Neutrino physics
Lecture 2

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- neutrino propagation in matter
  - constant matter density – long baseline experiments
  - varying matter density – solar neutrinos
- 3-flavour interpretation of experimental results
- the absolute neutrino mass scale
- the neutrino nature: neutrinoless double beta decay

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Neutrino propagation in matter

The interaction of neutrinos with matter (e-, p, n) affect their propagation $\Longrightarrow$ modified oscillation parameters + a new phenomenon: matter-induced flavour conversion in a medium with varying density

<u>Appropriate description</u>: Schrödinger-like equation

$$i\frac{d}{dt}|\nu(t)\rangle = H|\nu(t)\rangle$$

The Hamiltonian H contains a potential term describing the interactions of the neutrinos with the medium and can depend on t

It is convenient to write the Schrödinger equation in the flavour eigenstate basis $\{|\nu_\alpha\rangle, |\nu_\beta\rangle, \cdots\}$, in which $|\nu(t)\rangle = \sum_\beta \nu_\beta(t)|\nu_\beta\rangle$:

$$i\frac{d}{dt}\nu_\beta(t) = \sum_\gamma H_{\beta\gamma}\,\nu_\gamma(t)$$

$$\nu_\beta(t) = \langle\nu_\beta|\nu(t)\rangle$$

$$H_{\beta\gamma} = \langle\nu_\beta|H|\nu_\gamma\rangle$$

$\nu_\beta(t)$ is the probability amplitude to find the neutrino in the state $|\nu_\beta\rangle$ at t

if $|\nu(t=0)\rangle = |\nu_\alpha\rangle$, then $P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2$

Vacuum oscillations in the Schrödinger formalism (2-flavour case)

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = H_0 \begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} \qquad H_0 = U\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}U^\dagger$$

The Hamiltonian in vacuum H_0 is diagonalized by the PMNS matrix

One can check that this reproduces the standard oscillation formula (*)

It is convenient to subtract from H_0 a piece proportional to the unit matrix (which only affects the overall phase of the neutrino state vector $|\nu(t)\rangle$, leaving oscillations unchanged) to bring it to the form:

$$H_0 = \begin{pmatrix} -\frac{\Delta m^2}{2E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}$$

E = neutrino energy (negligible difference between E_1 and E_2 for ultra-relativistic neutrinos)

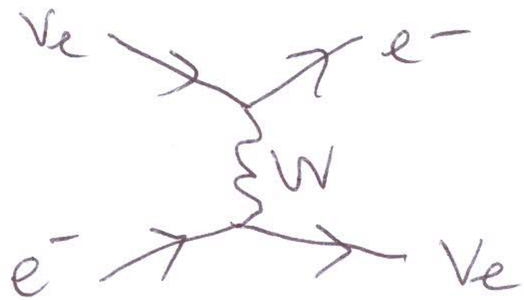
(*) <u>proof</u>: first note that $|\nu_\beta\rangle = \sum_i U^*_{\beta i}\,|\nu_i\rangle \;\Rightarrow\; \nu_\beta(t) = \sum_i U_{\beta i}\,\nu_i(t)$, where $\nu_i(t) \equiv \langle \nu_i|\nu(t)\rangle$ to compute the oscillation probability $P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2$, it is enough to solve the Schrödinger equation for $\nu_i(t)$, which gives $\nu_i(t) = e^{-iE_i t}\,\nu_i(0) = e^{-iE_i t}\sum_\gamma U^*_{\gamma i}\,\nu_\gamma(0)$, hence $\nu_\beta(t) = \sum_{i,\gamma} U_{\beta i}U^*_{\gamma i}\,e^{-iE_i t}\,\nu_\gamma(0)$, where $\nu_\gamma(0) = \langle\nu_\gamma|\nu(0)\rangle = \langle\nu_\gamma|\nu_\alpha\rangle = \delta_{\gamma\alpha}$, which reproduces the known oscillation formula

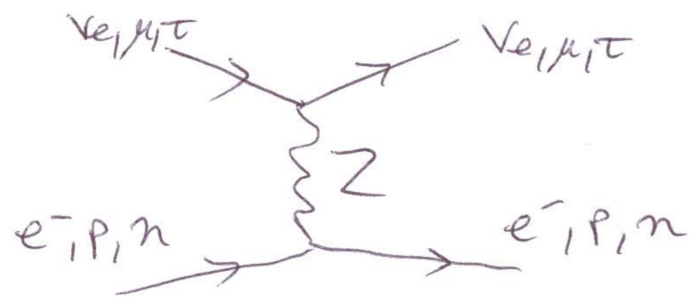
Neutrino Hamiltonian in matter

$$H_0 \;\rightarrow\; H_m = H_0 + V$$

V induced by interactions (anti-)neutrinos / e-, p, n of the medium

Relevant interactions: forward elastic scatterings ($\vec{p}_\nu$ unchanged)





CC – only for ν_e

NC – same for $\nu_{e,\mu,\tau}$ $\Rightarrow$ can be subtracted from Hm

$$V_{\rm CC} = \sqrt{2}\, G_F n_e(x) \qquad\qquad V_{\rm NC} = -\frac{G_F}{\sqrt{2}}\, n_n(x)$$

GF = Fermi constant

In the flavour eigenstate basis:

$$(H_m)_{\beta\gamma} \;=\; (H_0)_{\beta\gamma} + V_\beta\, \delta_{\beta\gamma} \qquad\qquad V_\beta = V^\beta_{\rm CC} + V^\beta_{\rm NC}$$

For anti-neutrinos, V has the opposite sign: $V \;\rightarrow\; -V$

For a sterile (= insensitive to weak interactions) neutrino: $V_\beta \;=\; 0$

Example with electron neutrinos and another flavour:

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = H_m \begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} \qquad \begin{cases}\alpha = e\\ \beta = \mu, \tau, s\end{cases}$$

$$H_m = \begin{pmatrix} -\frac{\Delta m^2}{2E}\cos 2\theta + \sqrt{2}\,G_F n & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}$$

$$n = \begin{cases} n_e(x) & \text{if } \beta = \mu, \tau \\ n_e(x) - \frac{1}{2}\,n_n(x) & \text{if } \beta = s \end{cases}$$

For antineutrinos, $+\sqrt{2}\,G_F n \;\rightarrow\; -\sqrt{2}\,G_F n$

Energy levels in matter and matter eigenstates

In vacuum, the PMNS matrix U relates the flavour eigenstates to the mass eigenstates (= eigenstates of H_0):

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \qquad \begin{matrix} \leftarrow \; E_1 = \sqrt{\vec{p}^{\,2} + m_1^2} \\ \leftarrow \; E_2 = \sqrt{\vec{p}^{\,2} + m_2^2} \end{matrix}$$

In matter, one defines matter eigenstates = eigenstates of Hm

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U_m^* \begin{pmatrix} |\nu_1^m\rangle \\ |\nu_2^m\rangle \end{pmatrix} \qquad \left.\begin{matrix} \leftarrow \; E_1^m \\ \leftarrow \; E_2^m \end{matrix}\right\} \begin{matrix} \text{eigenvalues of } H_m = \\ \text{energy levels in matter} \end{matrix}$$

Um contains the mixing angle in matter that diagonalizes Hm :

$$H_m = U_m \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} U_m^\dagger \qquad U_m = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$$

