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- Lecture 1: Introduction to flavor physics
- Lecture 2: Meson mixing, rare decays, universality tests
  - Neutral meson mixing
  - The structure of $\Delta F=2$ amplitudes
  - Rare $b \rightarrow s\, ll$ decays: generalities
  - Rare $b \rightarrow s\, ll$ decays: selected results
  - Tests of Lepton Flavor Universality
- Lecture 3: Flavor physics beyond the SM

University of Zurich UZH

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Recap from last lecture

$$\mathscr{L}_{\text{SM-EFT}} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{Higgs}} + \Sigma_i \frac{1}{\Lambda_i^{d-4}} O_i^{d\geq 5}$$

Large flavor symmetry

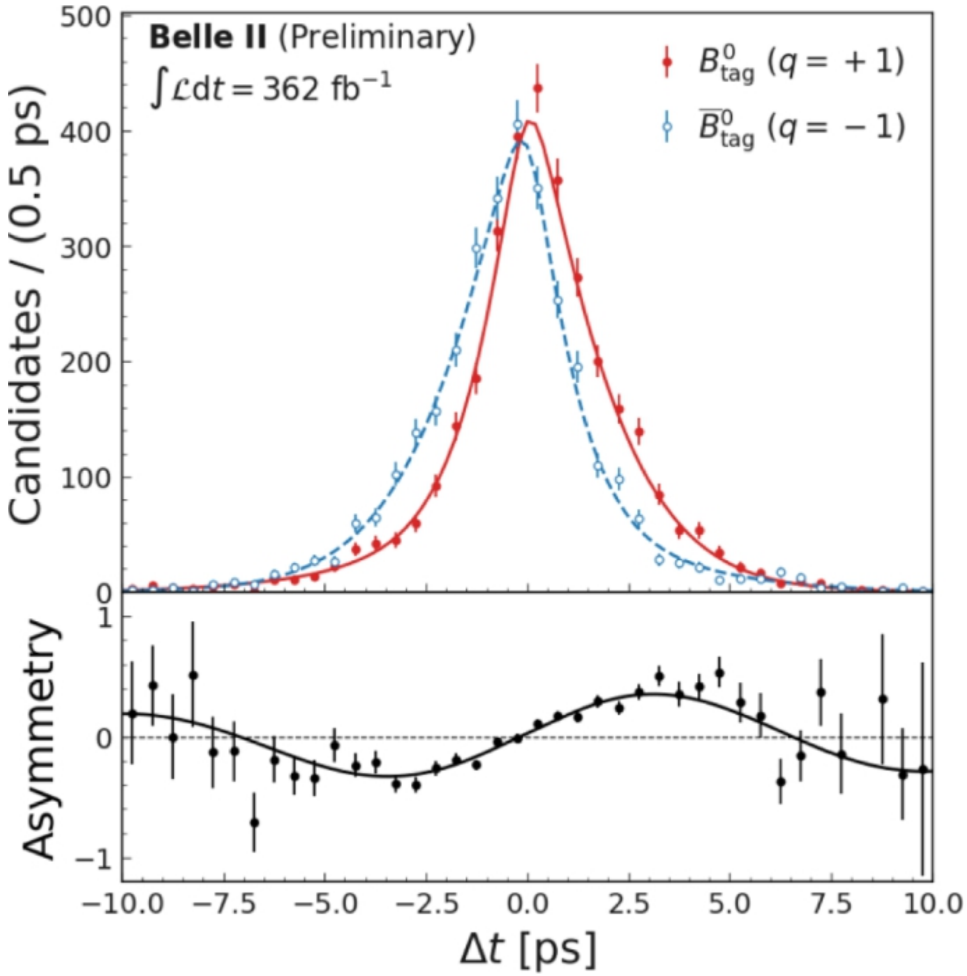
Yukawa interaction

Exact & approximate (*accidental* ?) symmetries

The great interest of precision measurements in flavor physics is the possibility to tests a large number of non-standard higher-dim. operators which may correspond to rather high-energy scales ↔ flavor structure BSM

Today we'll discuss in more detail some specific amplitudes & observables

Neutral meson mixing



▸ *Neutral meson mixing*

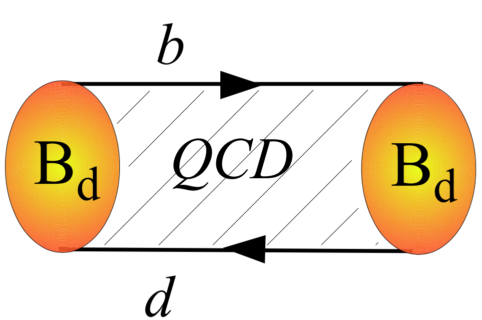
The lightest bound states (mesons) composed by a *quark-antiquark pair* of same charge but *different flavor* form very interesting systems: a pair of pseudo-scalar mesons with

- tiny mass difference (*due to 2nd order weak interactions*)
- mass eigenstates different from flavor eigenstates

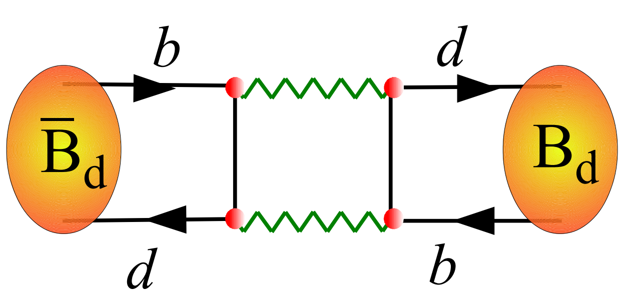
Four systems of this type: $K^0 = |\,\bar{s}\, d\,\rangle$, $B_d \equiv B^0 = |\,\bar{b}\, d\,\rangle$, $B_s = |\,\bar{b}\, s\,\rangle$, $D^0 = |\,\bar{c}\, s\,\rangle$

The interesting time-evolution of these systems has allowed to discover the phenomenon of CP violation in fundamental interactions (*observed for the first time in the neutral kaon system*).

E.g.:



$M_{Bd} = 5.279$ GeV



$\Delta M_{Bd} = 3.4\times10^{-13}$ GeV

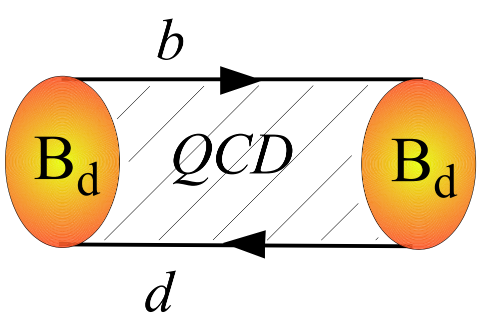
Neutral meson mixing

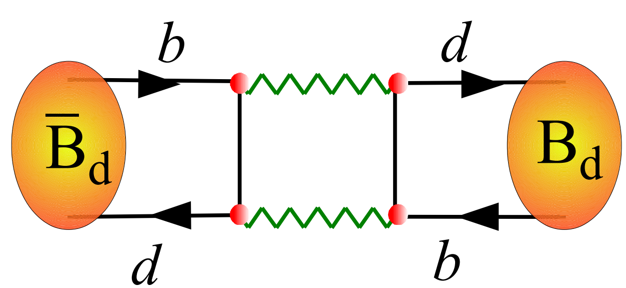
The effective Hamiltonian describing the ground state (i.e. the mass matrix) of these systems has a relatively simple structure:

$$i \frac{d}{dt} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix} = \begin{bmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{bmatrix} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix}$$

- The CPT theorem implies $M_{11} = M_{22} = M_0 = real$
- If CP were a good symmetry, then $M_{12} = M_{21} \rightarrow M_{12} = real$

  However, CP is violated in the SM (in the Yukawa sector) and the complex phase in the CKM matrix induces a complex phase in $M_{12} = |\Delta M/2|\, e^{i\phi_M}$





$M_0 = M_{Bd} = 5.279$ GeV

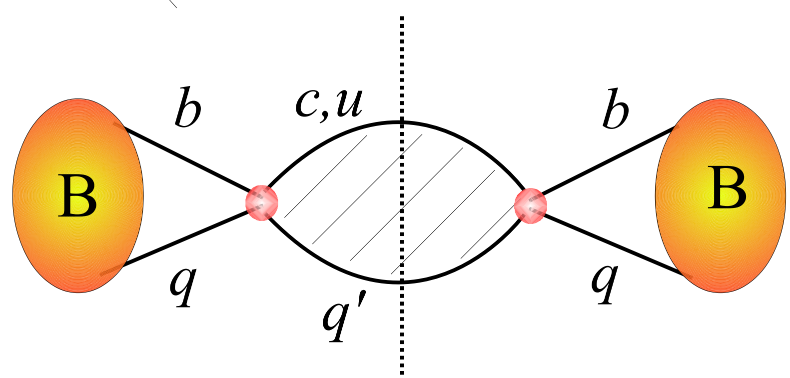
$2\,|\,M_{12}\,| = \Delta M_{Bd} = 3.4\times10^{-13}$ GeV

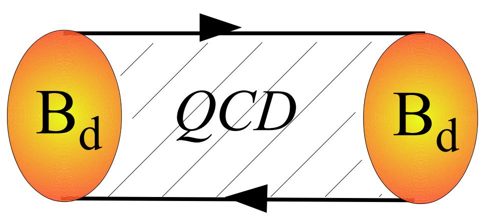
Neutral meson mixing

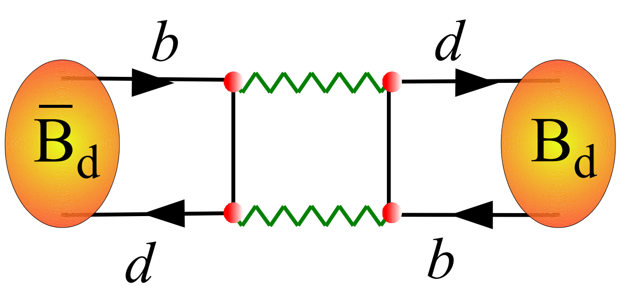
Taking into account the (weak) decay of the heavy quarks inside the mesons, the time evolution of the system is described by means of a non-Hermitian Hamiltonian:

$$i\,\frac{d}{dt}\begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix} = [\,M - i\,\Gamma/2\,]\begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix}$$

$$\begin{bmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{bmatrix}$$







$$|\,\Gamma_{12}/M_{12}\,| \ll 1$$

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$$\begin{bmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{bmatrix}$$

Mass eigenstates:

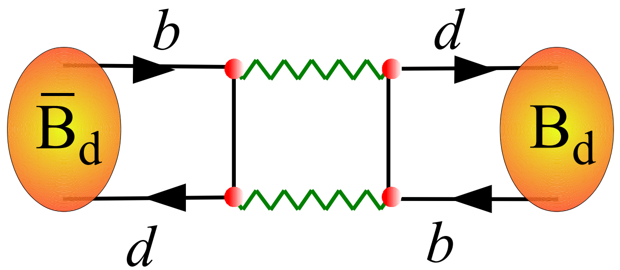
$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = q|B^0\rangle + p|\bar{B}^0\rangle$$

$$\frac{q}{p} \approx \arg(M_{12}) = e^{i\phi_{Bd}}$$

(in the limit $|\Gamma_{12}/M_{12}| << 1$)

Key observation: for B_d and B_s mesons both magnitude & phase of M_{12} can be computed precisely in the SM



Neutral meson mixing

The study of time-dependent decays of neutral B into CP eigenstates provides a marvelous tool to extract both phase & magnitude of the mixing amplitude:

$$
\begin{tikzcd}
\text{[t=0]}\quad B^0 \arrow[r, "q/p"] \arrow[rr, bend right, "A_f"'] & \bar{B}^0 \arrow[r, "\bar{A}_f"] & f \quad \text{[t]}
\end{tikzcd}
$$

$$\mathrm{CP}\,|f\rangle = \eta_f\,|f\rangle$$

$$\boxed{\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}}$$

Phase-convention independent quantity [$\leftrightarrow$ *observable*]

If $|\lambda_f| = 1$ (i.e. if A_f is dominated by a single weak phase) then :

$$\Gamma\left(B^0(t)\to f\right) \propto e^{-\Gamma_B t}\left[1-\eta_f\,\mathrm{Im}(\lambda_f)\,\sin\left(\Delta m_B t\right)\right]$$

$$\Gamma\left(\bar{B}^0(t)\to f\right) \propto e^{-\Gamma_B t}\left[1+\eta_f\,\mathrm{Im}(\lambda_f)\,\sin\left(\Delta m_B t\right)\right]$$

$$\mathrm{Im}(\lambda_f) = \sin(\underbrace{\phi_{Bd}}_{\textit{mixing phase}} - 2\,\underbrace{\phi_{A_f}}_{\textit{phase of } A_f})$$

Neutral meson mixing

The study of time-dependent decays of neutral B into CP eigenstates provides a marvelous tool to extract both phase & magnitude of the mixing amplitude.

Key points to successfully use this method:

- [EXP]: flavor tagging and time-dependent resolution are essential ingredients
- [TH]: identify final states such that A_f is dominated by a single weak phase

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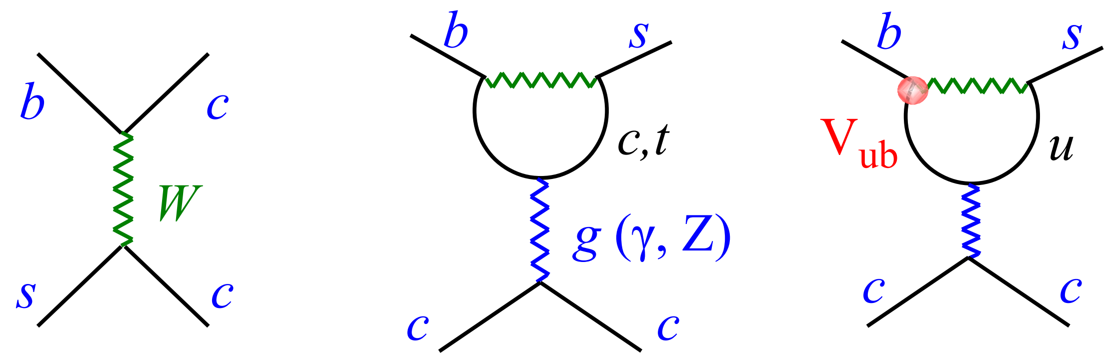
E.g.: $|B_d\rangle \to |\psi K_S\rangle$ $[\ b+d \to c\bar{c}s+d\ ]$



real $O(\lambda^2)$ real $O(\alpha_S\, \lambda^2)$ $O(\alpha_S\, \lambda^5)$

dominant amplitude pollution $\lesssim 1\ \%$

A_f = real (*in the standard CKM convention*)

Neutral meson mixing

The study of time-dependent decays of neutral B into CP eigenstates provides a marvelous tool to extract both phase & magnitude of the mixing amplitude.

