Forward Physics and QCD at the LHC and EIC
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The color glass condensate and forward physics at the LHC and EIC

Néstor Armesto
IGFAE, Universidade de Santiago de Compostela
nestor.armesto@usc.es

FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL "Unha maneira de facer Europa"

Contents:

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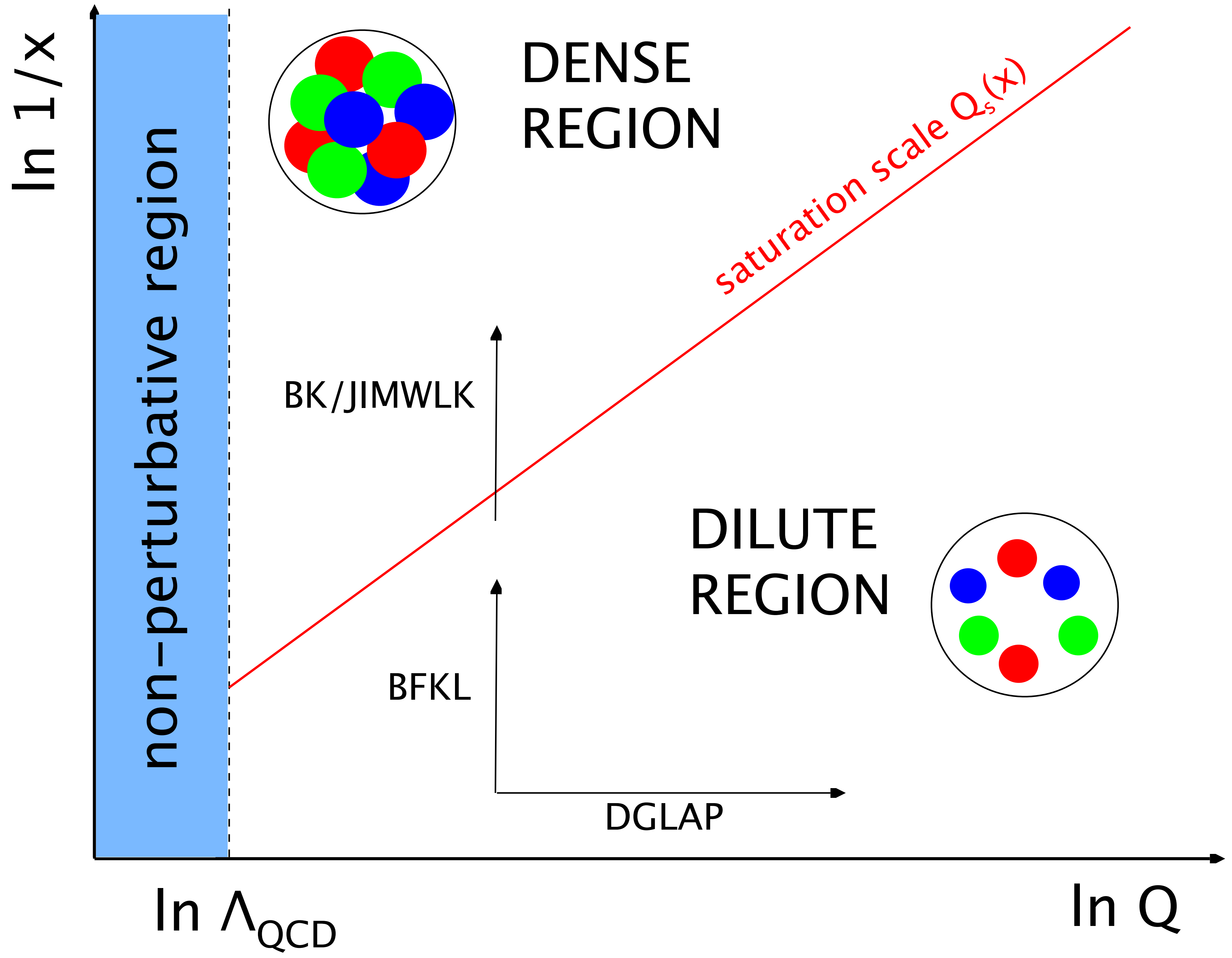
See the talk by Tolga Altinoluk at Initial Stages 2023 for more information.

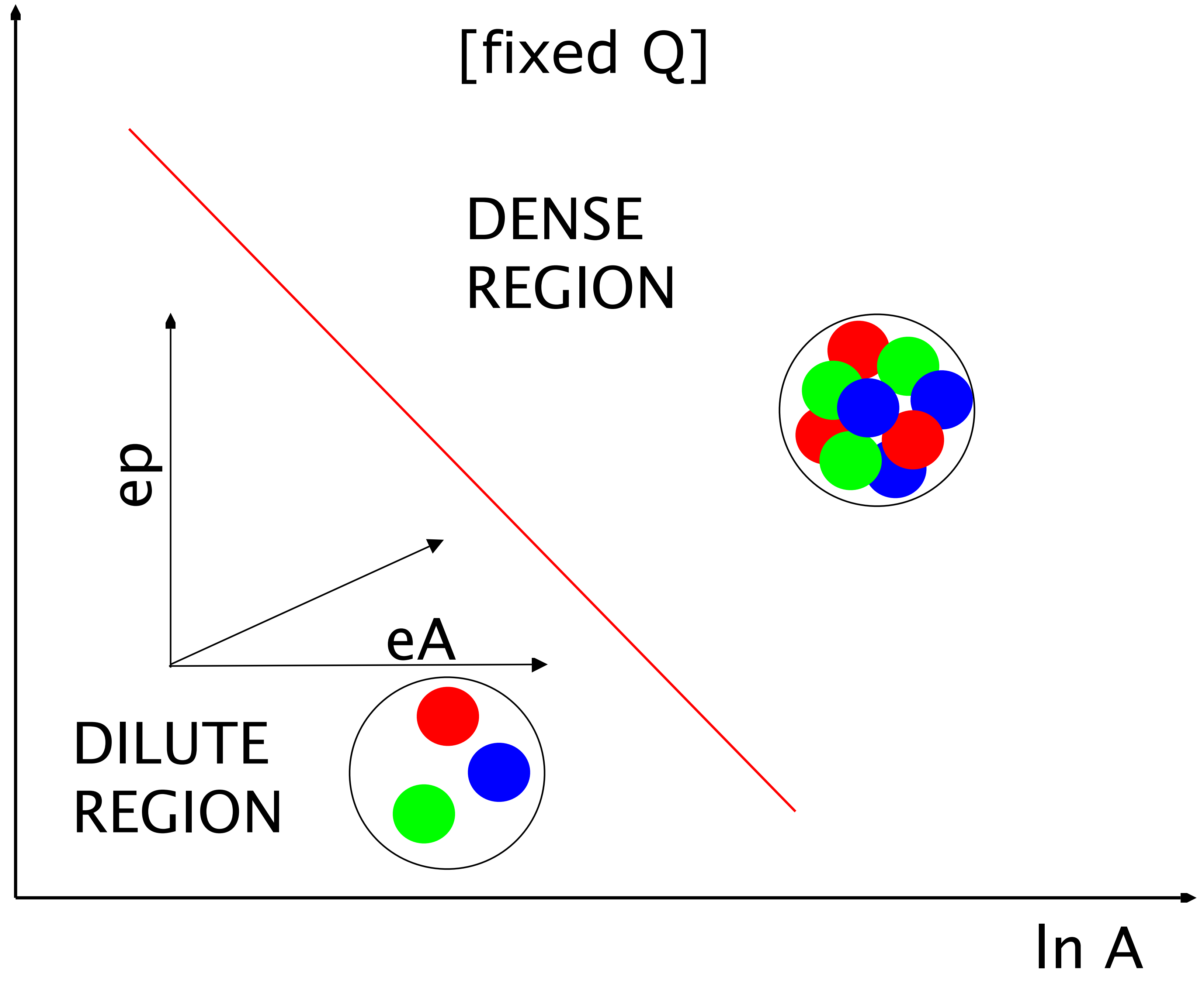
Small x:

- Standard fixed-order perturbation theory (DGLAP, linear evolution) must eventually fail:
  - ➜ Large logs, e.g., $\alpha_s \ln 1/x \sim 1$: resummation (BFKL,CCFM,ABF,CCSS).
  - ➜ High density ⇒ linear evolution cannot hold: saturation, either perturbative (CGC) or non-perturbative.

$$\frac{xG_A(x,Q_s^2)}{\pi R_A^2 Q_s^2} \sim 1 \Longrightarrow Q_s^2 \propto A^{1/3} x^{\sim -0.3}$$

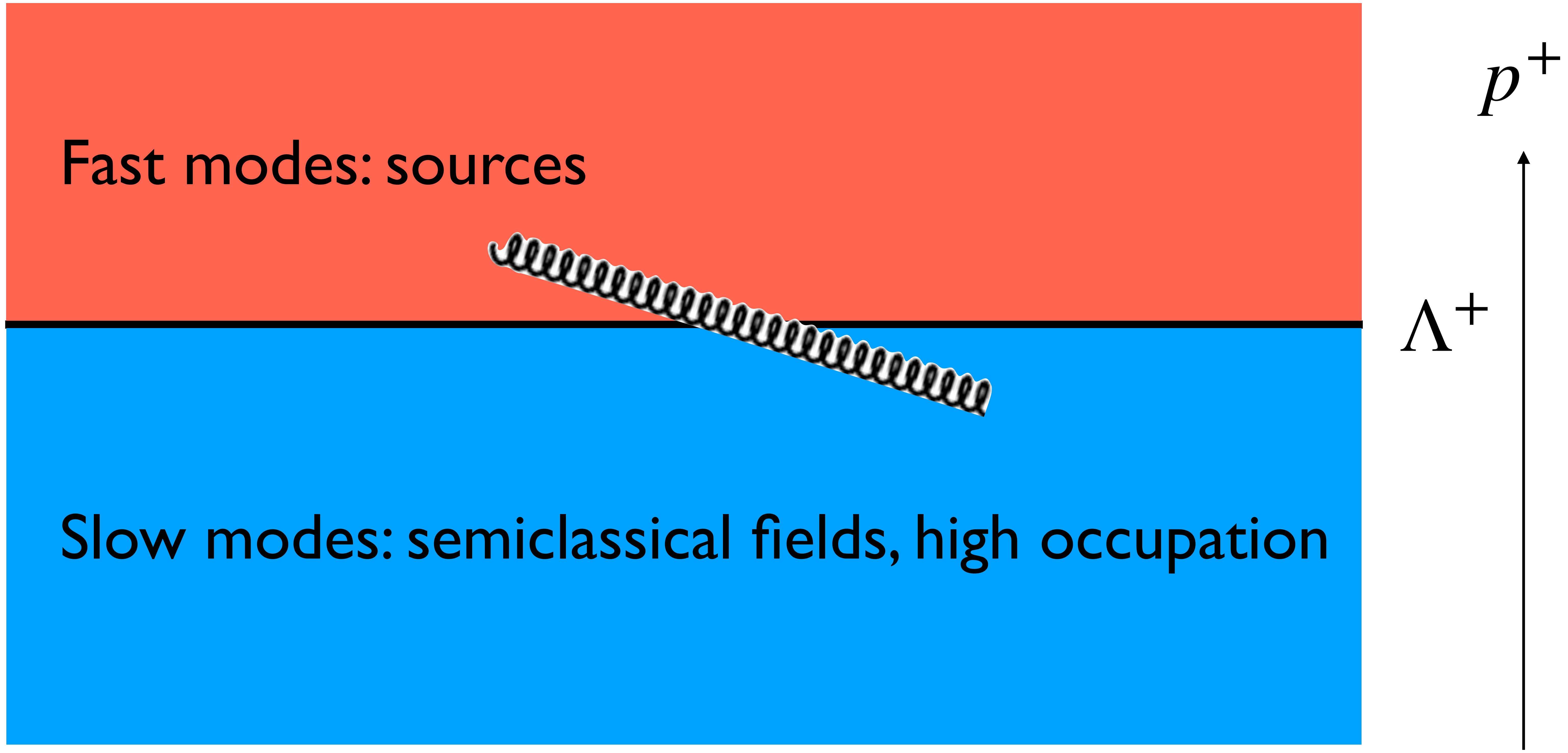
- Non-linear effects driven by density ⇒ 2-pronged approach: ↓x/↑A.





The CGC:

- The CGC is the effective field theory that describes high energy scattering in QCD in the Regge-Gribov limit (fixed $Q^2, x \to 0$).



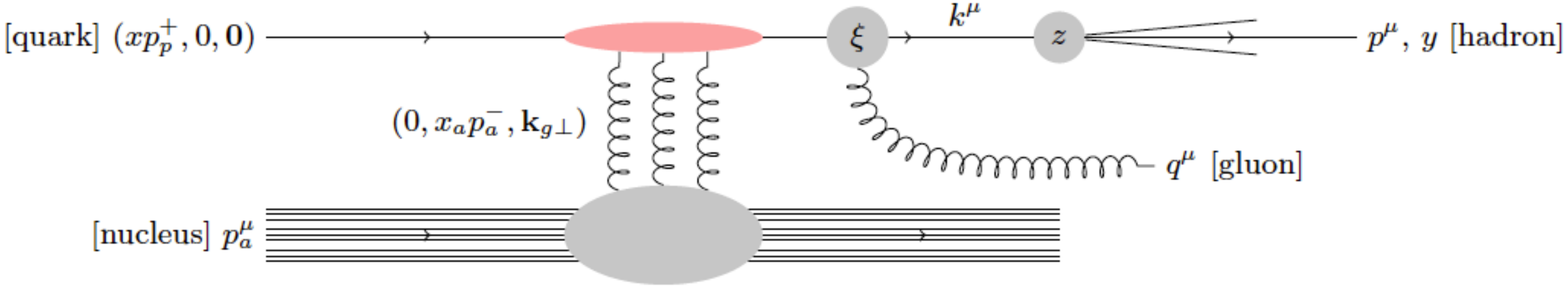
- Independence of the physical observables on the cut-off separating fast and slow modes leads to an RG-type equation which, for ensembles of Wilson lines describing the target and considering scattering of a dilute projectile on a dense target, is JIMWLK (BK).

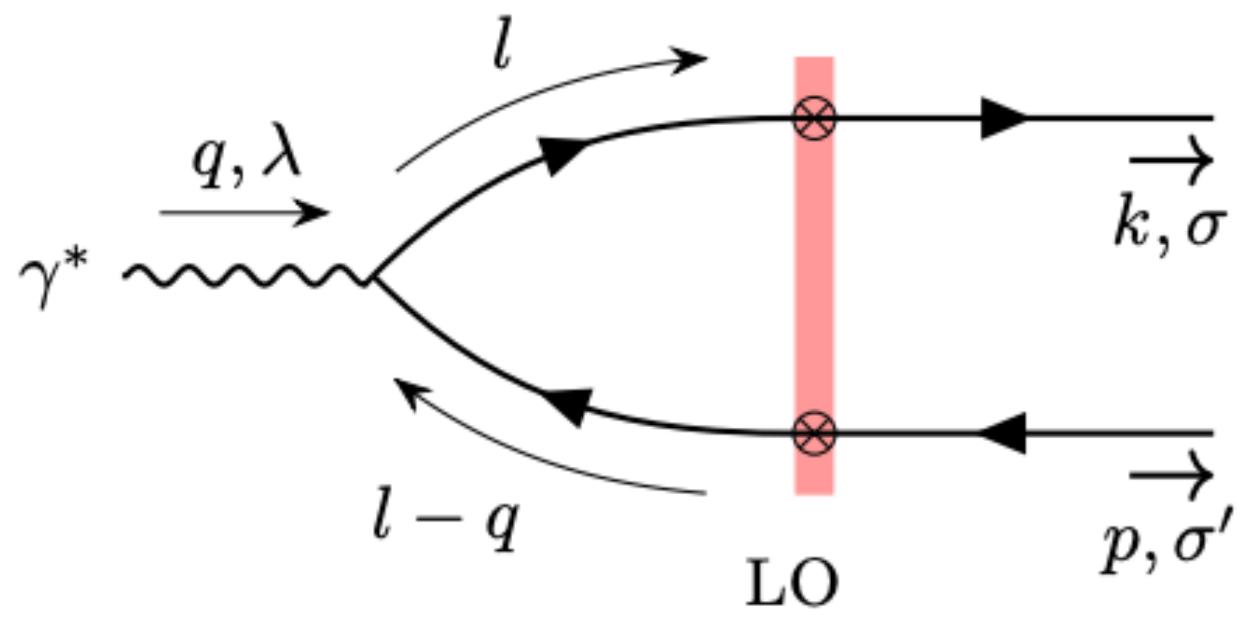
Dilute-dense scattering in the CGC:

- Compute the contributions relevant for the process from the projectile point of view (using equal or light-front quantization, covariant or light-cone gauges, Feynman diagrams or wave functions in Light Cone PT,…).

- Partons in the different contributions interact with the target through Wilson lines (usually at fixed transverse positions, eikonal approximation), that in the cross section appear as ensembles $\langle W\cdots W\rangle_T$.

$$W(x_\perp) = Pexp\left[-ig\int A\cdot dl\right]$$





- At NLO, collinear and soft divergencies appear, which must be shown to be absorbed in DGLAP-type evolution (of PDFs, FFs, jet functions,…) and JIMWLK-type evolution of $\langle W\cdots W\rangle_T$, respectively; additional large logarithms may appear (threshold, Sudakov,…).
- Models must be used for the non-perturbative input of object whose evolution we consider: PDFs, FFs, jet functions, $\langle W\cdots W\rangle_T$ (MV), Wigner functions,…

The path to precision:

- LO calculations: they show qualitative agreement with experimental data but lack precision to estimate uncertainties and establish clearly the existence of saturation.

- **NLO calculations**: burst of activity in recent years.

  ➜ Evolution equations: massive quarks in DIS, issues at NLO.

  ➜ eA: dijet, dihadron and single hadron.

  ➜ Forward pA: single hadron and jet production in hybrid factorization.

- Relation with TMDs and TMD factorization.

- Not addressed in this talk *(apologies!)*: production at central rapidities, diffraction, exclusive processes, particle correlations, non-eikonal corrections, models for averages,…

Contents:

1. Introduction.

2. Evolution equations.

3. Dijet production in DIS.

4. Forward production in pA.

5. Summary.

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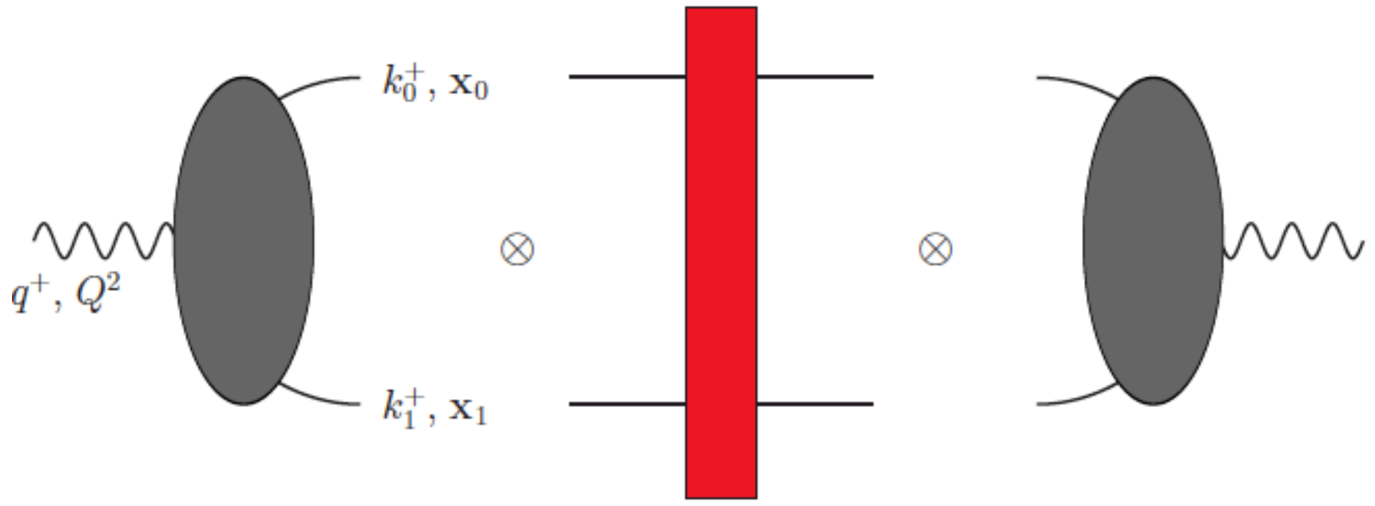
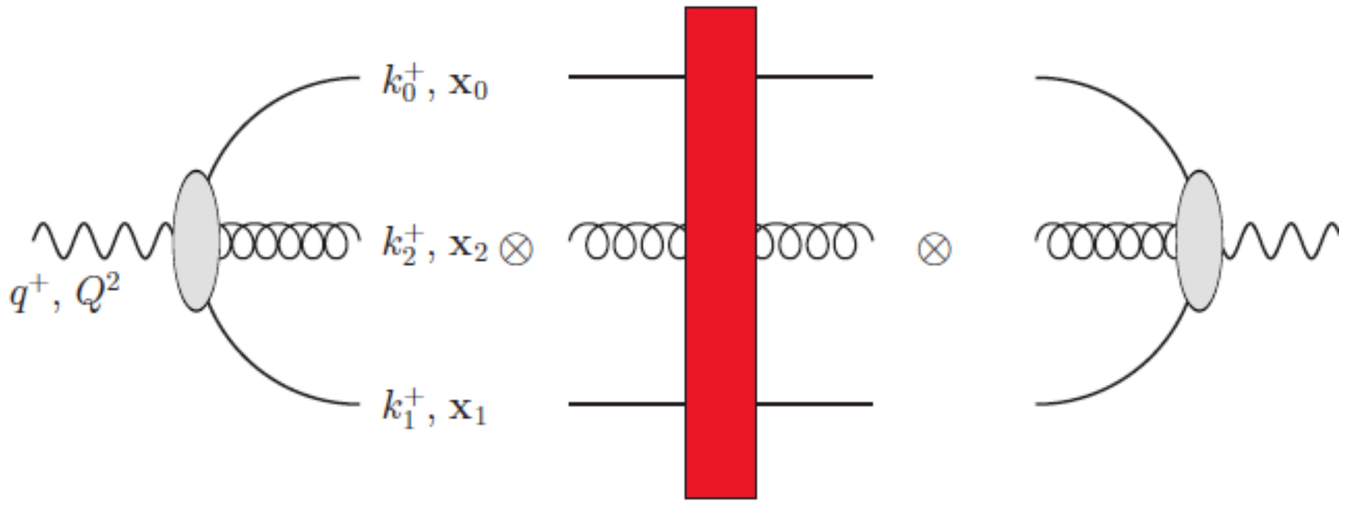
NLO evolution equations:

- NLO evolution equations available:
  - ➜ NLO BK (0710.4330, 1309.7644).
  - ➜ NLO JIMWLK (1310.0378, 1610.03453).





