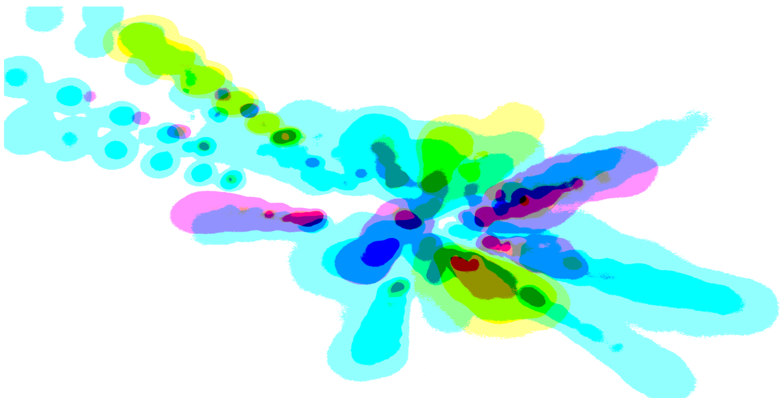
What's big about small x



Raju Venugopalan

Brookhaven National Laboratory

Bad Honnef, October 23-27, 2023

Outline of my talk

- Small x physics is intrinsically non-perturbative: “Classical” lumps from quantum coherence
- Small x and quantum information science: Goldstone modes and Bekenstein bound
- Small x and heavy-ion collisions: universality across energy scale and (nearly) instantaneous thermalization
- Gravity and QCD at small x: Lipatov double copy, gravitational waves in strong fields and the problem of Black Hole formation in trans-Planckian scattering
- Small x and spin: the role of topology and topological “sphaleron” transitions

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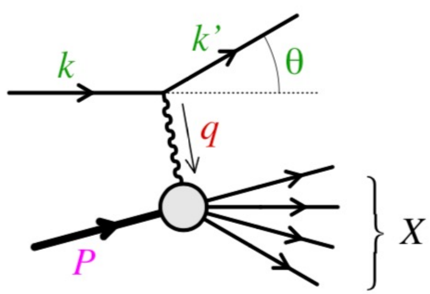
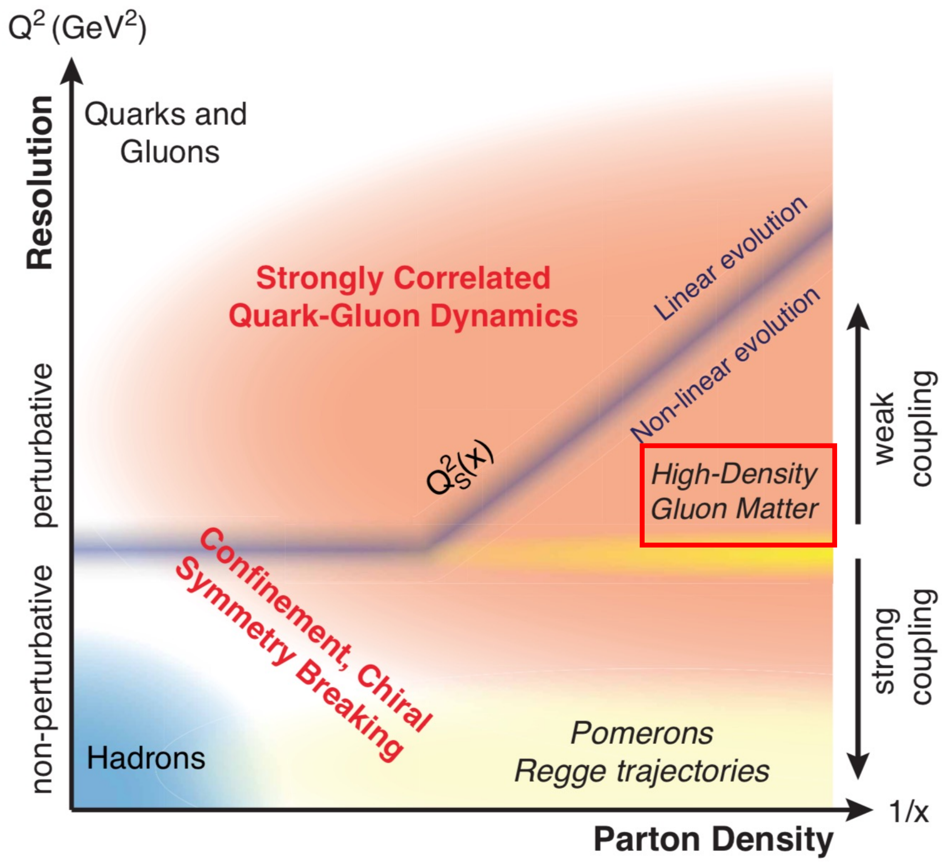
Tarasov, RV, arXiv:2008.08104, arXiv: 2109.10370
Bhattacharya, Hatta, Vogelsang, arXIv:2210.13419, arXiv:2305.09431

Mapping out terra incognita in the QCD landscape



Deeply inelastic scattering

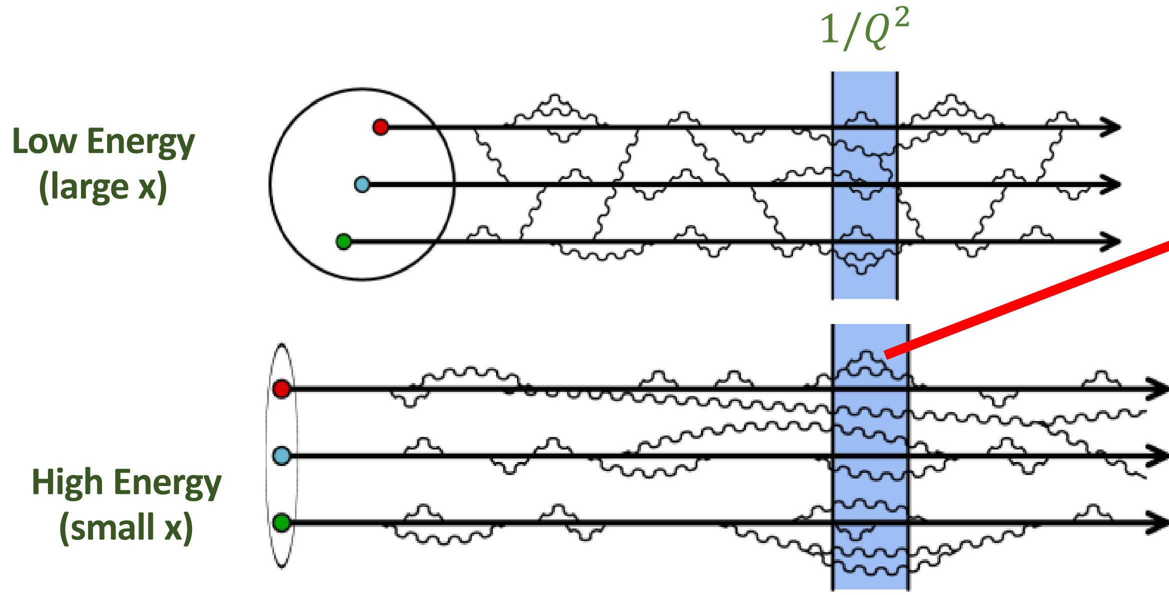


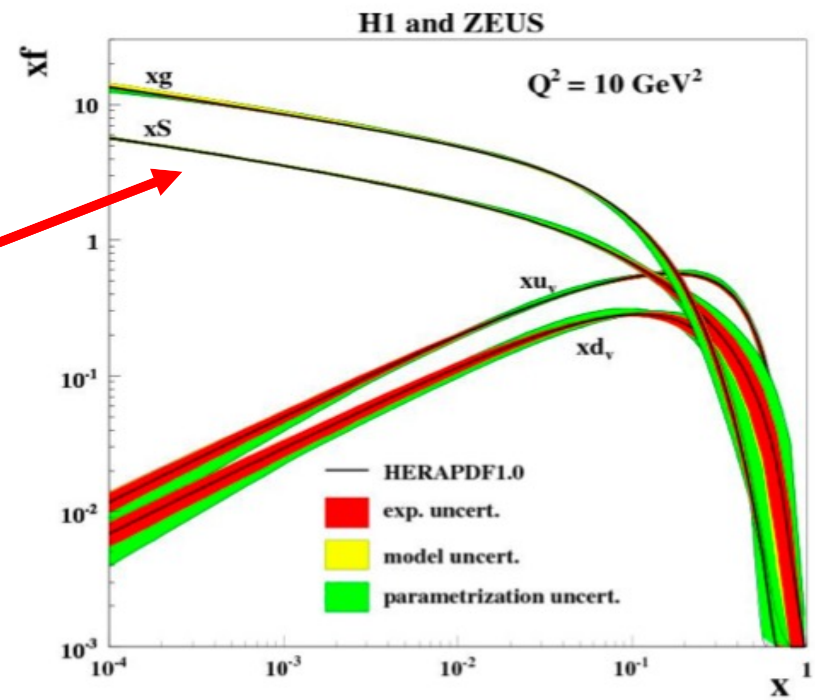
Aschenauer et al., arXiv:1708.01527
Rep.Prog. Phys. 82, 024301 (2019)

Many open questions: 3-D quark-gluon structure of the proton,
spin and orbital dynamics, many-body correlations, multi-particle production...

