Probing gluon saturation via photon-hadron interactions at high energy

Edmond lancu

IPhT, Université Paris-Saclay

with A.H. Mueller, D.N. Triantafyllopoulos, and S.-Y. Wei



Forward Physics & QCD, Bad Honnef 2023

Outline

- Color Dipole Picture for photon-hadron interactions at high energy
 - Gluon Saturation: brief introduction (cf. talk by Raju Venugopalan)
 - DIS: from the target picture to the dipole picture
 - Gluon Saturation: Inclusive vs. Diffractive DIS
- Diffractive jets in γ -hadron interactions
 - Exclusive dijets: higher twist
 - 2+1 jets: the dominant channel
 - TMD factorisation from the Color Dipole Picture
 - Unintegrated gluon distribution of the Pomeron
- Diffractive jets in Ultraperipheral Nucleus-Nucleus Collisions
 - opportunities (gluon saturation) & difficulties

Rise of gluon PDF

• Electron-proton DIS at HERA, two invariants: Q^2 and $x = \frac{Q^2}{2P \cdot \sigma}$



• $xG(x,Q^2)=\#$ of gluons with longitudinal momentum fraction x and transverse momentum $k_{\perp}\leq Q$

• Dominance of gluons at small $x \le 0.01$: QCD evolution

Soft gluons

• Soft gluon emission from a quark







 $\mathrm{d}\mathcal{P} \simeq \frac{\alpha_s}{2\pi} \frac{\mathrm{d}k_{\perp}^2}{k_{\perp}^2} \underbrace{P_{g \leftarrow q}(x)}_{\sim 2C_T/x} \mathrm{d}x$

 $\mathrm{d}\mathcal{P} \simeq \frac{\alpha_s}{2\pi} \frac{\mathrm{d}k_{\perp}^2}{k_{\perp}^2} \underbrace{P_{g \leftarrow g}(x)}_{} \mathrm{d}x$ $\sim 2N_c/x$

- Large emission probability when the gluon is soft/low-energy: $x \ll 1$
- By iterating soft gluon emissions => Gluon cascades (BFKL evolution)

BFKL evolution (Balitsky, Fadin, Kuraev, Lipatov, 1974-78)

• Two successive gluon emissions, which are strongly ordered



• $Y \equiv \ln(1/x)$: rapidity difference between parent quark and final gluon

• k+1 successive emissions, strongly ordered in x



• Linear evolution: gluons multiply, yet they do not interact

Evolution and the transverse density



- Increase k_{\perp} , or Q^2 (DGLAP) \Longrightarrow many small partons \Longrightarrow low density
- Increase 1/x (BFKL) \implies many partons of similar size \implies high density
- Overlapping gluons can interact with each other

Gluon occupancy

• The relevant quantity: the gluon occupation number in phase space



 $\bullet\,$ gluon area $\times\,$ gluon density

$$n(x,Q^2) \simeq \frac{1}{Q^2} \times \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

 $\bullet\,$ dilute systems have $n\ll 1$

- When $n \gtrsim 1$, gluons overlap, but their interactions are still suppressed by α_s
- Interactions become of $\mathcal{O}(1)$ when $n(x,Q^2) \sim 1/\alpha_s$
- Emergent transverse momentum scale: saturation momentum $Q_s(x, A)$

BK-JIMWLK evolution

• Evolution becomes non-linear (JIMWLK) and gluon occupancy saturates (Jalilian-Marian, lancu, McLerran, Weigert, Leonidov, and Kovner, 97–00)



schematically ("BK eq.")
 (Balitsky, 95; Kovchegov, 99)

$$\frac{\partial n}{\partial Y} = \alpha_s n - \alpha_s^2 n^2$$

• Fixed point
$$n \sim 1/\alpha_s$$

- Non-linear, functional, generalisation of BFKL: Wilson line operators
 - unitarity for scattering amplitudes, no infrared diffusion
- Known to NLO + collinear resummations (see talk by Nestor Armesto)

Saturation momentum



• $x \sim 10^{-3}$ (EIC): $Q_s^2 \sim 2 \text{ GeV}^2$ for Pb or Au

• $x \sim 10^{-5}$ (LHC): $Q_s^2 \sim 10 \text{ GeV}^2$ for Pb and $\sim 1 \text{ GeV}^2$ for a proton