By analogy with $\nu_\beta(t) = \langle \nu_\beta | \nu(t) \rangle$, one defines $\nu_i^m(t) = \langle \nu_i^m | \nu(t) \rangle$
(amplitude of probability to find the neutrino in the ith matter eigenstate at t)

then
$$\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

Medium with constant matter density

n(x) = n ⇒ Hm, hence the matter eigenstates $|\nu_i^m\rangle$, energy levels E_i^m and mixing matrix Um, do not depend on t

Using $\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$, one can rewrite the Schrödinger equation for the probability amplitudes $\nu_i^m(t)$

$$i \frac{d}{dt} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix} = \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

which is solved by $\nu_i^m(t) = e^{-iE_i^m t}\, \nu_i^m(0)$, giving

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2 = |\textstyle\sum_i (U_m)_{\beta i}\, \nu_i^m(t)|^2 = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

→ oscillations in matter with constant density are governed by the same formula as in vacuum, with the replacements

$$\theta \;\to\; \theta_m\,, \qquad \frac{\Delta m^2}{4E} \;\to\; \frac{(E_m^2 - E_m^1)}{2}$$

Oscillation parameters in matter

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E}\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2\cos^2 2\theta + \sin^2 2\theta}$$

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2\cos^2 2\theta + \sin^2 2\theta}}$$

$$\cos 2\theta_m = \frac{\left(1-\frac{n}{n_{\rm res}}\right)\cos 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2\cos^2 2\theta + \sin^2 2\theta}}$$

$$n_{\rm res} \equiv \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F E}$$

for antineutrinos,

$$n \to -n$$

(n = n_e if only active neutrinos)

MSW resonance (Mikheev-Smirnov-Wolfenstein):

$$\sin 2\theta_m = 1 \quad \text{for } n = n_{\rm res}$$

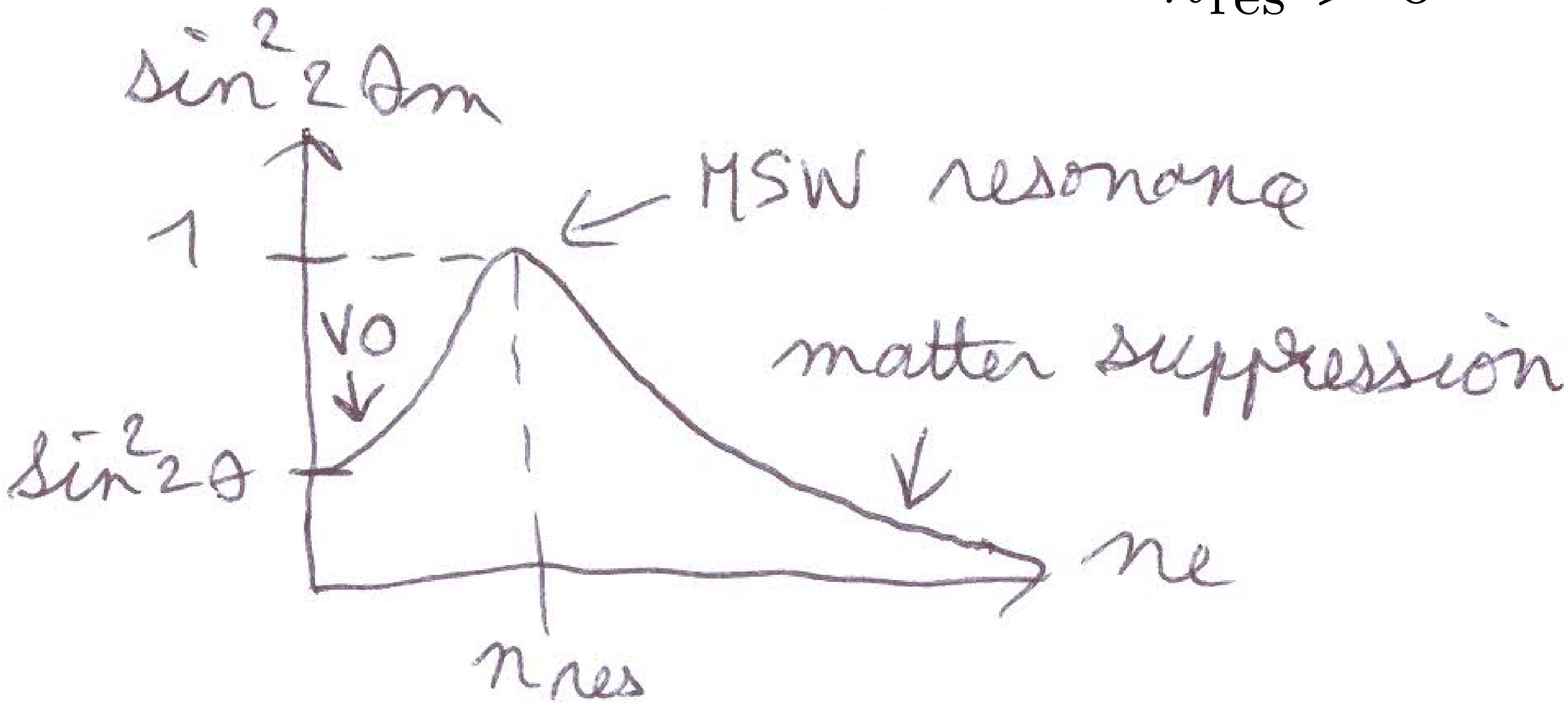
(irrespective of the value of the mixing angle in vacuum θ)

Resonance condition: $\begin{cases} \Delta m^2 \cos 2\theta > 0 & \text{for neutrinos} \\ \Delta m^2 \cos 2\theta < 0 & \text{for antineutrinos} \end{cases}$

When neutrino oscillations are enhanced, antineutrino oscillations are suppressed, and vice versa

Different regimes for oscillations in matter :

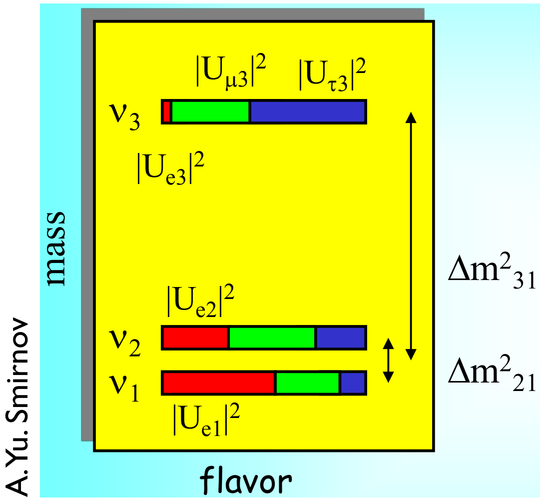
$n_{\mathrm{res}} > 0$



- low density ($n \ll n_{\mathrm{res}}$) : $\sin 2\theta_m \simeq \sin 2\theta$ $\Rightarrow$ vacuum oscillations
- resonance ($n = n_{\mathrm{res}}$) : $\sin 2\theta_m = 1$
- high density ($n \gg n_{\mathrm{res}}$) : $\sin 2\theta_m < (\ll) \sin 2\theta$ $\Rightarrow$ oscillations are suppressed by matter effects