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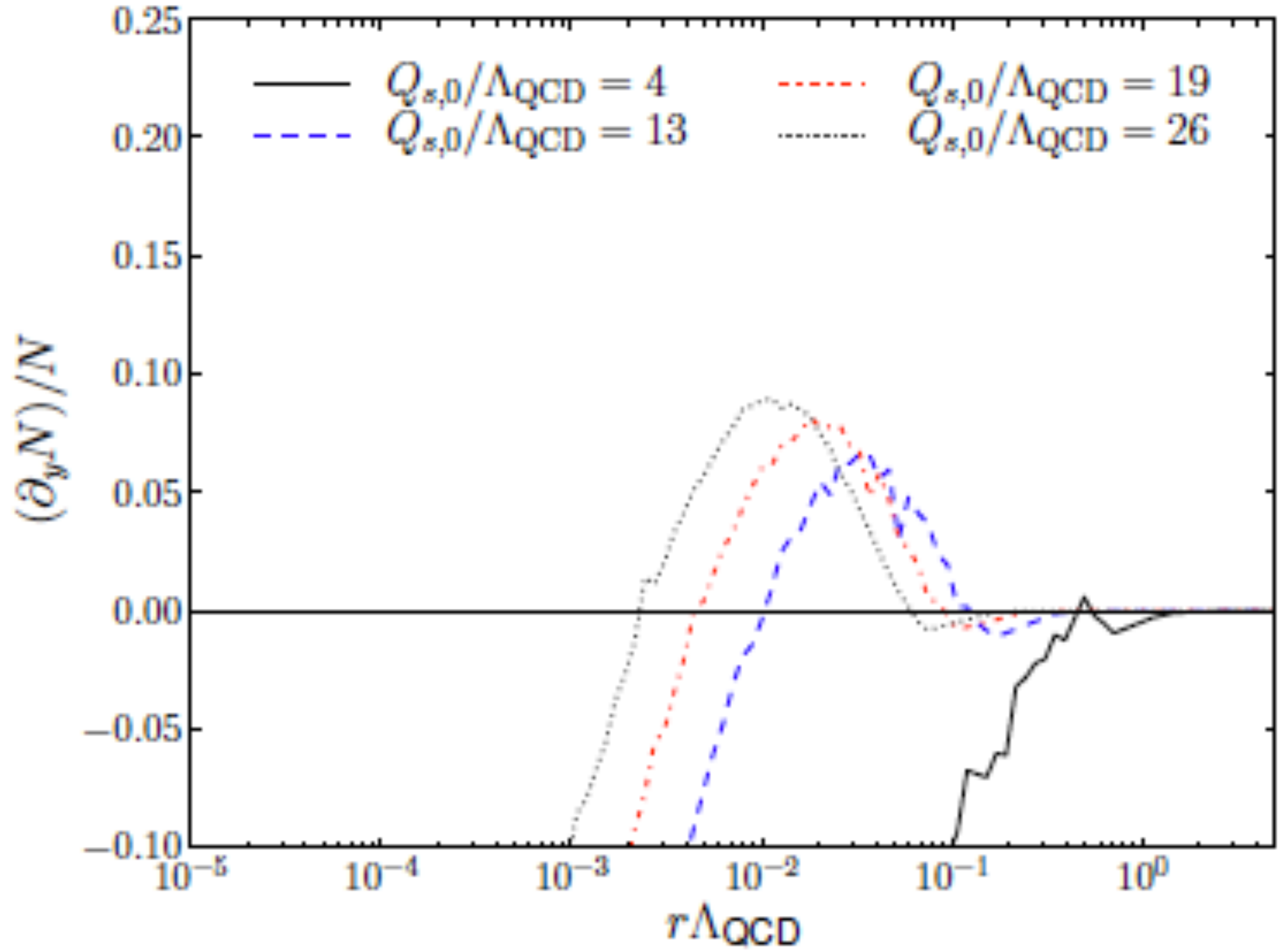

- Instabilities appeared (akin to those in NLO BFKL):
  - ➜ Kinematic constrains (1401.0313, 1902.06637).
  - ➜ Collinear improvements (1502.05642,1507.03651).
- Good fits to HERA data (but of similar quality to those with rcBK - LO impact factor, only running coupling corrections) (1507.07120).
- Recent discussions on scales (several choices possible):
  - ➜ Large transverse logs (from typical momenta of projectile to target) assigned to DGLAP instead of running coupling (2308.15545).
  - ➜ No Langevin implementation for NLO JIMWLK for most scale choices (2310.10738).

NLO evolution equations:

• NL

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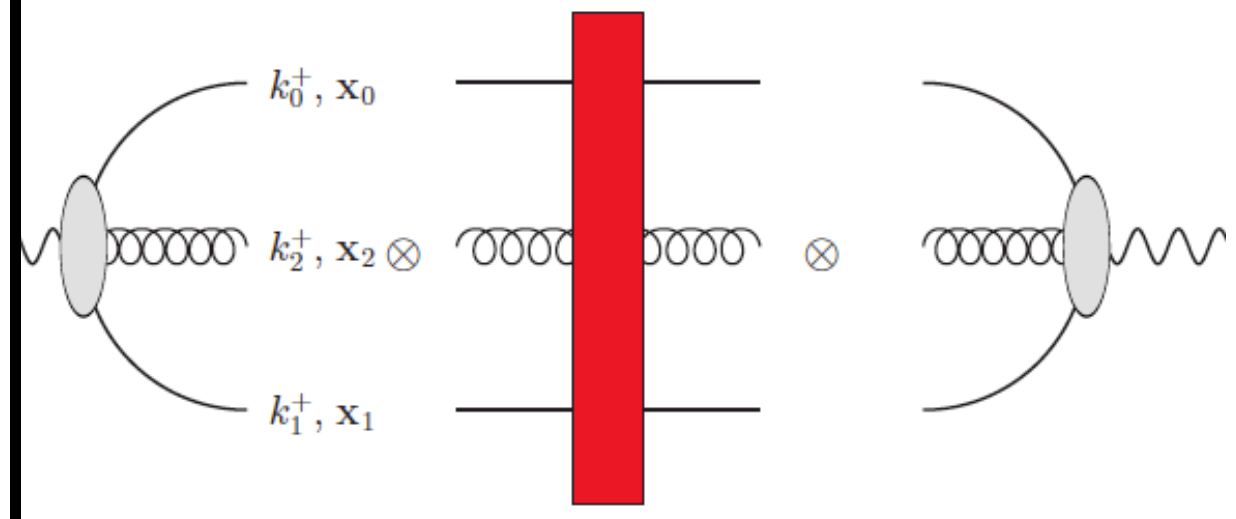

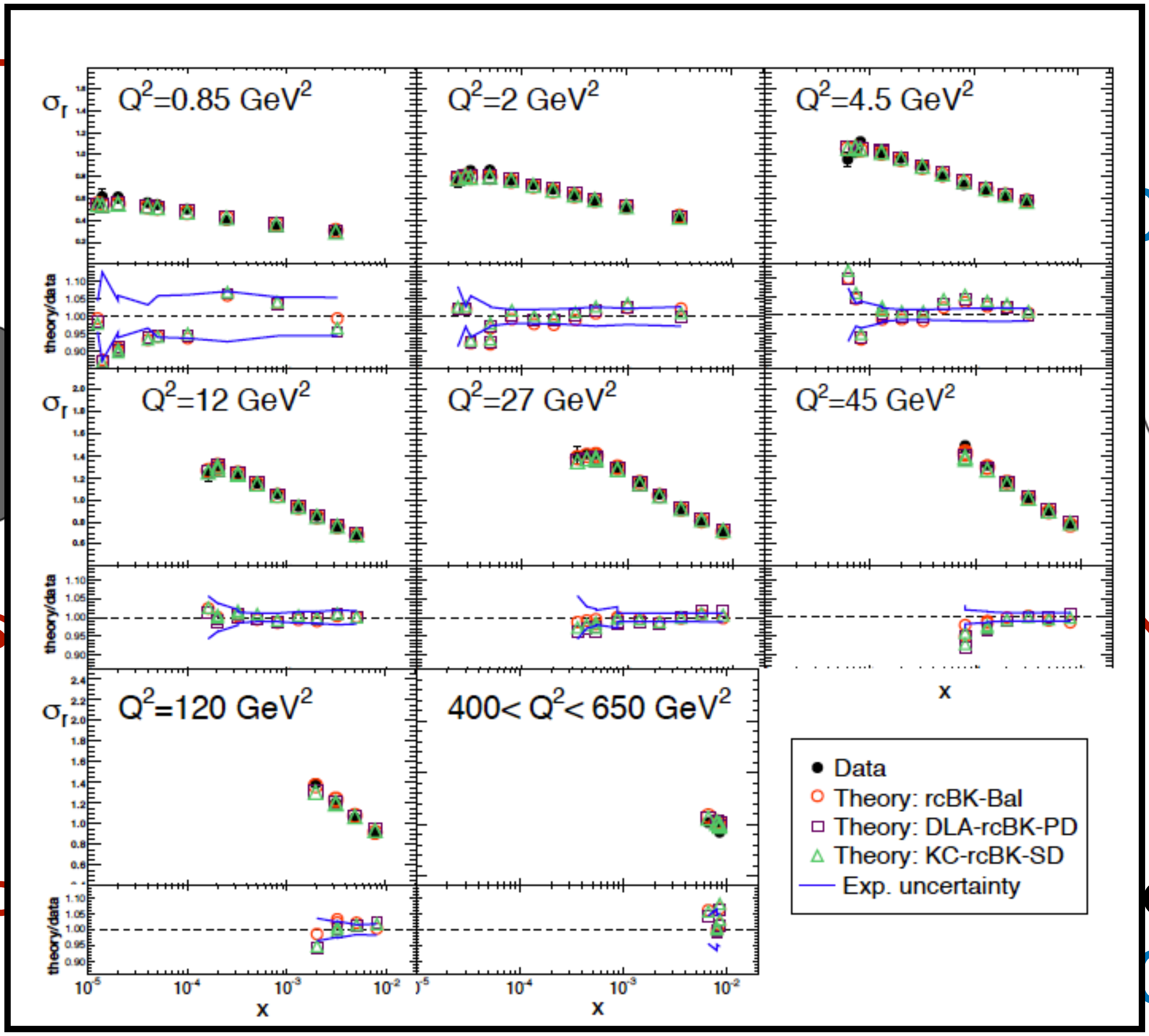

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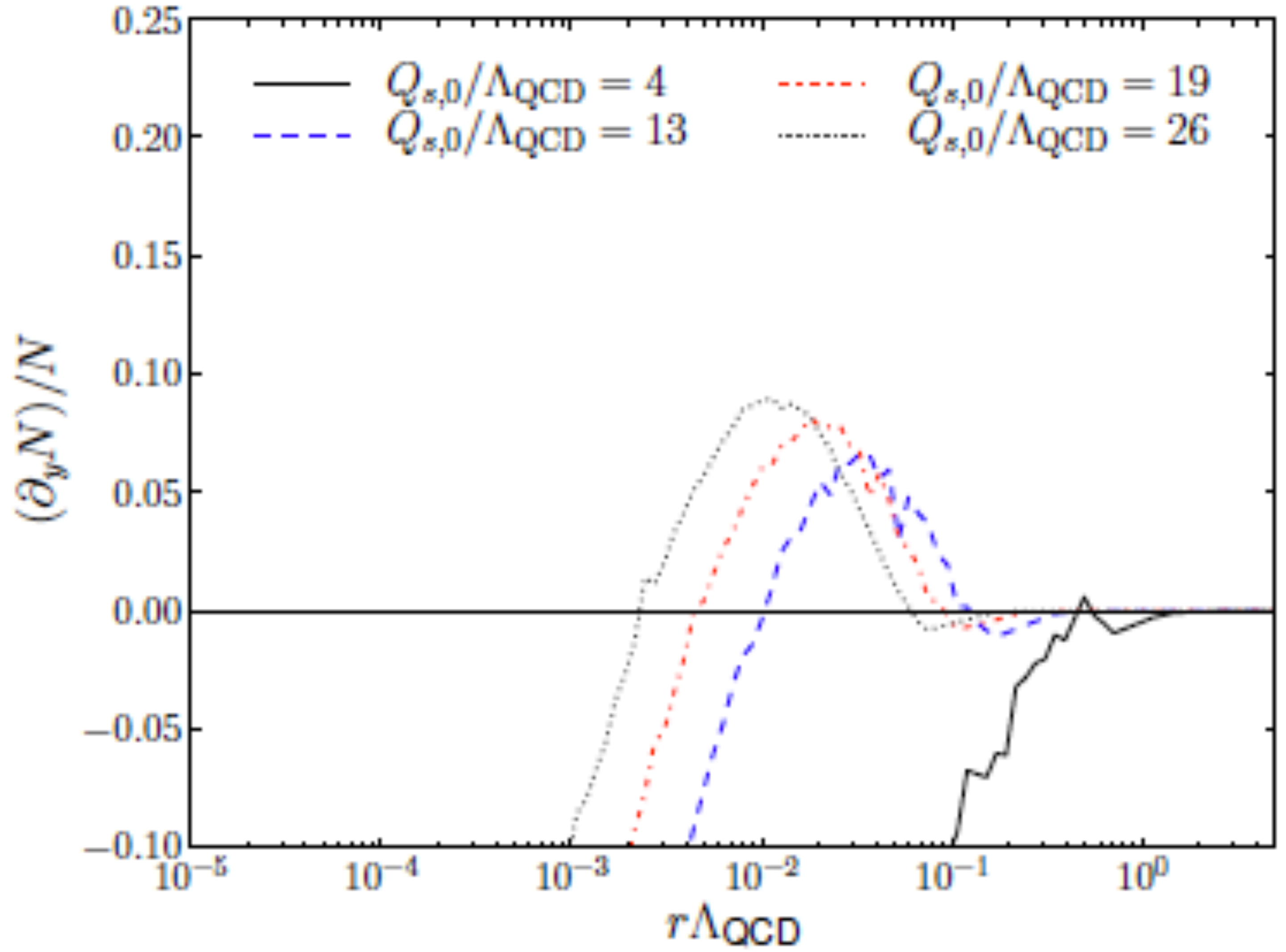

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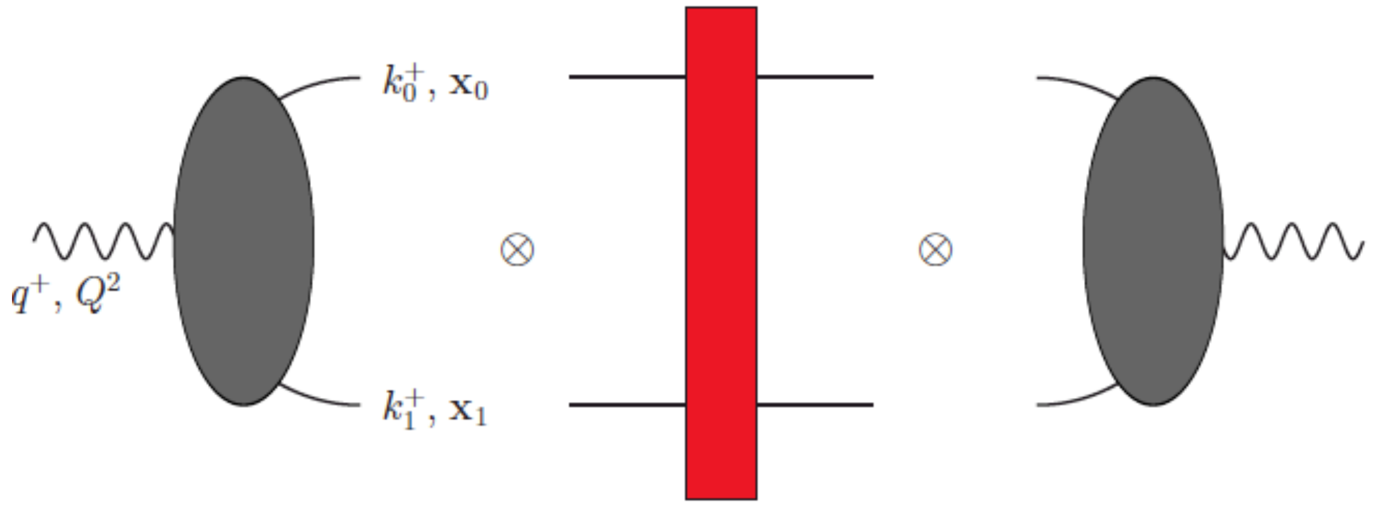