Dipole picture: Lorentz frame

- Usual parton picture: target infinite momentum frame $P_N^-
 ightarrow \infty$
- $x \ll 1$: boost to the dipole frame where the photon is energetic: $q^+ \gg Q$
- Goal: transfer the $q\bar{q}$ pair from the target to the projectile



$$x = \frac{Q^2}{2P_N^- q^+} \ll \frac{1}{A^{1/3}} \iff \Delta x^+ \simeq \frac{2q^+}{Q^2} \gg \frac{A^{1/3}}{P_N^-} \sim \frac{R_A}{\gamma}$$

- ${\, \bullet \,}$ the virtual photon fluctuates into a $q \bar q$ pair long before the scattering
- ${\, \bullet \,}$ the $q \bar{q}$ color dipole acts as a probe of the gluon distribution

Dipole picture: gauge choice

- Usual parton picture: target light-cone gauge $A^- = 0$
- $x \ll 1$: change to the light-cone gauge of the projectile (dipole): $A^+ = 0$
- Goal: transfer the high energy evolution & saturation from target to dipole
 - wavefunction of a relativistic system is well defined in its LC gauge



• building the wavefunction is simpler for a dilute object

Dipole picture: gauge choice

- Usual parton picture: target light-cone gauge $A^- = 0$
- $x \ll 1$: change to the light-cone gauge of the projectile (dipole): $A^+ = 0$
- Goal: transfer the high energy evolution & saturation from target to dipole
 - wavefunction of a relativistic system is well defined in its LC gauge



- building the wavefunction is simpler for a dilute object
- this becomes obvious after adding non-linear effects
- Gluon saturation in target picture \iff multiple scattering in dipole picture

Dipole picture for inclusive DIS

• Lorentz contraction: the dipole scatters of a shockwave

$$\sigma_{\gamma^*A}(Q^2, x) = \int \mathrm{d}^2 r \int_0^1 \mathrm{d}\vartheta \left| \Psi_{\gamma^* \to q\bar{q}}(r, \vartheta; Q^2) \right|^2 \underbrace{\sigma_{\mathrm{dipole}}(r, A, x)}_{2\pi R_A^2 T_A(r, x)}$$

- All QCD dynamics in the dipole amplitude $T_A(r, x)$
- Solution to BK/JIMWLK eqs, MV model for initial condition at $x_0 \sim 10^{-2}$

$$T_A(r,x) \simeq \begin{cases} r^2 Q_s^2(A,x) & \text{for } rQ_s \ll 1 \text{ (color transparency)} \\ 1 & \text{for } rQ_s \gtrsim 1 \text{ (black disk/saturation)} \end{cases}$$

Dipole picture for inclusive DIS



- Virtuality limits large dipoles: $r \lesssim 1/\bar{Q}$ with $\bar{Q}^2 \equiv \vartheta(1-\vartheta)Q^2$
- Saturation requires $r\gtrsim 1/Q_s$, hence $ar{Q}^2 \lesssim Q_s^2$
- When $Q^2 \gg Q_s^2$, dominant contribution from weak scattering $(rQ_s \ll 1)$
- Even weak scattering can be affected by saturation

Geometric scaling at HERA

(Staśto, Golec-Biernat, Kwieciński, 2000)



• In general, $\sigma_{\gamma^* p}(x, Q^2)$ involves 2 independent variables

- For $x \leq 10^{-2}$, it depends only upon $\tau \equiv Q^2/Q_s^2(x)$: "scaling"
- For larger $x > 10^{-2}$: no scaling

Forward Physics & QCD, Bad Honnef 2023

Gluon saturation in γA

Diffraction: F_{2D}

• Elastic scattering/diffraction is more sensitive to strong scattering



- Colourless exchange: 2-gluon ladder, (BFKL) Pomeron, rapidity gap
- F_{2D} controlled by the black disk limit $(T \sim 1)$ even when $Q^2 \gg Q_s^2$
 - asymmetric $q\bar{q}$ pairs, $\vartheta(1-\vartheta)\ll 1,$ with large size $r\sim 1/Q_s$
 - $\bullet\,$ would be non-perturbative, $r\sim 1/\Lambda,$ in absence of saturation

One vs two Pomerons... at HERA

• 2-gluon exchange: σ_{el} rises with $\frac{1}{r}$, or with A, like two Pomerons

$$T^2 \simeq \left[r^2 Q_s^2(A, x) \right]^2 \propto \frac{A^{2/3}}{r^{2\lambda_s}}$$



Figure 9: The ratio of $\sigma_{diff}/\sigma_{tot}$ versus the γ^*p energy W. The data is from ZEUS and the solid lines correspond to the results of the DGLAP improved model with massless quarks (FIT 2).