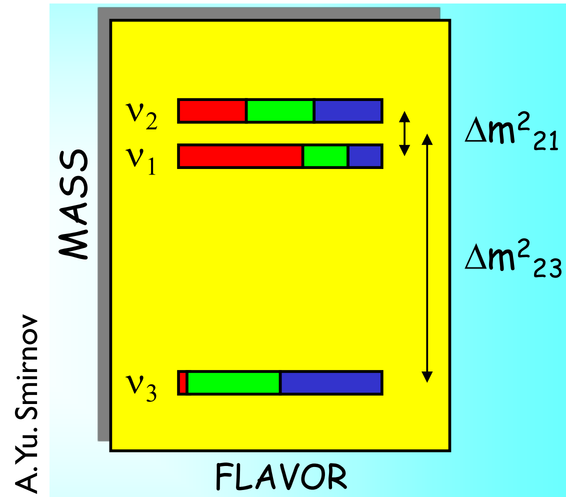
Application: determination of the mass hierarchy in long-baseline experiments

Two mass orderings allowed by experiments:



Normal hierarchy

$\Delta m^2_{31} > 0$



Inverted hierarchy

$\Delta m^2_{31} < 0$

In vacuum: $P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$

For long baselines (> several 100 km), matter effects cannot be neglected

$$n_{\rm res} = \frac{\Delta m^2_{31} \cos 2\theta_{13}}{2\sqrt{2}\, G_F E} \qquad \begin{cases} n_{\rm res} > 0 & \text{for normal hierarchy} \\ n_{\rm res} < 0 & \text{for inverted hierarchy} \end{cases}$$

If $n_{\rm res}$ is close to the Earth crust density, neutrino (antineutrino) oscillations are enhanced for NH (IH), while antineutrino (neutrino) oscillations are suppressed

[may have to disentangle CP violation from matter effect]

an outdated but informative plot

$$R = \frac{\overline{\nu}_e \to \overline{\nu}_\mu}{\nu_e \to \nu_\mu}$$

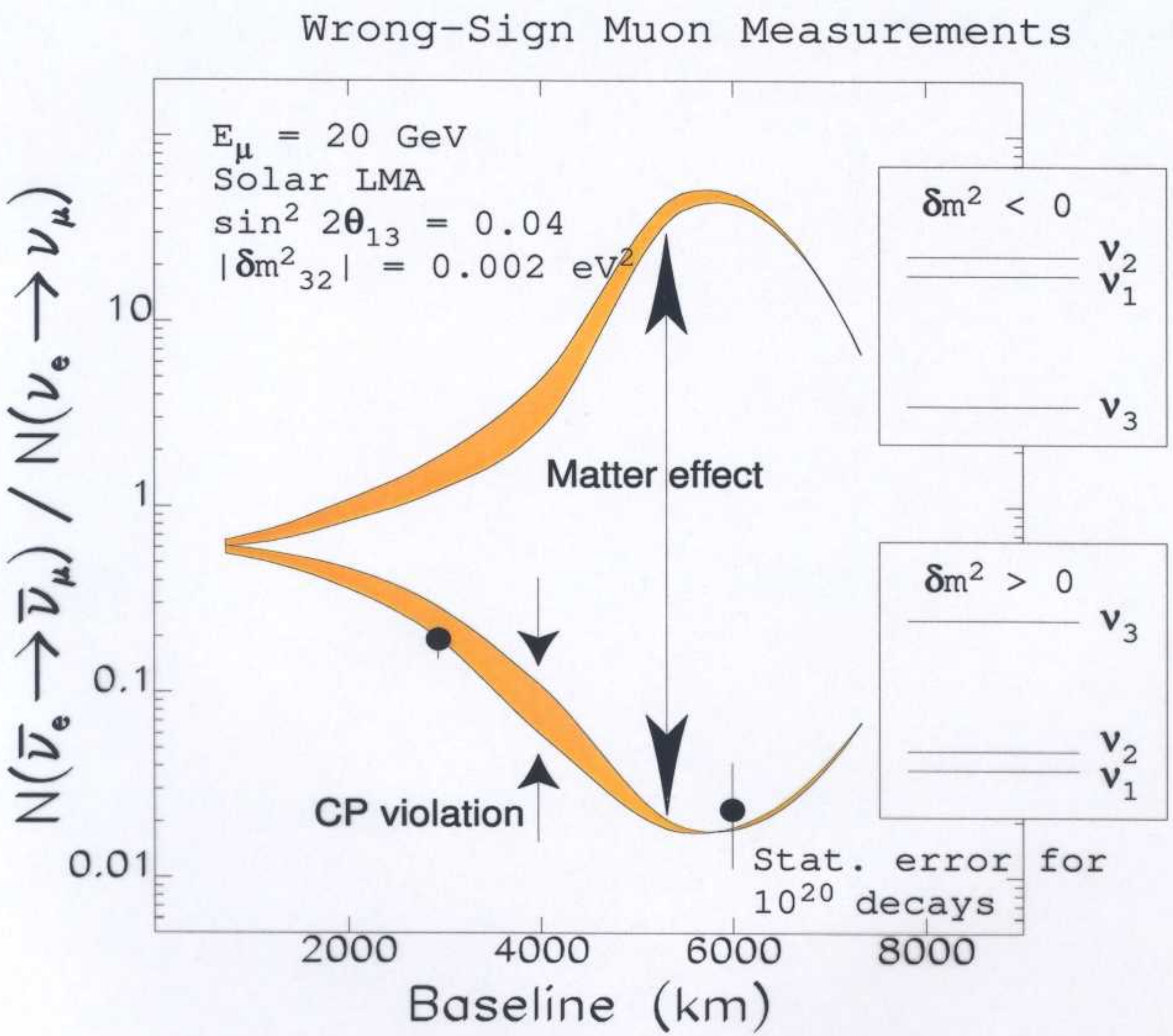


Figure 2: Predicted ratios of $\bar{\nu}_e \to \bar{\nu}_\mu$ to $\nu_e \to \nu_\mu$ rates at a 20 GeV neutrino factory. The statistical error shown corresponds to 10^{20} muon decays of each sign and a 50 kt detector.

- Un baseline de $L = \mathcal{O}$ (3000 km) est nécessaire/optimale

[Barger, Geer, Raja, Whisnant]

Medium of varying density (e.g. the Sun)

Now the matter eigenstates, energy levels and mixing angle depend on t

→ "instantaneous" matter eigenstates: $|\nu_i^m(t)\rangle \leftarrow E_i^m(t)$

$$H_m = U_m \begin{pmatrix} E_1^m(t) & 0 \\ 0 & E_2^m(t) \end{pmatrix} U_m^\dagger \qquad U_m = \begin{pmatrix} \cos\theta_m(t) & \sin\theta_m(t) \\ -\sin\theta_m(t) & \cos\theta_m(t) \end{pmatrix}$$