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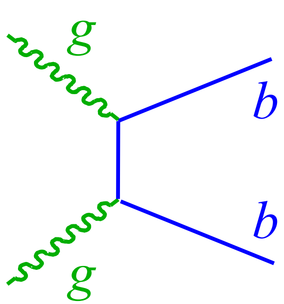
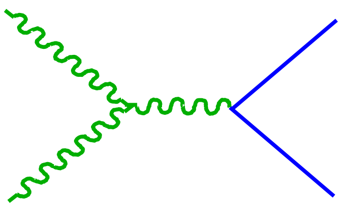
B factories:

$e^+ + e^- \rightarrow \Psi(4S) \rightarrow \bar{B}\ B$

- clean environment [$\sigma(B)/\sigma(bkg) \sim 0.3$]
- coherent quantum state → clean flavor tag from the opposite meson decay (e.g. $b \rightarrow c\, e^- \nu$)
- low stat. [~ 10^8 B pairs / 100 fb^{-1}]

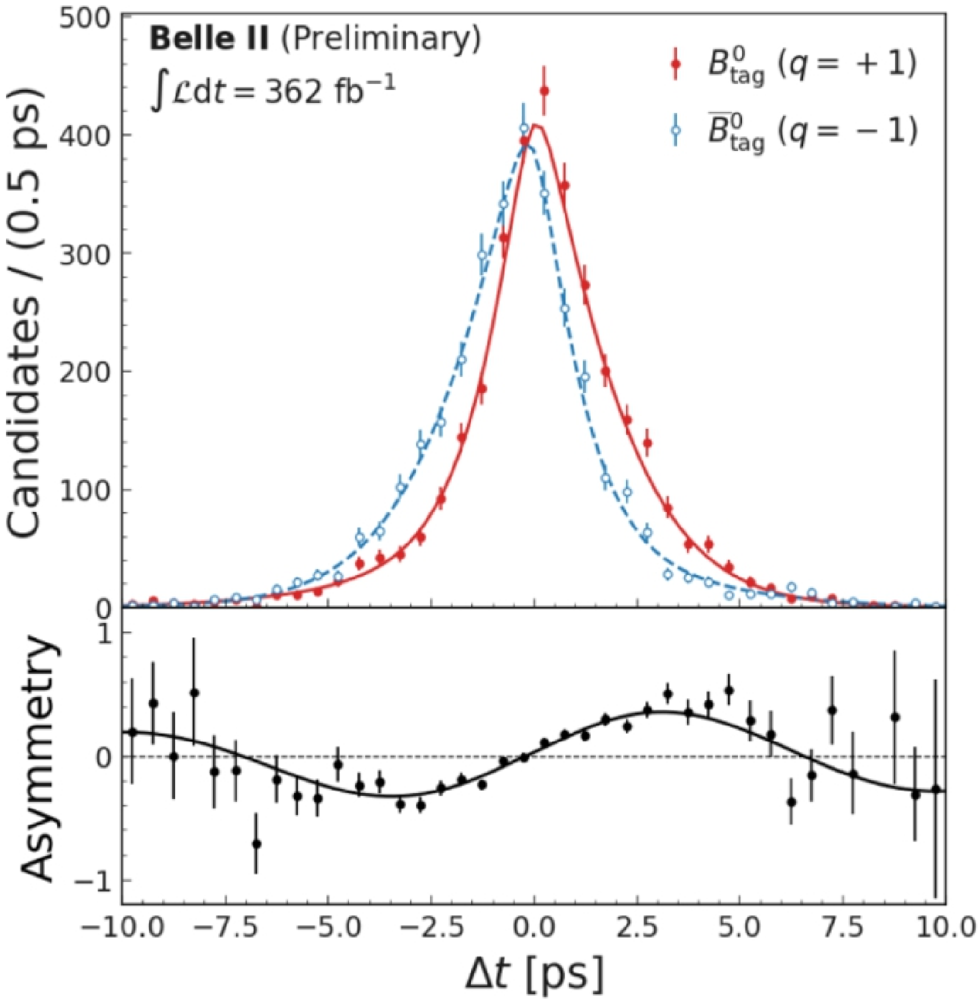
Hadron colliders:



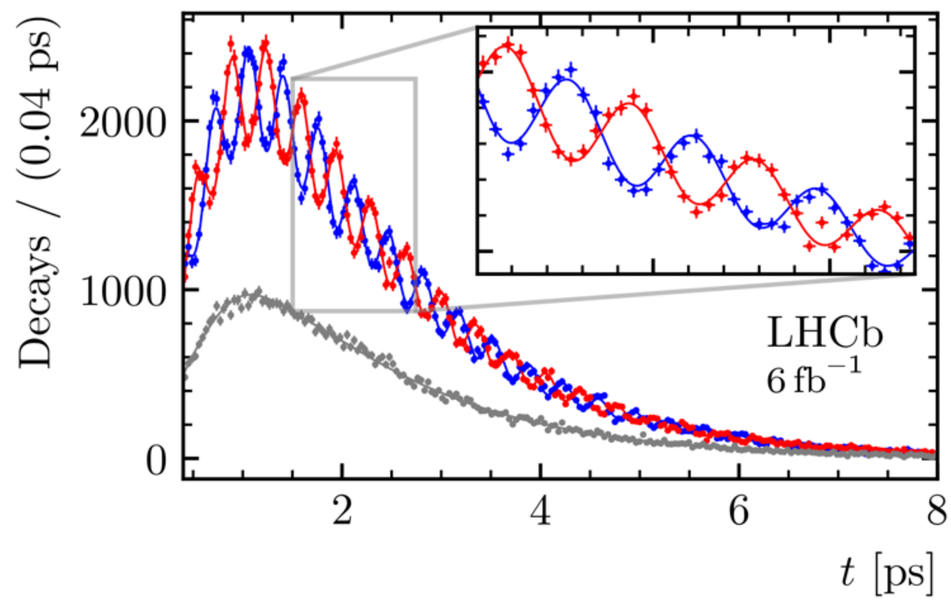


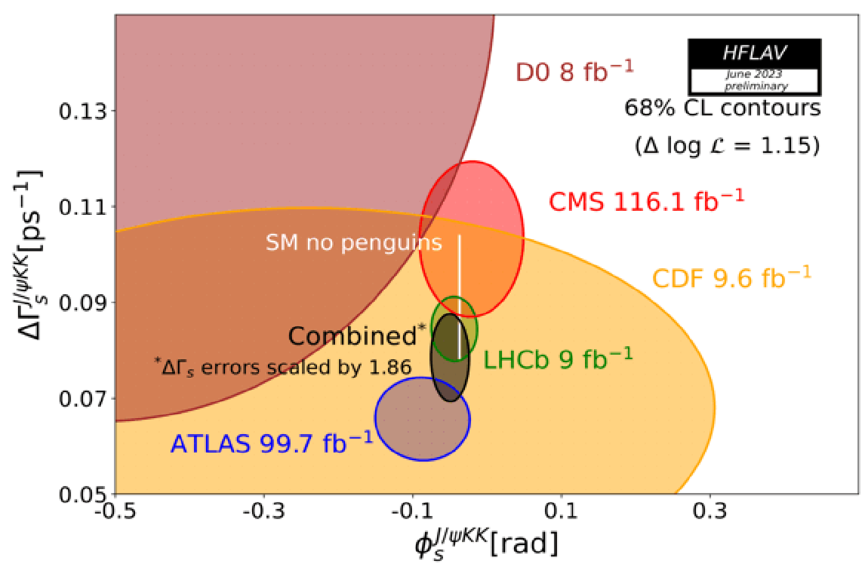
- dirty environment [$\sigma(B)/\sigma(bkg) < 0.01$]
- incoherent quantum state
- high stat. [~ 10^{12} B pairs / 1 fb^{-1}]
- all hadrons with b-quarks produced

Neutral meson mixing



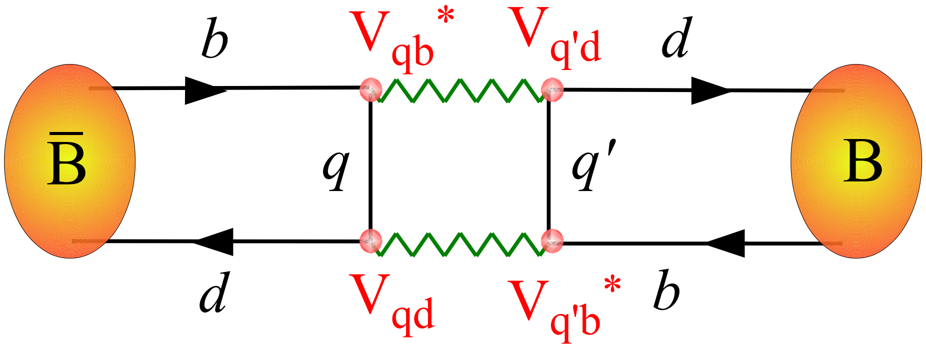
B_s mixing





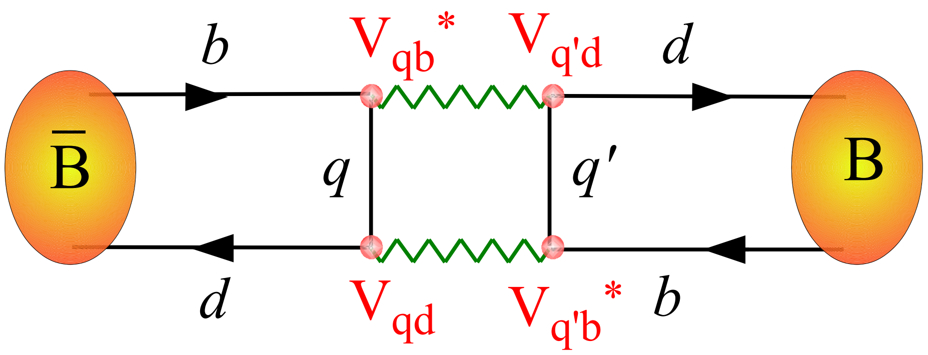
Nowadays we have an excellent exp. knowledge of both magnitude & phase of both B_d & B_s mixing amplitudes

The structure of ΔF=2 amplitudes



The structure of $\Delta F=2$ amplitudes

Let's give a closer look to the mixing amplitude:

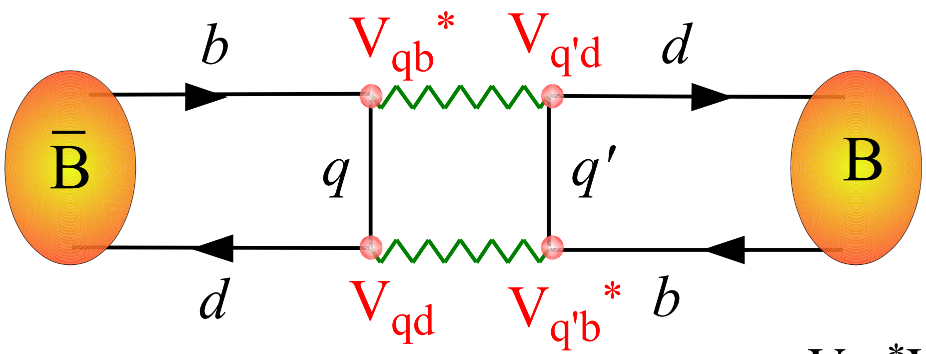


Highly suppressed amplitude potentially very sensitive physics beyond the SM

- No SM tree-level contribution
- Strong suppression within the SM due to the CKM hierarchy
- Measurable with good accuracy via the time evolution of the neutral meson systems
- Calculable with good accuracy since dominated by short-distance dynamics [power-like "GIM mechanism" → top-quark dominance]

The structure of ΔF=2 amplitudes

We can understand why the intermediate top-quark contribution is dominant (*i.e. the reason why it can be computed precisely*) from simple arguments:



$$A_{\Delta F=2} = \Sigma_{q,q'=u,c,t} (V_{qb}^{*} V_{qd}) (V_{q'b}^{*} V_{q'd}) \, A_{q'q}$$

$V_{ub}^{*} V_{ud} = - V_{tb}^{*} V_{td} - V_{cb}^{*} V_{cd}$ [CKM unitarity]

$$A_{\Delta F=2} = \Sigma_{q=u,c,t} (V_{qb}^{*} V_{qd}) \left[V_{tb}^{*} V_{td} (A_{tq} - A_{uq}) + V_{cb}^{*} V_{cd} (A_{cq} - A_{uq}) \right]$$

"GIM" cancellation

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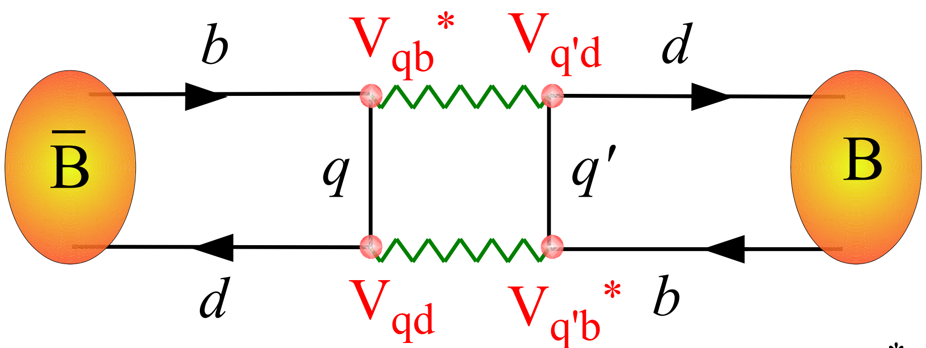

$$A_{\Delta F=2} = \Sigma_{q,q'=u,c,t} (V_{qb}^* V_{qd})\,(V_{q'b}^* V_{q'd})\, A_{q'q}$$

$$V_{ub}^* V_{ud} = - V_{tb}^* V_{td} - V_{cb}^* V_{cd} \qquad \text{[CKM unitarity]}$$

$$A_{\Delta F=2} = \Sigma_{q=u,c,t} (V_{qb}^* V_{qd})\,[\, V_{tb}^* V_{td}\,(A_{tq}-A_{uq}) + V_{cb}^* V_{cd}\,(A_{cq}-A_{uq})\,]$$

$$A_{qq'} \sim \frac{g^4}{16\pi^2 m_W^2} \left[\cancel{\text{Const.}} + \frac{m_q\, m_{q'}}{m_W^2} + \ldots \right] \langle \bar{B} |\, (\bar{b}_L \gamma_\mu d_L)^2\, | B \rangle$$

[expansion of the loop amplitude for small (internal) quark masses]

$$A_{\Delta F=2} \sim (V_{tb}^* V_{td})^2 \; \frac{g^4 m_t^2}{16\pi^2 m_W^4} + \ldots$$

The structure of ΔF=2 amplitudes

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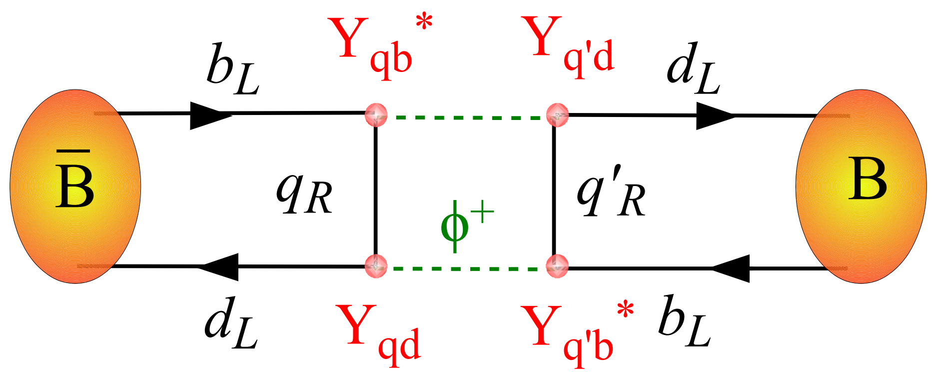

It is even more instructive (*and more correct...*) to compute the amplitude in the limit where we *switch-off* gauge interactions ("gauge-less limit")

$$Y_U = V^+ \times \mathrm{diag}(y_u, y_c, y_t)$$

$$\mathscr{L}_{\text{Yukawa}} \rightarrow \bar{d}_L{}^i \, Y_U{}^{ik} \, u_R{}^k \, \phi^- + h.c.$$

- *In the gauge-less limit there is no W field*
- *The three Goldstone bosons associated to the Higgs field remain massless*
- *The charged Goldstone bosons mediate flavor-changing interactions*

The structure of ΔF=2 amplitudes

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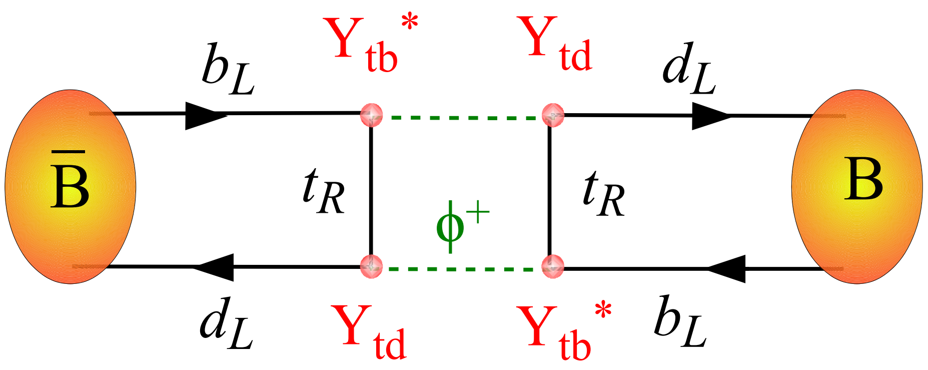

