams

UV singularities and how to cure them

1. Acceptance. UV singularities can appear in the intermediate stages of calculations.

2. Diagnosis. Infinities are given mathematical meaning. Achieved by introducing a regulator parameter in the expressions for the loops. The regulator makes the integrals well-defined, apart from a limit value of the regulator, for which the integral is singular.

   → **Regularisation**

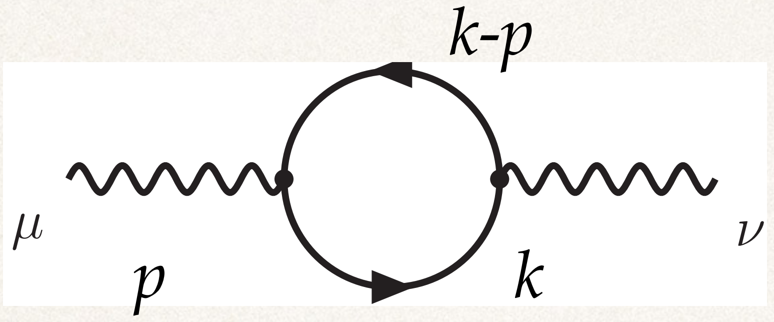
   Most often used: dimensional regularisation (DR).

   - Integrals are calculated in $d \neq 4$ dimensions. $\int d^4k \rightarrow \int d^dk$

     $g^2 \rightarrow g^2\mu^{4-d}$ to keep the action dimensionless, μ is an arbitrary scale
   - Singularities manifest as poles $\frac{1}{(4-d)^n}$
   - DR preserves gauge symmetry and can be used for regularisation of both, UV and IR singularities.

3. Treatment. The singularities are absorbed in the redefined parameters and fields of the theory. A finite number of redefinitions has to yield well-defined results for all observables at any order of perturbation theory. → **Renormalisation**

Example



$$\propto \int d^4k \frac{\mathrm{Tr}[\ldots]}{(k^2 - m^2)((p-k)^2 - m^2)} \sim \int \frac{d^4k}{k^4} \sim \int \frac{dk}{k}$$

logarithmic singularity

large k

Dimensional regularisation $(d = 4 - 2\epsilon)$

$$\int \frac{d^dk}{k^4} \to \int d\Omega_d \int_K^\infty dk \frac{k^{d-1}}{k^4} = \frac{(2\pi)^{2-\epsilon}}{\Gamma(2-\epsilon)} \left(-\frac{1}{2\epsilon}\right) \left[\frac{1}{k^{2\epsilon}}\bigg|_{k=\infty} - \frac{1}{k^{2\epsilon}}\bigg|_{k=K} \right] \propto \frac{1}{2\epsilon} - \log(K) + \mathcal{O}(\epsilon)$$

QED renormalisation

- Reinterpret the QED Lagrangian $\mathscr{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \bar{\psi}\left(i\gamma^\mu \partial_\mu - e\,\gamma^\mu A_\mu - m\right)\psi$

  as written in terms of bare (unrenormalised) parameters $e_0,\ m_0$ and fields ψ^0, A^0 and relate

$$\psi^0 = \sqrt{Z_2}\,\psi^R \qquad A^0_\mu = \sqrt{Z_3}\,A^R_\mu \qquad m_0 = Z_m m_R \qquad e_0 = Z_e e_R$$

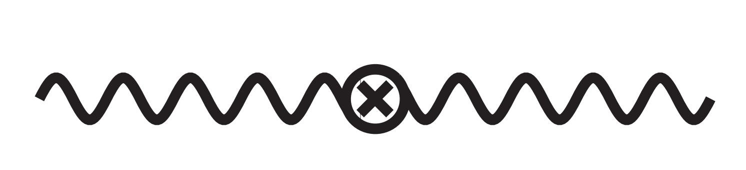
renormalisation constants Z_i

- With $Z_1 \equiv Z_e Z_2 \sqrt{Z_3}$ and $Z_i = 1 + (Z_i - 1)$ obtain **Lagrangian of the renormalised perturbation theory**

$$\mathscr{L} = -\frac{1}{4}(\partial_\mu A^R_\nu - \partial_\nu A^R_\mu)^2 + \bar{\psi}^R\left(i\gamma^\mu \partial_\mu - e_R\,\gamma^\mu A^R_\mu - m_R\right)\psi^R \; -\frac{1}{4}(Z_3 - 1)(\partial_\mu A^R_\nu - \partial_\nu A^R_\mu)^2 + i(Z_2 - 1)\bar{\psi}^R\gamma^\mu\partial_\mu\psi^R - e_R\,(Z_1 - 1)\bar{\psi}^R\gamma^\mu A_\mu \psi^R$$
$$- \left[(Z_m - 1) + (Z_2 - 1)\right] m_R \bar{\psi}^R\psi^R$$

counterterms

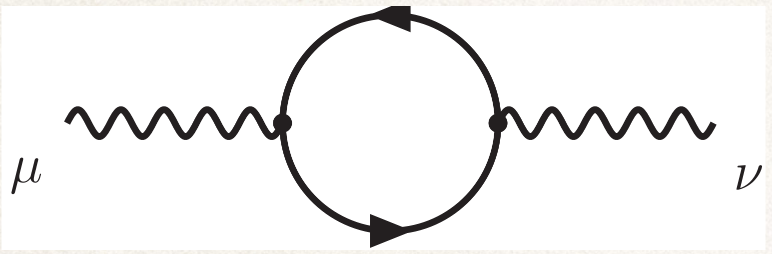
$\rightarrow$ additional Feynman rules for the counterterms, e.g.



$$-i(Z_3 - 1)\,(p^2 g^{\mu\nu} - p^\mu p^\nu)$$

- Fix **renormalisation conditions** defining Z_i's

Example ctnd.

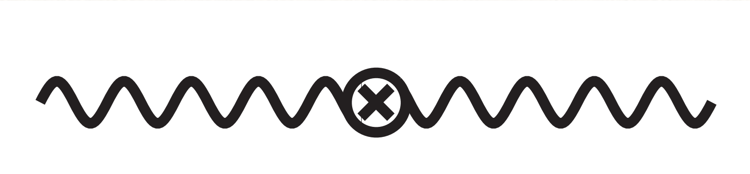


$$= -\,i(p^2 g^{\mu\nu} - p^\mu p^\nu e^2)\, e^2\, \Pi_2(p^2)$$

$$\Pi_2(p^2) = \frac{1}{2\pi^2}\int_0^1 dx\, x\,(1-x)\left[\frac{2}{\epsilon} + \log\left(\frac{\tilde{\mu}^2}{m^2 - p^2 x\,(1-x)}\right)\right]$$

$$\tilde{\mu}^2 = 4\pi e^{-\gamma_E}\,\mu^2$$

Counterterm contribution



$$-i(Z_3 - 1)\,(p^2 g^{\mu\nu} - p^\mu p^\nu)$$

Renormalisation condition $\quad \Pi(0) = 0$

where $-i(p^2 g^{\mu\nu} - p^\mu p^\nu)\,\Pi(p^2)$ is the sum of all 1PI contributions to the photon 2-point function

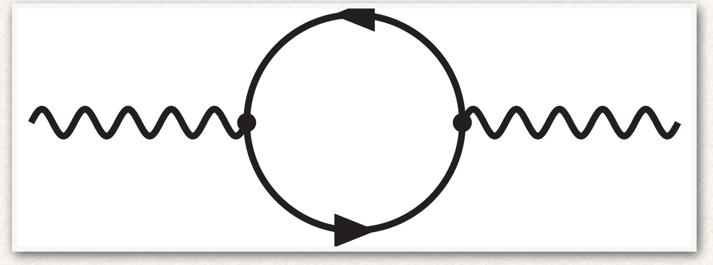
Up to order e^2 $\quad \Pi(p^2) = e^2\,\Pi_2(p^2) + (Z_3 - 1) + \ldots$ $\quad$ hence $\quad Z_3 - 1 = -\dfrac{e^2}{6\pi^2}\left[\dfrac{1}{\epsilon} + \dfrac{1}{2}\log\left(\dfrac{\tilde{\mu}^2}{m^2}\right)\right]$

$$\Rightarrow\ \Pi(p^2) = \frac{e^2}{2\pi^2}\int_0^1 dx\, x\,(1-x)\log\left(\frac{m^2}{m^2 - p^2 x\,(1-x)}\right) + \ldots$$

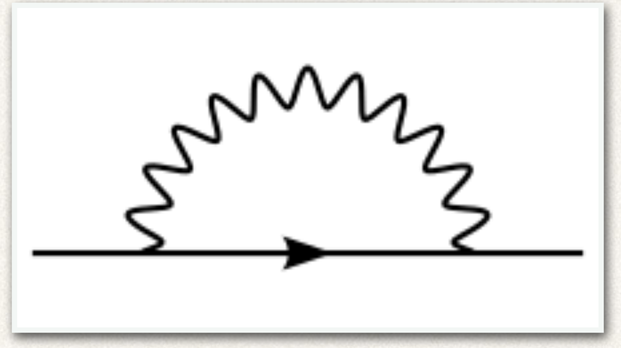
finite result!

Renormalisability (1)

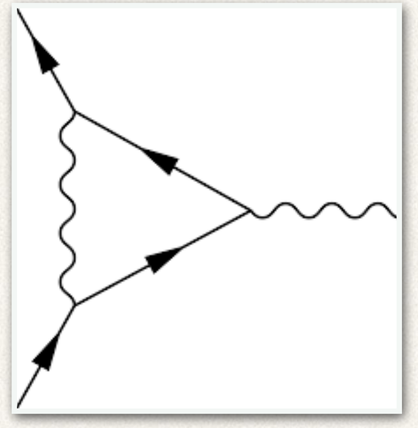
- Renormalisable theory: all UV divergencies can be cancelled with a finite number of counterterms to any order in perturbation theory
  - QED renormalisation program at 1 loop:



Z_3



Z_2, Z_m



Z_e

4 renormalisation conditions

- Normalisation of the fields not observables, so can be rescaled. Normalisation of the parameters set by measured quantities.
- The principle of renormalisability is an essential condition for any viable physical theory → observables are finite functions that can be in principle calculated in perturbation theory
- Renormalisation procedure introduces renormalisation scale μ as an artefact of the regularisation prescription

Renormalisability (2)

- Although Weinberg and Salam speculated that their theory is renormalizable, a proof of renormalizability of Yang-Mills theories with SSB was delivered a few years later, in 1971, by t'Hooft and Veltman  (Nobel Prize 1999)

-

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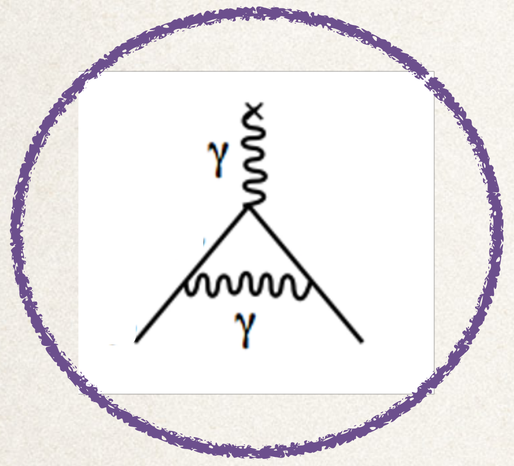
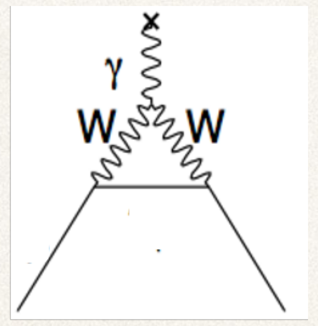
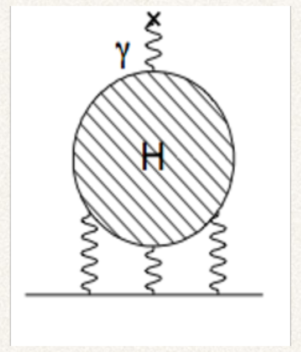
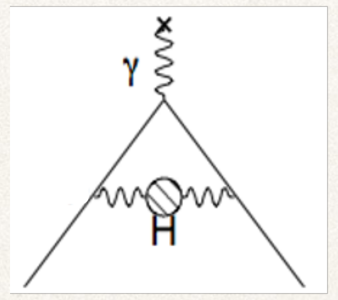
- Renormalised QED has proven to be spectacularly successful
  - anomalous magnetic moment of the electron









calculated up to 5 loops in QED

$$a_e^{\text{exp}} = 1159652.18059(13) \times 10^{-9}$$ *[Fan et al. Phys.Rev.Lett. 130 (2023) 7]*

agrees with the SM to 1 part in 10^{12}

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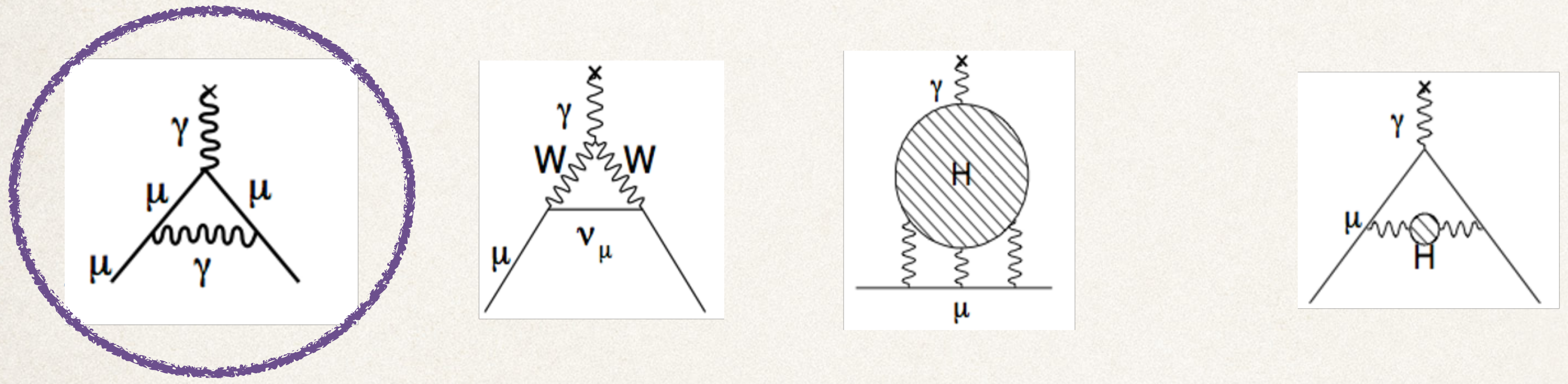
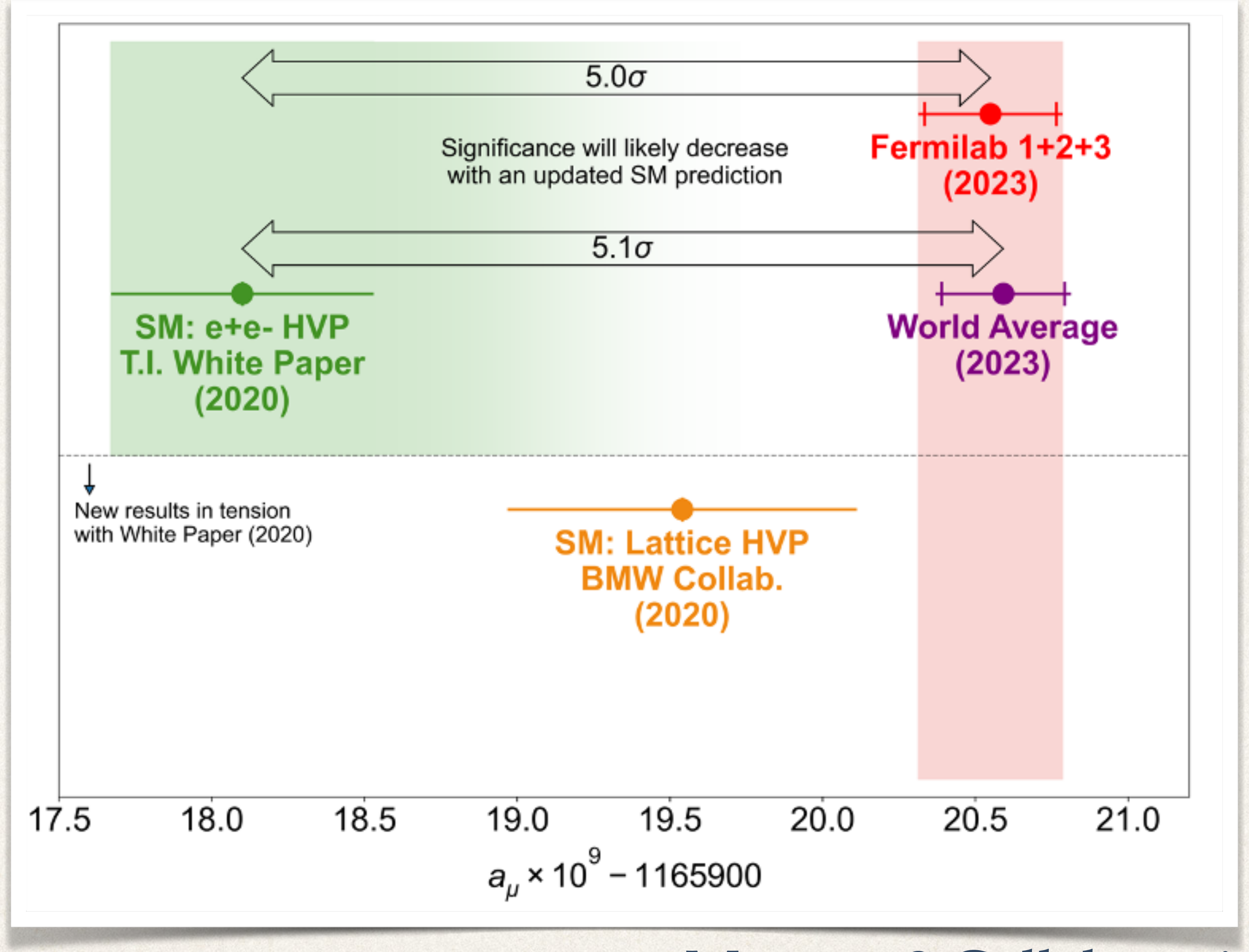
- Renormalised QED has proven to be spectacularly successful
  - anomalous magnetic moment of the muon





calculated up to 5 loops in QED

Muon g-2 Collaboration, August 2023

$$a_\mu^{\mathrm{QED}} = 1165847.1893(10) \times 10^{-9}$$

$$a_\mu^{\mathrm{SM}} = 1165918.100(430) \times 10^{-9}$$

$$a_\mu^{\mathrm{exp}} = 1165920.591(221) \times 10^{-9}$$ (world average 08'23)

Running coupling

- The renormalisation process and the measurable quantities must be independent of the arbitrary renormalisation scale → consequence: running coupling. Not being an observable, the coupling can depend on this scale.

Running coupling

- The renormalisation process and the measurable quantities must be independent of the arbitrary renormalisation scale → consequence: running coupling. Not being an observable, the coupling can depend on this scale.

- Consider a dimensionless observable R, e.g. a ratio of two cross sections. If only one scale Q is relevant for this observable, then from dimensional analysis after renormalisation $R = R(Q^2/\mu^2, \alpha(\mu^2))$ $\qquad \alpha = g^2/(4\pi)$

- Independence of R on μ implies

$$\mu^2 \frac{d}{d\mu^2} R(Q^2/\mu^2, \alpha(\mu^2)) = \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha}{\partial \mu^2} \frac{\partial}{\partial \alpha}\right] R(Q^2/\mu^2, \alpha(\mu^2)) = 0$$

With $\quad \beta(\alpha) \equiv \mu^2 \frac{\partial \alpha}{\partial \mu^2} \quad t \equiv \log\left(\frac{Q^2}{\mu^2}\right) \quad \alpha_\mu \equiv \alpha(\mu^2)$

renormalisation group equation

$$\left[-\frac{\partial}{\partial t} + \beta(\alpha_\mu) \frac{\partial}{\partial \alpha_\mu}\right] R(e^t, \alpha_\mu) = 0$$

Running coupling (2)

To solve
$$\left[-\frac{\partial}{\partial t}+\beta(\alpha_\mu)\frac{\partial}{\partial \alpha_\mu}\right] R(e^t,\alpha_\mu)=0 \qquad (*)$$

define (in an implicit way) a new function, the running coupling $\alpha(Q^2)$, through the equation
$$t=\int_{\alpha_\mu}^{\alpha(Q^2)}\frac{d\alpha}{\beta(\alpha)} \qquad (**)$$

By differentiating this equation wrt. t at fixed α_μ and wrt. α_μ at fixed t one finds $R(1,\alpha(Q^2))$ is a solution of (*)
→ entire scale dependence in R enters through $\alpha(Q^2)$

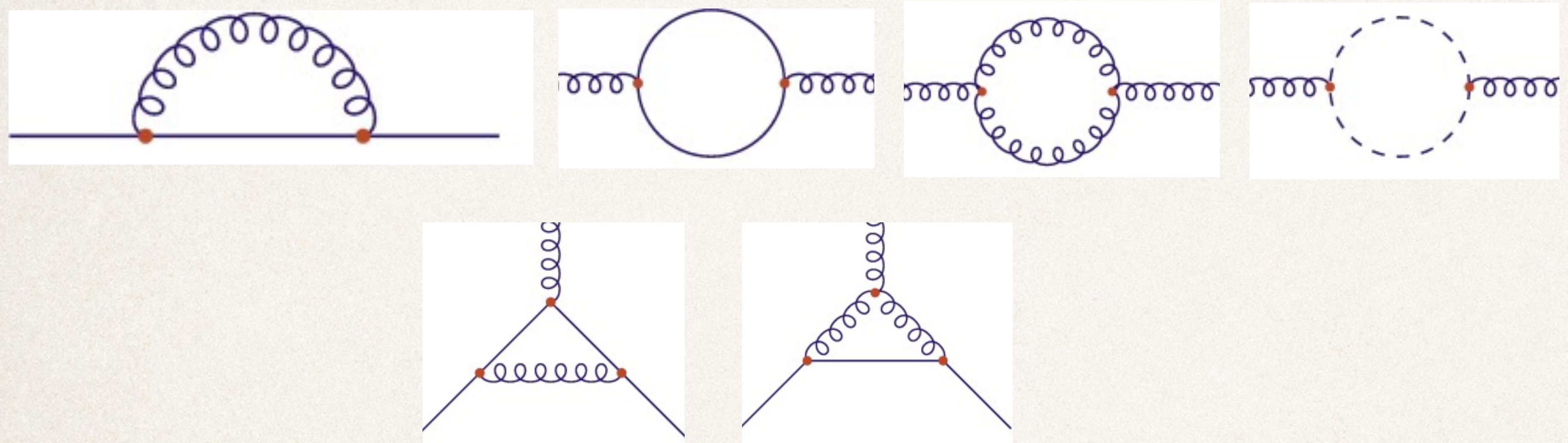
Using $\beta(\alpha)=-\alpha^2 b_0+\ldots$ integration of (**) gives

$$t=\frac{1}{b_0}\left(\frac{1}{\alpha(Q^2)}-\frac{1}{\alpha_\mu}\right) \qquad \Rightarrow \alpha(Q^2)=\frac{\alpha(\mu^2)}{1+\alpha(\mu^2)b_0\log(Q^2/\mu^2)}$$