16/34

• But it doesn't! Controlled by multiple scattering: $T\sim$ 1, or $r\sim 1/Q_s$

vs. $T \simeq r^2 Q_s^2(A, x) \propto \frac{A^{1/3}}{m^{\lambda_s}}$

• Almost the same scaling as σ_{tot} :

 $\frac{\sigma_{el}}{\sigma_{tot}} \, \sim \, \frac{1}{\ln \left(Q^2/Q_s^2\right)}$

- Weak *x*-dependence confirmed by HERA (Bartels, Golec-Biernat, Kowalski, 2002)
- Would be interesting to also check the *A*-dependence at the EIC

Exclusive production of hard dijets

- The simplest jet measurement in diffraction: a pair of back-to-back jets
- Leading order: a $q\bar{q}$ pair, which is hard and relatively symmetric:

• $k_{1\perp} \simeq k_{2\perp} \equiv P_{\perp} \sim Q \gg Q_s$ & $\vartheta \sim 1/2$

• Hard scale sets dipole size $r \sim 1/P_{\perp} \ll 1/Q_s \Longrightarrow$ weak scattering



• Rare events ("higher twist"), insensitive to saturation

Diffractive 2+1 jets

(E.I., A.H. Mueller, D.N. Triantafyllopoulos, PRL 128 (2022) 20)

- Can one have diffractive dijets at leading twist ? ($\sim 1/P_{\perp}^4)$
- $\bullet\,$ Yes, two hard jets $P_\perp \gg Q_s$ and one semi-hard $k_{3\perp} \sim Q_s \ll P_\perp$
- Third, semi-hard, jet provides dijet imbalance: $|{m k}_1+{m k}_2|=k_{3\perp}\ll P_\perp$



$$R \sim \frac{1}{Q_s} \, \gg \, r \sim \frac{1}{P_\perp}$$

- Effective gluon-gluon dipole
- $\left\{ Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}} \quad \bullet \text{ Strong scattering: } T_{gg}(R, Y_{\mathbb{P}}) \sim 1 \right\}$
 - $\mathcal{O}(\alpha_s)$, but leading-twist

• N.B. Saturation momentum evaluated at the rapidity gap $Y_{\mathbb{P}} = \ln \frac{1}{\tau_{\mathbb{P}}}$

TMD factorisation

• Scale separation \implies Factorisation \implies Gluon is part of the Pomeron

 $\frac{\mathrm{d}\sigma_{2+1}^{\gamma_{T,L}^*A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\mathbf{P}\mathrm{d}^2\mathbf{K}\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}(\vartheta_1,\vartheta_2,Q^2,P_{\perp}^2)\,\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_{\perp}^2)}{\mathrm{d}^2\mathbf{K}}$



• The hard factor: $\gamma^* \to q \bar{q}$ decay & the gluon emission

$$H_T = lpha_{
m em} lpha_s \left(\sum e_f^2\right) artheta_1 artheta_2 (artheta_1^2 + artheta_2^2) rac{1}{P_{\perp}^4} \quad {
m when} \, \, Q^2 \ll P_{\perp}^2$$

Forward Physics & QCD, Bad Honnef 2023

TMD factorisation

• Scale separation \implies Factorisation \implies Gluon is part of the Pomeron

 $\frac{\mathrm{d}\sigma_{2+1}^{\gamma_{T,L}^*A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\mathbf{P}\mathrm{d}^2\mathbf{K}\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}(\vartheta_1,\vartheta_2,Q^2,P_{\perp}^2)\,\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_{\perp}^2)}{\mathrm{d}^2\mathbf{K}}$



• Gluon diffractive TMD: unintegrated gluon distribution of the Pomeron

• x: energy fraction of the exchanged gluon with respect to the Pomeron

The Pomeron UGD

$$\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_{\perp}^{2})}{\mathrm{d}^{2}\boldsymbol{K}} = \frac{S_{\perp}(N_{c}^{2}-1)}{4\pi^{3}} \underbrace{\Phi_{g}(x,x_{\mathbb{P}},K_{\perp}^{2})}_{\mathrm{occupation number}}$$

• Given explicitly in terms of the gluon-gluon dipole amplitude $T_{gg}(R,Y_{\mathbb{P}})$

$$\Phi_g(x, x_{\mathbb{P}}, K_{\perp}^2) \simeq (1-x) \begin{cases} 1, & K_{\perp} \lesssim \tilde{Q}_s(x) \\ \\ \frac{\tilde{Q}_s^4(x)}{K_{\perp}^4}, & K_{\perp} \gg \tilde{Q}_s(x) \end{cases}$$