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$$\mathscr{L}_{\rm Yukawa} \rightarrow \bar{d}_L^{\,i}\, Y_U^{ik}\, u_R^{\,k}\, \phi^- + h.c.$$

$$A^{\text{gauge-less}}_{\rm DF=2} \sim (V_{tb}^* V_{td})^2 \frac{(y_t)^4}{16\pi^2 m_t^2} \sim (V_{tb}^* V_{td})^2 \frac{g^4 m_t^2}{16\pi^2 m_W^4}$$

$$m_t = y_t\, v/\sqrt{2}$$
$$m_W = g\, v/2$$

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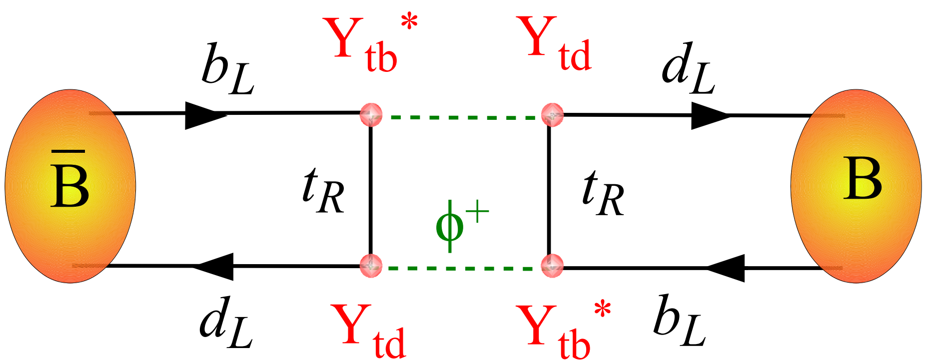

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$$m_t = y_t\, v/\sqrt{2}$$
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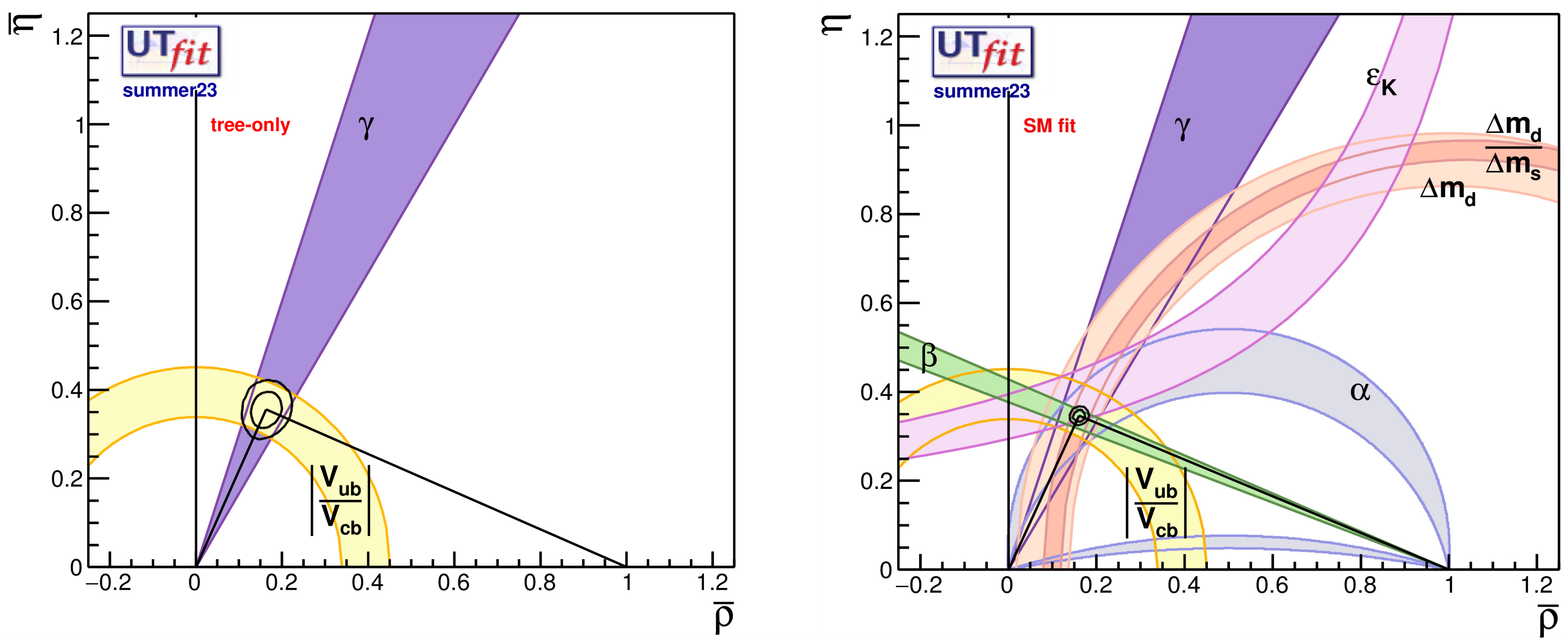
This procedure leads to the exact result in the limit $m_t \gg m_W$:

$$A^{\rm full}_{\rm DF=2} = A^{\text{gauge-less}}_{\rm DF=2} \times [\, 1 + O(g^2)\,]$$

Clear demonstration that flavor-changing processes originate from the Yukawa sector of the SM

The structure of ΔF=2 amplitudes & corresponding BSM bounds

Current data show no significant deviations from the SM (at the 5%-30% level, depending on the specific amplitude) on ΔF =2 observables (mass differences and CP-violating phases) → strong bounds on possible BSM contributions:



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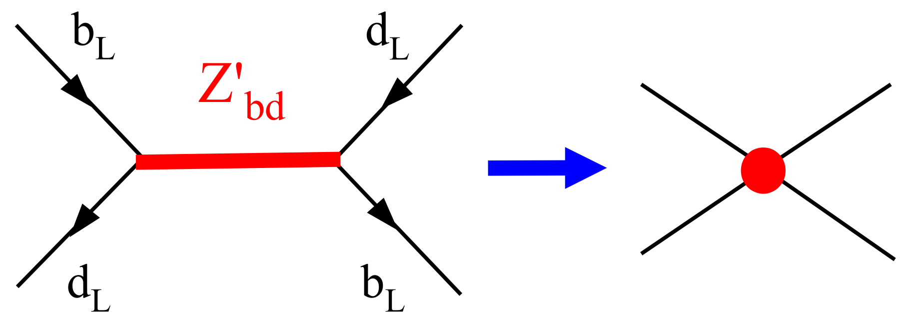
$$M(B_d - \bar{B}_d) \sim \frac{(y_t^2 V_{tb}^* V_{td})^2}{16\pi^2 m_t^2} + c_{NP} \frac{1}{\Lambda^2}$$

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM} + \sum \frac{c_n}{\Lambda^{d-4}} O_n^{(d)}$$

The list of dimension 6 ops. includes $(b_L \gamma_\mu d_L)^2$ that contributes to B_d mixing at the tree-level

Possible dynamical origin of this d=6 operator:



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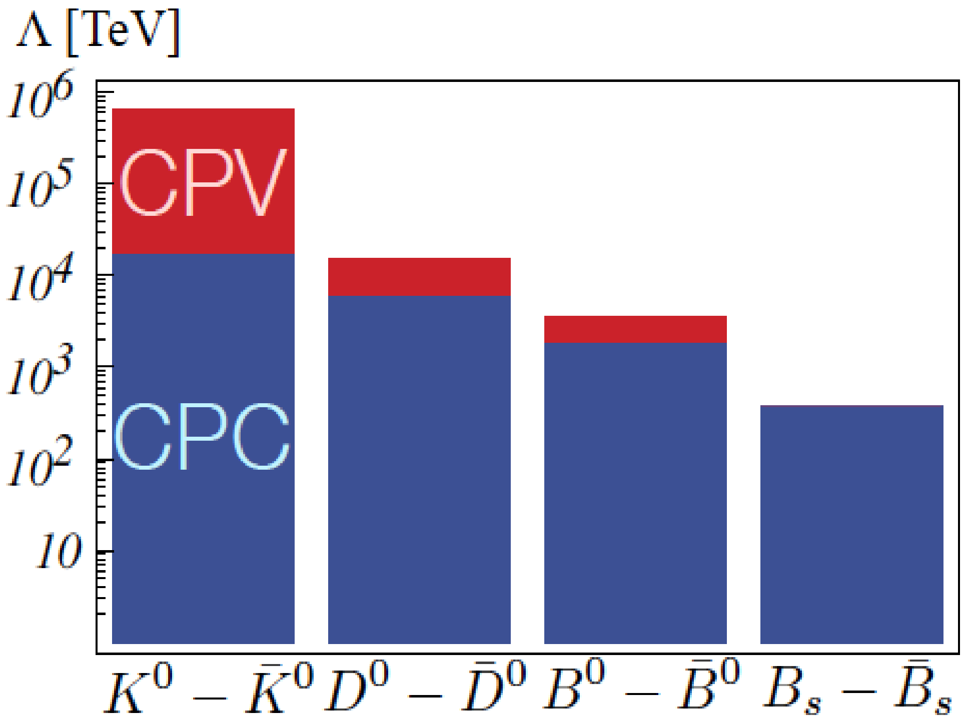
Operator	Bounds on Λ (TeV) Re	Im
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2	1.1×10^2
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	3.7×10^2	3.7×10^2



Quite discouraging at first sight...

However, remember the discussion about accidental symmetries: these seemingly high scales could well be a "mirage"...

The structure of ΔF=2 amplitudes & corresponding BSM bounds

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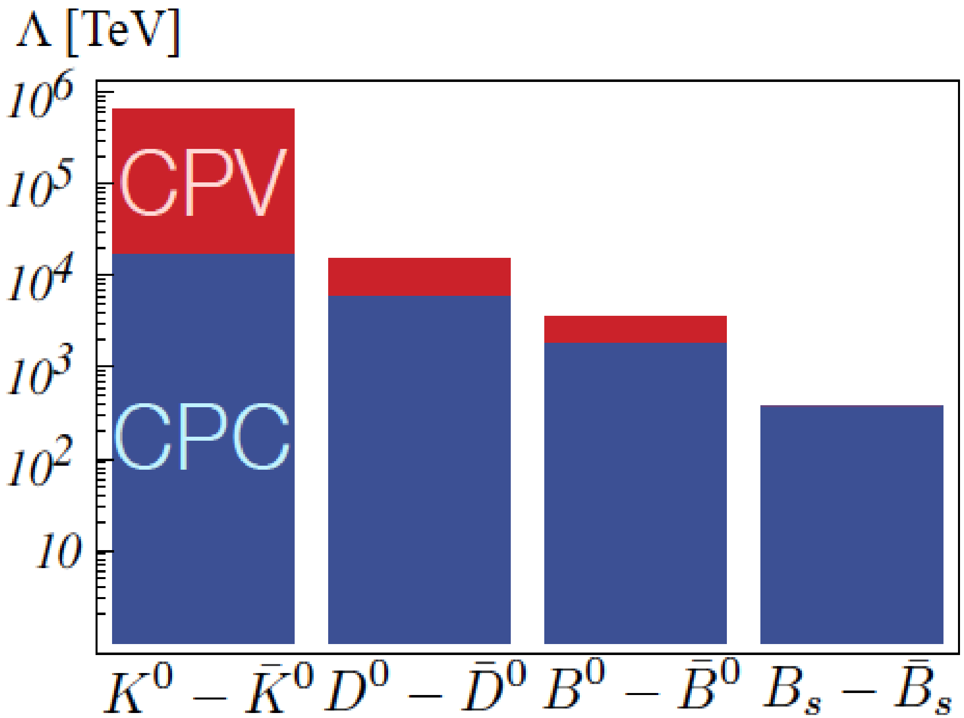

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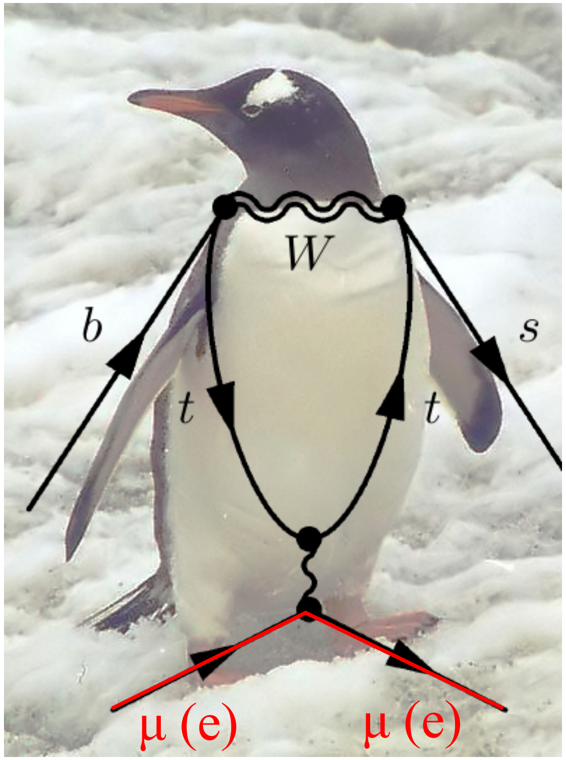
However, remember the discussion about accidental symmetries: these seemingly high scales could well be a "mirage"...

The only unambiguous message is:

No large breaking of the approximate $U(2)^n$ flavor symmetry at near-by energy scales

Rare $b \to s\, l^+ l^-$ decays: generalites



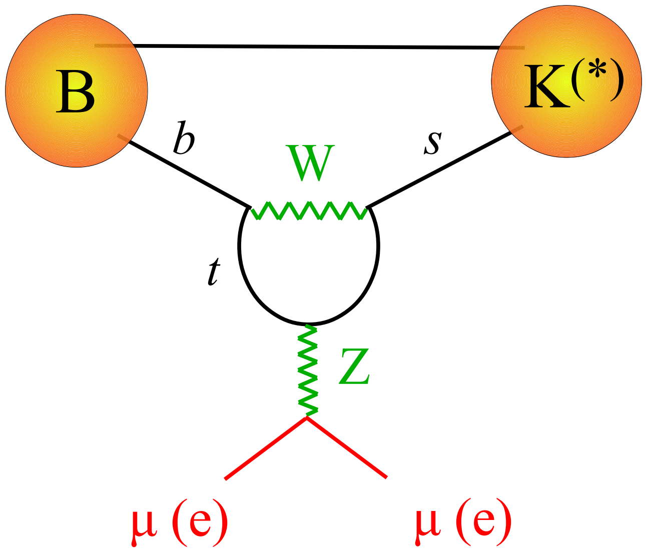
Rare $b \to sll$ decays: generalities

The ΔF=2 amplitudes are not the only suppressed flavor-changing amplitudes we are able to compute & measure precisely, enabling precise BSM tests.