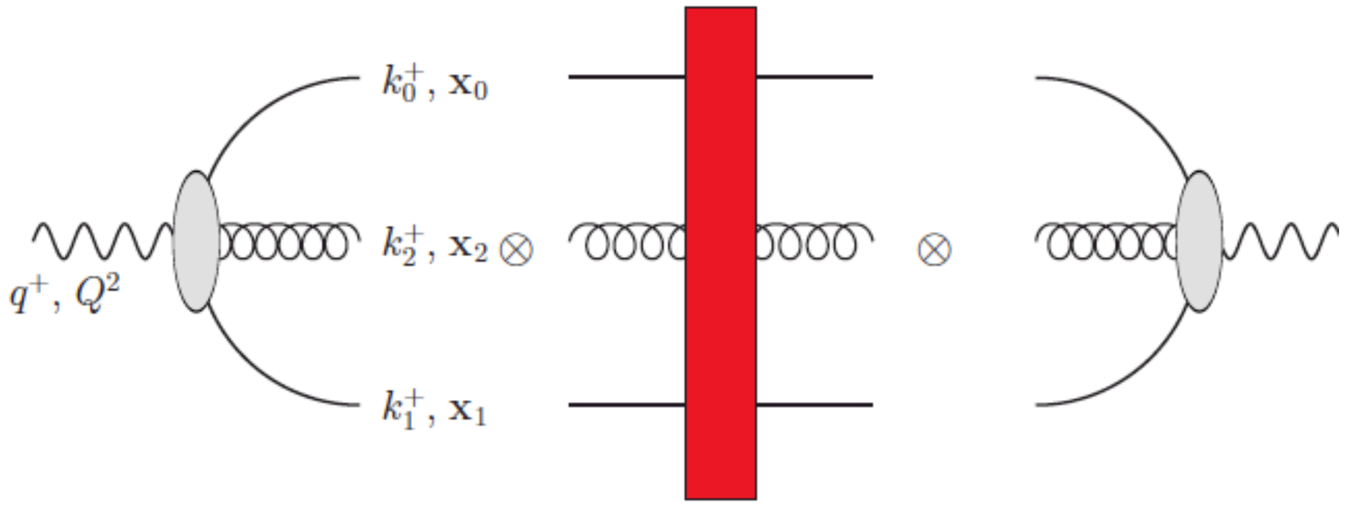

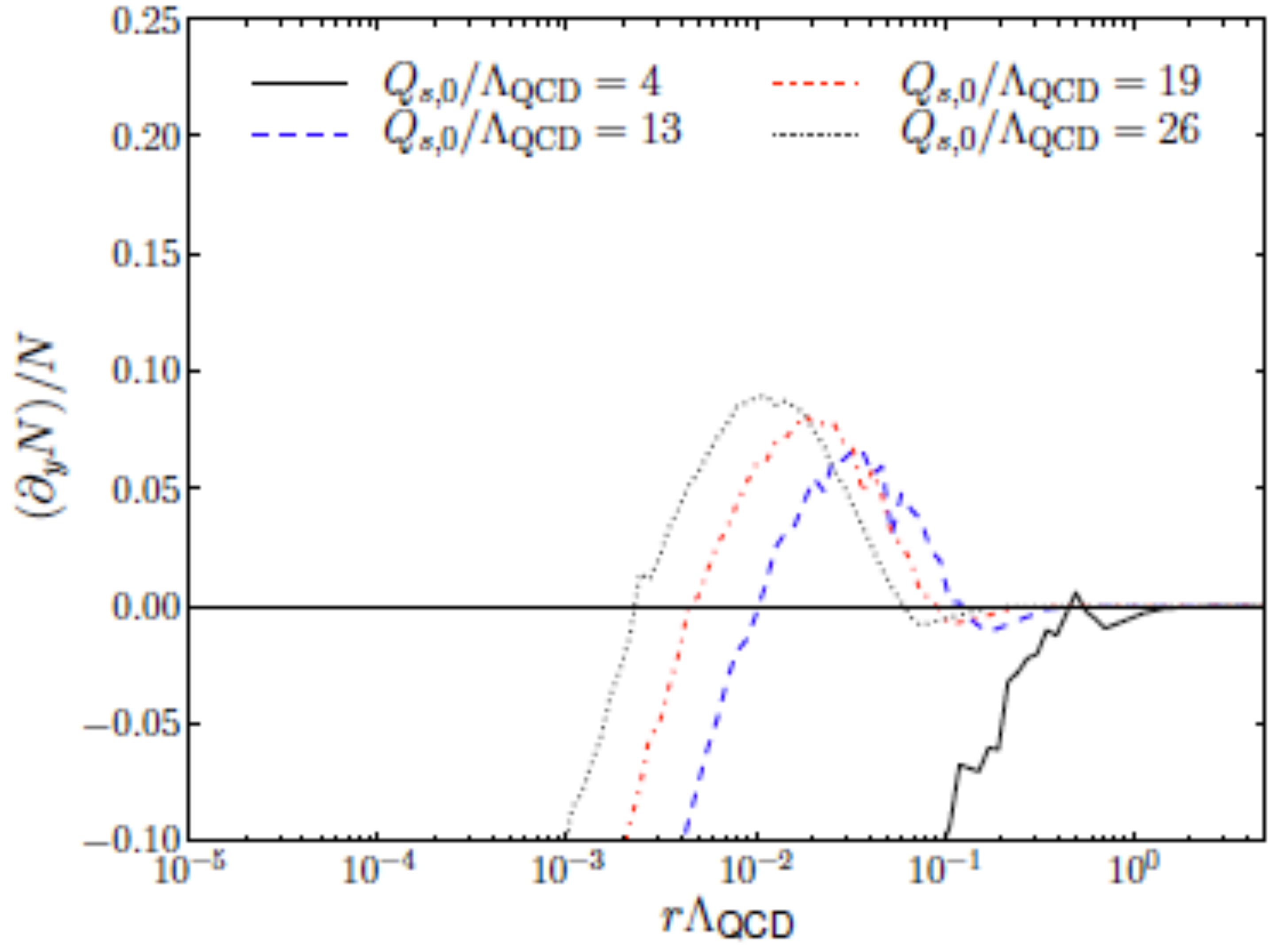

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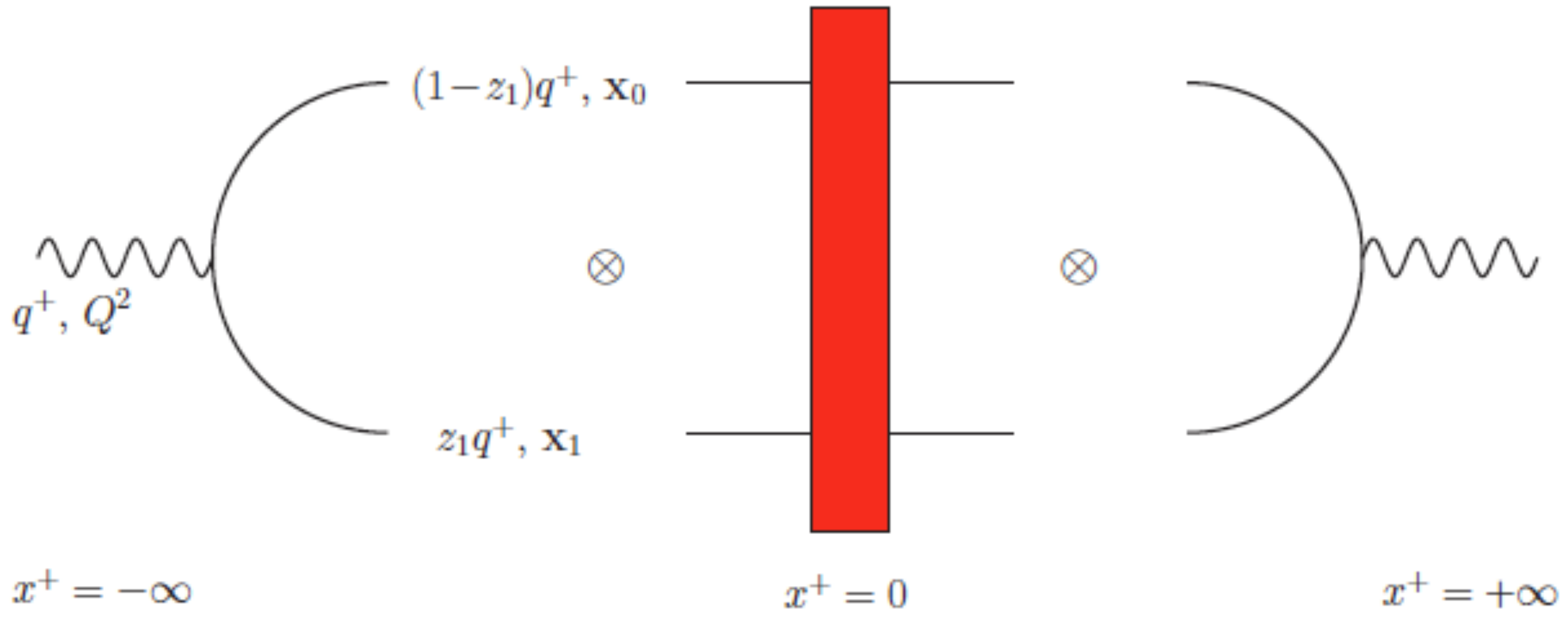
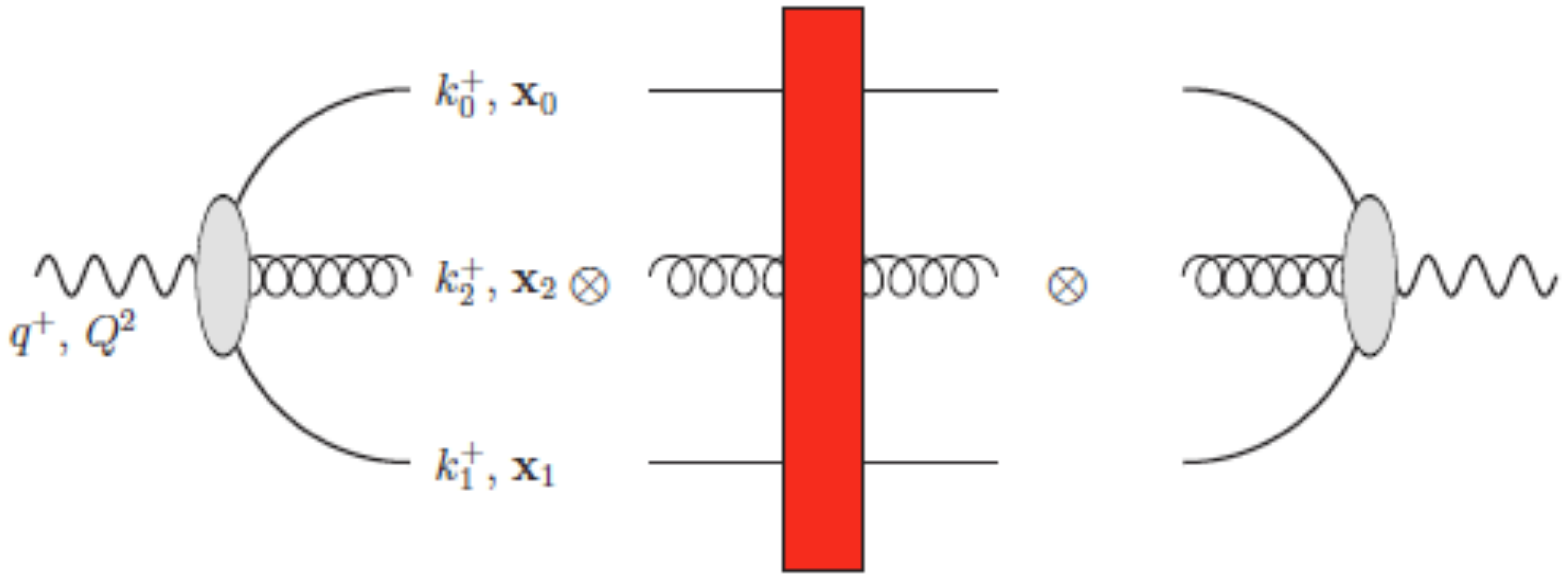
NLO impact factors for DIS:

- NLO impact factor for massless quarks (1009.4729, 1207.3844, 1112.4501, 1606.00777, 1708.06557, 1711.08207).



Dipole

$$\sigma_{T,L}^{\gamma A \to X}(x_{Bj}, Q^2) \propto \int_{\mathbf{x}_0,\mathbf{x}_1} \int_0^1 dz_1 \Phi_{T,L}^{q\bar{q},LO}(\mathbf{x}_{01}, z_1, Q^2)\left[1 - \langle s_{01} \rangle\right]$$



One loop

Tree level

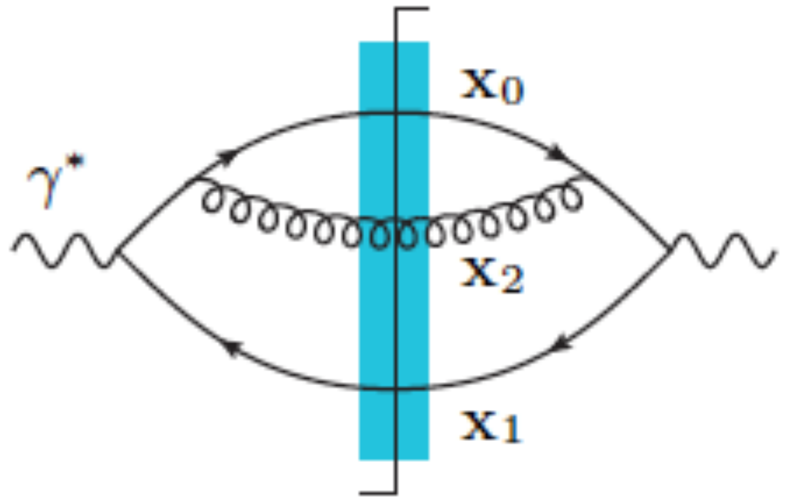
$$\sigma_{T,L}(x_{Bj}, Q^2) = \sum_{q\bar{q}\ st.} \left|\Psi_{q\bar{q}}^{\gamma_{T,L}^*}\right|^2 \left[1 - \langle s_{01} \rangle_0\right] + \sum_{q\bar{q}g\ st.} \left|\Psi_{q\bar{q}g}^{\gamma_{T,L}^*}\right|^2 \left[1 - \langle s_{012} \rangle_0\right]$$

- UV divergencies cancelled/renormalized, soft divergencies leading to small x evolution: BK/JIMWLK.

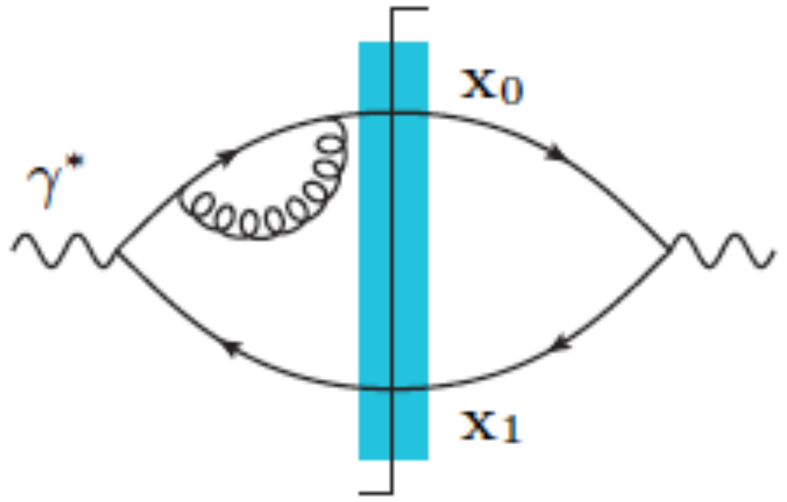
- NLO impact factor for massive quarks (2103.14549, 2112.03158, 2204.02486): clarification of mass renormalisation in Light-Cone PT.

NLO impact factors for DIS:

- Description of HERA data including massive contributions using NLO BK (2211.03504):



(a) $q\bar{q}g$



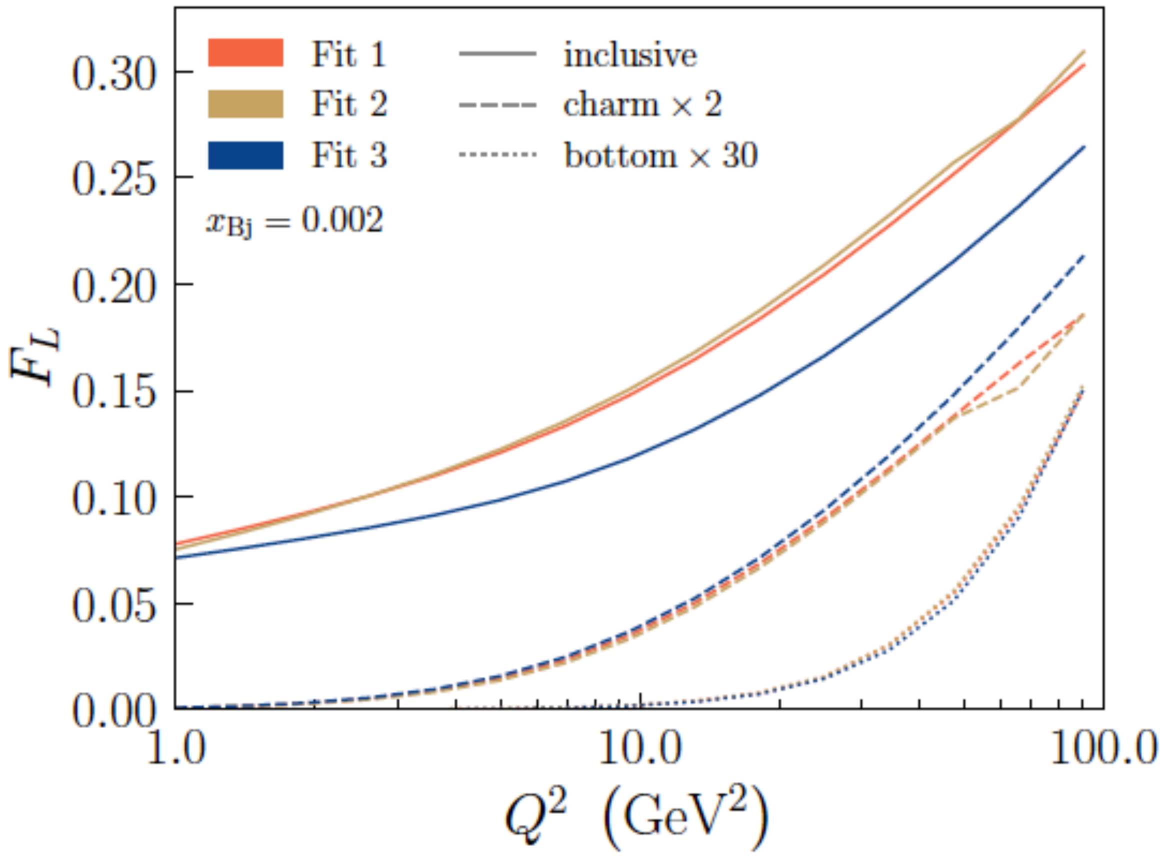
(b) $q\bar{q}$

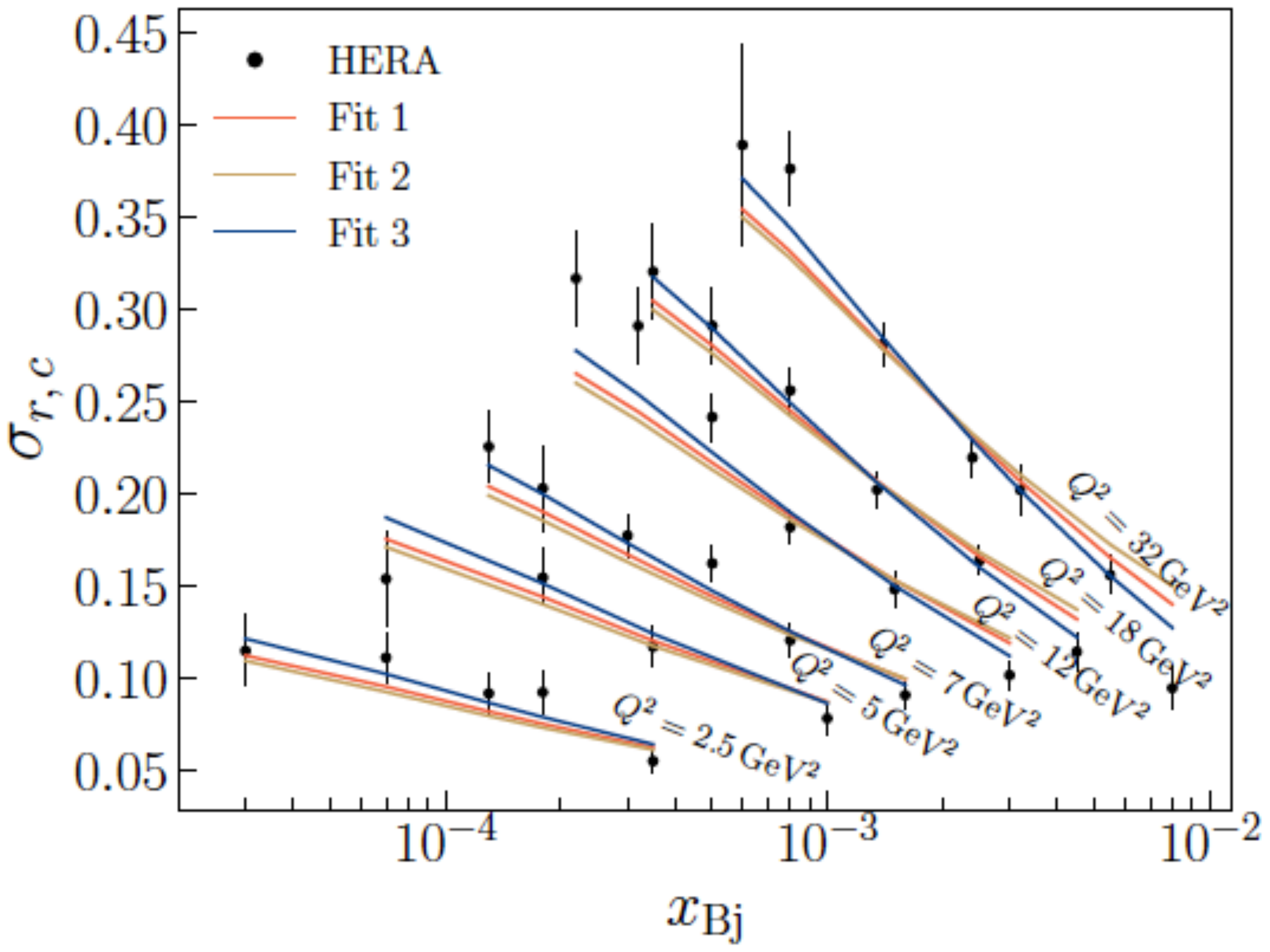
$$S_{01} = \frac{1}{N_c} \left\langle \mathrm{Tr}\{V(\mathbf{x}_0)V^\dagger(\mathbf{x}_1)\}\right\rangle,$$