Running coupling (3)

In QED $\quad b_0^{\text{QED}} = -\frac{1}{3\pi} \qquad \alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log(Q^2/\mu^2)}$

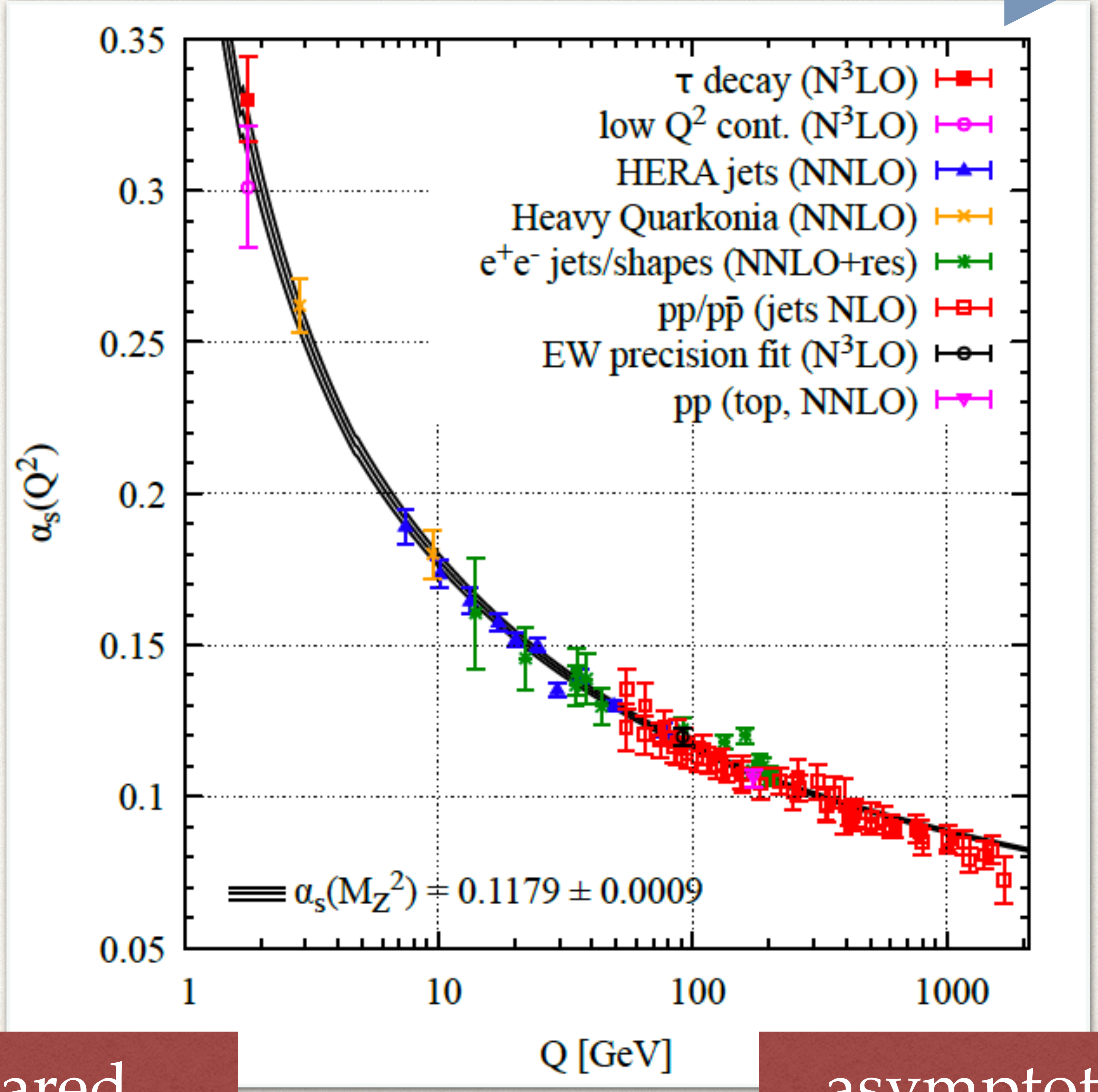
In QCD



$$b_0^{\text{QCD}} = \frac{11C_A - 2n_f}{12\pi} > 0 \quad \text{for} \;\; n_f < 17 \qquad \alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)(33-2n_f)}{12\pi}\log(Q^2/\mu^2)}$$

➜ see lectures by Xu Feng

non-perturbative → perturbative



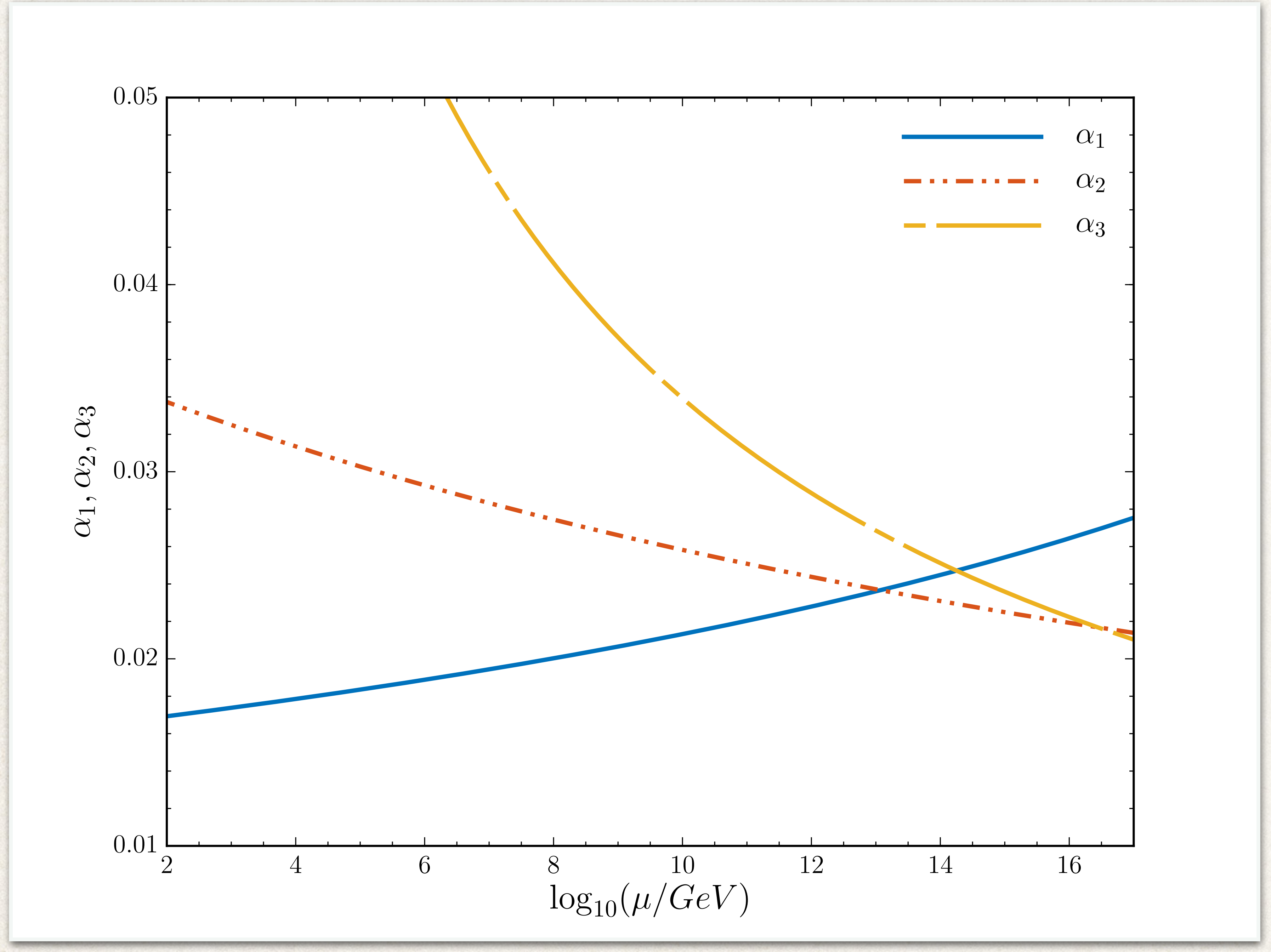
infrared slavery

asymptotic freedom

confinement, hadronization

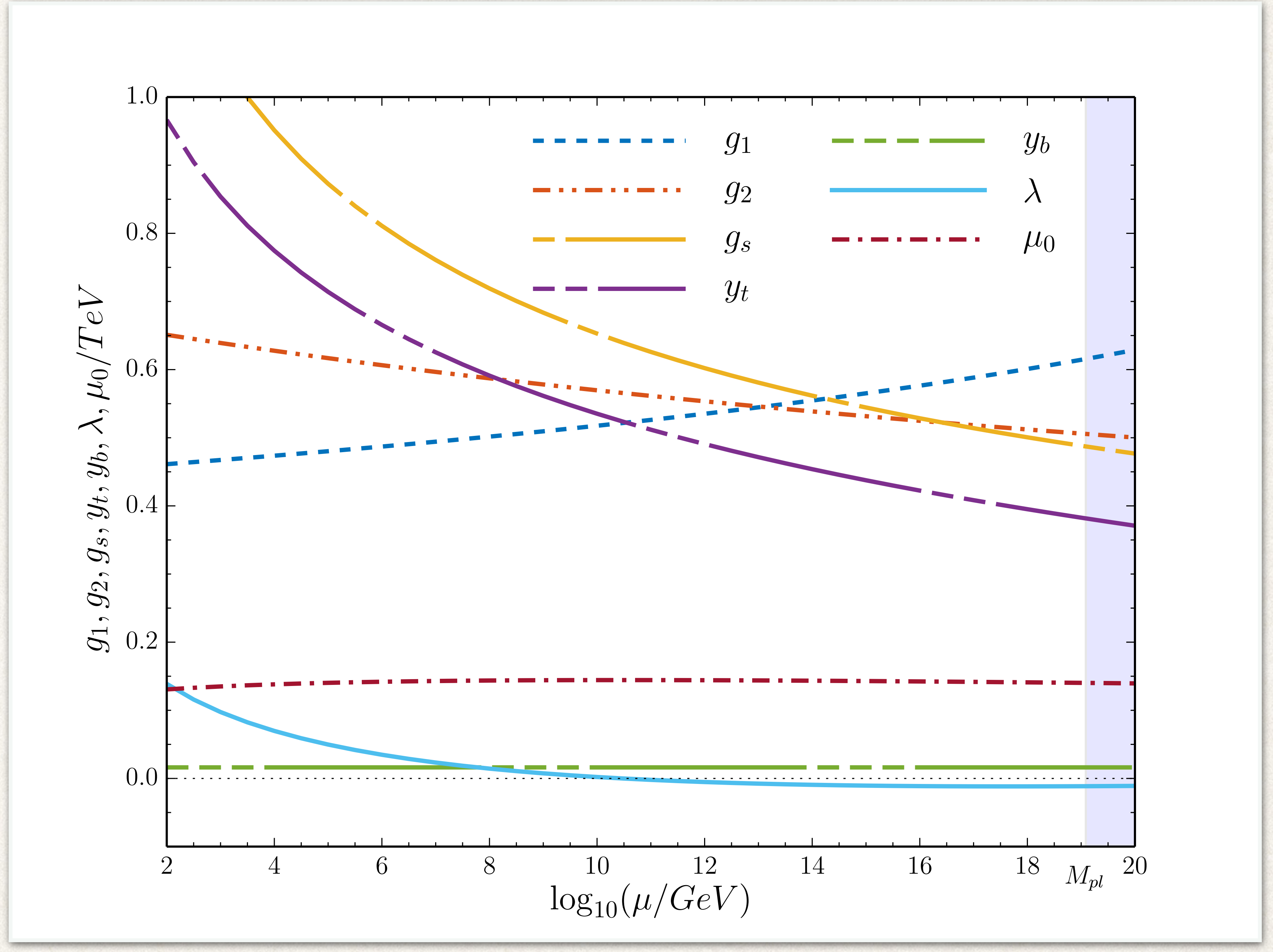
Running of the SM parameters

SM gauge couplings



Hint of unification?

together with Higgs and Yukawa couplings



arXiv:1601.08143

Anomalies

- Anomalies appear when a symmetry of a classical system is not preserved at the quantum level
- Noether's theorem: continuous symmetries imply conserved currents
- Currents associated with anomalous symmetries are not conserved → in QFT gauged symmetries must be anomaly free → consistency check (global anomalies not problematic)
- In the SM, axial currents $J^A_\mu = \bar{\psi}\gamma_\mu\gamma_5 T^a \psi$ receive non-zero corrections to $\partial^\mu J^A_\mu$ from one-loop triangle diagrams

- The anomaly comes with a factor $\mathrm{Tr}[T^a\{T^b, T^c\}]_L - \mathrm{Tr}[T^a\{T^b, T^c\}]_R$, where T^i are the generators of the considered symmetries in the SM and the traces are over all LH or RH fields
- In the SM only U(1)-SU(3)-SU(3), U(1)-SU(2)-SU(2), U(1)-U(1)-U(1) anomalies contribute

Anomalies (2)

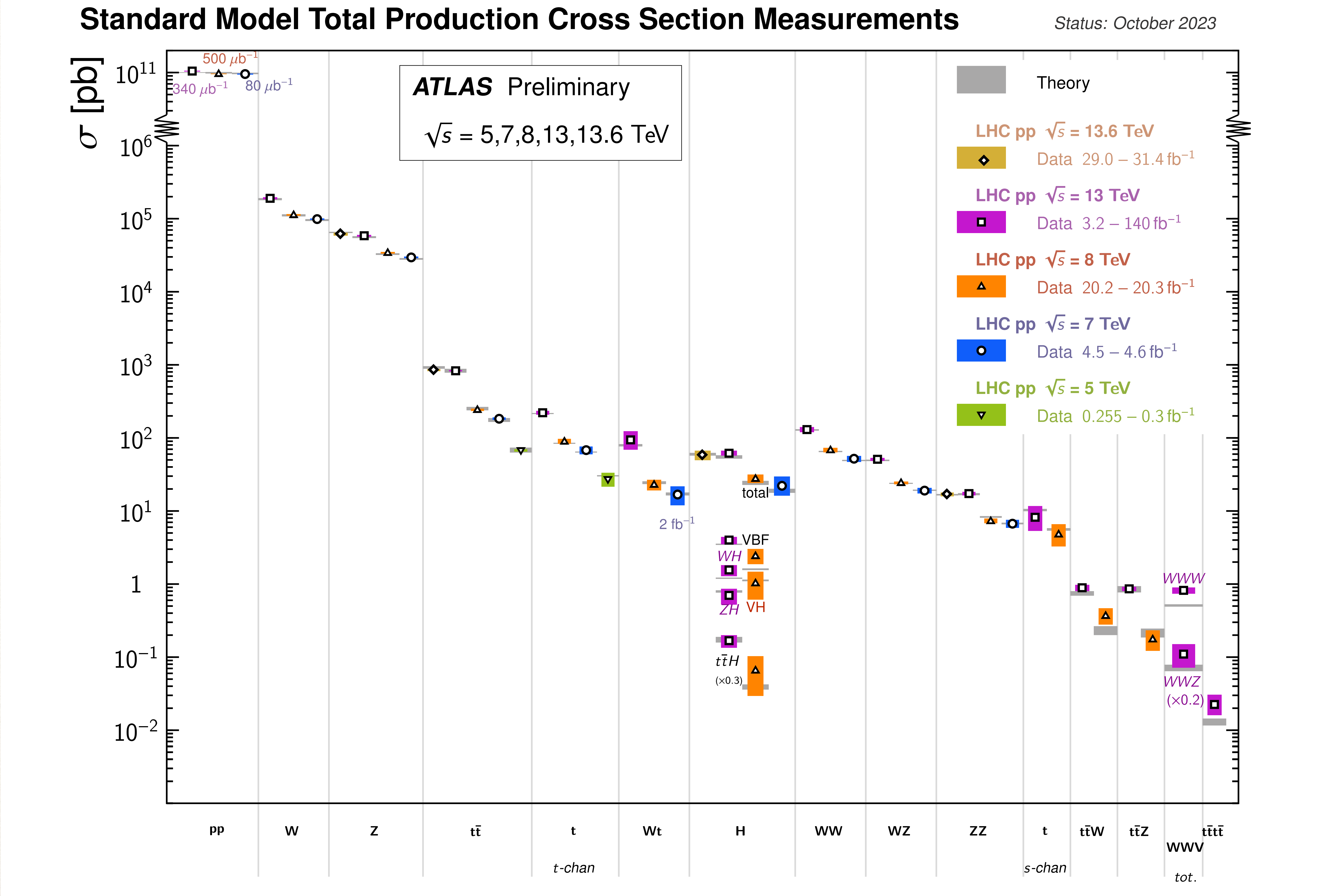
	ν_L	e_L	e_R	u_L	d_L	u_R	d_R
Y	-1	-1	-2	1/3	1/3	4/3	-2/3

$$\mathrm{Tr}[T^a\{T^b, T^c\}]_L - \mathrm{Tr}[T^a\{T^b, T^c\}]_R$$

- number of particles and their hypercharges conspire to cancel
- works within 1 generation
- anomaly cancellation provides a strong restriction on potential new particles

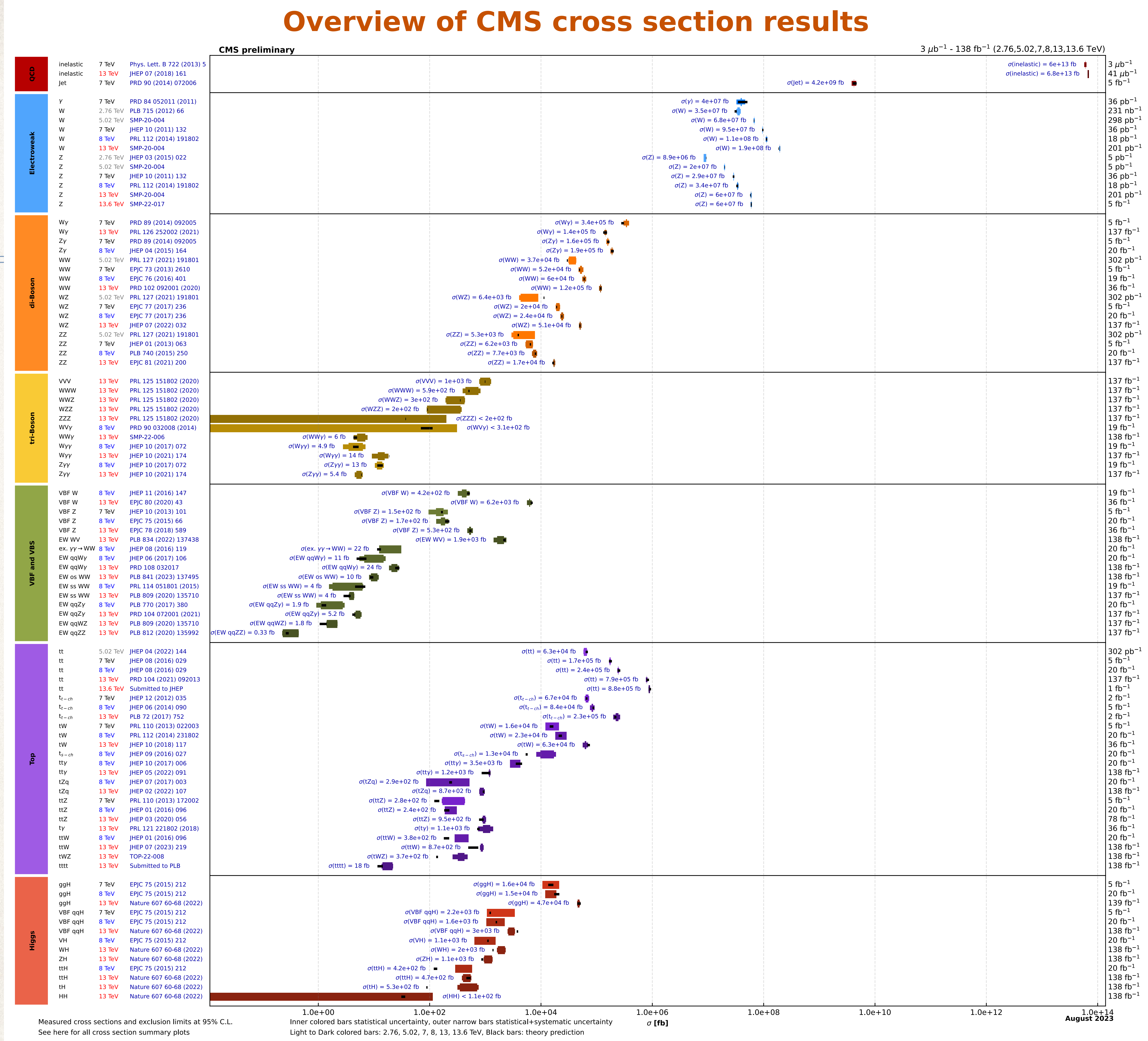
- U(1)-SU(3)-SU(3), only quarks $$3\left[2\left(\frac{1}{3}\right)+\frac{2}{3}-\frac{4}{3}\right]=0$$
- U(1)-SU(2)-SU(2), only doublets $$2\left[-1+3\left(\frac{1}{3}\right)\right]=0$$
- U(1)-U(1)-U(1) all fermions $$\left[2\,(-1)^3-(-2)^3+6\left(\frac{1}{3}\right)^3-3\left(-\frac{2}{3}\right)^3-3\left(\frac{4}{3}\right)^3\right]=0$$

SM@LHC (1)