- $\bullet\,$ Valid for large gap/small $x_{\mathbb{P}} \lesssim 10^{-2}$ and any $x \leq 1$
- Effective saturation momentum: $ilde{Q}_s^2(x,Y_{\mathbb{P}}) = (1-x)Q_s^2(Y_{\mathbb{P}})$
- The bulk of the distribution lies at saturation: $K_{\perp} \lesssim \tilde{Q}_s(x)$
- Gluon saturation in the Pomeron \leftrightarrow Black disk limit $T_{gg} = 1$

Numerical results

(E.I., A.H. Mueller, D.N. Triantafyllopoulos, S.-Y. Wei, arXiv:2207.06268)

- Left: McLerran-Venugopalan model. Right: BK evolution of T_{gg}
 - multiplied by K_{\perp} (from the measure $d^2 K_{\perp}$)
- Pronounced maximum at $K_{\perp}\simeq ilde{Q}_s$



• BK evolution: increasing $Q_s^2(Y_{\mathbb{P}})$, approximate geometric scaling

From TMD to diffractive PDF

- Sensitivity to saturation persists after integrating out the K_{\perp} -distribution
 - the integral is rapidly converging and effectively cut off at $K_\perp \sim \tilde{Q}_s(x)$

$$xG_{\mathbb{P}}(x,x_{\mathbb{P}},P_{\perp}^2) \equiv \int^{P_{\perp}} \mathrm{d}K_{\perp}^2 \,\Phi(x,x_{\mathbb{P}},K_{\perp}^2) \,\propto \,(1-x)^2 \,Q_s^2(Y_{\mathbb{P}})$$

• Large separation of scales $P_{\perp}^2 \gg Q_s^2$: one must include DGLAP evolution



- MV model at $x_{\mathbb{P}} = 10^{-2}$
- BK evolution with $Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}}$
- DGLAP evolution with initial condition at $P_{\perp}^2 \sim Q_s^2(Y_{\mathbb{P}})$

• Collinear factorisation from CGC

The gluon diffractive PDF



• DGLAP: increase for very small $x \le 0.01$, slight decrease for x > 0.05

• When $x \to 1$, the distribution vanishes even faster

Gluon saturation in γA

2+1 diffractive dijets in AA UPCs

- $b > R_A + R_B$: interactions mediated by quasi-real photons
- One nucleus source of photons, other nucleus hadronic target
- Coherent diffraction: photon gap + diffractive gap



Pb+Pb UPCs at the LHC (see talk by Brian Cole)

Recent measurements: ATLAS-CONF-2022-021 and CMS arXiv:2205.00045



• Several thousands of candidate-events for coherent diffraction

- no just $\gamma\gamma$ scattering: cross-section would be 10 times smaller
- Most likely: 2+1 jets ... but how to experimentally check ?

• $K_{\perp} \sim Q_s \sim 1 \div 2 \text{ GeV}$: not really a jet! Measure as a hadron (?)

A favourable event situation

- Assume the photon to be a right mover: it was emitted by nucleus B
- 3rd jet must lie within the hadronic detector: $|\eta_3| < |\eta_{max}| = 2.4$



• $\omega = 40 \text{GeV}, P_{\perp} = 10 \text{GeV}$

• $\eta_{1,2}\simeq 1.4$, $x_{\mathbb{P}}\simeq 0.002$

$$\Delta \eta_{\rm jet} \gtrsim \ln \frac{2P_{\perp}}{Q_s} \simeq 2.3$$

- Observing the 3rd jet would be extremely useful
 - $\bullet\,$ it propagates towards the nuclear target: lift the A vs. B ambiguity
 - ullet one can measure the diffractive rapidity gap and thus infer $x_{\mathbb P}$
 - ullet rapidity separation $\Delta\eta_{\rm jet}:$ a measure of the saturation momentum Q_s

A likely event at CMS

• Dijet events selected by CMS have larger $P_{\perp} \ge 30 \text{GeV} (arXiv:2205.00045)$



• $\omega = 40 \text{GeV}$, $P_{\perp} = 30 \text{GeV}$

•
$$\eta_{1,2}\simeq 0.3$$
, $x_{\mathbb{P}}\simeq 0.02$

$$\Delta \eta_{\rm jet} \gtrsim \ln \frac{2P_\perp}{Q_s} \simeq 3.4$$

• $|\eta_3| = 3.1 > |\eta_{\max}| = 2.4$: the 3rd jet is missed by the detector \bigotimes