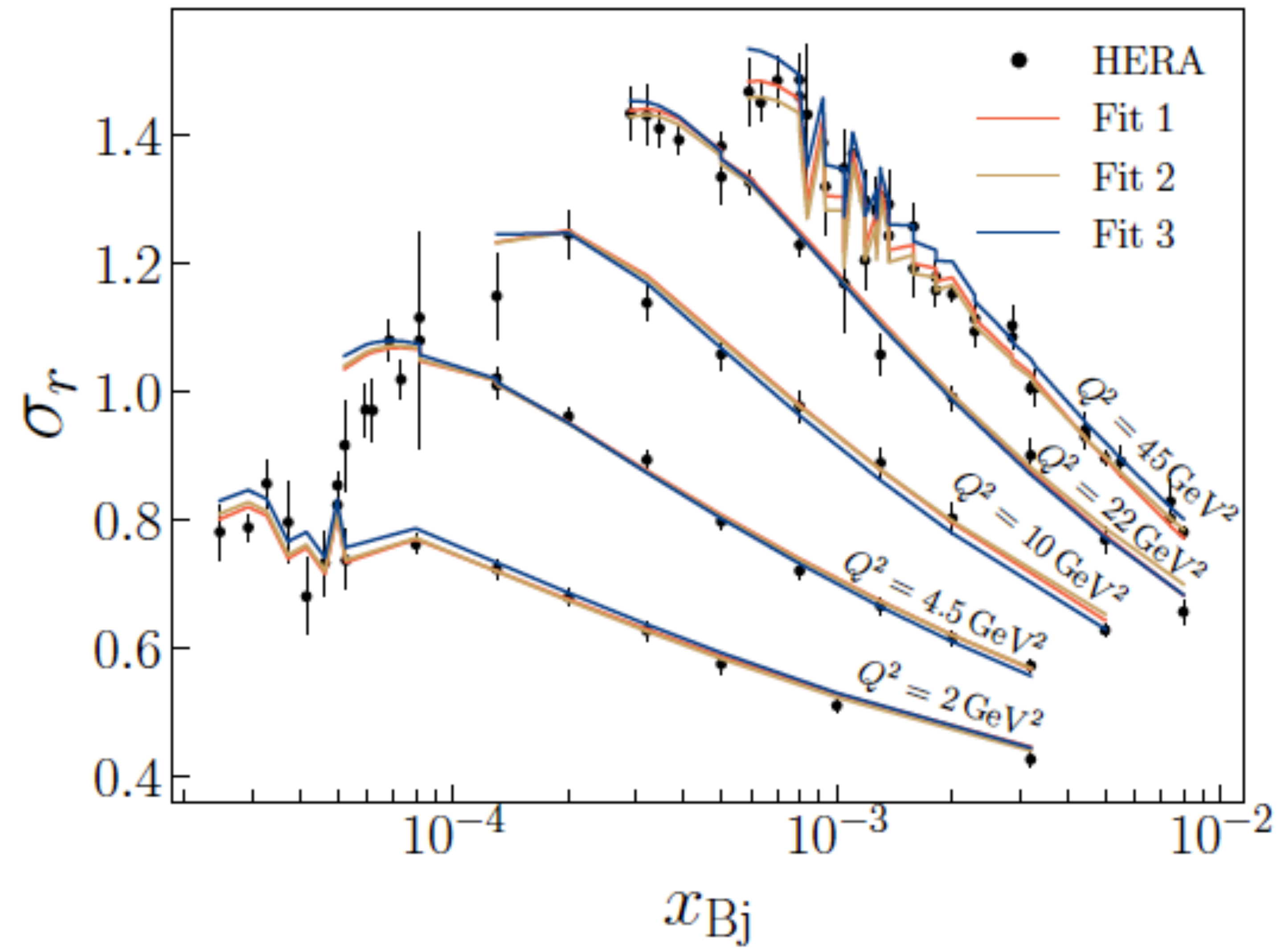
$$S_{012} = \frac{N_c}{2C_F}\left(S_{02}S_{21} - \frac{1}{N_c^2}S_{01}\right)$$

$$\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)$$

#	Resum. scheme	α_s	$Y_{0,\mathrm{BK}}$	m_c [GeV]	χ^2_c/N	m_b [GeV]	χ^2_b/N	χ^2_{tot}/N
1	ResumBK	PD	0	1.42	1.86	4.83	1.37	1.25
2	KCBK	PD	0	1.49	2.55	4.96	1.58	1.23
3	TBK	BSD	0	1.29	1.02	5.04	1.12	1.83







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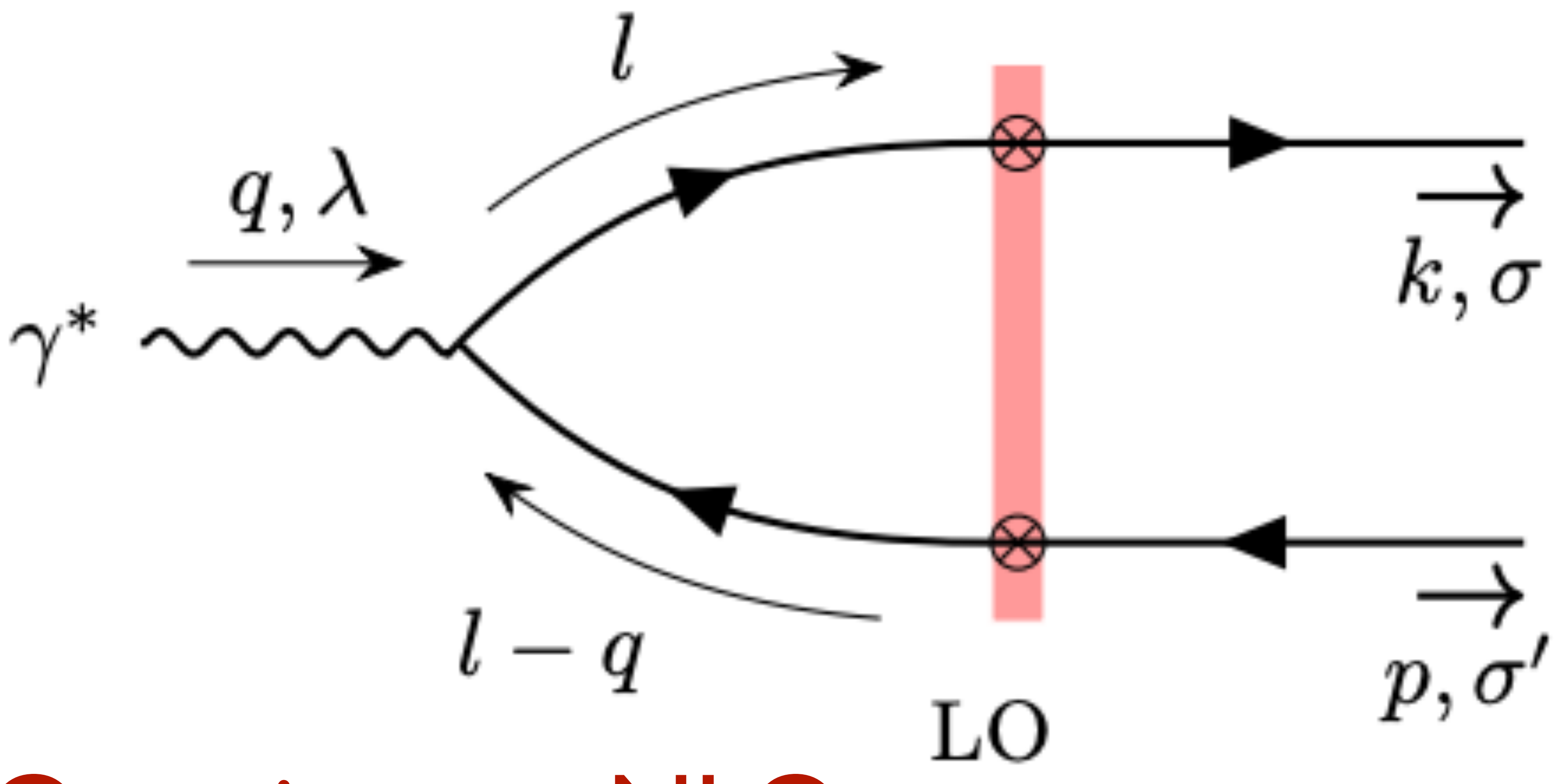
Dijets in DIS:

- Dijet production in DIS:
  - ➜ Ingredient of many calculations.
  - ➜ Dijet imbalance sensitive to radiation and, eventually, to saturation.
  - ➜ In the back-to-back (aka correlation) limit, ensembles of Wilson lines are related to TMDs at x=0 (different ones for different gauge links, 1503.03421).



$$K = k_\perp + p_\perp,\quad P = z_p k_\perp - z_k p_\perp,\quad K \ll P$$

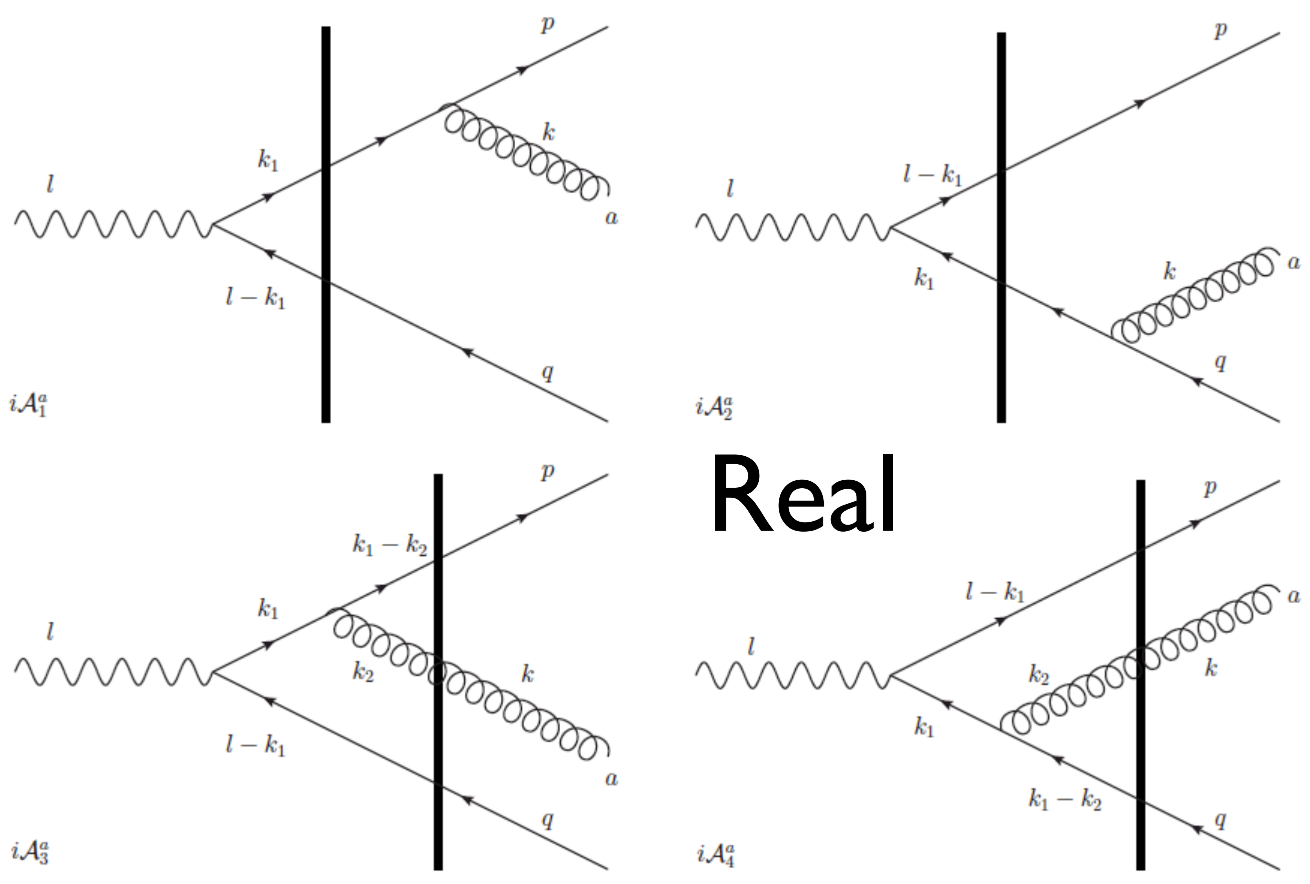
$\langle W^\dagger W W^\dagger W \rangle \rightarrow$ unpolarised and polarised WW gluon TMDs (1101.0715).

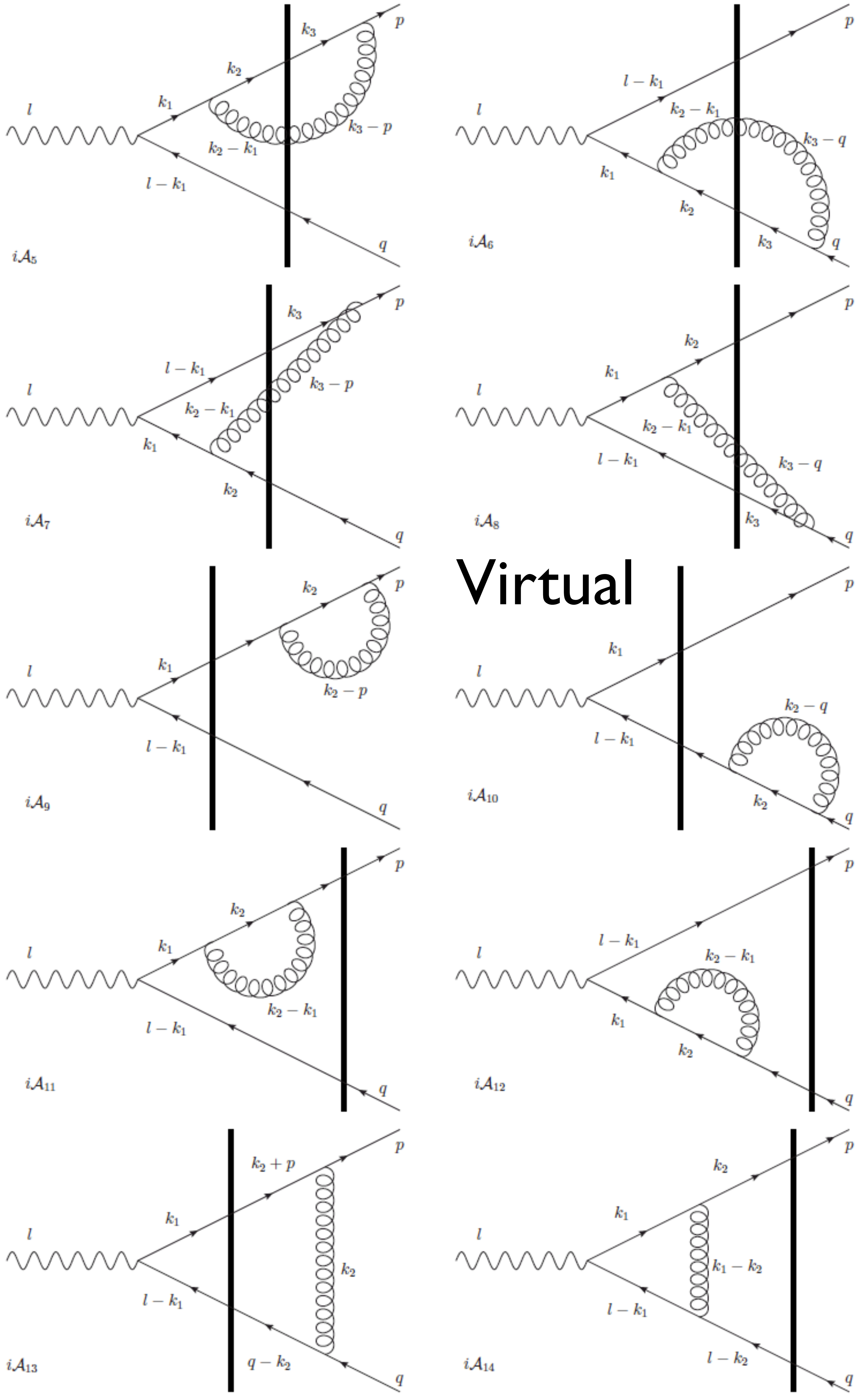
- Questions at NLO:
  - ➜ Does one get TMD factorization at NLO in the back-to-back limit? It was already done in several processes ($1 \rightarrow 2,\ 1 \rightarrow 3,\ 2 \rightarrow 2,\ 2 \rightarrow 3$) but at LO.
  - ➜ How to deal with large logarithms $\ln \frac{P^2}{K^2}$ (Sudakov) that appear at NLO (1308.2993).

NLO dijets in DIS:

- Calculations of the NLO diagrams for several observables:
  - ➜ Single hadron (2210.03208).
  - ➜ Dihadrons (2207.03606, 2301.03117, 2211.04837).
  - ➜ Dijets in photoproduction (2204.11650).
  - ➜ Dijets in DIS (2108.06347, 2208.13872, 2304.03304, 2308.00022 - BNL).

- 2204.11650: back-to-back limit studied, Sudakov double logs are obtained with the wrong (correct) sign when naive (kinematically improved) low-x LL evolution is performed.
- BNL group: using kinematically improved LL evolution, TMD factorization at NLO is probed, with an impact factor which resums both double and single Sudakov logs!