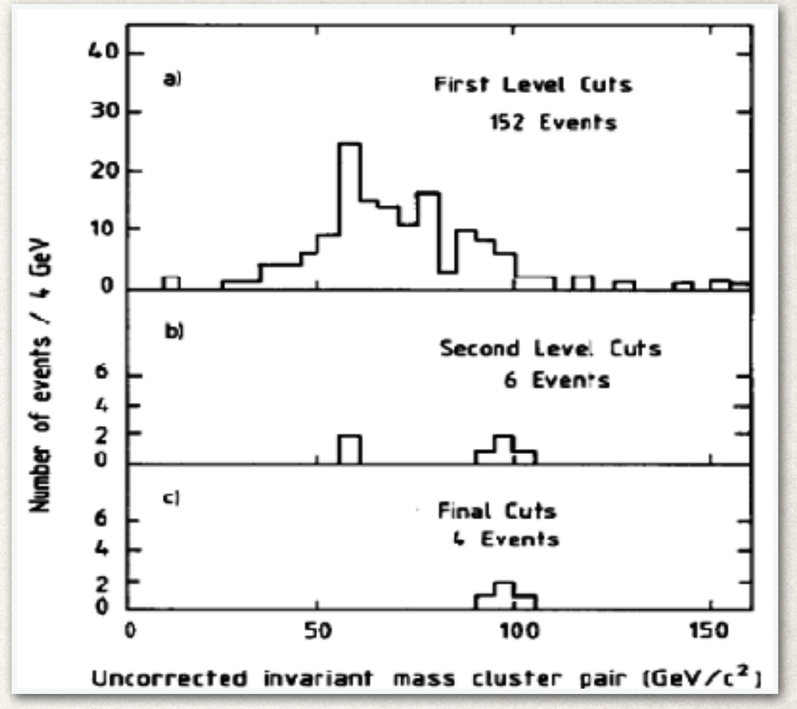
Standard Model Total Production Cross Section Measurements
Status: October 2023
ATLAS Preliminary
√s = 5,7,8,13,13.6 TeV
σ [pb]
Theory
LHC pp √s = 13.6 TeV
Data 29.0 – 31.4 fb⁻¹
LHC pp √s = 13 TeV
Data 3.2 – 140 fb⁻¹
LHC pp √s = 8 TeV
Data 20.2 – 20.3 fb⁻¹
LHC pp √s = 7 TeV
Data 4.5 – 4.6 fb⁻¹
LHC pp √s = 5 TeV
Data 0.255 – 0.3 fb⁻¹
500 μb⁻¹
340 μb⁻¹
80 μb⁻¹
2 fb⁻¹
total
VBF
WH
ZH
VH
ttH
(×0.3)
WWW
WWZ
(×0.2)
pp
W
Z
tt̄
t
t-chan
Wt
H
WW
WZ
ZZ
t
s-chan
tt̄W
tt̄Z
WWV
tot.
tt̄tt̄

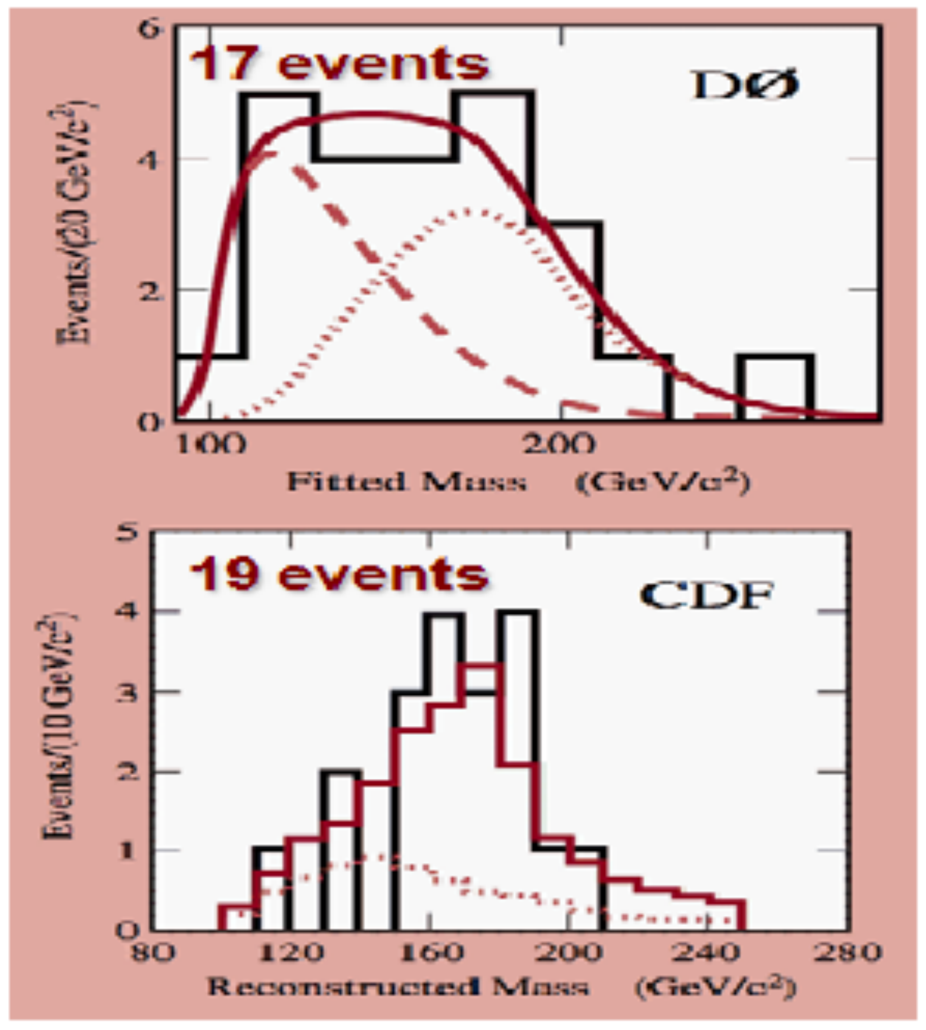
SM@LHC (2)


Overview of CMS cross section results
CMS preliminary
3 μb⁻¹ - 138 fb⁻¹ (2.76,5.02,7,8,13,13.6 TeV)
QCD
Electroweak
di-Boson
tri-Boson
VBF and VBS
Top
Higgs
σ [fb]
August 2023
Measured cross sections and exclusion limits at 95% C.L.
See here for all cross section summary plots
Inner colored bars statistical uncertainty, outer narrow bars statistical+systematic uncertainty
Light to Dark colored bars: 2.76, 5.02, 7, 8, 13, 13.6 TeV, Black bars: theory prediction

EW tests over the years

- LEP: e^+e^- collider, in operation 1989-2000
  - 4 experiments (ALEPH, DELPHI, L3, OPAL)
  - $\sqrt{s}$ from 90 GeV to 209 GeV
  - Two phases
    - LEP1: Z physics
    - LEP2: *W* physics, reaching the *WW* threshold and above
  - largest and most powerful e^+e^- collider to date
- SLC, e^+e^- collider, in operation 1989-1998
  - $\sqrt{s}$ ~ 90 GeV , polarised beams
- $Sp\bar{p}S$, in operation 1981-1990
  - $\sqrt{s}$ =540, 630 GeV
  - discovery of W and Z bosons (UA1 &UA2)
- Tevatron, $p\bar{p}$ collisions (1987-2011)
  - $\sqrt{s}$ =1.8, 1.96 TeV
  - top quark discovery, mass measurement
  - M_W measurements
- LHC, pp collisions (2008 -)
  - Higgs boson discovery
  - …





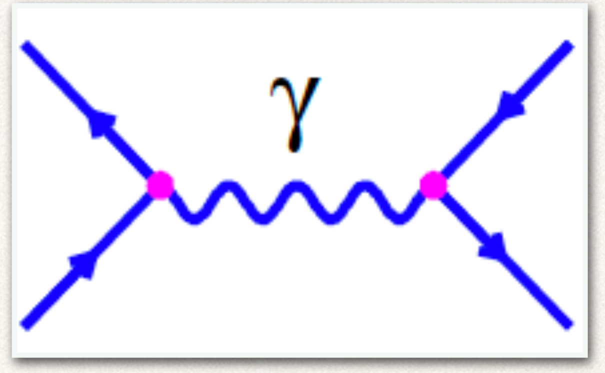
1995, CDF and DØ experiments, Fermilab

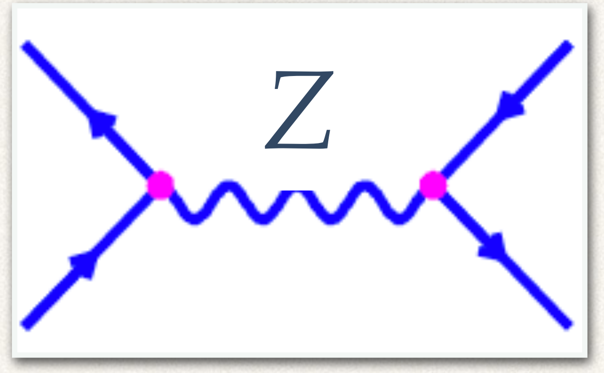
Testing the SM

- What can be tested?
  - Theoretical predictions for observables: cross sections, differential distributions, decay widths,,…
  - "Properties" of the theory
    - built-in assumptions, e.g. number of generations
    - existence of the particles appearing in the model and their properties
      → most famous: Higgs boson but also top quark, τ neutrino, and even earlier W, Z bosons or b and c quarks,..
    - existence of the interactions predicted by the model and their properties
      → e.g. Yukawa couplings, gauge boson interactions, gauge-fermion interactions
    - running of the coupling
    - …
  - Overall consistency → global precision fits

$e^+e^- \to f\bar{f}$

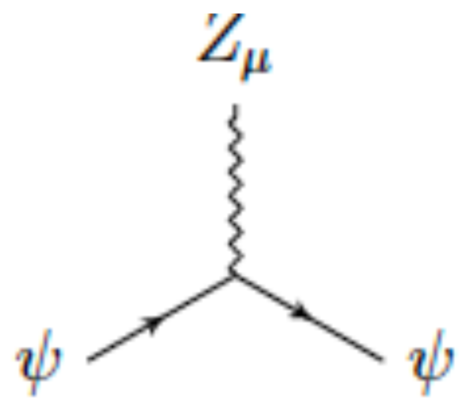
- $e^+e^- \to f\bar{f}$ cross section





at the Z resonance

$$\sigma = \frac{4\pi\alpha}{3s}\frac{1}{16 sin^4\theta_W \cos^4\theta_W}(c_V^{e2} + c_A^{e2})(c_V^{f2} + c_A^{f2})\frac{s}{(s-M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$



$$-\frac{ig}{2\cos\theta_W}\gamma^\mu(c_V^l - c_A^l\gamma_5)$$

$$c_V^f = T_3 - 2e_f \sin^2\theta_W$$
$$c_A^f = T_3$$

- Partial width

$$\Gamma_f = \frac{\alpha M_Z}{12\sin^2\theta_W\cos^2\theta_W}(c_V^{f2} + c_A^{f2})$$

$$\left(\Gamma_Z = \sum_f \Gamma_f\right)$$

$$\Rightarrow \qquad \sigma = 12\pi\frac{\Gamma_e\Gamma_f}{M_Z^2}\frac{s}{(s-M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$

and at $\sqrt{s} = M_Z$

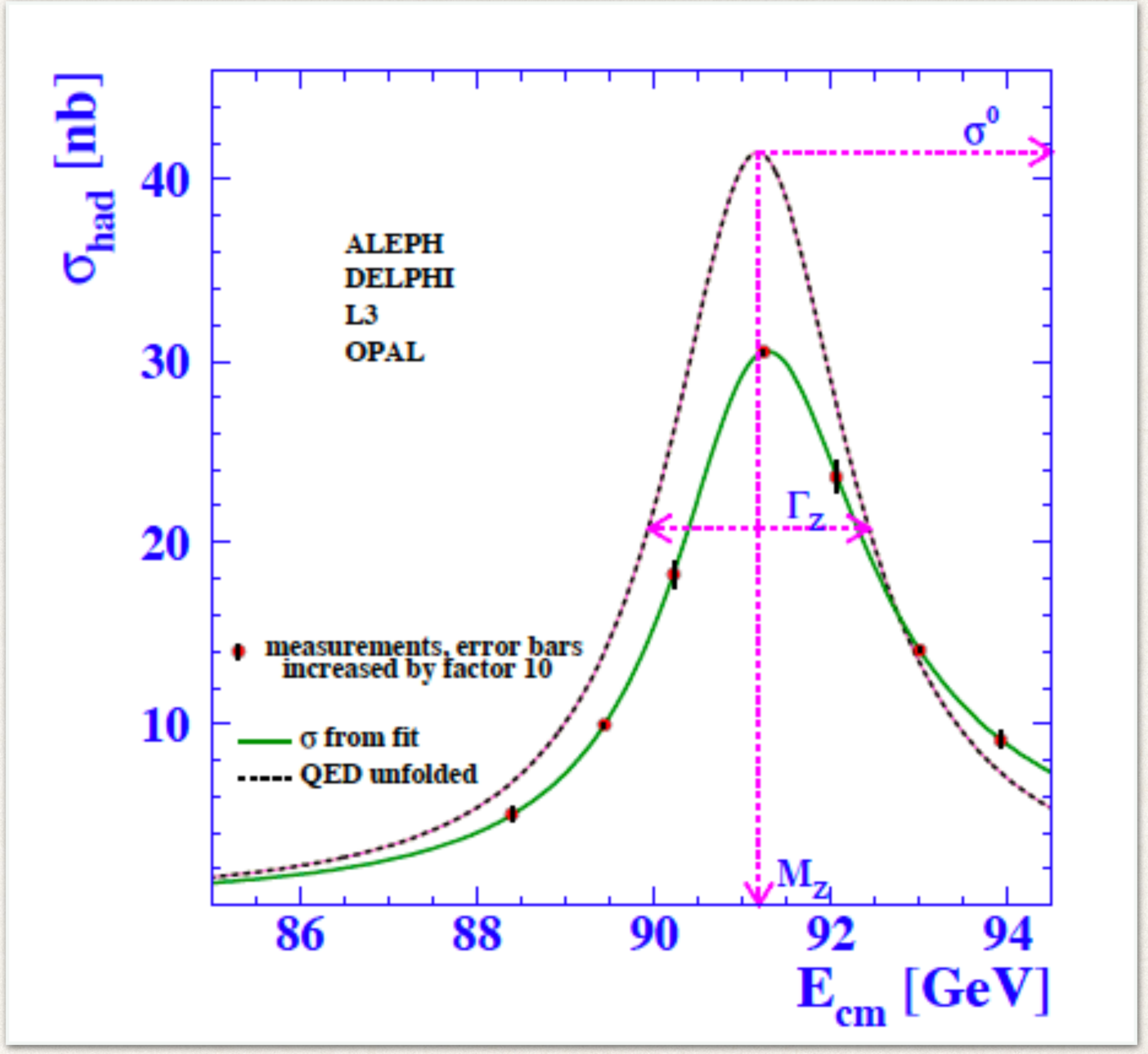
$$\sigma = \frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}$$

Z line shape

- Z resonance curve

$$\sigma = 12\pi \frac{\Gamma_e \Gamma_f}{M_Z^2} \frac{s}{(s - M_Z^2)^2 + (s\Gamma_Z/M_Z)^2} \text{ with peak at } \sigma_{\text{peak}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2}$$

  - Resonance location gives M_Z, width of the curve gives Γ_Z, height σ_{peak} gives $\Gamma_e \Gamma_f$



The formula receives QED (and QCD if hadrons in the final state are considered) corrections, most importantly from the QED initial state radiation

m_Z [GeV]	91.1891 ± 0.0031
Γ_Z [GeV]	2.4959 ± 0.0043
σ_h^0 [nb]	41.558 ± 0.057
R_e	20.690 ± 0.075
R_μ	20.801 ± 0.056
R_τ	20.708 ± 0.062

$R_l = \frac{\Gamma_{had}}{\Gamma_l}$

as measured by ALEPH

Test of lepton couplings' universality

- Determination of c_V^f, c_A^f from forward-backward asymmetry (→ differential distributions)

Number of light neutrino generations

- The relation $\Gamma_Z = \Gamma_{had} + 3\,\Gamma_l + \Gamma_{inv}$

  allows to determine the invisible partial width

  $$\Gamma_{inv} = N_\nu \Gamma_\nu \qquad \Gamma_\nu = \Gamma(Z \to \nu_i \bar{\nu}_i)$$

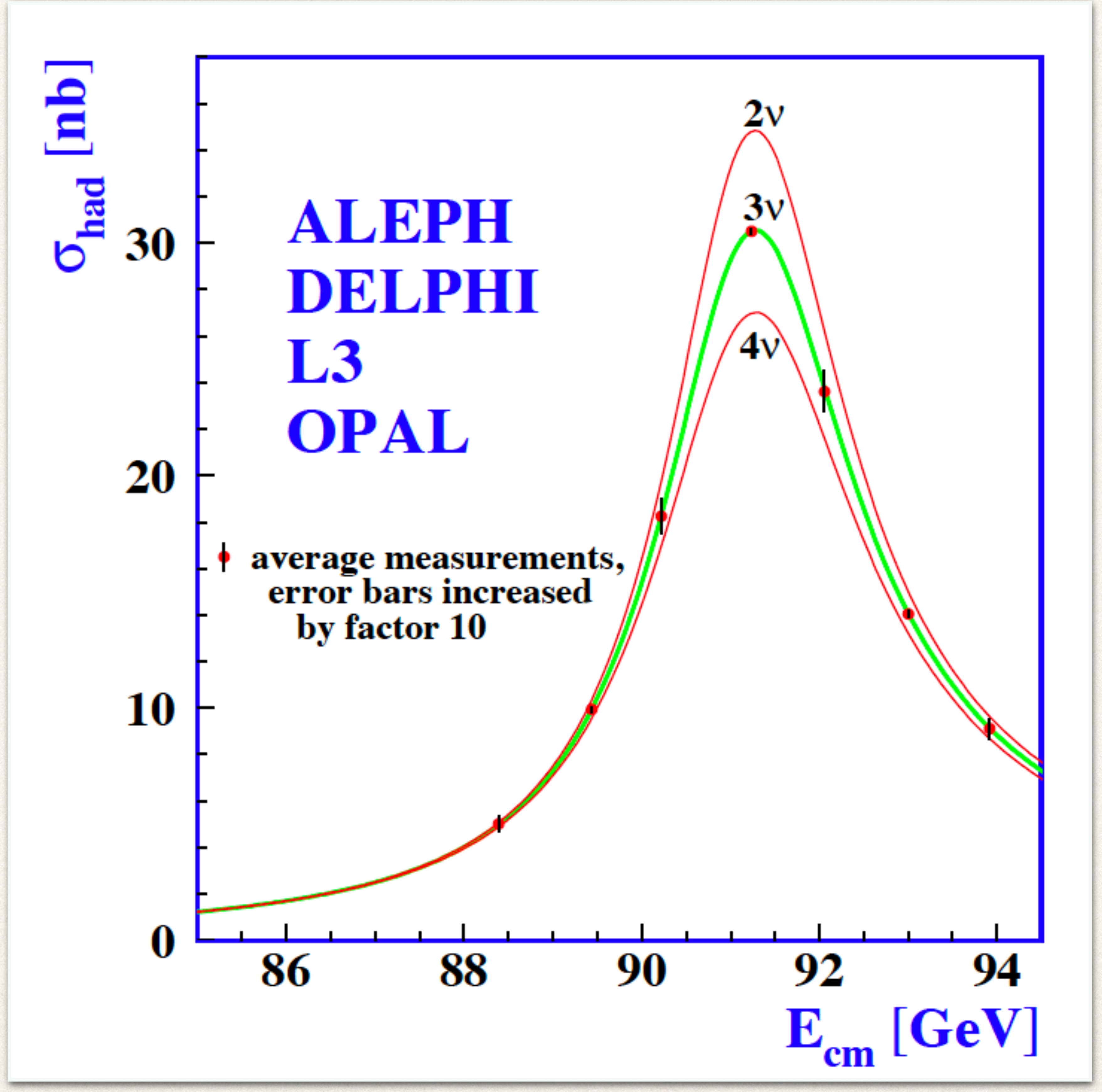
- The number of neutrinos

  $$N_\nu = \frac{\Gamma_{inv}}{\Gamma_\nu} = \left(\frac{\Gamma_{inv}}{\Gamma_l}\right)\left(\frac{\Gamma_l}{\Gamma_\nu}\right)$$