Wee partons ($x << 1$) are intrinsically non-perturbative

Spacetime picture of wee partons in a hadron

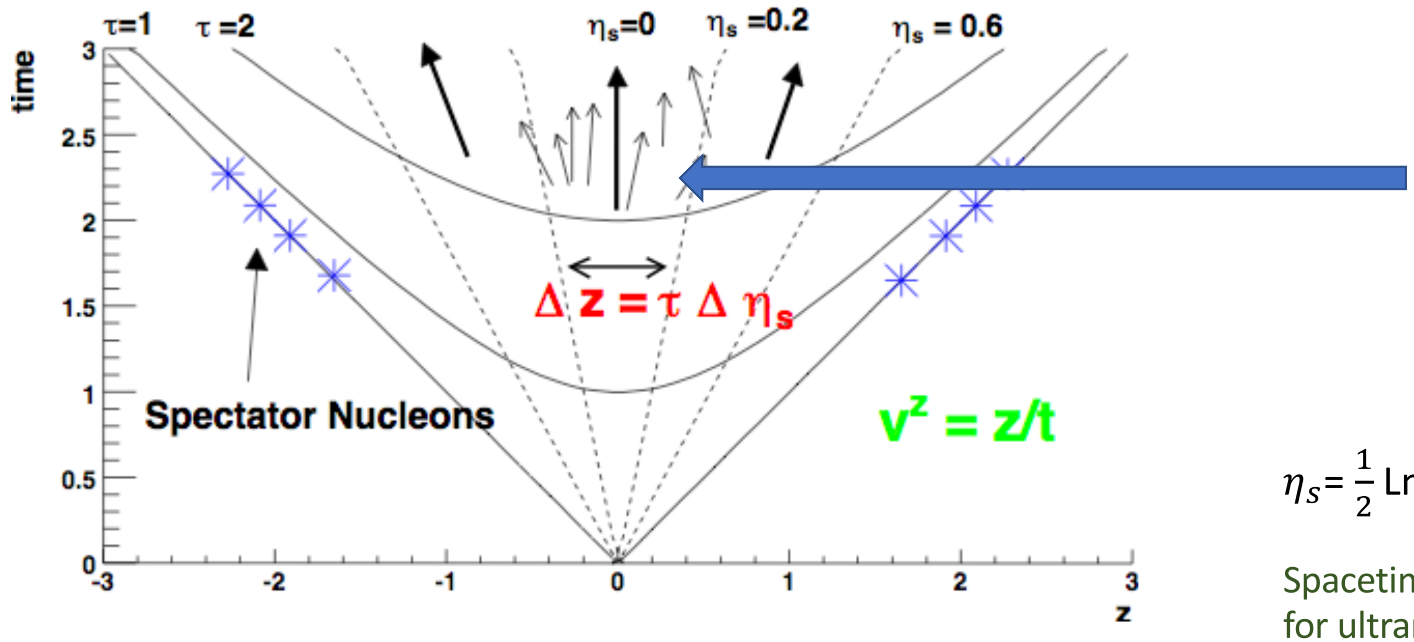


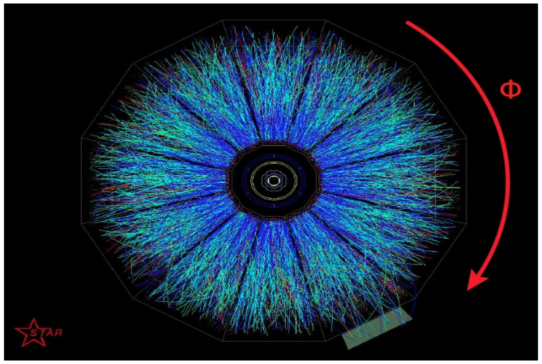


As the proton is boosted, "parton" fluctuations live longer -- released as Bremsstrahlung

Suppression in coupling compensated by large phase space for soft glue: $\alpha_S\, Ln\left(\frac{1}{x}\right) \sim 1$

Spacetime picture of a high energy hadron-hadron collision





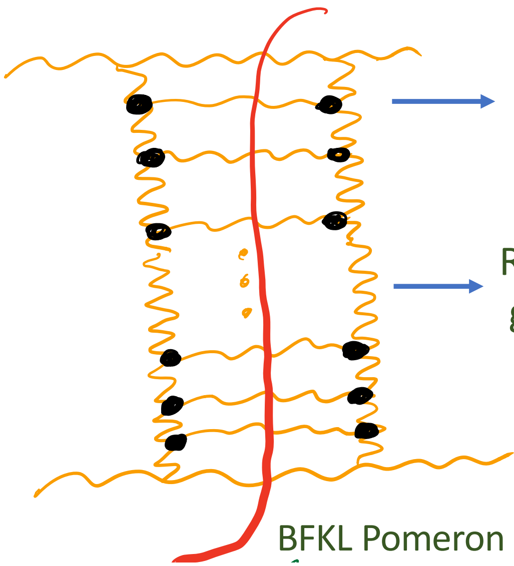
$$\eta_s = \frac{1}{2}\,\mathrm{Ln}\,\left(\frac{t+z}{t-z}\right) \approx Y$$

Spacetime rapidity ≈ Momentum rapidity
for ultrarelativistic particles

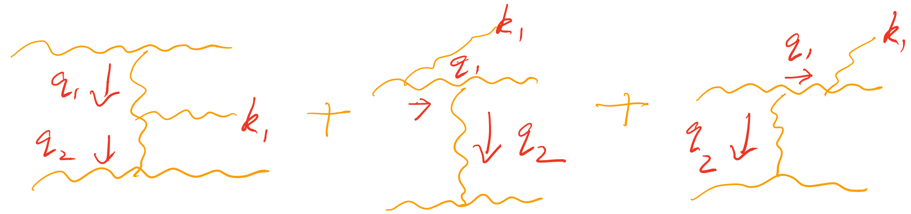
Fast “valence” partons populate fragmentation regions at large rapidities – “leading particle” effect
Slow “wee” partons populate central rapidities (mostly gluons and sea-quark pairs)

Wee partons in the BFKL Paradigm

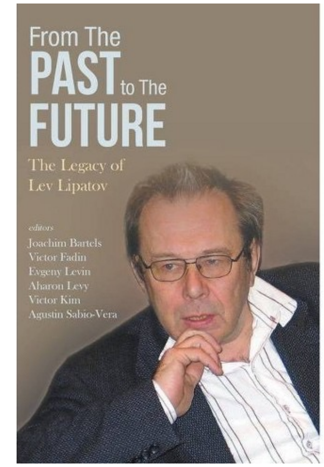
Sophisticated construction to describe 2→ N scattering in multi − Regge kinematics



Lipatov vertex



Reggeized gluon $\equiv \frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1}-y_i)}$



Lev Lipatov

Regge trajectory: $\alpha^{(2)}(t) = \kappa_\Gamma^2 \left(\frac{\mu^2}{-t}\right)^{2\epsilon} \left(\frac{\beta_0}{\epsilon^2} + \frac{\gamma_K^{(2)}}{8\epsilon} + \frac{\gamma_\wedge^{(2)}}{2} + \zeta_2\beta_0\right) + \mathcal{O}(\epsilon)$

$\gamma_K^{(2)}$: Two loop cusp anomalous dimension

$\gamma_\Lambda^{(2)}$: Two loop wedge anomalous dimension

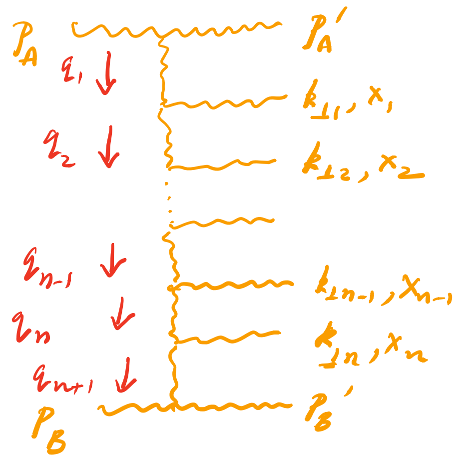
Fadin, hep-ph/9807528

$$\sigma_{tot} = 2\,\mathrm{Im}\,A(s, t=0)$$
$$= s^\lambda \text{ with } \lambda = \frac{4\alpha_s N_c \ln_e 2}{\pi}$$
$$\simeq 0.5 \text{ for } \alpha_s = 0.2$$