Saturation versus other logs:

- 2308.00022: predictions for dijets at DIS with/without saturation effects included.

$$\left\langle \mathrm{d}\sigma_{\rm LO}^{(0),\lambda} + \alpha_s \mathrm{d}\sigma_{\rm NLO}^{(0),\lambda} \right\rangle_{\eta_c} = \frac{1}{2}\mathcal{H}_{\rm LO}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0) \Bigg\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \Bigg[\underbrace{-\frac{N_c}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right)}_{\text{Sudakov double log}}$$

$$\underbrace{-s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right)}_{\text{Sudakov single logs}} + \frac{N_c}{2} f_1^\lambda(\chi, z_1, R) + \frac{1}{2N_c} f_2^\lambda(\chi, z_1, R) \Bigg] \Bigg\}$$

$$+ \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{\rm LO}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0) \left\{ \frac{N_c}{2}\left[1 + \ln(R^2)\right] - \frac{1}{2N_c}\ln(z_1 z_2 R^2) \right\}.$$

Hard factor, $\lambda = L, T$

Unpolarized WW gluon TMD

NLO coefficient functions

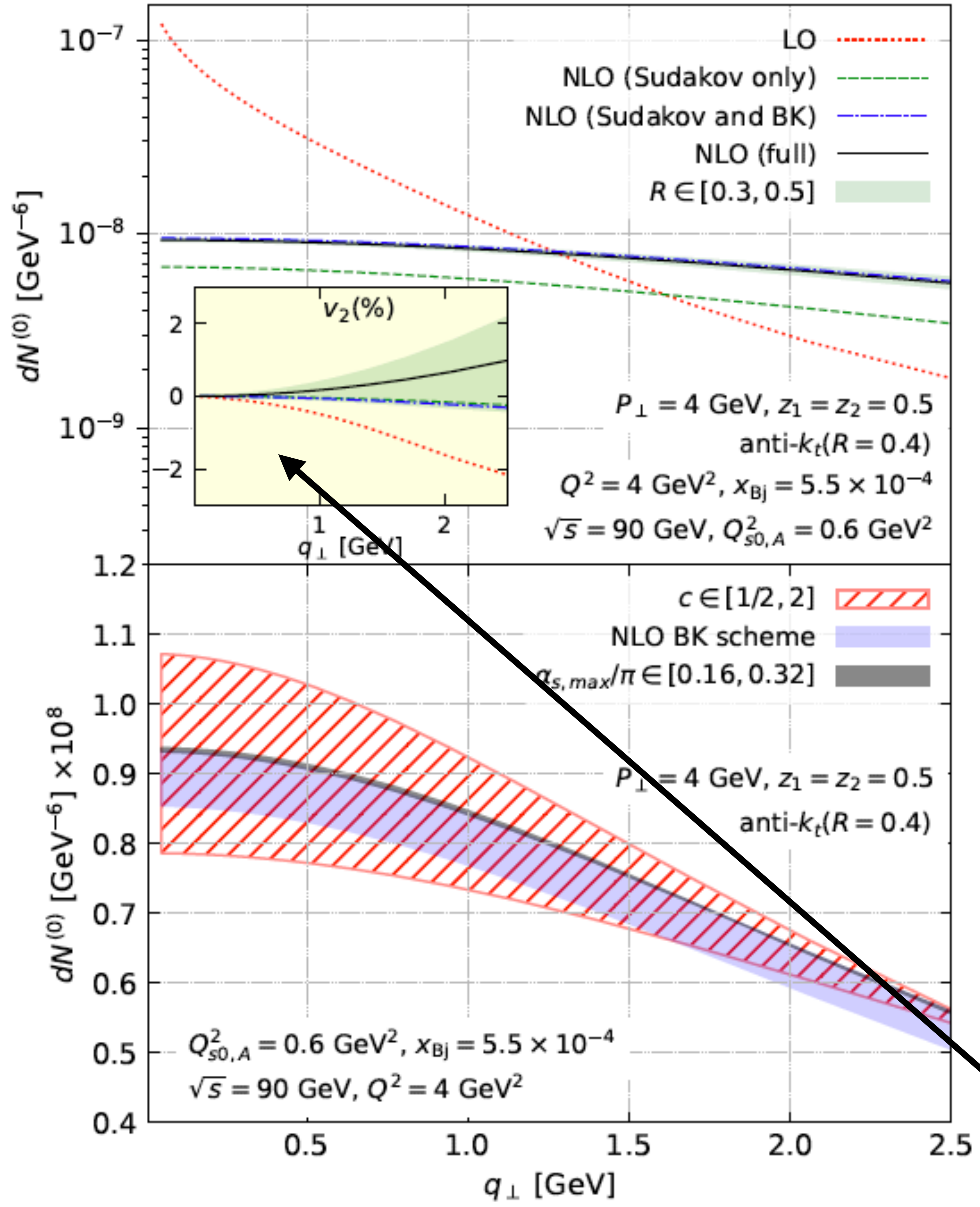
Jet radius

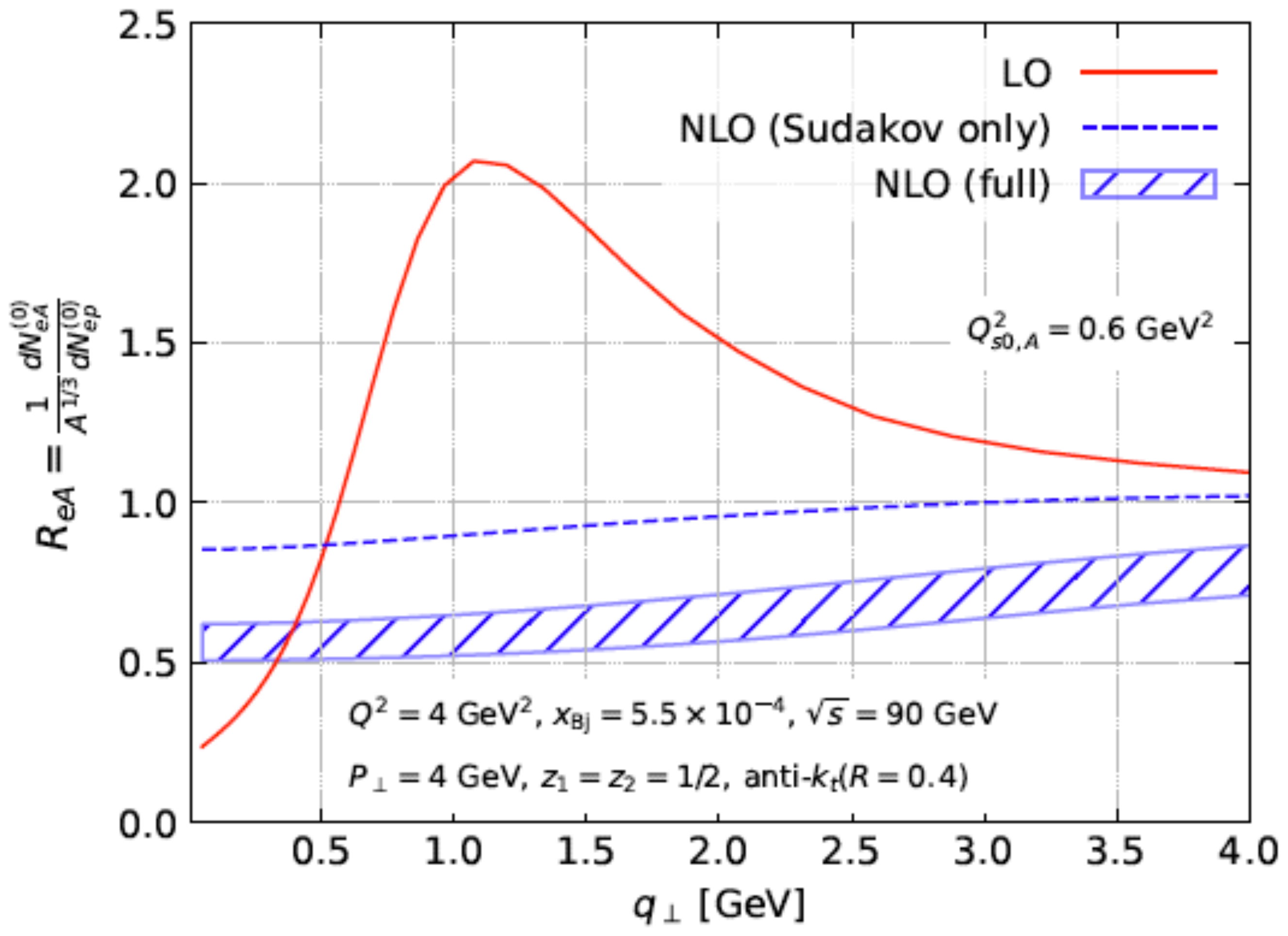
Linearly polarized WW gluon TMD

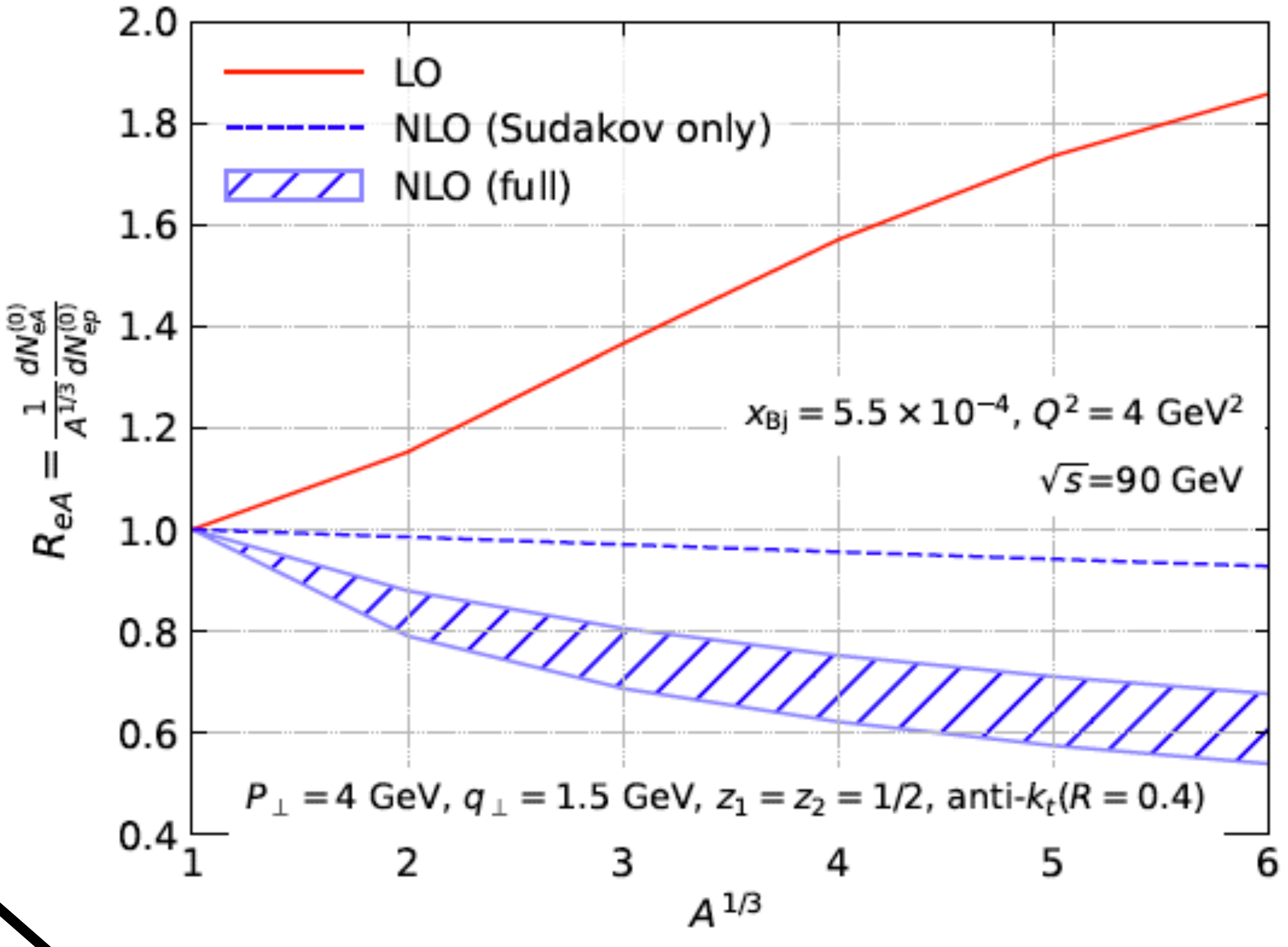
$$\frac{\partial \hat{G}^{ij}_{Y_f}}{\partial Y_f} = \frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2\boldsymbol{z}_\perp \Theta(-Y_f - \Delta_c) K_{\rm LLx} \otimes \hat{G}^{ij}_{Y_f}$$

Saturation versus other logs:

- 2308.00022: predictions for dijets at DIS with/without saturation effects included.



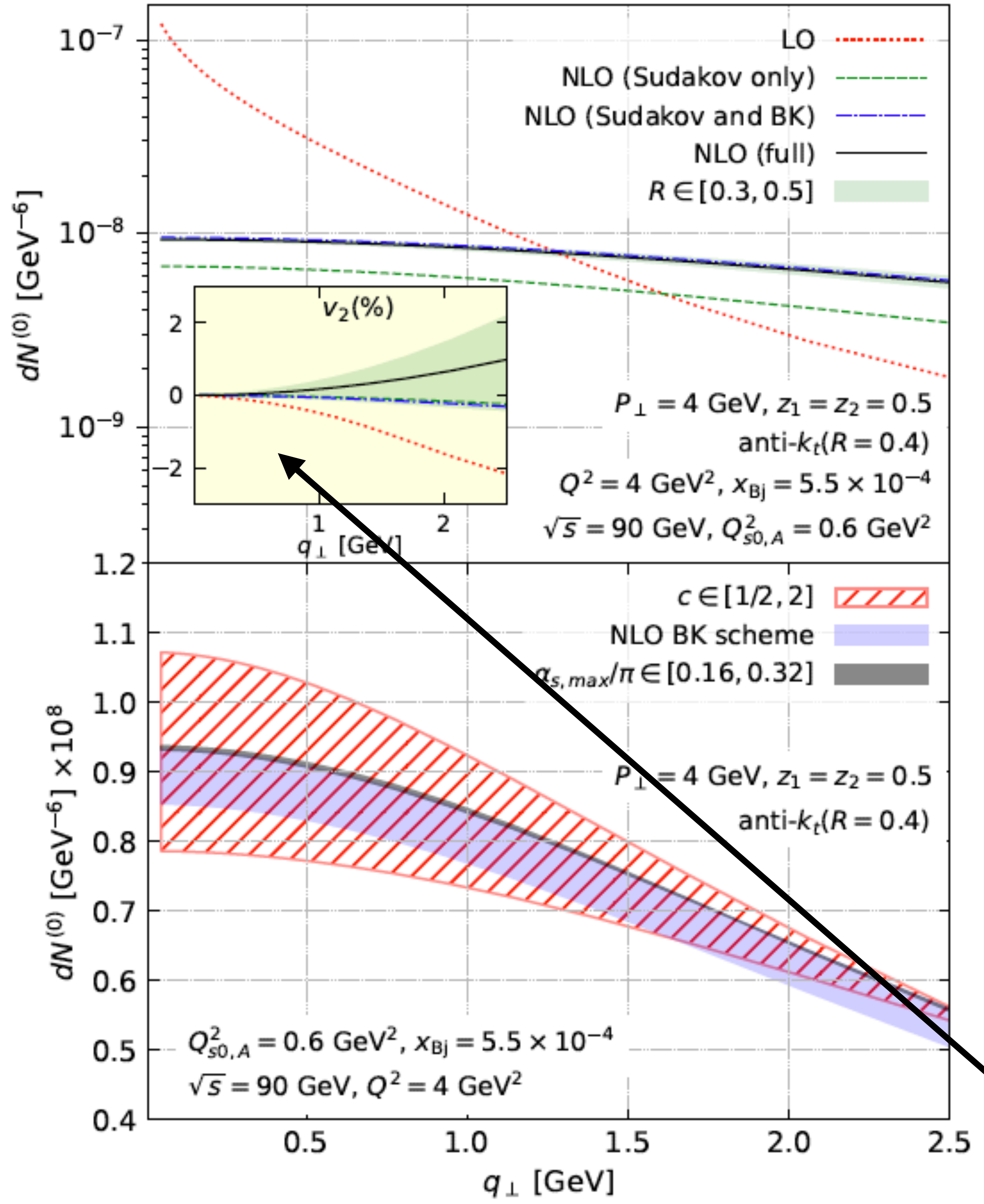


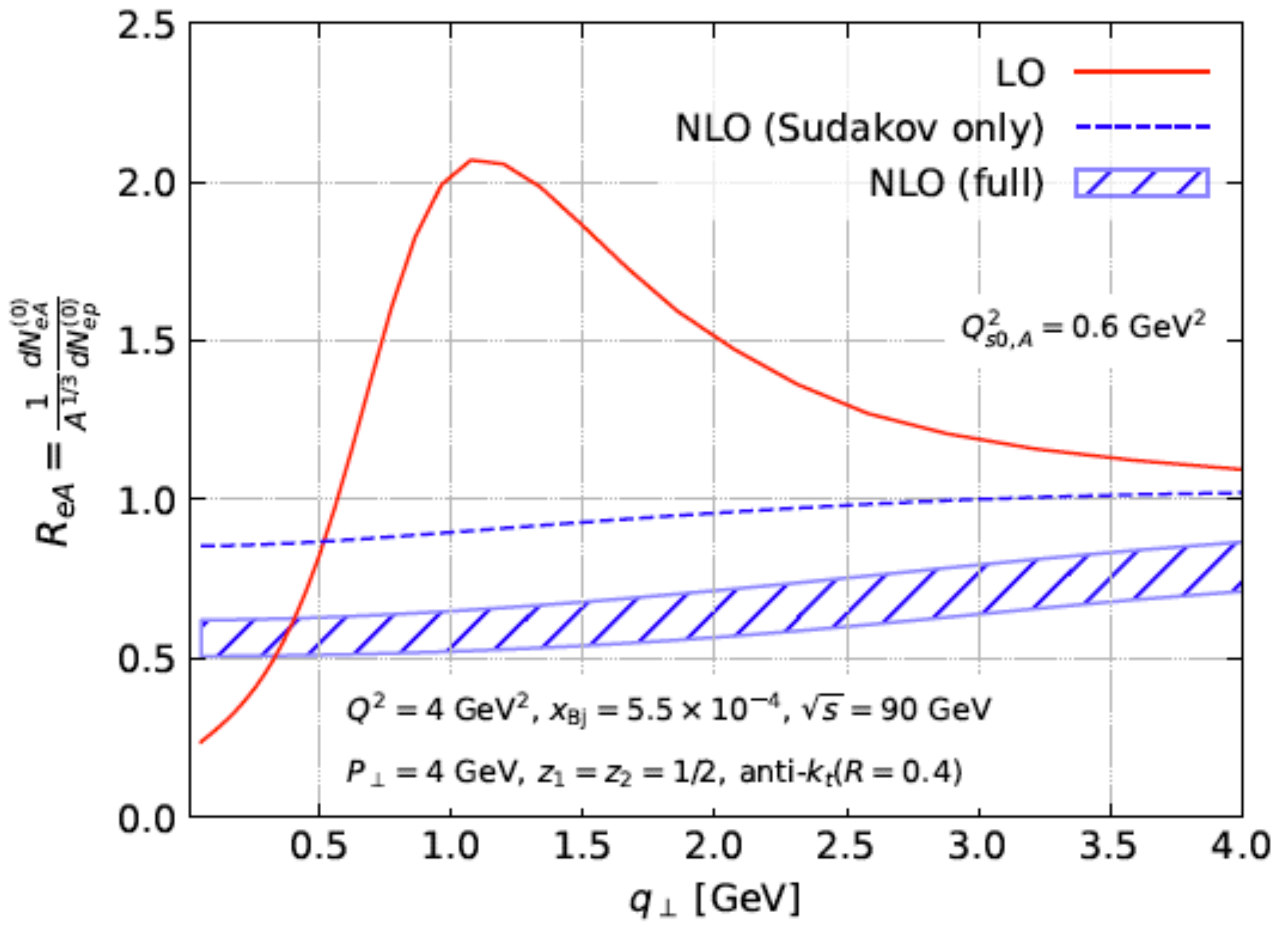


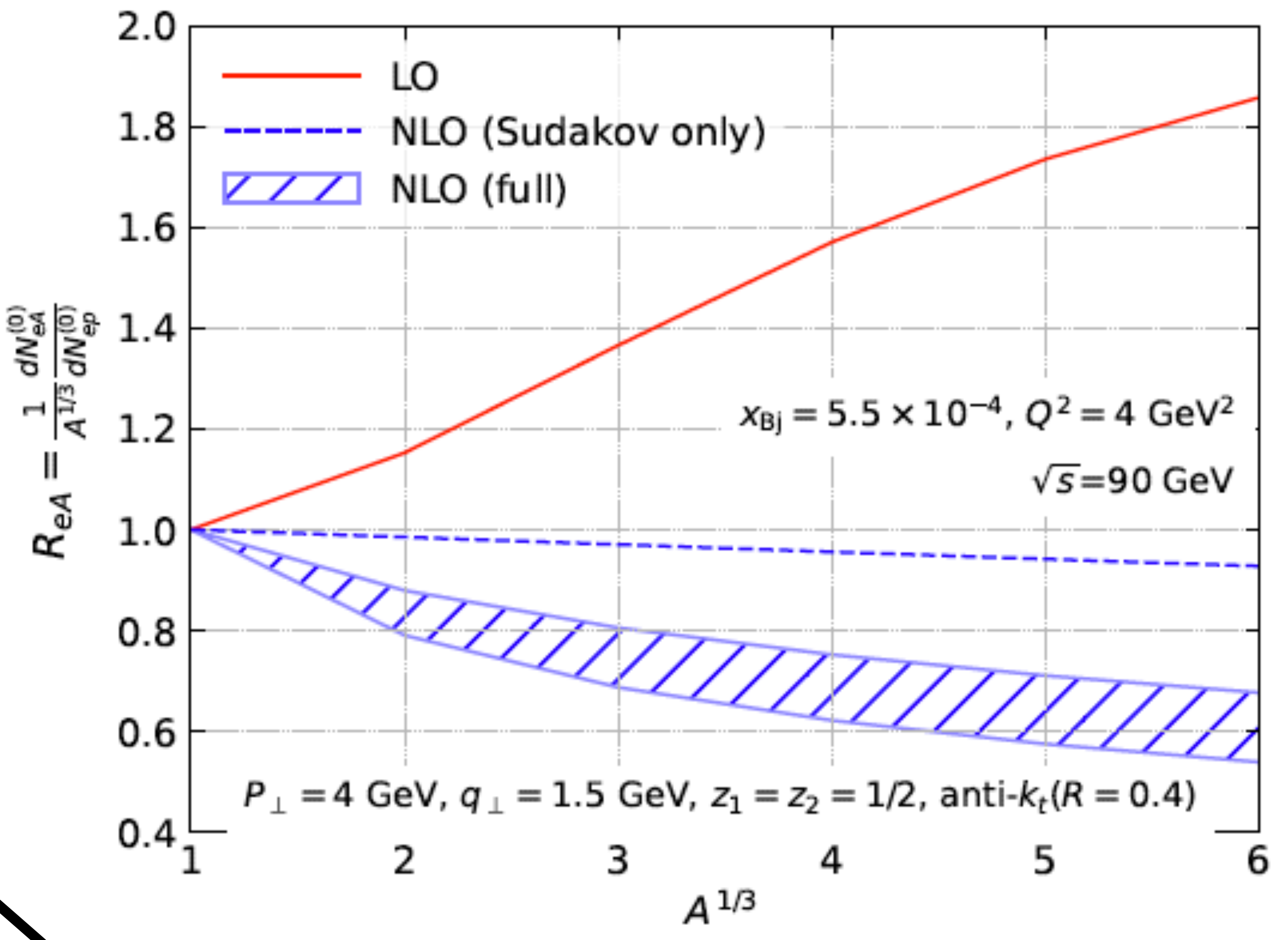
Angle between total momentum and imbalance

Saturation versus other logs:

- 2308.00022: predictions for dijets at DIS with/without saturation effects included.