An interesting complementary category are ΔF=1 Flavor Channing Neutral Current amplitudes, such as $b \to s\, l^+ l^-$

- No SM tree-level contribution
- Strong suppression within the SM because of CKM hierarchy
- Measurable with high accuracy (*in several cases*)
- More complicated interplay of QCD and weak interactions (*or, equivalently, interplay of short- and long-distance dynamics*)

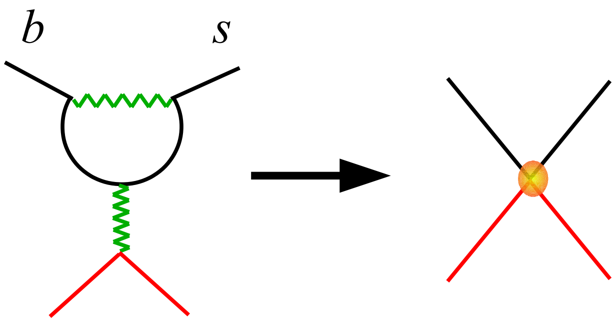
E.g.:



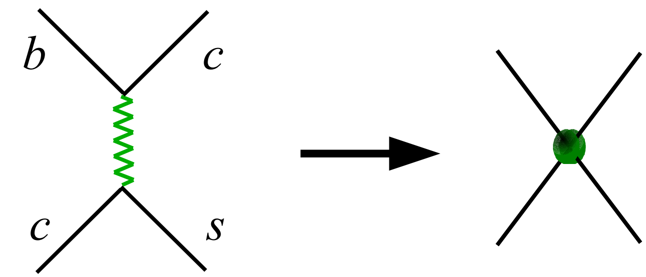
Rare $b \to sll$ decays: generalities

General procedure in 3 steps to separate the different energy scales of the problem

1st step: Construction of a local eff. Lagrangian at the electroweak scale integrating out all the heavy fields above & around m_W (*including the heavy SM fields*)



$$\mathscr{L}_{eff} = \Sigma_i\, C_i(M_W)\, Q_i$$



FCNC operators:

$$Q_6 = \sum_q (\bar{b}_L \gamma_\mu s_L)\bar{q}\gamma^\mu q \quad \text{[Gluon penguin]}$$

$$\vdots$$

$$Q_9 = (\bar{b}_L \gamma_\mu s_L)\bar{l}\gamma^\mu l \quad [\gamma \text{ \& Z penguin + box}]$$

$$Q_{10} = (\bar{b}_L \gamma_\mu s_L)\bar{l}\gamma^\mu \gamma_5 l \quad \text{[Z penguin + box]}$$

Four-quark (tree-level) ops.:

$$Q_1 = (\bar{b}_L \gamma_\mu s_L)(\bar{c}_L \gamma^\mu c_L)$$

$$Q_2 = (\bar{b}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu s_L)$$

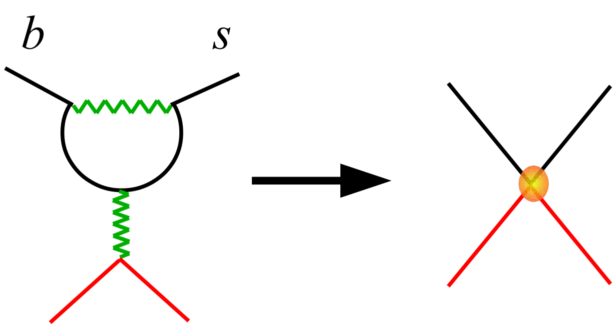
$$\vdots$$

The interesting short-distance info is encoded in the $C_i(M_W)$ (*initial conditions*) of the Wilson coefficients of the FCNC operators

Rare $b \to sll$ decays: generalities

General procedure in 3 steps to separate the different energy scales of the problem

1st step: Construction of a local eff. Lagrangian at the electroweak scale integrating out all the heavy fields above & around m_W (*including the heavy SM fields*)



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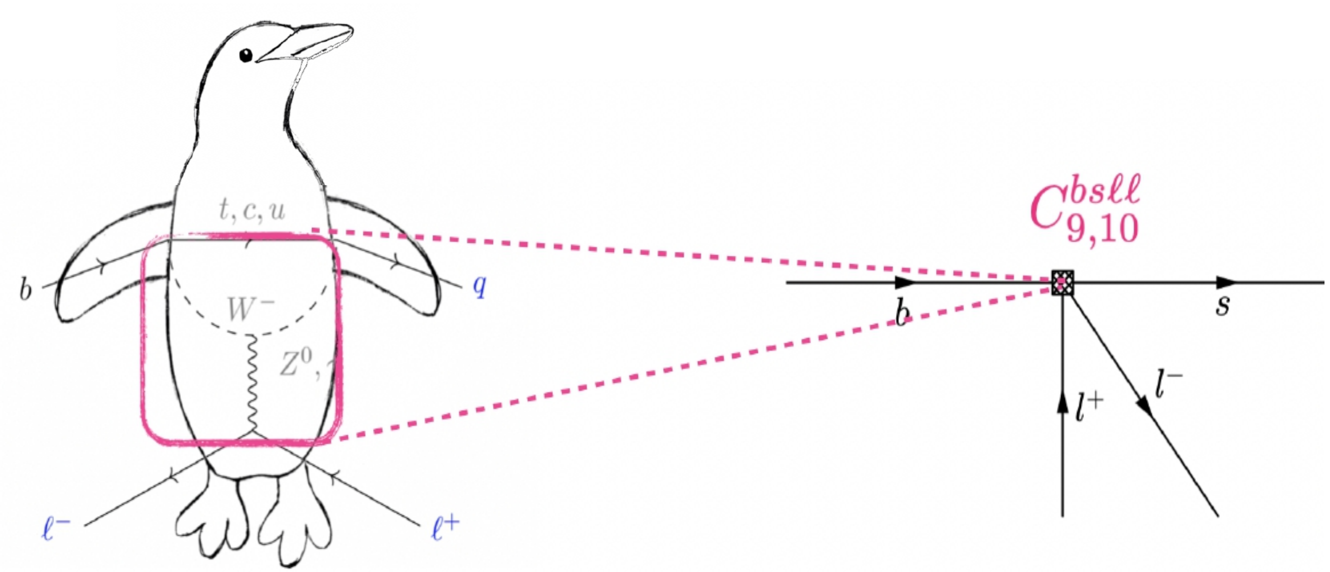

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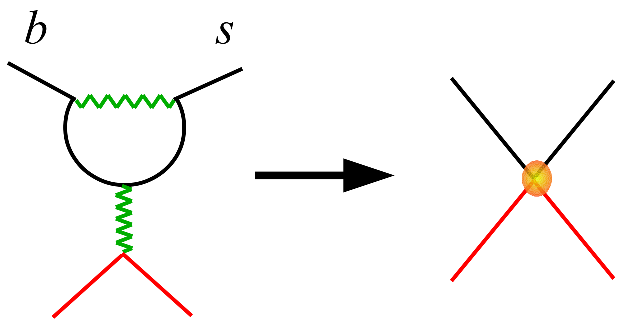

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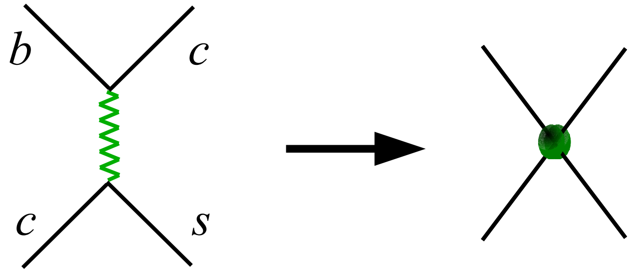

FCNC operators:

$$Q_6 = \sum_q (\bar{b}_L \gamma_\mu s_L)\bar{q}\gamma^\mu q$$ [Gluon penguin + NP...]

$\vdots$

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$\vdots$

N.B.: Non-standard effects due to heavy new particles result, in full generality, in modified values for the $C_i(M_W)$ (*possibly also when they are vanishing in SM*)

Rare $b \to sll$ decays: generalities

2nd step: Evolution of L_{eff} down to low scales using the renormalization group

Penguin operators:

$$Q_6 = \sum_q (\bar{b}_L \gamma_\mu s_L)\, \bar{q}\, \gamma^\mu q$$
$$\vdots$$
$$Q_9 = (\bar{b}_L \gamma_\mu s_L)\, \bar{l}\, \gamma^\mu l$$
$$Q_{10} = (\bar{b}_L \gamma_\mu s_L)\, \bar{l}\, \gamma^\mu \gamma_5 l$$

$$\boxed{\mathscr{L}_{eff} = \Sigma_i\, C_i(M_W)\, Q_i}$$
$$\downarrow$$
$$\boxed{\mathscr{L}_{eff} = \Sigma_i\, C_i(\mu \sim m_b)\, Q_i}$$

Four-quark (tree-level) ops.:

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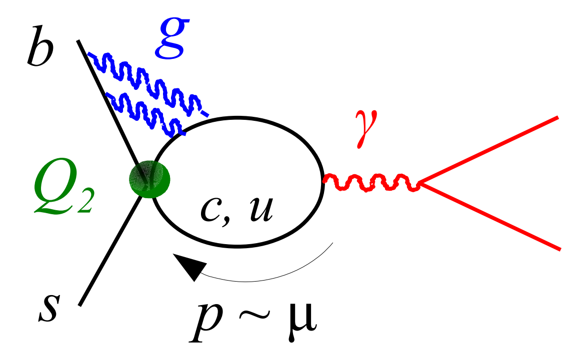
$$\vdots$$

Potential *dilution of the short-distance information*: due to the mixing of the four-quark Q_i into the FCNC Q_i [perturbative long-distance contribution]

e.g.:



- This effect is very small in operators which cannot be generated at low scales, such as Q_{10} ($\leftrightarrow$ Z penguin & W box) or the analog of $Q_{9,10}$ for $l=\nu$
- But is large for (most) other operators, such as Q_9

Rare $b \to sll$ decays: generalities

3rd step: Evaluation of the hadronic matrix elements

$$A(B \to f) = \Sigma_i\, C_i(\mu)\, \langle f |\, Q_i \,| B \rangle (\mu) \qquad [\, \mu \sim m_b \,]$$

Local matrix element of quark bilinears, such as

$$\langle K \,|\, \bar{b}\, \gamma_\mu\, s \,| B \rangle$$

- Good precision from Lattice QCD (*as in charged-curr. semileptonic decays*)

Non-local matrix of four-quark ops. involving charm ($Q_{1,2}$) $\to$ non-perturbative long-distance effects, very large in specific kinematic regions

- Irreducible theory error (*beside few exceptions*)

Rare $b \to sll$ decays: generalities

3rd step: Evaluation of the hadronic matrix elements

$$A(B \to f) = \Sigma_i\, C_i(\mu)\ \langle f \,|\, Q_i \,|B\, \rangle(\mu) \qquad [\ \mu \sim m_b\ ]$$

E.g.: $B \to K \mu^+\mu^-$

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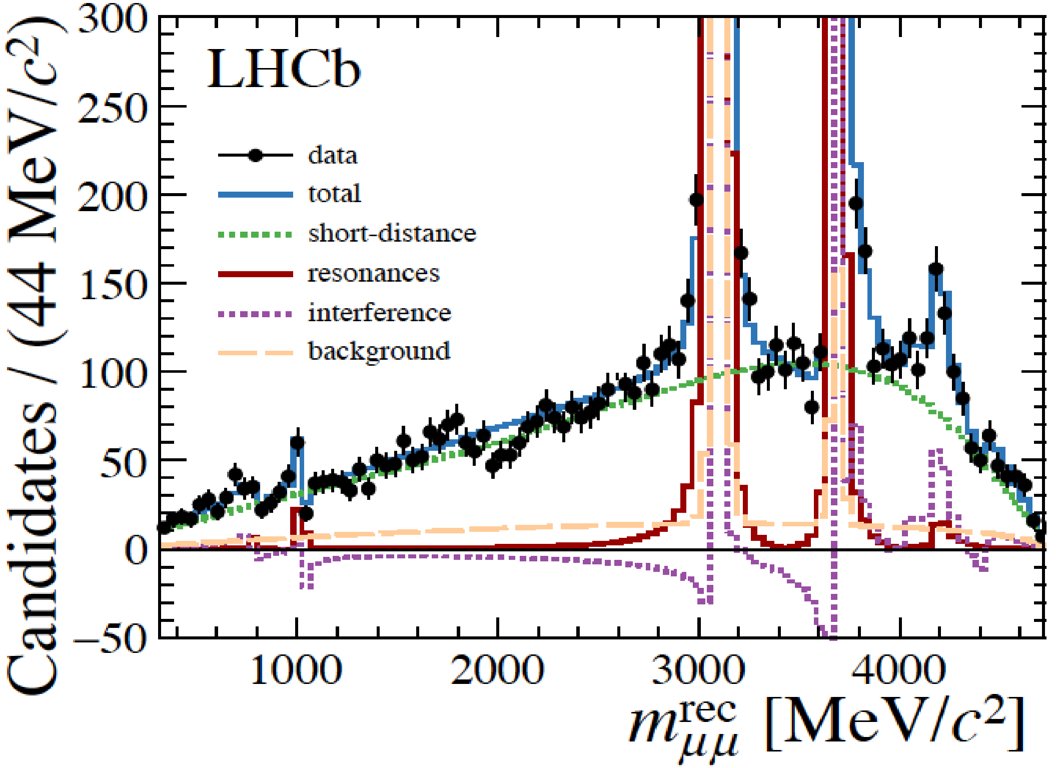
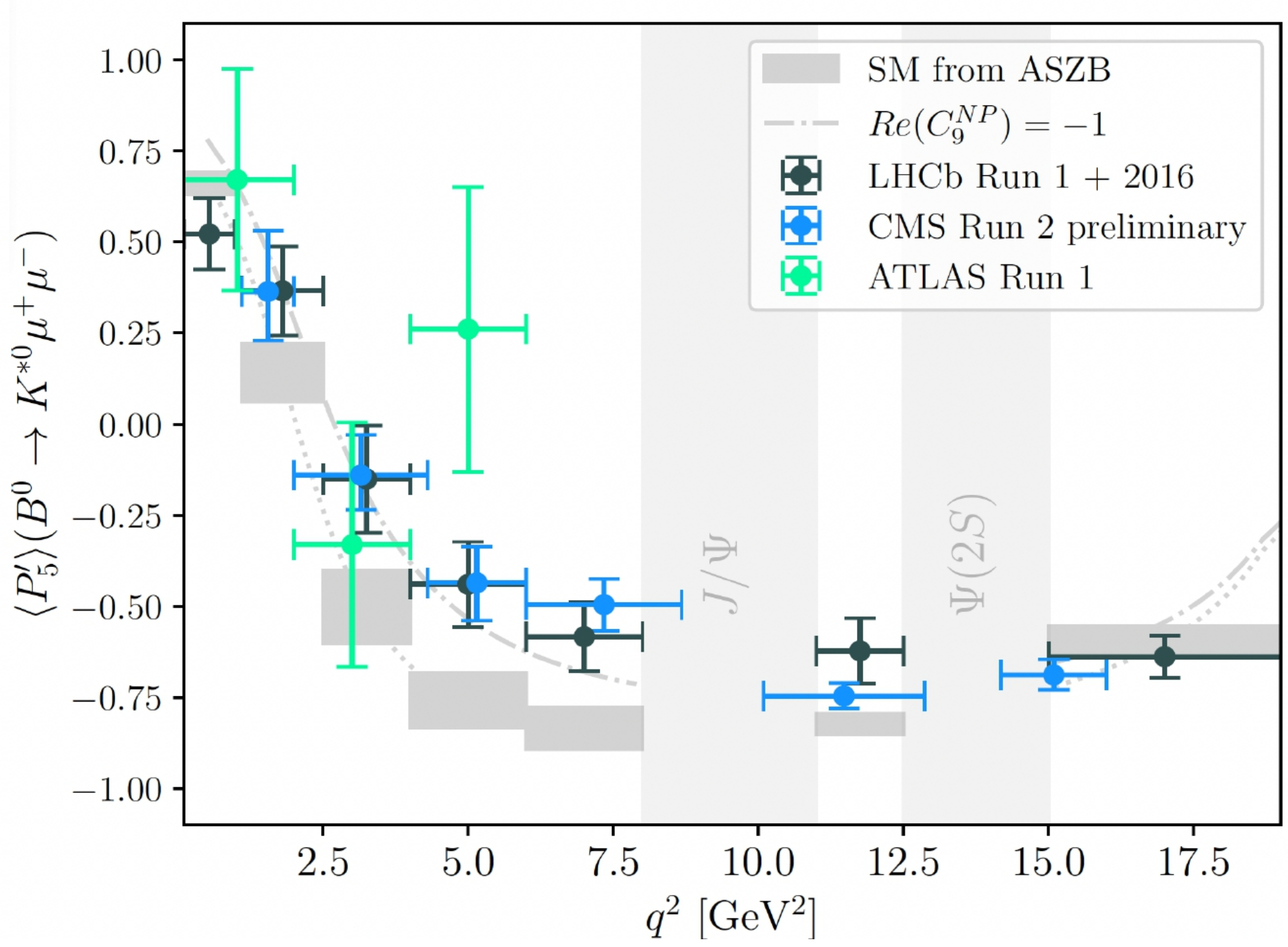