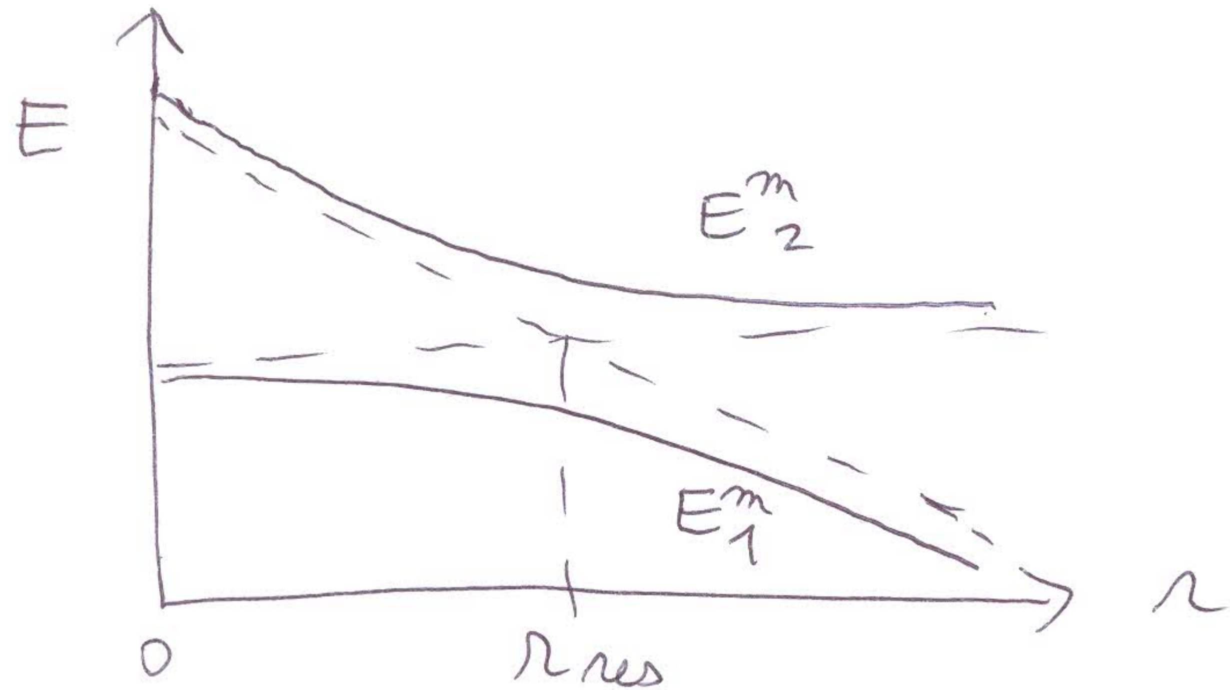
The Schrödinger equation now depends on the time variation of θm:

$$i\frac{d}{dt}\begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix} = \begin{pmatrix} E_1^m(t) & -i\dot{\theta}_m \\ i\dot{\theta}_m & E_2^m(t) \end{pmatrix} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

In most physical environments (including the Sun), the evolution is <u>adiabatic</u> (the neutrino state has the time to adjust to the variation of density) and one can neglect $\dot{\theta}_m$ in the Schrödinger equation. A neutrino produced in a given matter eigenstate will stay in the same matter eigenstate during its propagation, but its flavour composition will change

→ adiabatic flavour conversion

"Level crossing" in the Sun (case n_e (r=0) >> n_{res})



This is the case for high-energy solar neutrinos (E > 1 MeV)

$$n_e(r=0) \gg n_{\rm res} \;\Rightarrow\; \sin 2\theta_m^0 \simeq 0 \text{ and } \cos 2\theta_m^0 \simeq -1 \;\Rightarrow\; \theta_m^0 \simeq \pi/2$$

$$\Rightarrow\; |n_e\rangle = \cos\theta_m^0\,|n_1^m(r=0)\rangle + \sin\theta_m^0\,|n_2^m(r=0)\rangle \simeq |n_2^m(r=0)\rangle$$

⇒ a neutrino produced at the center of the Sun is a quasi pure matter eigenstate and, as it evolves adiabatically, exits the Sun in the same eigenstate

$$|\nu_2^m(r=R_{\rm Sun})\rangle = |\nu_2\rangle = \sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\beta\rangle \qquad (\beta = \mu,\tau)$$

⇒ a high-energy neutrino produced at the center of the Sun is a quasi pure matter eigenstate and exits the Sun in the eigenstate

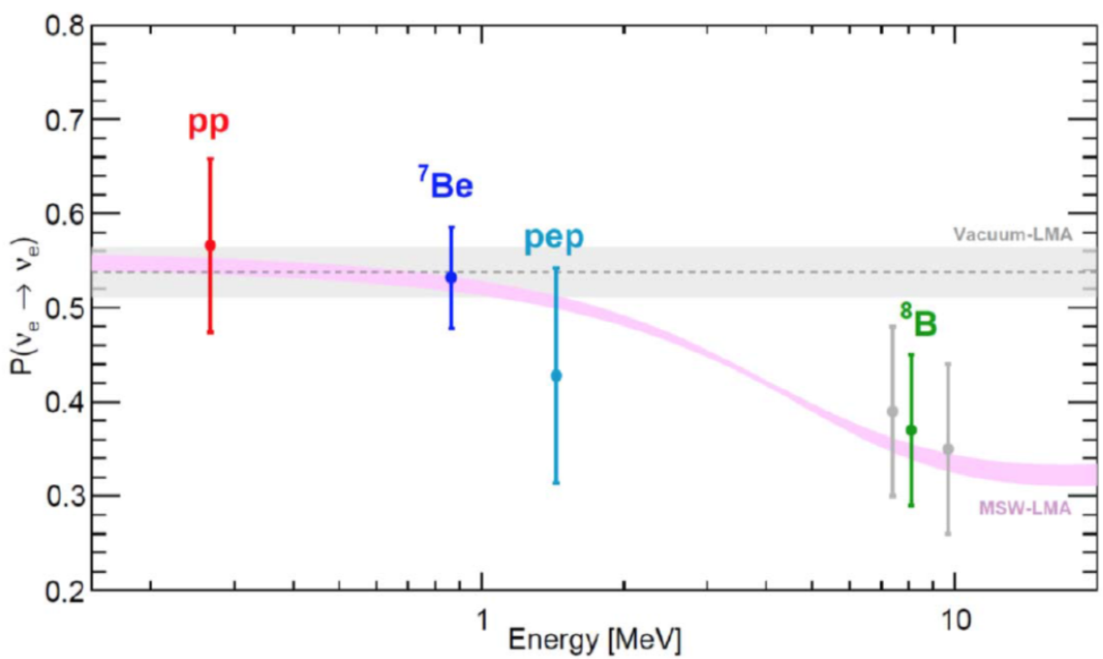
$$|\nu_2^m(r = R_{\rm Sun})\rangle = |\nu_2\rangle = \sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\beta\rangle \qquad (\beta = \mu, \tau)$$

⇒ reaches the Earth as a $|\nu_2\rangle$, giving (using the observed value of θ_{12} for θ)

$$P_{ee} \;=\; |\langle\nu_e|\nu_2\rangle|^2 \;=\; \sin^2\theta \;\simeq\; 0.3$$

For low-energy solar neutrinos, the level-crossing condition is not satisfied ($n_e(r=0) \ll n_{\rm res}$) and matter effects are small

⇒ averaged vacuum oscillations: $$P_{ee} \;=\; 1 - \frac{1}{2}\sin^2 2\theta \;\simeq\; 0.58$$



Borexino Phase II results

[talk at TAU2018, arXiv:1810.12967]