- Note: power corrections in K/P (iTMD) previously were considered in dijets in DIS in 2106.11301. Here $\sqrt{s}\gg P\gg K,Q_s$.

$$\mathrm{d}\sigma_{\mathrm{CGC}}=\underbrace{\mathrm{d}\sigma_{\mathrm{TMD}}+\overbrace{\mathcal{O}\left(\frac{k_\perp}{Q_\perp}\right)}^{\text{kinematic}}}_{\mathrm{d}\sigma_{\mathrm{ITMD}}}+\overbrace{\mathcal{O}\left(\frac{Q_s}{Q_\perp}\right)}^{\text{genuine}}$$

Angle between total momentum and imbalance

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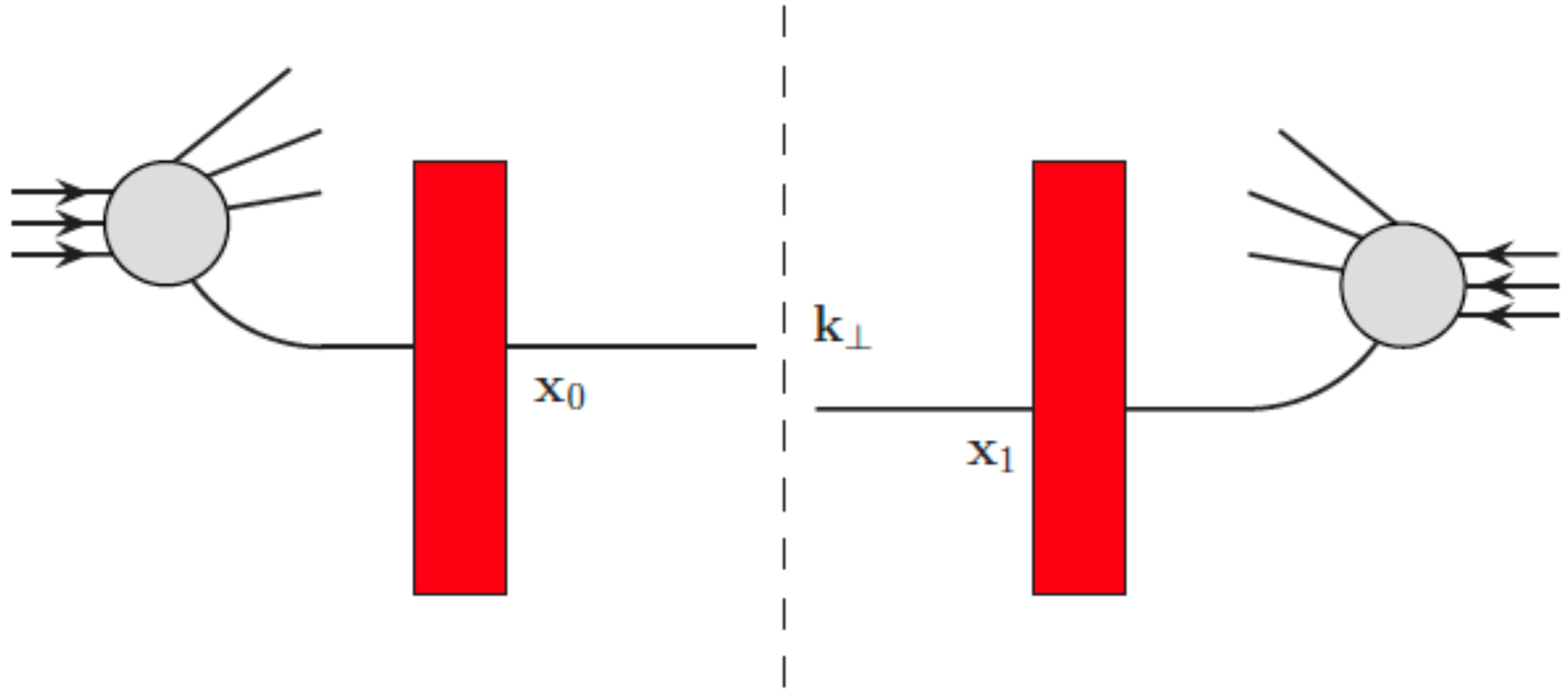
The hybrid model at LO:

• State of the art for forward particle production in pA collisions: hybrid model, proposed at LO in 2005 (hep-ph/0506308).



$$\frac{d\sigma^{q\to H}}{d^2k\, d\eta} = \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \int e^{ik(x_0-x_1)} \langle s(x_0,x_1)\rangle$$

• Wave function of the projectile proton treated in the spirit of collinear factorization (incoming parton with negligible transverse momentum).

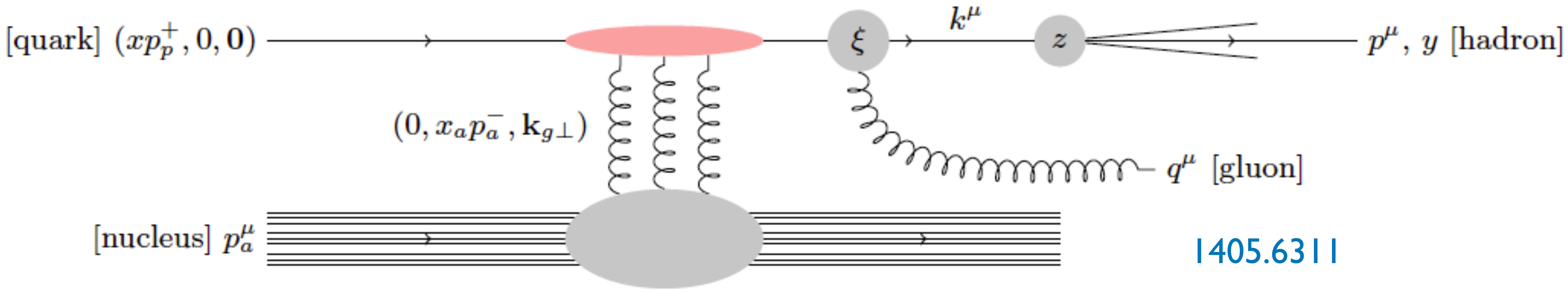
• Perturbative corrections to this wave function given by usual QCD (+QED for photons) perturbative processes.

• CGC treatment of the target as a collection of strong color fields that transfer transverse momentum to the partons rescattering on them.

• At LO, transverse momentum gained solely from rescattering.

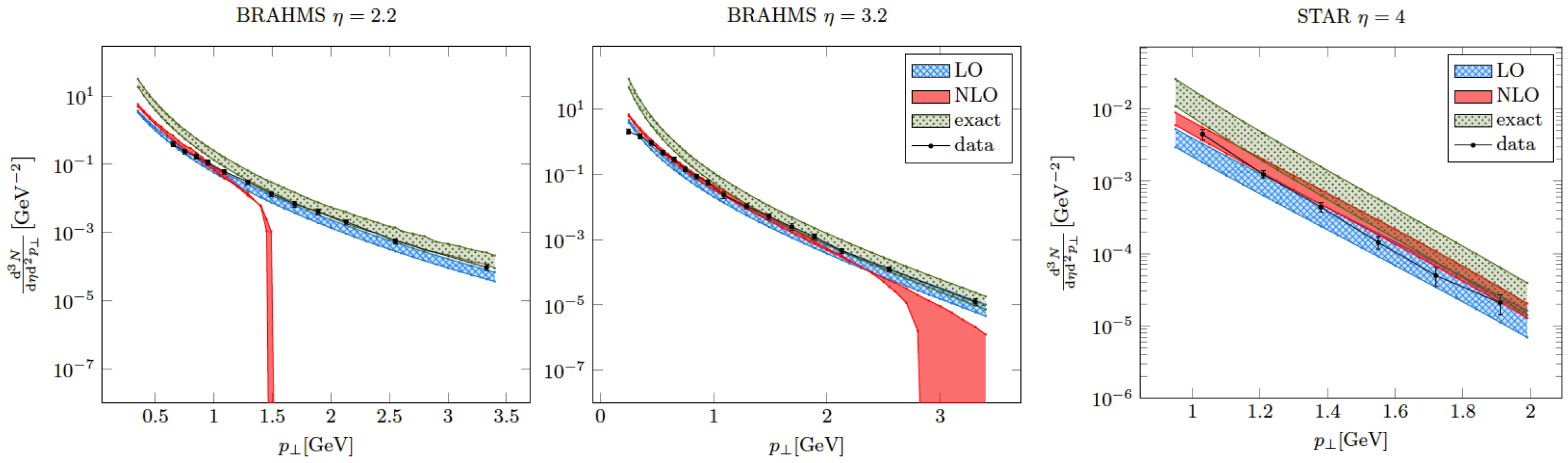
The hybrid model at NLO:

• Full NLO corrections in 2011 (1112.1061, 1203.6139): collinear divergencies absorbed in the DGLAP evolution of PDFs and FFs, rapidity divergencies in the BK evolution of $\langle W \cdots W \rangle_T$.



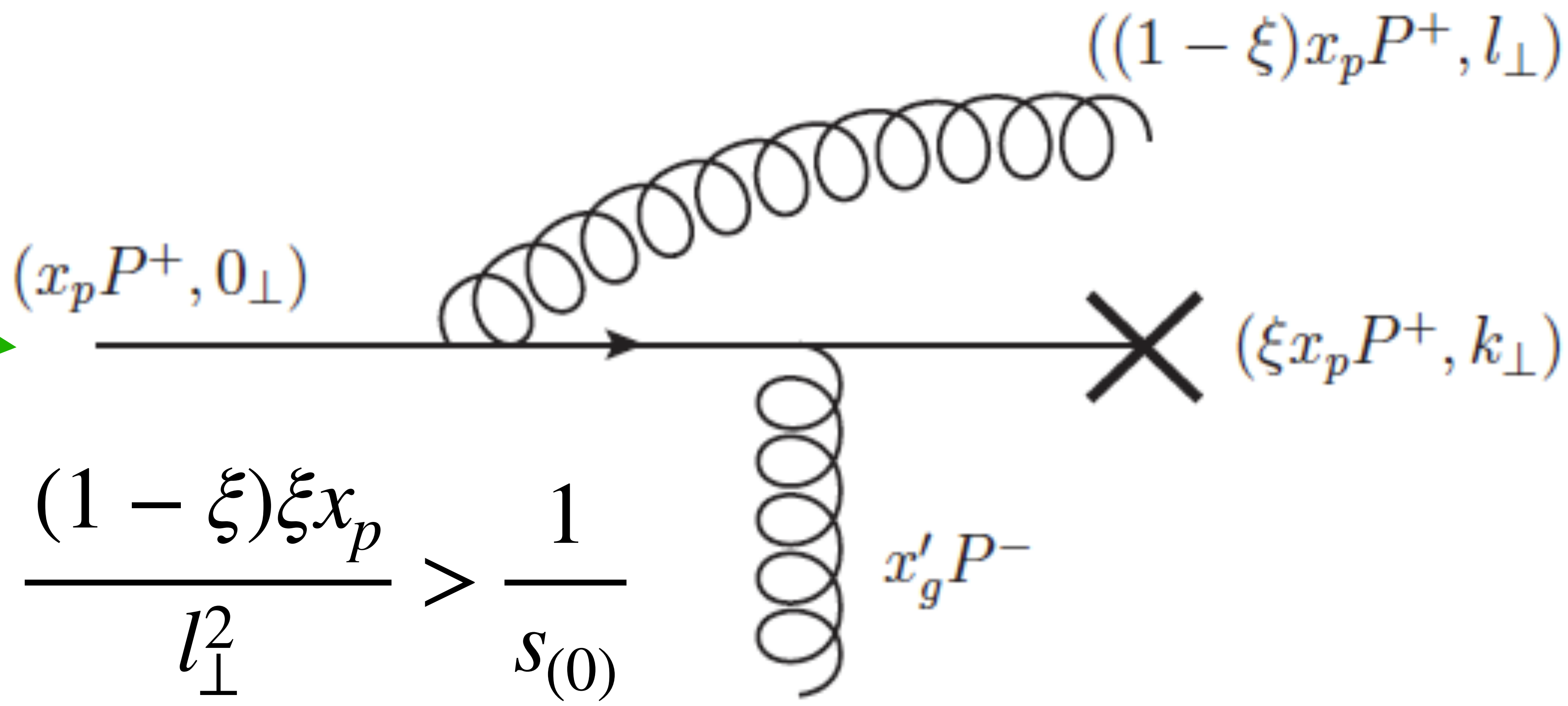
1405.6311

• Numerical analysis (1405.6311): cross sections turned out to be negative at large transverse momentum, a problem alleviated at larger rapidities or energies.



The problem (I):

- Several solutions proposed along the years:
  ➜ Kinematic constraints (1505.05183)/Ioffe time restriction (1411.2869) leading to new, BK-like terms.
  ➜ Choice of rapidity scales (1403.5221,1407.6314,1608.05293, 1712.07480).
  ➜ Threshold (2004.11990) and Sudakov (2112.06975) resummation.



$$\frac{(1-\xi)\xi x_p}{l_\perp^2} > \frac{1}{s_{(0)}}$$

$$\mu_r \propto r_\perp^{-2}, \quad \Lambda \gg \Lambda_{QCD}$$

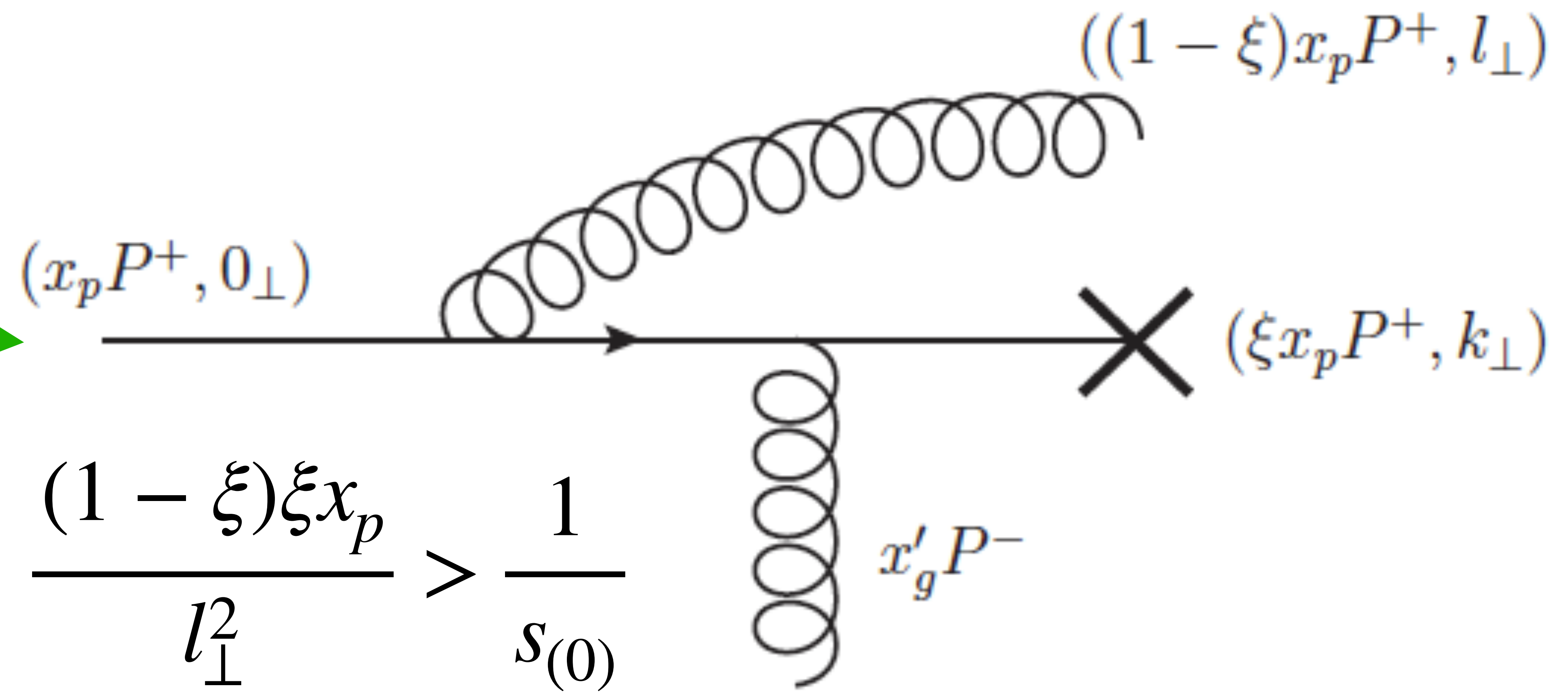
$$\ln\frac{k_\perp^2}{\mu_r^2}\ ,\quad \ln\frac{\mu^2}{\mu_r^2}\ ,\quad \ln^2\frac{k_\perp^2}{\mu_r^2} \longrightarrow \ln\frac{k_\perp^2}{\Lambda^2} + I_1(\Lambda)\ ,\quad \ln\frac{\mu^2}{\Lambda^2} + I_1(\Lambda)\ ,\quad \ln^2\frac{k_\perp^2}{\Lambda^2} + I_2(\Lambda)$$

Sudakov form factor

Threshold resummations (additional DGLAP of PDFs and FFs)

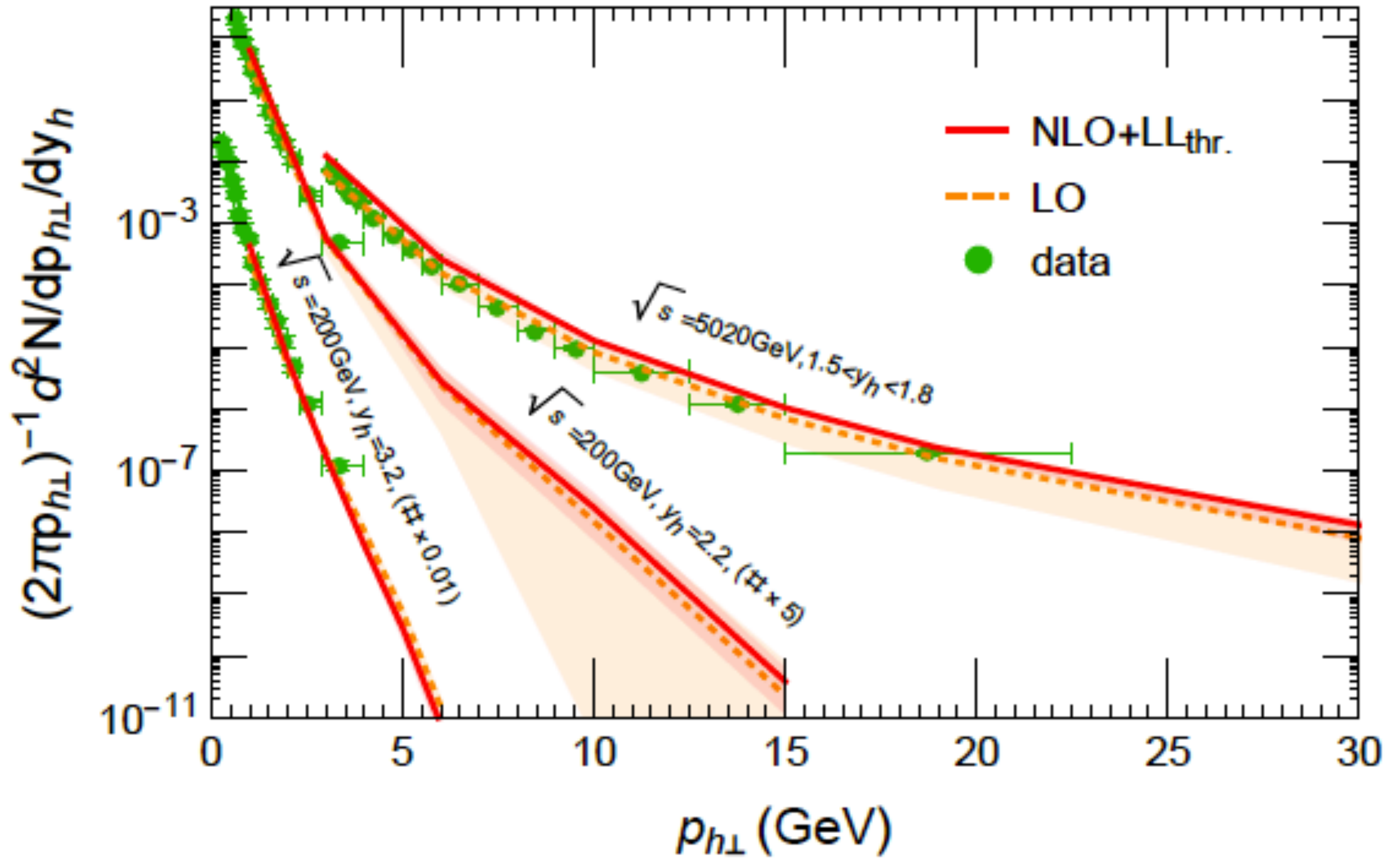
The problem (I):

- Several solutions proposed along the years:
  ➜ Kinematic constraints (1505.05183)/Ioffe time restriction (1411.2869) leading to new, BK-like terms.
  ➜ Choice of rapidity scales (1403.5221,1407.6314,1608.05293, 1712.07480).
  ➜ Threshold (2004.11990) and Sudakov (2112.06975) resummation.