- Lessons: Trigger on rare events with high photon energy ω
- Use a hadronic detector with larger rapidity coverage η_{max}
- Measure jets with lower $P_{\perp} \leq 15 \text{GeV}$

Conclusions

- Diffraction in γA (EIC, UPC): best laboratory to study gluon saturation
- For sufficiently small $x_{\mathbb{P}} \lesssim 10^{-2}$ and/or large $A \sim 200$, diffractive TMDs and PDFs can be computed from first principles
- Collinear factorisation for hard diffraction emerging from CGC
- Due to saturation, diffractive dijets are dominated by (2+1)-jet events
- AA UPCs: Measuring the semi-hard, 3rd, jet is tough, but very useful
 - measure dijets (or dihadrons) with lower $P_{\perp} \leq 10 \, {\rm GeV}$
 - hadronic detectors with larger rapidity coverage
 - $\bullet\,$ what about diffractive hadrons in UPCs at ALICE ?
- Kinematical conditions should be more favorable at the EIC

Soft gluon and TMD factorisation

• The third jet is relatively soft: $k_3^+ = \vartheta_3 q^+$ with $\vartheta_3 \sim \frac{Q_s^2}{Q^2} \ll 1$

• gluon formation time must be small enough to scatter: $rac{k_3^+}{k_{a+}^2}\lesssim rac{q^+}{Q^2}$

• Scale separation \implies Factorisation \implies Gluon is part of the Pomeron



• x: energy fraction of the exchanged gluon with respect to the Pomeron

• gluon absorption puts the hard dijet system on-shell

Soft gluon and TMD factorisation (2)

- The strong ordering in both k_{\perp} and k^+ is essential for factorisation
- The dipole picture holds in the projectile light cone gauge $A^+ = 0$
 - ${\, \bullet \,}$ right moving partons couple to the A^- component of the target field



- The TMD picture holds in the target light cone gauge $A^- = 0$
 - only the soft gluon couples to the target field: $v^i A^i$ with $v^i = k^i/k^+$

Energy cutoff

- Energy is not that high:
 - LHC: $\sqrt{s_{\scriptscriptstyle NN}}=2E_N=5\,{\rm TeV}$, yet $\sqrt{s_{\scriptscriptstyle \gamma N}}=\sqrt{4\omega_{\rm max}E_N}\simeq 650{\rm GeV}$
 - upper energy cutoff: $b \sim \frac{1}{Q} > 2R_A \Rightarrow \omega < \frac{\gamma}{2R_A} \equiv \omega_{\max} \simeq 40 \text{ GeV}$
- 10^{-3} 10^{-2} 10^{-1} 10^{0} B 10^{2} Y_{γ} $P_{\perp} \gg O$. 10^{1} $\frac{dN}{dr}$ 10^{0} $0000000 K_1 \sim Q_s$ Z = 82 10^{-1} $Y_{\mathbb{P}}$ $R_A = R_B = 6 \text{ fm}$ $P_N^ 10^{-2}$ 10^{-1} 10^{0} 10^{1} 10^{2} ω (GeV)

• exponential suppression for $\omega > \omega_{\max}$

$x_{\mathbb{P}}$ is not that small

- Limited energy and relatively hard dijets $P_{\perp} \geq 15 \, {\rm GeV}$
 - relatively large $x_{\mathbb{P}}$: $x_{\mathbb{P}} \gtrsim 5 \times 10^{-3}$
 - one cannot probe the high energy evolution of the Pomeron



- Not the ideal "small- $x_{\mathbb{P}}$ " set-up! Similar in that sense to the EIC
- Decreasing P_{\perp} would greatly help !

Final-state radiation

- Dijet momentum imbalance dominated by final-state radiation
 - additional gluons with transverse momenta $Q_s \ll k_\perp \ll P_\perp$



• LHC: dijet imbalance $Q_T = |\mathbf{k}_1 + \mathbf{k}_2| \sim 10 \text{ GeV} \gg Q_s$

- consistent with final state radiation (Hatta et al, 2010.10774)
- insensitive to the 3rd jet

2+1 jets with a hard gluon

• The third (semi-hard) jet can also be a quark: same-order



• TMD factorisation: quark unintegrated distribution of the Pomeron