BFKL Hamiltonian. Remarkable properties: holomorphic separability; generalization to integrable model; beautiful work in N=4 SUSY

State-of-the art review of NLL BFKL and beyond: Del Duca, Dixon, arXiv:2203.13026
Fine print: talk by Sabio Vera

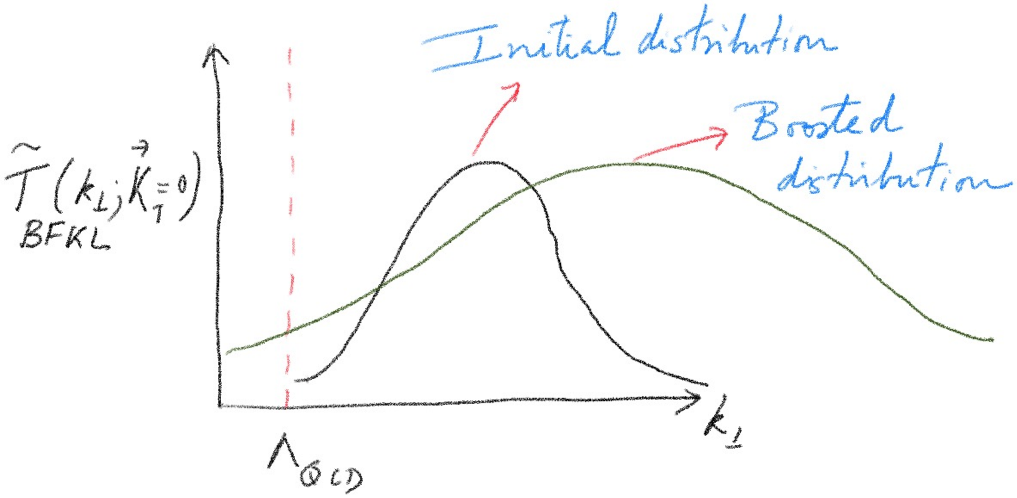
Wee partons in perturbative QCD: $2 \rightarrow N$ amplitudes



BFKL Eqn: LLx "Leading log" all-order resummation

$(\ \alpha_n \mathrm{Ln}(\frac{1}{x})\ )^n$ of real and virtual graphs all orders in α_n

Soln. of BFKL equation exhibits infrared diffusion



For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

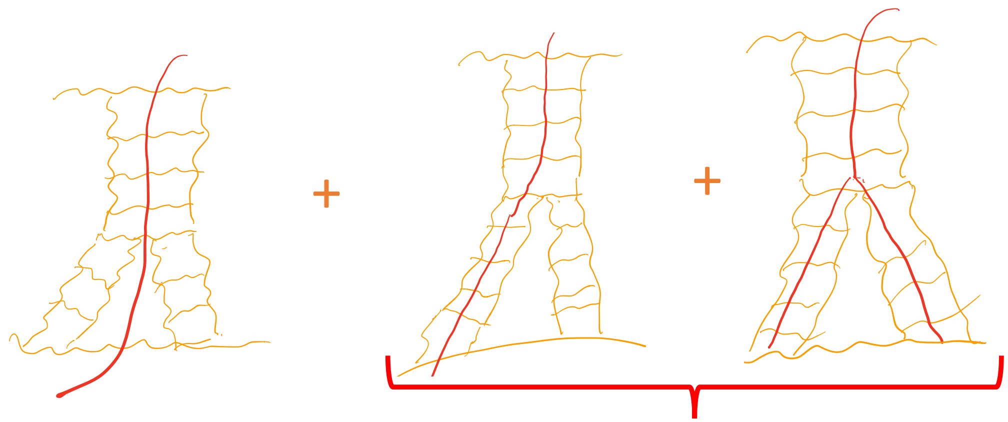
significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251

NLL BFKL does not cure infrared diffusion

Wee partons in pQCD: many-body dynamics in $2 \rightarrow N$ amplitudes

Gribov,Levin,Ryskin (1983)
Mueller,Qiu (1986)



Recombination

Screening

$$= -\frac{[\alpha_s G(x,Q^2)]^2}{Q^2R^2}$$

“The elimination of such complicated interlocking infrared divergences would certainly be a Herculean task …might not even be possible”

S. Weinberg, PRD (1965)

"A fascinating equilibrium of (gluon) splitting and recombination should eventually result: it’s a considerable theoretical challenge…”

F. Wilczek, Nature (1999)

Emergent saturation scale Q_S suppresses infrared diffusion in transverse momentum

All-twist power corrections equally important when

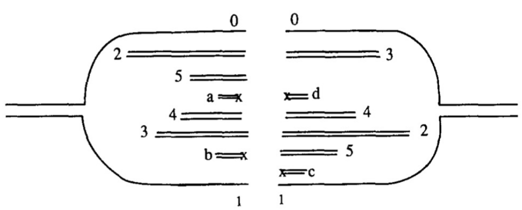
$$N \equiv \frac{xG_A(x,Q_S^2)}{2(N_c^2-1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$$

Classicalization when $\alpha_S(Q_S) \ll 1$ for $Q_S \gg \Lambda_{QCD}$

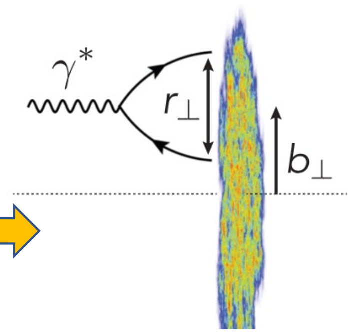
Classicalization and perturbative unitarization: gluon saturation

s-channel "dipole" scattering picture

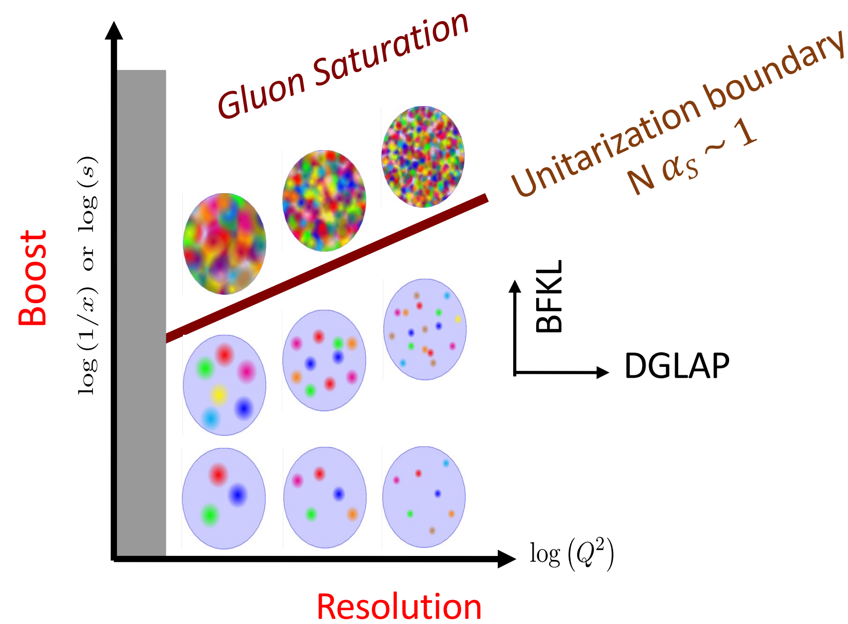
Mueller, NPB415 (1994) 373
Mueller, Patel, hep-ph/9403256



Boosted projectile



Boosted target



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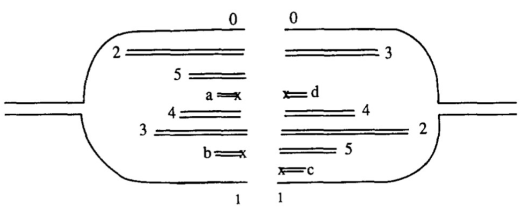

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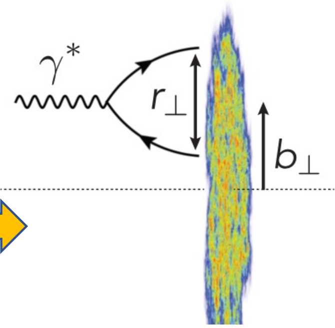