3-flavour interpretation of experimental results

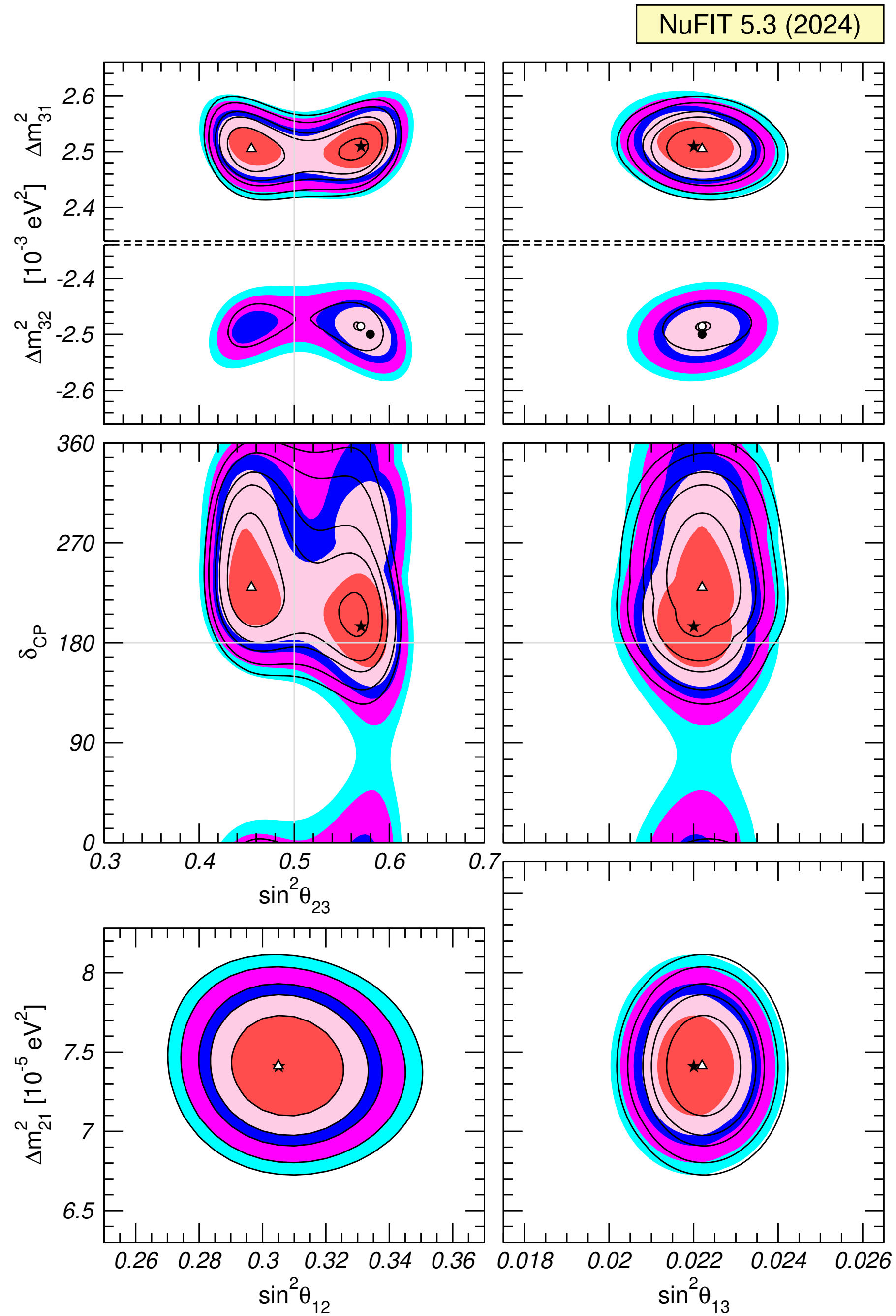
(aka « 3-flavour global fit »)

I. Esteban et al., JHEP 09 (2020) 178
NuFIT 5.3 (2024), www.nu-fit.org

(based on data available in March 2024)

All experimental data (leaving aside a few anomalies) is very well described in the 3-flavour framework, and the determination of oscillation parameters is becoming more and more precise

Other fits by F. Capozzi et al. (2021) and P. F. de Salas et al. (2020) find similar results



The different contours correspond to 1σ, 90%, 2σ, 99%, 3σ CL (2 dof).

Allowed ranges for the oscillation parameters (March 2024)

[I. Esteban et al., NuFIT 5.3 (2024) , JHEP 09 (2020) 178]

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.454^{+0.019}_{-0.016}$	$0.411 \to 0.606$	$0.568^{+0.016}_{-0.021}$	$0.412 \to 0.611$
	$\theta_{23}/^\circ$	$42.3^{+1.1}_{-0.9}$	$39.9 \to 51.1$	$48.9^{+0.9}_{-1.2}$	$39.9 \to 51.4$
	$\sin^2\theta_{13}$	$0.02224^{+0.00056}_{-0.00057}$	$0.02047 \to 0.02397$	$0.02222^{+0.00069}_{-0.00057}$	$0.02049 \to 0.02420$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.13}_{-0.11}$	$8.23 \to 8.95$
	$\delta_{\mathrm{CP}}/^\circ$	232^{+39}_{-25}	$139 \to 350$	273^{+24}_{-26}	$195 \to 342$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.505^{+0.024}_{-0.026}$	$+2.426 \to +2.586$	$-2.487^{+0.027}_{-0.024}$	$-2.566 \to -2.407$

3σ uncertainty around 15% for θ_{12} and Δm^2_{21} , less than 10% for θ_{13} and $\Delta m^2_{3\ell}$

The best known parameters are θ_{13} and Δm^2_{31} (Δm^2_{32} in the case of inverted ordering), with 3σ uncertainties below 10%, and θ_{12} and Δm^2_{21}, with 3σ uncertainties around 15%

By contrast, θ_{23} (first [$\theta_{23} < \pi/4$] or second [$\theta_{23} > \pi/4$] octant ?), the mass ordering (or mass hierarchy), and the CP-violating phase δ depend on subleading 3-flavour effects and are poorly known

$\Rightarrow$ not yet statistically significant

Inverted mass ordering is disfavoured at 3σ when including atmospheric neutrino data (1.5σ otherwise, due to the interplay between LBL accelerator and SBL reactor experiments, which are sensitive to different combinations of Δm^2_{32} and Δm^2_{31})

CP violation: CP conservation is excluded at about 90% C.L. and the best fit for the CP-violating phase is around 230°. Assuming the true ordering is inverted (which is disfavoured), CP conservation is excluded at 3σ

θ_{23} octant: no significant statistical preference for either of the two octants

The absolute neutrino mass scale

Oscillation experiments measure only mass squared differences
→ information on the neutrino mass scale from beta decay or cosmology

Cosmology

Upper bound on sum of neutrino masses from CMB and large structure data
[eV-scale SM neutrinos would be hot dark matter and affect structure formation, leading to fewer small structures than observed ⇒ must be a subdominant DM component]

$$\sum m_\nu < 0.12 \text{ eV}$$ (95 %, *Planck* TT,TE,EE+lowE +lensing+BAO). [Planck 2018]

[adding Lyman-a, Palanque-Delabrouille et al. obtain < 0.09 eV, 95% CL (JCAP04 (2020) 038)]

Kinematic measurements (beta decay)

The non-vanishing neutrino mass leads to a distortion of the Ee spectrum close to the endpoint

Best bound (KATRIN) : $m_\nu < 0.8 \text{ eV} \quad (90\% \text{ C.L.})$

[Nature Phys. 18 (2022) 160]