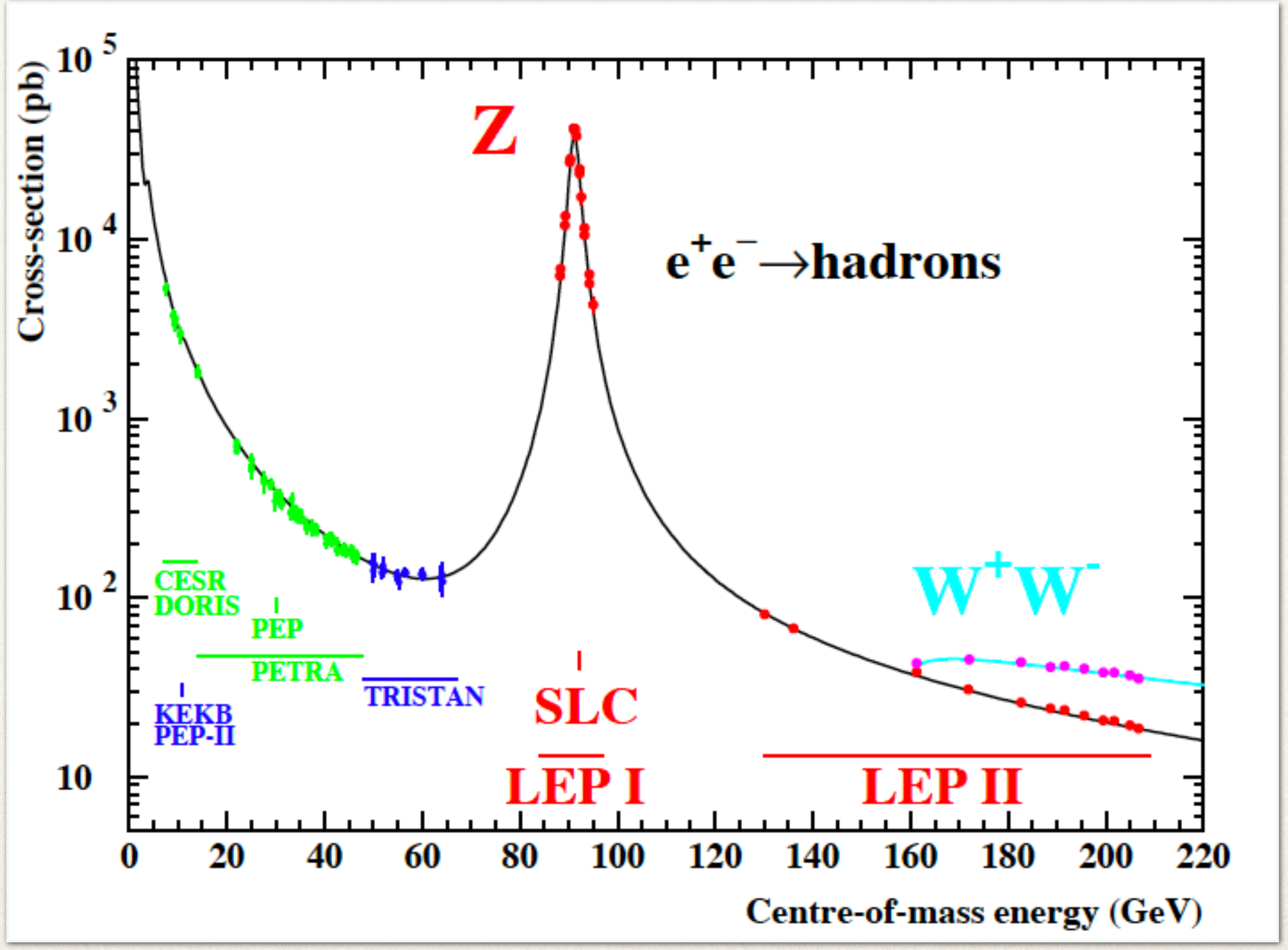
  ↑ from exp. ↑ from SM

- Combined result from the four LEP experiments

  $$N_\nu = 2.984 \pm 0.008$$

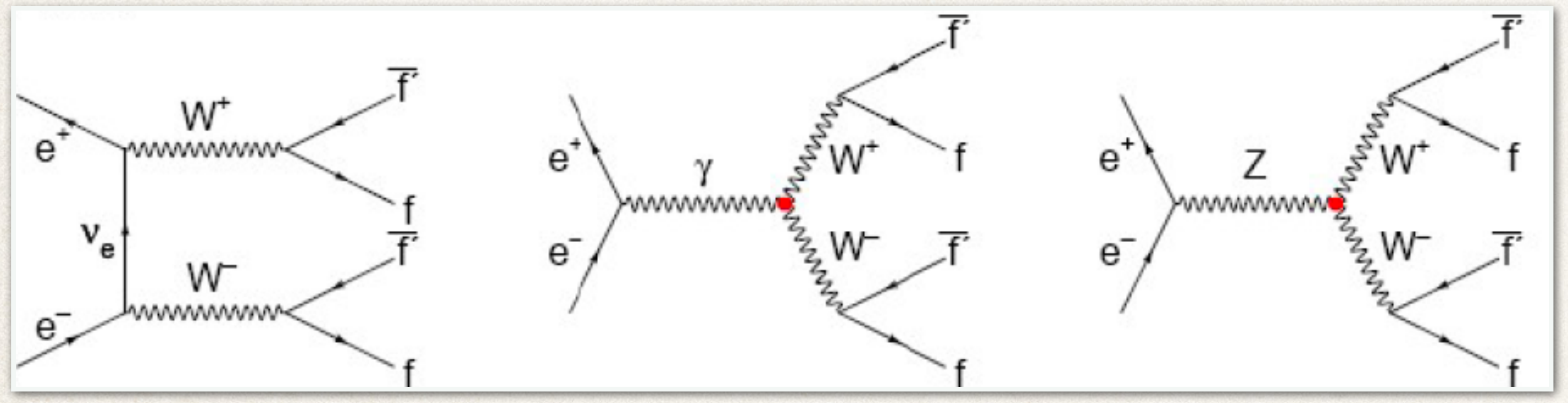


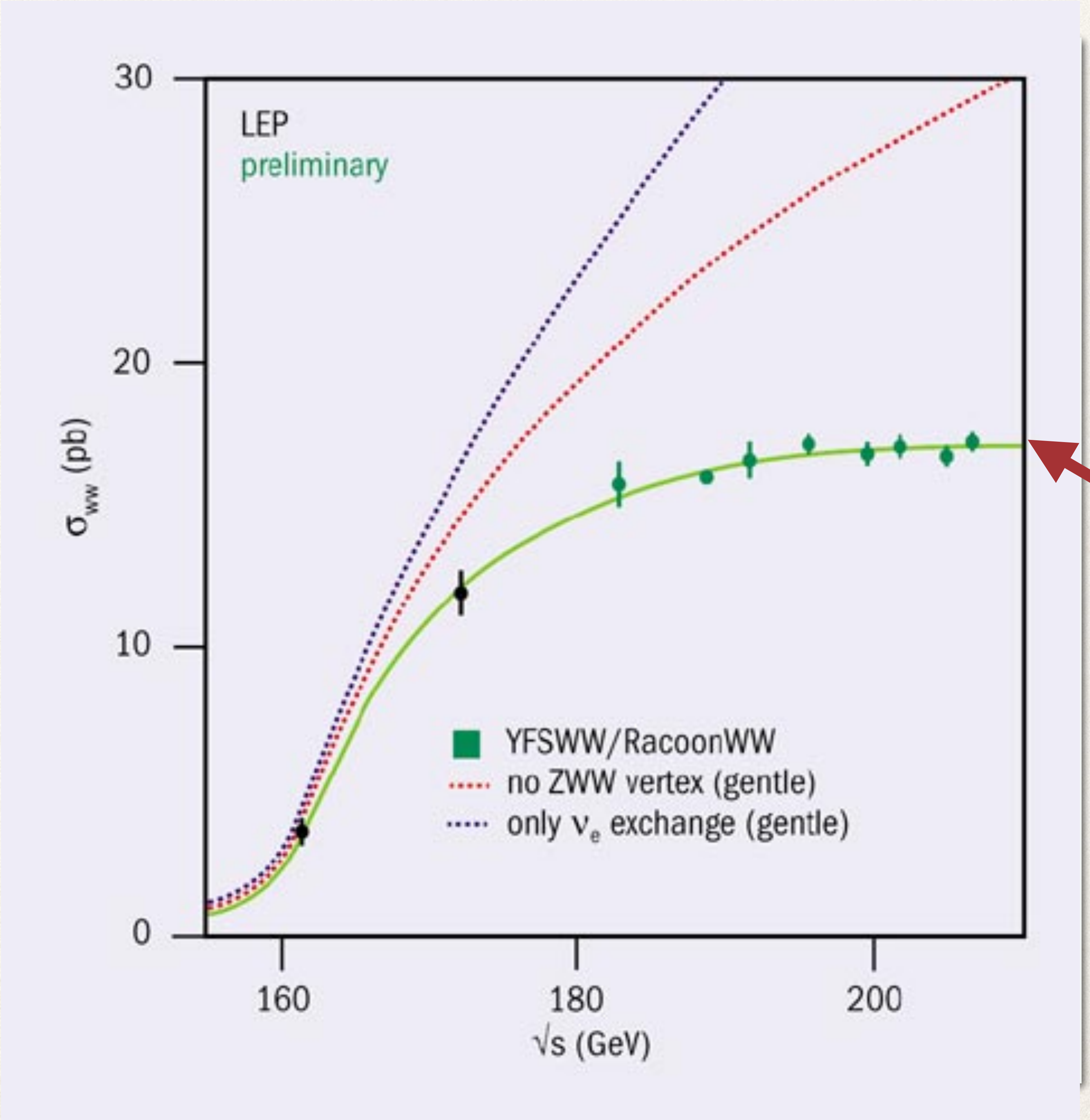
Away from the Z pole

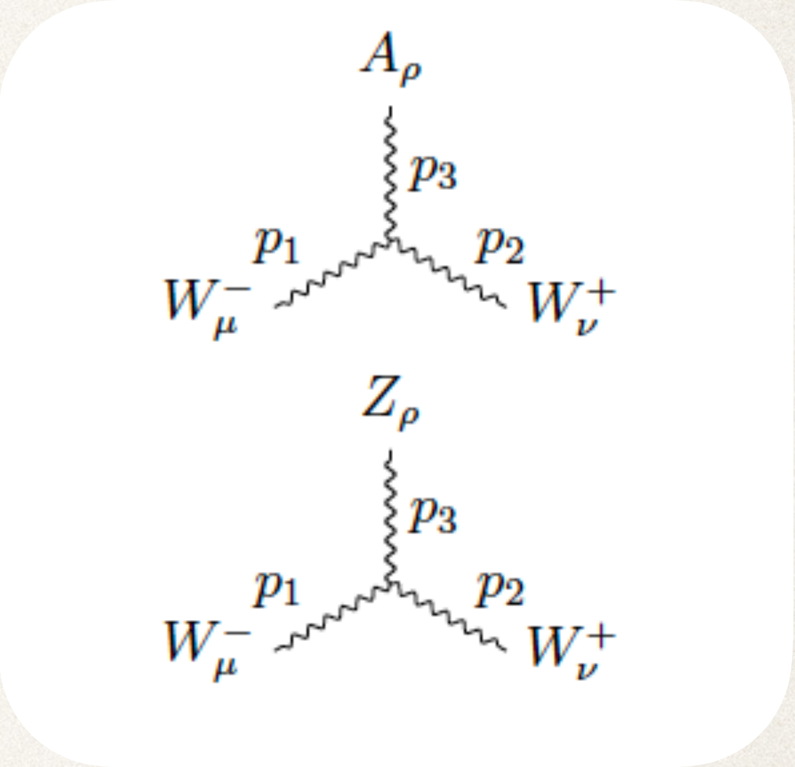


WW threshold

- Measurements of $e^+e^- \to W^+W^- \to 4$ fermions allow to
  - probe gauge boson interactions
  - determine M_W from the dependence of the WW production threshold behaviour on M_W)







0.5 % precision
of theoretical predictions

Global fits

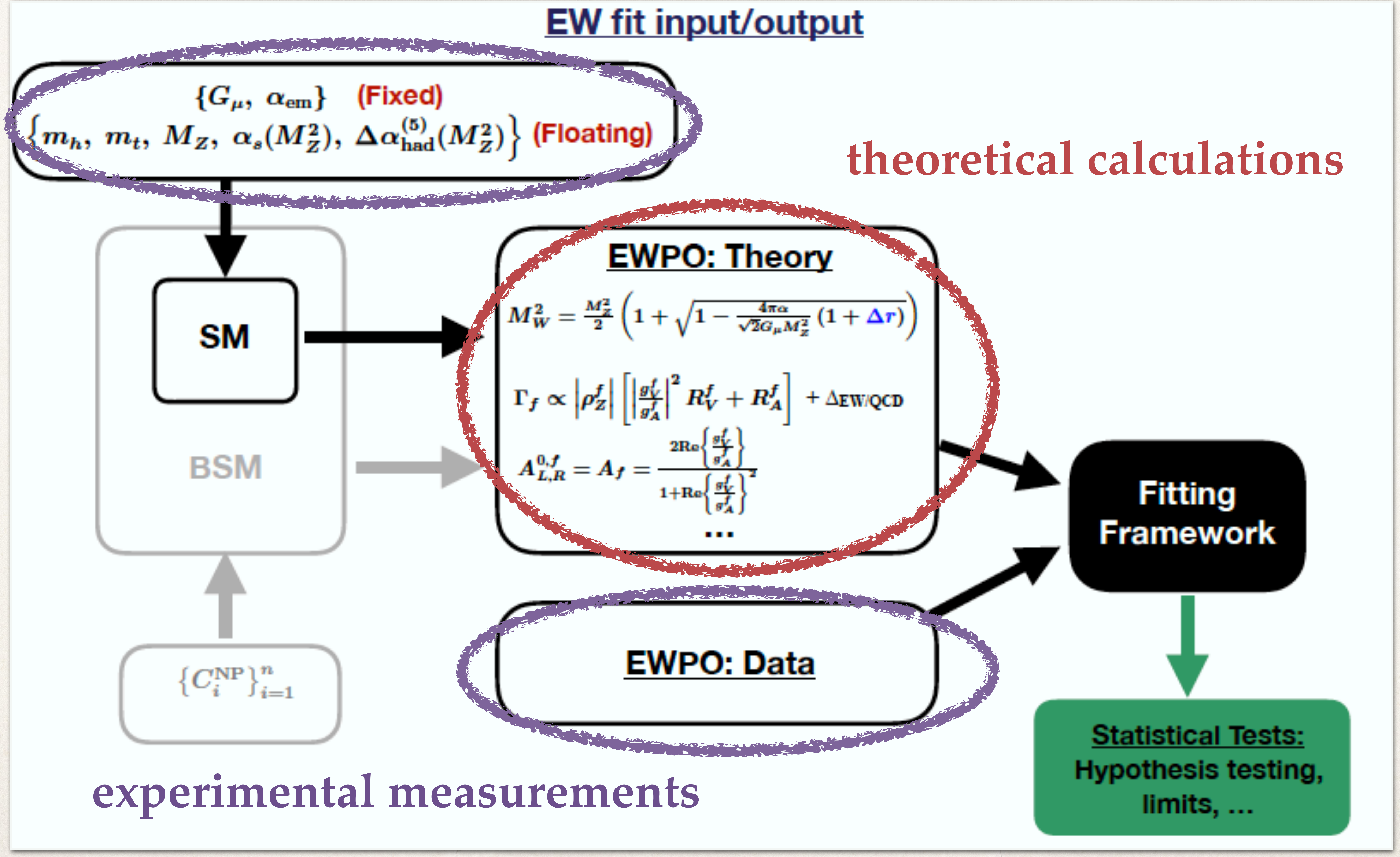
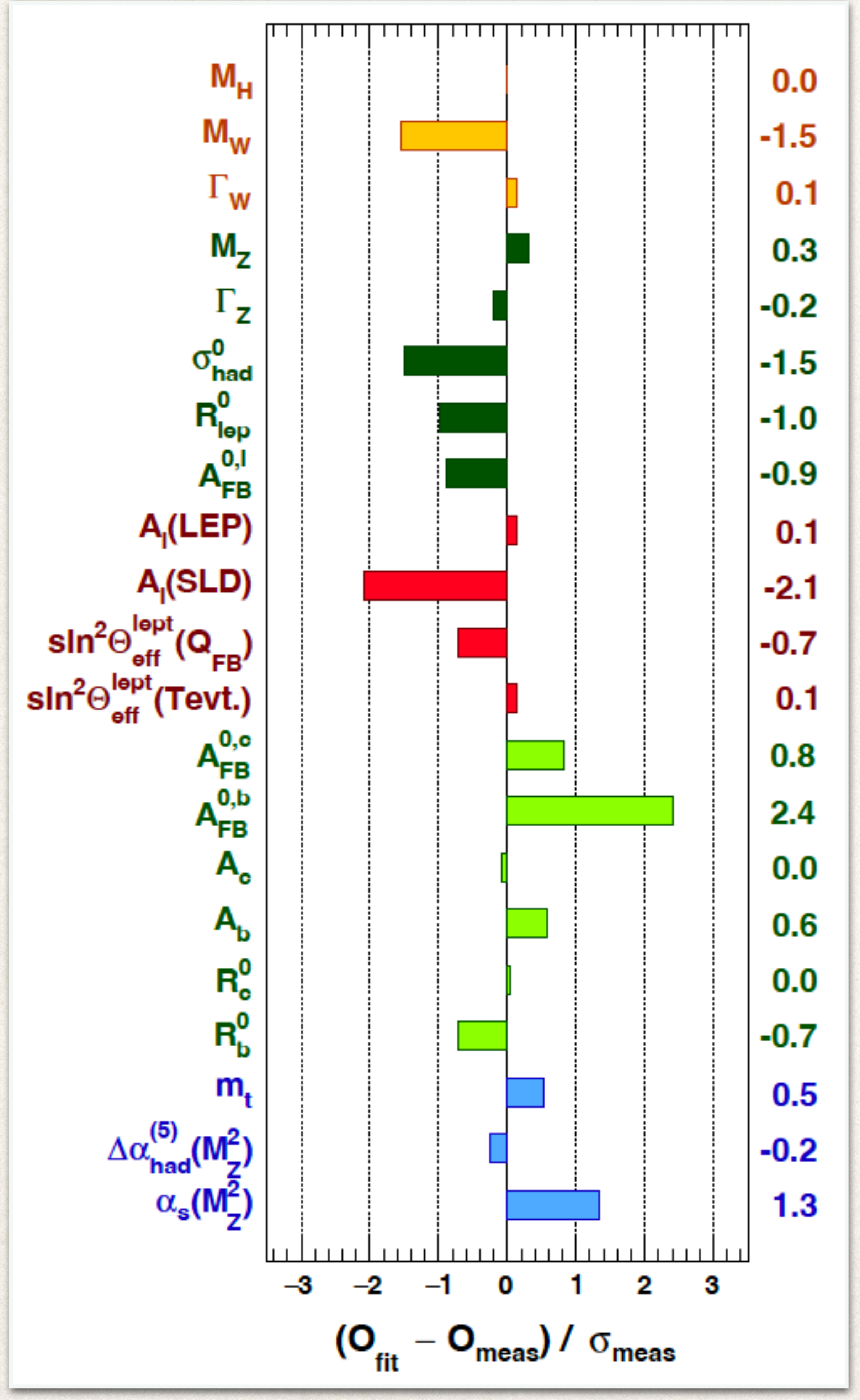
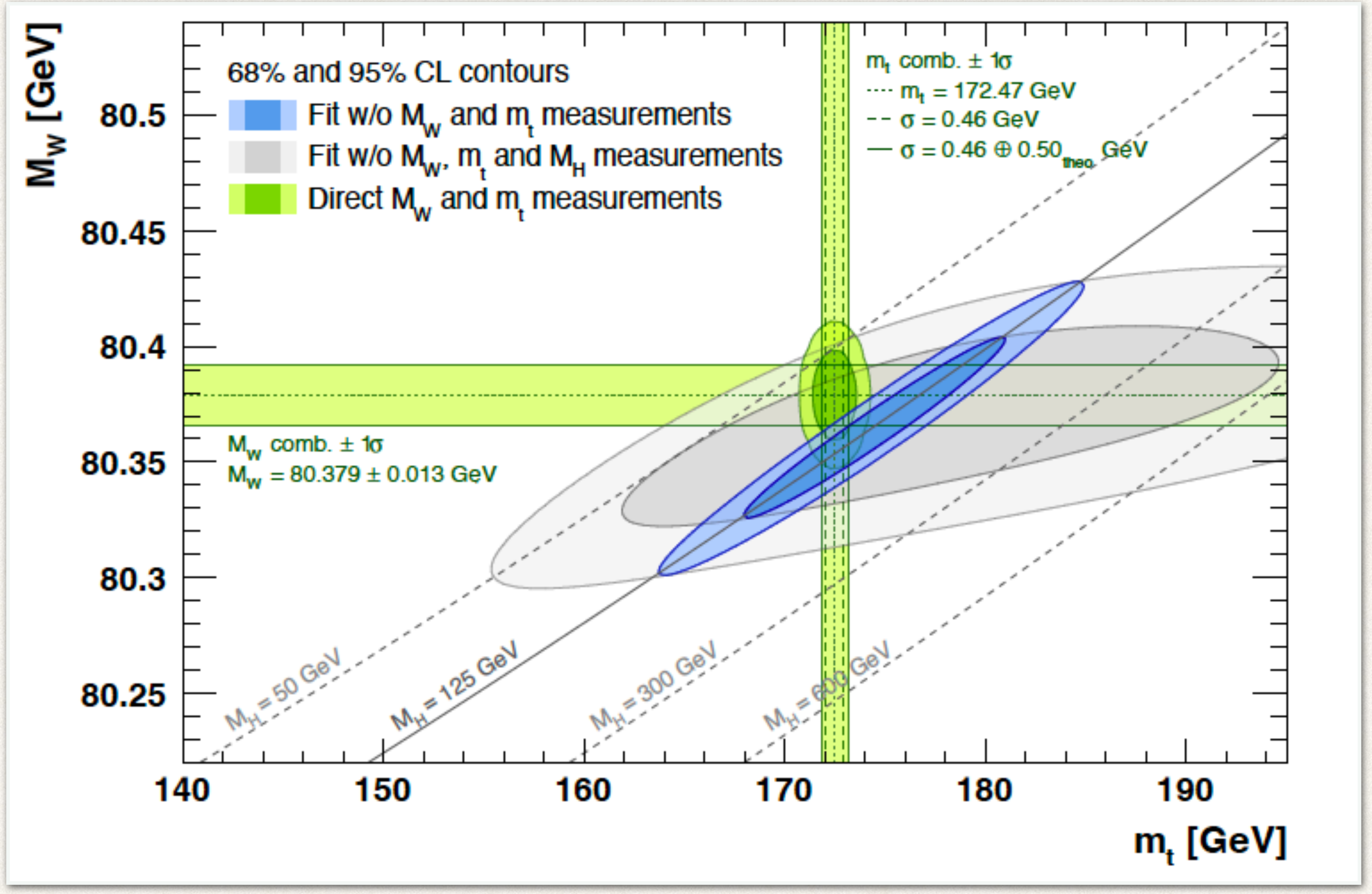


image credit: J. de Blas

Global consistency of the SM





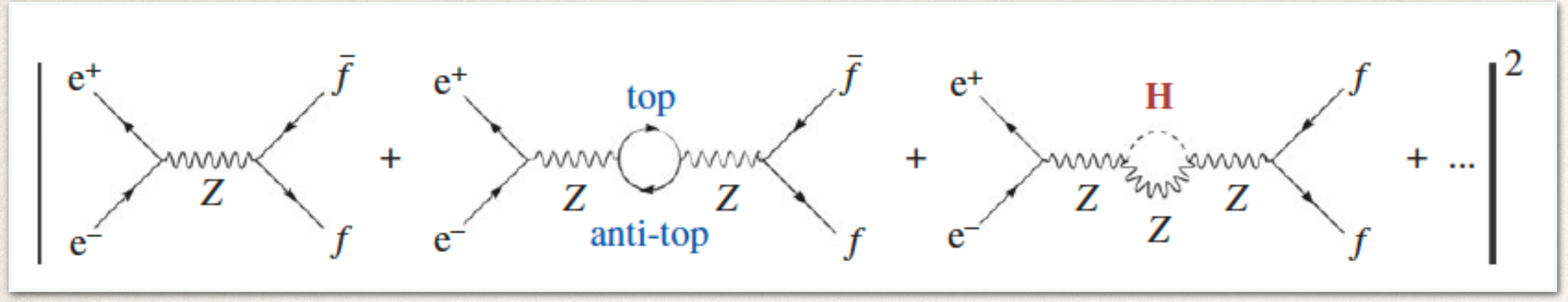
1803.01853

- Input data from many experiments: at LEP, SLC (e^+e^-), Tevatron ($p\bar{p}$), LHC (pp), …
- Global fits can be similarly used to put constraints on BSM models /SMEFT parameters