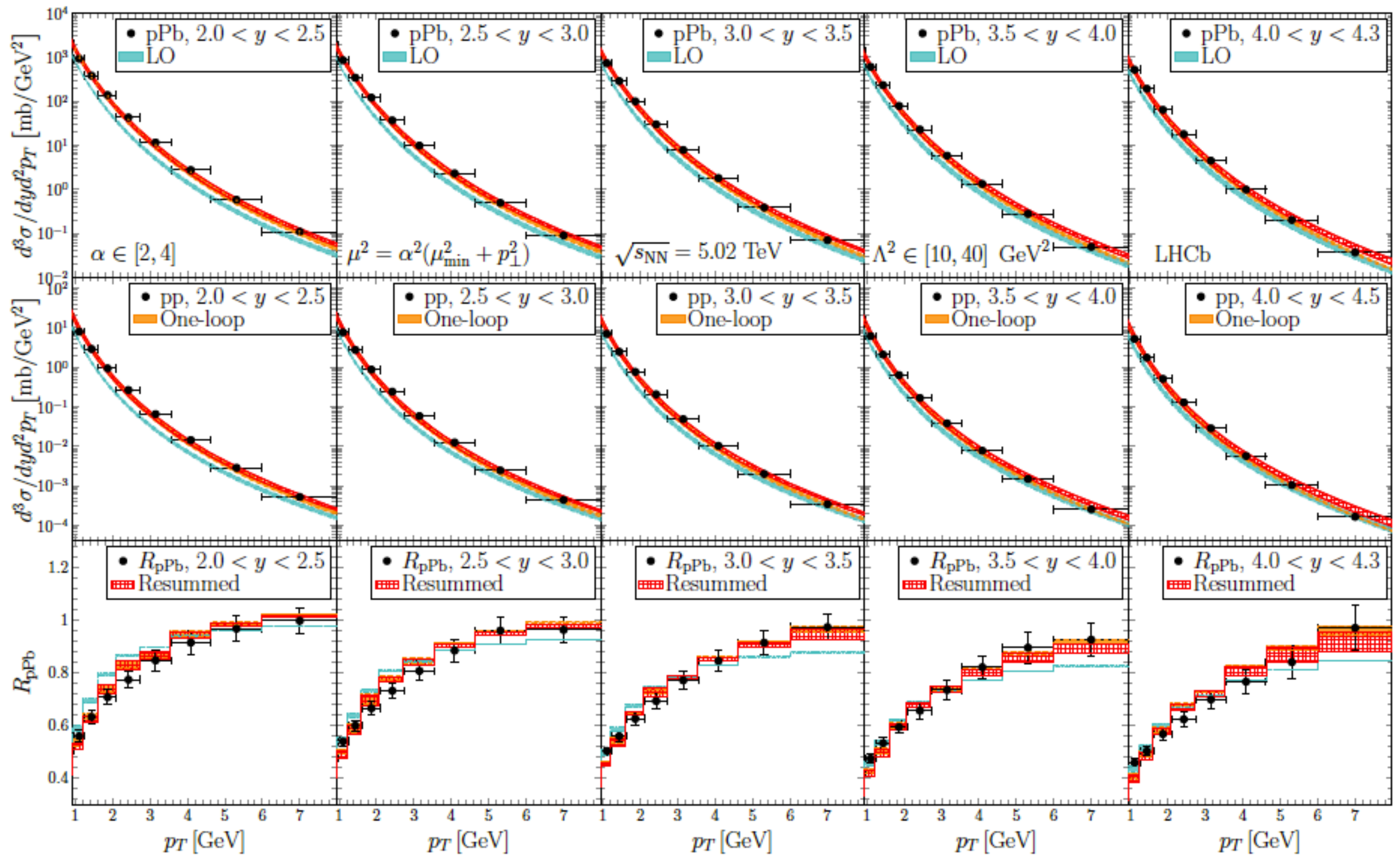


$$\frac{(1-\xi)\xi x_p}{l_\perp^2} > \frac{1}{s_{(0)}}$$

- They lead to a successful description of data but lack of understanding of what was or still is wrong, or of guidance on how to rectify it.



2004.11990



2112.06975

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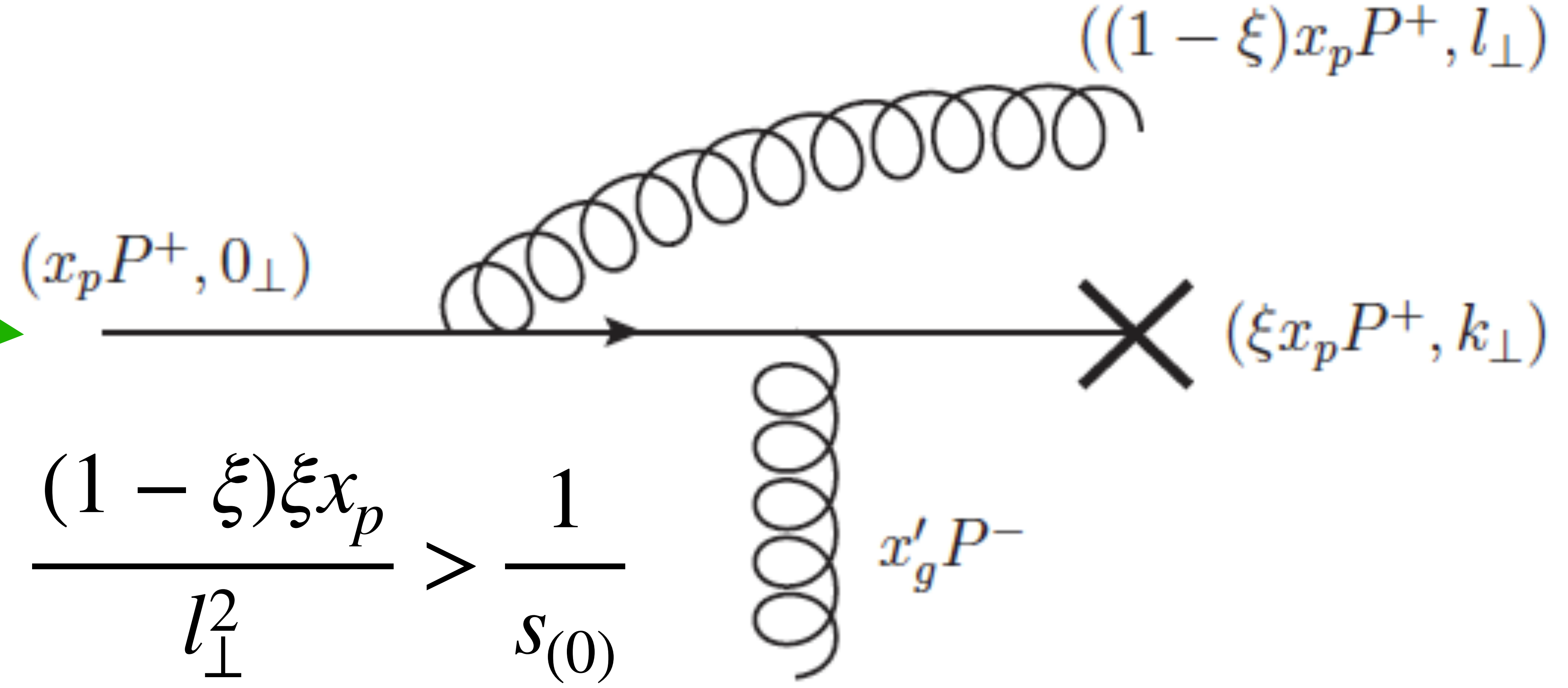

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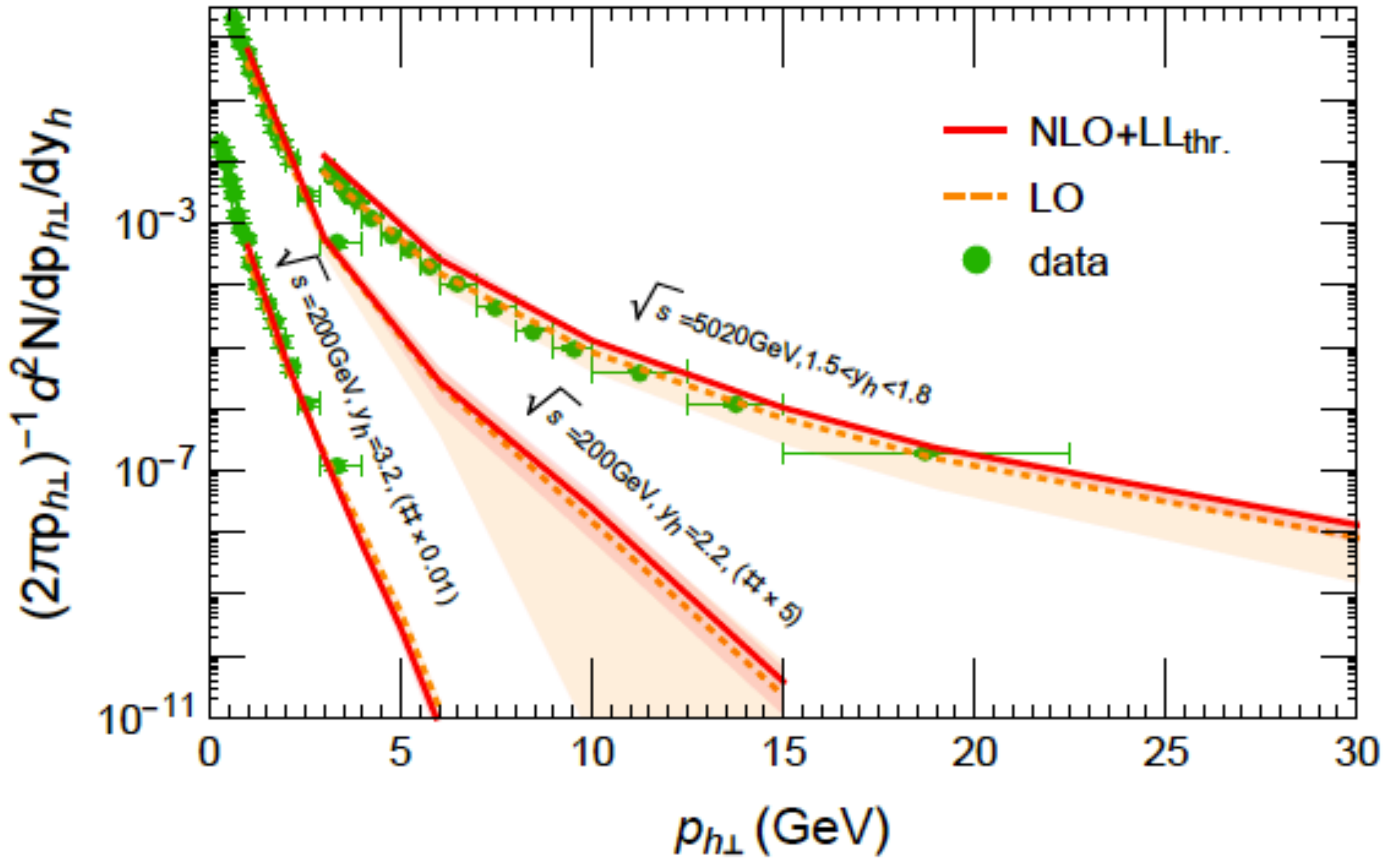

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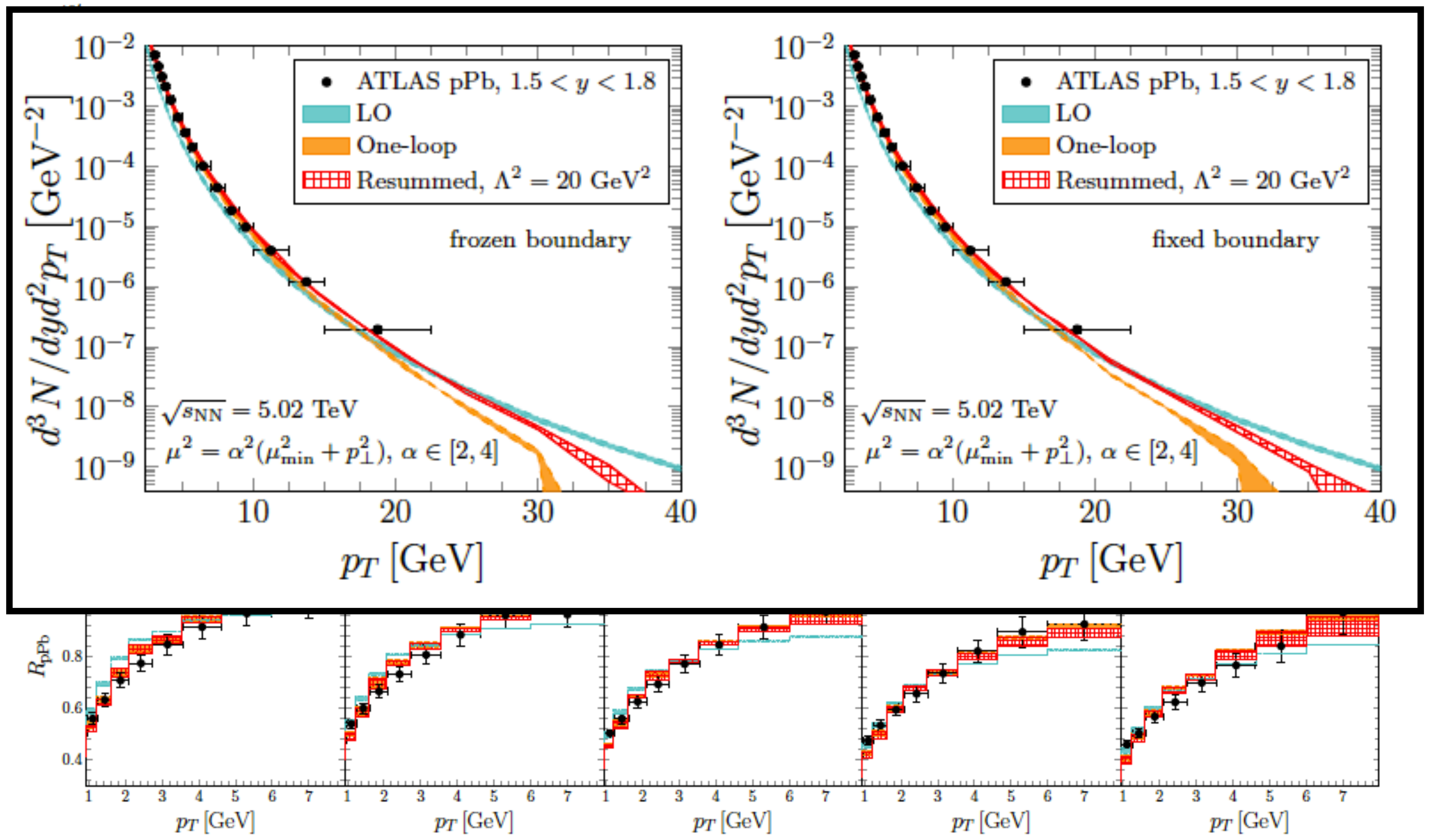

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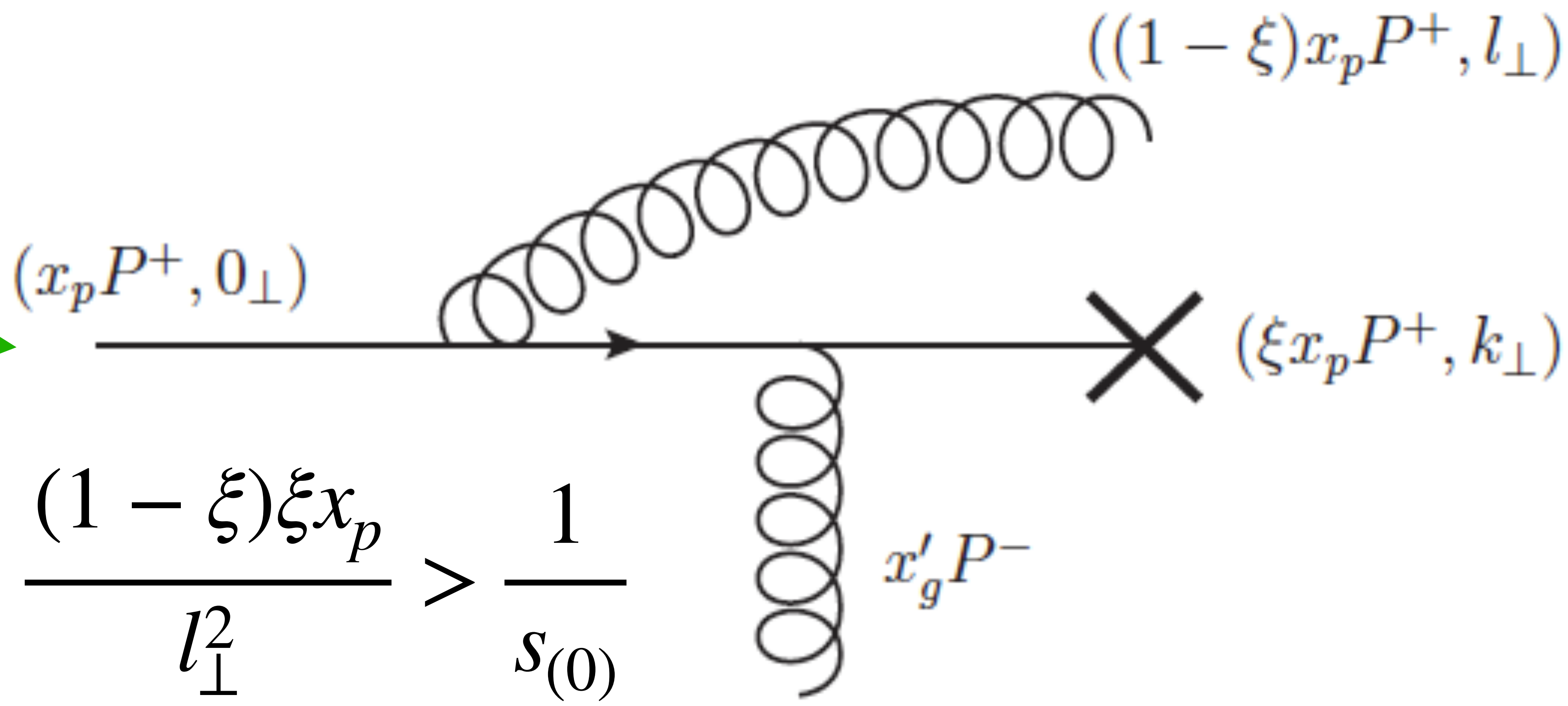