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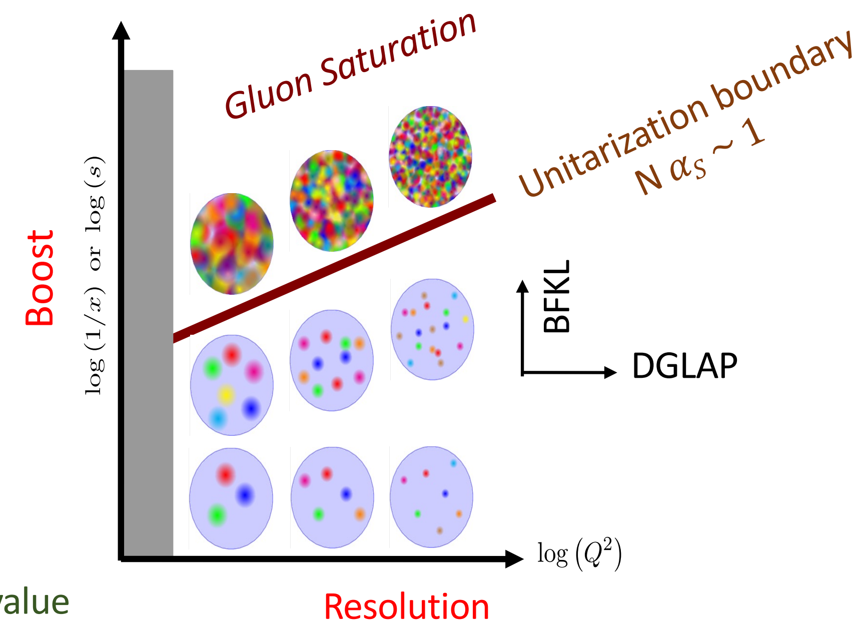
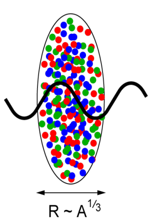


Golec-Biernat-Wusthoff model:

$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0\,\left[1 - \exp\left(-r_\perp^2\, Q_s^2(x)\right)\right]$$

Emergent semi-hard scale $Q_s^2(x) = Q_0^2\left(\frac{x_0}{x}\right)^{\lambda}$ → BFKL eigenvalue



Quantum coherence of large # of color charges

for x << $A^{-1/3} \rightarrow Q_{S,A}^2\ /\ Q_{s,p}^2\ \propto\ A^{\frac{1}{3}}$

$\sim constant\ when\ Y \rightarrow \infty$

Color transparency for $r_\perp^2 Q_S^2$ << 1 ($\sigma \propto A$)

Color opacity (”black disk”) for $r_\perp^2 Q_S^2$ >> 1 ($\sigma \propto\ A^{2/3}$)

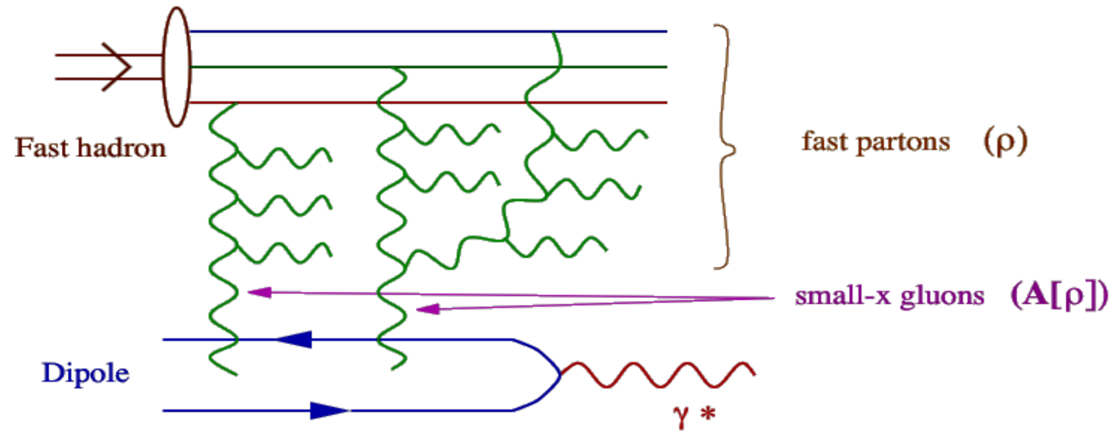
QCD picture of observed “shadowing” at small x

The Color Glass Condensate: classical EFT for Regge asymptotics

Born-Oppenheimer separation between fast and slow light-front modes

Large x (P^+) modes: static, strong ($\sim 1/g$) color sources ρ^a

Small x ($k^+ << P^+$) modes: fully dynamical gauge fields A_μ^a



$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\delta(A^+) e^{iS_{\Lambda^+}[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\delta(A^+) e^{iS_{\Lambda^+}[A,\rho]}} \right\}$$

$W_{\Lambda^+}[\rho]$: nonpert. gauge inv. weight functional defined at initial $x_0 = \Lambda^+ / P^+$

$S_{\Lambda^+}[A, \rho]$: Yang-Mills action + gauge-inv. coupling of sources to fields (Wilson line)

JIMWLK RG describes the nonlinear evolution of sources and fields with change of rapidity scale

JIMWLK= Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner (1997-2021)
CGC review: Gelis,Iancu,Jalilian-Marian,RV:arXiv 1002.0333

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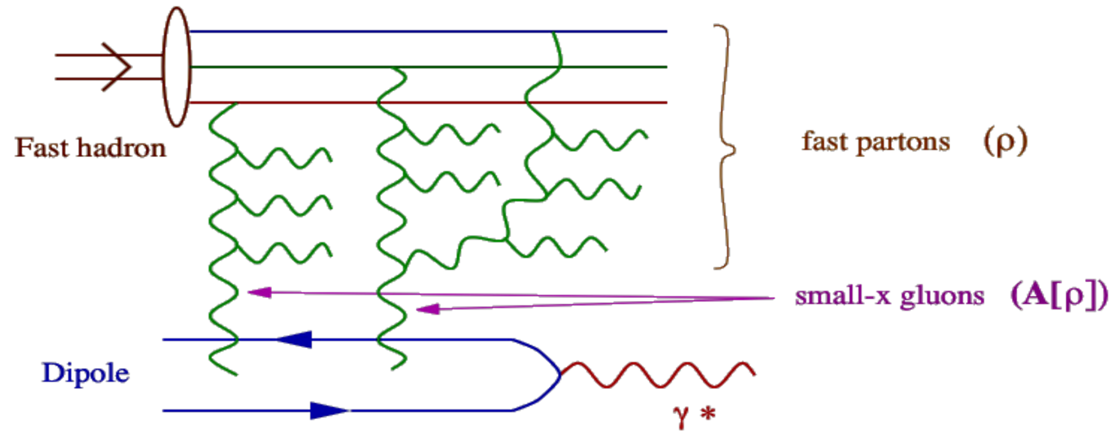


Explicit construction for large nuclei (large number of coherent sources of color charge at small x-large "Ioffe time")

$$W_{\Lambda^+}[\rho] \rightarrow \int [d\rho] \exp\left(-\int d^2x_\perp \left[\frac{\rho^a\rho^a}{2\mu_A^2} - \frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

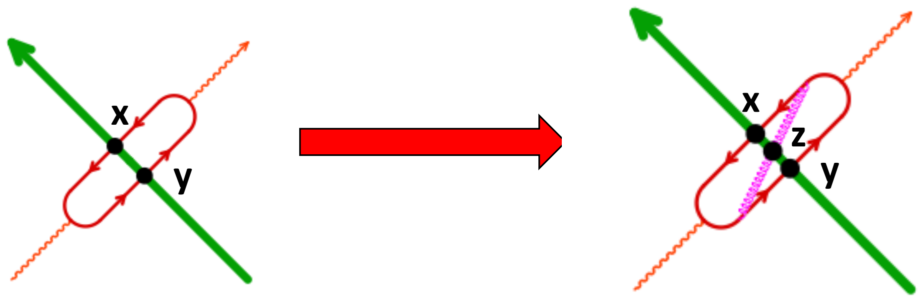
Pomeron configurations

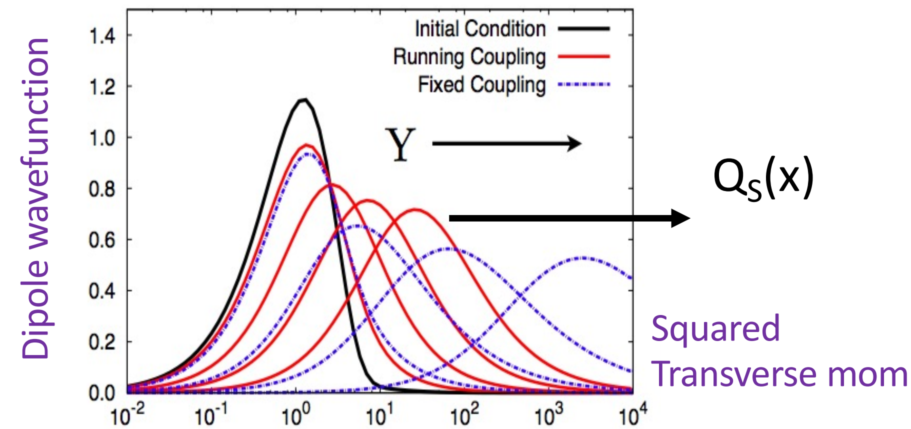
Odderon configurations

For A>>1, $\mu_A^2 \sim Q_S^2 \propto A^{1/3} >> \Lambda_{QCD}^2$
weak coupling EFT for large parton densities!