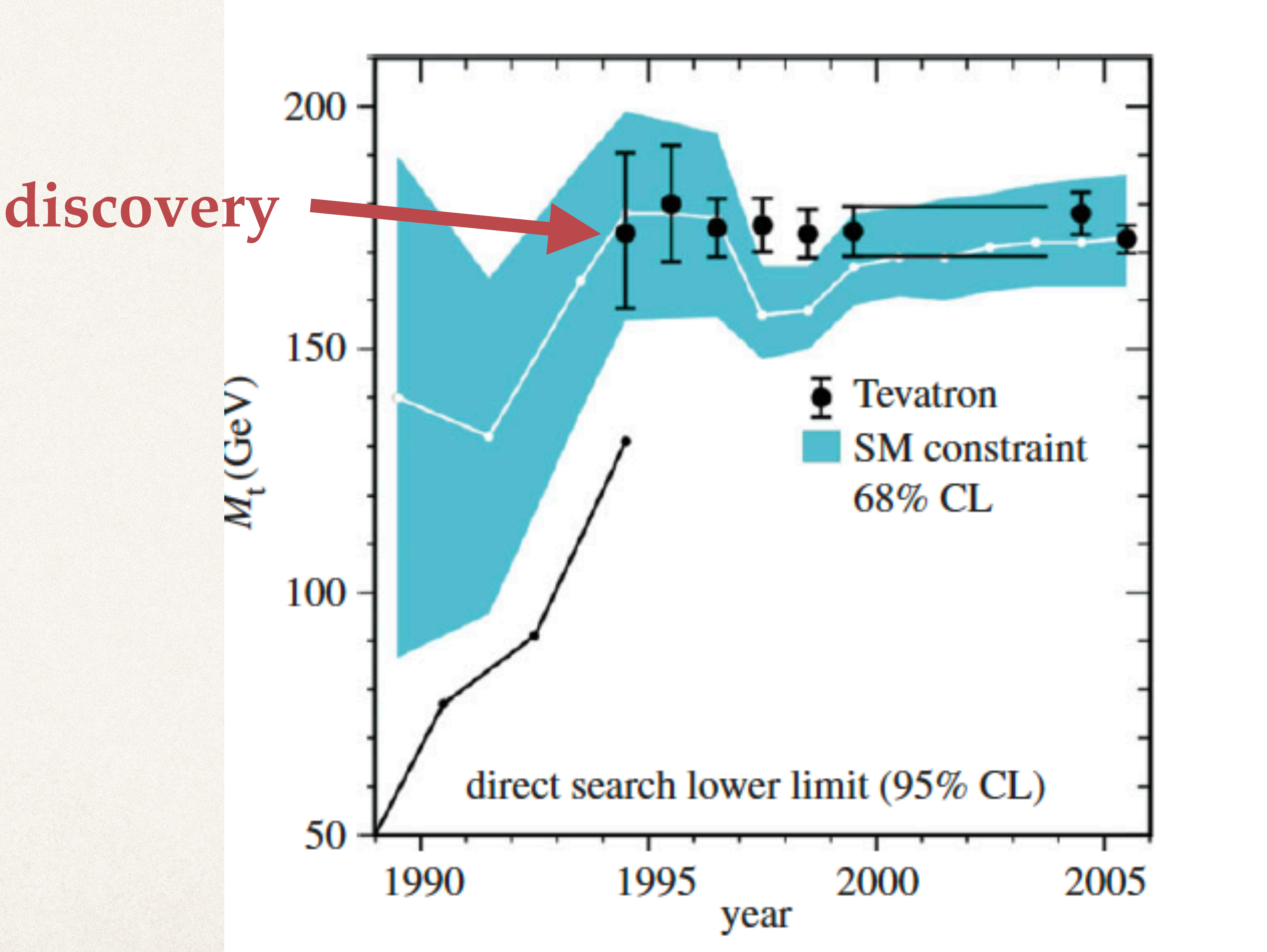
→ see lectures by J. Guimaraes da Costa

Predictions from global fits

- First direct indication of the predictive power of global fits: top mass prediction on basis of measurements at LEP and SLC



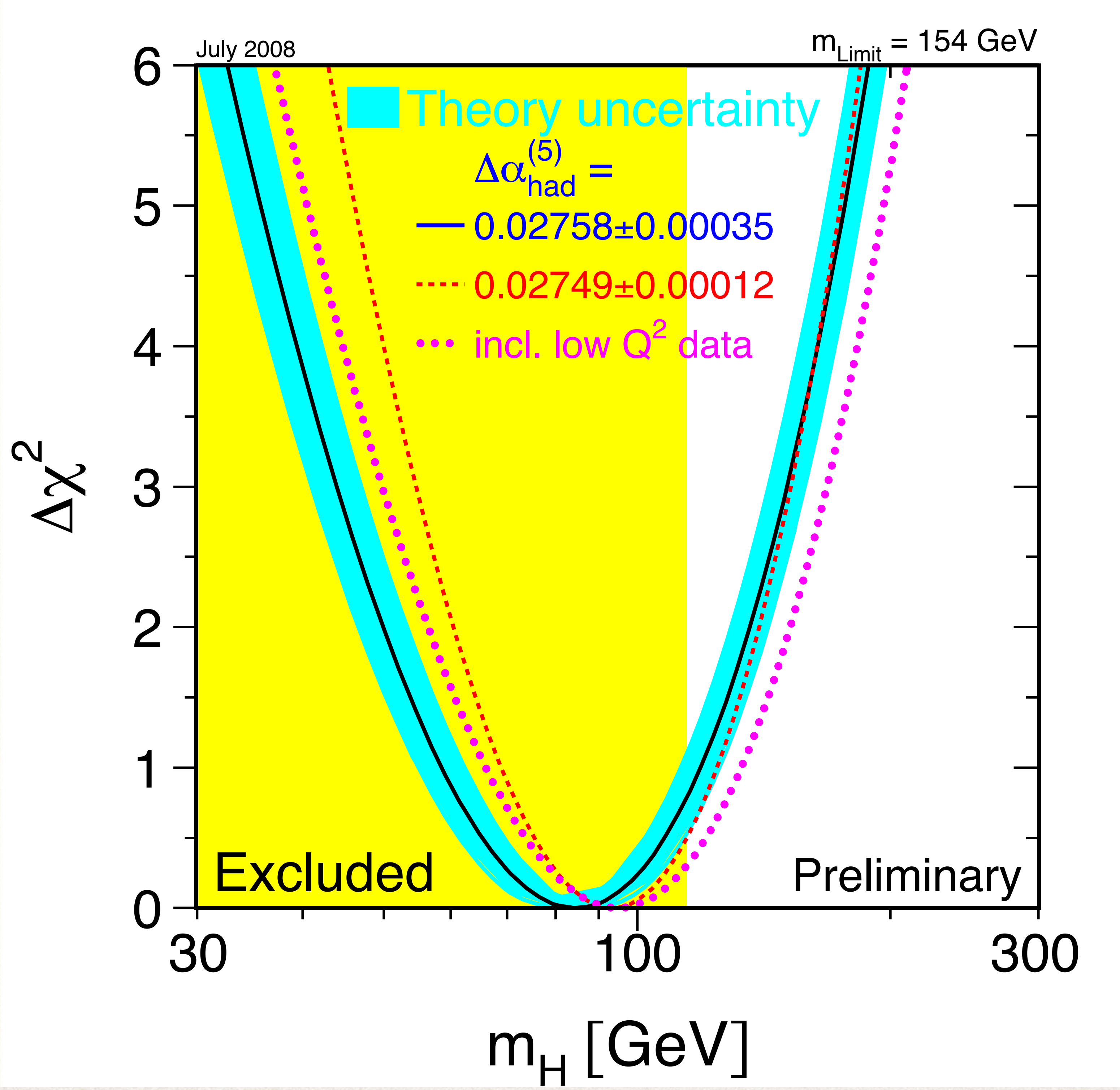
- Good agreement between the indirect prediction of m_t and the directly measured value confirms the validity of SM radiative corrections



Indirect Higgs searches

- With the top mass measured, one could make prediction for the Higgs mass → the famous "blue band" plot

  Just before turning on the LHC…



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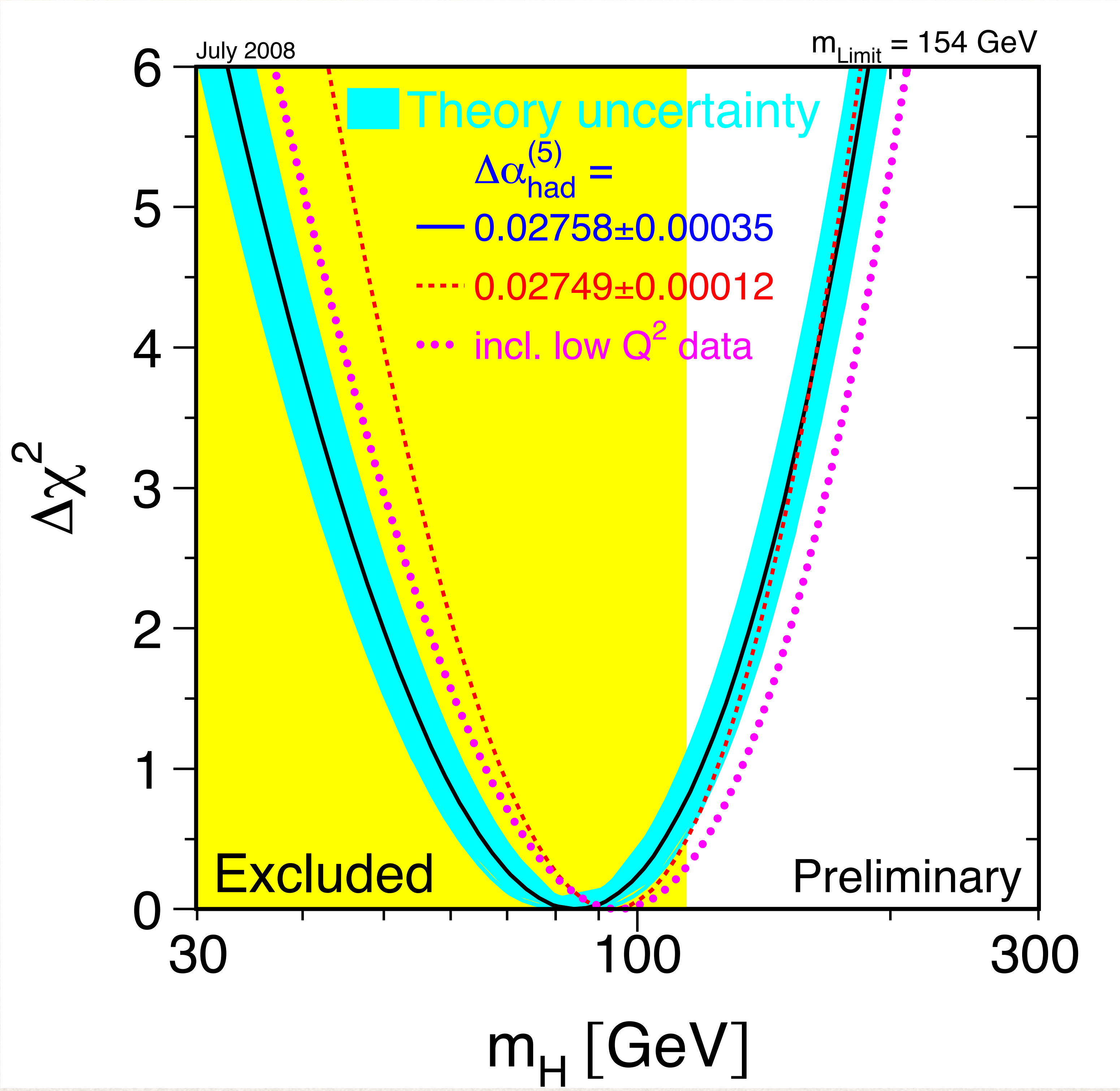

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No sign of the Higgs boson

5 December 2001

NewScientist HALF PRICE

The legendary particle that physicists thought explained why matter has mass probably does not exist. So say researchers who have spent a year analysing data from the LEP accelerator at the CERN nuclear physics lab near Geneva.

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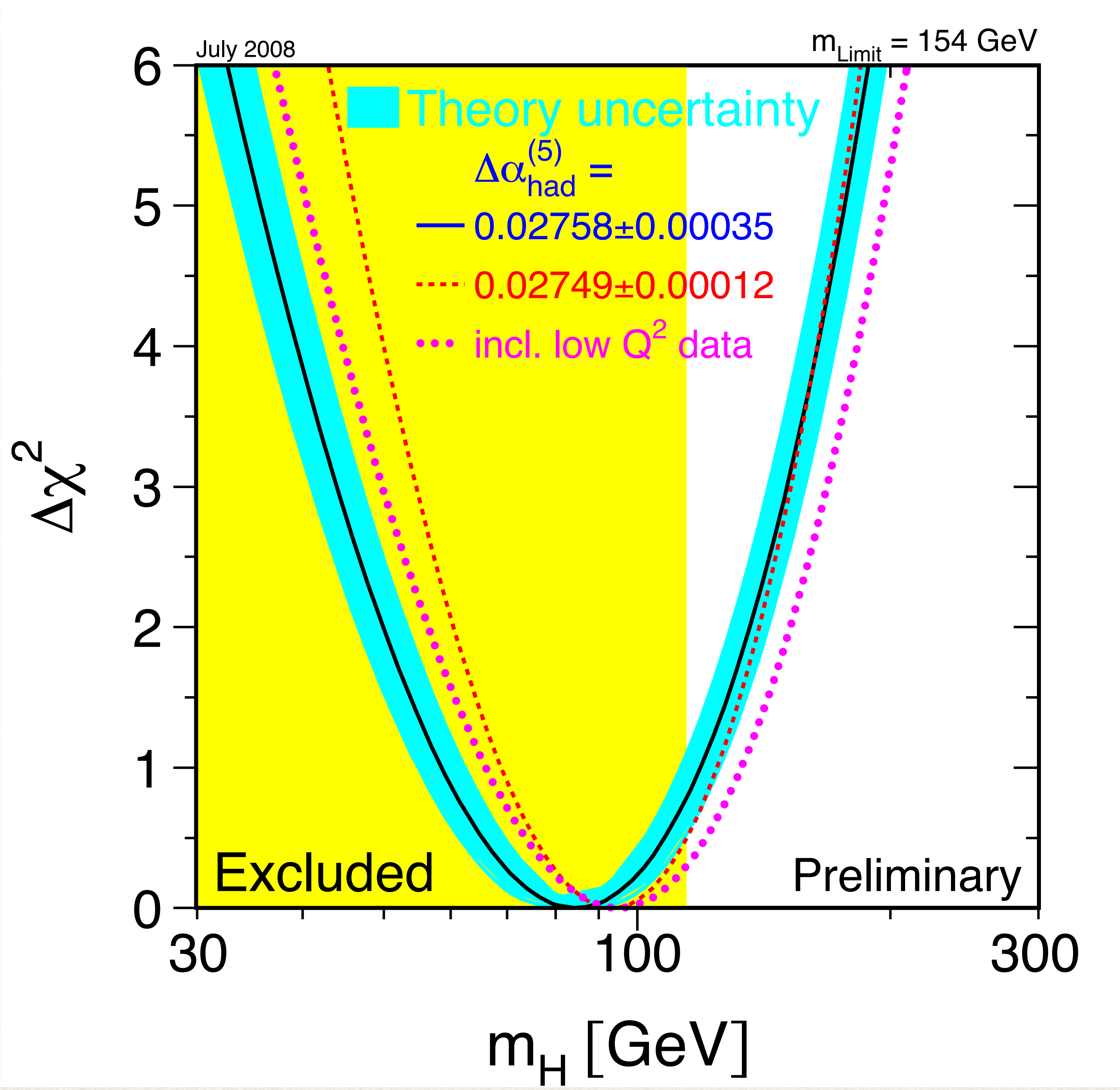

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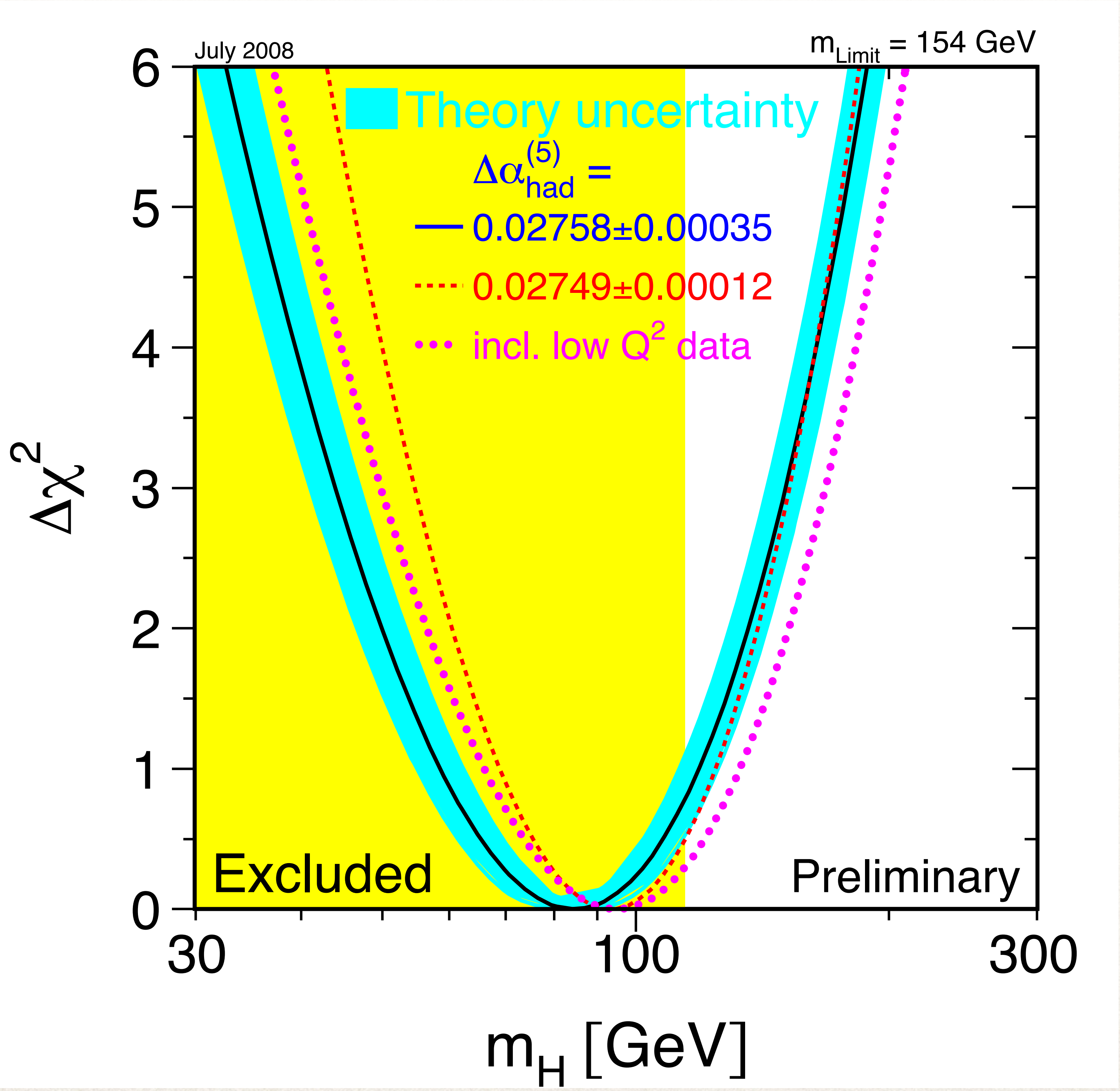
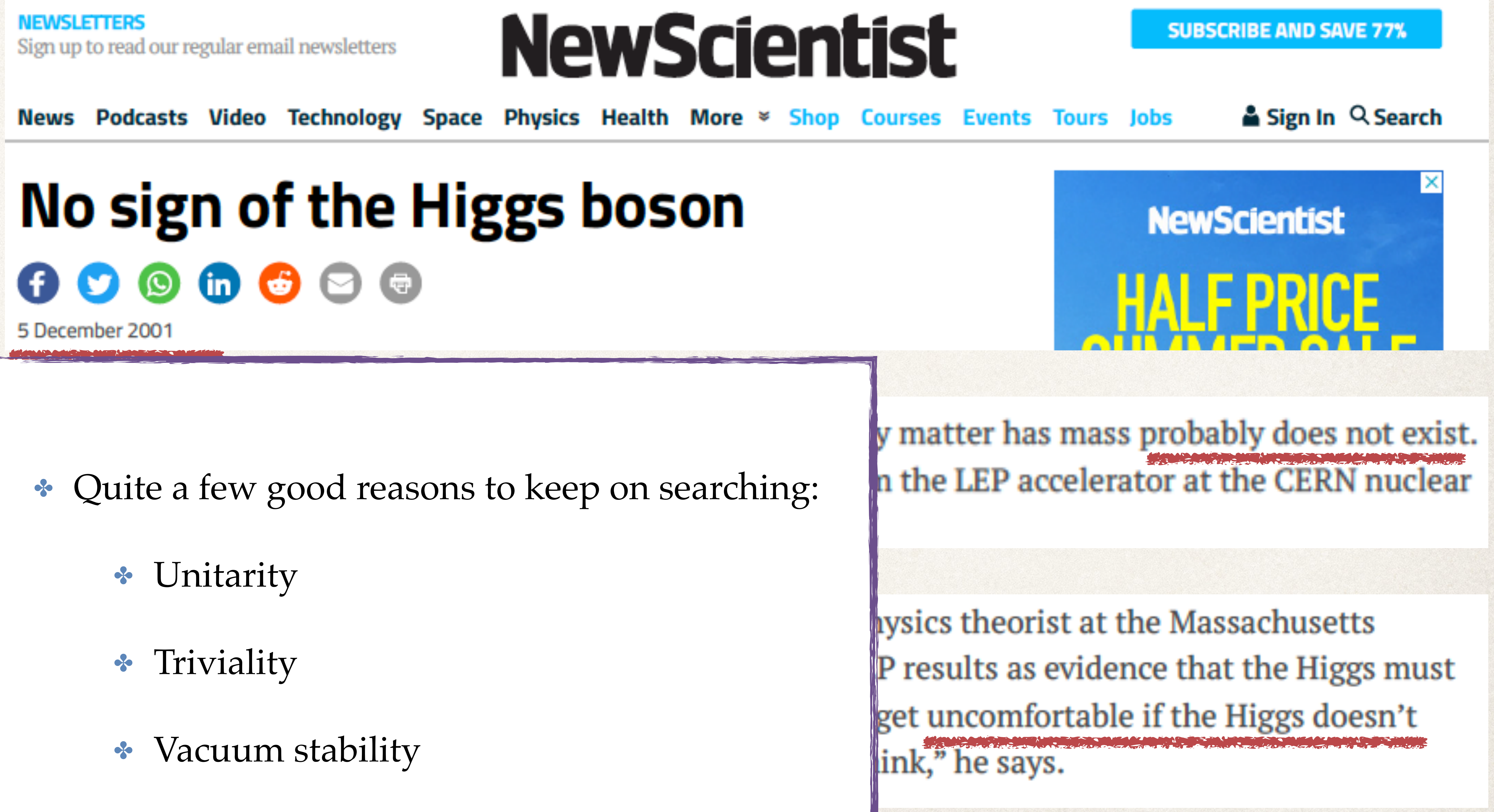
Not everyone is too bothered, yet. Frank Wilczek, a particle physics theorist at the Massachusetts Institute of Technology, points out that you could take the LEP results as evidence that the Higgs must be sitting at an improbably high energy. He says he’ll start to get uncomfortable if the Higgs doesn’t show up by about 130 GeV. “Then I would have a good long think,” he says.