Rare $b \to s\ l^+l^-$ decays: selected results



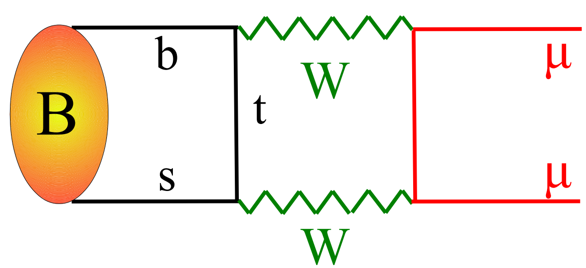
Rare $b \to sll$ decays: selected results

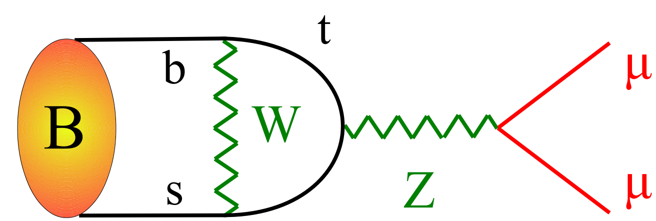
I. $B_s \to \mu^+\mu^-$

This decay is a special case among exclusive *B* decays:

- The kinematics (*lepton pair in J=0*) forbids vector-current contributions → fully dominated by short-distance [only the Q_{10} op. contributes in the SM]
- Hadronic matrix element particularly simple: $\langle 0 | \bar{b}\, \gamma_\mu \gamma_5\, s \,|\, B_s(p) \rangle = i f_{B_s} p_\mu$

Leading SM diagrams (unitary gauge):





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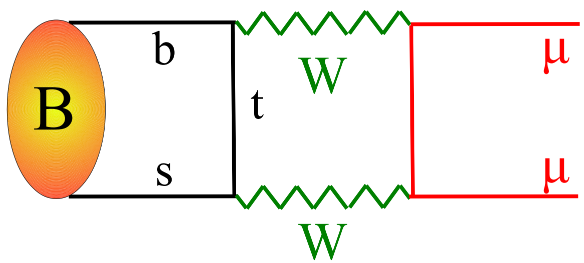
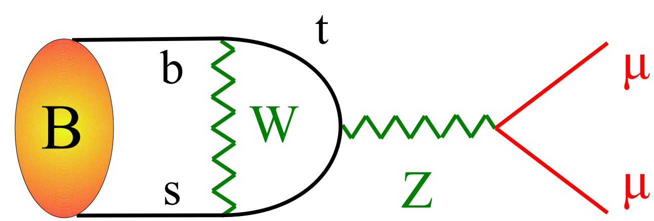
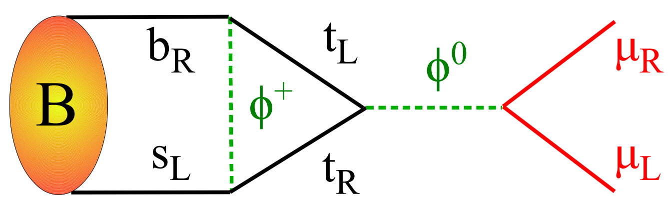




gauge-less limit



good approx. to the full SM amplitude

Very clean probe of the Yukawa mechanism
(sensitive probe of possible extended Higgs sectors)

Rare $b \to sll$ decays: selected results

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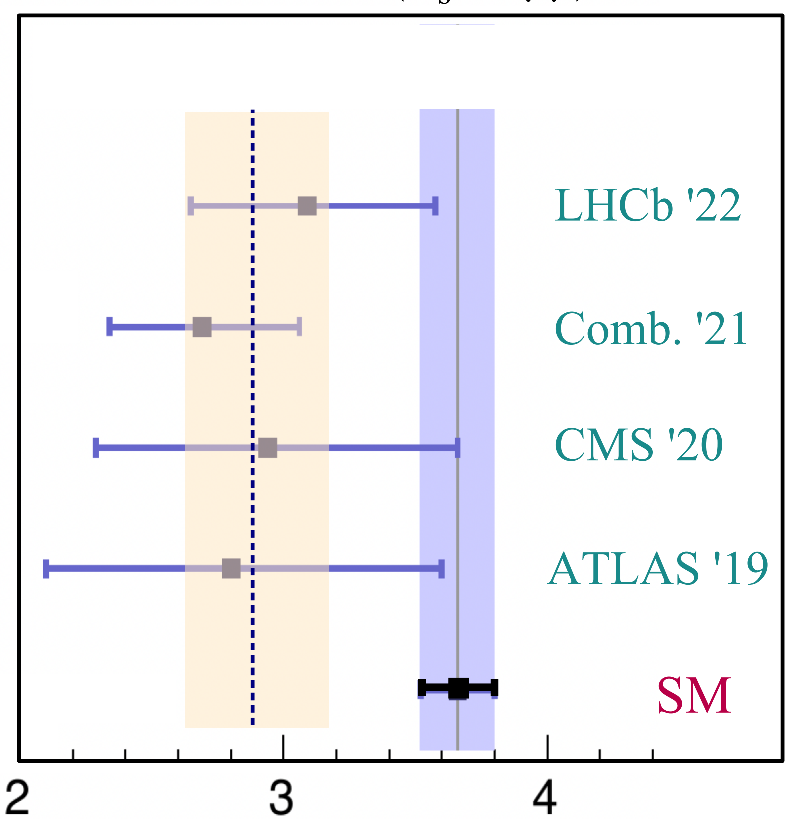
$10^9 \times B(B_s \to \mu\mu)$



For a few years an interesting (mild) tension was observed.

$$BR_{SM} = (3.66 \pm 0.14) \times 10^{-9}$$

Beneke *et al.* '19

Rare $b \to sll$ decays: selected results

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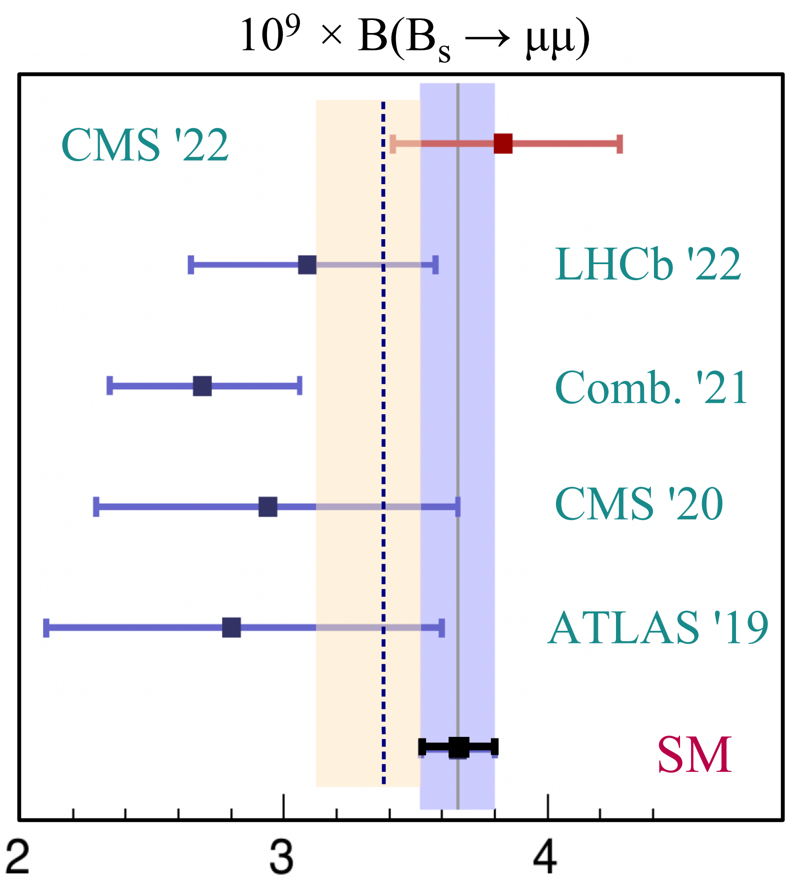


For a few years an interesting (mild) tension was observed, *but it disappeared at the end of 2022...*

However, the experimental error is still largely dominant → *room for improvement*

$$BR_{SM} = (3.66 \pm 0.14) \times 10^{-9}$$

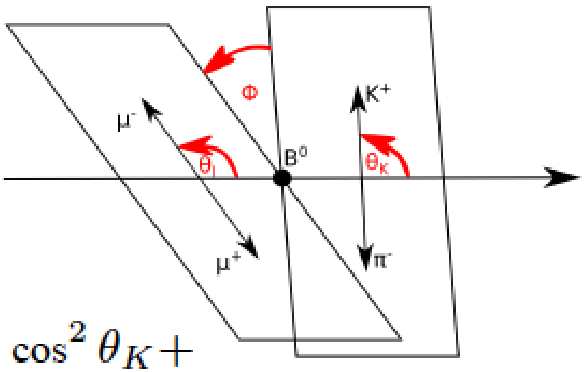
Beneke *et al.* '19

Rare $b \to sll$ decays: selected results

II. Differential distributions in $B \to K^* \mu^+\mu^-$

The process $B^0 \to K^{0*} (\to K^+\pi^-)\, \mu^+\mu^-$ is characterized by 3 independent angles and the invariant mass $q^2 = m_{\mu\mu}$

General decomposition of the differential distribution:



$$\frac{d^4(\Gamma+\bar{\Gamma})}{d\cos\theta_\ell\, d\cos\theta_K\, d\phi\, dq^2} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K +$$
$$\frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell +$$
$$S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi +$$
$$S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + S_6\sin^2\theta_K\cos\theta_\ell +$$
$$S_7\sin 2\theta_K\sin\theta_\ell\sin\phi +$$
$$S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

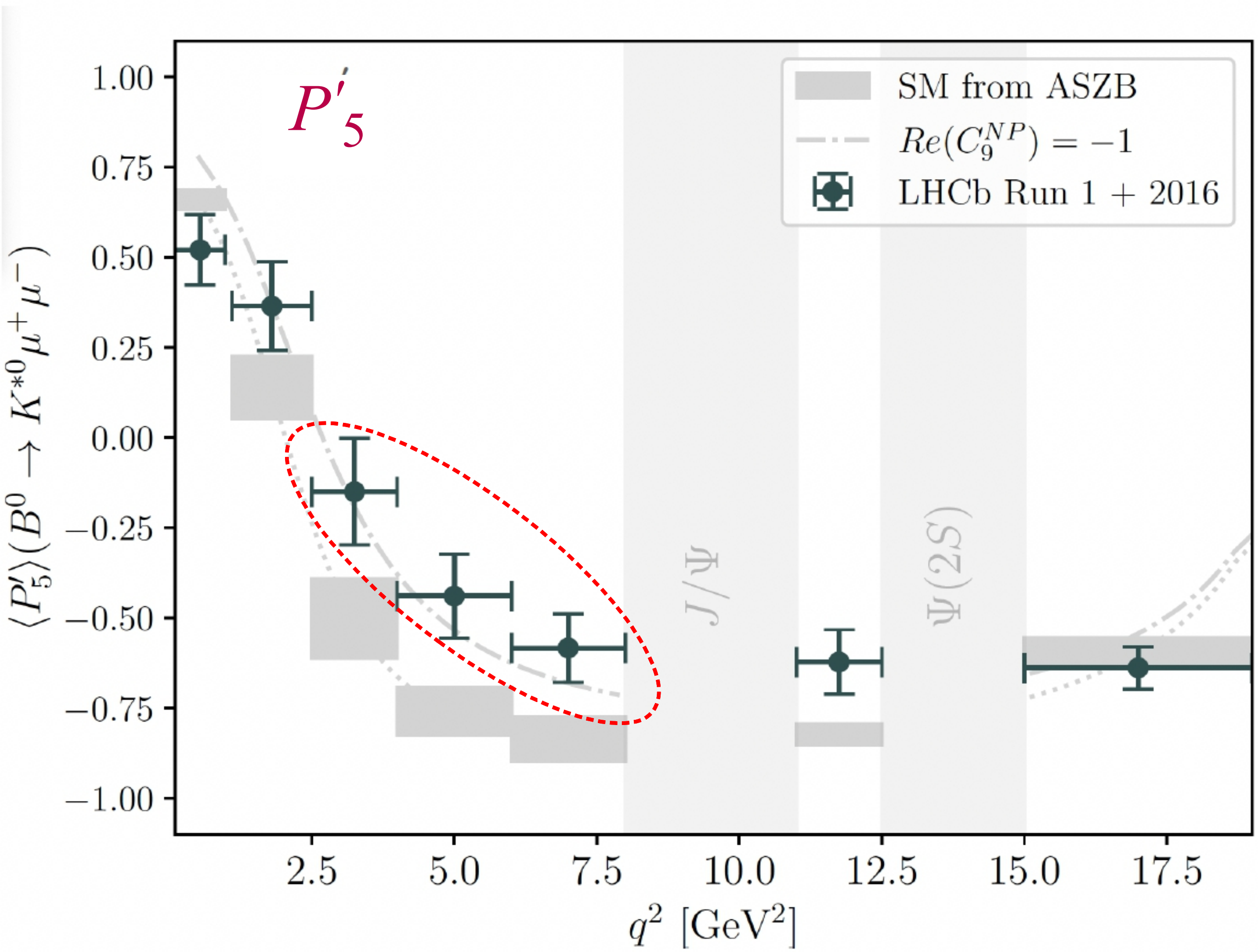
$$P'_5 = \frac{S_5}{\sqrt{F_L(1-F_L)}}$$

observable designed to cancel form factor dependence in the heavy-quark limit

Descotes-Genon, Matias, Ramon, Virto '12

Rare $b \to sll$ decays: selected results

II. Differential distributions in $B \to K^* \mu^+\mu^-$



Since 2013 a significant tension with *"reference" SM predictions* has been observed by LHCb, consistently over the <u>whole q^2 spectrum.</u>