McLerran,RV (1994)

Inclusive DIS: dipole evolution in gluon shockwave background

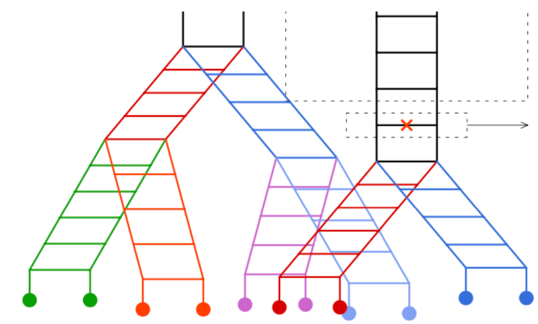




$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

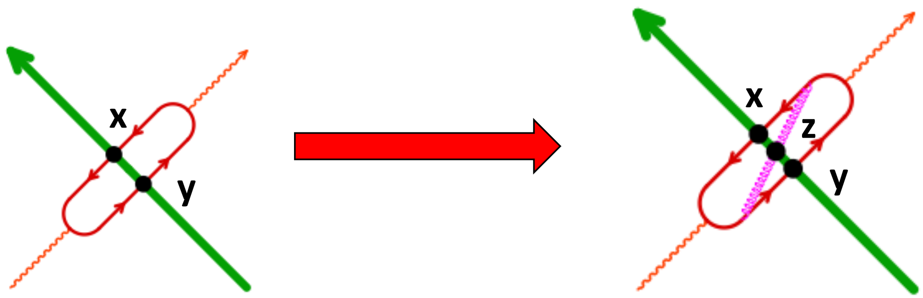
Y = Ln(1/x)

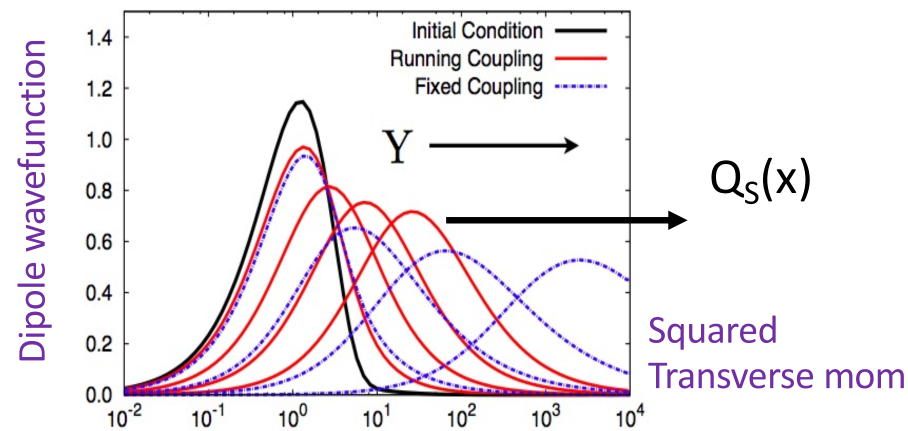
Closed form expression for A >>1, Nc→ ∞: non-linear Balitsky-Kovchegov (BK) eqn.



“Fan multi-Pomeron” diagrams

Inclusive DIS: dipole evolution in gluon shockwave background





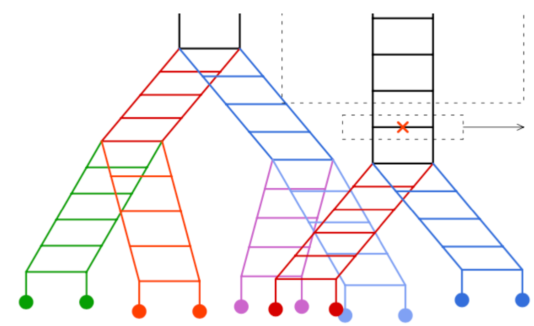
B-JIMWLK RG eqn. for dipole Wilson-line correlator:

$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\, \langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\, \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Closed form expression for A >>1, $N_c \to \infty$: non-linear Balitsky-Kovchegov (BK) eqn.

Fixed point of evolution saturates cross-section for fixed impact parameter
- this defines the close packing scale $Q_S(x)$ – cure for IR diffusion

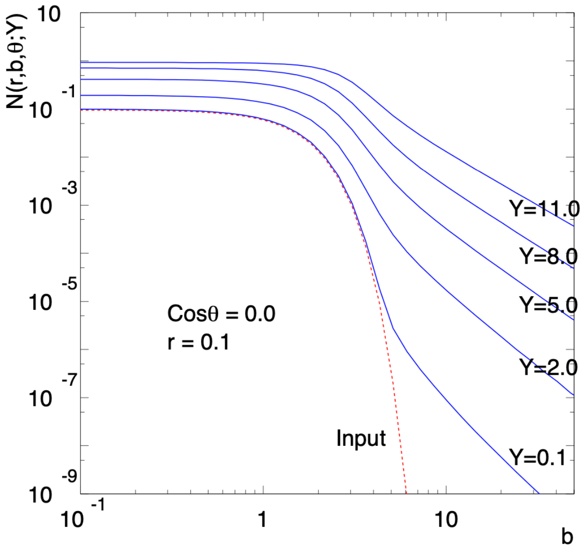
The BFKL equation is the low density $V \approx 1 - ig\rho/\nabla T^2$ limit of the BK equation…



Multi-Pomeron diagrams → BFKL ladder

The elephant in the room

Dipole amplitude as function of imp. parameter

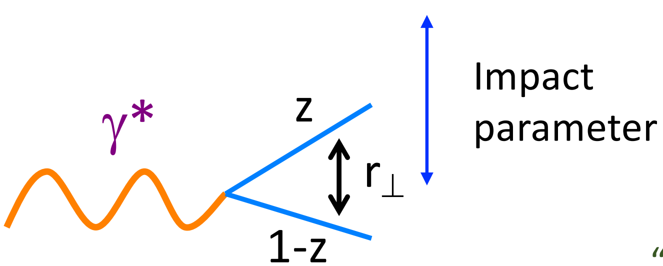


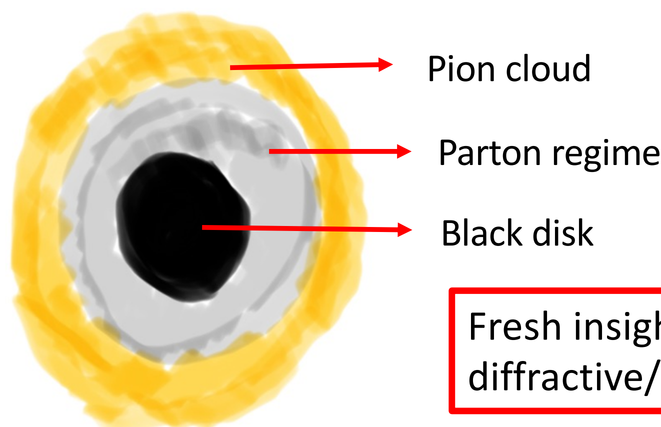
Golec-Biernat, Stasto, hep-ph/0306279

At fixed large b, exponential growth with rapidity

At small b, more modest growth characteristic of gluon saturation

Even with sharp exponential suppression of the initial dist. with b, perturbative Coulomb tail emerges quickly with rapidity evolution -reflective of missing physics of gluon/quark confinement



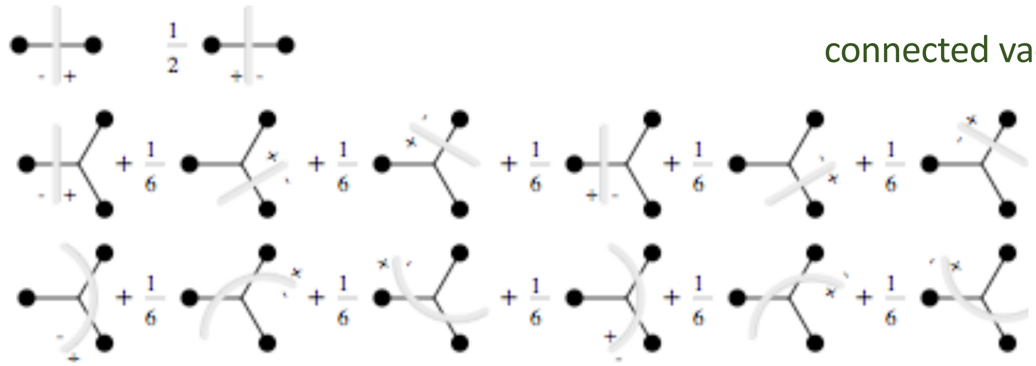


"Gribov" diffusion of the proton

Fresh insight from diffractive/exclusive final states

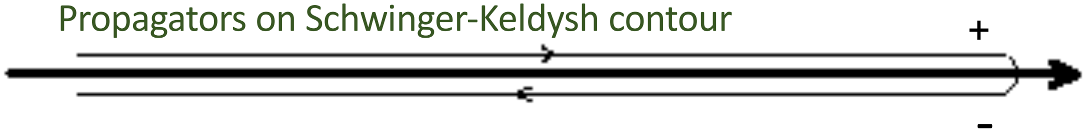
General all-order formalism: Cutkosky's rules in strong fields

$$2\,\mathrm{Im}\sum_{\mathrm{conn.}} V =$$



connected vacuum graphs in $\lambda\phi^3$

Propagators on Schwinger-Keldysh contour



Well-known example: Schwinger pair production in strong field QED

Simple understanding of "AGK cutting rules" of Reggeon Field Theory: combinatorics of cut and uncut sub-graphs contributing to a given multiplicity