Indirect Higgs searches

- With the top mass measured, one could make prediction for the Higgs mass → the famous "blue band" plot

Just before turning on the LHC…

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y matter has mass probably does not exist.
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P results as evidence that the Higgs must
get uncomfortable if the Higgs doesn't
ink," he says.

- Quite a few good reasons to keep on searching:
  - Unitarity
  - Triviality
  - Vacuum stability

Unitarity bound

- Consider $2 \to 2$ elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} \left(2l+1\right) P_l(\cos\theta) a_l$$

- a_i are the spin l partial wave amplitudes

  $P_l(\cos\theta)$ are Legendre polynomials: $\int_{-1}^{1} dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$

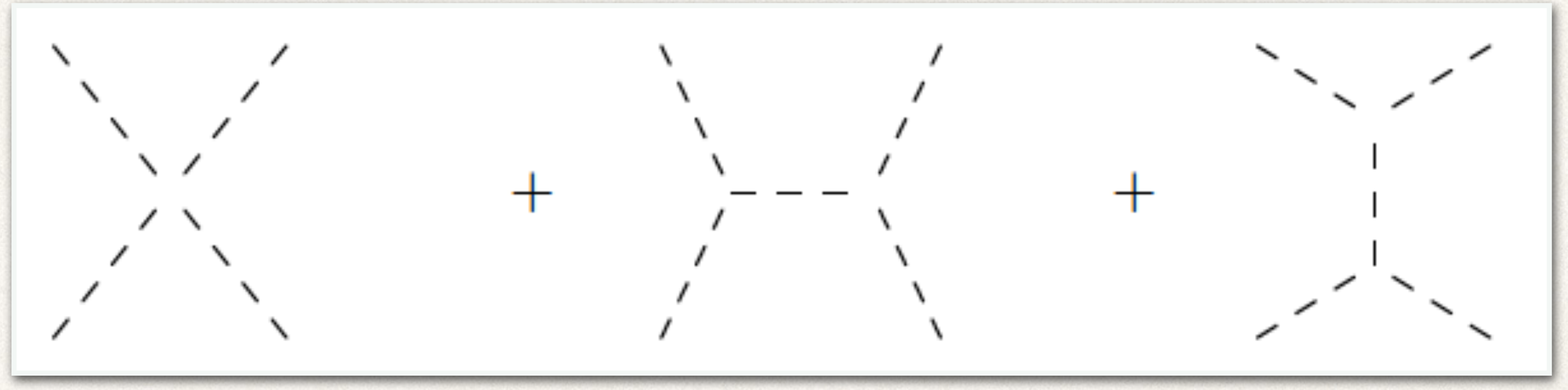
$$\Rightarrow \sigma = \frac{8\pi}{s} \sum_{l=0}^{\infty} \left(2l+1\right) \sum_{l'=0}^{\infty} \left(2l'+1\right) a_l a_{l'}^* \int_{-1}^{1} d\cos\theta P_l(\cos\theta) P_{l'}(\cos\theta) = \frac{16\pi}{s} \sum_{l=0}^{\infty} \left(2l+1\right) |a_l|^2$$

Direct consequence of the unitarity of the S matrix: the optical theorem

$\sigma = \frac{1}{s} \mathrm{Im}\left(A(\theta=0)\right)$ so for each l $\mathrm{Im}\left(a_l\right) = |a_l|^2$ and $\left|\mathrm{Re}\left(a_l\right)\right| \leq \frac{1}{2}$

Unitarity bound (2)

- Highly relativistic (boosted in the z-direction) gauge bosons dominated by their longitudinal polarisation
- Longitudinal degree of freedom ↔ Goldstone mode
- Equivalence theorem: at very high energies, $s \gg M_W^2$, the scattering amplitude for the longitudinally polarised W bosons can be approximated by the scattering amplitude for the Goldstone bosons, up to $\mathcal{O}(M_W^2/E^2)$ corrections
- Consider $A(W_L^+ W_L^- \to W_L^+ W_L^-) \sim A(\rho^+\rho^- \to \rho^+\rho^-)$



$$A = \frac{M_H^2}{v^2}\left[2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2}\right] \quad \text{and it follows} \quad a_0 = \frac{M_H^2}{16\pi v^2}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right] \xrightarrow{s \gg M_H^2} \frac{M_H^2}{8\pi v^2}$$

Hence for $s \gg M_H^2$ from $|\mathrm{Re}\, a_0| < \frac{1}{2}$ it follows $M_H < 2\sqrt{\pi} v = 870$ GeV

Triviality bound

- Running of the Higgs self coupling

$$\frac{d\lambda}{d\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + \left(g_2^2 + g_1^2\right)^2\right)\right]$$

- In the limit of the strong coupling λ

$$\frac{d\lambda}{d\log Q^2} = \frac{3}{4\pi^2}\lambda^2$$

with a solution

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2}\lambda(v)\log\left(\frac{Q^2}{v^2}\right)}$$

(Landau) pole at $1 = \frac{3}{4\pi^2}\lambda(v)\log\left(\frac{Q^2}{v^2}\right)$

Requesting finite λ at a given $Q = \Lambda$, i.e. $\lambda(\Lambda)^{-1} > 0$ leads to scale-dependent condition $M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}$

Vacuum stability bound

$$\frac{d\lambda}{d\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + \left(g_2^2 + g_1^2\right)^2\right)\right]$$

- In the limit of a weak Higgs self-coupling at the EW scale, the negative terms in the equation above can in principle lead to a negative value of the coupling → unstable Higgs potential
- Considering only the dominant term with the top Yukawa coupling y_t

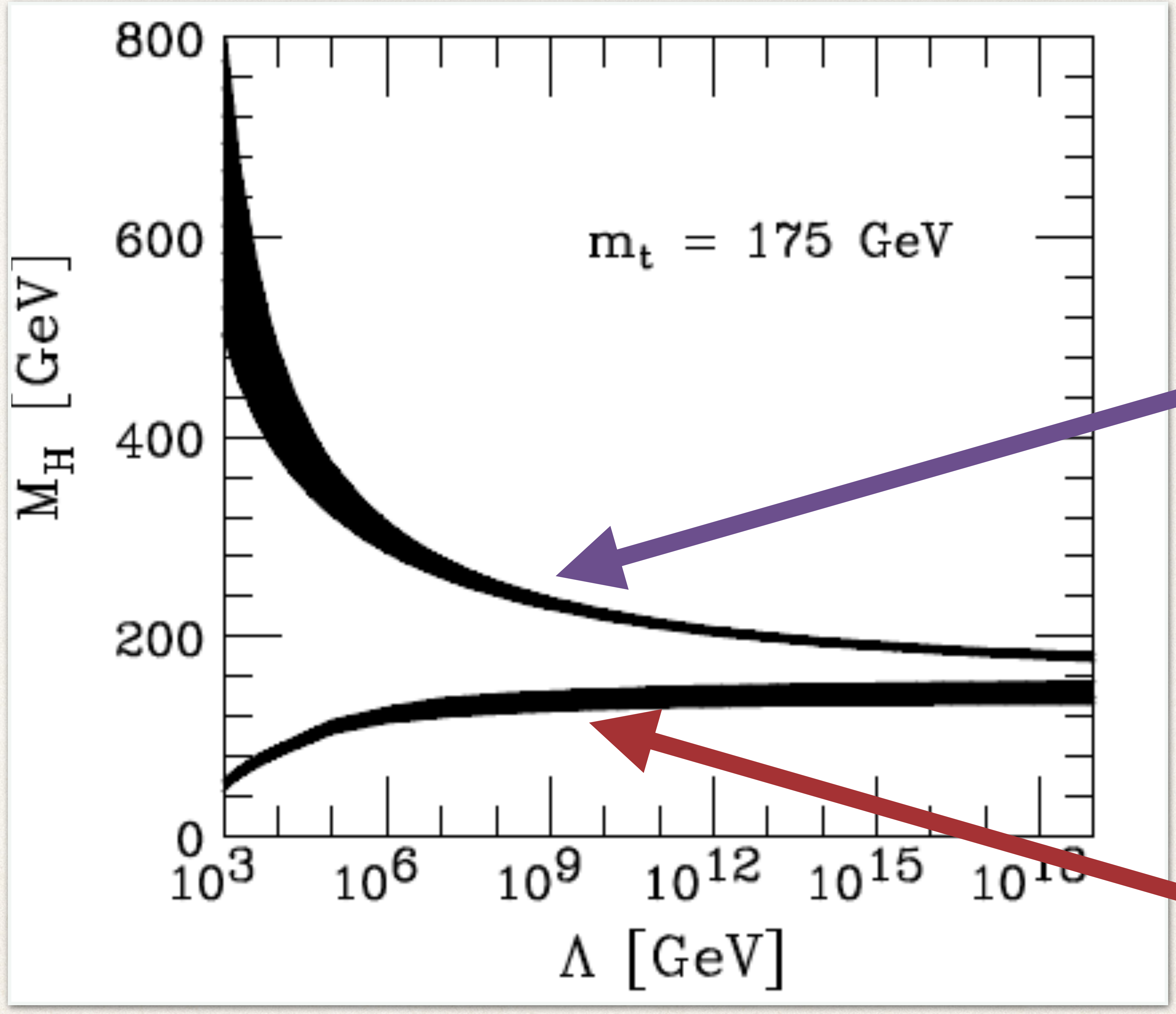
$$\frac{d\lambda}{d\log Q^2} = -\frac{3m_t^4}{4\pi^2 v^4}$$

with the solution $\lambda(Q^2) = \lambda(v^2) - \frac{3m_t^4}{4\pi^2 v^4}\log\left(\frac{Q^2}{v^2}\right)$

- Imposing $\lambda(\Lambda) > 0$ leads to $M_H^2 > \frac{3m_t^4}{2\pi^2 v^2}\log\left(\frac{\Lambda^2}{v^2}\right)$

Constraints on M_H in the SM

➜ see also lectures by J. Ellis

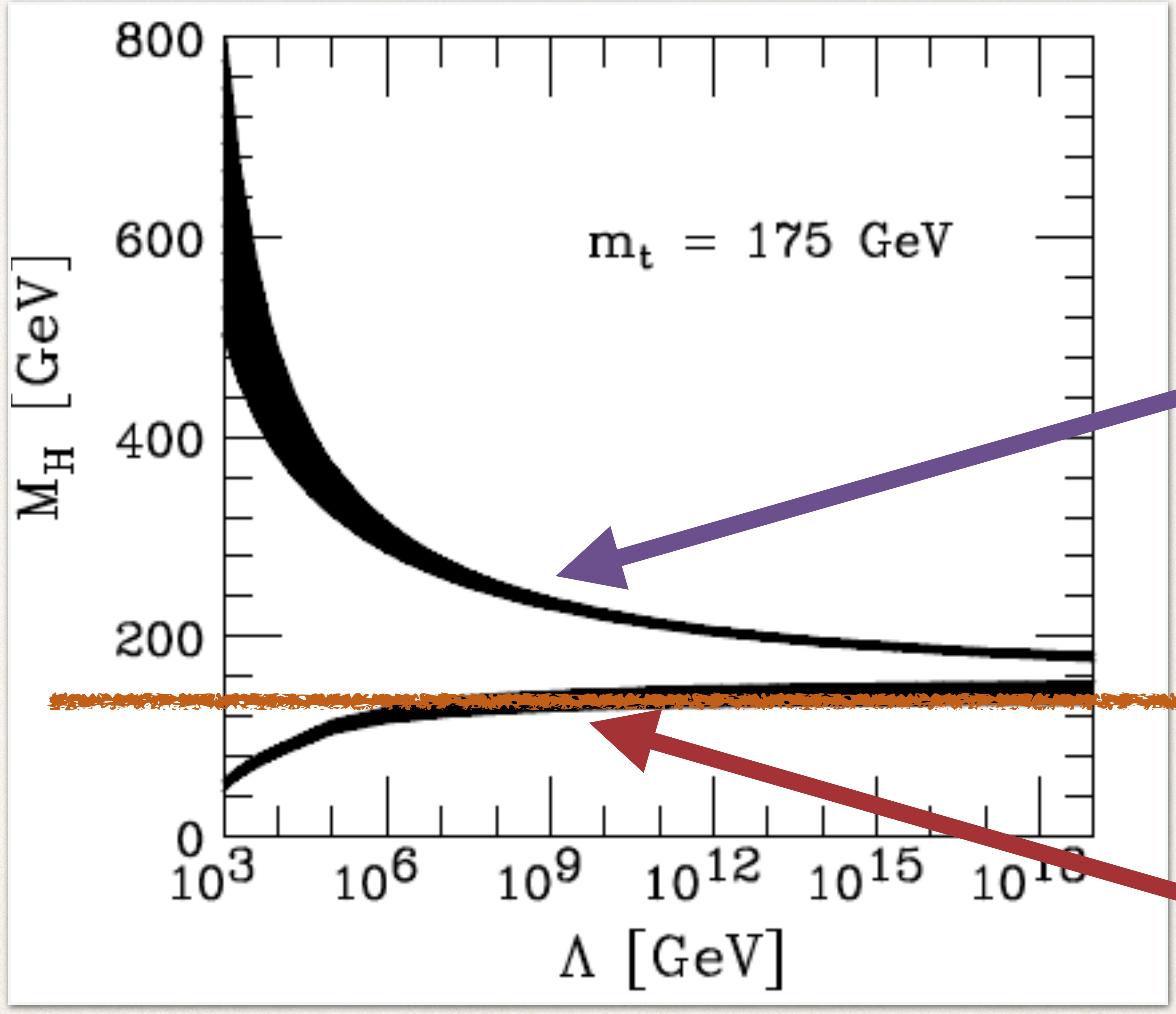


Perturbativity (triviality)

Stability of the potential

Constraints on M_H in the SM

➜ see also lectures by J. Ellis



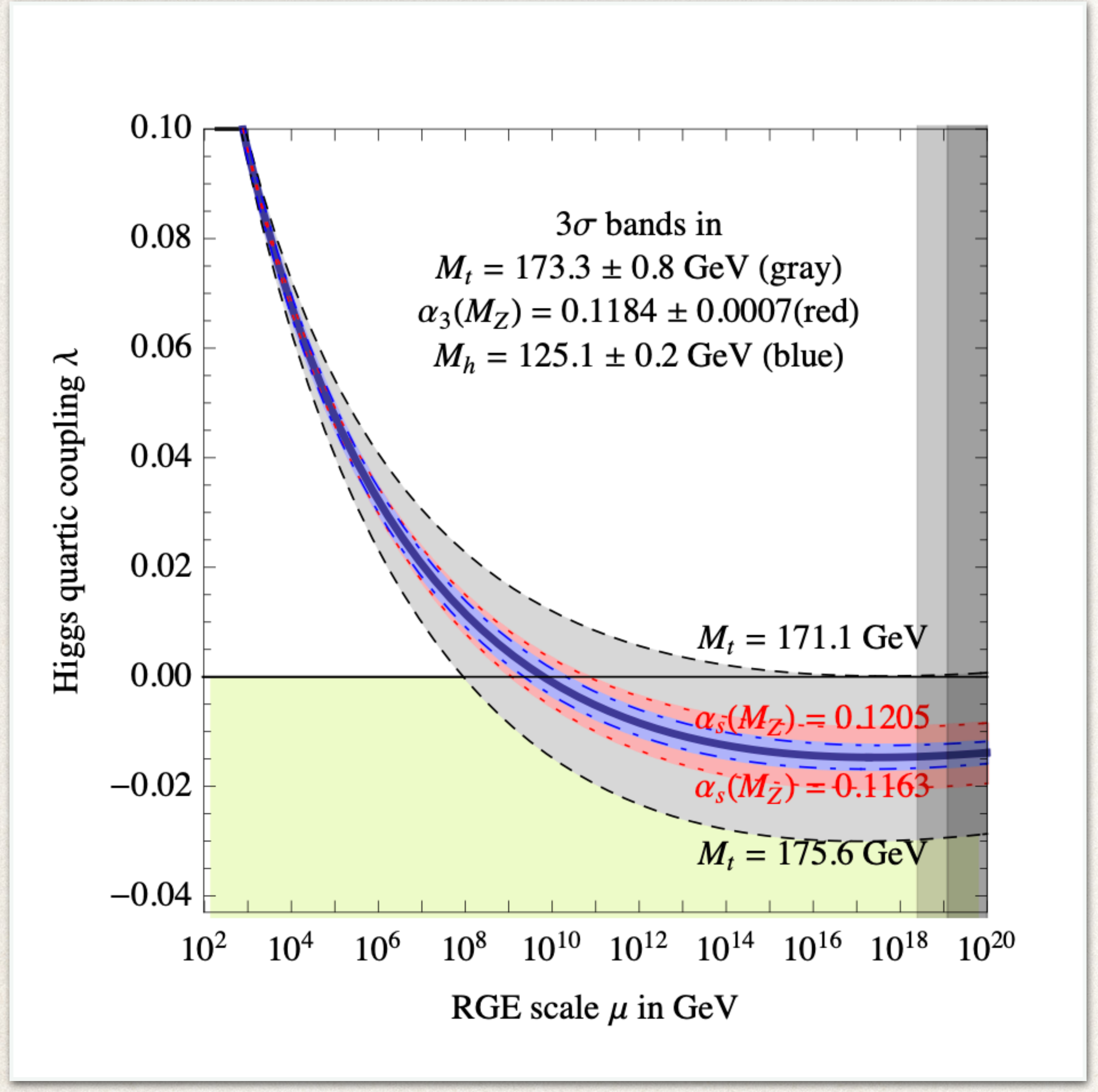
Perturbativity (triviality)