Tritium beta decay

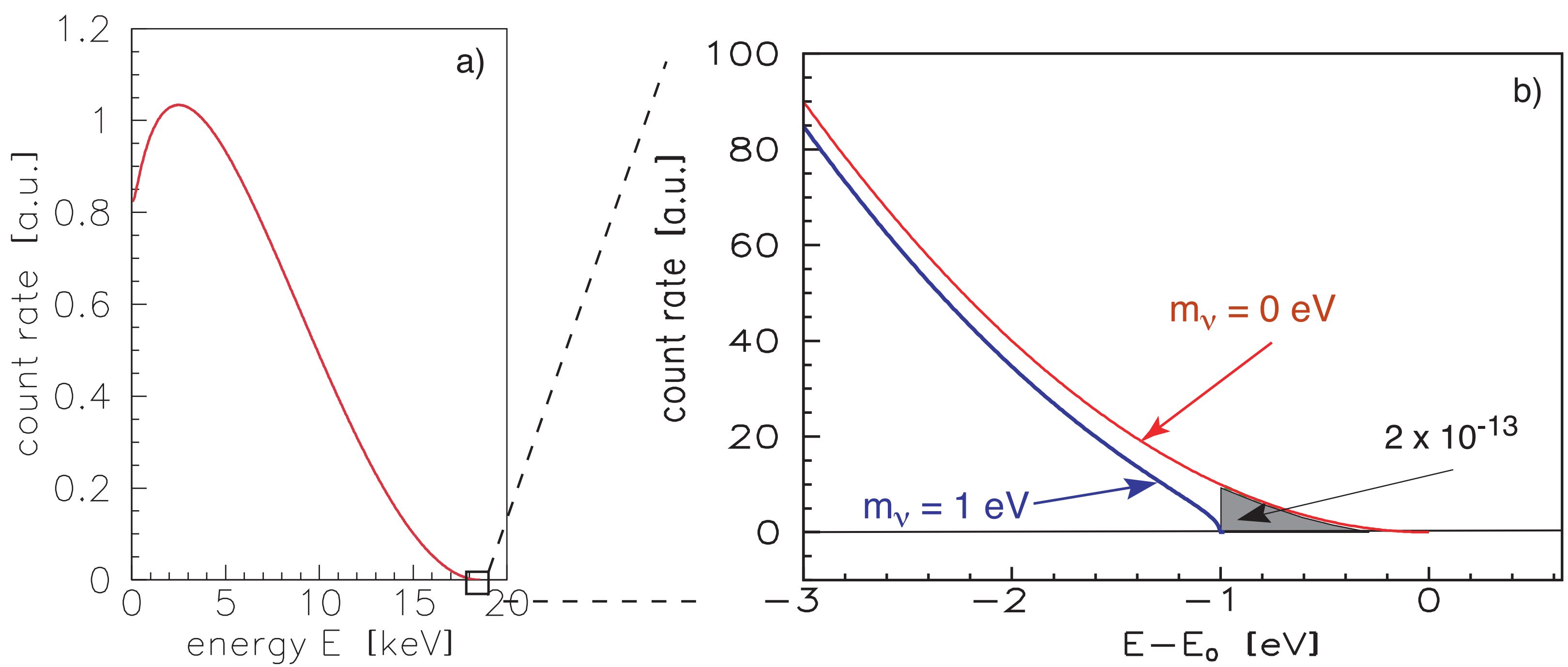
$$^{3}H \rightarrow {}^{3}H_{e} + e^{-} + \bar{\nu}_{e} \qquad\qquad E_0 = m_{^3H} - m_{^3H_e}$$

The electron energy spectrum is given by:

$$\frac{dN}{dE_e} = R(E_e)\sqrt{(E_0 - E_e)^2 - m_\nu^2} \qquad\qquad E_e = E_0 - E_\nu$$

Effect of the non-vanishing neutrino mass: $E_e^{max} = E_0 \rightarrow E_0 - m_\nu$

$\Rightarrow$ distorsion of the Ee spectrum close to the endpoint



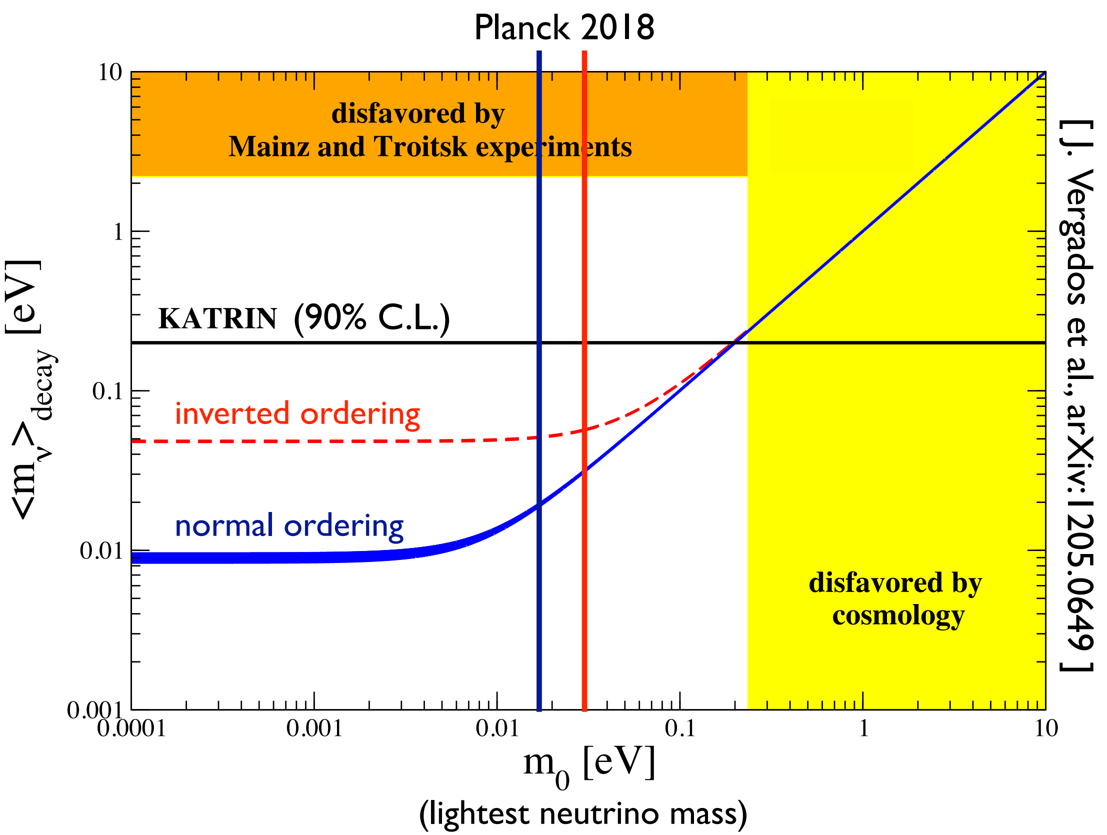
[Katrin Collaboration, hep-ex/0109033]

Present bound (KATRIN) : $m_\nu < 0.8 \text{ eV} \quad (90\% \text{ C.L.})$

KATRIN will reach a final sensitivity of about 0.3 eV (95% CL)
(5σ discovery potential 0.35 eV)

In pratice, there is no electron neutrino mass, but 3 strongly mixed mass eigenstates. However the energy resolution does not allow to resolve them, and what is measured is the effective mass

$$m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$



KATRIN will only test the degenerate case (already excluded by cosmology)

Future experiments like Project 8 aim at the 40 meV level (just below IO)