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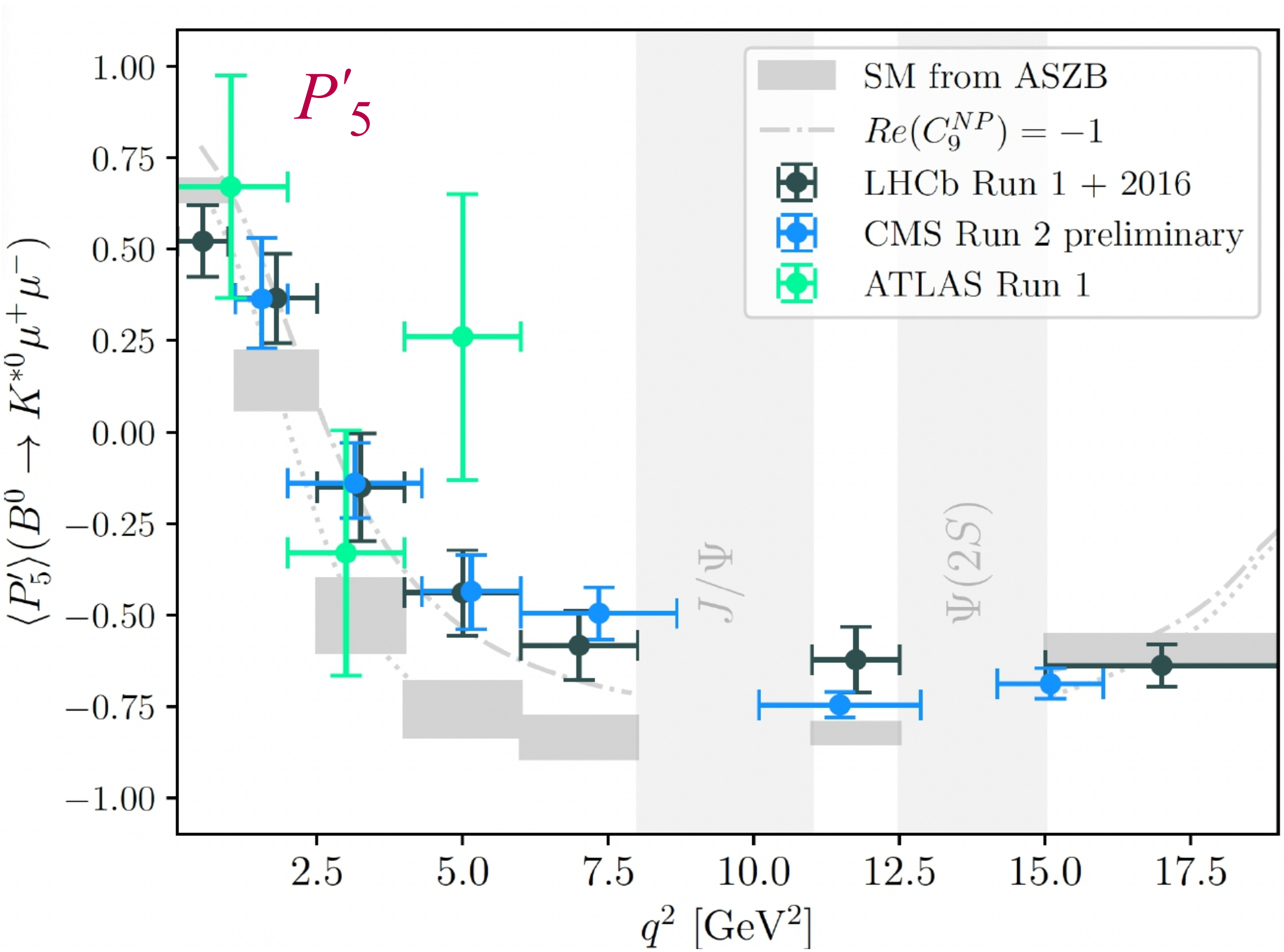

Since 2013 a significant tension with *"reference" SM predictions* has been observed by LHCb, consistently over the <u>whole q^2 spectrum</u>. Recent confirmation of the same effect by CMS

Rare $b \longrightarrow sll$ decays: selected results

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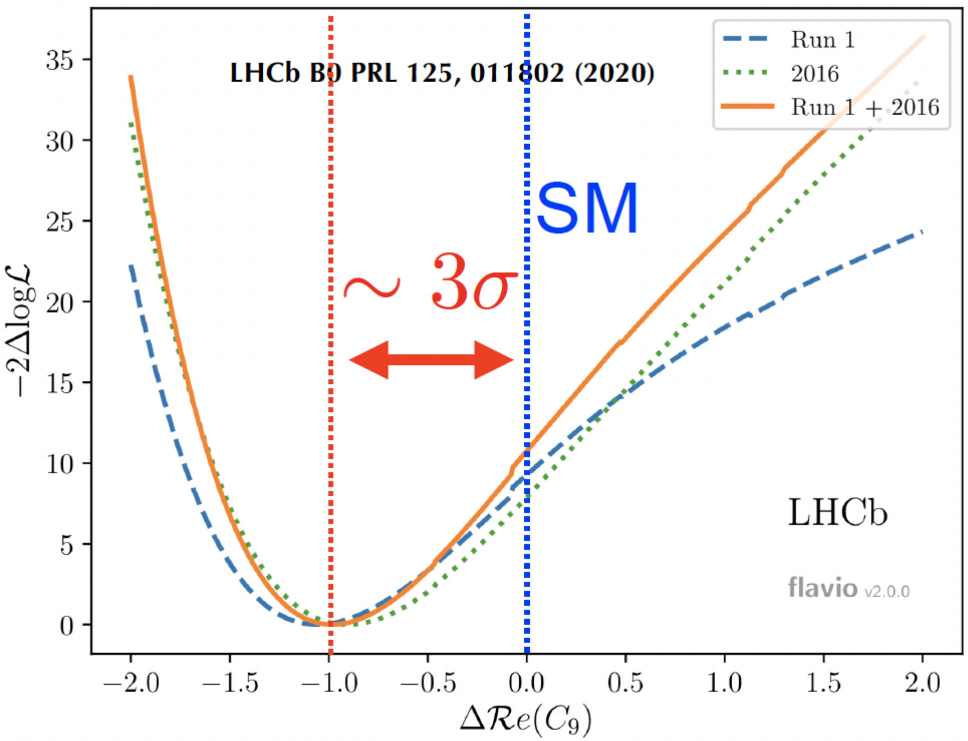
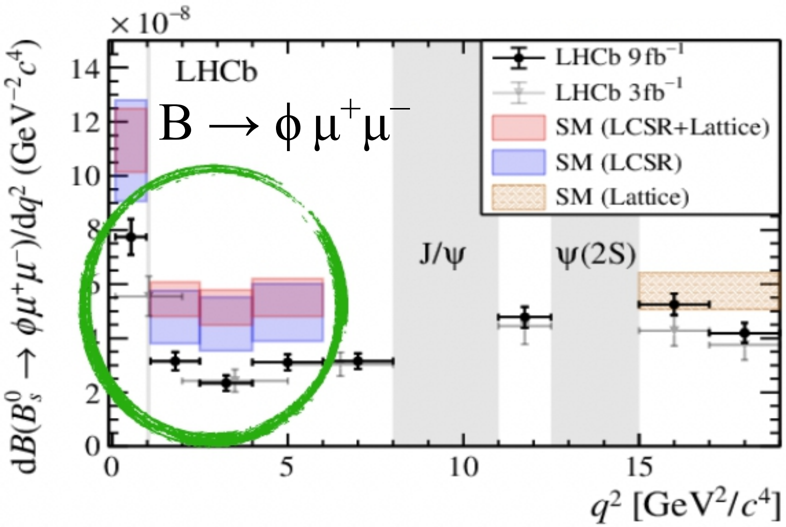


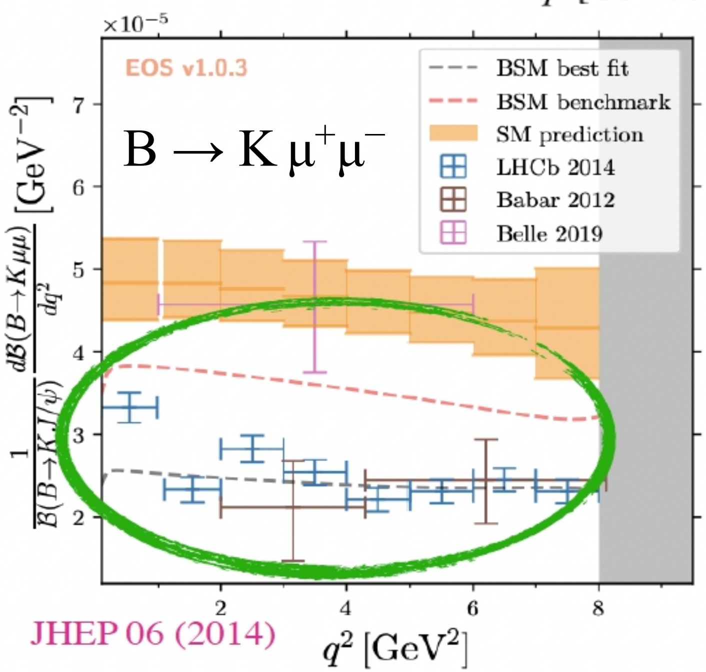
Data seem to be well described by a shift in C_9 with respect to its SM value. *However…*

- Remember long-distance effects can *pollute* the determination of C_9 (*SM theory error underestimated?*)

▸ *Rare $b \to sll$ decays: selected results*

III. Differential distributions in $B \to K^* \mu^+\mu^-$ & related observables





JHEP 06 (2014)

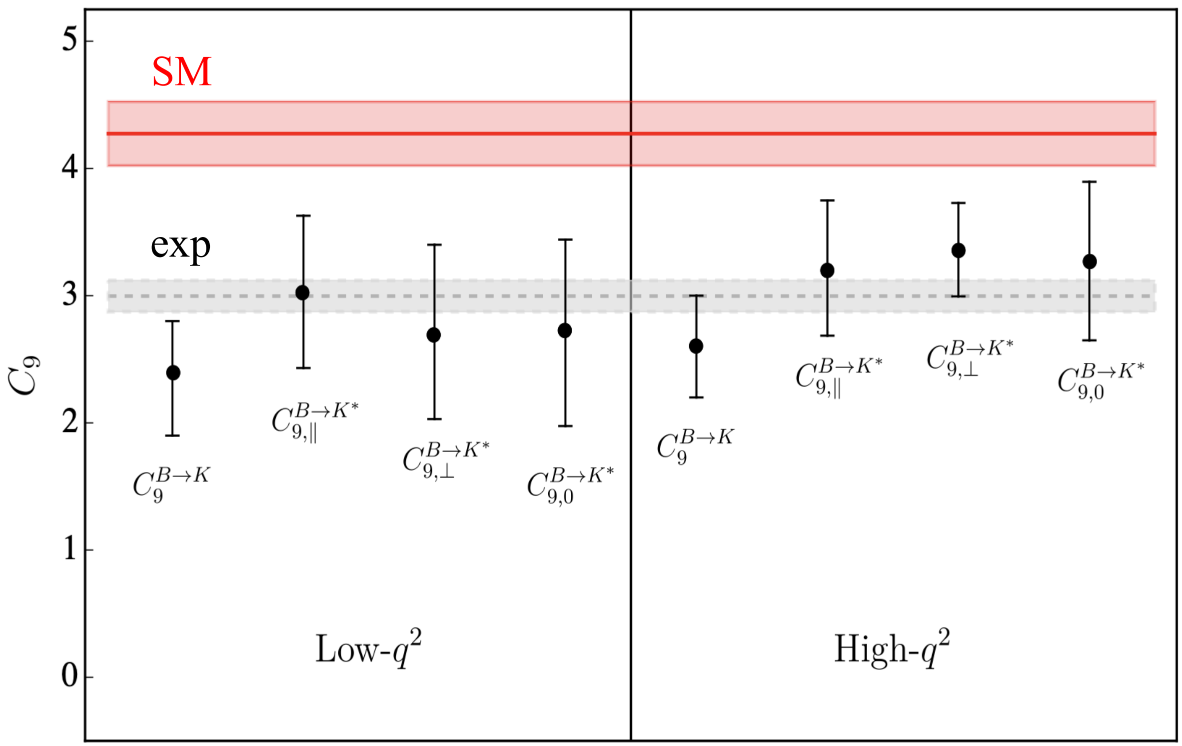
Data seem to be well described by a shift in C_9 with respect to its SM value. *However...*

- Remember long-distance effects can *pollute* the determination of C_9 (*SM theory error underestimated?*)
- On the other hand, similar pattern observed by other observables in related processes.

Rare $b \to sll$ decays: selected results

III. Differential distributions in $B \to K^* \mu^+\mu^-$ & related observables

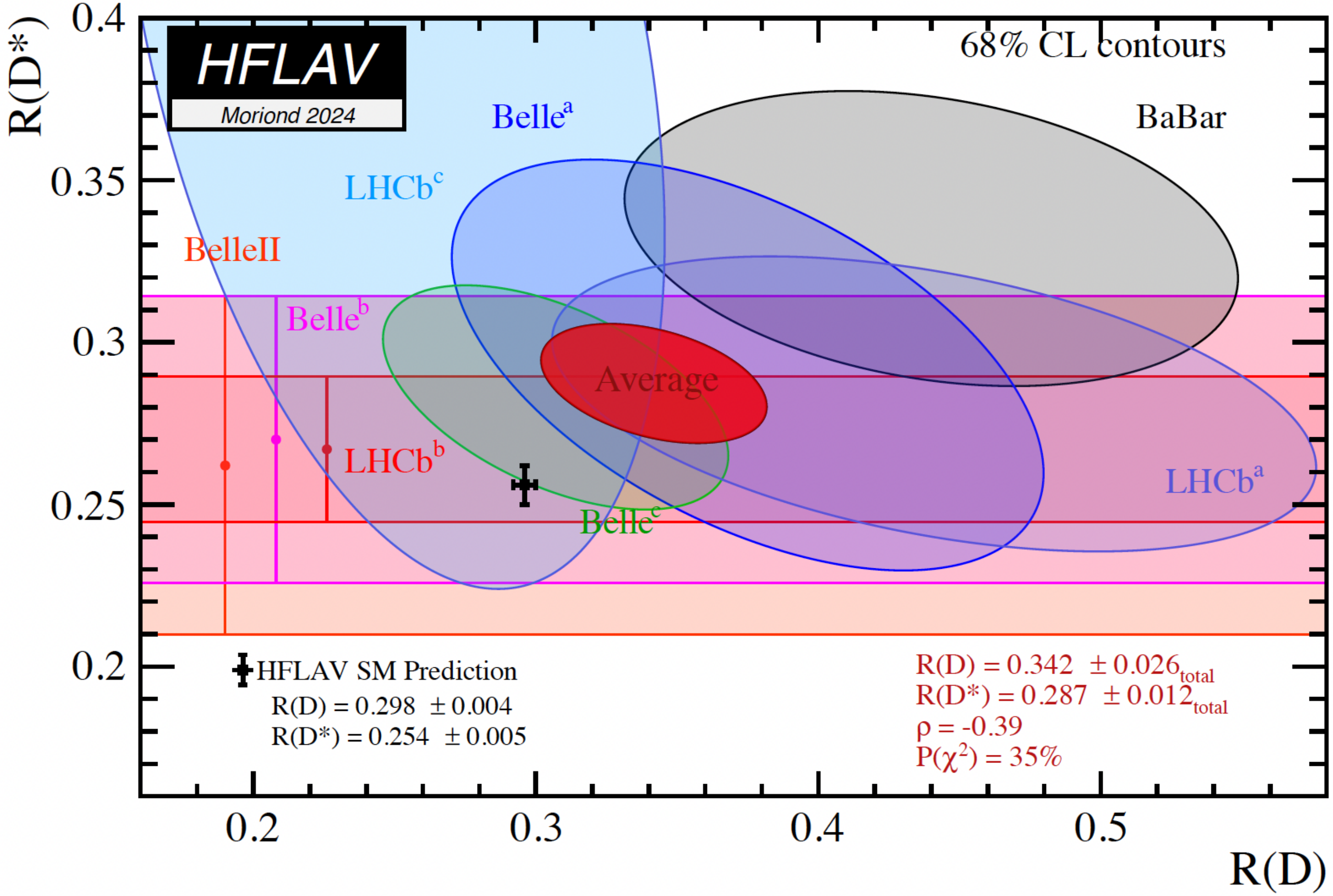
Taking a conservative approach, we cannot claim (*yet...*) that what has been observed is an unambiguous evidence of physics beyond the SM. However, the situation is quite intriguing.



Bordone, GI, Maecler, Tinari '24

- Differential distributions help us to disentangle short- vs. long-distance effects [short-distance should be "flat" in q^2 & "channel independent"]
- In the near future, with the help of high-statistics data, we could be able to disentangle "QCD pollution" from short-distance dynamics

Tests of Lepton Flavor Universality



General considerations on LFU

LFU [= *identical behavior of the 3 charged leptons*] is part of the approximate accidental flavor symmetries of the SM Lagrangian

LFU is badly broken in the Yukawa sector: $y_e \sim 3\times10^{-6}$, $y_\mu \sim 3\times10^{-4}$, $y_\tau \sim 10^{-2}$

However, all the lepton Yukawa couplings are small compared to SM gauge couplings, giving rise to the (*approximate*) universality of decay amplitudes which differ only by the different lepton species involved

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LFU has been verified with extremely high accuracy in several systems:

- $Z \to ll$ decays [~ 0.1%]
- $\tau \to l\nu\nu$ decays [~ 0.1%]
- $K \to (\pi)l\nu$ decays [~ 0.1%] & $\pi \to l\nu$ decays [~ 0.01%]

This is why it has been often assumed as a "sacred principle"...