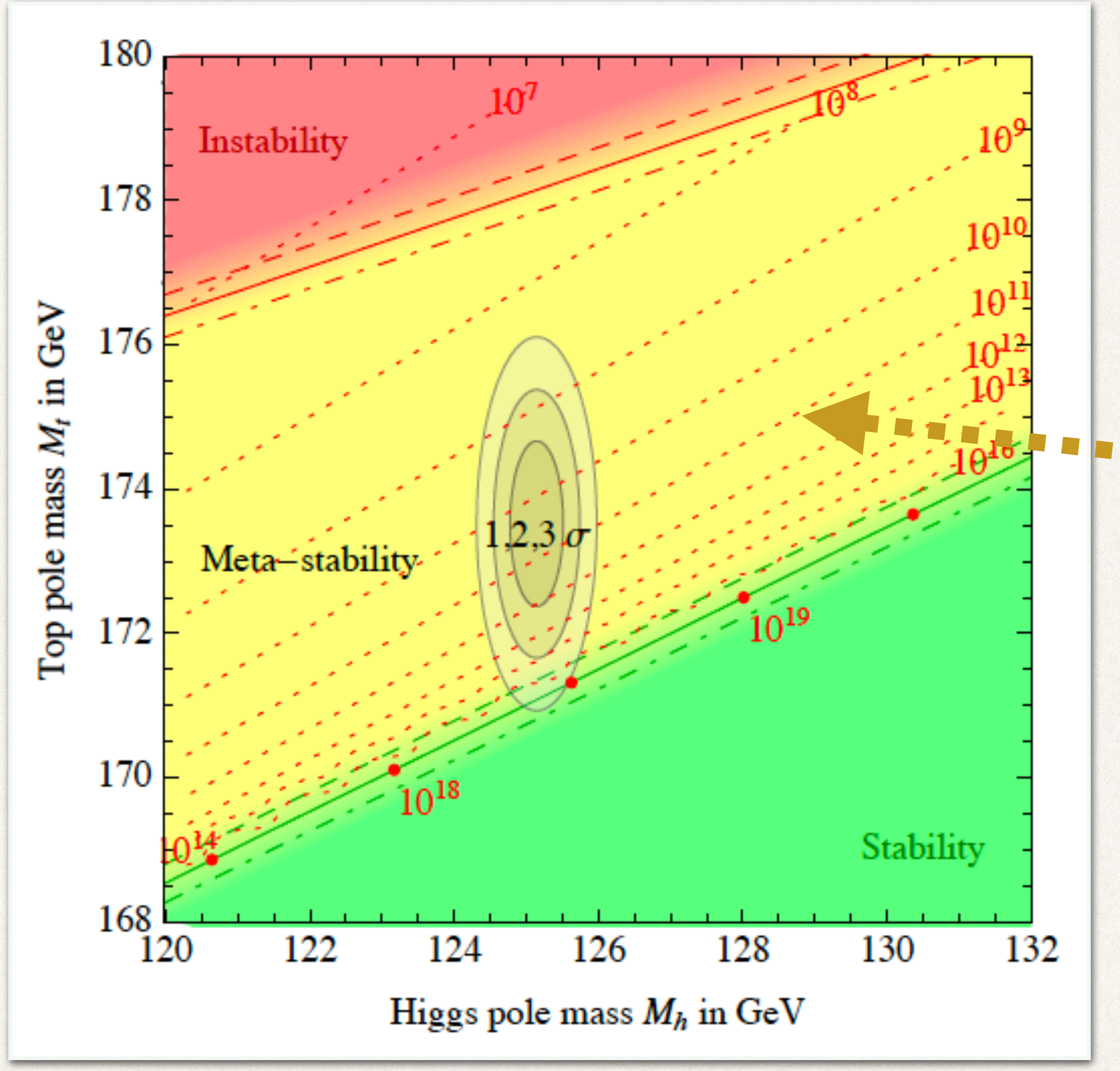
$M_H = 125$ GeV

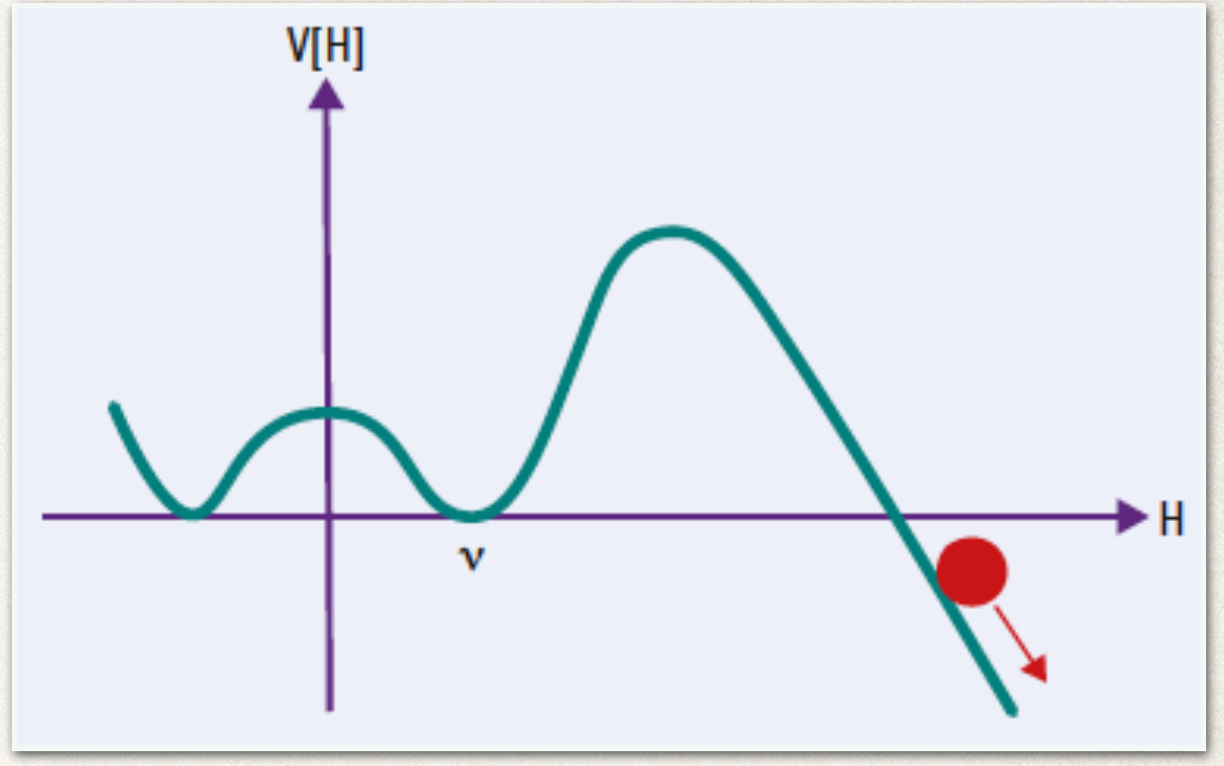
Stability of the potential

Living on the edge

arXiv:1307.3536



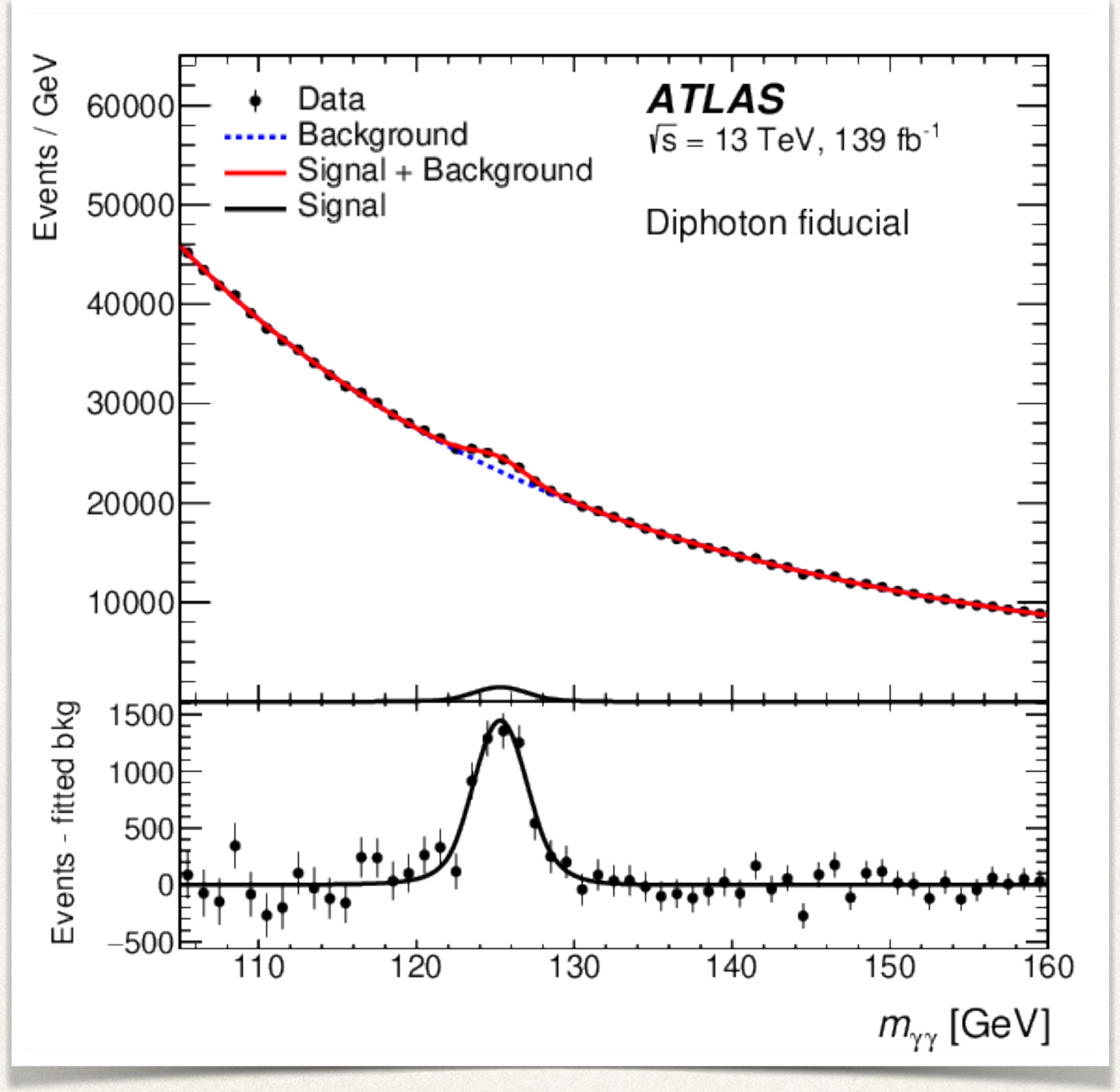


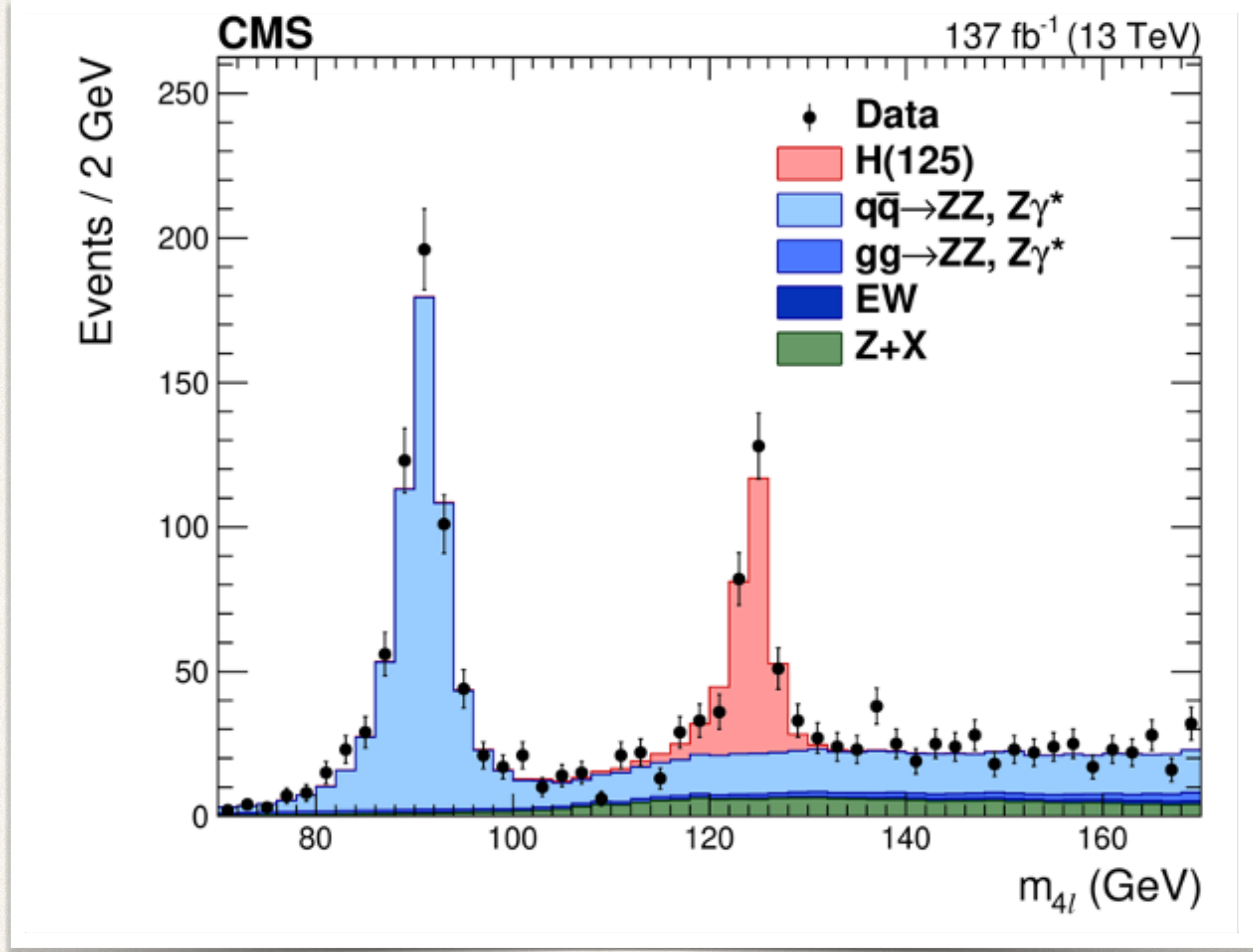


Precise knowledge of the top mass and the top Yukawa coupling (its running, measurement) crucial

12 years with the Higgs @ LHC

➜ see also lectures by J. Ellis





Global signal strength $\mu = \sigma/\sigma_{\text{SM}}$

$\mu = 1.05 \pm 0.06$ (ATLAS)

$\mu = 1.00 \pm 0.06$ (CMS)

How to make the SM Higgs at the LHC

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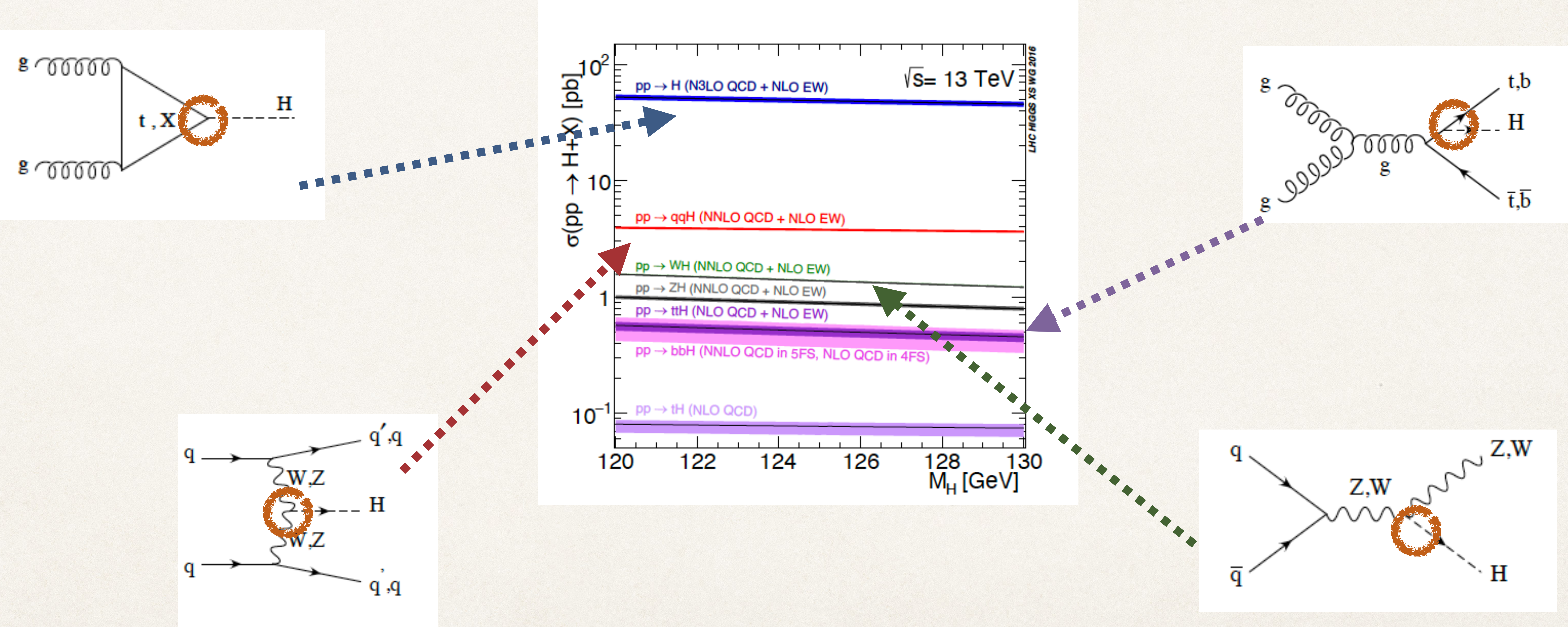
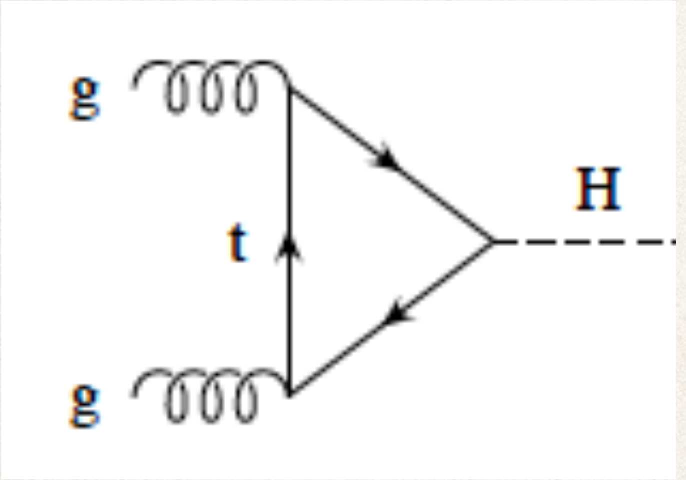
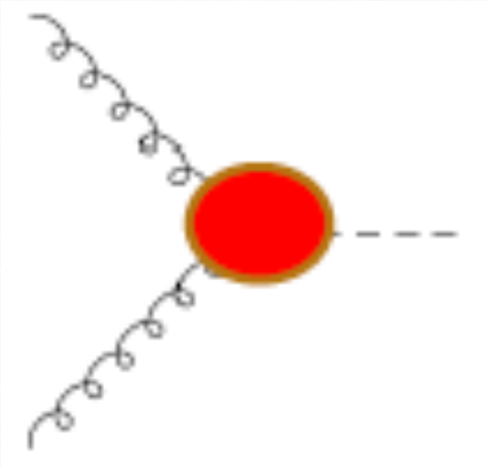


Gluon fusion

- Higgs boson does not couple directly to gluons
- At LO the gluon fusion process is mediated via loop, with the biggest contribution from the top quark ($y_t \propto m_t$)
- Effective (loop-induced) ggH coupling in the heavy top mass ($m_t \to \infty$) limit

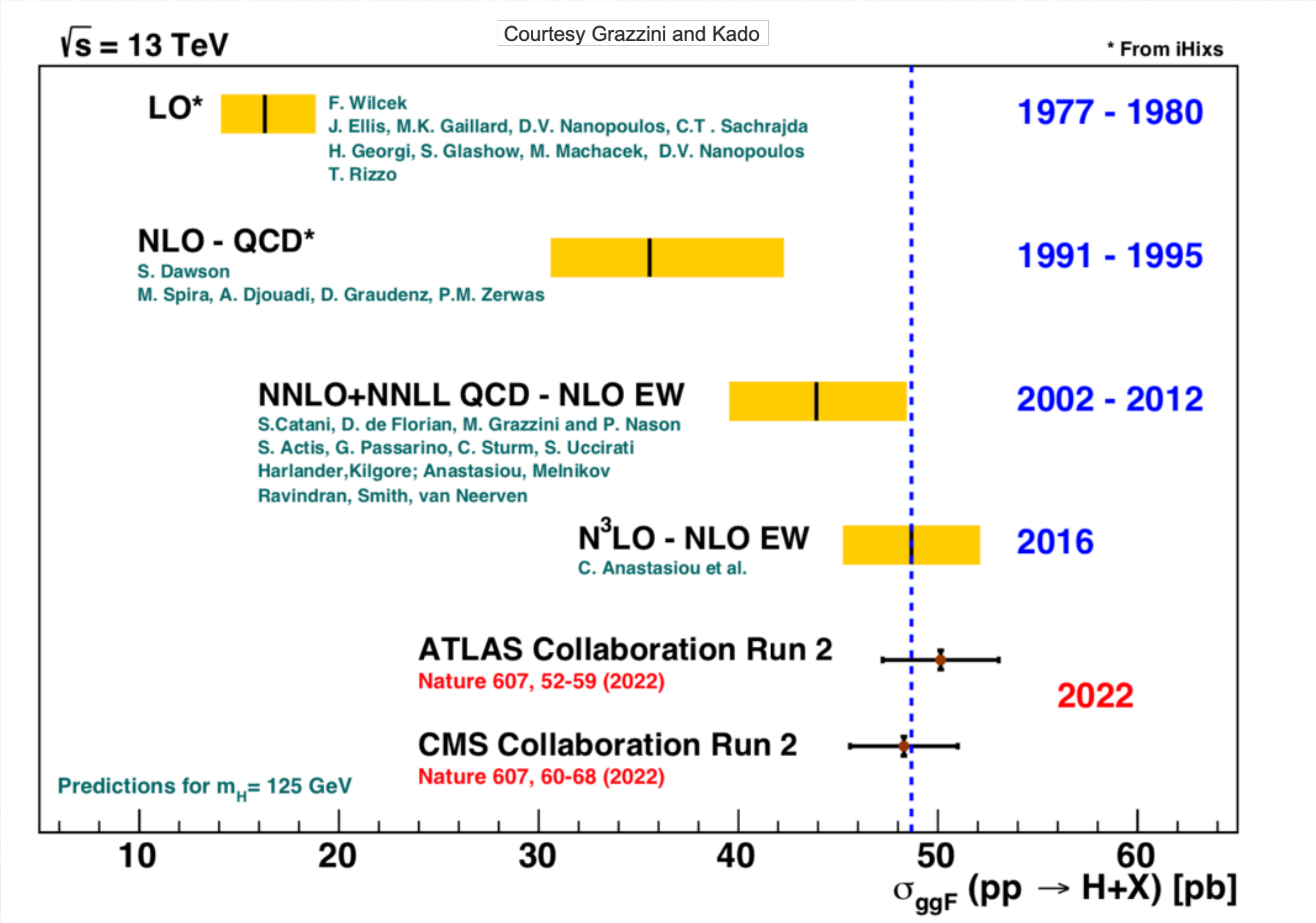
$$\mathscr{L}_{ggH,\,eff} = -\frac{1}{4v} C_H\, H G^{\mu\nu} G_{\mu\nu}$$