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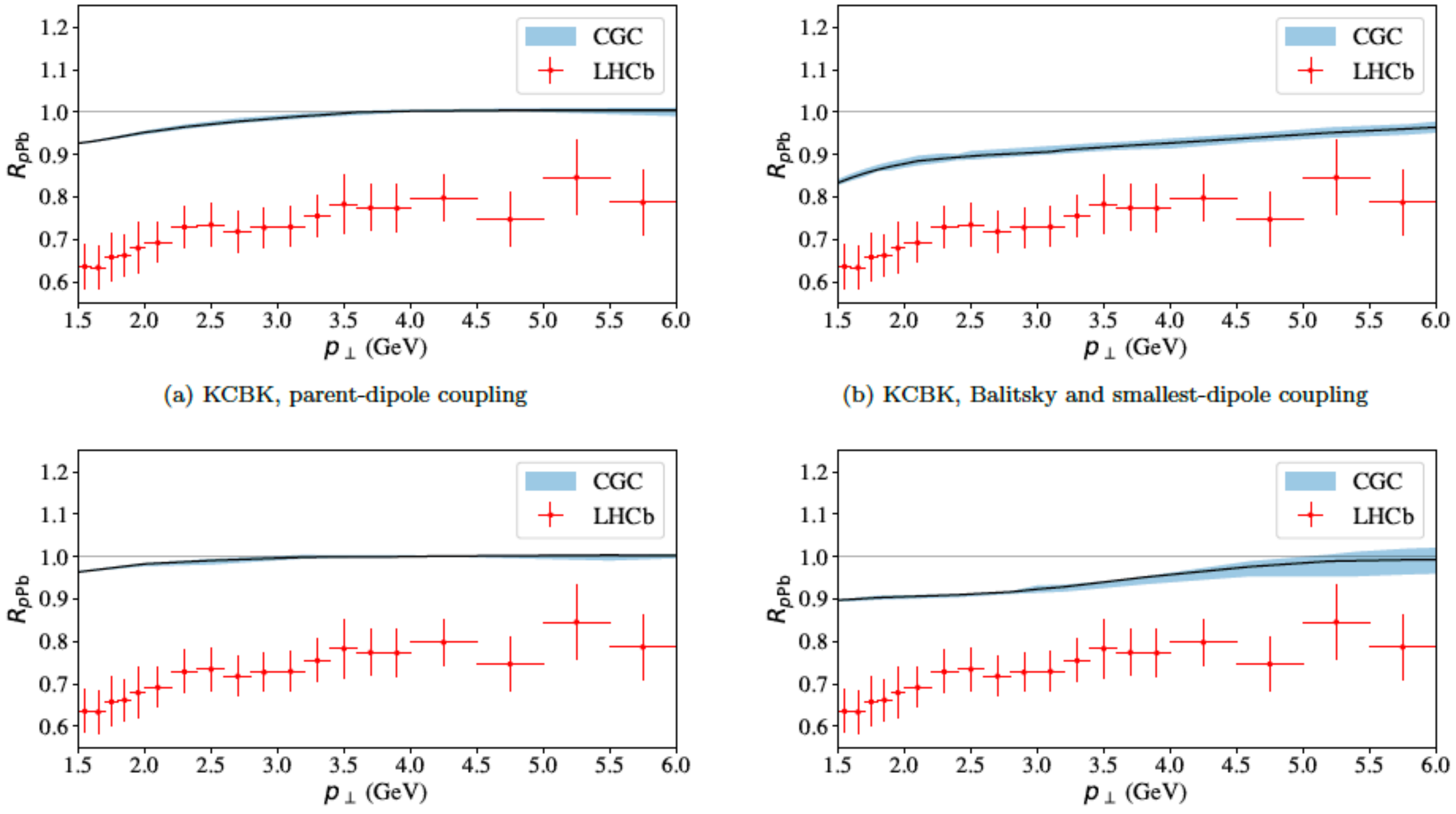
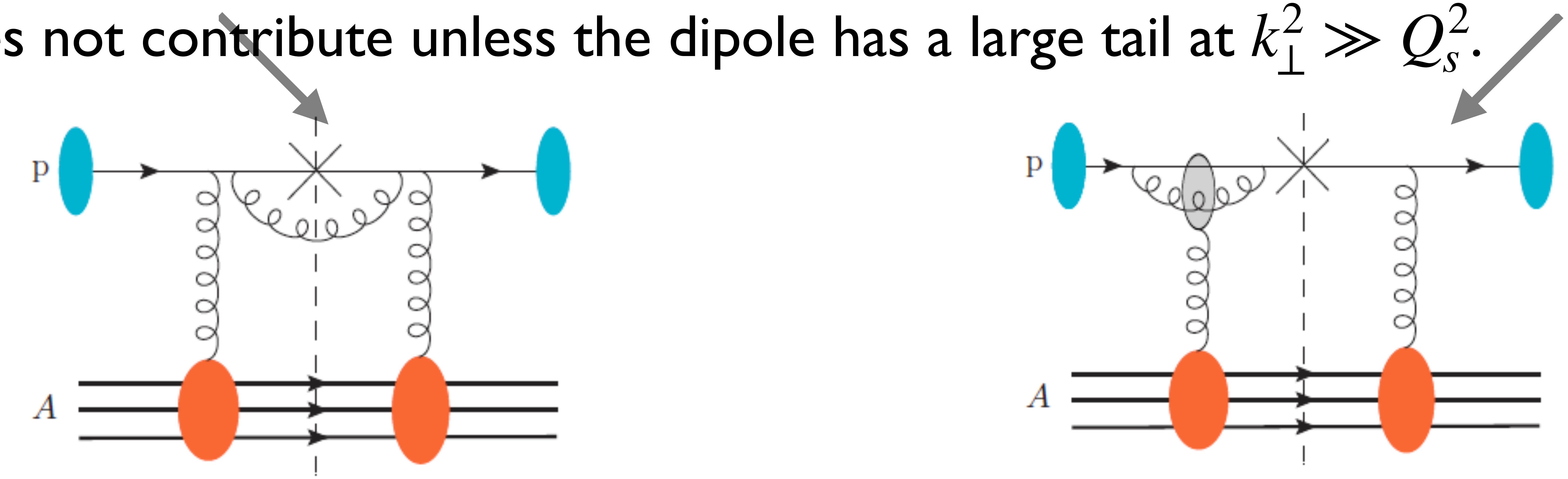
- 2310.06640: compatibility with DIS fits at NLO, effects of different evolution schemes.



(a) KCBK, parent-dipole coupling
(b) KCBK, Balitsky and smallest-dipole coupling
(c) ResumBK, parent-dipole coupling
(d) TBK, parent-dipole coupling

The problem (II):

• Should any eventual problem of negativity at NLO come not from large transverse momentum?: inelastic (real NLO) contribution squared (1102.5327), the elastic one (LO+virtual NLO) does not contribute unless the dipole has a large tail at $k_\perp^2 \gg Q_s^2$.



2112.06975

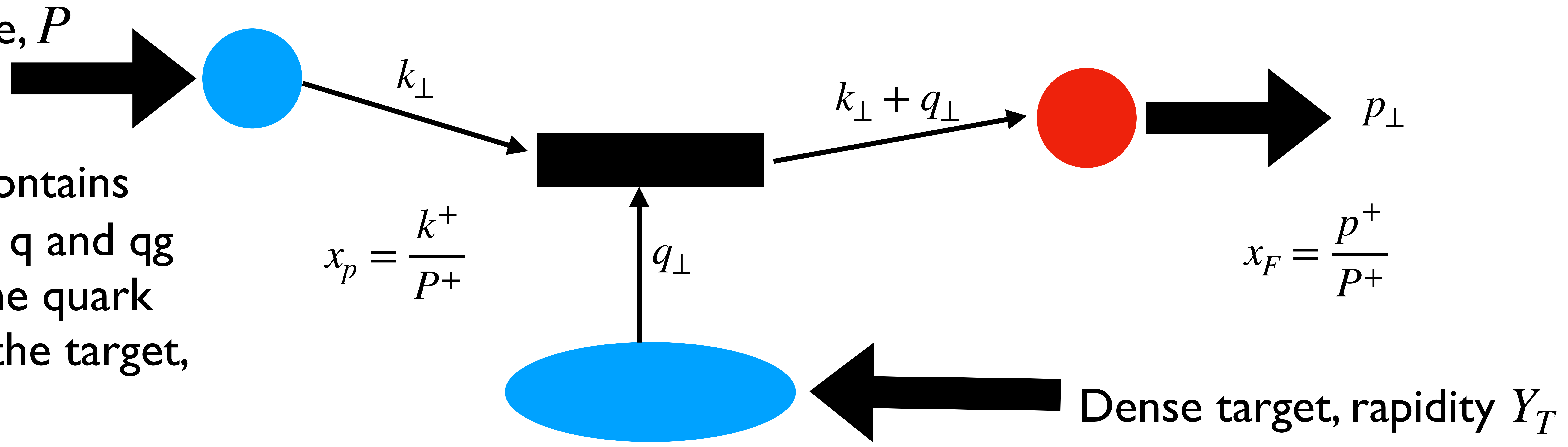
• The reason for the negativity is seemingly an over subtraction: the NLO is extracted collinear pieces that go to the DGLAP evolution of the collinear PDFs and FFs, and a soft piece (through the plus prescription) that goes into the BK evolution of the dipole scattering matrix. The remainder turns out to become negative at large transverse momentum (1505.05183).

• Altinoluk, NA, Beuf, Czajka, Kovner, Lublinsky, 2307.14922 and in progress: a reorganisation of the calculation in 1411.2869 leads to conclude that the correct framework to resum all large logarithms is not collinear factorization but TMD factorization, for the projectile.

The setup in 2307.14922:

- We work in a frame in which the target nucleus moves fast. We find a TMD-factorized parton model expression:

$$\int \frac{d\zeta}{\zeta^2} \int d^2k_\perp d^2q_\perp\, T\left(\frac{x_F}{\zeta}, k_\perp; \mu_T^2\right) P(k_\perp, q_\perp)\, F\left(\zeta; p_\perp, (k_\perp + q_\perp); \mu_F^2\right) + \text{NLO remainders} + \mathcal{O}\left(\frac{p_\perp^2, k_\perp^2, Q_s^2, \mu^2}{s_0}\right)$$



- $P(k_\perp, q_\perp)$ contains rescattering of q and qg systems (for the quark channel) with the target, Wilson lines.

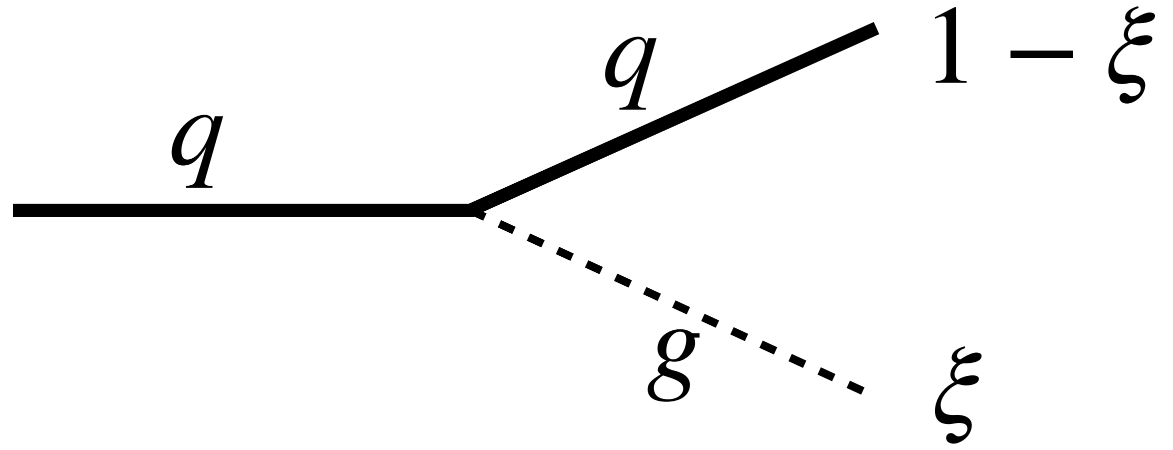
- Our scales are

$$\mu_T^2 = \max\left\{k_\perp^2, q_\perp^2, Q_s^2\right\} \approx \max\left\{(k_\perp + q_\perp)^2, Q_s^2\right\},\ \mu_F^2 = \left((q_\perp + k_\perp) - p_\perp/\zeta)^2\right) \approx \max\left\{(q_\perp + k_\perp)^2, (p_\perp/\zeta)^2\right\}$$

TMD distributions: one flavor PDFs

- TMD PDFs (single parton species to start with) are generated from collinear ones (**large** k):

$$x\mathcal{T}_q(x,k^2;k^2;\xi_0)=\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1}d\xi\frac{1+(1-\xi)^2}{\xi}\,\frac{x}{1-\xi}\,f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$



- Evolution (diagonal in parton species and momentum fraction; the second term corresponds to a loss due to the increase in resolution):

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)=\theta(\mu^2-k^2)\left[x\mathcal{T}_q(x,k^2;k^2;\xi_0)-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^{1}d\xi\frac{1+(1-\xi)^2}{\xi}x\,\mathcal{T}_q\left(x,k^2;l^2;\xi_0\right)\right]$$

- $xf^q_{\mu^2}(x)=\int_0^{\mu^2}\pi dk^2\,x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)$ follows DGLAP, definition independent of $\xi_0\ll 1$.
- TMD FFs are defined analogously; they can be generalised to n_f massless $q,\ \bar{q},\ g$.
- $\xi_0\propto\mu^2/s_0$, with s_0 an energy scale that comes from the Ioffe time restriction (1411.2869).
- Our definitions and evolution equations lead to (LO perturbative) CSS evolution equations and the Sudakov expression of TMDs for the CS variable $\zeta\propto s_0^2/\mu^2$ $(\xi_0^2=\mu^2/\zeta)$.

$q \to q \to H$ channel:

- Our dilute projectile contains quarks with transverse momentum smaller than $\mu_0 \sim \Lambda_{QCD}$.
- The dense target sits at some rapidity with no need of further evolution (no large rapidity logarithms found).

$$s(k) = \int_r \frac{1}{(2\pi)^2} e^{-ik\cdot r} s(r) \Longrightarrow s(r) = \int_l e^{il\cdot r} s(l) \Longrightarrow s(r=0) = 1 = \int_l s(l)$$

$$\frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \frac{d\bar{\sigma}^{q\to q}}{d^2k d\eta}\left(\frac{p}{\zeta}, \frac{x_F}{\zeta}\right)$$

$$\frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) s\left(\frac{p}{\zeta}\right)$$

$$\frac{d\bar{\sigma}^{q\to q}}{d^2k d\eta}(k, x_p) = \frac{d\bar{\sigma}_0^{q\to q}}{d^2k d\eta}(k, x_p) + \frac{d\bar{\sigma}_{1,r}^{q\to q}}{d^2k d\eta}(k, x_p) + \frac{d\bar{\sigma}_{1,v}^{q\to q}}{d^2k d\eta}(k, x_p)$$

Real terms provide PDF and FF TMDs with transverse momentum $\mu_0^2 < l^2 < \mu^2$, plus non log-enhanced reminders.

Virtual terms evolve LO PDF and FF TMDs to μ^2, plus non log-enhanced reminders.

- All channels $q \to q \to H$, $q \to g \to H$, $g \to g \to H$, $g \to q \to H$ included for full consistency.

Summary:

- I have revised some recent developments in CGC:
  - ➜ Evolution equations.
  - ➜ Dijet production in eA.
  - ➜ Single forward particle production in pA.

- There has been large progress in understanding the structure of the calculations and the different divergencies and large logarithms that appear ⇒ **road to precision** at the LHC and the EIC for unambiguously establishing the role of saturation/non-linear QCD dynamics in such collisions.

- Interesting connections with the TMD field: TMDs for target and projectile, FFs, and TMD-like factorization.

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Thanks a lot to you for your attention, to Tolga Altinoluk for feedback and to the organisers for their invitation!!!

Backup:

TMD distributions: CSS

• Our definitions and evolution equations lead exactly to (LO perturbative) CSS and the Sudakov expression of TMDs (see e.g. 2304.03302 or Collins' book) for the CS variable $\zeta \propto s_0^2/\mu^2$ ($\xi_0^2 = \mu^2/\zeta$).

$$\frac{\partial \ln \mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial \ln \mu^2} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi} \approx -\frac{\alpha_s}{2\pi}N_c\left[\ln\frac{1}{\xi_0}-\frac{3}{4}\right],$$

$$\frac{\partial \ln \mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial \ln \frac{1}{\xi_0}} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\left(1+(1-\xi_0)^2\right)\ln\frac{\mu^2}{k^2} \approx -\frac{\alpha_s}{2\pi}N_c \ln\frac{\mu^2}{k^2}.$$

$$\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = e^{-\frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\left(\ln^2\frac{\bar{s}_0}{k^2}-\ln^2\frac{\bar{s}_0}{\mu^2}\right)-\frac{3}{4}\ln\frac{\mu^2}{k^2}\right]}\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

$$= e^{-\frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\left(2\ln\frac{\bar{s}_0}{\mu^2}\ln\frac{\mu^2}{k^2}+\ln^2\frac{\mu^2}{k^2}\right)-\frac{3}{4}\ln\frac{\mu^2}{k^2}\right]}\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

• Taking $\bar{s}_0 = \mu^2 = Q^2$ (the hard scale), we get the leading and subleading logs in the Sudakov.

$q \to q \to H$: final expression

- Neglecting terms $\mathcal{O}\left(\frac{p^2, k^2, Q_s^2, \mu^2}{s_0}\right)$, $\mathcal{O}(\alpha_s^2)$, we get a parton model-like expression:

$$\frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int d\xi \int d^2l \int d^2k\, \mathcal{F}_H^q\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$
$$\times \frac{x_F}{\zeta(1-\xi)} \mathcal{T}_q\left(\frac{x_F}{\zeta(1-\xi)}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \mathcal{P}(\xi, \zeta; k+l; p, s_0, \mu^2, \mu_0^2),$$

$$\mathcal{P}(\xi, \zeta; k+l; p, s_0, \mu^2, \mu_0^2) = \int d\lambda \int_m \left\{\delta(\lambda)\delta(\xi-\lambda) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right)\right.$$
$$+\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\frac{1+(1-\lambda)^2}{\lambda}\theta(1-\lambda)$$
$$\left[\delta(\lambda-\xi)\theta\left(\xi-\frac{m^2\zeta}{x_F s_0}\right)\int_q s(m)s(q)\left[\frac{p/\zeta-m}{(p/\zeta-m)^2}-\frac{p/\zeta-(1-\xi)m}{(p/\zeta-(1-\xi)m)^2}\right]\left[\frac{p/\zeta-q}{(p/\zeta-q)^2}-\frac{p/\zeta-(1-\xi)q}{(p/\zeta-(1-\xi)q)^2}\right]\right.$$
$$\left.\left.-2\delta(\xi)\theta\left(\lambda-\frac{\mu^2\zeta}{x_F s_0}\right)\theta(m^2-\mu_0^2)\, s\left(\frac{p}{\zeta}\right) s\left(m+(1-\lambda)\frac{p}{\zeta}\right)\int_{\mu^2}^{\min[m^2,\lambda\bar{s}_0]}\frac{d^2q}{q^2}\right]\right\}.$$

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quark scattering

qg scattering due to $q \to qg$

The other channels:

- TMD PDFs:
  - ➜ For quark: it gets contributions from $q \to q$ and $g \to q$.
  - ➜ For antiquark: it gets contributions from $\bar{q} \to \bar{q}$ and $g \to \bar{q}$.
  - ➜ For gluon: it gets contributions from $g \to g, q \to g$ and $\bar{q} \to g$.

- TMD FFs:
  - ➜ For quark: it gets contributions from $q \to q \to H$ and $q \to g \to H$.
  - ➜ For antiquark: it gets contributions from $\bar{q} \to \bar{q} \to H$ and $\bar{q} \to g \to H$.
  - ➜ For gluon: it gets contributions from $g \to g \to H, g \to q \to H$ and $g \to \bar{q} \to H$.

- The complete quark piece of the parton-like formula contains 2 dipoles in the fundamental representation (we work at large N_c), and keeps the form with additional NLO remainders.

- The gluon piece of the parton-like formula contains 3 dipoles in the fundamental representation (we work at large N_c), and additional NLO remainders.