But there is no deep reason, to assume it holds BSM.

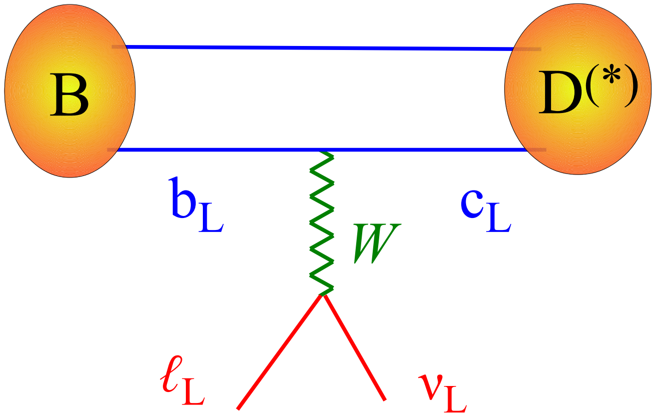
As we shall see, there are also no strong experimental tests in semileptonic processes involving 3^{rd} generation quarks, which actually show intriguing hints of LFU violations

LFU tests in $b \to c$ transitions

The way we test LFU in charged-current $b \to c$ transitions is via the ratios

$$R_{12}(H_c) = \frac{\Gamma(B \to H_c\, \ell_1 \nu)}{\Gamma(B \to H_c\, \ell_2 \nu)}$$

$$H_c = D \text{ or } D^*$$



We are not able to compute very precisely, separately, numerators and denominators in these ratios because of hadronic uncertainties...

E.g.: $A(B \to D\, \ell\nu)_{SM} = G_{eff}\, V_{cb}\, \underbrace{\langle D |\, \bar{b}_L \gamma_\mu c_L\, | B \rangle}\ \bar{\ell}\gamma^\mu\nu$

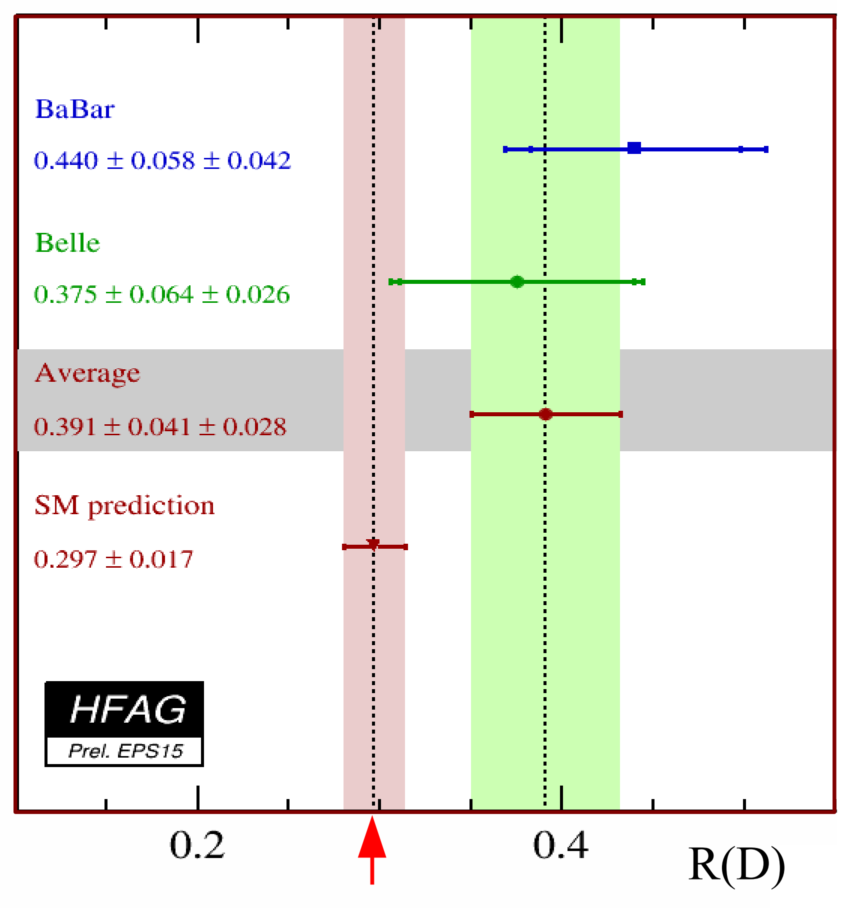
$\to f_+(q^2)\,(p_B+p_D)_\mu + f_-(q^2)\,(p_B-p_D)_\mu$

But these uncertainties cancels (to a very good accuracy) in the ratios

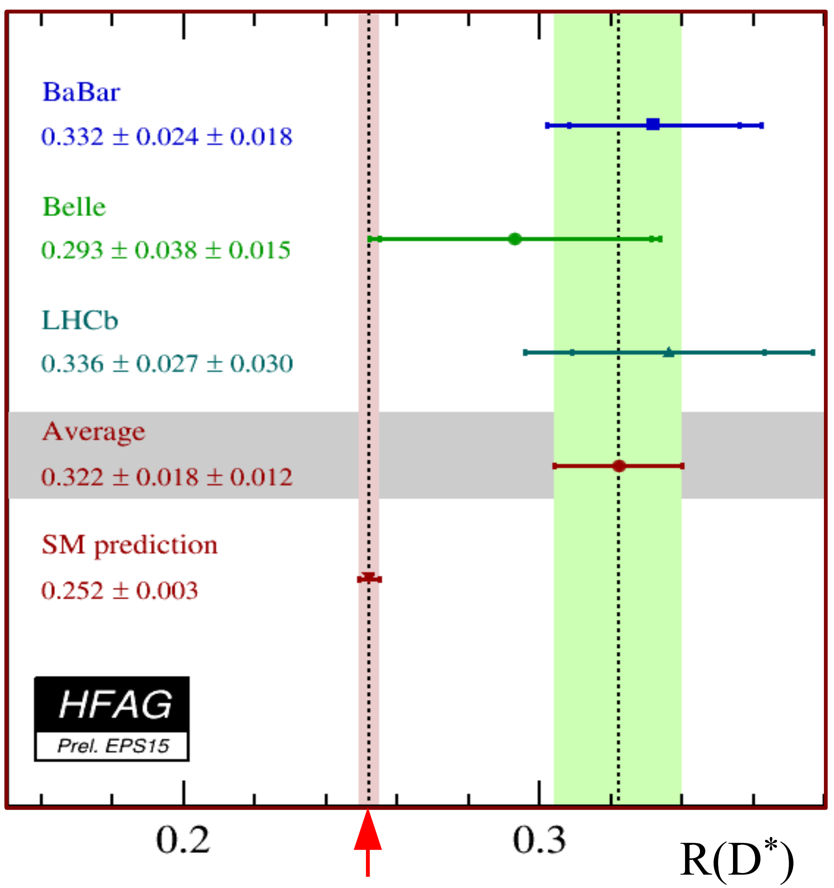
The "anomaly" appears when comparing τ vs. light leptons (μ, e)

LFU tests in $b \to c$ transitions

LFU tests in $b \to c$ transitions [τ vs. light leptons (μ, e)]:

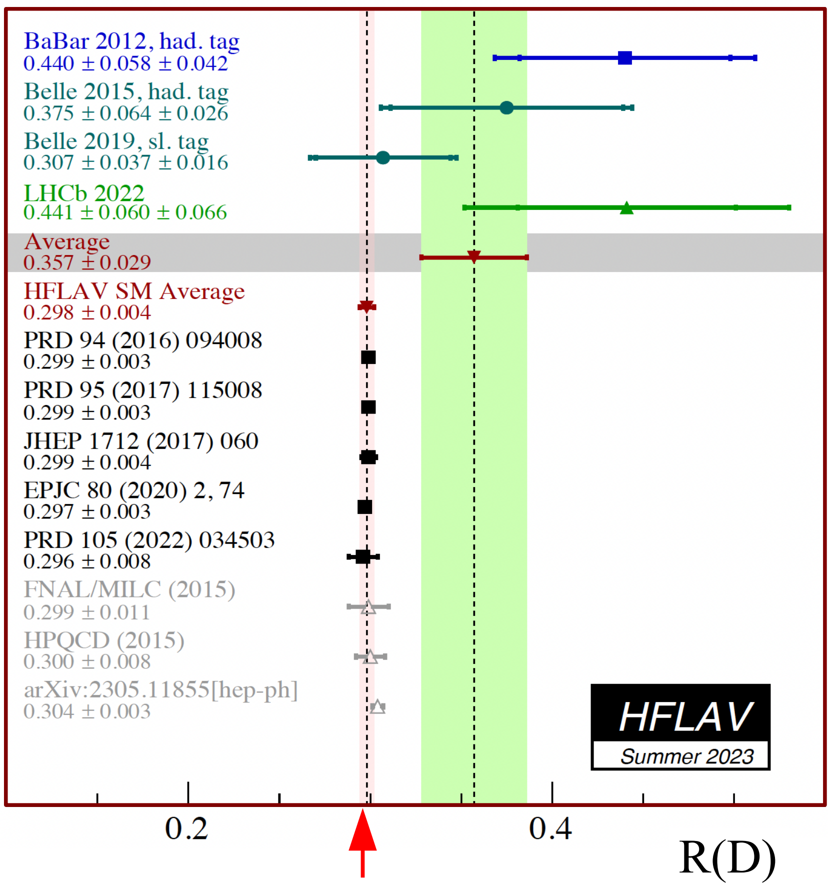


2015

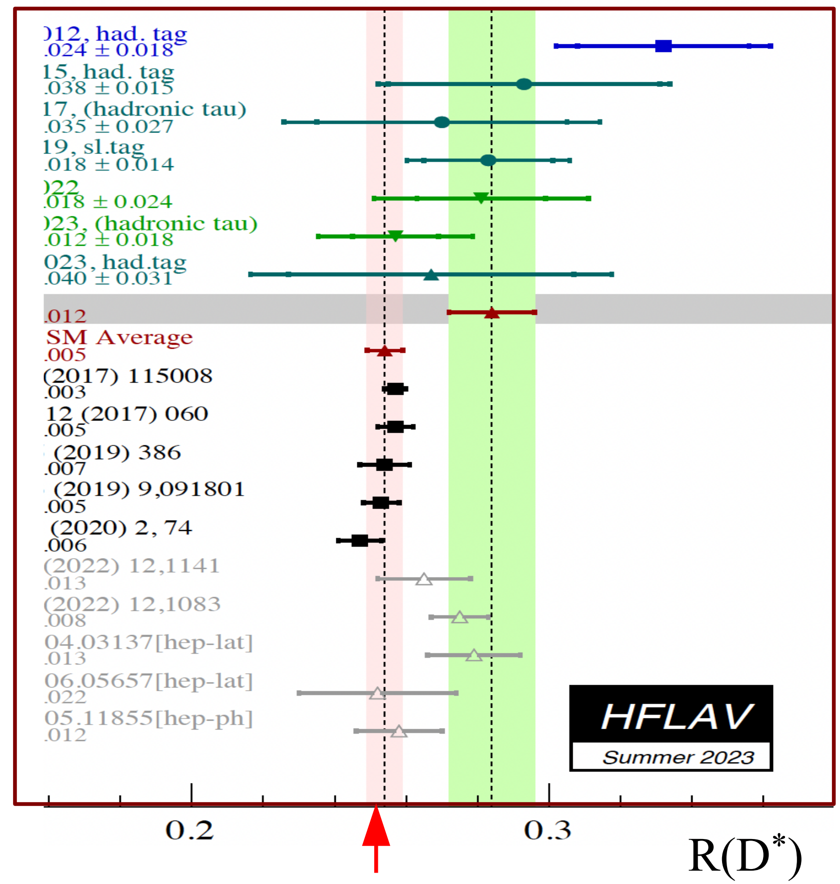


LFU tests in b → c transitions

LFU tests in b → c transitions [τ vs. light leptons (μ, e)]:



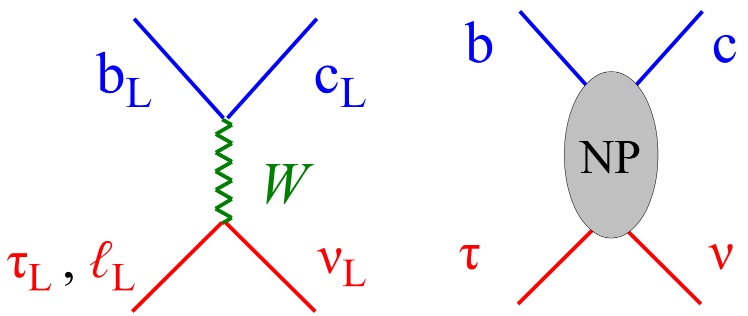
2023

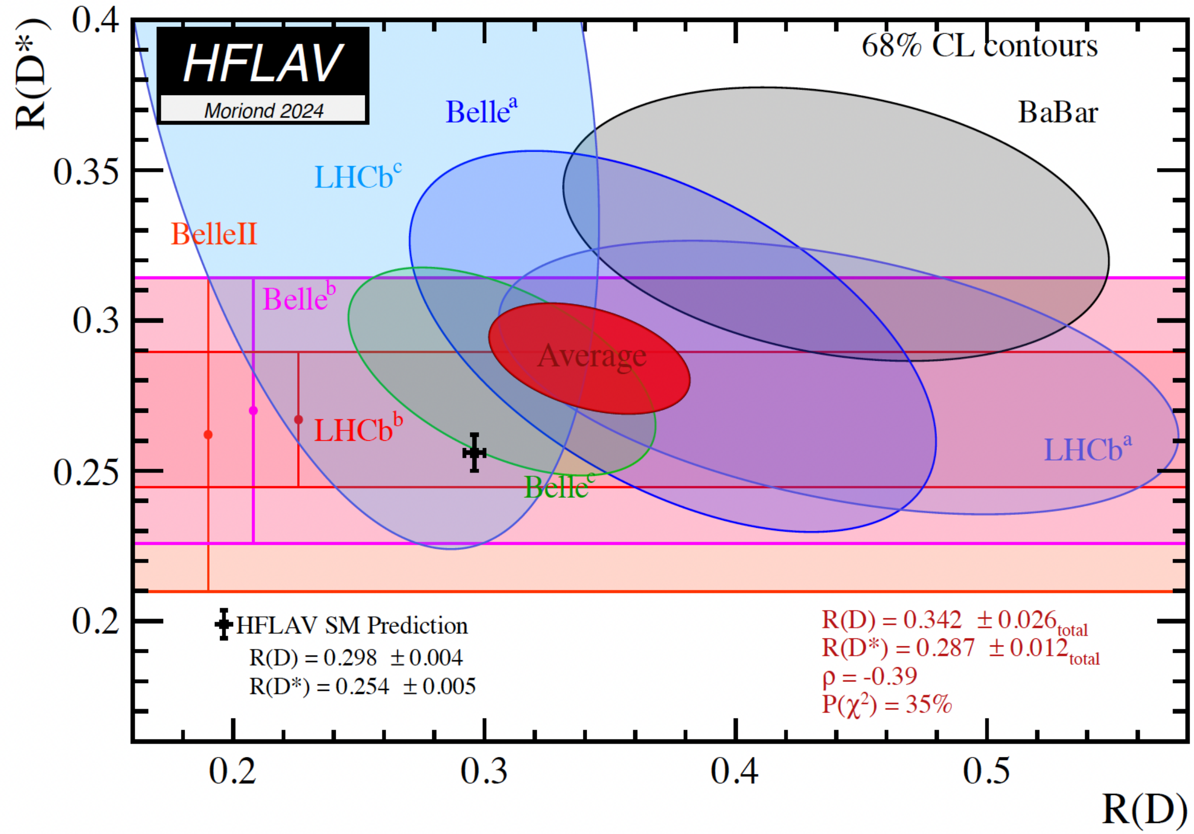


LFU tests in b → c transitions

LFU tests in b → c transitions [τ vs. light leptons (μ, e)]:

$$R(H_c) = \frac{\Gamma(B \to H_c\, \tau\nu)}{\Gamma(B \to H_c\, \ell\nu)}$$





- No single experimental results deviates significantly from the SM, but data are all well compatible and their combination leads to 3.1σ deviation vs. the SM