In practice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates, and

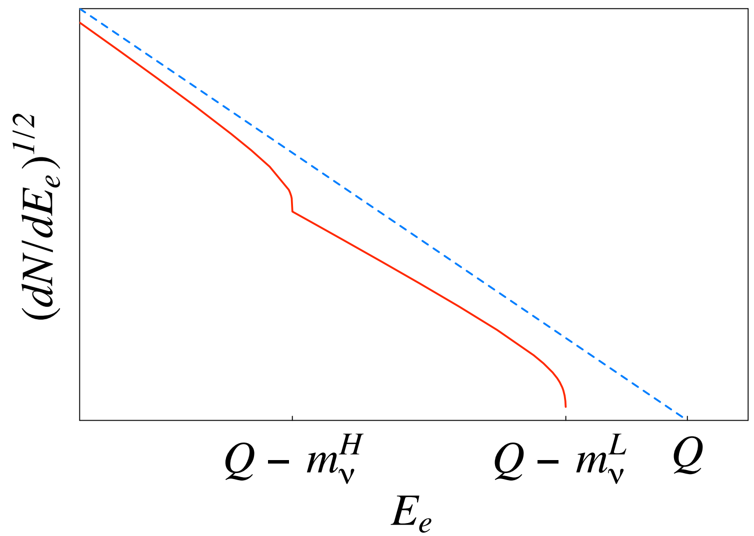
$$\frac{dN}{dE_e} = R(E_e) \sum_i |U_{ei}|^2 \sqrt{(E_0 - E_e)^2 - m_i^2}\, \Theta(E_0 - E_e - m_i)$$

If all mi's are smaller than the energy resolution, this can be rewritten as:

$$\frac{dN}{dE_e} = R(E_e) \sqrt{(E_0 - E_e)^2 - m_\beta^2} \qquad m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$

If there is an eV-scale sterile neutrino (comparable to the energy resolution of KATRIN), its mass may be resolved:

$$\frac{1}{R(E_e)} \frac{dN}{dE_e} = (1 - |U_{e4}|^2) \sqrt{(E_0 - E_e)^2 - m_\beta^2}$$
$$+ |U_{e4}|^2 \sqrt{(E_0 - E_e)^2 - m_4^2}\, \Theta(E_0 - E_e - m_4)$$

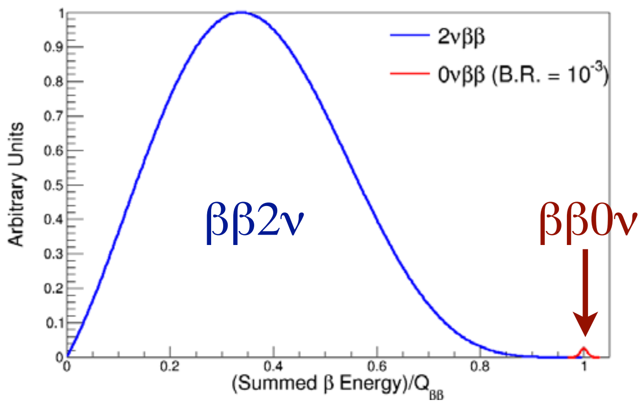


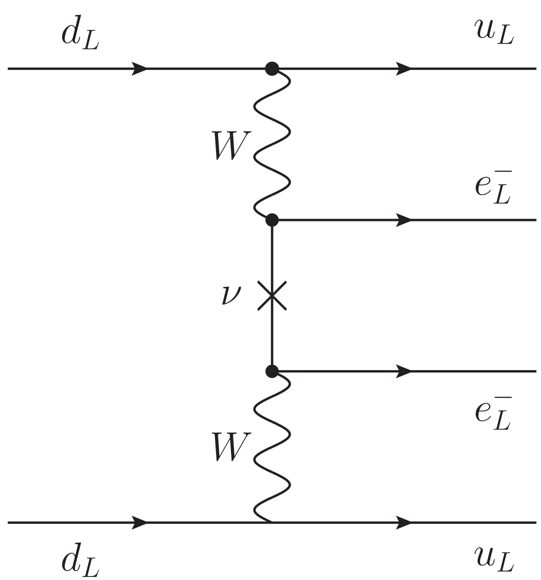
The neutrino nature: neutrinoless double beta decay

$$(A, Z) \;\rightarrow\; (A, Z+2) + e^- + e^-$$

violates lepton number by 2 units

$\Rightarrow$ possible only for Majorana neutrinos





$$Q_{\beta\beta} \equiv M_i - M_f - 2m_e = T_{e_1} + T_{e_2}$$

Half-life: $$\left[T^{0\nu}_{1/2}\right]^{-1} = \Gamma_{0\nu} = G_{0\nu}(Q_{\beta\beta}, Z)\; |M_{0\nu}|^2 \; |m_{\beta\beta}|^2$$

integrated phase-space factor

nuclear matrix element (NME)
(large theoretical uncertainty)

Sensitive to the effective mass parameter:

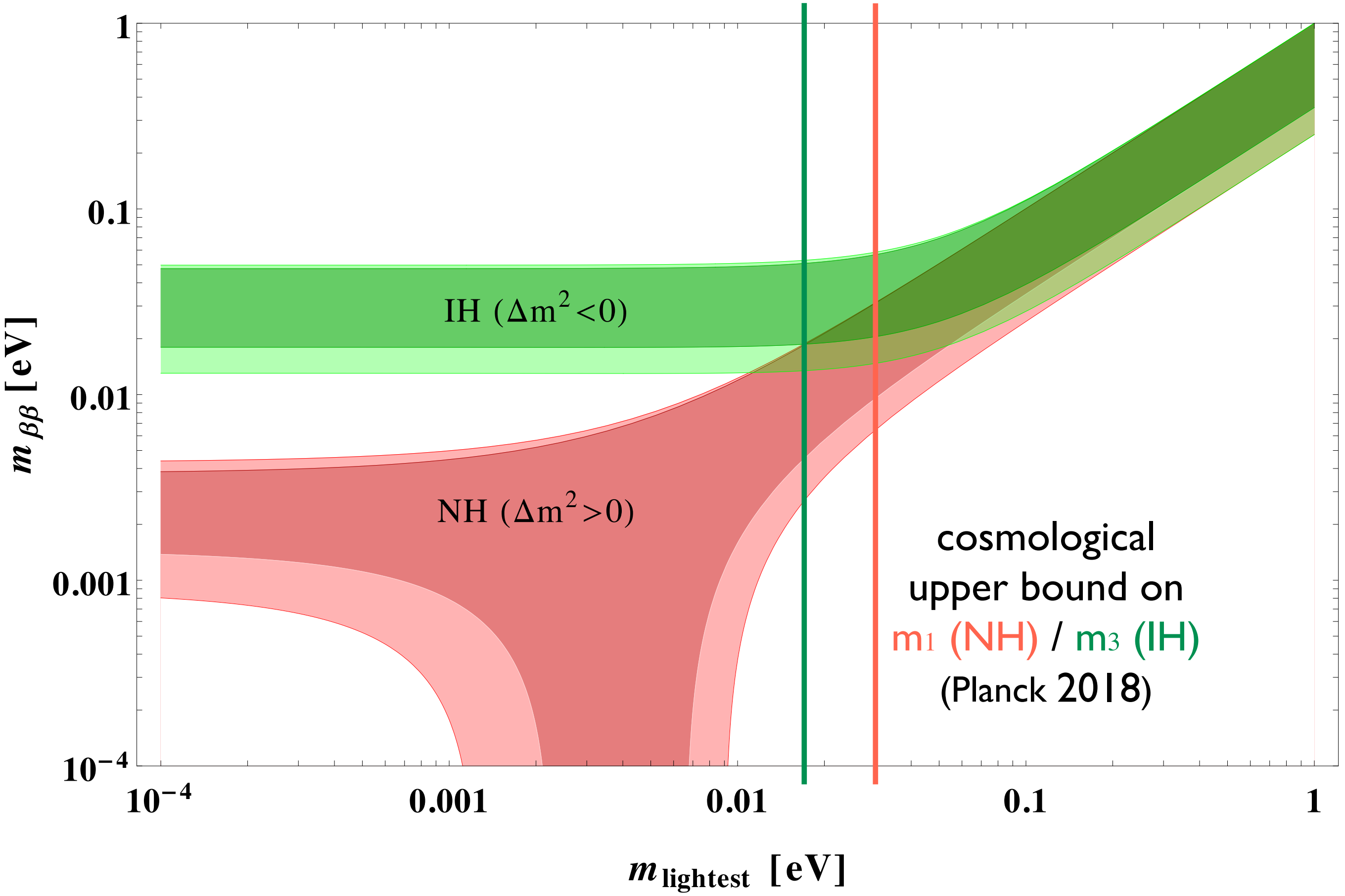
$$m_{\beta\beta} \equiv \sum_i m_i U_{ei}^2 = m_1 c_{13}^2 c_{12}^2 e^{2i\alpha_1} + m_2 c_{13}^2 s_{12}^2 e^{2i\alpha_2} + m_3 s_{13}^2$$

possible cancellations in the sum (Majorana phases α_1, α_2 in U)