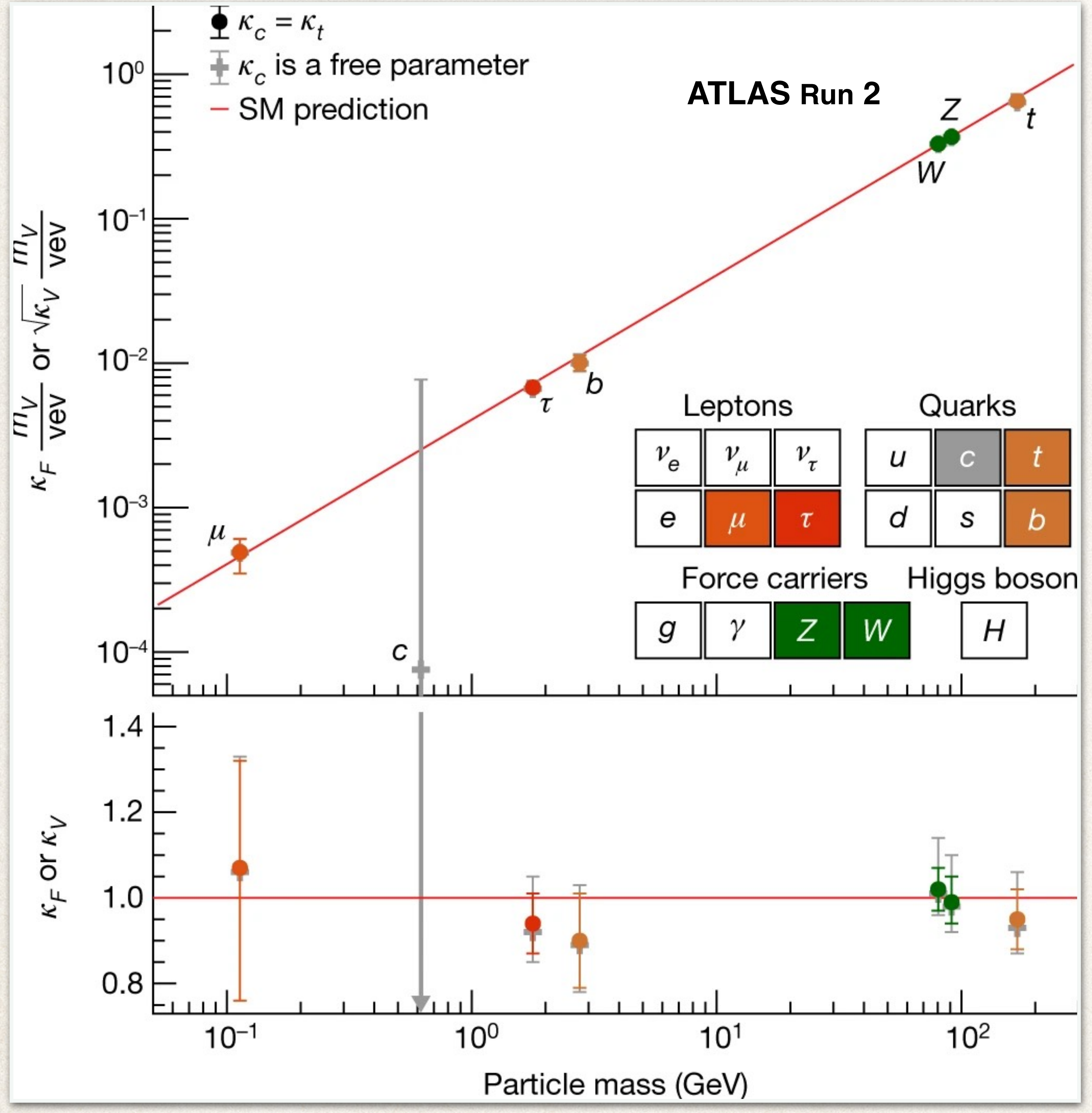


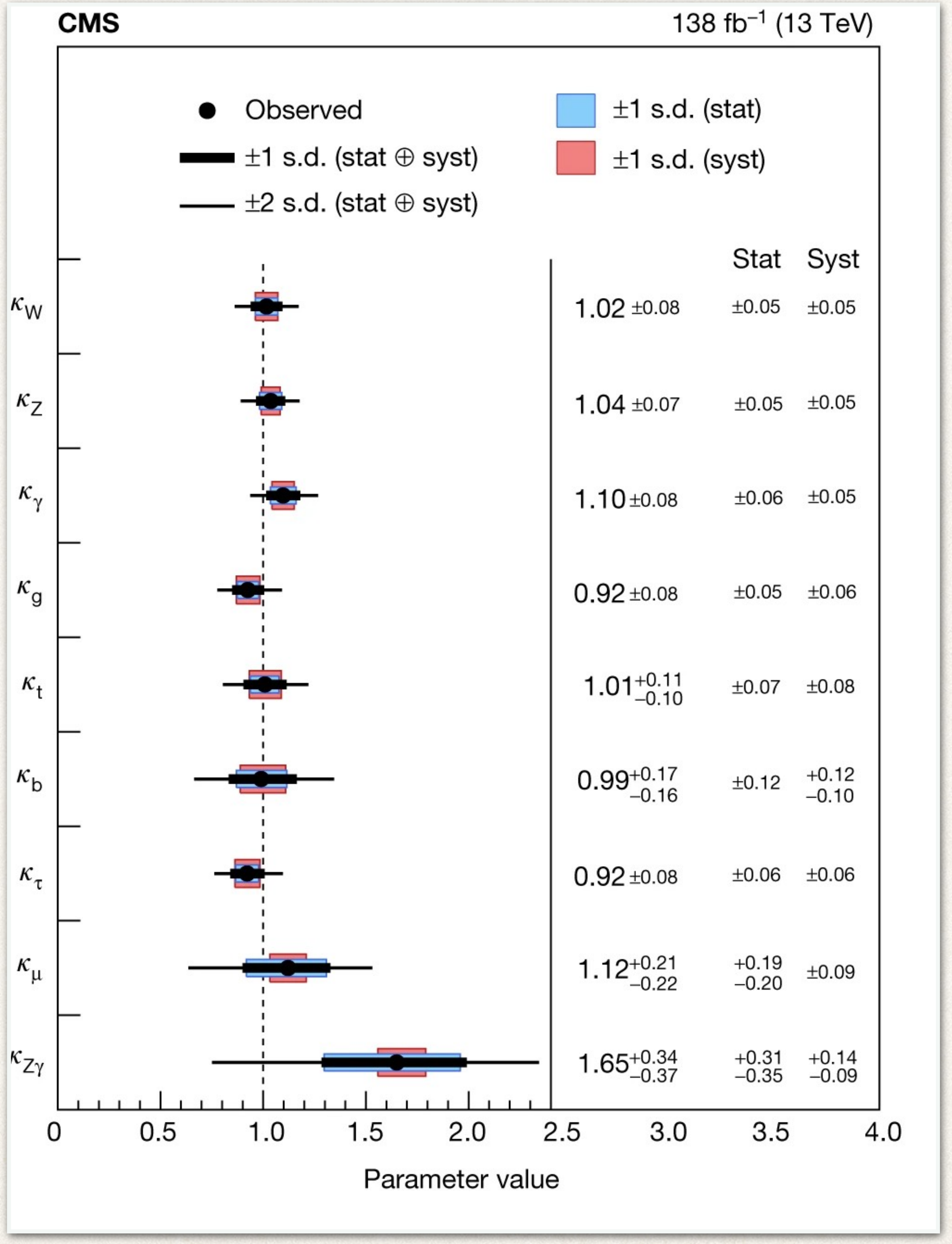
- In the heavy top limit approximation analogous to the Drell-Yan process!

Calculations of the inclusive rate

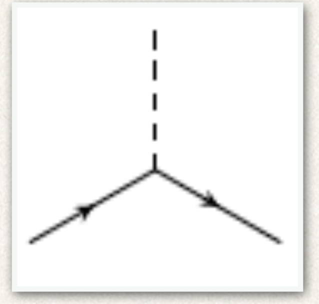


Higgs boson interactions

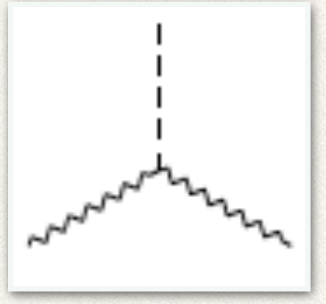




- κ 's are rescaling factors of the SM couplings



$$-ig\frac{m_f}{M_W}\kappa_f$$



$$-igM_V g_{\mu\nu}\,\kappa_V$$

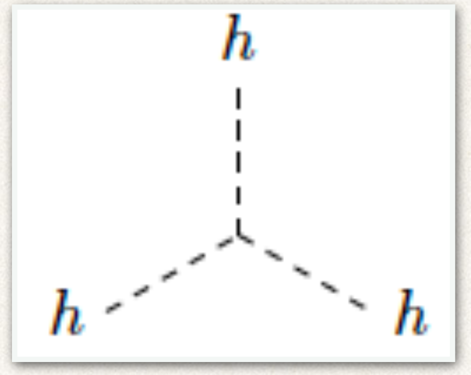
- "kappa-framework" is very / too simple (problems with gauge invariance, kinematic information, consistency at higher orders,..) , better approach is provided by effective field theory

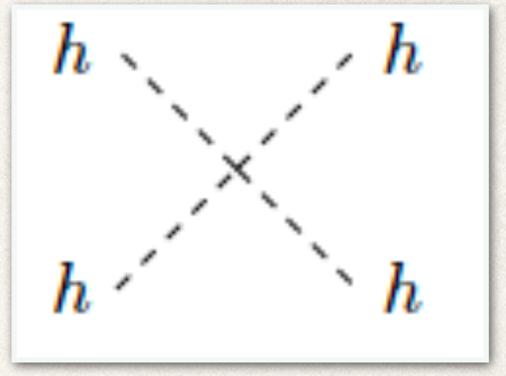
- Higgs-gauge boson couplings → test of the EWSB mechanism, Yukawa couplings → mass of elementary particles
- The scaling of the couplings with the mass of the particles is a central prediction of the theory

Higgs self-coupling

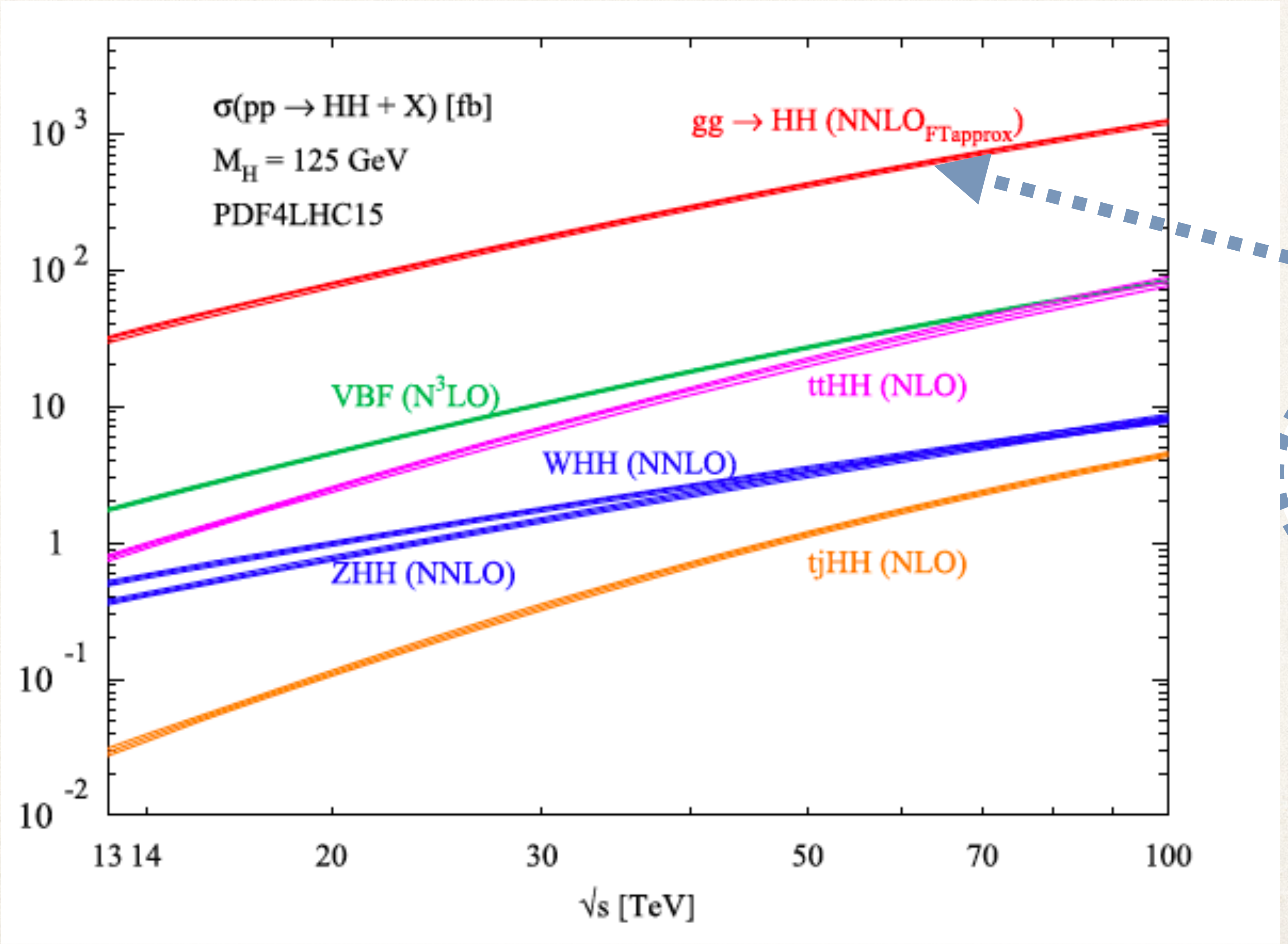
$$V = \frac{M_H^2}{2}H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

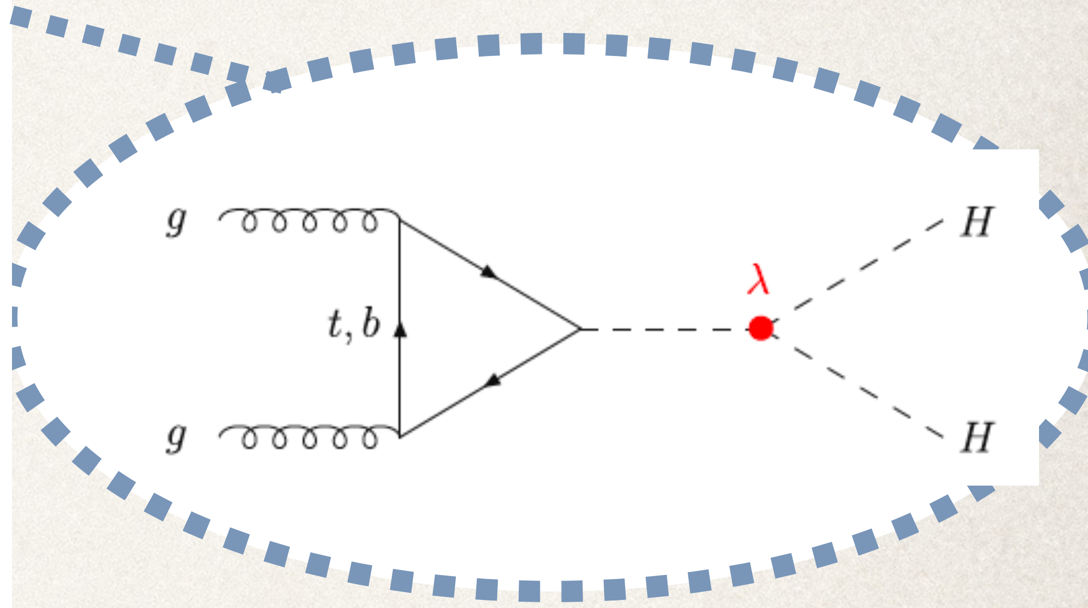
$$\left(\lambda = \frac{M_H^2}{2v^2}\right)$$





- Value of λ decides the shape of the SM scalar potential
- λ can be probed at hadron colliders through the Higgs pair-production processes
- Extremely low cross section at the LHC (1000 times smaller than single Higgs production)





arXiv:1910.00012

SM in EFT

- SM "works" fantastically well → a very good approximation of an unknown BSM theory chosen by Nature → all new physics at a scale $\Lambda \gg v$ → SM as lowest order in EFT expansion of the full theory

$$\mathscr{L}_{SMEFT} = \mathscr{L}_{SM} + \frac{1}{\Lambda}\mathscr{L}_5 + \frac{1}{\Lambda}\mathscr{L}_6 + \ldots$$

$$\mathscr{L}_n = \sum_k c_K^{(n)} \mathscr{O}_k^{(n)}$$

SM fields

BSM effects

- All the principles same as behind SM (QFT, local gauge symmetries, matter content and the quantum numbers, Higgs mechanism of EWSB via single SU(2) doublet field) apart from renormalizability: order-by-order

- Dim-5: only 1 operator (Weinberg) $\mathscr{O}_5 = \frac{v^2}{2}\bar{\nu}_L^c \nu_L$ → Majorana mass term

- Dim-6: 59 operators ("Warsaw basis"), 2499 if flavour structure considered

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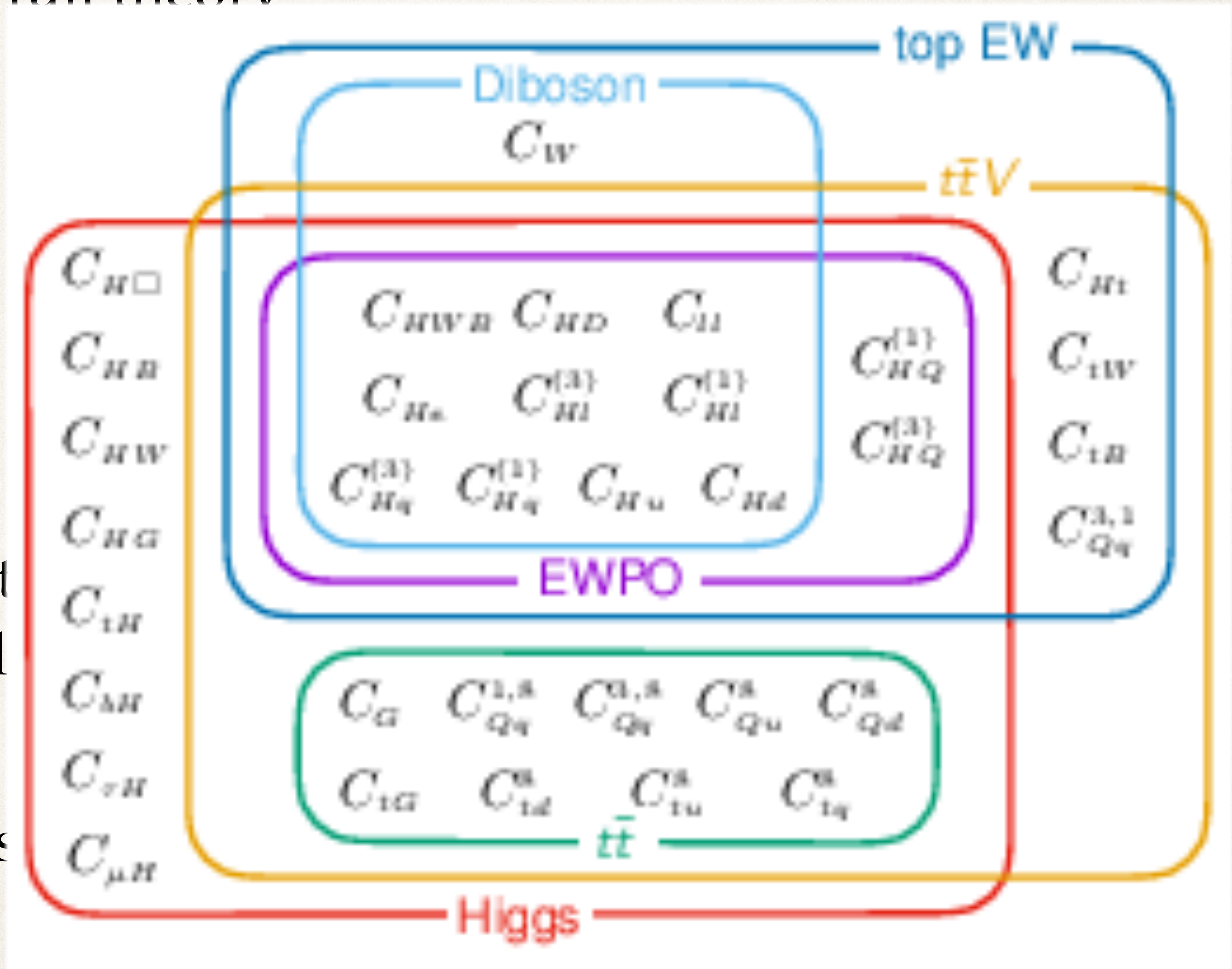

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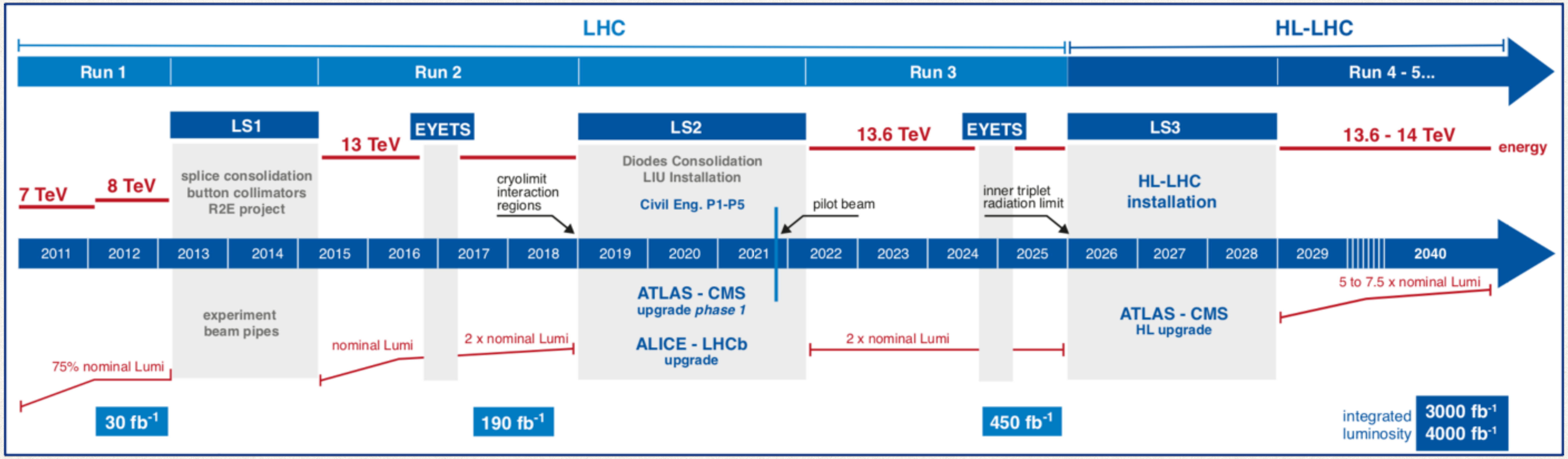
The future is bright

- Next stage of the LHC, after current Run 3: High-Luminosity LHC (HL-LHC): 20 times larger data sample and improved detectors



- However, precision measurements of the full Higgs sector: triple and quartic self-interactions, Yukawa couplings to light fermions etc. will require next generation colliders such as e.g. FCC (-ee).

Summary

- With the discovery of a new boson 12 years ago, and then subsequent confirmation of it as the Higgs boson, the EW SM is now complete
- Results of collider physics experiments are in agreement with the EWSM — it “works” amazingly well
- Yet, it can only be an effective theory — the evidence for BSM physics is there
  - dark matter
  - neutrino masses
  - matter-antimatter symmetry
  - ….
- The Higgs sector still needs to be fully tested
- The search for new physics relies on developments in both experiment and theory

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- where does the Higgs potential come from?
- why 3 generations? Why the observed mass pattern?
- what protects the Higgs mass (naturalness problem)?
- …

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Good news: there is a lot to understand and discover!