- Very general consequence of unitarity in strong fields
- Independent of the language of Pomerons and Reggeons

AGK: Abramovsy,Gribov,Kancheli

Paradigm shift? Perhaps Pomerons best viewed as simplest constructions enforcing strong field unitarity rather than fundamental objects

Gelis,RV: hep-ph/0601209, hep-ph/0608117

From LO+LLx to NLO+NLLx

State of the art:

Small x evolution:

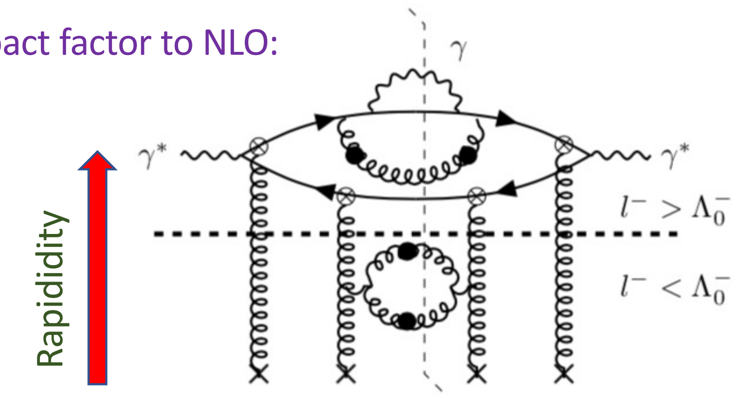
NLO BFKL: Fadin, Lipatov (1998)
NLO JIMWLK: Balitsky, Chirilli, arXiv:1309.7644, Grabovsky, arXiv:1307.5414
Caron-Huot, arXiv:1309.6521, Kovner,Lublinsky,Mulian, arXiv:1310.0378,
Lublinsky, Mulian, arXiv:1610.03453
NNLO BK (SYM): Caron-Huot, Herranen (2018)

Resummed NLLx:
Salam (1999); Ciafaloni,Colferai,Salam,Stasto (1999-2004)
Ducloue,Iancu,Madrigal,Mueller,Soyez,Triantaffyllopoulos (2015-2019)

NLO impact factors:

Inclusive DIS: Balitsky,Chirilli (2013)
Diffractive DIS: Boussarie,Szymanowski,Wallon (2016)
Massive quarks: Beuf,Lappi,Paatelainen (2021)
p+A forward di-jets: Iancu,Mulian (2021)
Photon+di-jet in DIS: Roy,RV (2020)
DIS di-jets/di-hadrons: Caucal,Salazar,RV (2021); Caucal, Salazar, Schenke, RV (2022)
Taels, Altinoluk,Beuf, Marquet, arXiv:2204.11650; Bergabo, Jalilian-Marian, arXiv:2207.03606
+ 20 odd papers this year

Impact factor to NLO:

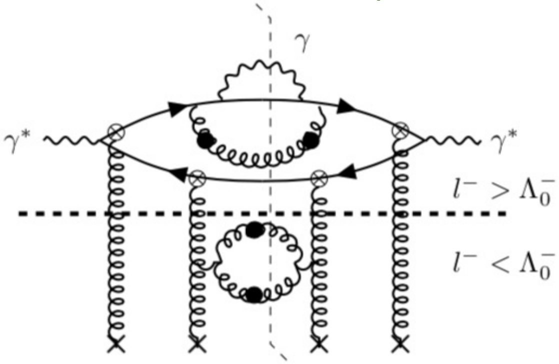


Evolution to NLLx:
$O(\alpha_S^2 \ \mathrm{Ln}(\frac{1}{x}))$

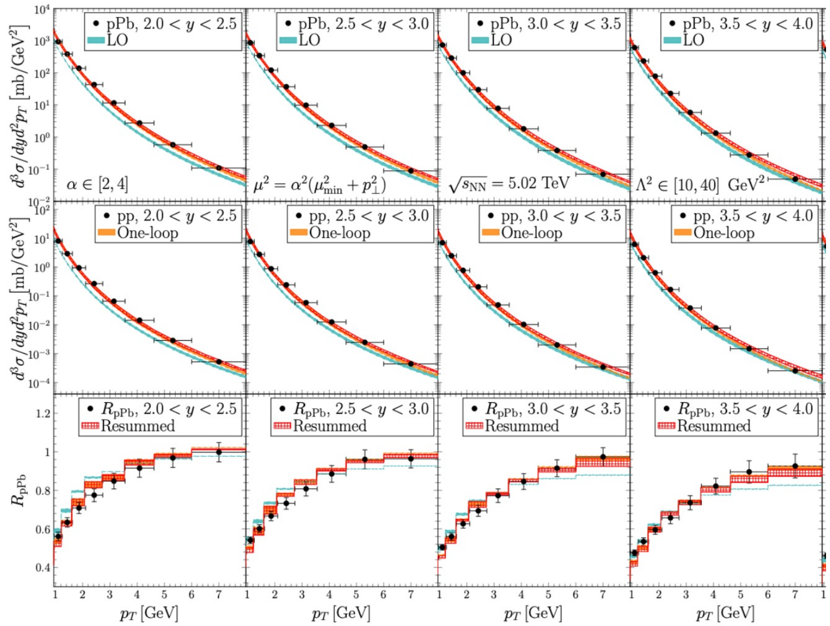
(Dressed "shockwave" propagators include coherent multiple scatterings to all orders)

CGC state of the art: global analysis of DIS+hadron-hadron collisions

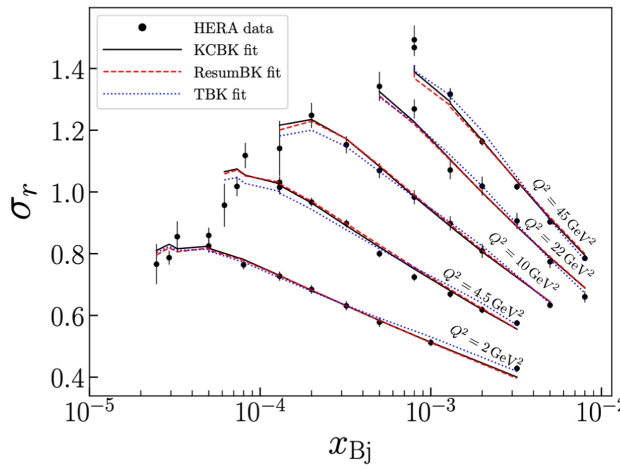
Precision CGC EFT computations



Single inclusive hadron distributions at the LHC



HERA DIS structure functions



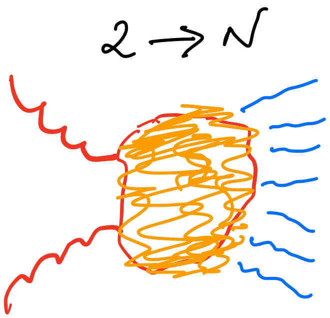
Beuf,Hanninen,Lappi,Mantysaari, arXiv:2007.01645

Shi,Wang,Wei,Xiao, arXiv:2112.06975

SURGE DOE Topical Theory Collaboration: 22 PI’s from 16 institutions (2022-2027)

Small x and quantum information science

Classicalization+unitarization – saturates the Bekenstein bound



$$P_{2\to N} \sim e^{S} \alpha_s^N N! \quad \text{If } N \sim \frac{1}{\alpha_s}$$

$$P_{2\to N} \sim e^{S} \alpha_s^N \left(\frac{1}{\alpha_s}\right)^N e^{-1/\alpha_s}$$

Exponential suppression of "classical lumps" unless $S = \frac{1}{\alpha_S}$ $\rightarrow P_{2\rightarrow N} = O(1)$