[Dell'Oro et al., arXiv:1404.2616]

IH

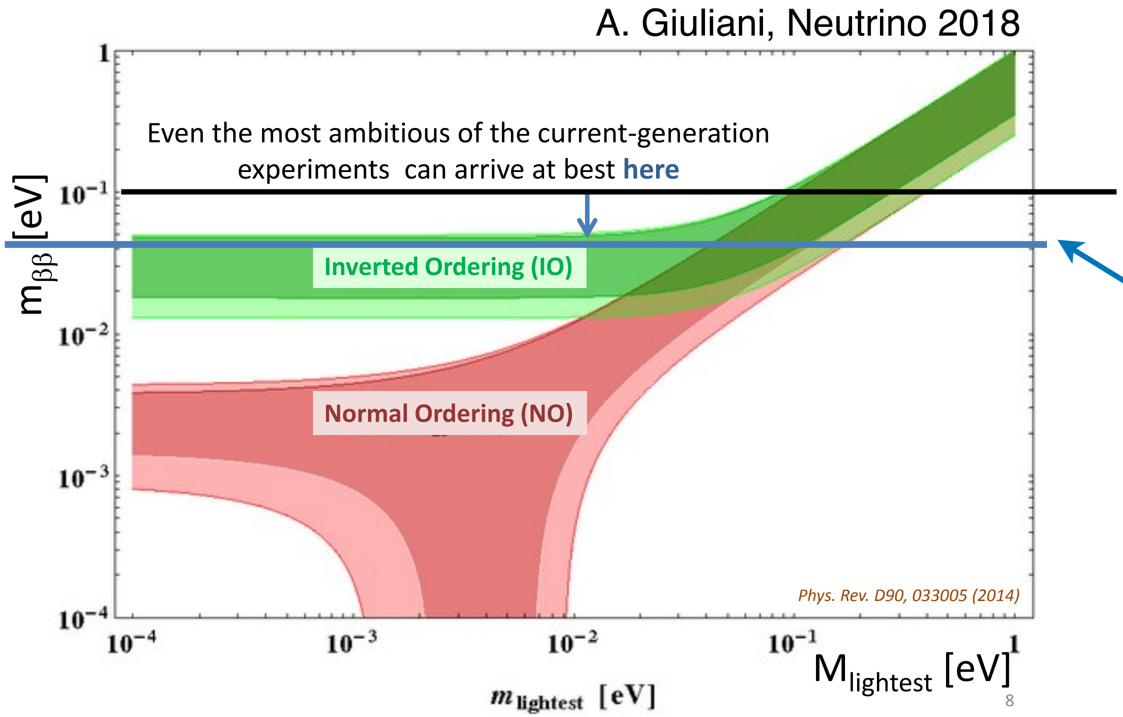
NH



dark shaded areas = best fit values of oscillation parameters (only α_1, α_2 vary)

light shaded areas = 3σ regions due to uncertainties on oscillation parameters (+ dependence on α_i)

- need to reach 10 meV to exclude IH (lower bound on $m_{\beta\beta}$)
- need to reach few meV to test NH (if no mass degeneracy)
- if unlucky (m_1 ~ 1-10 meV), may not observe ββ0ν even if neutrinos are Majorana (cancellation in $m_{\beta\beta}$ due to α_1, α_2)



currently here, around 100 meV
(experimental upper bounds
depend on NME calculations
⇒ 2 - 4 uncertainty factor)

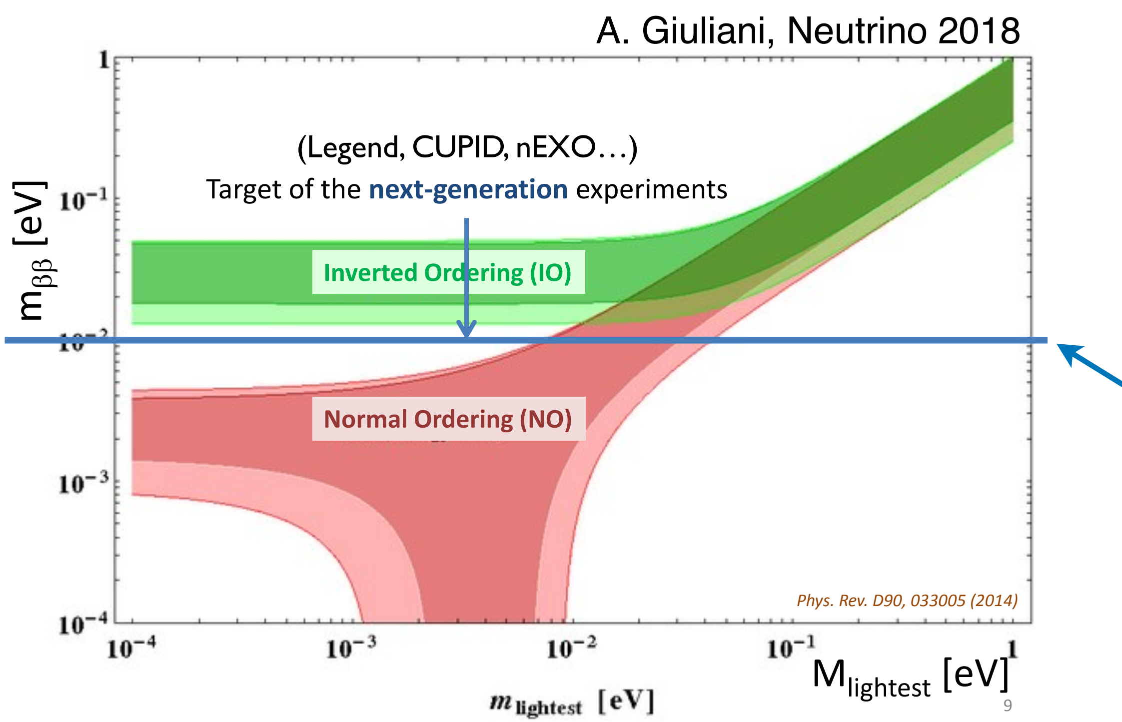
around 40 meV

Current best limit (90% C.L.) :
KamLAND-Zen (2022)
^{136}Xe-loaded liquid scintillator

$$T^{0\nu}_{1/2} > 2.3 \times 10^{26}\,\mathrm{yr}$$

$$m_{\beta\beta} < (36-156)\ \mathrm{meV}$$

(uncertainty from NMEs)



around 10 meV