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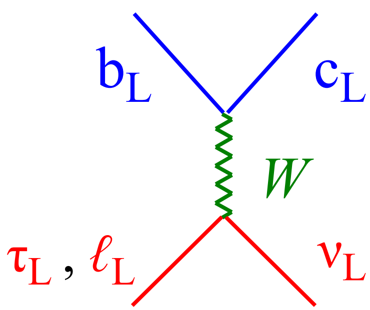
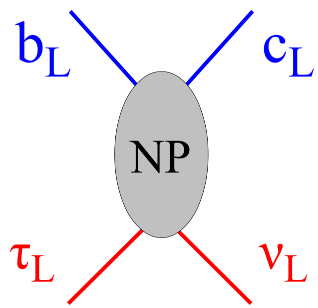
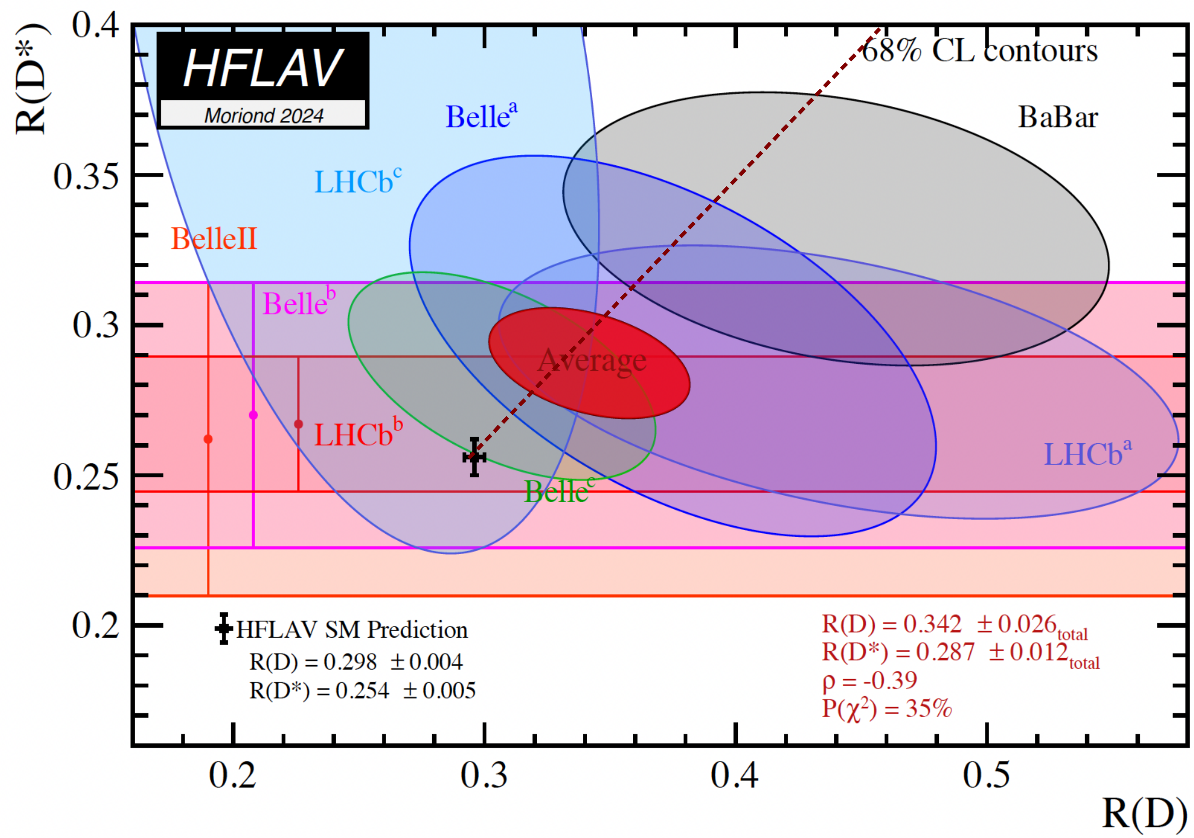


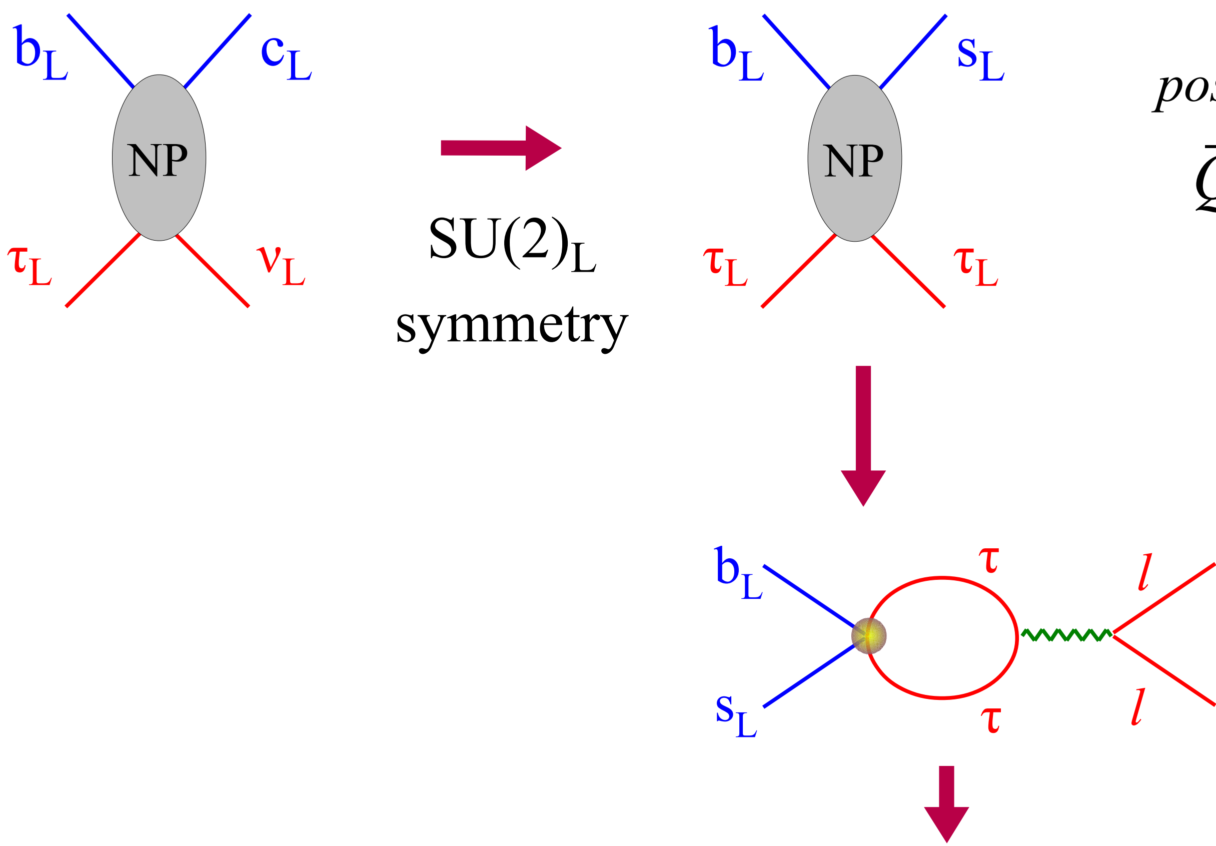




- No single experimental results deviates significantly from the SM, but data are all well compatible and their combination leads to 3.1σ deviation vs. the SM
- The two channels are consistent with a universal enhancement (~10 – 20%) of the SM $b_L \to c_L\, \tau_L\, \nu_L$ amplitude

LFU tests in b → c transitions – possible connection to the bsll anomaly?

A rather interesting aspect of the LFU anomaly observed in $b \to c\tau\nu$
is a possible connection to C_9 – anomaly observed in $b \to s\ell\ell$



possible effective operator:

$$\bar{Q}_L^{\,3} \gamma_\mu Q_L^{\,2} \;\; \bar{L}_L^{\,3} \gamma^\mu L_L^{\,3}$$

$$Q_L^{\,i} = \begin{bmatrix} u_L^{\,i} \\ d_L^{\,i} \end{bmatrix}$$

$$L_L^{\,i} = \begin{bmatrix} \nu_L^{\,i} \\ e_L^{\,i} \end{bmatrix}$$

$$\Delta C_9^{e,\mu} \approx -0.5$$

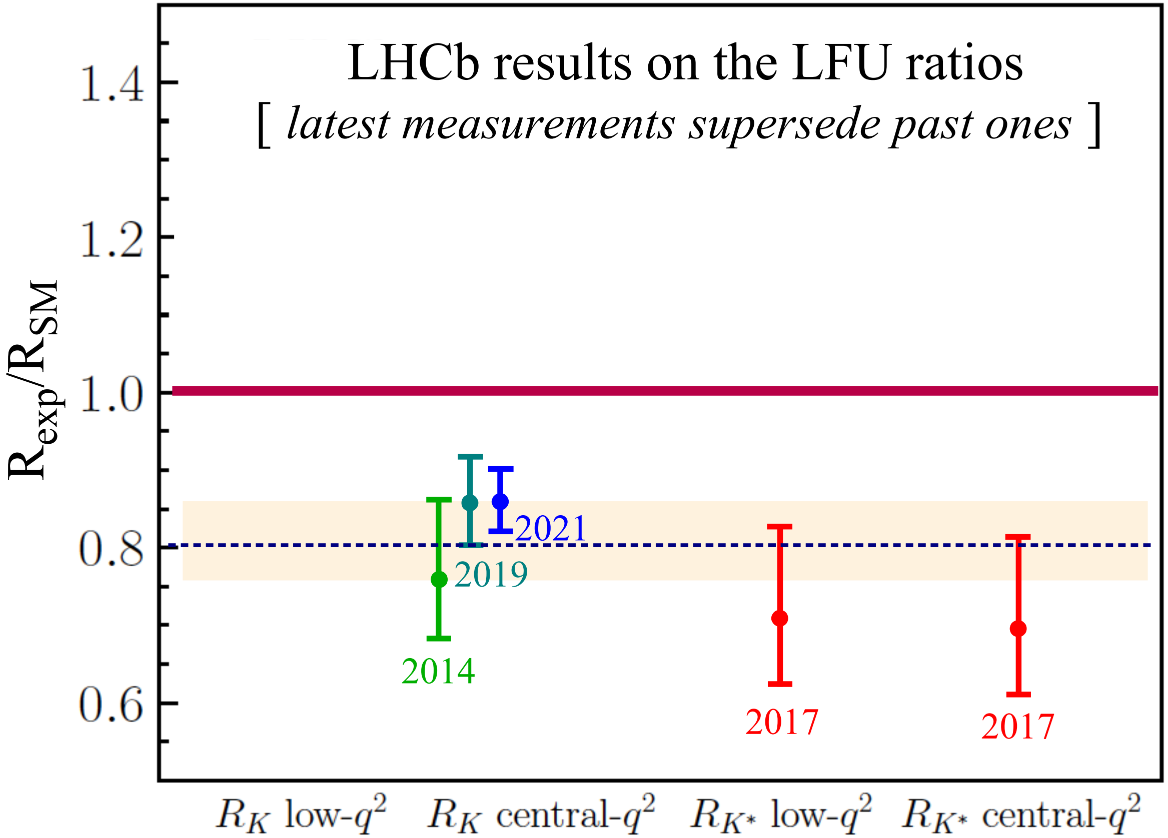
[*correct sign & size...*]

Bobeth & Haisch '11
Crivellin *et al.* '18
Alguero *et al.* '18

LFU tests in $b \to s$ transitions

Last but not least, LFU tests have been performed recently also in the neutral-current $b \to s\ell\ell$ decays, probing in this case the μ vs. e universality.

The situation was very exciting till the end of 2022...



$$R_H = \frac{\int d\Gamma(B \to H\,\mu\mu)}{\int d\Gamma(B \to H\,ee)}$$

Very clean SM predictions
[theory uncertainty below 1%]

▶ LFU tests in $b \to s$ transitions

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The situation was very exciting till the end of 2022, when a more refined experimental analysis changed the picture...

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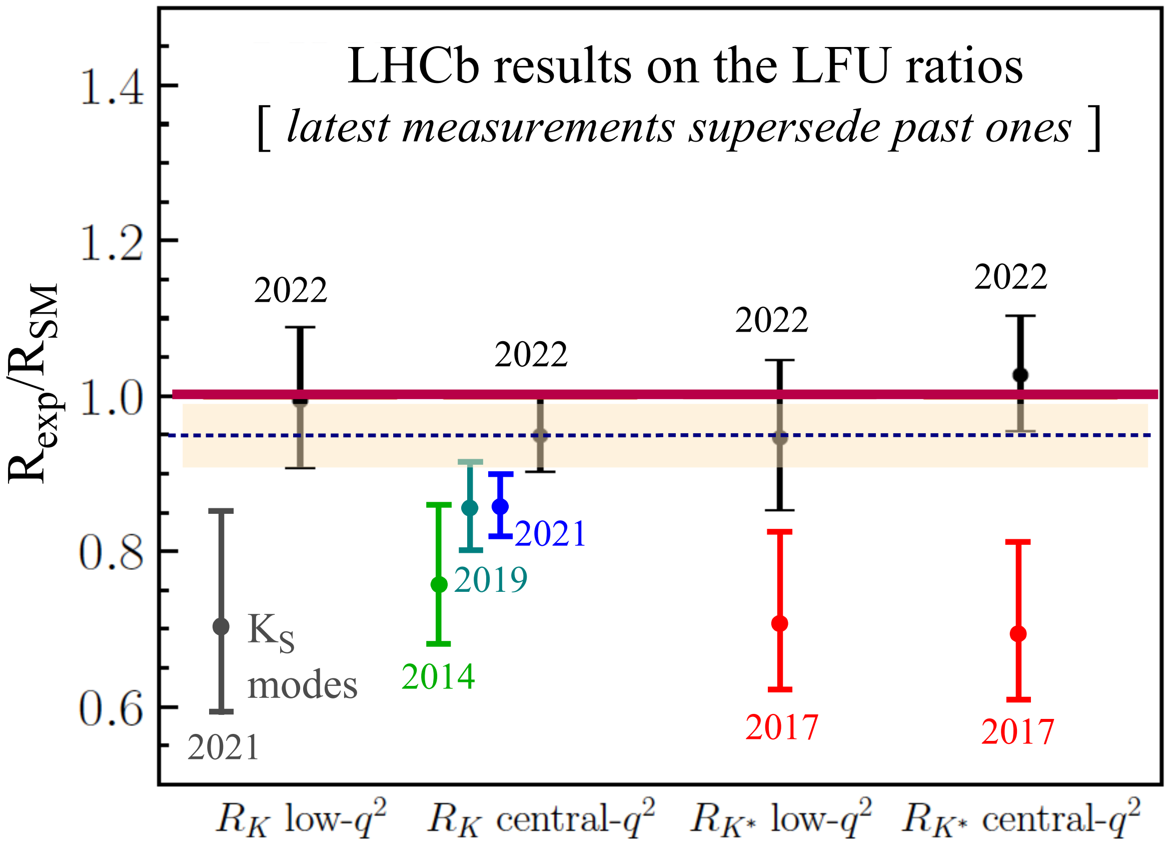

Very clean SM predictions
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This was a good reminder we should be very cautions in interpreting "anomalies"...

However, as for $B(B_s \to \mu\mu)$, note that the exp. error is still largely dominant $\to$ *large room for improvements*