This entropy saturates the Bekenstein bound $S \leq 2\pi E R/\hbar$

on the maximal information in a given spacetime region

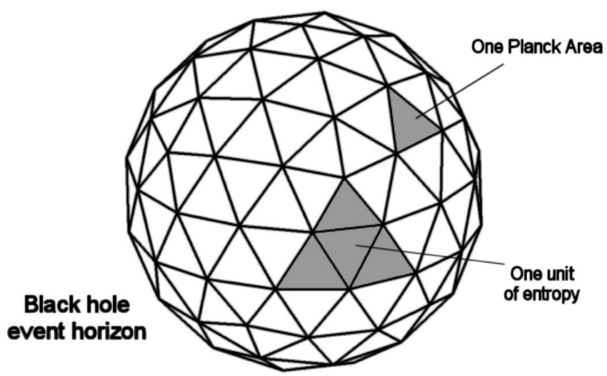
If we define $E = N\,Q_S$ as the energy in a critically packed volume = R_S^3 of quanta ("qubits") saturating unitarity (maximal information) and $Q_S = 1/R_S$

$$when\ N = \frac{1}{\alpha_S} \rightarrow S_{Bek} = \frac{1}{\alpha_S}$$

Dvali, RV,
arXiv:2106.11989, PRD (2022)

Critical packing saturates Bekenstein-Hawking area law

Bekenstein-Hawking bound



**(for a nice discussion,
see Bousso, arxIv:1810.01880)**

The entropy can be expressed in terms of a Goldstone decay constant f_G

- spontaneous breaking of Poincare invariance and a global sub-group (corresponding to large gauge transformations) by the gluon shockwave

Decay constant of Goldstone field ϕ is $f_G = R_S\,\partial_x\phi = \frac{\sqrt{N}}{R_S} = \sqrt{N}\;Q_S$

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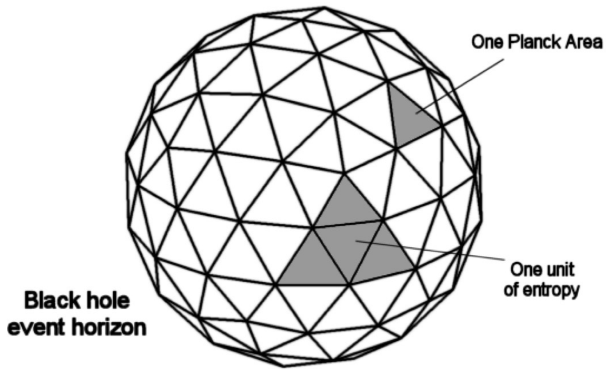

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Hence one can equivalently express the entropy as

$S = \frac{1}{\alpha_S} = \text{Area} \times f_G^2$

Dvali, arXiv:1907.07332

In gravity, $f_G^2 = M_{Planck}^2 = \frac{1}{G}$ so one recovers the Hawking-Bekenstein bound on the entropy

What are the physical consequences for classical lumps in QCD ?

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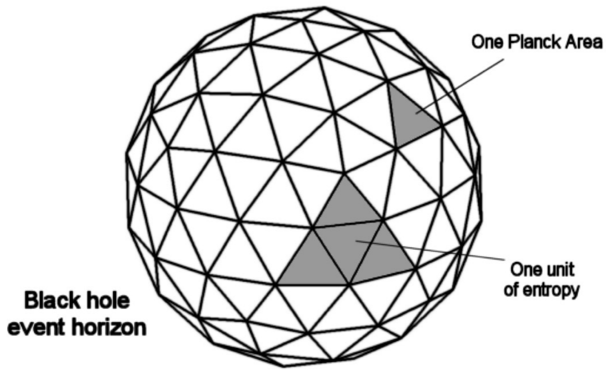

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In DIS, scattering of Goldstones causes CGC shockwave to decay on time scale $\tau_{Goldstone} = \frac{1}{\alpha_S}\frac{1}{Q_S} \ll \tau_{Eikonal} \sim \sqrt{s}$

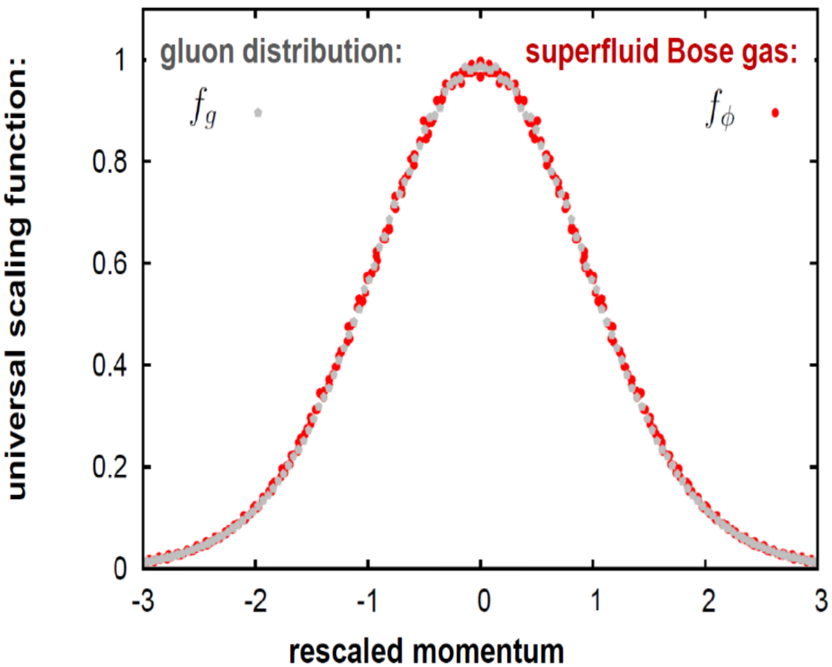
Such **final state** effects can influence emission of soft radiation on momentum scales $\sim \alpha_s\ Q_S$

In heavy-ion collisions, this descrption is nothing but the early-time bottom-up thermalization scenario

Universality: saturated glue and overoccupied ultracold atoms

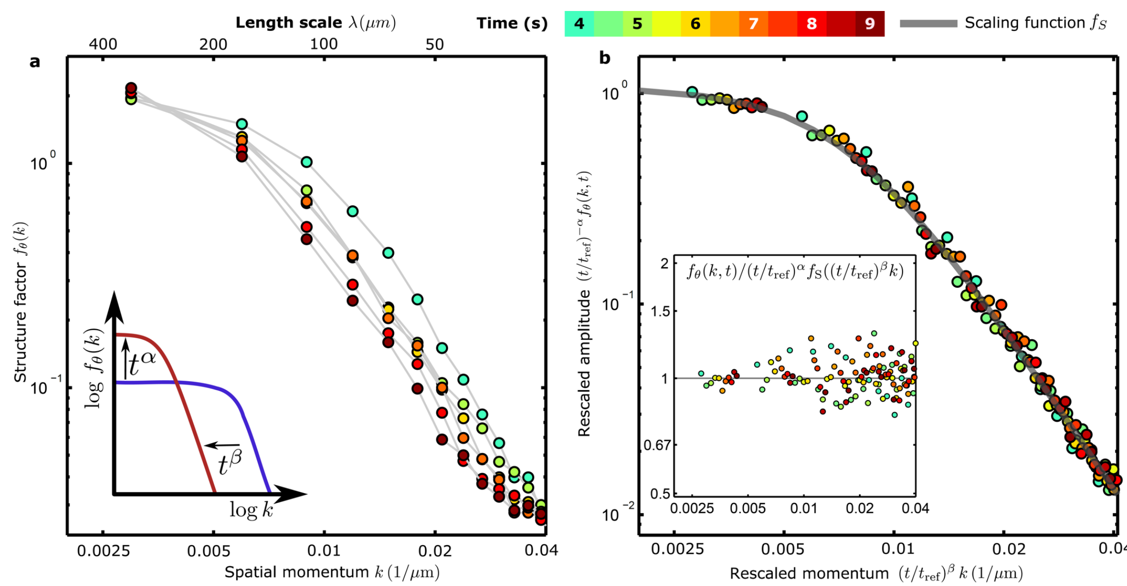
Overoccupied expanding Yang-Mills fields and self-interacting scalars described by the same non-thermal attractor

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$



Berges,Boguslavski,Schlichting,RV, PRL (2015) Editor's suggestion

^{87}Rb BEC in a quasi 1D optical trap



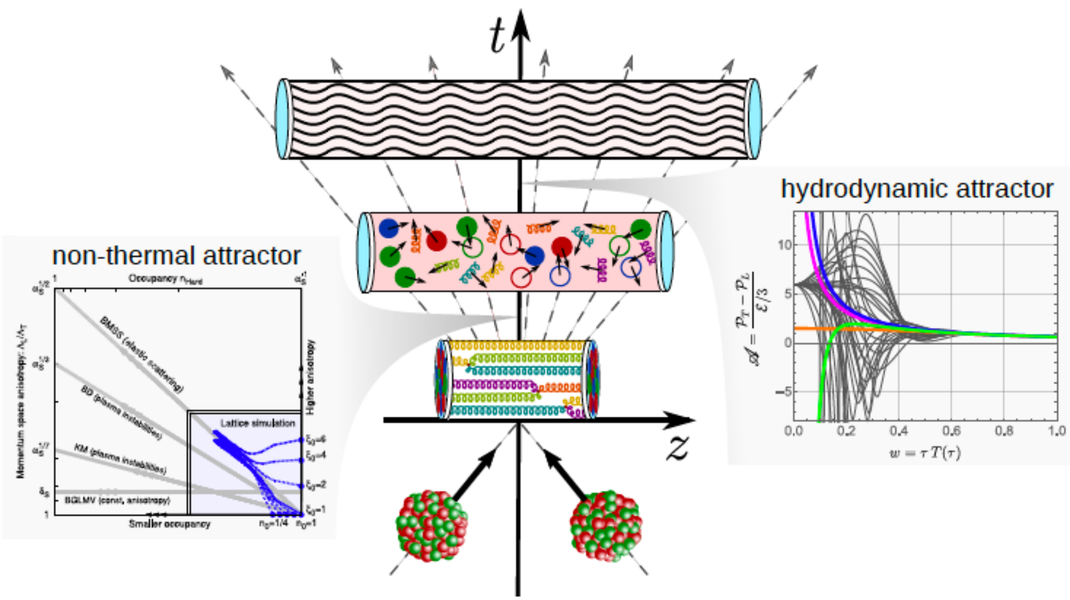
Oberthaler BEC Labs, Prüfer et al, arXiv:1805.11881, *Nature* (2018)

Scalable cold-atom quantum simulator for overoccupied features of gauge theories?

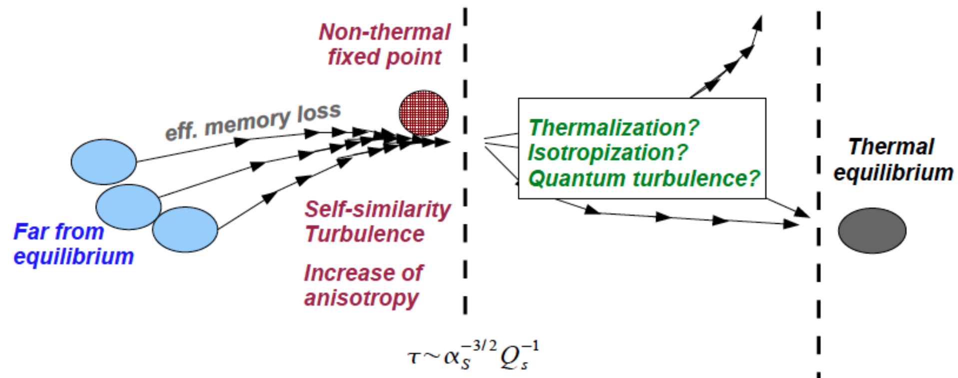
R. Ott et al., arXiv:2012.10432

Spacetime evolution of a heavy-ion collision: bottom-up thermalization

Quark-Gluon Plasma undergoing
hydrodynamic expansion



Collision of overoccupied Color Glass Condensate shockwaves



Thermal soft gluon bath for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

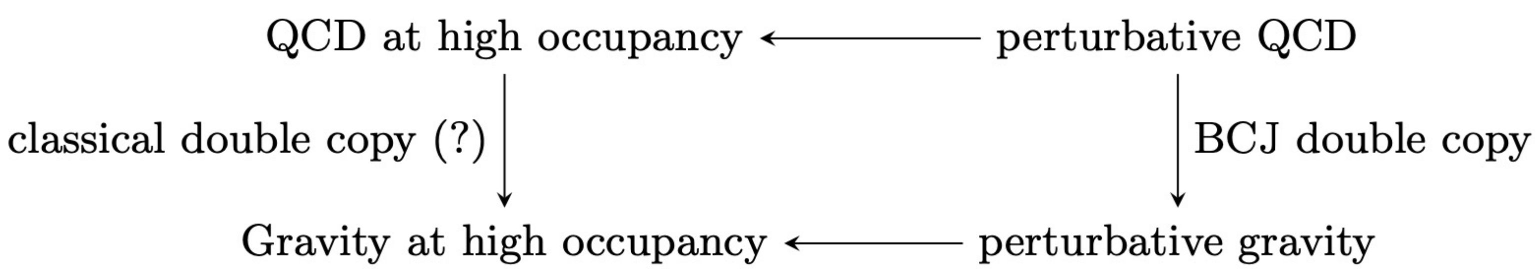
Thermalization temperature: $T_i = \alpha_S^{2/5} Q_S$

Very rapid thermalization
as $\alpha_S(Q_S) \rightarrow 0$ *and* $Q_S \rightarrow \infty$

Baier, Mueller, Schiff, Son,
hep-ph/0009237

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

Double Copy: gluon → gravitational radiation in shockwave collisions



Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Bern, Carrasco, Johannson,
arXiv: 1004.0476

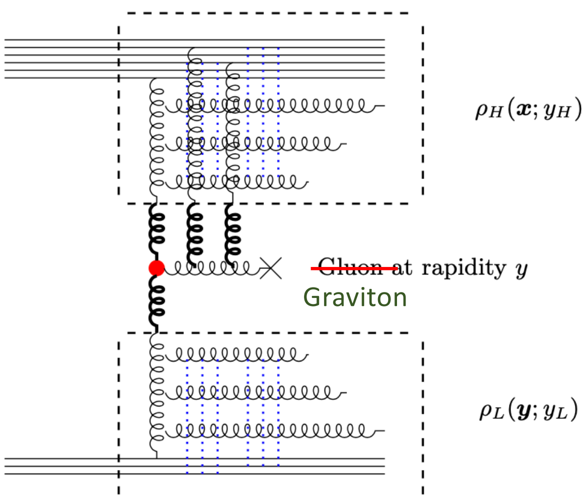
Review:
Bern et al., arXiv:1909.01358

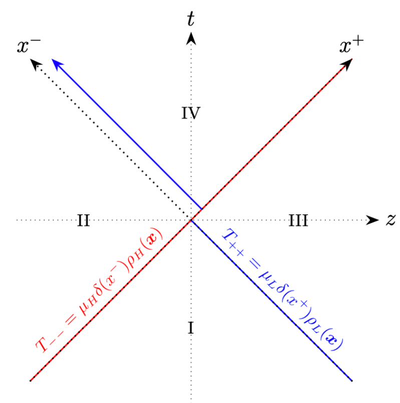
Double Copy: gluon → gravitational radiation in shockwave collisions

Classical lumps spring into action !

In QCD, in the CGC EFT, strong field semi-classical methods powerful alternative to amplitudes approaches
- RG equations in rapidity allow for quantitative study of gluon saturation and the approach to it

Can we do the same for gravity in the strong field regime of trans-Planckian scattering?
Can we compute gravitational wave radiation with varying frequency and impact parameter to extract quantum features of GR, and obtain insight into BH formation?



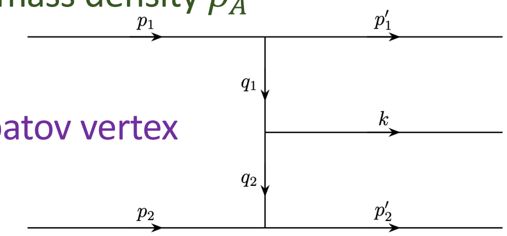


Derivation of Lipatov double copy from Einstein's eqns.

Solve Einstein's eqns. for linearized perturbations $h_{\mu\nu}$ around strong field shockwave metric for $R_S < b$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Shockwave A with mass density ρ_A



GR Lipatov vertex

Shockwave B with mass density ρ_B

Radiation amplitude:

$$-\boldsymbol{k}^2 \tilde{h}_{ij} = \int \frac{d\boldsymbol{q}_2}{(2\pi)^2} \frac{\rho_A(\boldsymbol{q}_1)}{\boldsymbol{q}_1^2} \frac{\rho_B(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} \Gamma_{ij}$$

where

$$\Gamma_{ij} = 2\left[\left(q_{2i} - k_i \frac{\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^2}\right)\left(q_{2j} - k_j \frac{\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^2}\right) - k_i k_j \frac{\boldsymbol{q}_{1\perp}^2 \boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^4}\right]$$

Lipatov double copy

$$\Gamma_{\mu\nu} \equiv \frac{1}{2} C_\mu C_\nu - \frac{1}{2} N_\mu N_\nu$$

C_μ is the QCD Lipatov vertex and N_μ is the QED Bremsstrahlung vertex

A semi-classical computation in GR (completely analogous to prior QCD YM demonstration) can reproduce Lipatov's result obtained by Feynman diagram computations in multi-Regge kinematics

Reggeization a la Lipatov proceeds analogously though there are subtle important differences

Himanshu Raju, RV, in preparation.
In QCD, analogous derivation of Lipatov vertex from gluon shock wave collisions:
Blaizot, Gelis, RV, hep-ph/0402256, Gelis, Mehtar-Tani, hep-ph/0512079

Concluding remarks

Significant progress in understanding realtime dynamics in QCDs Regge limit using strong field weak coupling methods.

May inspire a novel way to think about strong field dynamics at large coupling

Rich interdisciplinary connections – heavy-ion collisions, cold atoms, and now GR.